ECON7960

Problem Set #3

Due March 25

ANALYTICAL

1. Consider the following (3×3) transition matrix

0.0	0.15	0.3
p_1	0.0	p_3
0.5	p_2	0

   a) What values of p_1, p_2 and p_3 make this matrix a proper left stochastic transition matrix?

   b) Use the Chapman-Kolmogorov equation to find the probability mass function of the system after 10 time steps using the initial conditions $\pi[0] = (0, .3, .7)$.

   c) Find the stationary (ergodic) distribution of this Markov process using the Perron-Frobenius theorem.

   d) Is a Markov process defined by this transition matrix irreducible, that is, do all states communicate with each other and is it aperiodic?

R

1. Construct a Gibbs Sampler (not using any package) to analyze the posterior $p[\mu, \sigma|x]$ for $n = 100$ simulated observations from a Normal distribution with $\sigma = 1.5$ and $\mu = 25$ using conjugate priors.

   a) Diagnose your sampler using traceplots, autocorrelation plots, and Geweke statistics and discuss why your output looks good/bad.

b) Compute the posterior mode and 95% HPD region of μ and σ.

c) Set the same model up in STAN and compare your results.

2. Consider Darwin's dataset containing 15 differences of the heights of cross- and self-fertilized plants. Due to the presence of outliers model these data as a sample from a Cauchy likelihood with location parameter μ and scale parameter σ:

$$p[x|\mu,\sigma] = \frac{1}{\pi\sigma\left(1+\frac{x-\mu}{\sigma}\right)^2} \tag{0.1}$$

a) Access Darwin's data in R from the package "LearnBayes" using the command *data(darwin)*.

b) Using the noninformative prior $p[\mu,\sigma] \propto \frac{1}{\sigma}$ construct a Metropolis random walk algorithm (not using any package) to simulate 10000 draws from the posterior density. You may need to tune your proposal density standard deviation. (Hint: use summary statistics to set initial conditions. You may find transformations of σ to be better behaved).

c) Diagnose your sampler using traceplots, autocorrelation plots, and Geweke statistics and discuss why your output looks good/bad.

d) Compute the posterior mode and 95% HPD region of μ and σ.

e) Set the same model up in STAN and compare your results.