A Critical Look at the Automatic SAS® Forecasting System

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Abstract

The goal of this effort is to study the SAS® automatic model selection and forecasting procedure for studying time series data. This procedure generates various time series model and suggests the best choice model based on either minimum mean squared error (MSE), minimum root mean square error (RMSE), minimum mean absolute error (MAE), and minimum mean absolute percent error (MAPE), along with several other accuracy measures. This study focuses on using the classic time series data sets given in Appendix B of *Introduction to Time Series Analysis and Forecasting* (2nd ed., p 581-626) by Douglas C. Montgomery, Cheryl L. Jennings & Murat Kulahci.

Contents

Introduction

All time series forecasting problems involve the use of time series data; however, the SAS(AFS) system can produce automatic forecasts and accuracy statistics for any input time series, and without human interaction. Additionally, this system can suggest the "best model" to use in practice, based on various measures of forecast accuracy, such as MSE, MAE and MAPE. To study this "best model", Table B.1 – B.20 in total 20 data sets, which are given in Appendix B of *Introduction to Time Series Analysis and Forecasting*, are focuses of this paper.

Individually each data set was split into training data and testing data. SAS(AFS) suggested "best model" for specific data set was checked by model validation techniques, where it was monitored that can SAS(AFS) suggestive model forecast the training data close to testing data. Moreover, different measures of forecast accuracy were compared among full data, training data and holdout data to see the competence of the SAS(AFS) suggestive "best model". Autocorrelation Function (ACF) plot of errors was also observed to see where there are any structure present in the errors as well as in the data set which was not captured by the model. Based on different techniques, the final observation was given whether SAS(AFS) suggestive "best model" for specific data set produce results accurately. The same procedures were applied in Table B.1 – B.20, all 20 data sets and at the end, I tried to give a final conclusion based on the outcomes of these data sets results.

TABLE B.1 (MARKET YIELD ON US TREASURY SECURITIES AT 10-YEAR CONSTANT MATURITY) ANALYSIS

Table B.1 comprises data on the market yield on US Treasury Securities at 10-year constant maturity from April 1953 through February 2007, where the percentage rate variable is collected monthly. The data set excludes last 12 monthly values as a holdout set in order to analyze the accuracy of SAS(AFS), which suggested a damped trend exponential smoothing model as the "best model" for this training, or initialization, data set. SAS(AFS) also produced the predicted values for the next 12 months. I have calculated different types of forecast errors based on the hold out data and predicted data.

The model equation is

$$Y_t = \mu_t + \beta_t\, t + \varepsilon_t$$

where,

Y_t = observed value i.e. securities rate at time t

μ_t = smoothed level and $\mu_t = 4.56983$

β_t = smoothed trend and $\beta_t = 0.14964$

Now, the k-step prediction equation is

$$\hat{Y}_t(k) = L_t + \sum_{i=1}^{k} \phi^t\, T_t$$

and the smoothing equations are

$$L_t = \alpha\, Y_t + (1-\alpha)\,(L_{t-1} + \phi\, T_{t-1})$$

and $T_t = \gamma\,(L_t - L_{t-1}) + (1-\gamma)\,\phi\, T_{t-1}$

where,

L_t = Level estimate and $L_0 = \mu_t = 4.56983$

T_t = Trend estimate and $T_0 = \beta_t = 0.14964$

α = level smoothing and $\alpha = 0.999$

γ = trend smoothing and $\gamma = 0.999$ and

ϕ = damping smoothing and $\phi = 0.31412$

The ARIMA model equivalency to damped-trend linear exponential smoothing is the ARIMA (1, 1, 2) model,

$(1-\phi B)(1-B)\, Y_t = (1-\theta_1 B - \theta_2 B^2)\, \varepsilon_t$

$\theta_1 = 1 + \phi - \alpha - \alpha\gamma\phi$

$\theta_2 = (\alpha - 1)\,\phi$

Table B.1: Market yield on US Treasury Securities at 10-year constant maturity

Month	Rate, %	Month	Rate, %	Month	Rate, %	Month	Rate, %
Apr-1953	2.83	Oct-1966	5.01	Apr-1980	11.47	Oct-1993	5.33
May-1953	3.05	Nov-1966	5.16	May-1980	10.18	Nov-1993	5.72
Jun-1953	3.11	Dec-1966	4.84	Jun-1980	9.78	Dec-1993	5.77
Jul-1953	2.93	Jan-1967	4.58	Jul-1980	10.25	Jan-1994	5.75
Aug-1953	2.95	Feb-1967	4.63	Aug-1980	11.10	Feb-1994	5.97
Sep-1953	2.87	Mar-1967	4.54	Sep-1980	11.51	Mar-1994	6.48
Oct-1953	2.66	Apr-1967	4.59	Oct-1980	11.75	Apr-1994	6.97

Month	Rate, %	Month	Rate, %	Month	Rate, %	Month	Rate, %
Nov-1953	2.68	May-1967	4.85	Nov-1980	12.68	May-1994	7.18
Dec-1953	2.59	Jun-1967	5.02	Dec-1980	12.84	Jun-1994	7.10
Jan-1954	2.48	Jul-1967	5.16	Jan-1981	12.57	Jul-1994	7.30
Feb-1954	2.47	Aug-1967	5.28	Feb-1981	13.19	Aug-1994	7.24
Mar-1954	2.37	Sep-1967	5.30	Mar-1981	13.12	Sep-1994	7.46
Apr-1954	2.29	Oct-1967	5.48	Apr-1981	13.68	Oct-1994	7.74
May-1954	2.37	Nov-1967	5.75	May-1981	14.10	Nov-1994	7.96
Jun-1954	2.38	Dec-1967	5.70	Jun-1981	13.47	Dec-1994	7.81
Jul-1954	2.30	Jan-1968	5.53	Jul-1981	14.28	Jan-1995	7.78
Aug-1954	2.36	Feb-1968	5.56	Aug-1981	14.94	Feb-1995	7.47
Sep-1954	2.38	Mar-1968	5.74	Sep-1981	15.32	Mar-1995	7.20
Oct-1954	2.43	Apr-1968	5.64	Oct-1981	15.15	Apr-1995	7.06
Nov-1954	2.48	May-1968	5.87	Nov-1981	13.39	May-1995	6.63
Dec-1954	2.51	Jun-1968	5.72	Dec-1981	13.72	Jun-1995	6.17
Jan-1955	2.61	Jul-1968	5.50	Jan-1982	14.59	Jul-1995	6.28
Feb-1955	2.65	Aug-1968	5.42	Feb-1982	14.43	Aug-1995	6.49
Mar-1955	2.68	Sep-1968	5.46	Mar-1982	13.86	Sep-1995	6.20
Apr-1955	2.75	Oct-1968	5.58	Apr-1982	13.87	Oct-1995	6.04
May-1955	2.76	Nov-1968	5.70	May-1982	13.62	Nov-1995	5.93
Jun-1955	2.78	Dec-1968	6.03	Jun-1982	14.30	Dec-1995	5.71
Jul-1955	2.90	Jan-1969	6.04	Jul-1982	13.95	Jan-1996	5.65
Aug-1955	2.97	Feb-1969	6.19	Aug-1982	13.06	Feb-1996	5.81
Sep-1955	2.97	Mar-1969	6.30	Sep-1982	12.34	Mar-1996	6.27
Oct-1955	2.88	Apr-1969	6.17	Oct-1982	10.91	Apr-1996	6.51
Nov-1955	2.89	May-1969	6.32	Nov-1982	10.55	May-1996	6.74
Dec-1955	2.96	Jun-1969	6.57	Dec-1982	10.54	Jun-1996	6.91
Jan-1956	2.90	Jul-1969	6.72	Jan-1983	10.46	Jul-1996	6.87
Feb-1956	2.84	Aug-1969	6.69	Feb-1983	10.72	Aug-1996	6.64
Mar-1956	2.96	Sep-1969	7.16	Mar-1983	10.51	Sep-1996	6.83
Apr-1956	3.18	Oct-1969	7.10	Apr-1983	10.40	Oct-1996	6.53
May-1956	3.07	Nov-1969	7.14	May-1983	10.38	Nov-1996	6.20
Jun-1956	3.00	Dec-1969	7.65	Jun-1983	10.85	Dec-1996	6.30
Jul-1956	3.11	Jan-1970	7.79	Jul-1983	11.38	Jan-1997	6.58
Aug-1956	3.33	Feb-1970	7.24	Aug-1983	11.85	Feb-1997	6.42
Sep-1956	3.38	Mar-1970	7.07	Sep-1983	11.65	Mar-1997	6.69
Oct-1956	3.34	Apr-1970	7.39	Oct-1983	11.54	Apr-1997	6.89
Nov-1956	3.49	May-1970	7.91	Nov-1983	11.69	May-1997	6.71
Dec-1956	3.59	Jun-1970	7.84	Dec-1983	11.83	Jun-1997	6.49
Jan-1957	3.46	Jul-1970	7.46	Jan-1984	11.67	Jul-1997	6.22
Feb-1957	3.34	Aug-1970	7.53	Feb-1984	11.84	Aug-1997	6.30
Mar-1957	3.41	Sep-1970	7.39	Mar-1984	12.32	Sep-1997	6.21
Apr-1957	3.48	Oct-1970	7.33	Apr-1984	12.63	Oct-1997	6.03
May-1957	3.60	Nov-1970	6.84	May-1984	13.41	Nov-1997	5.88
Jun-1957	3.80	Dec-1970	6.39	Jun-1984	13.56	Dec-1997	5.81
Jul-1957	3.93	Jan-1971	6.24	Jul-1984	13.36	Jan-1998	5.54
Aug-1957	3.93	Feb-1971	6.11	Aug-1984	12.72	Feb-1998	5.57
Sep-1957	3.92	Mar-1971	5.70	Sep-1984	12.52	Mar-1998	5.65
Oct-1957	3.97	Apr-1971	5.83	Oct-1984	12.16	Apr-1998	5.64
Nov-1957	3.72	May-1971	6.39	Nov-1984	11.57	May-1998	5.65
Dec-1957	3.21	Jun-1971	6.52	Dec-1984	11.50	Jun-1998	5.50
Jan-1958	3.09	Jul-1971	6.73	Jan-1985	11.38	Jul-1998	5.46
Feb-1958	3.05	Aug-1971	6.58	Feb-1985	11.51	Aug-1998	5.34

Month	Rate, %	Month	Rate, %	Month	Rate, %	Month	Rate, %
Mar-1958	2.98	Sep-1971	6.14	Mar-1985	11.86	Sep-1998	4.81
Apr-1958	2.88	Oct-1971	5.93	Apr-1985	11.43	Oct-1998	4.53
May-1958	2.92	Nov-1971	5.81	May-1985	10.85	Nov-1998	4.83
Jun-1958	2.97	Dec-1971	5.93	Jun-1985	10.16	Dec-1998	4.65
Jul-1958	3.20	Jan-1972	5.95	Jul-1985	10.31	Jan-1999	4.72
Aug-1958	3.54	Feb-1972	6.08	Aug-1985	10.33	Feb-1999	5.00
Sep-1958	3.76	Mar-1972	6.07	Sep-1985	10.37	Mar-1999	5.23
Oct-1958	3.80	Apr-1972	6.19	Oct-1985	10.24	Apr-1999	5.18
Nov-1958	3.74	May-1972	6.13	Nov-1985	9.78	May-1999	5.54
Dec-1958	3.86	Jun-1972	6.11	Dec-1985	9.26	Jun-1999	5.90
Jan-1959	4.02	Jul-1972	6.11	Jan-1986	9.19	Jul-1999	5.79
Feb-1959	3.96	Aug-1972	6.21	Feb-1986	8.70	Aug-1999	5.94
Mar-1959	3.99	Sep-1972	6.55	Mar-1986	7.78	Sep-1999	5.92
Apr-1959	4.12	Oct-1972	6.48	Apr-1986	7.30	Oct-1999	6.11
May-1959	4.31	Nov-1972	6.28	May-1986	7.71	Nov-1999	6.03
Jun-1959	4.34	Dec-1972	6.36	Jun-1986	7.80	Dec-1999	6.28
Jul-1959	4.40	Jan-1973	6.46	Jul-1986	7.30	Jan-2000	6.66
Aug-1959	4.43	Feb-1973	6.64	Aug-1986	7.17	Feb-2000	6.52
Sep-1959	4.68	Mar-1973	6.71	Sep-1986	7.45	Mar-2000	6.26
Oct-1959	4.53	Apr-1973	6.67	Oct-1986	7.43	Apr-2000	5.99
Nov-1959	4.53	May-1973	6.85	Nov-1986	7.25	May-2000	6.44
Dec-1959	4.69	Jun-1973	6.90	Dec-1986	7.11	Jun-2000	6.10
Jan-1960	4.72	Jul-1973	7.13	Jan-1987	7.08	Jul-2000	6.05
Feb-1960	4.49	Aug-1973	7.40	Feb-1987	7.25	Aug-2000	5.83
Mar-1960	4.25	Sep-1973	7.09	Mar-1987	7.25	Sep-2000	5.80
Apr-1960	4.28	Oct-1973	6.79	Apr-1987	8.02	Oct-2000	5.74
May-1960	4.35	Nov-1973	6.73	May-1987	8.61	Nov-2000	5.72
Jun-1960	4.15	Dec-1973	6.74	Jun-1987	8.40	Dec-2000	5.24
Jul-1960	3.90	Jan-1974	6.99	Jul-1987	8.45	Jan-2001	5.16
Aug-1960	3.80	Feb-1974	6.96	Aug-1987	8.76	Feb-2001	5.10
Sep-1960	3.80	Mar-1974	7.21	Sep-1987	9.42	Mar-2001	4.89
Oct-1960	3.89	Apr-1974	7.51	Oct-1987	9.52	Apr-2001	5.14
Nov-1960	3.93	May-1974	7.58	Nov-1987	8.86	May-2001	5.39
Dec-1960	3.84	Jun-1974	7.54	Dec-1987	8.99	Jun-2001	5.28
Jan-1961	3.84	Jul-1974	7.81	Jan-1988	8.67	Jul-2001	5.24
Feb-1961	3.78	Aug-1974	8.04	Feb-1988	8.21	Aug-2001	4.97
Mar-1961	3.74	Sep-1974	8.04	Mar-1988	8.37	Sep-2001	4.73
Apr-1961	3.78	Oct-1974	7.90	Apr-1988	8.72	Oct-2001	4.57
May-1961	3.71	Nov-1974	7.68	May-1988	9.09	Nov-2001	4.65
Jun-1961	3.88	Dec-1974	7.43	Jun-1988	8.92	Dec-2001	5.09
Jul-1961	3.92	Jan-1975	7.50	Jul-1988	9.06	Jan-2002	5.04
Aug-1961	4.04	Feb-1975	7.39	Aug-1988	9.26	Feb-2002	4.91
Sep-1961	3.98	Mar-1975	7.73	Sep-1988	8.98	Mar-2002	5.28
Oct-1961	3.92	Apr-1975	8.23	Oct-1988	8.80	Apr-2002	5.21
Nov-1961	3.94	May-1975	8.06	Nov-1988	8.96	May-2002	5.16
Dec-1961	4.06	Jun-1975	7.86	Dec-1988	9.11	Jun-2002	4.93
Jan-1962	4.08	Jul-1975	8.06	Jan-1989	9.09	Jul-2002	4.65
Feb-1962	4.04	Aug-1975	8.40	Feb-1989	9.17	Aug-2002	4.26
Mar-1962	3.93	Sep-1975	8.43	Mar-1989	9.36	Sep-2002	3.87
Apr-1962	3.84	Oct-1975	8.14	Apr-1989	9.18	Oct-2002	3.94
May-1962	3.87	Nov-1975	8.05	May-1989	8.86	Nov-2002	4.05
Jun-1962	3.91	Dec-1975	8.00	Jun-1989	8.28	Dec-2002	4.03

Month	Rate, %	Month	Rate, %	Month	Rate, %	Month	Rate, %
Jul-1962	4.01	Jan-1976	7.74	Jul-1989	8.02	Jan-2003	4.05
Aug-1962	3.98	Feb-1976	7.79	Aug-1989	8.11	Feb-2003	3.90
Sep-1962	3.98	Mar-1976	7.73	Sep-1989	8.19	Mar-2003	3.81
Oct-1962	3.93	Apr-1976	7.56	Oct-1989	8.01	Apr-2003	3.96
Nov-1962	3.92	May-1976	7.90	Nov-1989	7.87	May-2003	3.57
Dec-1962	3.86	Jun-1976	7.86	Dec-1989	7.84	Jun-2003	3.33
Jan-1963	3.83	Jul-1976	7.83	Jan-1990	8.21	Jul-2003	3.98
Feb-1963	3.92	Aug-1976	7.77	Feb-1990	8.47	Aug-2003	4.45
Mar-1963	3.93	Sep-1976	7.59	Mar-1990	8.59	Sep-2003	4.27
Apr-1963	3.97	Oct-1976	7.41	Apr-1990	8.79	Oct-2003	4.29
May-1963	3.93	Nov-1976	7.29	May-1990	8.76	Nov-2003	4.30
Jun-1963	3.99	Dec-1976	6.87	Jun-1990	8.48	Dec-2003	4.27
Jul-1963	4.02	Jan-1977	7.21	Jul-1990	8.47	Jan-2004	4.15
Aug-1963	4.00	Feb-1977	7.39	Aug-1990	8.75	Feb-2004	4.08
Sep-1963	4.08	Mar-1977	7.46	Sep-1990	8.89	Mar-2004	3.83
Oct-1963	4.11	Apr-1977	7.37	Oct-1990	8.72	Apr-2004	4.35
Nov-1963	4.12	May-1977	7.46	Nov-1990	8.39	May-2004	4.72
Dec-1963	4.13	Jun-1977	7.28	Dec-1990	8.08	Jun-2004	4.73
Jan-1964	4.17	Jul-1977	7.33	Jan-1991	8.09	Jul-2004	4.50
Feb-1964	4.15	Aug-1977	7.40	Feb-1991	7.85	Aug-2004	4.28
Mar-1964	4.22	Sep-1977	7.34	Mar-1991	8.11	Sep-2004	4.13
Apr-1964	4.23	Oct-1977	7.52	Apr-1991	8.04	Oct-2004	4.10
May-1964	4.20	Nov-1977	7.58	May-1991	8.07	Nov-2004	4.19
Jun-1964	4.17	Dec-1977	7.69	Jun-1991	8.28	Dec-2004	4.23
Jul-1964	4.19	Jan-1978	7.96	Jul-1991	8.27	Jan-2005	4.22
Aug-1964	4.19	Feb-1978	8.03	Aug-1991	7.90	Feb-2005	4.17
Sep-1964	4.20	Mar-1978	8.04	Sep-1991	7.65	Mar-2005	4.50
Oct-1964	4.19	Apr-1978	8.15	Oct-1991	7.53	Apr-2005	4.34
Nov-1964	4.15	May-1978	8.35	Nov-1991	7.42	May-2005	4.14
Dec-1964	4.18	Jun-1978	8.46	Dec-1991	7.09	Jun-2005	4.00
Jan-1965	4.19	Jul-1978	8.64	Jan-1992	7.03	Jul-2005	4.18
Feb-1965	4.21	Aug-1978	8.41	Feb-1992	7.34	Aug-2005	4.26
Mar-1965	4.21	Sep-1978	8.42	Mar-1992	7.54	Sep-2005	4.20
Apr-1965	4.20	Oct-1978	8.64	Apr-1992	7.48	Oct-2005	4.46
May-1965	4.21	Nov-1978	8.81	May-1992	7.39	Nov-2005	4.54
Jun-1965	4.21	Dec-1978	9.01	Jun-1992	7.26	Dec-2005	4.47
Jul-1965	4.20	Jan-1979	9.10	Jul-1992	6.84	Jan-2006	4.42
Aug-1965	4.25	Feb-1979	9.10	Aug-1992	6.59	Feb-2006	4.57
Sep-1965	4.29	Mar-1979	9.12	Sep-1992	6.42	Mar-2006	4.72
Oct-1965	4.35	Apr-1979	9.18	Oct-1992	6.59	Apr-2006	4.99
Nov-1965	4.45	May-1979	9.25	Nov-1992	6.87	May-2006	5.11
Dec-1965	4.62	Jun-1979	8.91	Dec-1992	6.77	Jun-2006	5.11
Jan-1966	4.61	Jul-1979	8.95	Jan-1993	6.60	Jul-2006	5.09
Feb-1966	4.83	Aug-1979	9.03	Feb-1993	6.26	Aug-2006	4.88
Mar-1966	4.87	Sep-1979	9.33	Mar-1993	5.98	Sep-2006	4.72
Apr-1966	4.75	Oct-1979	10.30	Apr-1993	5.97	Oct-2006	4.73
May-1966	4.78	Nov-1979	10.65	May-1993	6.04	Nov-2006	4.60
Jun-1966	4.81	Dec-1979	10.39	Jun-1993	5.96	Dec-2006	4.56
Jul-1966	5.02	Jan-1980	10.80	Jul-1993	5.81	Jan-2007	4.76
Aug-1966	5.22	Feb-1980	12.41	Aug-1993	5.68	Feb-2007	4.72
Sep-1966	5.18	Mar-1980	12.75	Sep-1993	5.36		

The collection of Figures and Tables that follow are individually identified below.

List of Figures:

List of Tables:

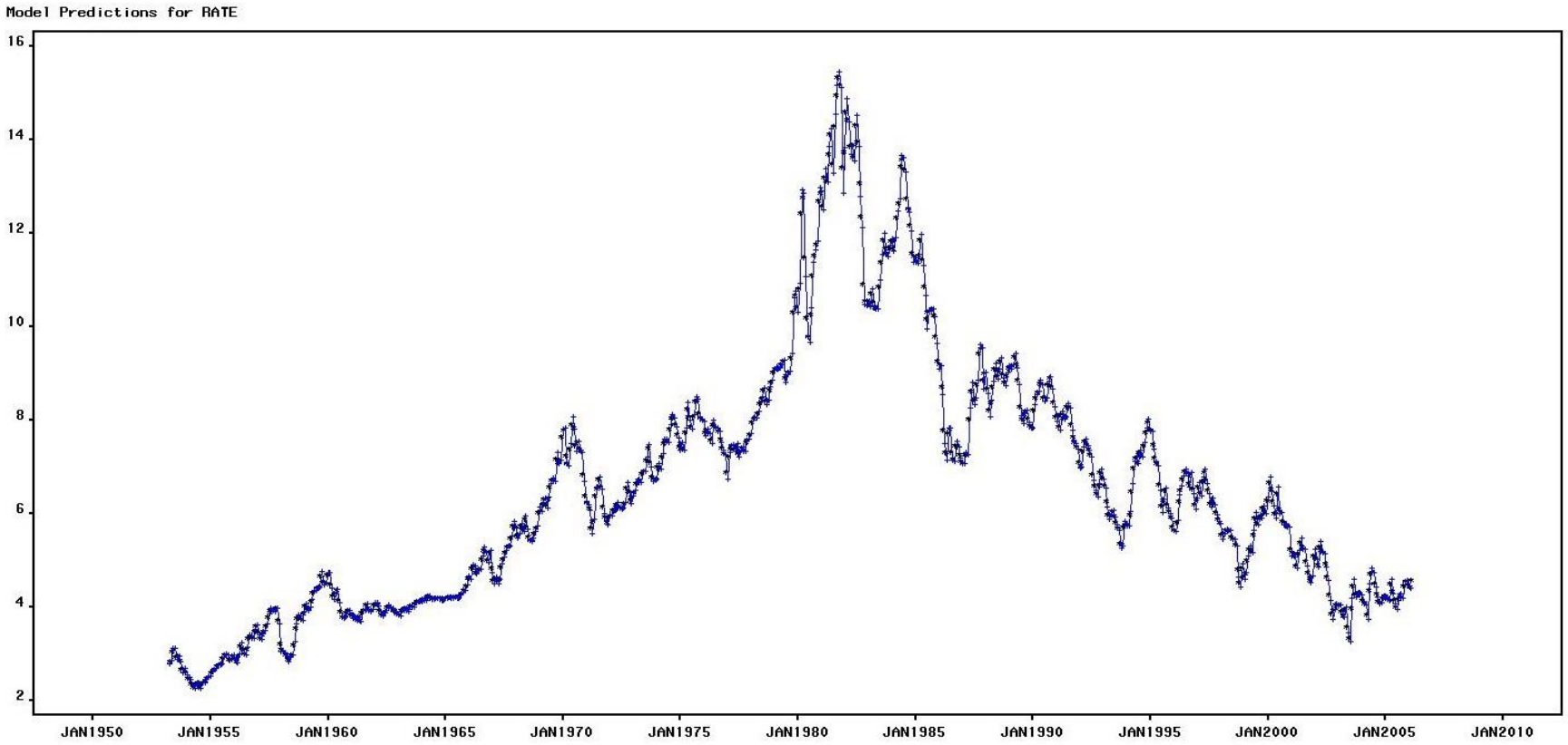

Figure 1 depicts a time series graph for predicted values ((+) signs) using the training data ((*) signs) set for Rate variable, as reported by SAS(AFS).

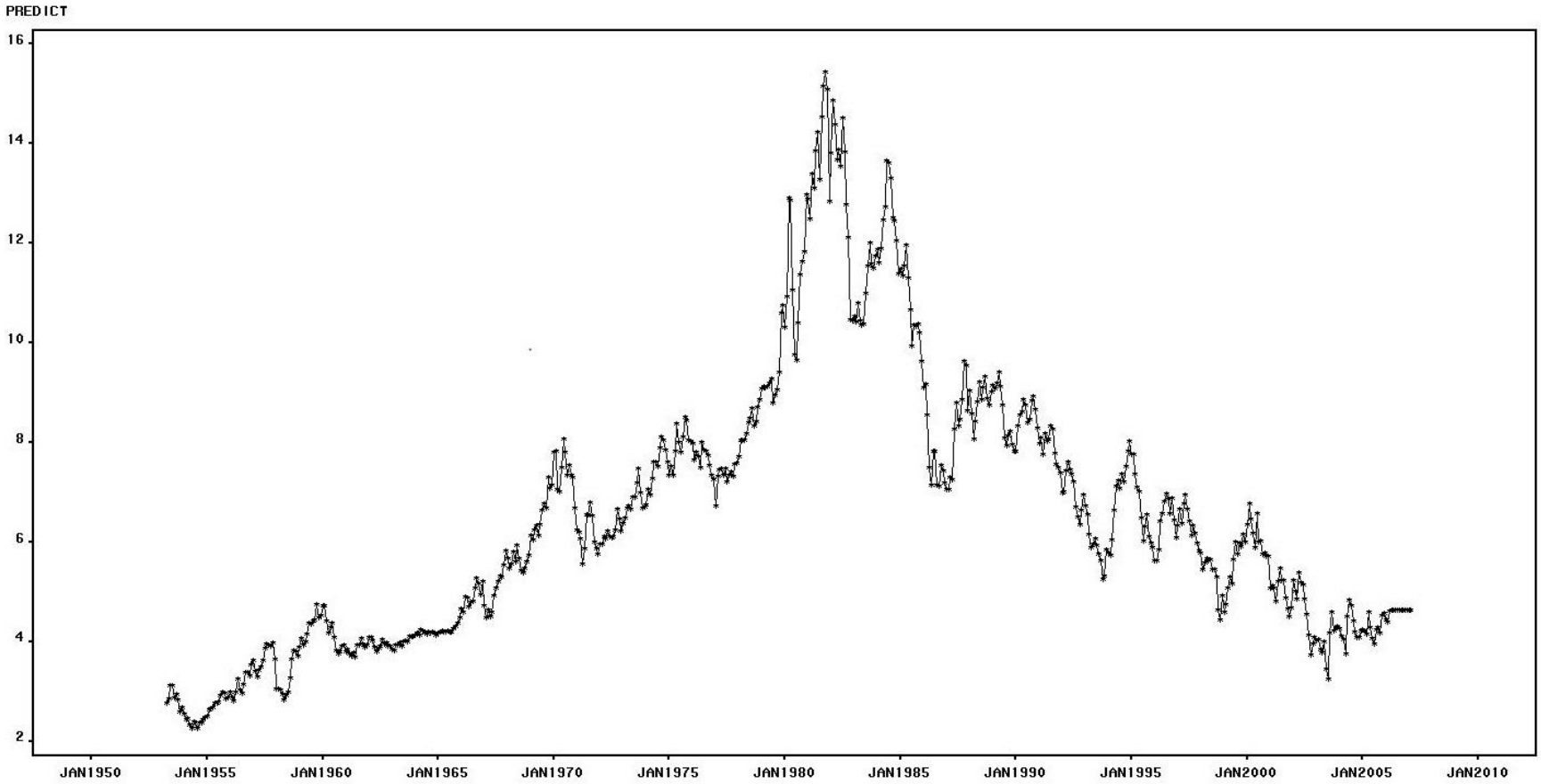

Figure 2 shows a time series graph for predicted values ((*) signs) for Rate variable as reported by SAS(AFS), along with 12-month forecasted values for the Rate variable. Here, predicted and forecast values are plotted using ((*) signs).

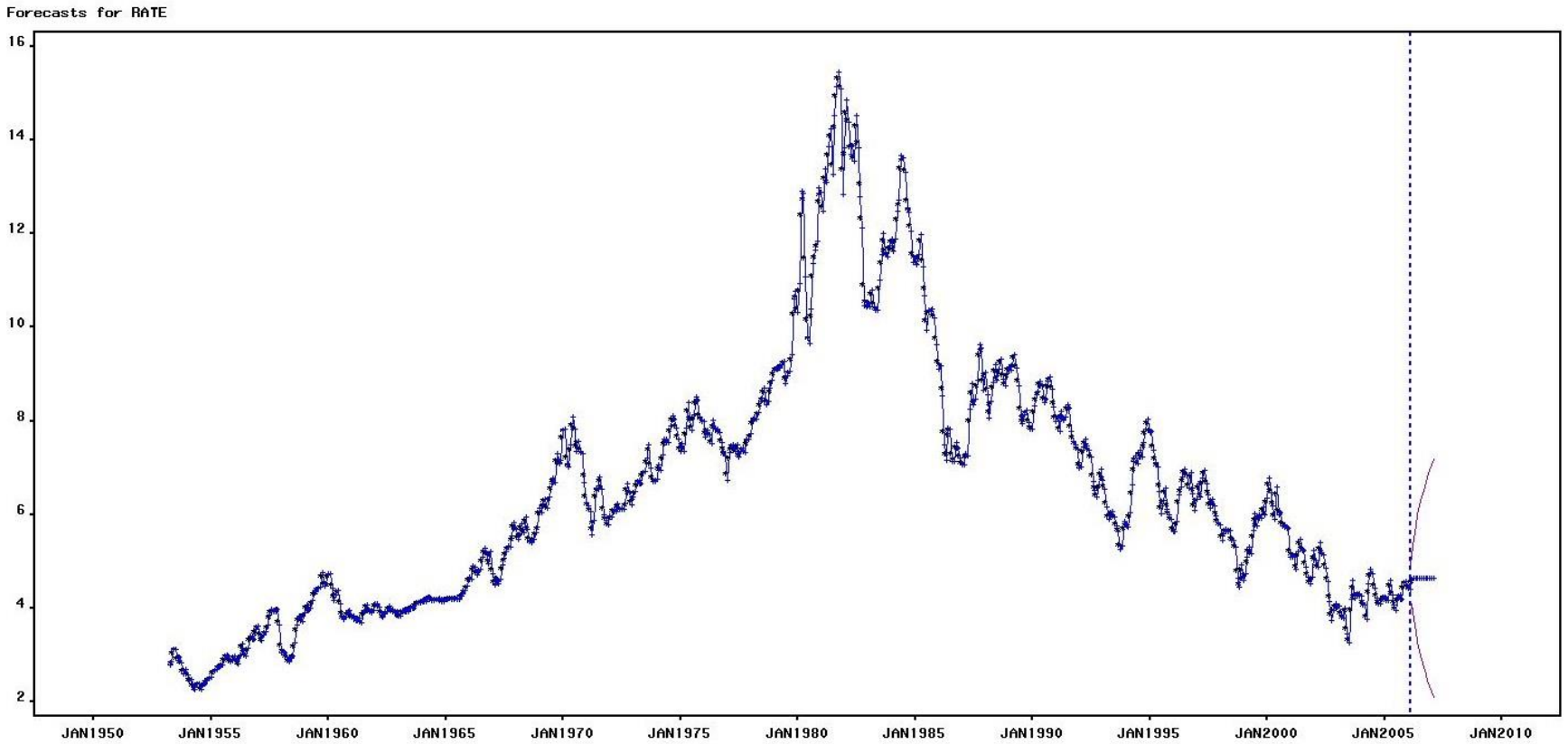

Figure 3a depicts a time series graph for predicted values ((+) signs) and the training data ((*) signs) for the Rate variable, as reported by SAS(AFS), along with 12-month forecasted values and prediction limits.

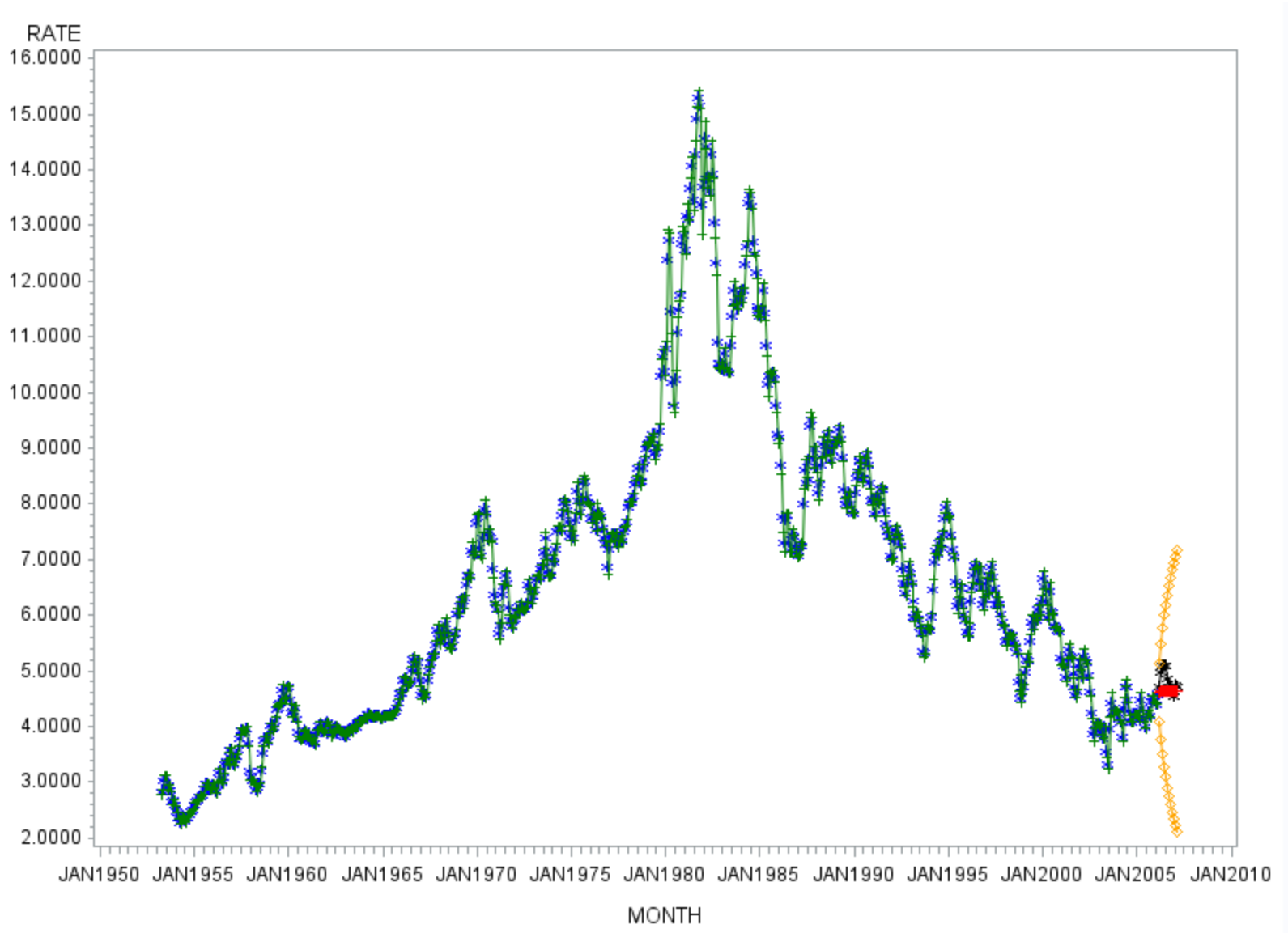

Figure 3b shows a time series graph for predicted values (green-colored (+) signs) for the Rate variable as reported by SAS(AFS), along with 12-month forecasted values, which are plotted using red-colored plus (+) signs. The training data are plotted using blue-colored star (*) signs, and testing data are plotted using black-colored star (*) signs, while the 12-month confidence limits are yellow-colored.

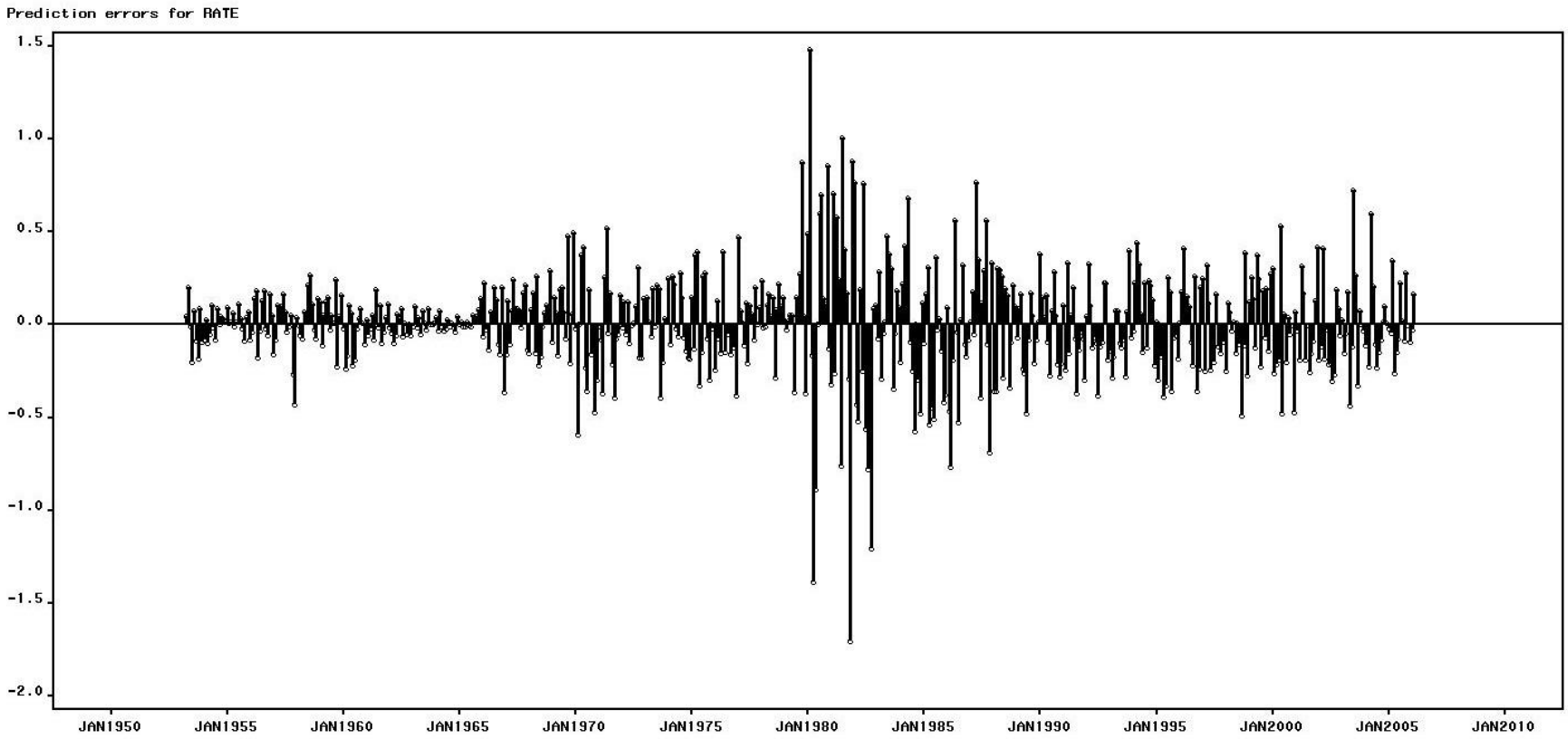

Figure 4. Prediction errors by time series values for the training data set.

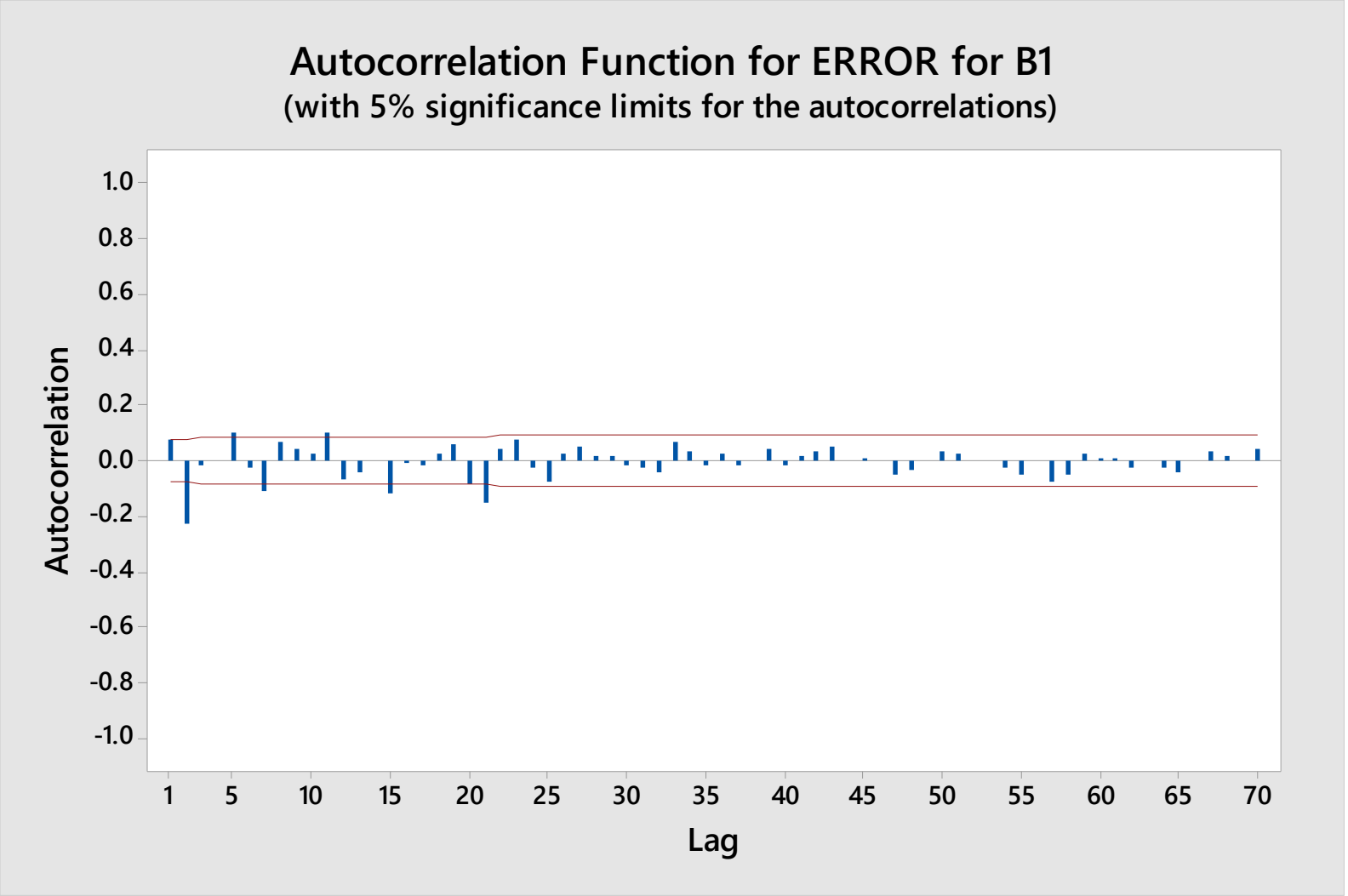

Figure 5. Autocorrelation Function (ACF) for the training data set.

Accuracy Measure for Table B.1 data.

Actual y_t	Predict $\widehat{y_t}$(t-1)	Forecast error $e_t(1) = y_t - \widehat{y_t}$(t-1)	Absolute error $\lvert e_t(1)\rvert$	Square error $[e_t(1)]^2$	Relative error $re_t(1) = 100(e_t(1)/y_t)$	Absolute percentage error $\lvert re_t(1)\rvert$
4.72	4.6168	0.1032	1.26824	0.010650	2.18644	2.18644
4.99	4.6316	0.3584	2.28244	0.128451	7.18236	7.18236
5.11	4.6362	0.4738	2.97715	0.224486	9.27202	9.27202
5.11	4.6377	0.4723	2.96688	0.223067	9.24266	9.24266
5.09	4.6382	0.4518	2.83009	0.204123	8.87623	8.87623
4.88	4.6383	0.2417	1.74462	0.058419	4.95287	4.95287
4.72	4.6383	0.0817	1.20698	0.006675	1.73093	1.73093
4.73	4.6384	0.0916	1.23481	0.008391	1.93658	1.93658
4.60	4.6384	-0.0384	0.91538	0.001475	-0.83478	0.83478
4.56	4.6384	-0.0784	0.83483	0.006147	-1.71930	1.71930
4.76	4.6384	0.1216	1.32312	0.014787	2.55462	2.55462
4.72	4.6384	0.0816	1.20670	0.006659	1.72881	1.72881

Table A. Error Measures table B1 time series data.

Data set	**MSE**	$\widehat{\sigma_{e(1)}}$	**MAD**	**MPE**	**MAPE**
Full data (647 data)	0.06934	0.2633	0.18037	0.02040	2.70924
Training data (635 data)	0.07034	0.2652	0.18184	0.01961	2.72020
Holdout data (12 data)	0.0744440	0.2728	0.216208	3.92579	4.35147

Table B. Accuracy Measures table B1 time series data.

Table B provides SAS (AFS)® forecast errors for the full data set, the training data set, and the holdout (or test) data set. The damped trend exponential smoothing model was suggested as the "best model" for this data set. SAS (AFS)® and MINITAB® produced the accuracy measures shown in Table B, which comprise the Mean Square Error (MSE), the Standard Deviation of the Forecast Errors (SDFE), the Mean Absolute Deviation (MAD), the Mean Percent Forecast Error (MPE), and Mean Absolute Percent Error (MAPE).

Here, the accuracy statistics are computed as follows:

Mean square error:

$$\textbf{MSE} = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = 0.0744440$$

Standard Deviation of Forecast Error (Root Mean Square Error):

$$\widehat{\sigma_{e(1)}} = \sqrt{MSE} = \sqrt{0.0744440} = 0.273$$

Mean Absolute Deviation (Mean Absolute Error):

$$\textbf{MAD} = \frac{1}{n}\sum_{t=1}^{n} |e_t(1)| = 0.216$$

Mean Percent Forecast Error:

$$\textbf{MPE} = \frac{1}{n}\sum_{t=1}^{n} re_t(1) = 3.92$$

Mean Absolute Percent Error:

$$\textbf{MAPE} = \frac{1}{n}\sum_{t=1}^{n} |re_t(1)| = 4.35$$

Worksheet 1 ***

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	Actual	Predict	Error	Abs_Error	Sq_Error	Relative_Error	Abs_Percentage_error	MSE	Std_Dev	MAD	MPE	MAPE
1	4.72	4.6168	0.1032	0.1032	0.010650	2.18644	2.18644	0.0744440	0.272844	0.216208	3.92579	4.35147
2	4.99	4.6316	0.3584	0.3584	0.128451	7.18236	7.18236					
3	5.11	4.6362	0.4738	0.4738	0.224486	9.27202	9.27202					
4	5.11	4.6377	0.4723	0.4723	0.223067	9.24266	9.24266					
5	5.09	4.6382	0.4518	0.4518	0.204123	8.87623	8.87623					
6	4.88	4.6383	0.2417	0.2417	0.058419	4.95287	4.95287					
7	4.72	4.6383	0.0817	0.0817	0.006675	1.73093	1.73093					
8	4.73	4.6384	0.0916	0.0916	0.008391	1.93658	1.93658					
9	4.60	4.6384	-0.0384	0.0384	0.001475	-0.83478	0.83478					
10	4.56	4.6384	-0.0784	0.0784	0.006147	-1.71930	1.71930					
11	4.76	4.6384	0.1216	0.1216	0.014787	2.55462	2.55462					
12	4.72	4.6384	0.0816	0.0816	0.006659	1.72881	1.72881					

Table C. MINITAB® 17 Calculation of Forecast Errors for Holdout Data.

The first two columns of Table C consist of the actual values of the holdout data set and the SAS (AFS)® forecasted values, based on training data. The remaining columns give forecast errors, th absolute forecast errors, the squared forecast errors, the relative forecast errors and absolute forecast percentage errors using the MINITAB® calculator function.

Obs	DATE	ACTUAL	PREDICT	UPPER	LOWER	ERROR	STD	NERROR	_LEVEL_	_TREND_
636	MAR2006	.	4.6168	5.1379	4.0958	.	0.2659	.	4.6168	0.0470
637	APR2006	.	4.6316	5.4914	3.7718	.	0.4387	.	4.6316	0.0148
638	MAY2006	.	4.6362	5.7675	3.5050	.	0.5772	.	4.6362	0.004638
639	JUN2006	.	4.6377	5.9957	3.2797	.	0.6929	.	4.6377	0.001457
640	JUL2006	.	4.6382	6.1926	3.0837	.	0.7931	.	4.6382	0.000458
641	AUG2006	.	4.6383	6.3677	2.9089	.	0.8824	.	4.6383	0.000144
642	SEP2006	.	4.6383	6.5268	2.7499	.	0.9635	.	4.6383	0.0000452
643	OCT2006	.	4.6384	6.6735	2.6032	.	1.0383	.	4.6384	0.0000142
644	NOV2006	.	4.6384	6.8103	2.4664	.	1.1081	.	4.6384	4.4553E-6
645	DEC2006	.	4.6384	6.9390	2.3377	.	1.1738	.	4.6384	1.3995E-6
646	JAN2007	.	4.6384	7.0609	2.2159	.	1.2360	.	4.6384	4.396E-7
647	FEB2007	.	4.6384	7.1769	2.0998	.	1.2952	.	4.6384	1.3809E-7

Table D. Forecasted values for the last 12 months of Holdout Data.

Table D contains forecasted values for the final twelve months for the Rate variable, and the corresponding lower and upper 95% prediction limits. The Root Mean Square Prediction Error, Level and Trend smoothing components are also shown..

Model Parameter	Estimate	Std. Error	T	Prob>|T|
LEVEL Smoothing Weight	0.99900	0.2930	3.4091	0.0007
TREND Smoothing Weight	0.99900	2.0662	0.4835	0.6289
DAMPING Smoothing Weight	0.31412	0.2778	1.1307	0.2586
Residual Variance (sigma squared)	0.07068	.	.	.
Smoothed Level	4.56983	.	.	.
Smoothed Trend	0.14964	.	.	.

Table E. Parameter Estimates for Training Data using the Damped Trend Exponential Smoothing model suggested by SAS (AFS)®.

Results

In Figure 3b, it seems the model fits the data appropriately, however, the SAS(AFS) could not forecast values with acceptably small prediction error. There are a few spikes in the ACF function, which means forecast errors may have additional structure not captured by the model. The MSE, RMSE and MAD are similar for both the training and holdout data sets; that is, the variability in forecast errors are approximately similar.

TABLE B.2 (PHARMACEUTICAL PRODUCT SALES) ANALYSIS

Table B.2 comprises data on the Pharmaceutical product sales for 120 weeks, where the sale variable (in thousands) is collected weekly. The data set excludes last 12 weekly values as a holdout set in order to analyze the accuracy of SAS(AFS), which suggested a mean model as the "best model" for this training, or initialization, data set. SAS(AFS) also produced the predicted values for the next 12 weeks. I have calculated different types of forecast errors based on the hold out data and predicted data.

The model equation is

$$Y_t = \alpha$$

where,

α = intercept and $\hat{\alpha} = \bar{y} = 10374$

Table B.2: Pharmaceutical Product Sales

Week	Sales (In Thousands)	Week	Sales (In Thousands)	Week	Sales (In Thousands)	Week	Sales (In Thousands)
1	10618.1	31	10334.5	61	10538.2	91	10375.4
2	10537.9	32	10480.1	62	10286.2	92	10123.4
3	10209.3	33	10387.6	63	10171.3	93	10462.7
4	10553.0	34	10202.6	64	10393.1	94	10205.5
5	9934.9	35	10219.3	65	10162.3	95	10522.7
6	10534.5	36	10382.7	66	10164.5	96	10253.2
7	10196.5	37	10820.5	67	10327.0	97	10428.7
8	10511.8	38	10358.7	68	10365.1	98	10615.8
9	10089.6	39	10494.6	69	10755.9	99	10417.3
10	10371.2	40	10497.6	70	10463.6	100	10445.4
11	10239.4	41	10431.5	71	10080.5	101	10690.6
12	10472.4	42	10447.8	72	10479.6	102	10271.8
13	10827.2	43	10684.4	73	9980.9	103	10524.8
14	10640.8	44	10176.5	74	10039.2	104	9815.0
15	10517.8	45	10616.0	75	10246.1	105	10398.5
16	10154.2	46	10627.7	76	10368.0	106	10553.1
17	9969.2	47	10684.0	77	10446.3	107	10655.8
18	10260.4	48	10246.7	78	10535.3	108	10199.1
19	10737.0	49	10265.0	79	10786.9	109	10416.6
20	10430.0	50	10090.4	80	9975.8	110	10391.3
21	10689.0	51	9881.1	81	10160.9	111	10210.1
22	10430.4	52	10449.7	82	10422.1	112	10352.5
23	10002.4	53	10276.3	83	10757.2	113	10423.8
24	10135.7	54	10175.2	84	10463.8	114	10519.3
25	10096.2	55	10212.5	85	10307.0	115	10596.7
26	10288.7	56	10395.5	86	10134.7	116	10650.0
27	10289.1	57	10545.9	87	10207.7	117	10741.6
28	10589.9	58	10635.7	88	10488.0	118	10246.0
29	10551.9	59	10265.2	89	10262.3	119	10354.4
30	10208.3	60	10551.6	90	10785.9	120	10155.4

The collection of Figures and Tables that follow are individually identified below.

List of Figures:

List of Tables:

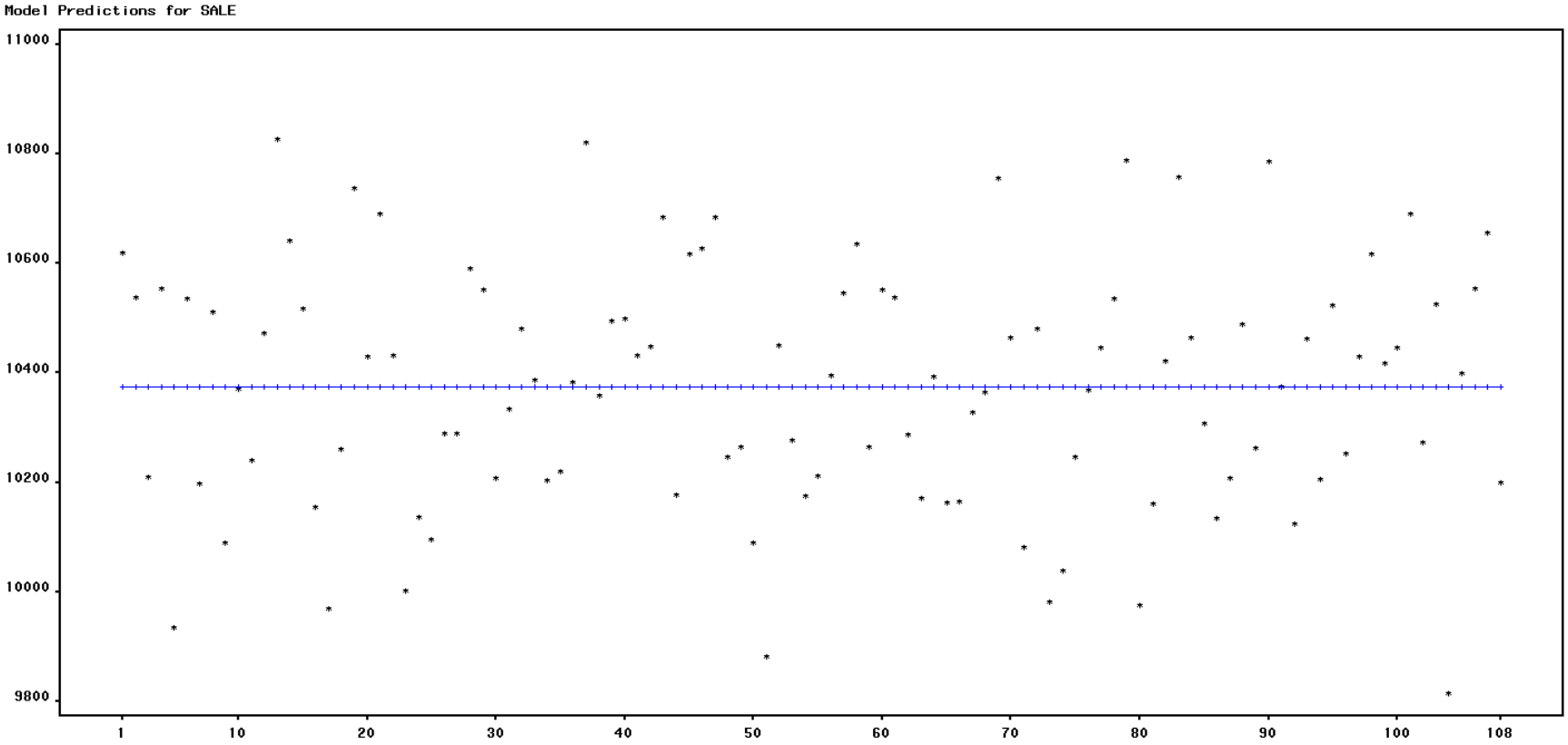

Figure 1 depicts a time series graph for predicted values ((+) signs) using the training data ((*) signs) set for Sale variable, as reported by SAS(AFS).

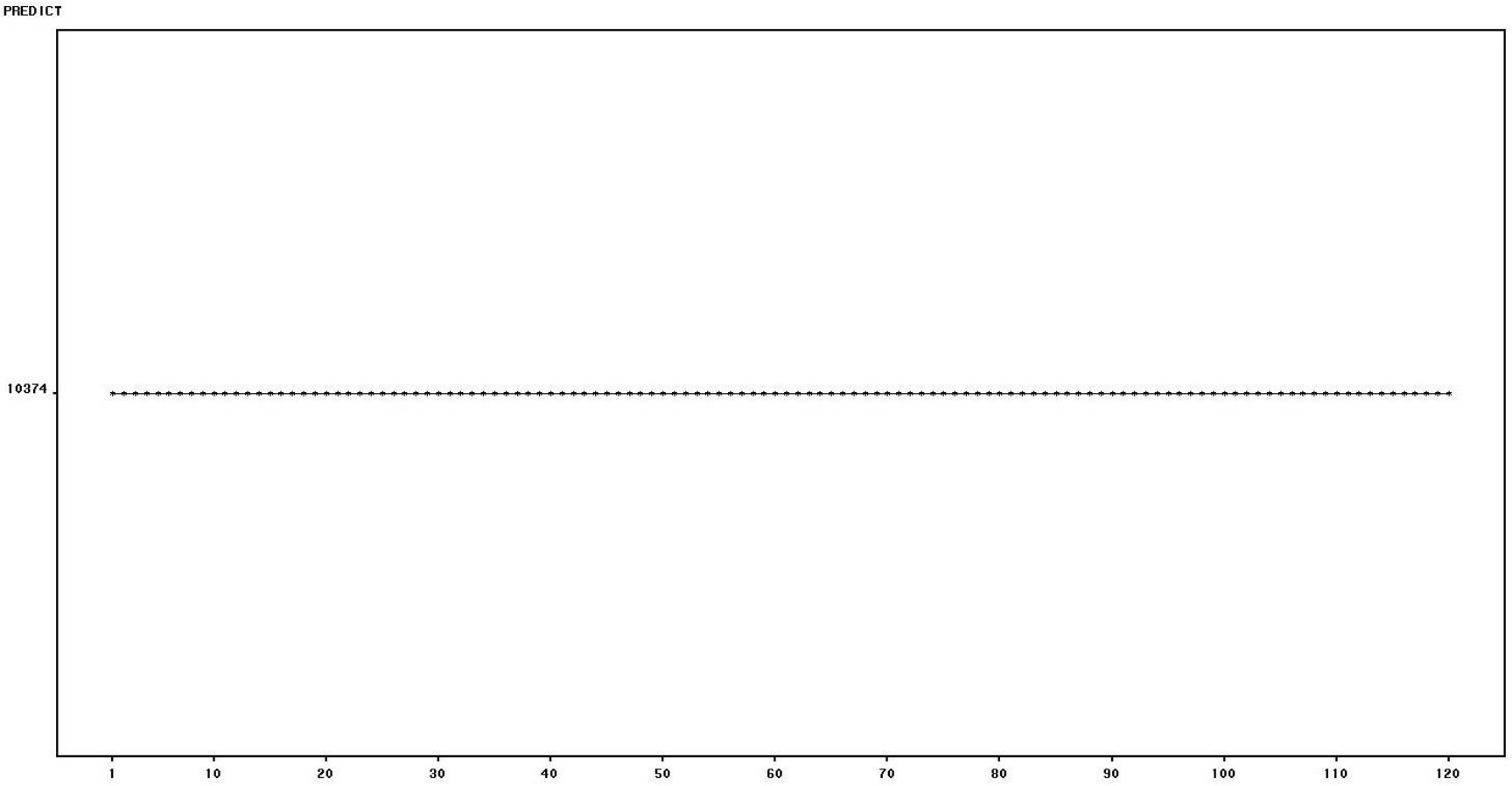

Figure 2 shows a time series graph for predicted values ((*) signs) for Sale variable as reported by SAS(AFS), along with 12-week forecasted values for the Sale variable. Here, predicted and forecast values are plotted using ((*) signs).

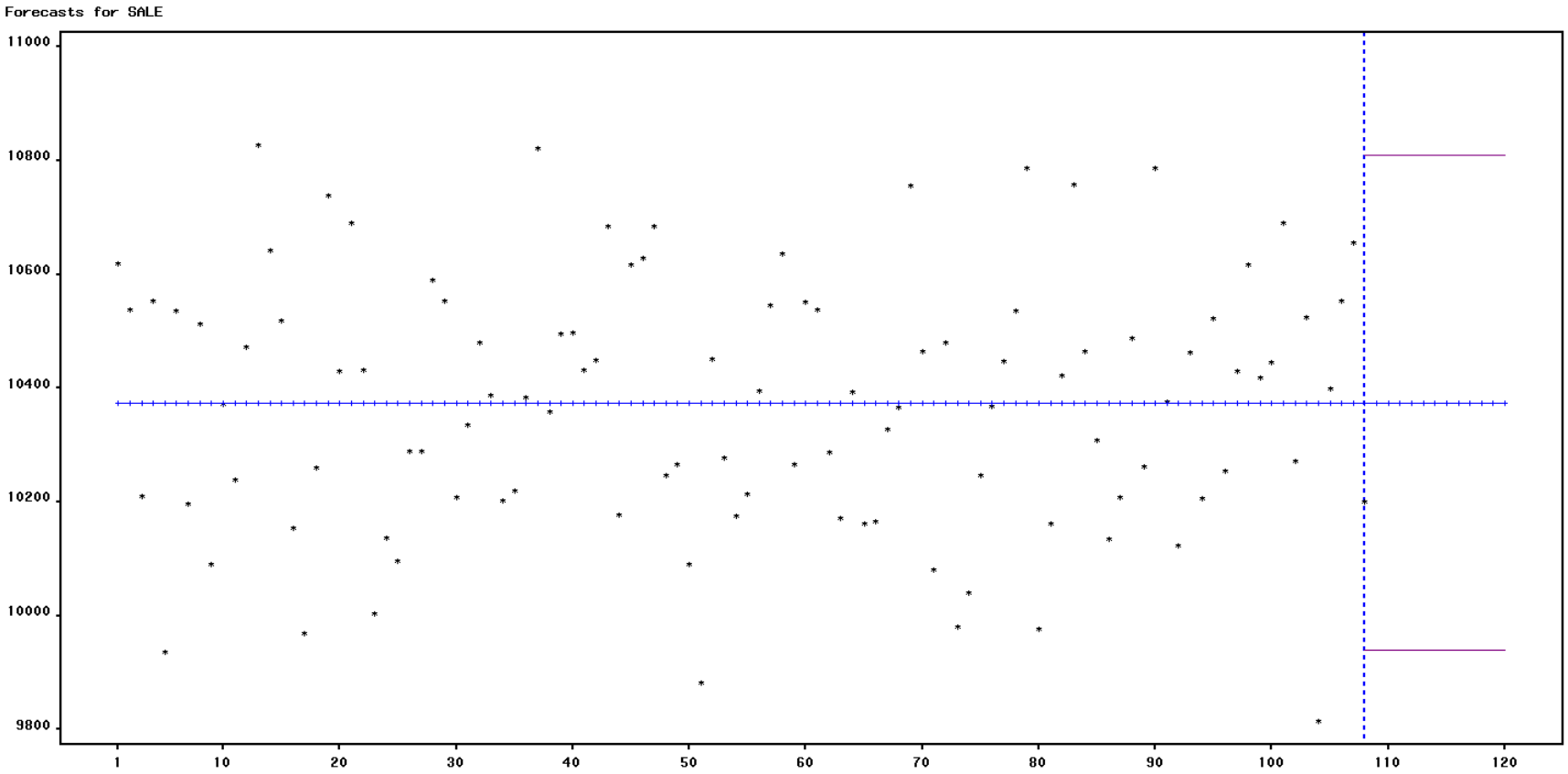

Figure 3a depicts a time series graph for predicted values ((+) signs) and the training data ((*) signs) for the Sale variable, as reported by SAS(AFS), along with 12-week forecasted values and prediction limits.

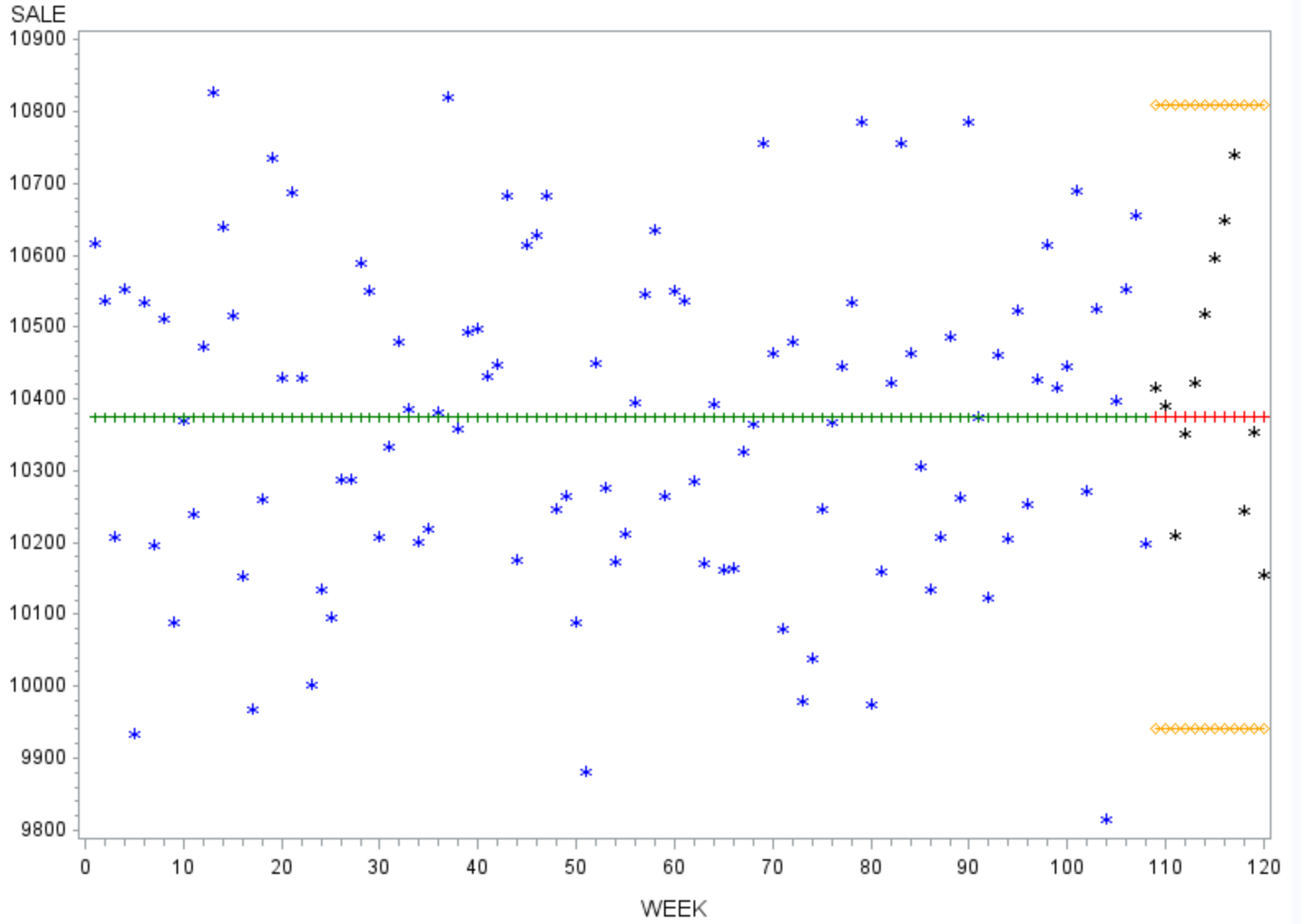

Figure 3b shows a time series graph for predicted values (green-colored (+) signs) for the Sale variable as reported by SAS(AFS), along with 12-week forecasted values, which are plotted using red-colored plus (+) signs. The training data are plotted using blue-colored star (*) signs, and testing data are plotted using black-colored star (*) signs, while the 12-week confidence limits are yellow-colored.

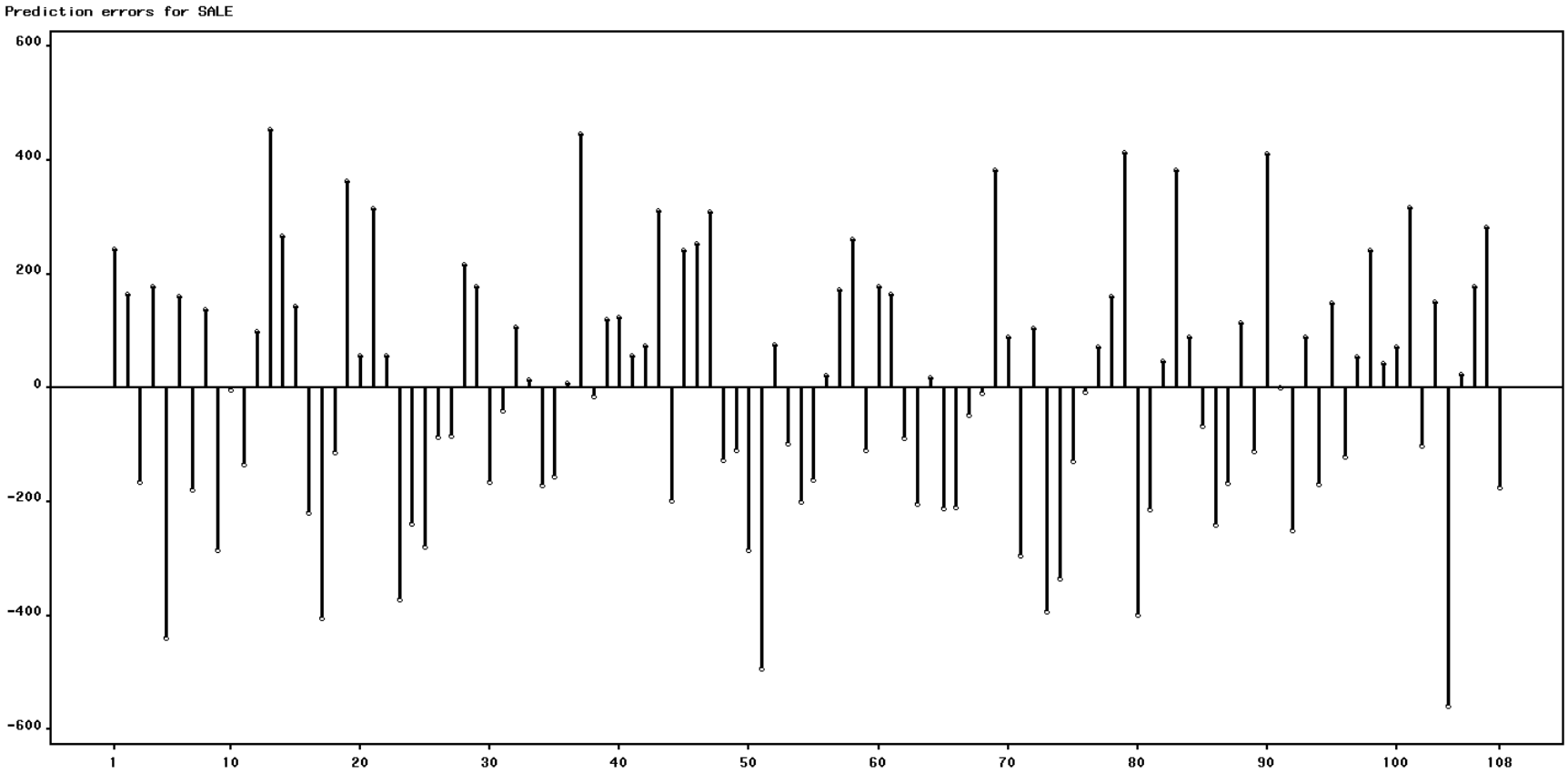

Figure 4. Prediction errors by time series values for the training data set.

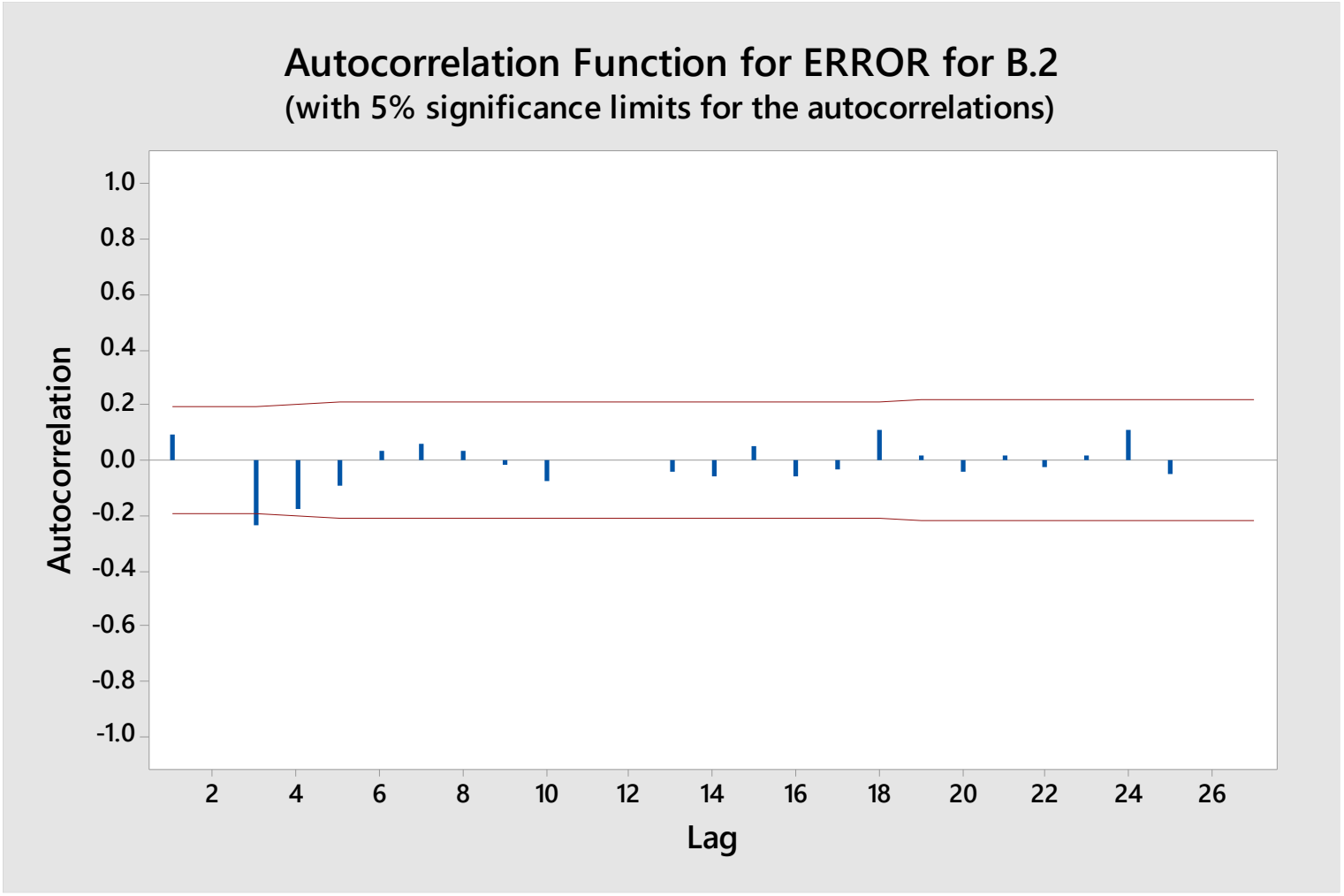

Figure 5. Autocorrelation Function (ACF) for the training data set.

Accuracy Measure for Table B.2 data.

Actual y_t	Predict $\widehat{y_t}$(t-1)	Forecast error $e_t(1) = y_t - \widehat{y_t}$(t-1)	Absolute error $\lvert e_t(1)\rvert$	Square error $[e_t(1)]^2$	Relative error $re_t(1) = 100(e_t(1)/y_t)$	Absolute percentage error $\lvert re_t(1)\rvert$
10416.6	10374	42.6	42.6	1815	0.40896	0.40896
10391.3	10374	17.3	17.3	299	0.16649	0.16649
10210.1	10374	-163.9	163.9	26863	-1.60527	1.60527
10352.5	10374	-21.5	21.5	462	-0.20768	0.20768
10423.8	10374	49.8	49.8	2480	0.47775	0.47775
10519.3	10374	145.3	145.3	21112	1.38127	1.38127
10596.7	10374	222.7	222.7	49595	2.10160	2.10160
10650.0	10374	276.0	276.0	76176	2.59155	2.59155
10741.6	10374	367.6	367.6	135130	3.42221	3.42221
10246.0	10374	-128.0	128.0	16384	-1.24927	1.24927
10354.4	10374	-19.6	19.6	384	-0.18929	0.18929
10155.4	10374	-218.6	218.6	47786	-2.15255	2.15255

Table A. Error Measures table B2 time series data.

Data set	**MSE**	$\widehat{\sigma_{e(1)}}$	**MAD**	**MPE**	**MAPE**
Full data (120 data)	46877.5	216.51216	177.66683	-0.04366	1.71443
Training data (108 data)	48610.7	220.47830	182.20509	-0.04532	1.75910
Holdout data (12 data)	31540.6	177.597	139.408	0.428814	1.32949

Table B. Accuracy Measures table B2 time series data.

Table B provides SAS (AFS)® forecast errors for the full data set, the training data set, and the holdout (or test) data set. The mean model was suggested as the "best model" for this data set. SAS (AFS)® and MINITAB® produced the accuracy measures shown in Table B, which comprise the Mean Square Error (MSE), the Standard Deviation of the Forecast Errors (SDFE), the Mean Absolute Deviation (MAD), the Mean Percent Forecast Error (MPE), and Mean Absolute Percent Error (MAPE).

Here, the accuracy statistics are computed as follows:

Mean square error:

$$\textbf{MSE} = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = 31540.6$$

Standard Deviation of Forecast Error (Root Mean Square Error):

$$\widehat{\sigma_{e(1)}} = \sqrt{MSE} = \sqrt{31540.6} = 177.597$$

Mean Absolute Deviation (Mean Absolute Error):

$$\textbf{MAD} = \frac{1}{n}\sum_{t=1}^{n} |e_t(1)| = 139.408$$

Mean Percent Forecast Error:

$$\textbf{MPE} = \frac{1}{n}\sum_{t=1}^{n} re_t(1) = 0.428814$$

Mean Absolute Percent Error:

$$\textbf{MAPE} = \frac{1}{n}\sum_{t=1}^{n} |re_t(1)| = 1.32949$$

Worksheet 1 ***

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	Actual	Predict	Forecast_Error	Abs_Error	Sq_Error	Relative_Error	Abs_Percentage_Error	MSE	Std_Dev	MAD	MPE	MAPE
1	10416.6	10374	42.6	42.6	1815	0.40896	0.40896	31540.6	177.597	139.408	0.428814	1.32949
2	10391.3	10374	17.3	17.3	299	0.16649	0.16649					
3	10210.1	10374	-163.9	163.9	26863	-1.60527	1.60527					
4	10352.5	10374	-21.5	21.5	462	-0.20768	0.20768					
5	10423.8	10374	49.8	49.8	2480	0.47775	0.47775					
6	10519.3	10374	145.3	145.3	21112	1.38127	1.38127					
7	10596.7	10374	222.7	222.7	49595	2.10160	2.10160					
8	10650.0	10374	276.0	276.0	76176	2.59155	2.59155					
9	10741.6	10374	367.6	367.6	135130	3.42221	3.42221					
10	10246.0	10374	-128.0	128.0	16384	-1.24927	1.24927					
11	10354.4	10374	-19.6	19.6	384	-0.18929	0.18929					
12	10155.4	10374	-218.6	218.6	47786	-2.15255	2.15255					

Table C. MINITAB® 17 Calculation of Forecast Errors for Holdout Data.

The first two columns of Table C consist of the actual values of the holdout data set and the SAS (AFS)® forecasted values, based on training data. The remaining columns give forecast errors, the absolute forecast errors, the squared forecast errors, the relative forecast errors and absolute forecast percentage errors using the MINITAB® calculator function.

Obs	DATE	ACTUAL	PREDICT	UPPER	LOWER	ERROR	STD	NERROR
109	109	.	10374	10809	9940	.	221.5062	.
110	110	.	10374	10809	9940	.	221.5062	.
111	111	.	10374	10809	9940	.	221.5062	.
112	112	.	10374	10809	9940	.	221.5062	.
113	113	.	10374	10809	9940	.	221.5062	.
114	114	.	10374	10809	9940	.	221.5062	.
115	115	.	10374	10809	9940	.	221.5062	.
116	116	.	10374	10809	9940	.	221.5062	.
117	117	.	10374	10809	9940	.	221.5062	.
118	118	.	10374	10809	9940	.	221.5062	.
119	119	.	10374	10809	9940	.	221.5062	.
120	120	.	10374	10809	9940	.	221.5062	.

Table D. Forecasted values for the last 12 weeks of Holdout Data.

Table D contains forecasted values for the final twelve weeks for the Sale variable, and the corresponding lower and upper 95% prediction limits. The Root Mean Square Prediction Error, Level and Trend smoothing components are also shown..

Model Parameter	Estimate	Std. Error	T	Prob>|T|
Intercept	10374	21.3144	486.7322	<.0001
Model Variance (sigma squared)	49065	.	.	.

Table E. Parameter Estimates for Training Data using the Mean model suggested by SAS (AFS)®.

Results

In Figure 3b, it seems the model could not fit the data appropriately as well as failed to forcast values accurately. The MSE, RMSE and MAD are not similar among the training and holdout data sets; that is, the variability in forecast errors are exists. However, Mean Absolute Percent Error are reasonaly small in all cases.

TABLE B.3 (CHEMICAL PROCESS VISCOSITY READING) ANALYSIS

Table B.3 comprises data on the Chemical Process Viscosity Reading for 100 time periods, where the reading variable is collected. The data set excludes last 12 periodical values as a holdout set in order to analyze the accuracy of SAS(AFS), which suggested a log linear (Holt) exponential smoothing model as the "best model" for this training, or initialization, data set. SAS(AFS) also produced the predicted values for the next 12 time periods. I have calculated different types of forecast errors based on the hold out data and predicted data.

The model equation is

$$w_t = \ln(y_t) = \mu_t + \beta_t\, t + \varepsilon_t$$

where,

Y_t = observed value i.e. reading at time t

μ_t = smoothed level and $\mu_t = 4.41141$

β_t = smoothed trend and $\beta_t = -0.0003328$

Now, the k-step prediction equation is

$$\hat{y}_t(k) = \exp(\hat{w}_t(k))$$

where, $\hat{w}_t(k) = L_t + K\, T_t$

and the smoothing equations are

$$L_t = \alpha\, Y_t + (1-\alpha)(L_{t-1} + T_{t-1})$$

and $T_t = \gamma (L_t - L_{t-1}) + (1-\gamma)\, T_{t-1}$

where,

L_t = Level estimate and $L_0 = \mu_t = 4.41141$

T_t = Trend estimate and $T_0 = \beta_t = -0.0003328$

α = level smoothing and $\alpha = 0.83914$

γ = trend smoothing and $\gamma = 0.00100$

Table B.3: Chemical Process Viscosity Reading

Time period	Reading	Time period	Reading	Time period	Reading	Time period	Reading
1	86.7418	26	87.2397	51	85.5722	76	84.7052
2	85.3195	27	87.5219	52	83.7935	77	83.8168
3	84.7355	28	86.4992	53	84.3706	78	82.4171
4	85.1113	29	85.6050	54	83.3762	79	83.0420
5	85.1487	30	86.8293	55	84.9975	80	83.6993
6	84.4775	31	84.5004	56	84.3495	81	82.2033
7	84.6827	32	84.1844	57	85.3395	82	82.1413
8	84.6757	33	85.4563	58	86.0503	83	81.7961
9	86.3169	34	86.1511	59	84.8839	84	82.3241
10	88.0006	35	86.4142	60	85.4176	85	81.5316
11	86.2597	36	86.0498	61	84.2309	86	81.7280
12	85.8286	37	86.6642	62	83.5761	87	82.5375
13	83.7500	38	84.7289	63	84.1343	88	82.3877
14	84.4628	39	85.9523	64	82.6974	89	82.4159
15	84.6476	40	86.8473	65	83.5454	90	82.2102

Time period	Reading	Time period	Reading	Time period	Reading	Time period	Reading
16	84.5751	41	88.4250	66	86.4714	91	82.7673
17	82.2473	42	89.6481	67	86.2143	92	83.1234
18	83.3774	43	87.8566	68	87.0215	93	83.2203
19	83.5385	44	88.4997	69	86.6504	94	84.4510
20	85.1620	45	87.0622	70	85.7082	95	84.9145
21	83.7881	46	85.1973	71	86.1504	96	85.7609
22	84.0421	47	85.0767	72	85.8032	97	85.2302
23	84.1023	48	84.4362	73	85.6197	98	86.7312
24	84.8495	49	84.2112	74	84.2339	99	87.0048
25	87.6416	50	85.9952	75	83.5737	100	85.0572

The collection of Figures and Tables that follow are individually identified below.

List of Figures:

List of Tables:

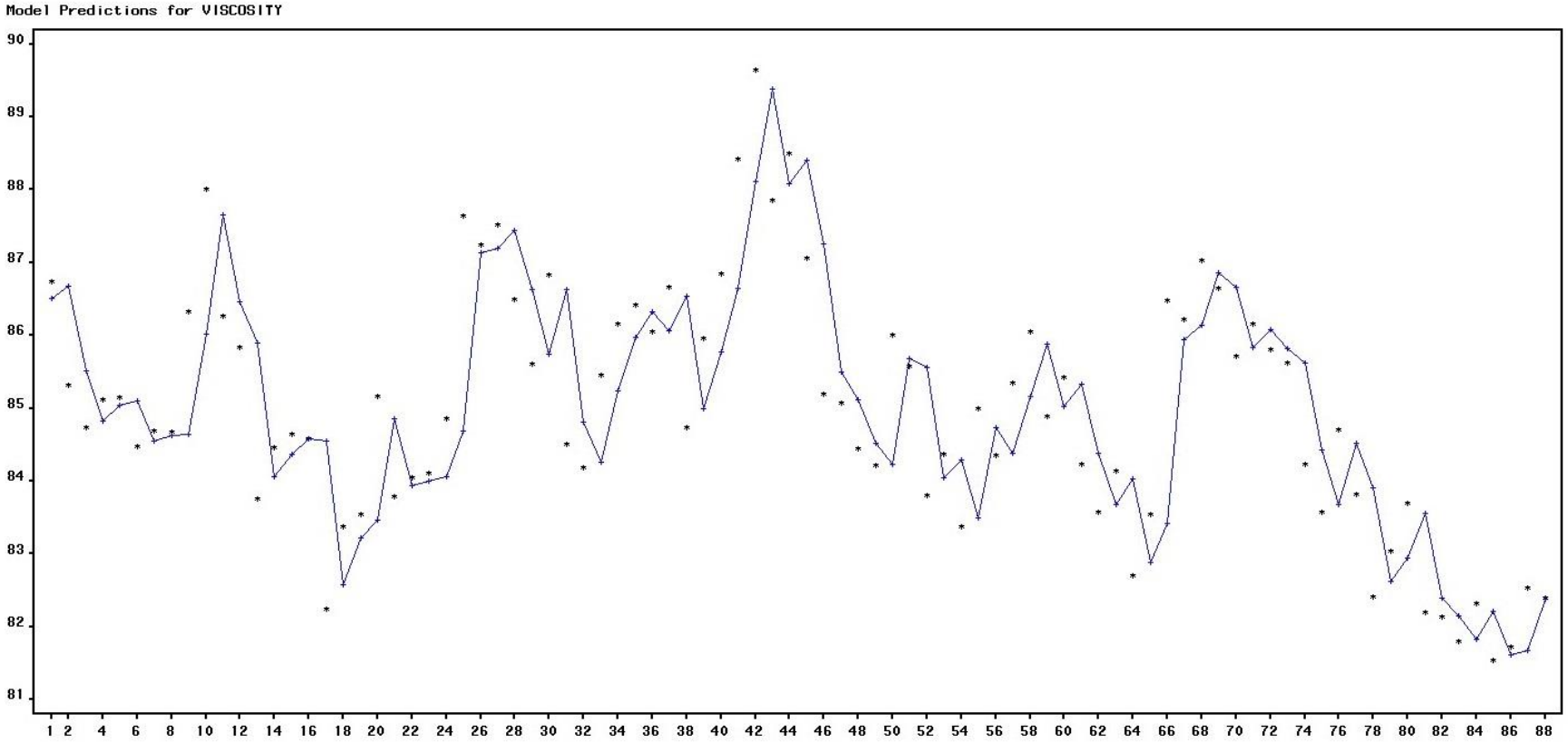

Figure 1 depicts a time series graph for predicted values ((+) signs) using the training data ((*) signs) set for Reading variable, as reported by SAS(AFS).

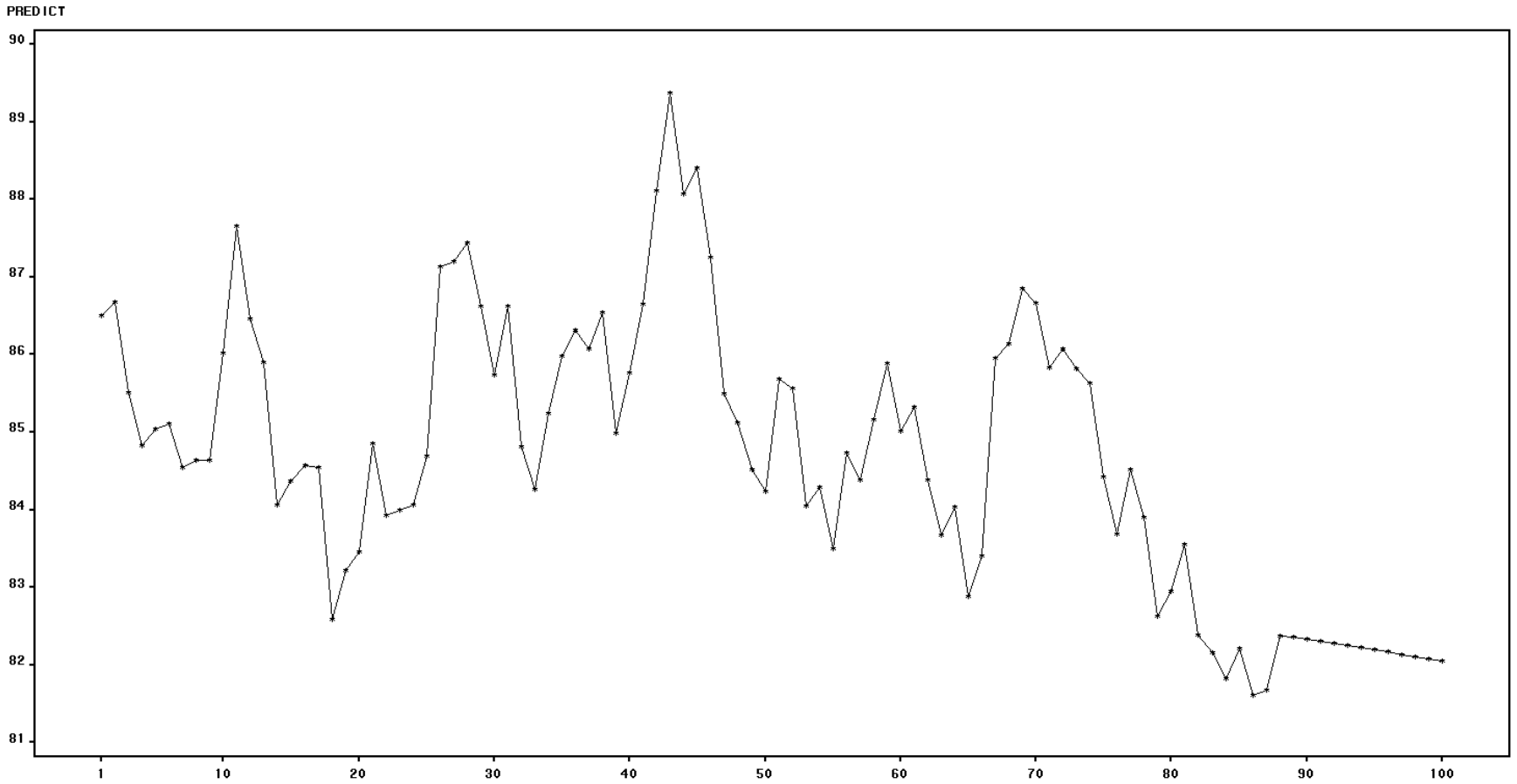

Figure 2 shows a time series graph for predicted values ((*) signs) for Reading variable as reported by SAS(AFS), along with 12-period forecasted values for the Reading variable. Here, predicted and forecast values are plotted using ((*) signs).

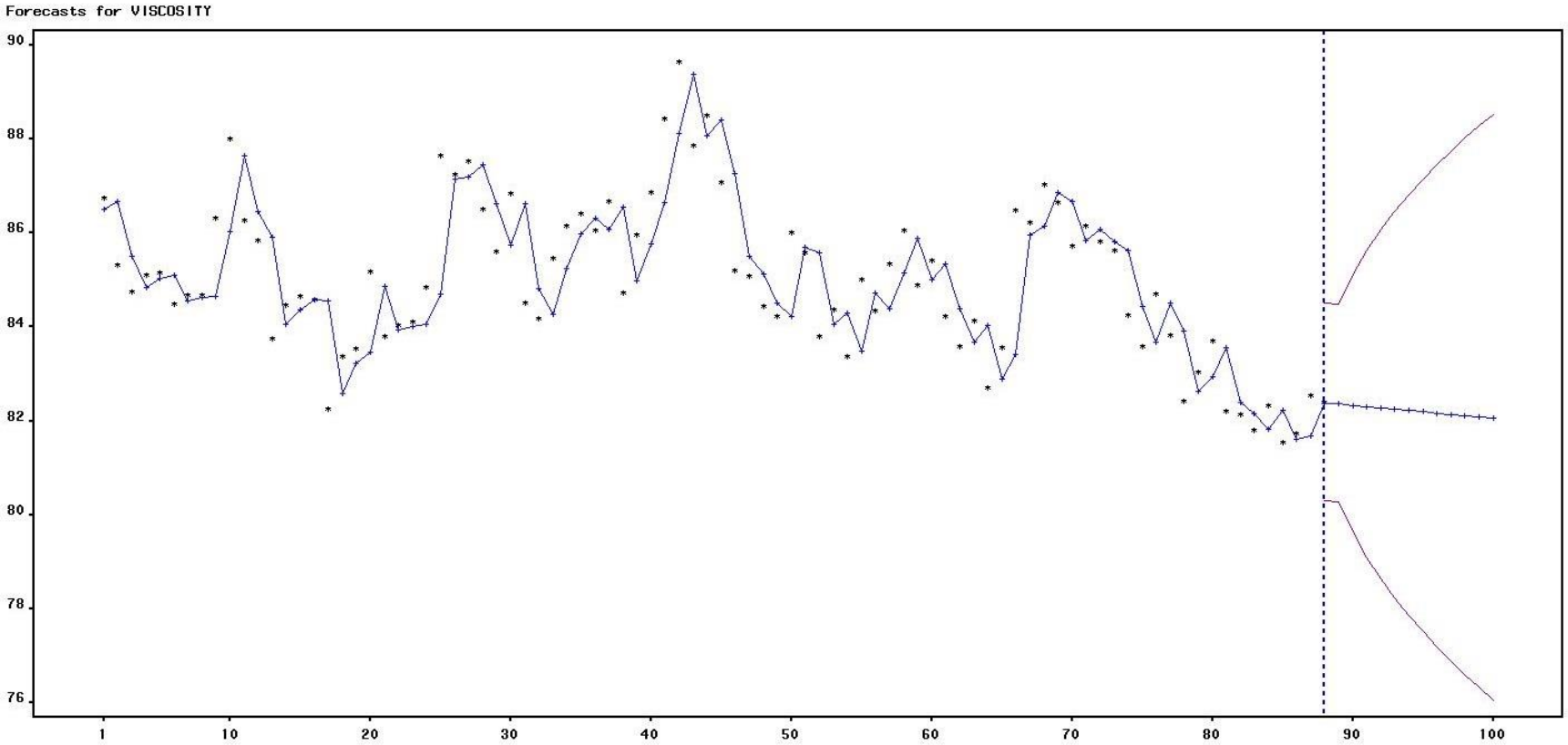

Figure 3a depicts a time series graph for predicted values ((+) signs) and the training data ((*) signs) for the Reading variable, as reported by SAS(AFS), along with 12-period forecasted values and prediction limits.

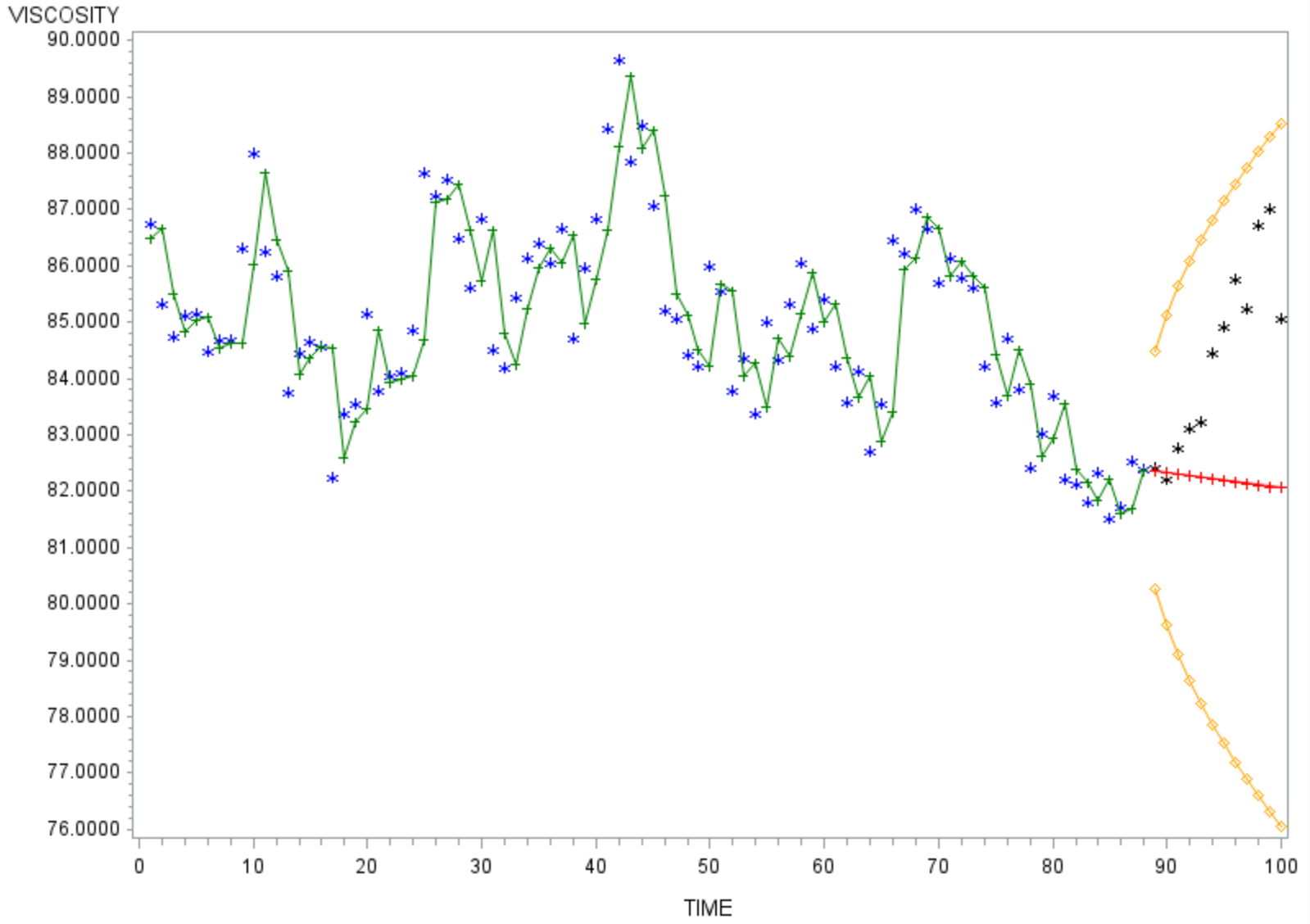

Figure 3b shows a time series graph for predicted values (green-colored (+) signs) for the Viscosity variable as reported by SAS(AFS), along with 12-time forecasted values, which are plotted using red-colored plus (+) signs. The training data are plotted using blue-colored star (*) signs, and testing data are plotted using black-colored star (*) signs, while the 12-time confidence limits are yellow-colored.

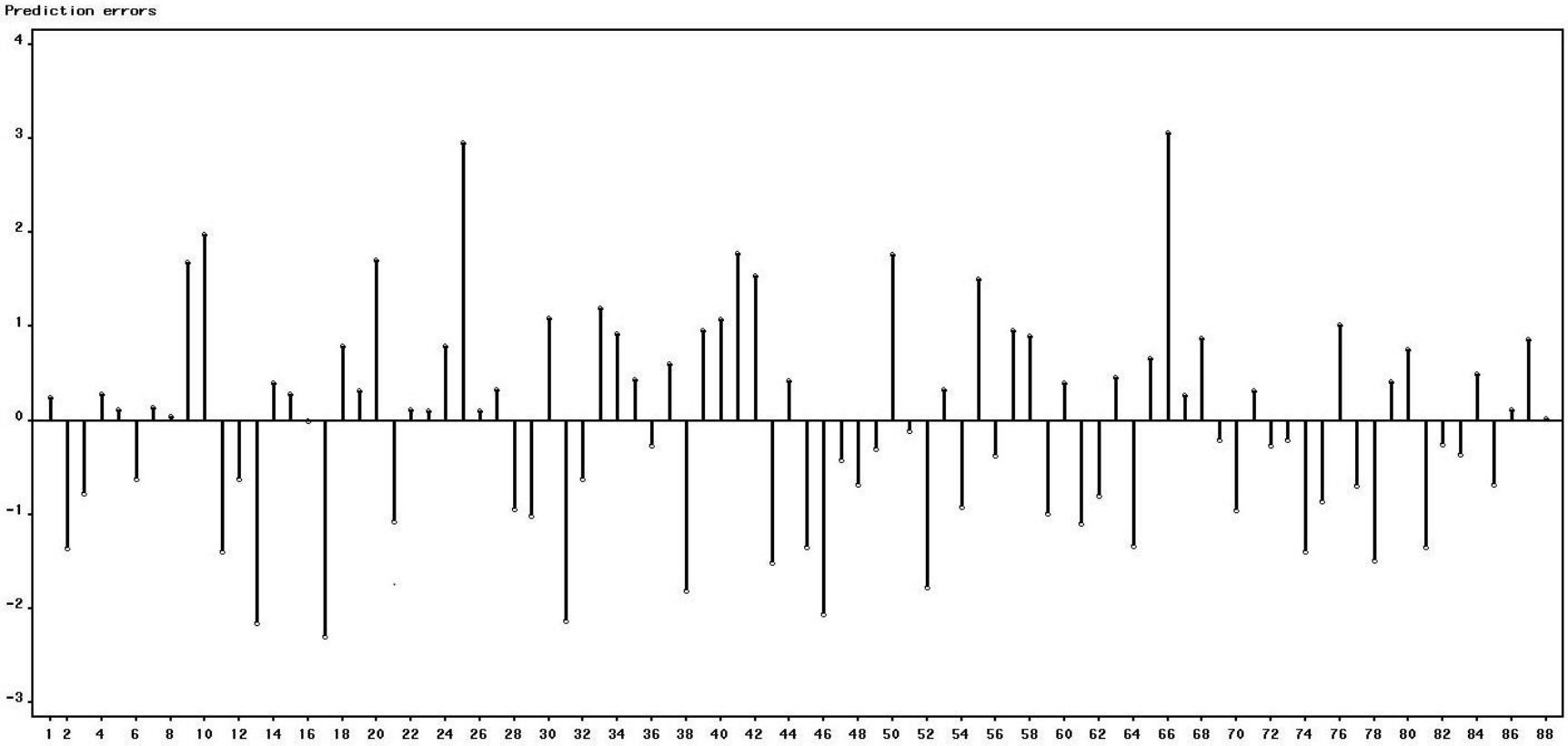

Figure 4. Prediction errors by time series values for the training data set.

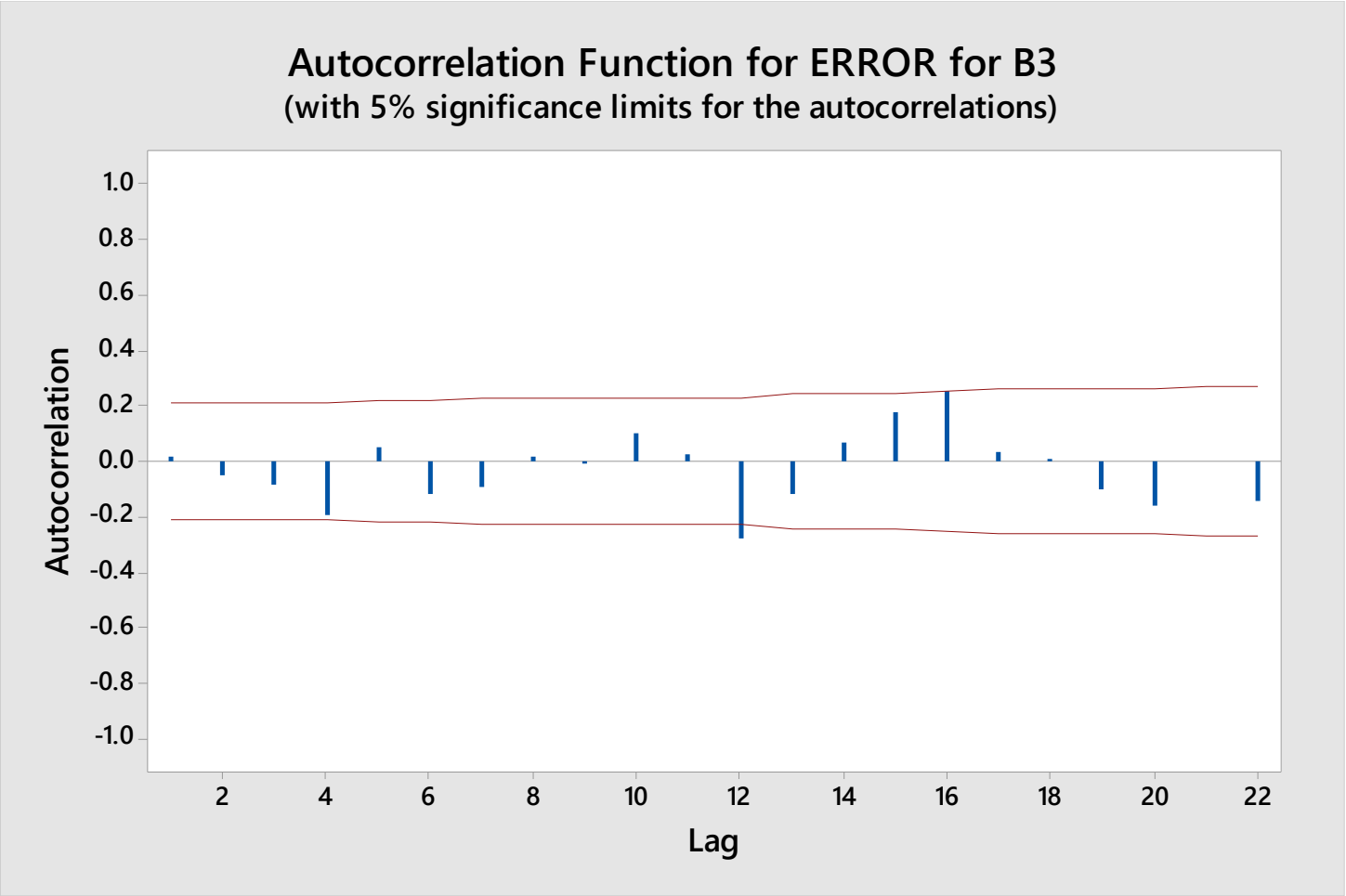

Figure 5. Autocorrelation Function (ACF) for the training data set.

Accuracy Measure for Table B.3 data.

Actual y_t	Predict $\widehat{y_t}$(t-1)	Forecast error $e_t(1) = y_t - \widehat{y_t}(t-1)$	Absolute error $\lvert e_t(1)\rvert$	Square error $[e_t(1)]^2$	Relative error $re_t(1) = 100(e_t(1)/y_t)$	Absolute percentage error $\lvert re_t(1)\rvert$
82.4159	82.3577	0.0582	0.0582	0.0034	0.07062	0.07062
82.2102	82.3303	-0.1201	0.1201	0.0144	-0.14609	0.14609
82.7673	82.3029	0.4644	0.4644	0.2157	0.56109	0.56109
83.1234	82.2755	0.8479	0.8479	0.7189	1.02005	1.02005
83.2203	82.2482	0.9721	0.9721	0.9450	1.16810	1.16810
84.4510	82.2208	2.2302	2.2302	4.9738	2.64082	2.64082
84.9145	82.1934	2.7211	2.7211	7.4044	3.20452	3.20452
85.7609	82.1661	3.5948	3.5948	12.9226	4.19165	4.19165
85.2302	82.1387	3.0915	3.0915	9.5574	3.62724	3.62724
86.7312	82.1114	4.6198	4.6198	21.3426	5.32657	5.32657
87.0048	82.0841	4.9207	4.9207	24.2133	5.65566	5.65566
85.0572	82.0568	3.0004	3.0004	9.0024	3.52751	3.52751

Table A. Error Measures table B3 time series data.

Data set	**MSE**	$\widehat{\sigma_{e(1)}}$	**MAD**	**MPE**	**MAPE**
Full data (100 data) – damped trend exponential smoothing	1.15794	1.07608	0.85307	-0.02534	1.00181
Training data (88 data) – log linear (Holt) exponential smoothing	1.20730	1.09877	0.87535	-0.03717	1.02758
Holdout data (12 data)	7.60948	2.75853	2.2201	2.57065	2.59499

Table B. Accuracy Measures table B3 time series data.

Table B provides SAS (AFS)® forecast errors for the full data set, the training data set, and the holdout (or test) data set. The log linear (Holt) exponential smoothing model was suggested as the "best model" for this data set. SAS (AFS)® and MINITAB® produced the accuracy measures shown in Table B, which comprise the Mean Square Error (MSE), the Standard Deviation of the Forecast Errors (SDFE), the Mean Absolute Deviation (MAD), the Mean Percent Forecast Error (MPE), and Mean Absolute Percent Error (MAPE).

Here, the accuracy statistics are computed as follows:

Mean square error:

$$\textbf{MSE} = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = 7.6094$$

Standard Deviation of Forecast Error (Root Mean Square Error):

$$\widehat{\sigma_{e(1)}} = \sqrt{MSE} = \sqrt{7.60948} = 2.75853$$

Mean Absolute Deviation (Mean Absolute Error):

$$\textbf{MAD} = \frac{1}{n}\sum_{t=1}^{n} |e_t(1)| = 2.2201$$

Mean Percent Forecast Error:

$$\textbf{MPE} = \frac{1}{n}\sum_{t=1}^{n} re_t(1) = 2.57065$$

Mean Absolute Percent Error:

$$\textbf{MAPE} = \frac{1}{n}\sum_{t=1}^{n} |re_t(1)| = 2.59499$$

Worksheet 1 ***

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	Actual	Predict	Forecast_Error	Abs_Error	Sq_Error	Relative_Error	Abs_Percentage_Error	MSE	Std_Dev	MAD	MPE	MAPE
1	82.4159	82.3577	0.0582	0.0582	0.0034	0.07062	0.07062	7.60948	2.75853	2.2201	2.57065	2.59499
2	82.2102	82.3303	-0.1201	0.1201	0.0144	-0.14609	0.14609					
3	82.7673	82.3029	0.4644	0.4644	0.2157	0.56109	0.56109					
4	83.1234	82.2755	0.8479	0.8479	0.7189	1.02005	1.02005					
5	83.2203	82.2482	0.9721	0.9721	0.9450	1.16810	1.16810					
6	84.4510	82.2208	2.2302	2.2302	4.9738	2.64082	2.64082					
7	84.9145	82.1934	2.7211	2.7211	7.4044	3.20452	3.20452					
8	85.7609	82.1661	3.5948	3.5948	12.9226	4.19165	4.19165					
9	85.2302	82.1387	3.0915	3.0915	9.5574	3.62724	3.62724					
10	86.7312	82.1114	4.6198	4.6198	21.3426	5.32657	5.32657					
11	87.0048	82.0841	4.9207	4.9207	24.2133	5.65566	5.65566					
12	85.0572	82.0568	3.0004	3.0004	9.0024	3.52751	3.52751					

Table C. MINITAB® 17 Calculation of Forecast Errors for Holdout Data.

The first two columns of Table C consist of the actual values of the holdout data set and the SAS (AFS)® forecasted values, based on training data. The remaining columns give forecast errors, th absolute forecast errors, the squared forecast errors, the relative forecast errors and absolute forecast percentage errors using the MINITAB® calculator function.

Obs	DATE	ACTUAL	PREDICT	UPPER	LOWER	ERROR	STD	NERROR	RESIDUAL	RESSTD	NRESID	_LEVEL_	_TREND_
89	89	.	82.3577	84.4876	80.2815	.	1.0730	.	.	.	.	4.4111	-0.000333
90	90	.	82.3303	85.1219	79.6303	.	1.4010	.	.	.	.	4.4107	-0.000333
91	91	.	82.3029	85.6325	79.1028	.	1.6659	.	.	.	.	4.4104	-0.000333
92	92	.	82.2755	86.0716	78.6469	.	1.8943	.	.	.	.	4.4101	-0.000333
93	93	.	82.2482	86.4627	78.2390	.	2.0982	.	.	.	.	4.4097	-0.000333
94	94	.	82.2208	86.8187	77.8664	.	2.2842	.	.	.	.	4.4094	-0.000333
95	95	.	82.1934	87.1476	77.5209	.	2.4563	.	.	.	.	4.4091	-0.000333
96	96	.	82.1661	87.4546	77.1973	.	2.6173	.	.	.	.	4.4087	-0.000333
97	97	.	82.1387	87.7437	76.8918	.	2.7691	.	.	.	.	4.4084	-0.000333
98	98	.	82.1114	88.0176	76.6015	.	2.9132	.	.	.	.	4.4081	-0.000333
99	99	.	82.0841	88.2785	76.3243	.	3.0506	.	.	.	.	4.4077	-0.000333
100	100	.	82.0568	88.5281	76.0585	.	3.1822	.	.	.	.	4.4074	-0.000333

Table D. Forecasted values for the last 12 periods of Holdout Data.

Table D contains forecasted values for the final twelve periods for the Reading variable, and the corresponding lower and upper 95% prediction limits. The Root Mean Square Prediction Error, Level and Trend smoothing components are also shown..

Model Parameter	Estimate	Std. Error	T	Prob>|T|
LEVEL Smoothing Weight	0.83914	0.0758	11.0767	<.0001
TREND Smoothing Weight	0.00100	0.0294	0.0340	0.9729
Residual Variance (sigma squared)	0.0001697	.	.	.
Smoothed Level	4.41141	.	.	.
Smoothed Trend	-0.0003328	.	.	.

Table E. Parameter Estimates for Training Data using the Log Linear (Holt) Exponential Smoothing model suggested by SAS (AFS)®.

Results

In Figure 3b, it seems the model fits the data appropriately, however, the forecast values produced by this model are very far from the testing values. There are a spikes in the ACF function at large lag, which means forecast errors may have additional structure not captured by the model. The MSE, RMSE and MAD are not similar among the training and holdout data sets; that is, the variability in forecast errors are exists.

TABLE B.4 (US PRODUCTION OF BLUE AND GORGONZOLA CHEESES) ANALYSIS

Table B.4 comprises data on the US production of Blue and Gorgonzola cheeses from 1950 through 1997, where the production variable (in 10^3 lb) is collected yearly. The data set excludes last 12 yearly values as a holdout set in order to analyze the accuracy of SAS(AFS), which suggested a damped trend exponential smoothing model as the "best model" for this training, or initialization, data set. SAS(AFS) also produced the predicted values for the next 12 years. I have calculated different types of forecast errors based on the hold out data and predicted data.

The model equation is

$$Y_t = \mu_t + \beta_t\, t + \varepsilon_t$$

where,

Y_t = observed value i.e. production at time t

μ_t = smoothed level and $\mu_t = 33533$

β_t = smoothed trend and $\beta_t = 697.43945$

Now, the k-step prediction equation is

$$\hat{Y}_t(k) = L_t + \sum_{i=1}^{k} \phi^t\, T_t$$

and the smoothing equations are

$$L_t = \alpha\, Y_t + (1-\alpha)\,(L_{t-1} + \phi\, T_{t-1})$$

and $T_t = \gamma\,(L_t - L_{t-1}) + (1-\gamma)\,\phi\, T_{t-1}$

where,

L_t = Level estimate and $L_0 = \mu_t = 33533$

T_t = Trend estimate and $T_0 = \beta_t = 697.43945$

α = level smoothing and $\alpha = 0.91885$

γ = trend smoothing and $\gamma = 0.00100$ and

ϕ = damping smoothing and $\phi = 0.99687$

The ARIMA model equivalency to damped-trend linear exponential smoothing is the ARIMA (1, 1, 2) model,

$(1-\phi B)(1-B)\, Y_t = (1-\theta_1 B - \theta_2 B^2)\, \varepsilon_t$

$\theta_1 = 1 + \phi - \alpha - \alpha\gamma\phi$

$\theta_2 = (\alpha - 1)\,\phi$

Table B.4: US production of Blue and Gorgonzola cheeses

Year	Production (10^3 lb)	Year	Production (10^3 lb)	Year	Production (10^3 lb)	Year	Production (10^3 lb)
1950	7,657	1962	14,507	1974	28,262	1986	34,198
1951	5,451	1963	15,400	1975	28,506	1987	35,863
1952	10,883	1964	16,800	1976	33,885	1988	37,789
1953	9,554	1965	19,000	1977	34,776	1989	34,561
1954	9,519	1966	20,198	1978	35,347	1990	36,434
1955	10,047	1967	18,573	1979	34,628	1991	34,371
1956	10,663	1968	19,375	1980	33,043	1992	33,307

Year	Production (10^3 lb)	Year	Production (10^3 lb)	Year	Production (10^3 lb)	Year	Production (10^3 lb)
1957	10,864	1969	21,032	1981	30,214	1993	33,295
1958	11,447	1970	23,250	1982	31,013	1994	36,514
1959	12,710	1971	25,219	1983	31,496	1995	36,593
1960	15,169	1972	28,549	1984	34,115	1996	38,311
1961	16,205	1973	29,759	1985	33,433	1997	42,773

The collection of Figures and Tables that follow are individually identified below.

List of Figures:

List of Tables:

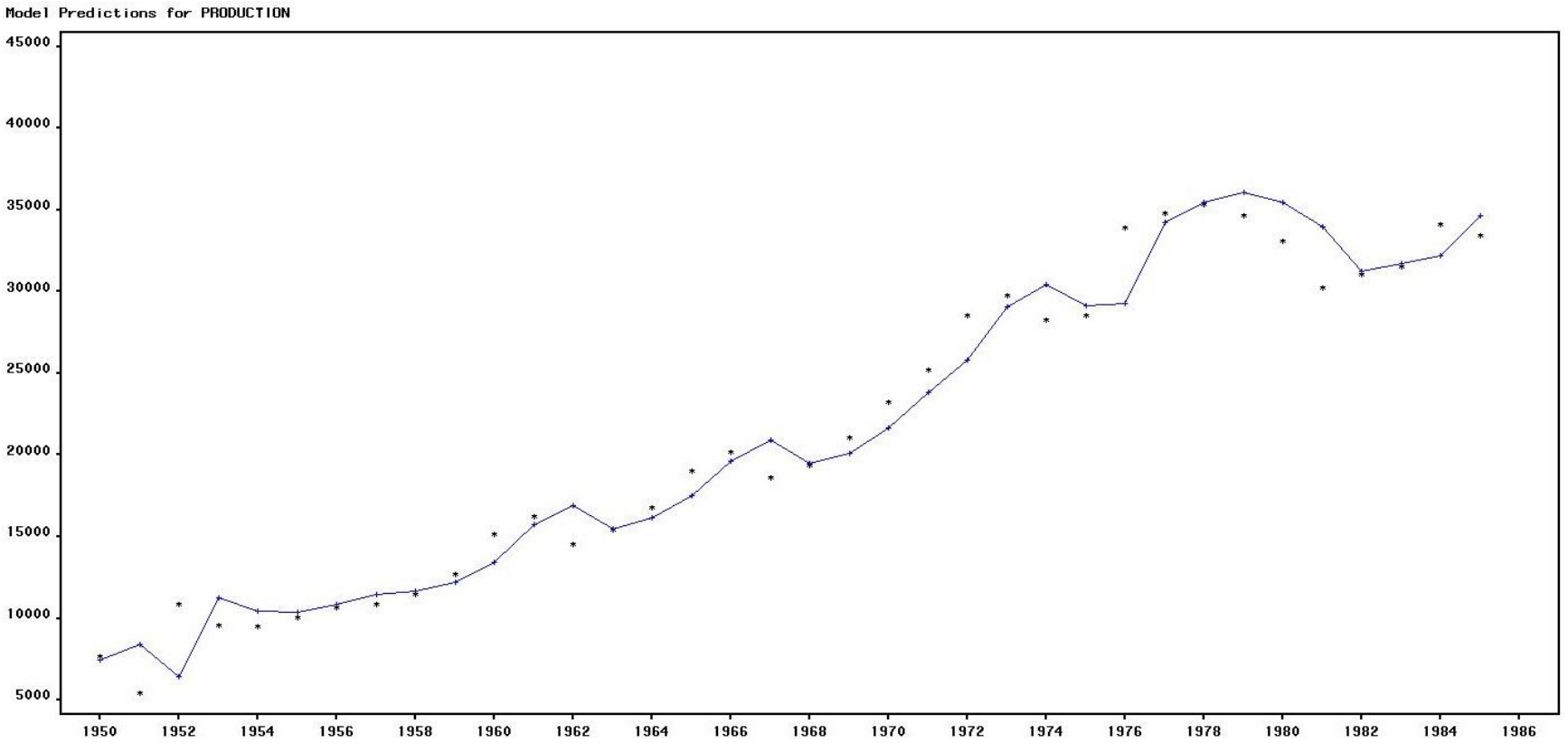

Figure 1 depicts a time series graph for predicted values ((+) signs) using the training data ((*) signs) set for Production variable, as reported by SAS(AFS).

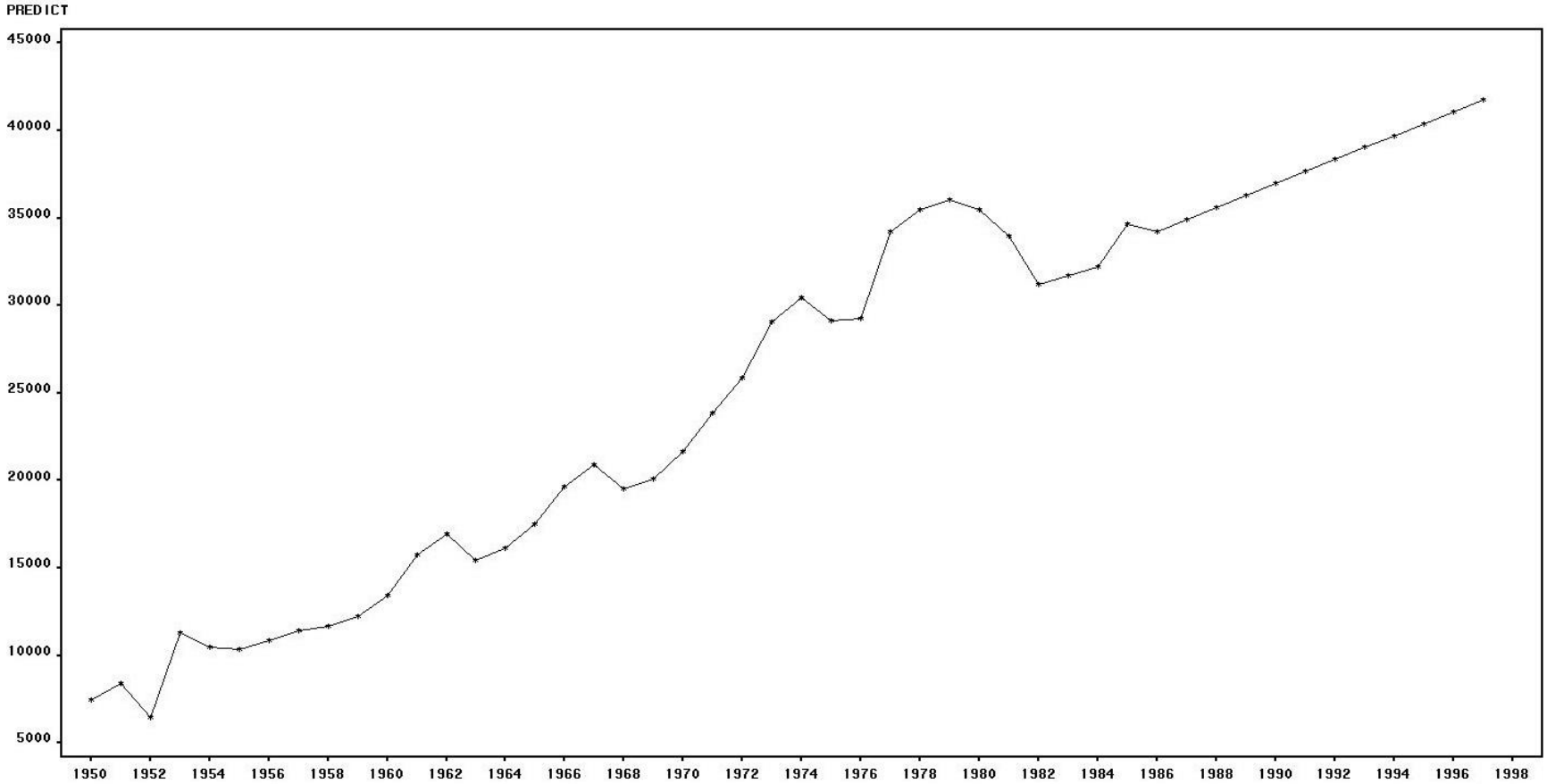

Figure 2 shows a time series graph for predicted values ((*) signs) for Production variable as reported by SAS(AFS), along with 12-year forecasted values for the Production variable. Here, predicted and forecast values are plotted using ((*) signs).

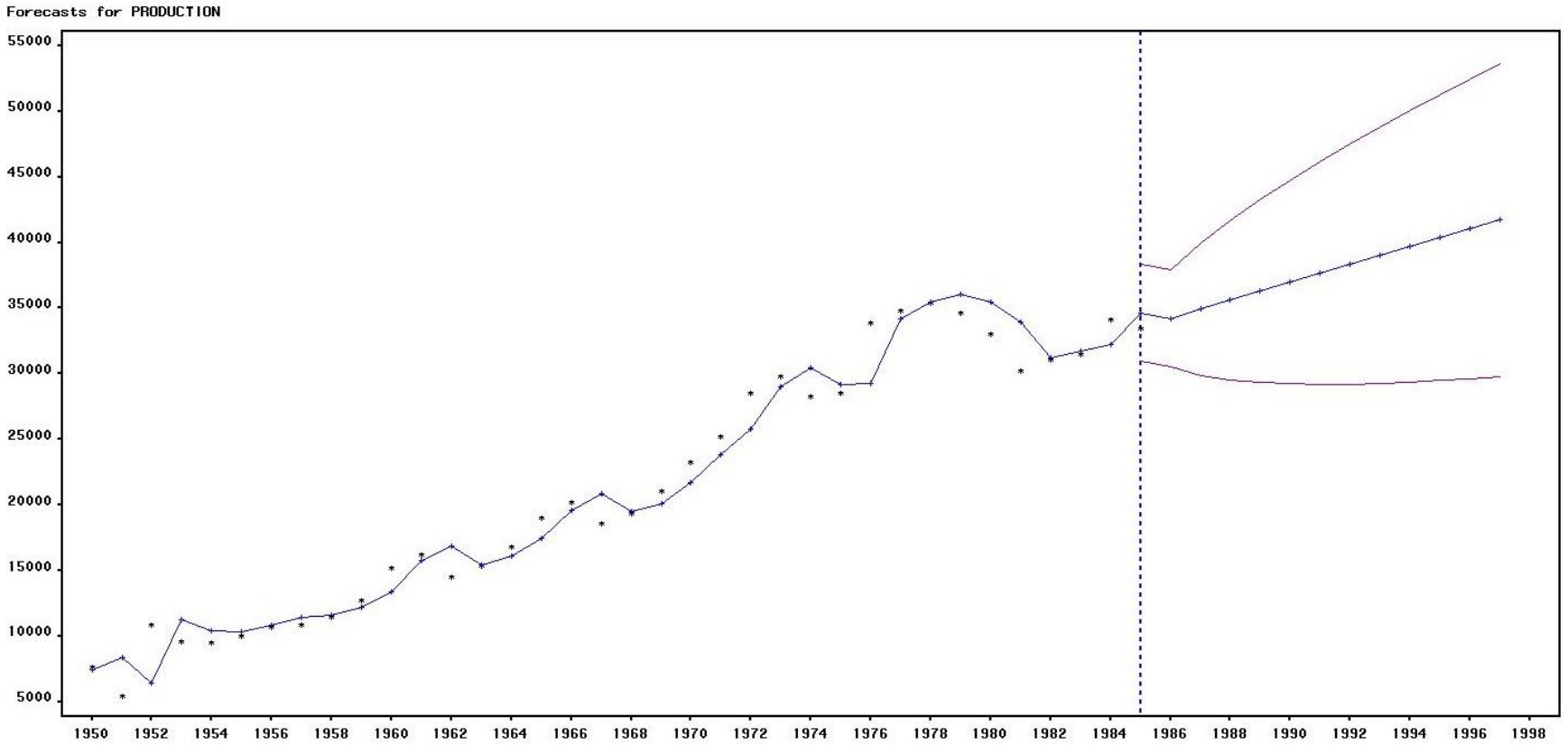

Figure 3a depicts a time series graph for predicted values ((+) signs) and the training data ((*) signs) for the Production variable, as reported by SAS(AFS), along with 12-year forecasted values and prediction limits.

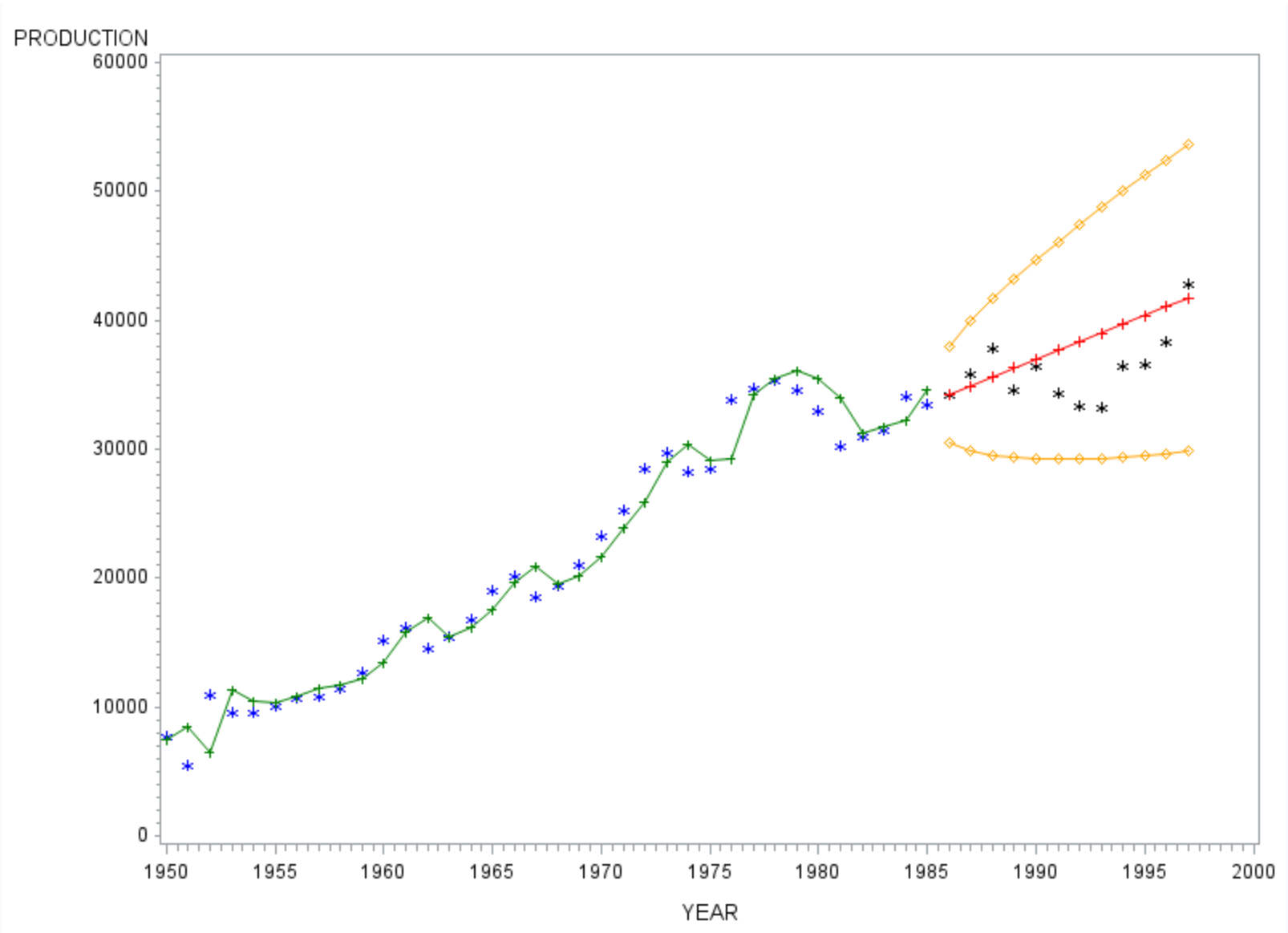

Figure 3b shows a time series graph for predicted values (green-colored (+) signs) for the Production variable as reported by SAS(AFS), along with 12-year forecasted values, which are plotted using red-colored plus (+) signs. The training data are plotted using blue-colored star (*) signs, and testing data are plotted using black-colored star (*) signs, while the 12-year confidence limits are yellow-colored.

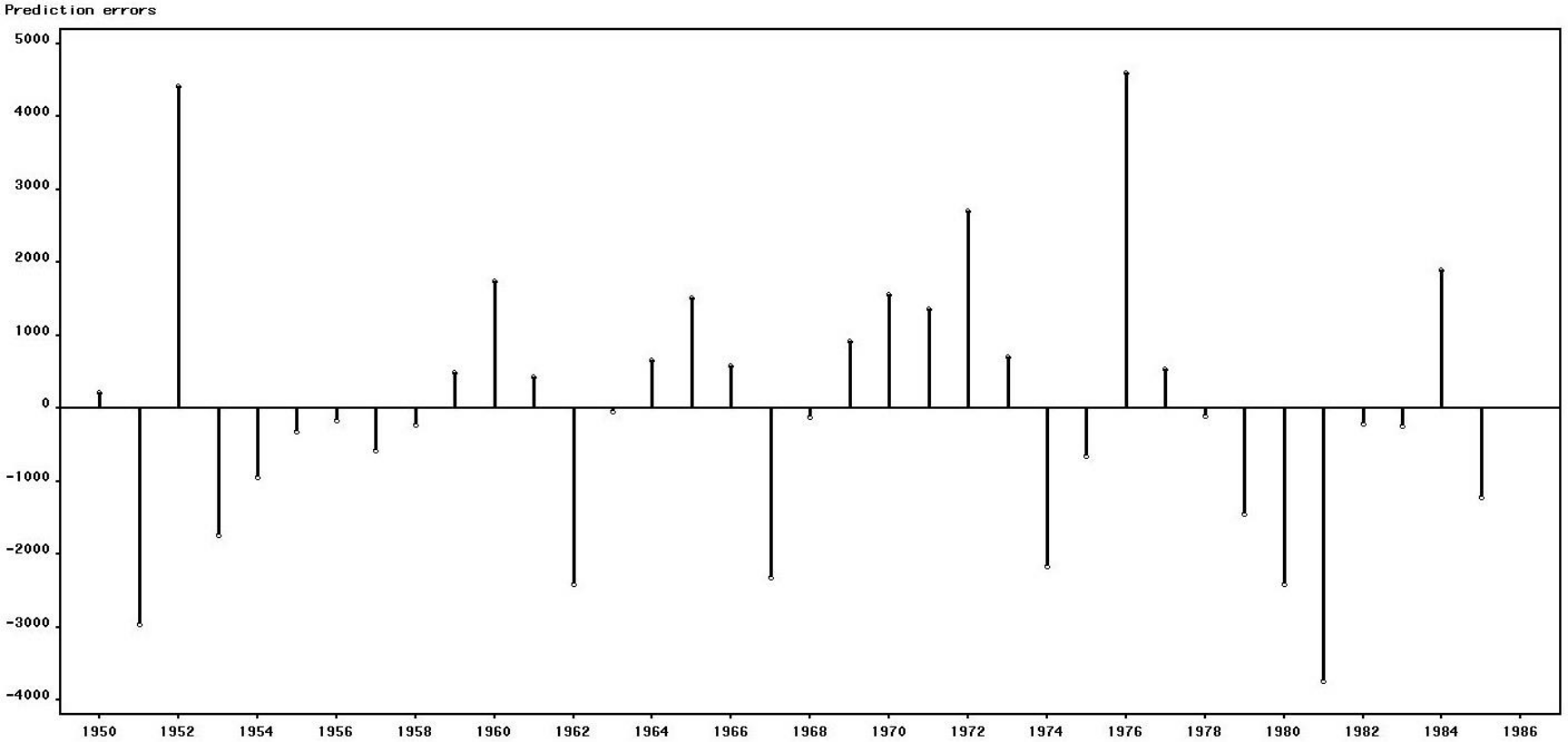

Figure 4. Prediction errors by time series values for the training data set.

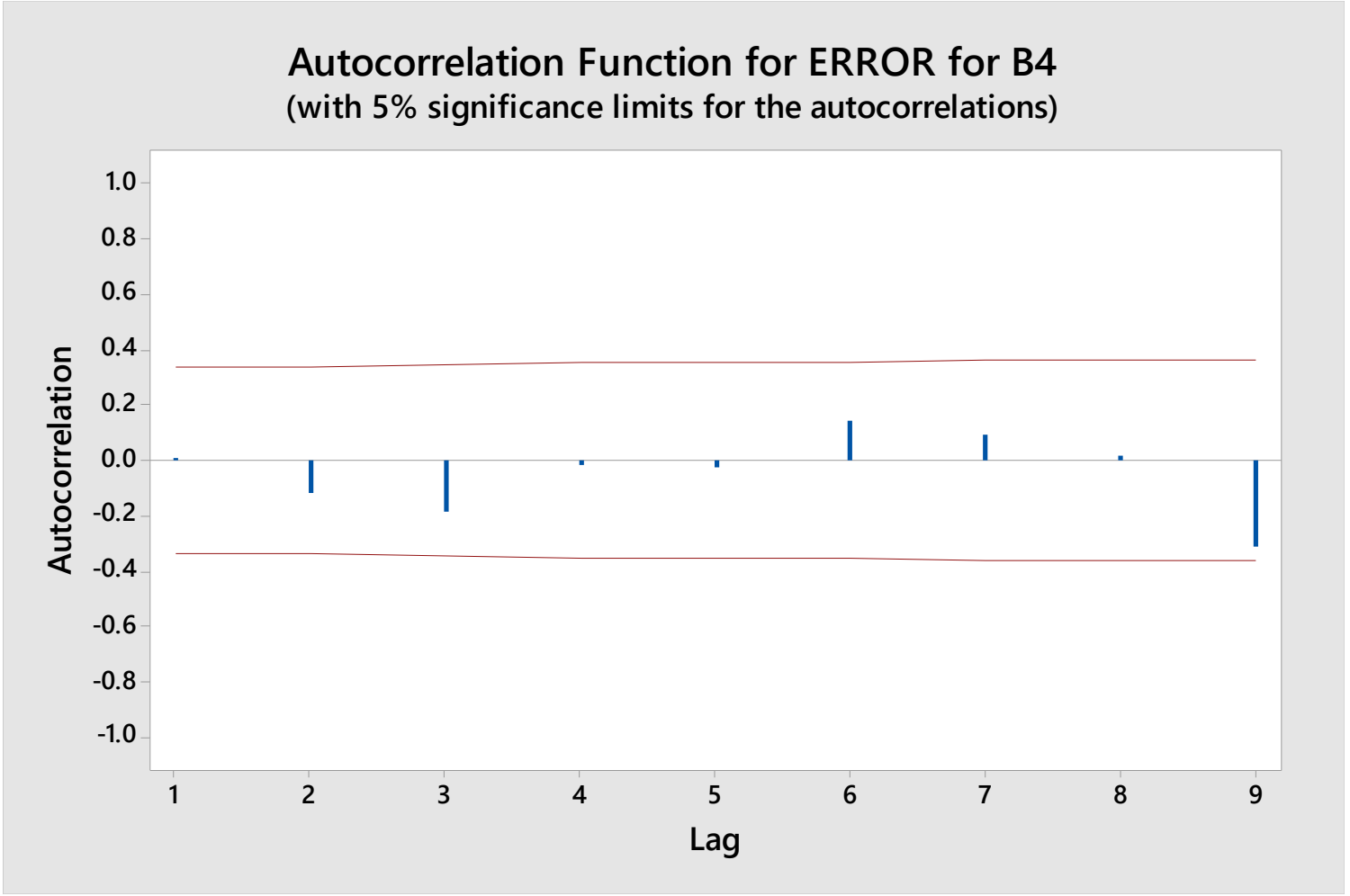

Figure 5. Autocorrelation Function (ACF) for the training data set.

Accuracy Measure for Table B.4 data.

Actual y_t	Predict $\widehat{y_t}$(t-1)	Forecast error $e_t(1) = y_t - \widehat{y_t}$(t-1)	Absolute error $\lvert e_t(1)\rvert$	Square error $[e_t(1)]^2$	Relative error $re_t(1) = 100(e_t(1)/y_t)$	Absolute percentage error $\lvert re_t(1)\rvert$
34198	34228	-30	30	900	-0.0877	0.0877
35863	34921	942	942	887364	2.6267	2.6267
37789	35612	2177	2177	4739329	5.7609	5.7609
34561	36301	-1740	1740	3027600	-5.0346	5.0346
36434	36987	-553	553	305809	-1.5178	1.5178
34371	37672	-3301	3301	10896601	-9.6040	9.6040
33307	38354	-5047	5047	25472209	-15.1530	15.1530
33295	39034	-5739	5739	32936121	-17.2368	17.2368
36514	39712	-3198	3198	10227204	-8.7583	8.7583
36593	40388	-3795	3795	14402025	-10.3708	10.3708
38311	41062	-2751	2751	7568001	-7.1807	7.1807
42773	41734	1039	1039	1079521	2.4291	2.4291

Table A. Error Measures table B4 time series data.

Data set	**MSE**	$\widehat{\sigma_{e(1)}}$	**MAD**	**MPE**	**MAPE**
Full data (48 data) – Linear (Holt) exponential smoothing	3526585.2	1877.9	1432.6	-0.75171	7.14770
Training data (36 data) – damped trend exponential smoothing	3257536.5	1804.9	1343.7	-1.06781	8.02606
Holdout data (12 data)	9295224	3048.81	2526	-5.34392	7.14671

Table B. Accuracy Measures table B4 time series data.

Table B provides SAS (AFS)® forecast errors for the full data set, the training data set, and the holdout (or test) data set. The damped trend exponential smoothing model was suggested as the "best model" for this data set. SAS (AFS)® and MINITAB® produced the accuracy measures shown in Table B, which comprise the Mean Square Error (MSE), the Standard Deviation of the Forecast Errors (SDFE), the Mean Absolute Deviation (MAD), the Mean Percent Forecast Error (MPE), and Mean Absolute Percent Error (MAPE).

Here, the accuracy statistics are computed as follows:

Mean square error:

$$\textbf{MSE} = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = 9295224$$

Standard Deviation of Forecast Error (Root Mean Square Error):

$$\widehat{\sigma_{e(1)}} = \sqrt{MSE} = \sqrt{9295224} = 3048.81$$

Mean Absolute Deviation (Mean Absolute Error):

$$\textbf{MAD} = \frac{1}{n}\sum_{t=1}^{n} |e_t(1)| = 2526$$

Mean Percent Forecast Error:

$$\textbf{MPE} = \frac{1}{n}\sum_{t=1}^{n} re_t(1) = -5.34392$$

Mean Absolute Percent Error:

$$\textbf{MAPE} = \frac{1}{n}\sum_{t=1}^{n} |re_t(1)| = 7.14671$$

Worksheet 1 ***

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	Actual	Predict	Forecast_Error	Abs_Error	Sq_Error	Relative_Error	Abs_Percentage_Error	MSE	Root_MSE	MAD	MPE	MAPE
1	34198	34228	-30	30	900	-0.0877	0.0877	9295224	3048.81	2526	-5.34392	7.14671
2	35863	34921	942	942	887364	2.6267	2.6267					
3	37789	35612	2177	2177	4739329	5.7609	5.7609					
4	34561	36301	-1740	1740	3027600	-5.0346	5.0346					
5	36434	36987	-553	553	305809	-1.5178	1.5178					
6	34371	37672	-3301	3301	10896601	-9.6040	9.6040					
7	33307	38354	-5047	5047	25472209	-15.1530	15.1530					
8	33295	39034	-5739	5739	32936121	-17.2368	17.2368					
9	36514	39712	-3198	3198	10227204	-8.7583	8.7583					
10	36593	40388	-3795	3795	14402025	-10.3708	10.3708					
11	38311	41062	-2751	2751	7568001	-7.1807	7.1807					
12	42773	41734	1039	1039	1079521	2.4291	2.4291					

Table C. MINITAB® 17 Calculation of Forecast Errors for Holdout Data.

The first two columns of Table C consist of the actual values of the holdout data set and the SAS (AFS)® forecasted values, based on training data. The remaining columns give forecast errors, th absolute forecast errors, the squared forecast errors, the relative forecast errors and absolute forecast percentage errors using the MINITAB® calculator function.

Obs	DATE	ACTUAL	PREDICT	UPPER	LOWER	ERROR	STD	NERROR	_LEVEL_	_TREND_
37	1986	.	34228	37923	30533	.	1885	.	34228	695.2599
38	1987	.	34921	39941	29901	.	2561	.	34921	693.0872
39	1988	.	35612	41676	29548	.	3094	.	35612	690.9213
40	1989	.	36301	43255	29346	.	3548	.	36301	688.7621
41	1990	.	36987	44732	29242	.	3952	.	36987	686.6097
42	1991	.	37672	46135	29209	.	4318	.	37672	684.4640
43	1992	.	38354	47480	29228	.	4656	.	38354	682.3250
44	1993	.	39034	48779	29289	.	4972	.	39034	680.1927
45	1994	.	39712	50041	29384	.	5270	.	39712	678.0671
46	1995	.	40388	51270	29506	.	5552	.	40388	675.9481
47	1996	.	41062	52471	29653	.	5821	.	41062	673.8357
48	1997	.	41734	53648	29820	.	6079	.	41734	671.7300

Table D. Forecasted values for the last 12 years of Holdout Data.

Table D contains forecasted values for the final twelve years for the Production variable, and the corresponding lower and upper 95% prediction limits. The Root Mean Square Prediction Error, Level and Trend smoothing components are also shown..

Model Parameter	Estimate	Std. Error	T	Prob>|T|
LEVEL Smoothing Weight	0.91885	0.1246	7.3754	<.0001
TREND Smoothing Weight	0.00100	0.0719	0.0139	0.9890
DAMPING Smoothing Weight	0.99687	0.0062	160.8341	<.0001
Residual Variance (sigma squared)	3553676	.	.	.
Smoothed Level	33533	.	.	.
Smoothed Trend	697.43945	.	.	.

Table E. Parameter Estimates for Training Data using the Damped Trend Exponential Smoothing model suggested by SAS (AFS)®.

Results

In Figure 3b, it seems the model fits the data appropriately, however, the forecast values produced by this model are very far from the testing values. There are no spikes in the ACF function at large lag, which means forecast errors are uncorrelated of this model. The MSE, RMSE and MAD are not similar among the training and holdout data sets; that is, the variability in forecast errors are exists. Moreover, MAPEs are very big in all cases.

TABLE B.5 (US BEVERAGE MANUFACTURER PRODUCT SHIPMENTS, UNADJUSTED) ANALYSIS

Table B.5 comprises data on the US Beverage Manufacturer Product Shipments, Unadjusted from January 1992 through December 2006, where the dollar variable (in Millions) is collected monthly. The data set excludes last 12 monthly values as a holdout set in order to analyze the accuracy of SAS(AFS), which suggested a Holt Winters – Additive model model as the "best model" for this training, or initialization, data set. SAS(AFS) also produced the predicted values for the next 12 months. I have calculated different types of forecast errors based on the hold out data and predicted data.

The model equation is

$$Y_t = (\beta_0 + \beta_t\, t) + SN_t + IR_t$$

where,

Y_t = observed value i.e. dollar at time t

$\beta_0 + \beta_t\, t$ = smoothed level and $\beta_0 + \beta_t\, t = 6492$

β_t = smoothed trend and $\beta_t = 12.25806$

SN_t = seasonal factor

Now, the point forecast made at time T for y_{T+p} is

$$\hat{Y}_{T+p}(k) = l_T + pb_T + sn_{T+p-L}$$

and the smoothing equations are

$$l_T = \alpha\,(Y_t - sn_{T-L}) + (1-\alpha)\,(l_{T-1} + b_{T-1})$$

$$b_T = \gamma\,(l_T - l_{T-1}) + (1-\gamma)\,b_{T-1}$$

and $sn_T = \delta\,(Y_T - l_T) + (1-\delta)\,sn_{T-L}$

where,

l_T = Level estimate and $l_0 = \beta_0 + \beta_t\, t = 6492$

b_T= Trend estimate and $b_0 = \beta_t = 12.25806$

Smoothed seasonal factor 1 = -714.85496
Smoothed seasonal factor 2 = -512.78178
Smoothed seasonal factor 3 = -71.54367
Smoothed seasonal factor 4 = -3.00790
Smoothed seasonal factor 5 = 453.67586
Smoothed seasonal factor 6 = 584.86648
Smoothed seasonal factor 7 = 238.84724
Smoothed seasonal factor 8 = 439.68429
Smoothed seasonal factor 9 = 134.45167
Smoothed seasonal factor 10 = -46.56763
Smoothed seasonal factor 11 = -198.01668
Smoothed seasonal factor 12 = -304.39465

α = level smoothing and $\alpha = 0.40720$

γ = trend smoothing and $\gamma = 0.00100$ and

δ = seasonal smoothing and $\delta = 0.00100$

Table B.5: US Beverage Manufacturer Product Shipments, Unadjusted

Month	Dollars (In Millions)	Month	Dollars (In Millions)	Month	Dollars (In Millions)	Month	Dollars (In Millions)
Jan-1992	3,519	Oct-1995	4,681	Jul-1999	5,339	Apr-2003	5,576
Feb-1992	3,803	Nov-1995	4,466	Aug-1999	5,474	May-2003	6,160
Mar-1992	4,332	Dec-1995	4,463	Sep-1999	5,278	Jun-2003	6,121
Apr-1992	4,251	Jan-1996	4,217	Oct-1999	5,184	Jul-2003	5,900
May-1992	4,661	Feb-1996	4,322	Nov-1999	4,975	Aug-2003	5,994
Jun-1992	4,811	Mar-1996	4,779	Dec-1999	4,751	Sep-2003	5,841
Jul-1992	4,448	Apr-1996	4,988	Jan-2000	4,600	Oct-2003	5,832
Aug-1992	4,451	May-1996	5,383	Feb-2000	4,718	Nov-2003	5,505
Sep-1992	4,343	Jun-1996	5,591	Mar-2000	5,218	Dec-2003	5,573
Oct-1992	4,067	Jul-1996	5,322	Apr-2000	5,336	Jan-2004	5,331
Nov-1992	4,001	Aug-1996	5,404	May-2000	5,665	Feb-2004	5,355
Dec-1992	3,934	Sep-1996	5,106	Jun-2000	5,900	Mar-2004	6,057
Jan-1993	3,652	Oct-1996	4,871	Jul-2000	5,330	Apr-2004	6,055
Feb-1993	3,768	Nov-1996	4,977	Aug-2000	5,626	May-2004	6,771
Mar-1993	4,082	Dec-1996	4,706	Sep-2000	5,512	Jun-2004	6,669
Apr-1993	4,101	Jan-1997	4,193	Oct-2000	5,293	Jul-2004	6,375
May-1993	4,628	Feb-1997	4,460	Nov-2000	5,143	Aug-2004	6,666
Jun-1993	4,898	Mar-1997	4,956	Dec-2000	4,842	Sep-2004	6,383
Jul-1993	4,476	Apr-1997	5,022	Jan-2001	4,627	Oct-2004	6,118
Aug-1993	4,728	May-1997	5,408	Feb-2001	4,981	Nov-2004	5,927
Sep-1993	4,458	Jun-1997	5,565	Mar-2001	5,321	Dec-2004	5,750
Oct-1993	4,004	Jul-1997	5,360	Apr-2001	5,290	Jan-2005	5,122
Nov-1993	4,095	Aug-1997	5,490	May-2001	6,002	Feb-2005	5,398
Dec-1993	4,056	Sep-1997	5,286	Jun-2001	5,811	Mar-2005	5,817
Jan-1994	3,641	Oct-1997	5,257	Jul-2001	5,671	Apr-2005	6,163
Feb-1994	3,966	Nov-1997	5,002	Aug-2001	6,102	May-2005	6,763
Mar-1994	4,417	Dec-1997	4,897	Sep-2001	5,482	Jun-2005	6,835
Apr-1994	4,367	Jan-1998	4,577	Oct-2001	5,429	Jul-2005	6,678
May-1994	4,821	Feb-1998	4,764	Nov-2001	5,356	Aug-2005	6,821
Jun-1994	5,190	Mar-1998	5,052	Dec-2001	5,167	Sep-2005	6,421
Jul-1994	4,638	Apr-1998	5,251	Jan-2002	4,608	Oct-2005	6,338
Aug-1994	4,904	May-1998	5,558	Feb-2002	4,889	Nov-2005	6,265
Sep-1994	4,528	Jun-1998	5,931	Mar-2002	5,352	Dec-2005	6,291
Oct-1994	4,383	Jul-1998	5,476	Apr-2002	5,441	Jan-2006	5,540
Nov-1994	4,339	Aug-1998	5,603	May-2002	5,970	Feb-2006	5,822
Dec-1994	4,327	Sep-1998	5,425	Jun-2002	5,750	Mar-2006	6,318
Jan-1995	3,856	Oct-1998	5,177	Jul-2002	5,670	Apr-2006	6,268
Feb-1995	4,072	Nov-1998	4,792	Aug-2002	5,860	May-2006	7,270
Mar-1995	4,563	Dec-1998	4,776	Sep-2002	5,449	Jun-2006	7,096
Apr-1995	4,561	Jan-1999	4,450	Oct-2002	5,401	Jul-2006	6,505
May-1995	4,984	Feb-1999	4,659	Nov-2002	5,240	Aug-2006	7,039
Jun-1995	5,316	Mar-1999	5,043	Dec-2002	5,229	Sep-2006	6,440
Jul-1995	4,843	Apr-1999	5,233	Jan-2003	4,770	Oct-2006	6,446
Aug-1995	5,383	May-1999	5,423	Feb-2003	5,006	Nov-2006	6,717
Sep-1995	4,889	Jun-1999	5,814	Mar-2003	5,518	Dec-2006	6,320

The collection of Figures and Tables that follow are individually identified below.

List of Figures:

List of Tables:

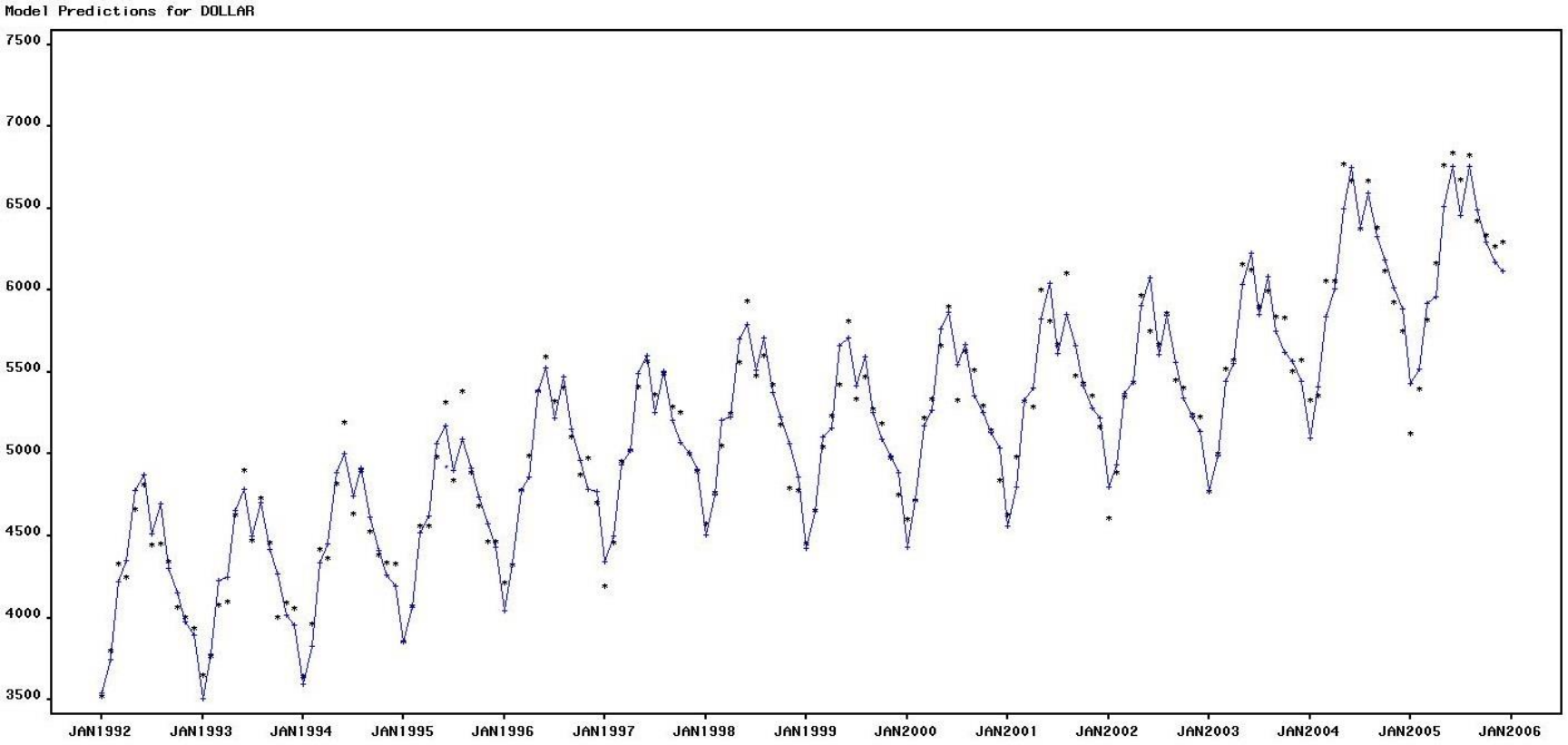

Figure 1 depicts a time series graph for predicted values ((+) signs) using the training data ((*) signs) set for Dollar variable, as reported by SAS(AFS).

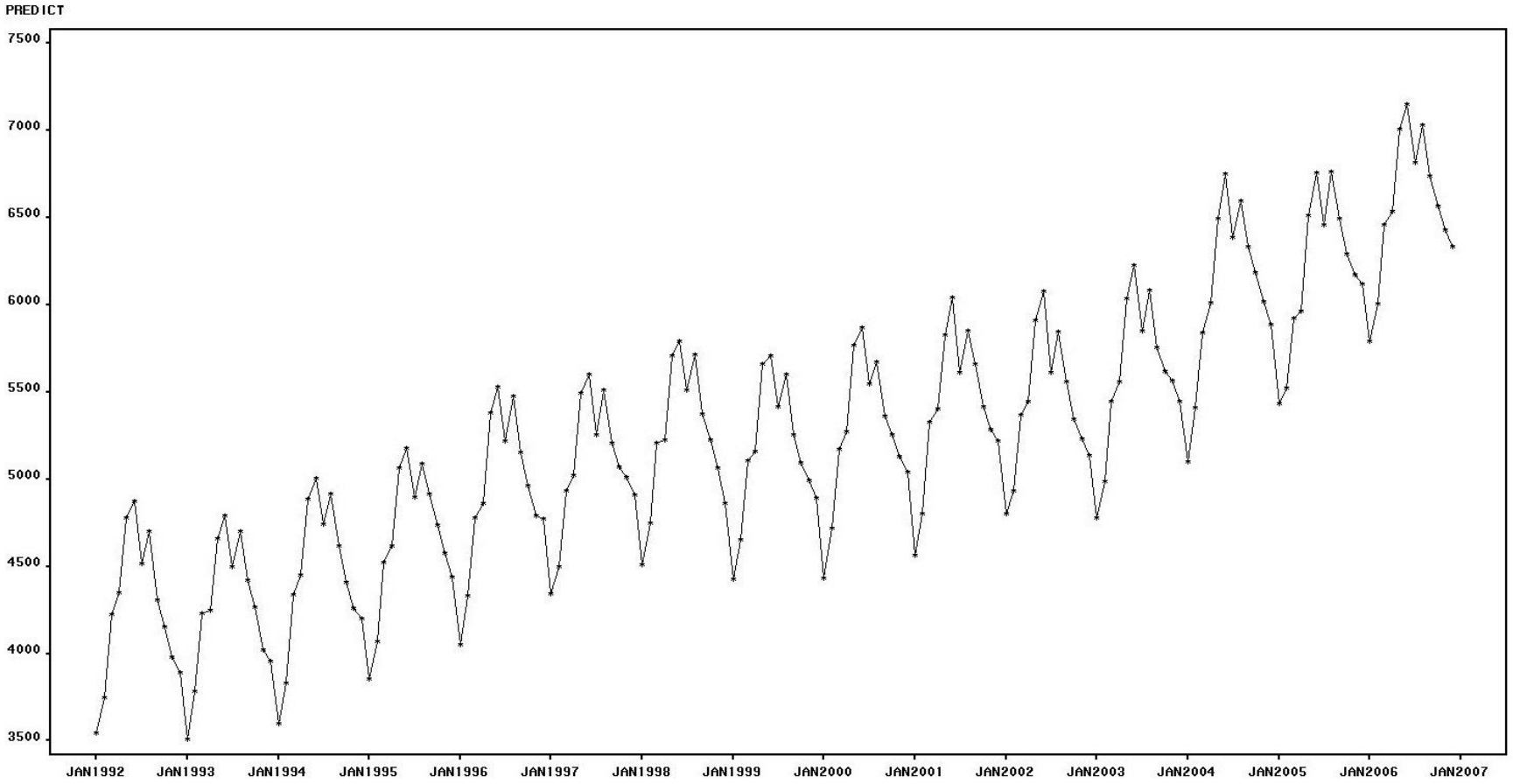

Figure 2 shows a time series graph for predicted values ((*) signs) for Dollar variable as reported by SAS(AFS), along with 12-month forecasted values for the Dollar variable. Here, predicted and forecast values are plotted using ((*) signs).

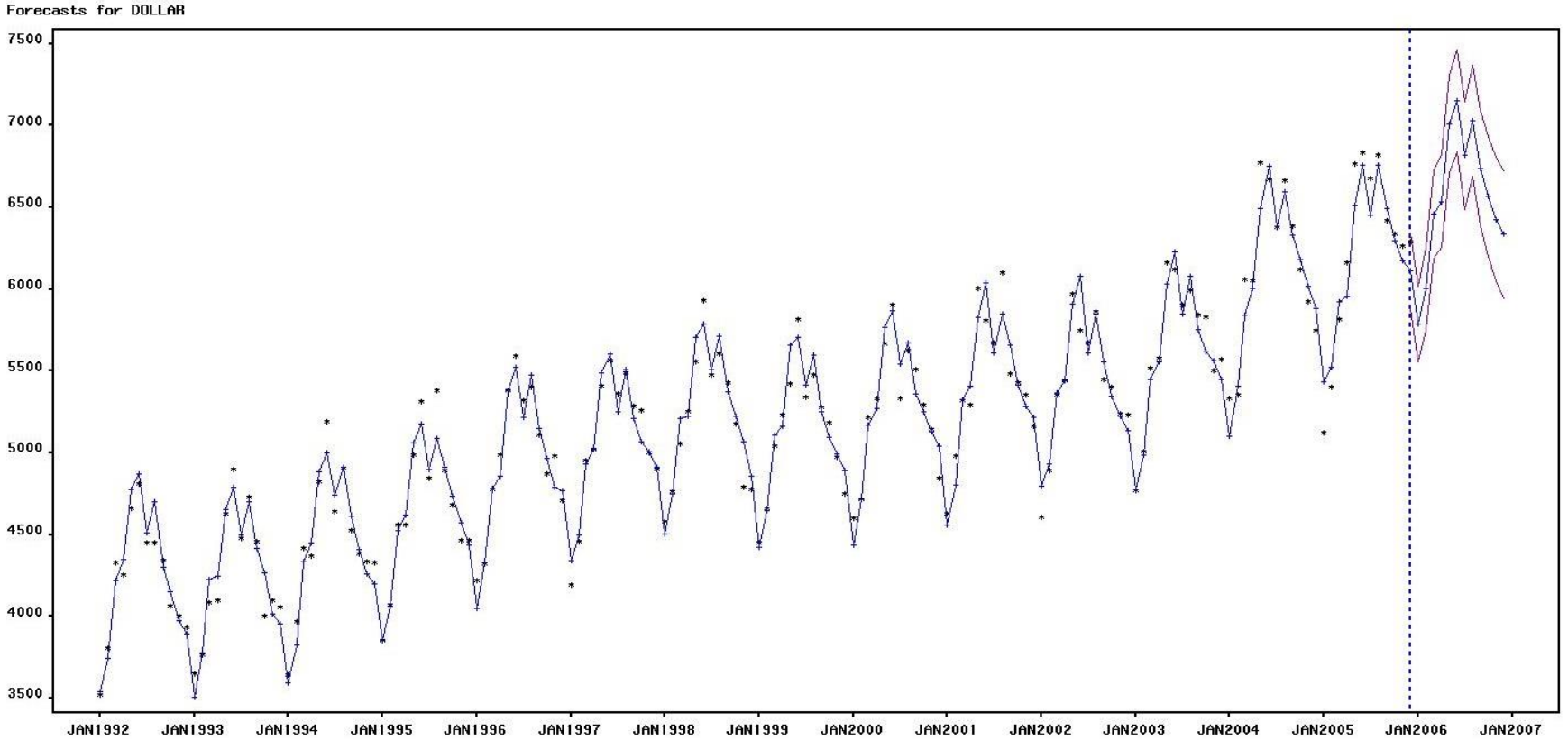

Figure 3a depicts a time series graph for predicted values ((+) signs) and the training data ((*) signs) for the Dollar variable, as reported by SAS(AFS), along with 12-month forecasted values and prediction limits.

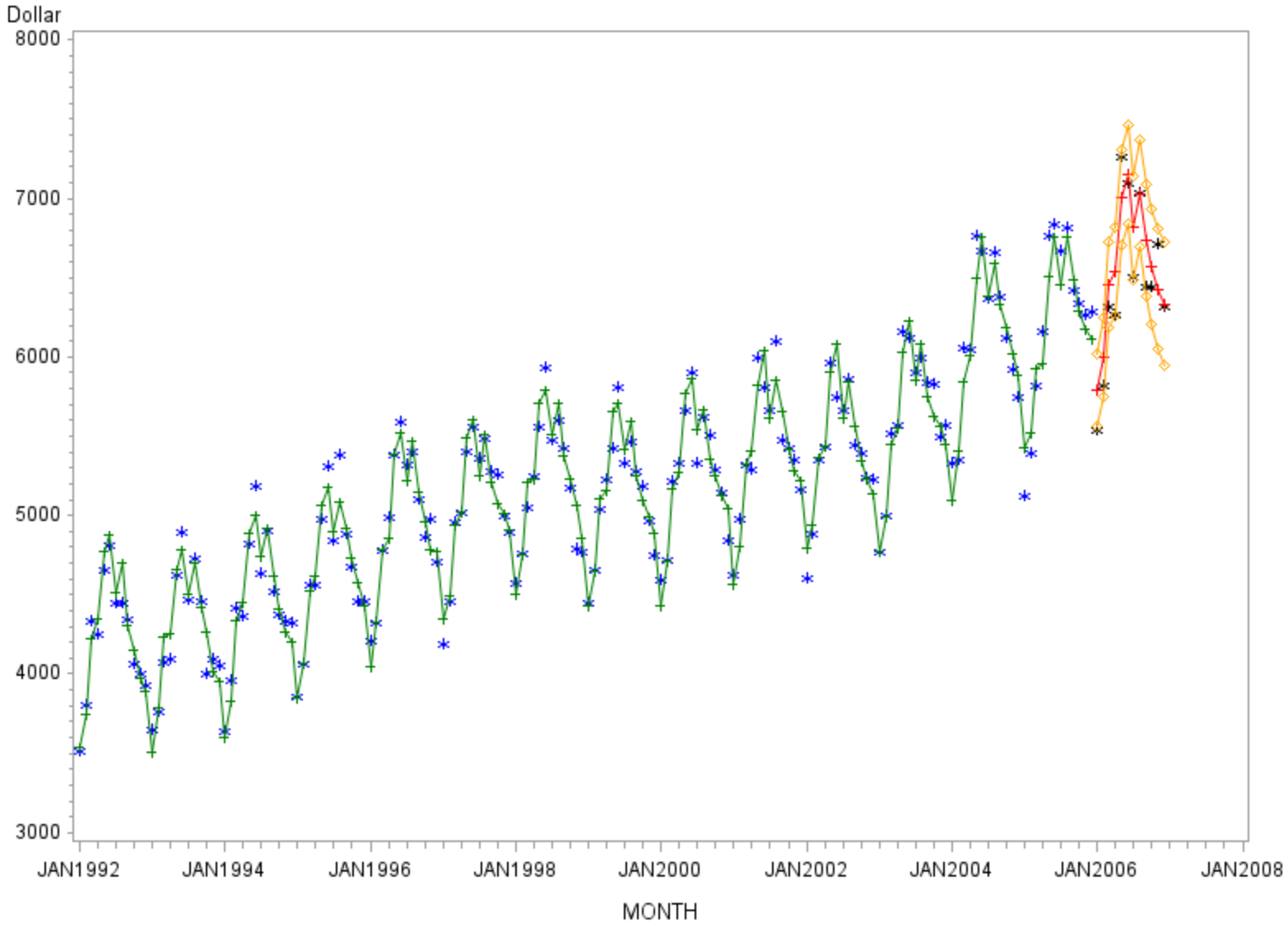

Figure 3b shows a time series graph for predicted values (green-colored (+) signs) for the Dollar variable as reported by SAS(AFS), along with 12-month forecasted values, which are plotted using red-colored plus (+) signs. The training data are plotted using blue-colored star (*) signs, and testing data are plotted using black-colored star (*) signs, while the 12-month confidence limits are yellow-colored.

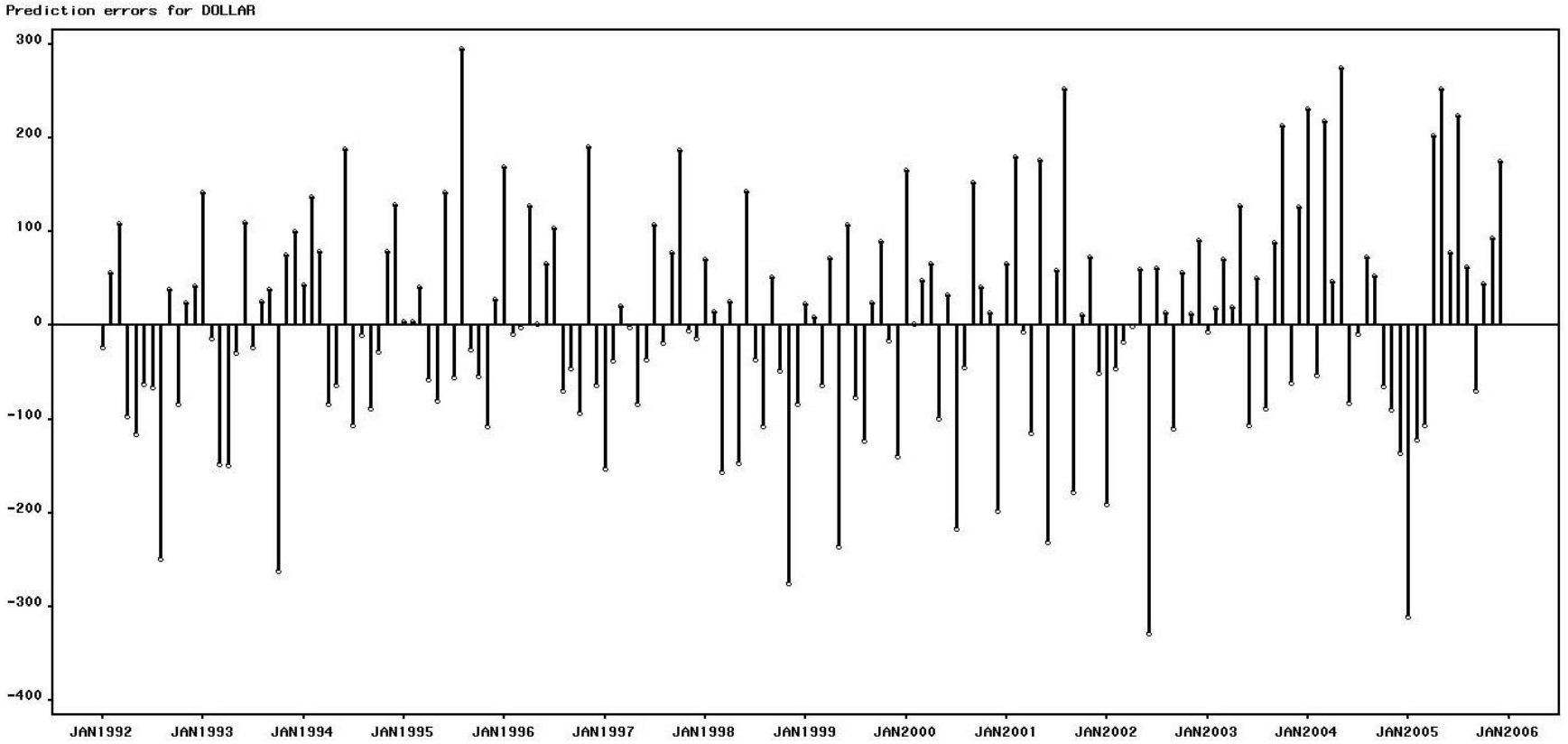

Figure 4. Prediction errors by time series values for the training data set.

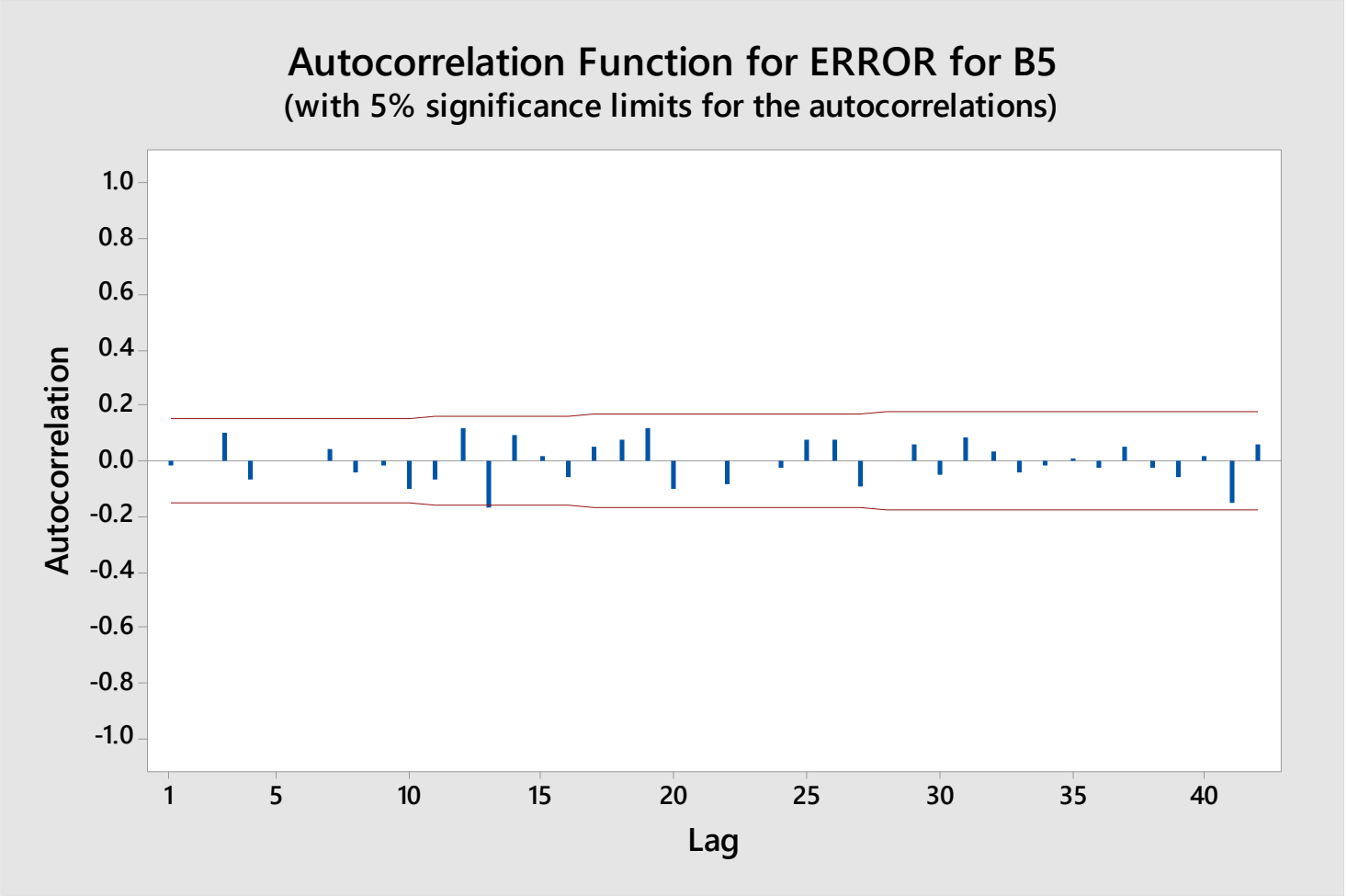

Figure 5. Autocorrelation Function (ACF) for the training data set.

Accuracy Measure for Table B.5 data.

Actual y_t	Predict $\widehat{y_t}$(t-1)	Forecast error $e_t(1) = y_t - \widehat{y_t}$(t-1)	Absolute error $\lvert e_t(1)\rvert$	Square error $[e_t(1)]^2$	Relative error $re_t(1) = 100(e_t(1)/y_t)$	Absolute percentage error $\lvert re_t(1)\rvert$
5540	5789	-249	249	62001	-4.49458	4.49458
5822	6003	-181	181	32761	-3.10890	3.10890
6318	6457	-139	139	19321	-2.20006	2.20006
6268	6538	-270	270	72900	-4.30759	4.30759
7270	7007	263	263	69169	3.61761	3.61761
7096	7150	-54	54	2916	-0.76099	0.76099
6505	6816	-311	311	96721	-4.78094	4.78094
7039	7029	10	10	100	0.14207	0.14207
6440	6736	-296	296	87616	-4.59627	4.59627
6446	6568	-122	122	14884	-1.89265	1.89265
6717	6428	289	289	83521	4.30252	4.30252
6320	6334	-14	14	196	-0.22152	0.22152

Table A. Error Measures table B5 time series data.

Data set	**MSE**	$\widehat{\sigma_{e(1)}}$	**MAD**	**MPE**	**MAPE**
Full data (180 data) – Log Winters additive method	15749.3	125.496	93.867	0.03149	1.77268
Training data (168 data) – Holt Winters Additive method model	13639	116.787	91.348	0.01675	1.77321
Holdout data (12 data)	45175.5	212.545	183.167	-1.52511	2.86881

Table B. Accuracy Measures table B5 time series data.

Table B provides SAS (AFS)® forecast errors for the full data set, the training data set, and the holdout (or test) data set. The Holt Winters – Additive model was suggested as the "best model" for this data set. SAS (AFS)® and MINITAB® produced the accuracy measures shown in Table B, which comprise the Mean Square Error (MSE), the Standard Deviation of the Forecast Errors (SDFE), the Mean Absolute Deviation (MAD), the Mean Percent Forecast Error (MPE), and Mean Absolute Percent Error (MAPE).

Here, the accuracy statistics are computed as follows:

Mean square error:

$$\textbf{MSE} = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = 45175.5$$

Standard Deviation of Forecast Error (Root Mean Square Error):

$$\widehat{\sigma_{e(1)}} = \sqrt{MSE} = \sqrt{45175.5} = 212.545$$

Mean Absolute Deviation (Mean Absolute Error):

$$\textbf{MAD} = \frac{1}{n}\sum_{t=1}^{n} |e_t(1)| = 183.167$$

Mean Percent Forecast Error:

$$\textbf{MPE} = \frac{1}{n}\sum_{t=1}^{n} re_t(1) = -1.52511$$

Mean Absolute Percent Error:

$$\textbf{MAPE} = \frac{1}{n}\sum_{t=1}^{n} |re_t(1)| = 2.86881$$

Worksheet 1 ***

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	Actual	Predict	Forecast_Error	Abs_Error	Sq_Error	Relative_Error	Abs_Percentage_Error	MSE	Root_MSE	MAD	MPE	MAPE
1	5540	5789	-249	249	62001	-4.49458	4.49458	45175.5	212.545	183.167	-1.52511	2.86881
2	5822	6003	-181	181	32761	-3.10890	3.10890					
3	6318	6457	-139	139	19321	-2.20006	2.20006					
4	6268	6538	-270	270	72900	-4.30759	4.30759					
5	7270	7007	263	263	69169	3.61761	3.61761					
6	7096	7150	-54	54	2916	-0.76099	0.76099					
7	6505	6816	-311	311	96721	-4.78094	4.78094					
8	7039	7029	10	10	100	0.14207	0.14207					
9	6440	6736	-296	296	87616	-4.59627	4.59627					
10	6446	6568	-122	122	14884	-1.89265	1.89265					
11	6717	6428	289	289	83521	4.30252	4.30252					
12	6320	6334	-14	14	196	-0.22152	0.22152					

Table C. MINITAB® 17 Calculation of Forecast Errors for Holdout Data.

The first two columns of Table C consist of the actual values of the holdout data set and the SAS (AFS)® forecasted values, based on training data. The remaining columns give forecast errors, th absolute forecast errors, the squared forecast errors, the relative forecast errors and absolute forecast percentage errors using the MINITAB® calculator function.

Obs	DATE	ACTUAL	PREDICT	UPPER	LOWER	ERROR	STD	NERROR	_LEVEL_	_TREND_	_SFACTOR
169	JAN2006	.	5789	6020	5558	.	117.8443	.	6504	12.2581	-714.8550
170	FEB2006	.	6003	6253	5754	.	127.2578	.	6516	12.2581	-512.7818
171	MAR2006	.	6457	6724	6190	.	136.0385	.	6528	12.2581	-71.5437
172	APR2006	.	6538	6821	6255	.	144.3017	.	6541	12.2581	-3.0079
173	MAY2006	.	7007	7305	6708	.	152.1319	.	6553	12.2581	453.6759
174	JUN2006	.	7150	7463	6837	.	159.5929	.	6565	12.2581	584.8665
175	JUL2006	.	6816	7143	6490	.	166.7342	.	6577	12.2581	238.8472
176	AUG2006	.	7029	7370	6689	.	173.5953	.	6590	12.2581	439.6843
177	SEP2006	.	6736	7090	6383	.	180.2082	.	6602	12.2581	134.4517
178	OCT2006	.	6568	6933	6202	.	186.5993	.	6614	12.2581	-46.5676
179	NOV2006	.	6428	6806	6051	.	192.7908	.	6627	12.2581	-198.0167
180	DEC2006	.	6334	6724	5945	.	198.8012	.	6639	12.2581	-304.3947

Table D. Forecasted values for the last 12 months of Holdout Data.

Table D contains forecasted values for the final twelve months for the Dollar variable, and the corresponding lower and upper 95% prediction limits. The Root Mean Square Prediction Error, Level and Trend smoothing components are also shown..

Model Parameter	Estimate	Std. Error	T	Prob>|T|
LEVEL Smoothing Weight	0.40720	0.0450	9.0391	<.0001
TREND Smoothing Weight	0.00100	0.0148	0.0677	0.9461
SEASONAL Smoothing Weight	0.00100	0.0368	0.0272	0.9784
Residual Variance (sigma squared)	13887	.	.	.
Smoothed Level	6492	.	.	.
Smoothed Trend	12.25806	.	.	.
Smoothed Seasonal Factor 1	-714.85496	.	.	.
Smoothed Seasonal Factor 2	-512.78178	.	.	.
Smoothed Seasonal Factor 3	-71.54367	.	.	.
Smoothed Seasonal Factor 4	-3.00790	.	.	.
Smoothed Seasonal Factor 5	453.67586	.	.	.
Smoothed Seasonal Factor 6	584.86648	.	.	.
Smoothed Seasonal Factor 7	238.84724	.	.	.
Smoothed Seasonal Factor 8	439.68429	.	.	.
Smoothed Seasonal Factor 9	134.45167	.	.	.
Smoothed Seasonal Factor 10	-46.56763	.	.	.
Smoothed Seasonal Factor 11	-198.01668	.	.	.
Smoothed Seasonal Factor 12	-304.39465	.	.	.

Table E. Parameter Estimates for Training Data using the Holt Winters – Additive model suggested by SAS (AFS)®.

Results

Figure 3b gives the impression that the model fits the data appropriately well and it can forecast future values successfully. The ACF function in figure 5 suggests that the forecast errors are structureless. Although the MSE, RMSE and MAD appear large for the holdout data when compared to the training data, note that the MAPE (mean average percent error) is still quite small in all cases.

TABLE B.6 (GLOBAL MEAN AIR SURFACE TEMPERATURE ANOMALY AND GLOBAL CO_2 CONCENTRATION) ANALYSIS

Table B.6 comprises data on the Global Mean Air Surface Temperature Anomaly and Global CO_2 Concentration from 1880 through 2004, where the temperature anomaly variable is collected yearly. The data set excludes last 12 yearly values as a holdout set in order to analyze the accuracy of SAS(AFS), which suggested a linear (Holt) exponential smoothing model as the "best model" for this training, or initialization, data set. SAS(AFS) also produced the predicted values for the next 12 years. I have calculated different types of forecast errors based on the hold out data and predicted data.

The model equation is

$$Y_t = \mu_t + \beta_t\, t + \varepsilon_t$$

where,

Y_t = observed value i.e. temperature anomaly at time t

μ_t = smoothed level and $\mu_t = 0.32054$

β_t = smoothed trend and $\beta_t = 0.00434$

Now, the k-step prediction equation is

$$\hat{Y}_t(k) = L_t + K\, T_t$$

and the smoothing equations are

$$L_t = \alpha\, Y_t + (1-\alpha)\,(L_{t-1} + T_{t-1})$$

and $T_t = \gamma\,(L_t - L_{t-1}) + (1-\gamma)\, T_{t-1}$

where,

L_t = Level estimate and $L_0 = \mu_t = 0.32054$

T_t = Trend estimate and $T_0 = \beta_t = 0.00434$

α = level smoothing and $\alpha = 0.26140$

γ = trend smoothing and $\gamma = 0.00100$

Table B.6: Global Mean Air Surface Temperature Anomaly and Global CO_2 Concentration

Year	Anomaly, C	CO2, ppmv	Year	Anomaly, C	CO2, ppmv	Year	Anomaly, C	CO2, ppmv
1880	-0.11	290.7	1922	-0.09	303.8	1964	-0.25	319.2
1881	-0.13	291.2	1923	-0.16	304.1	1965	-0.15	320.0
1882	-0.01	291.7	1924	-0.11	304.5	1966	-0.07	321.1
1883	-0.04	292.1	1925	-0.15	305.0	1967	-0.02	322.0
1884	-0.42	292.6	1926	0.04	305.4	1968	-0.09	322.9
1885	-0.23	293.0	1927	-0.05	305.8	1969	0.00	324.2
1886	-0.25	293.3	1928	0.01	306.3	1970	0.04	325.2
1887	-0.45	293.6	1929	-0.22	306.8	1971	-0.10	326.1
1888	-0.23	293.8	1930	-0.03	307.2	1972	-0.05	327.2
1889	0.04	294.0	1931	0.03	307.7	1973	0.18	328.8
1890	-0.22	294.2	1932	0.04	308.2	1974	-0.06	329.7
1891	-0.55	294.3	1933	-0.11	308.6	1975	-0.02	330.7
1892	-0.40	294.5	1934	0.05	309.0	1976	-0.21	331.8

Year	Anomaly, C	CO2, ppmv	Year	Anomaly, C	CO2, ppmv	Year	Anomaly, C	CO2, ppmv
1893	-0.39	294.6	1935	-0.08	309.4	1977	0.16	333.3
1894	-0.32	294.7	1936	0.01	309.8	1978	0.07	334.6
1895	-0.32	294.8	1937	0.12	310.0	1979	0.13	336.9
1896	-0.27	294.9	1938	0.15	310.2	1980	0.27	338.7
1897	-0.15	295.0	1939	-0.02	310.3	1981	0.40	339.9
1898	-0.21	295.2	1940	0.14	310.4	1982	0.10	341.1
1899	-0.25	295.5	1941	0.11	310.4	1983	0.34	342.8
1900	-0.05	295.8	1942	0.10	310.3	1984	0.16	344.4
1901	-0.05	296.1	1943	0.06	310.2	1985	0.13	345.9
1902	-0.30	296.5	1944	0.10	310.1	1986	0.19	347.2
1903	-0.35	296.8	1945	-0.01	310.1	1987	0.35	348.9
1904	-0.42	297.2	1946	0.01	310.1	1988	0.42	351.5
1905	-0.25	297.6	1947	0.12	310.2	1989	0.28	352.9
1906	-0.15	298.1	1948	-0.03	310.3	1990	0.49	354.2
1907	-0.41	298.5	1949	-0.09	310.5	1991	0.44	355.6
1908	-0.30	298.9	1950	-0.17	310.7	1992	0.16	356.4
1909	-0.31	299.3	1951	-0.02	311.1	1993	0.18	357.0
1910	-0.21	299.7	1952	0.03	311.5	1994	0.31	358.9
1911	-0.25	300.1	1953	0.12	311.9	1995	0.47	360.9
1912	-0.33	300.4	1954	-0.09	312.4	1996	0.36	362.6
1913	-0.28	300.8	1955	-0.09	313.0	1997	0.40	363.8
1914	-0.02	301.1	1956	-0.18	313.6	1998	0.71	366.6
1915	0.06	301.4	1957	0.08	314.2	1999	0.43	368.3
1916	-0.20	301.7	1958	0.10	314.9	2000	0.41	369.5
1917	-0.46	302.1	1959	0.05	315.8	2001	0.56	371.0
1918	-0.33	302.4	1960	-0.02	316.6	2002	0.70	373.1
1919	-0.09	302.7	1961	0.10	317.3	2003	0.66	375.6
1920	-0.15	303.0	1962	0.05	318.1	2004	0.60	377.4
1921	-0.04	303.4	1963	0.03	318.7			

The collection of Figures and Tables that follow are individually identified below.

List of Figures:

List of Tables:

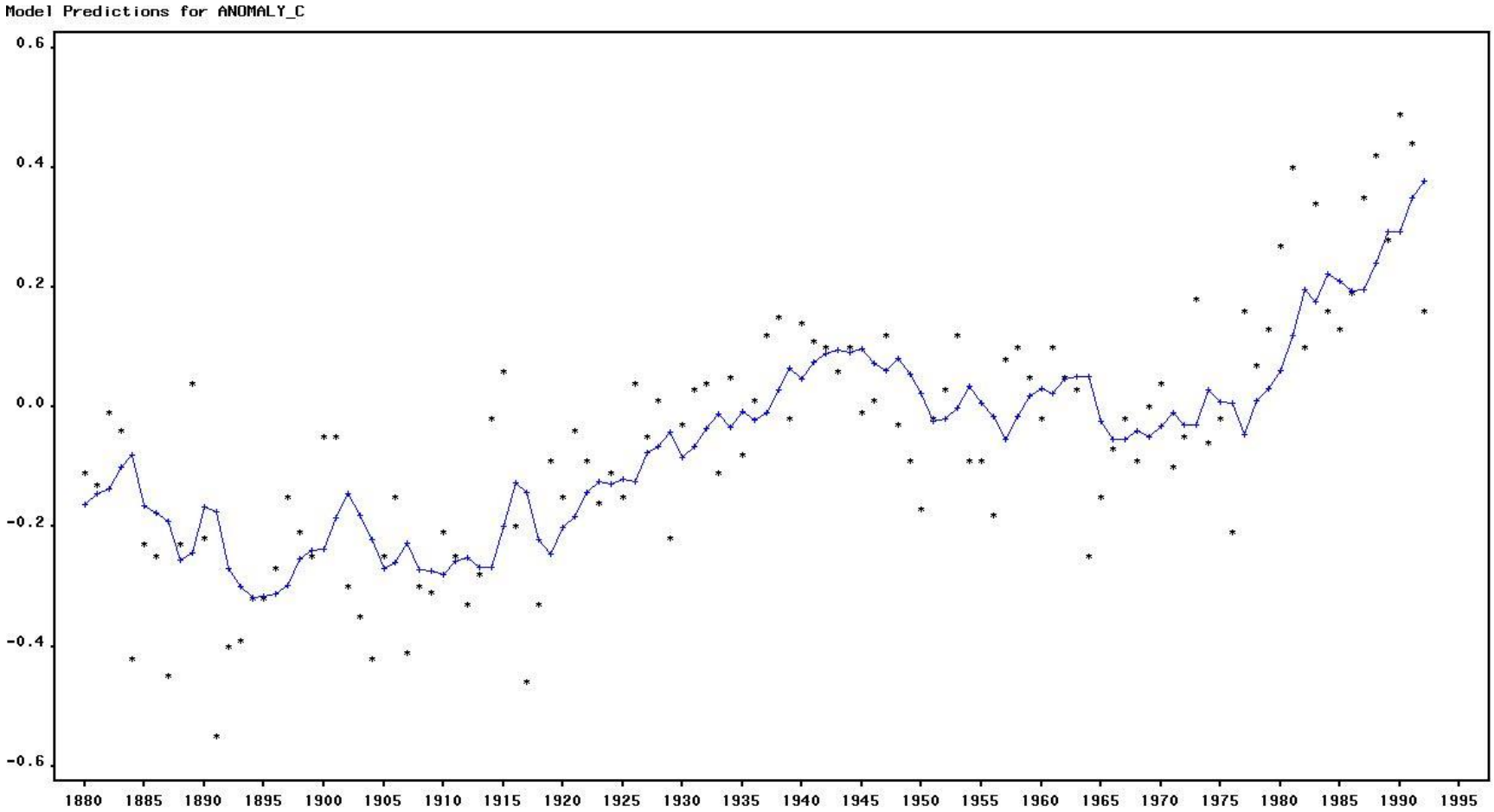

Figure 1 depicts a time series graph for predicted values ((+) signs) using the training data ((*) signs) set for Anomaly variable, as reported by SAS(AFS).

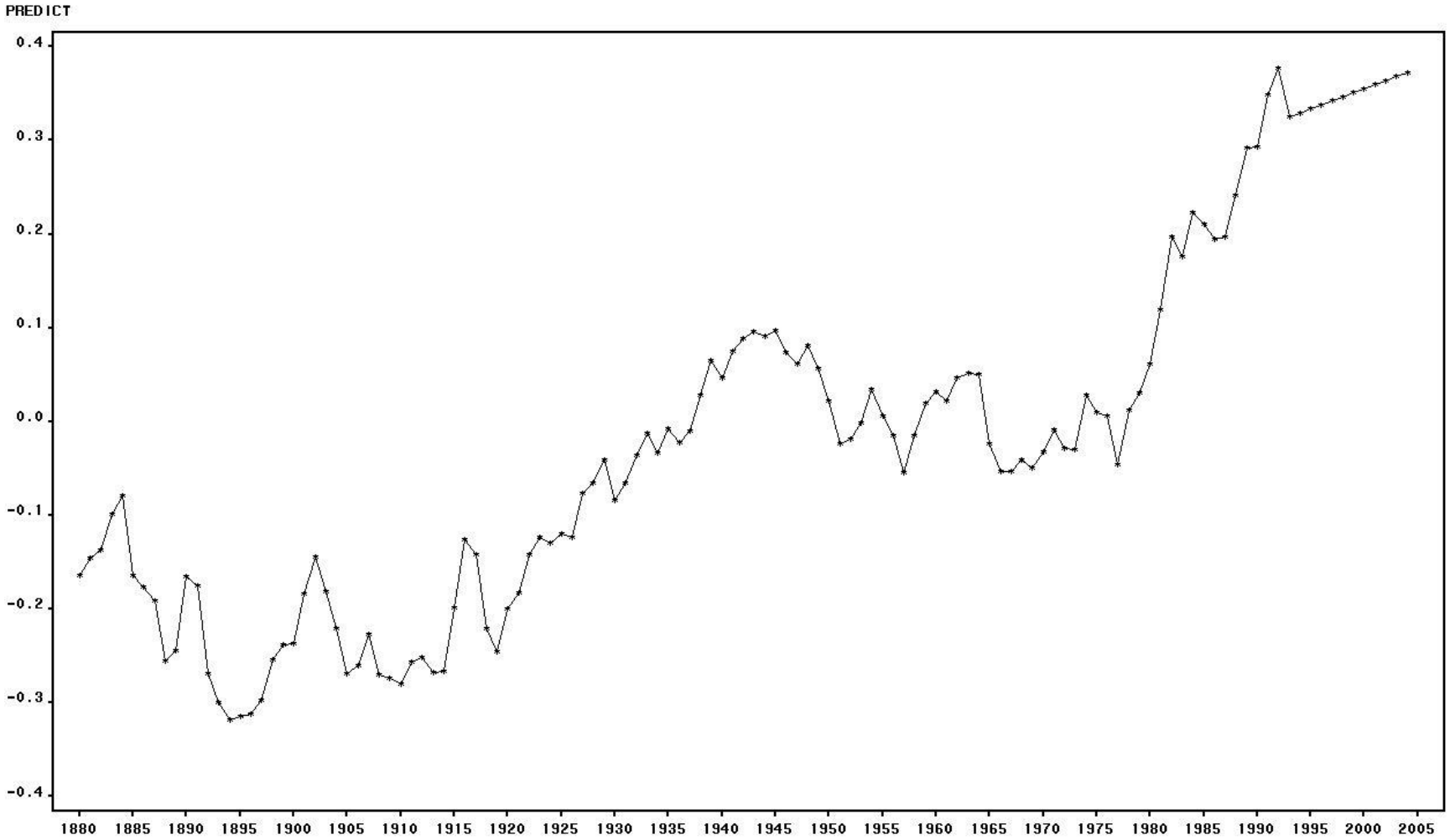

Figure 2 shows a time series graph for predicted values ((*) signs) for Anomaly variable as reported by SAS(AFS), along with 12-year forecasted values for the Anomaly variable. Here, predicted and forecast values are plotted using ((+) signs).

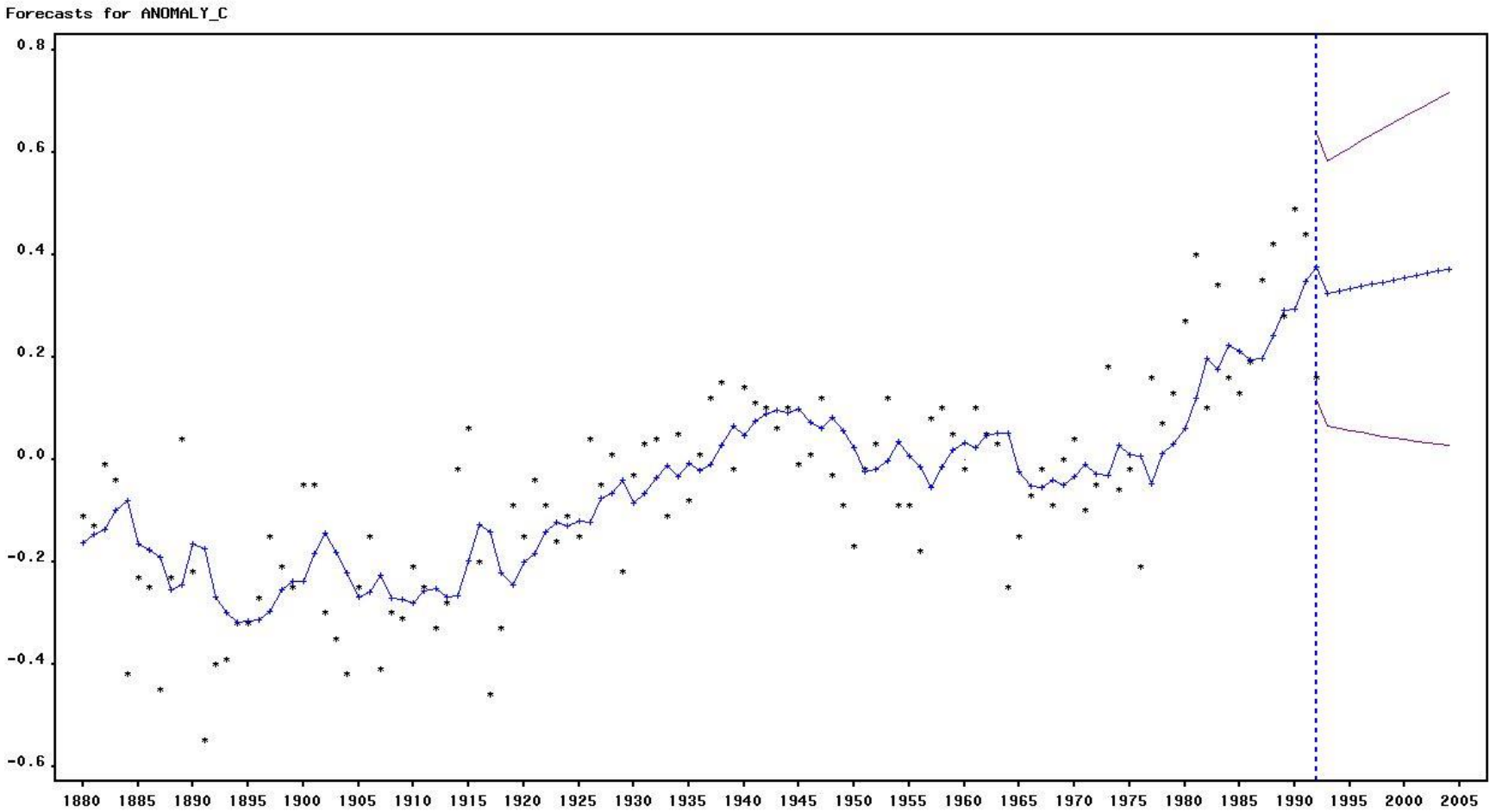

Figure 3a depicts a time series graph for predicted values ((+) signs) and the training data ((*) signs) for the Anomaly variable, as reported by SAS(AFS), along with 12-year forecasted values and prediction limits.

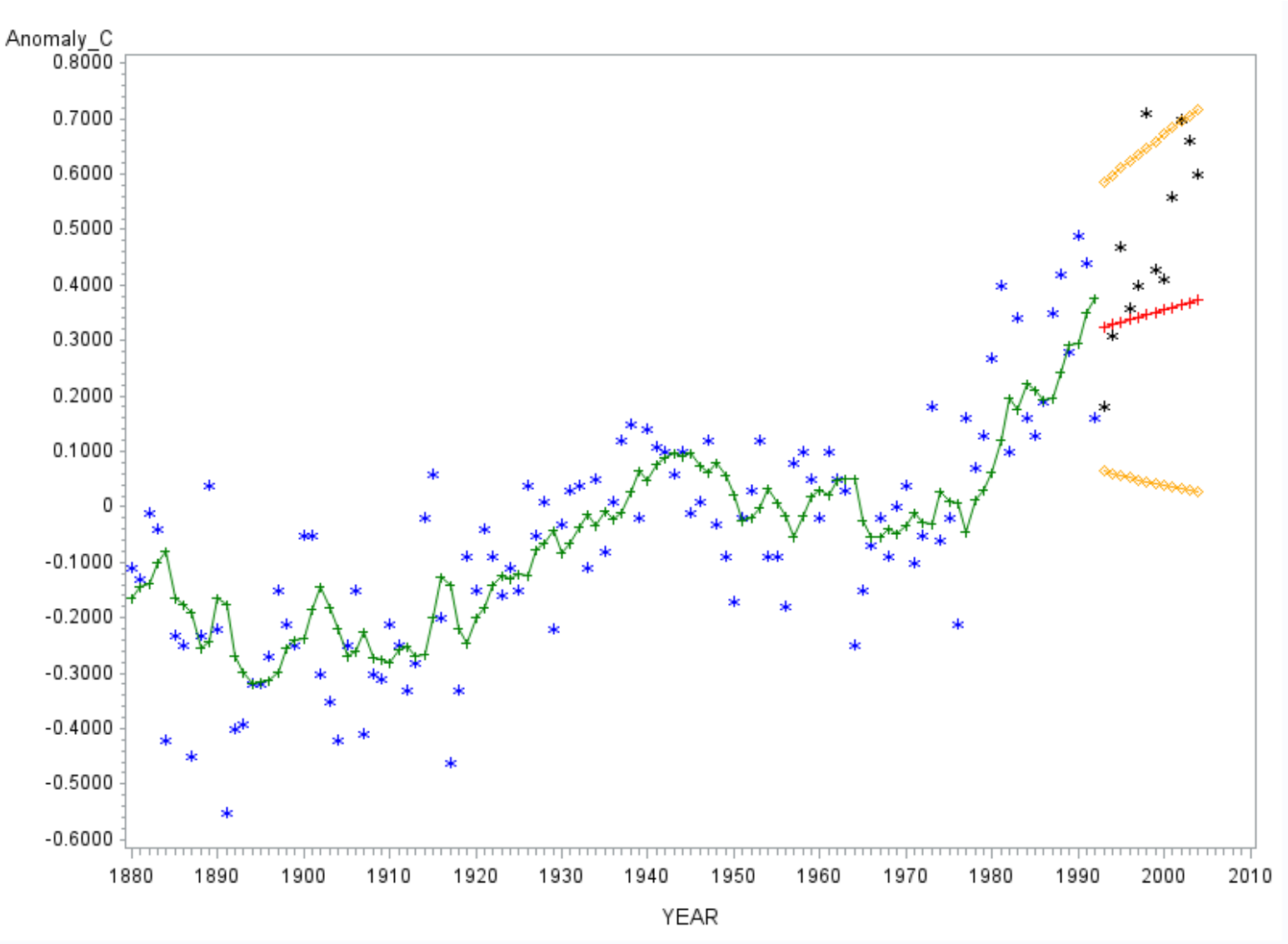

Figure 3b shows a time series graph for predicted values (green-colored (+) signs) for the Anomaly_C variable as reported by SAS(AFS), along with 12-year forecasted values, which are plotted using red-colored plus (+) signs. The training data are plotted using blue-colored star (*) signs, and testing data are plotted using black-colored star (*) signs, while the 12-year confidence limits are yellow-colored.

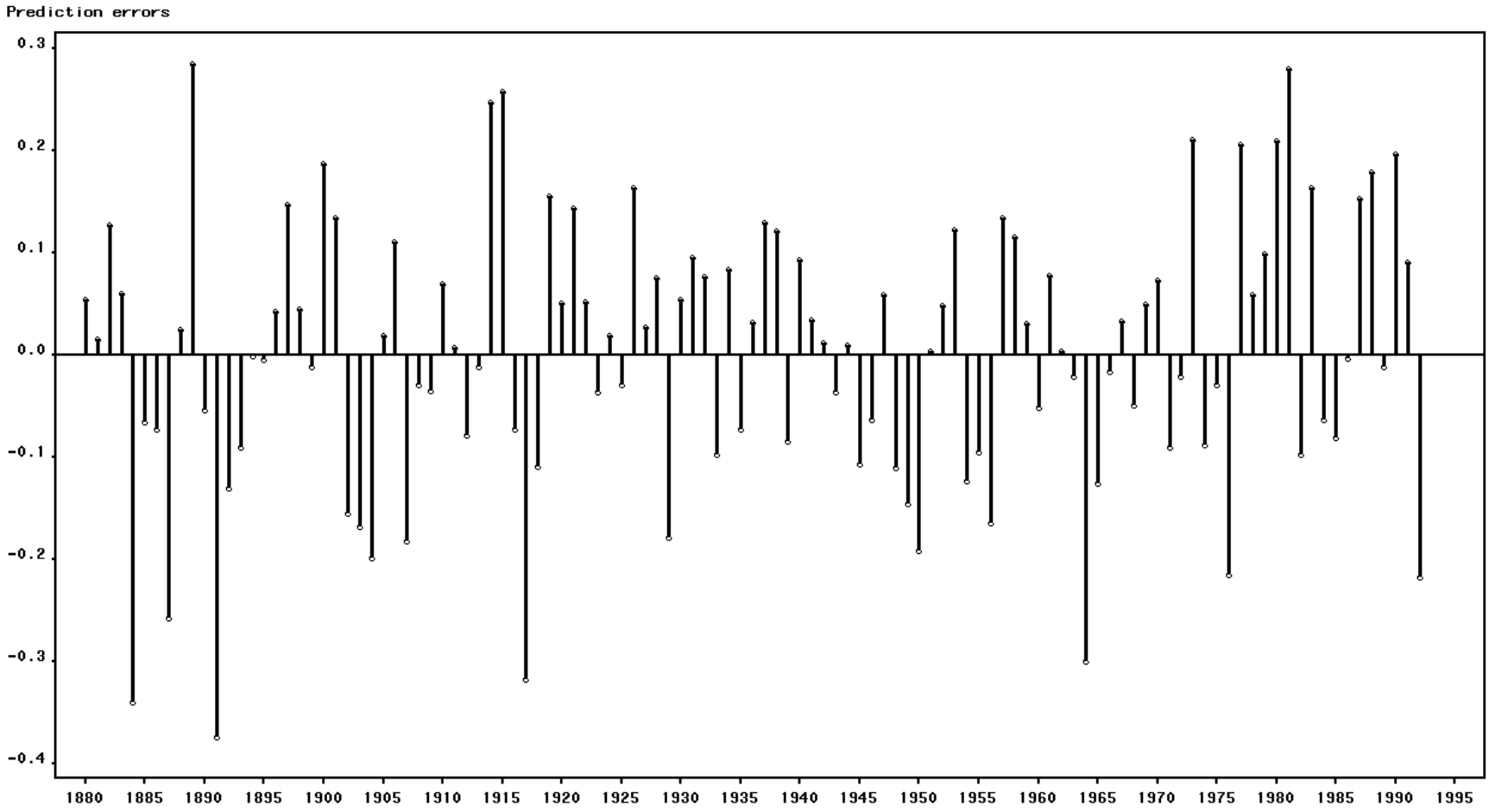

Figure 4. Prediction errors by time series values for the training data set.

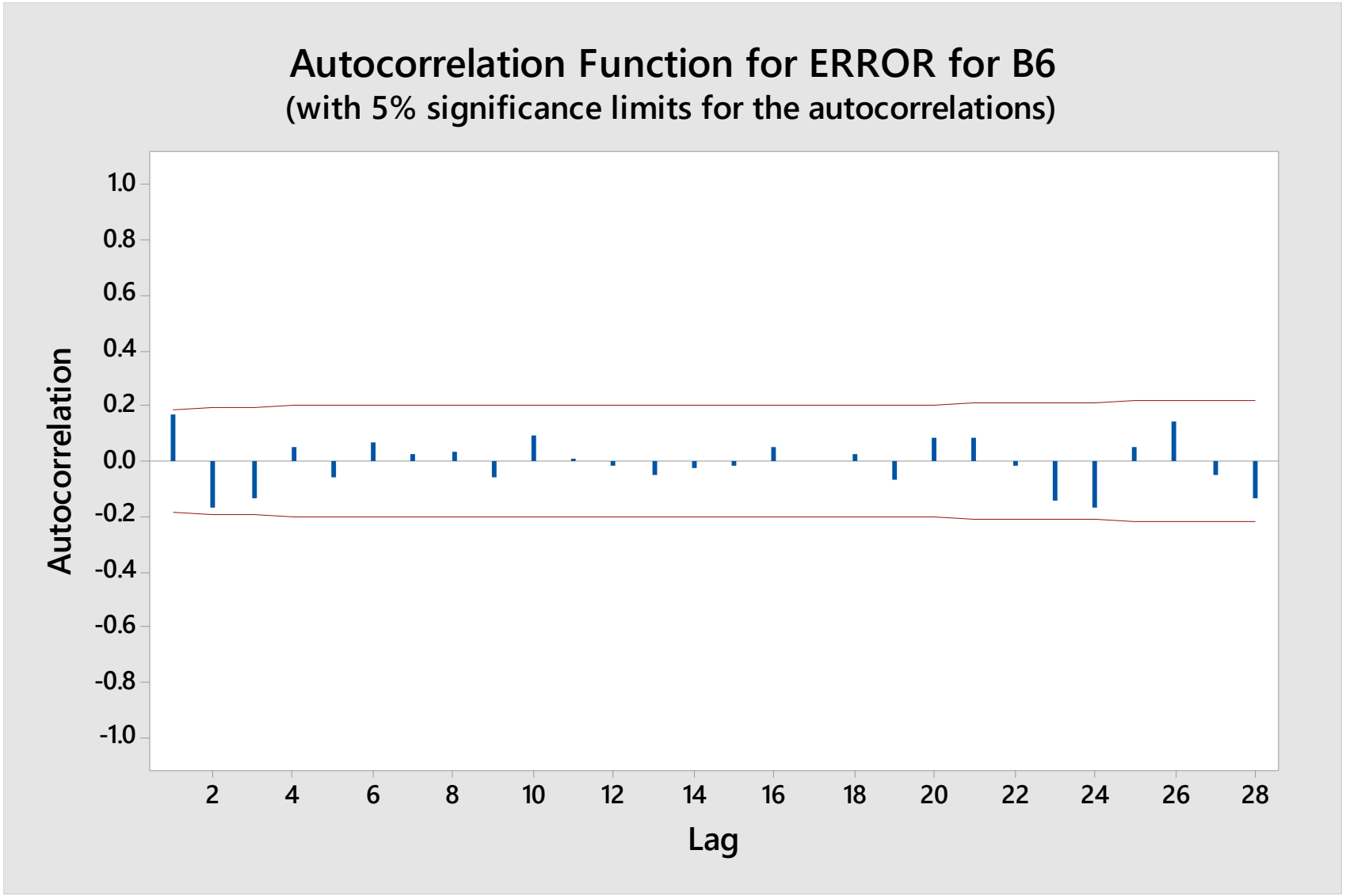

Figure 5. Autocorrelation Function (ACF) for the training data set.

Accuracy Measure for Table B.6 data.

Actual y_t	Predict $\widehat{y_t}$(t-1)	Forecast error $e_t(1) = y_t - \widehat{y_t}(t-1)$	Absolute error $\lvert e_t(1)\rvert$	Square error $[e_t(1)]^2$	Relative error $re_t(1) = 100(e_t(1)/y_t)$	Absolute percentage error $\lvert re_t(1)\rvert$
0.18	0.3249	-0.1449	0.1449	0.020996	-80.5000	80.5000
0.31	0.3292	-0.0192	0.0192	0.000369	-6.1935	6.1935
0.47	0.3336	0.1364	0.1364	0.018605	29.0213	29.0213
0.36	0.3379	0.0221	0.0221	0.000488	6.1389	6.1389
0.40	0.3422	0.0578	0.0578	0.003341	14.4500	14.4500
0.71	0.3466	0.3634	0.3634	0.132060	51.1831	51.1831
0.43	0.3509	0.0791	0.0791	0.006257	18.3953	18.3953
0.41	0.3552	0.0548	0.0548	0.003003	13.3659	13.3659
0.56	0.3596	0.2004	0.2004	0.040160	35.7857	35.7857
0.70	0.3639	0.3361	0.3361	0.112963	48.0143	48.0143
0.66	0.3683	0.2917	0.2917	0.085089	44.1970	44.1970
0.60	0.3726	0.2274	0.2274	0.051711	37.9000	37.9000

Table A. Error Measures table B6 time series data.

Data set	**MSE**	$\widehat{\sigma_{e(1)}}$	**MAD**	**MPE**	**MAPE**
Full data (125 data)	0.01748	0.13221	0.10299	33.55773	127.76260
Training data (113 data)	0.01726	0.13136	0.10308	36.43114	139.80537
Holdout data (12 data)	0.0395868	0.198964	0.161108	17.6465	32.0954

Table B. Accuracy Measures table B6 time series data.

Table B provides SAS (AFS)® forecast errors for the full data set, the training data set, and the holdout (or test) data set. The linear (Holt) exponential smoothing model was suggested as the "best model" for this data set. SAS (AFS)® and MINITAB® produced the accuracy measures shown in Table B, which comprise the Mean Square Error (MSE), the Standard Deviation of the Forecast Errors (SDFE), the Mean Absolute Deviation (MAD), the Mean Percent Forecast Error (MPE), and Mean Absolute Percent Error (MAPE).

Here, the accuracy statistics are computed as follows:

Mean square error:

$$\textbf{MSE} = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = 0.0395868$$

Standard Deviation of Forecast Error (Root Mean Square Error):

$$\widehat{\sigma_{e(1)}} = \sqrt{MSE} = \sqrt{0.0395868} = 0.198964$$

Mean Absolute Deviation (Mean Absolute Error):

$$\textbf{MAD} = \frac{1}{n}\sum_{t=1}^{n} |e_t(1)| = 0.161108$$

Mean Percent Forecast Error:

$$\textbf{MPE} = \frac{1}{n}\sum_{t=1}^{n} re_t(1) = 17.6465$$

Mean Absolute Percent Error:

$$\textbf{MAPE} = \frac{1}{n}\sum_{t=1}^{n} |re_t(1)| = 32.0954$$

Worksheet 1 ***

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	Actual	Predict	Forecast_Error	Abs_Error	Sq_Error	Relative_Error	Abs_Percentage_Error	MSE	Root_MSE	MAD	MPE	MAPE
1	0.18	0.3249	-0.1449	0.1449	0.020996	-80.5000	80.5000	0.0395868	0.198964	0.161108	17.6465	32.0954
2	0.31	0.3292	-0.0192	0.0192	0.000369	-6.1935	6.1935					
3	0.47	0.3336	0.1364	0.1364	0.018605	29.0213	29.0213					
4	0.36	0.3379	0.0221	0.0221	0.000488	6.1389	6.1389					
5	0.40	0.3422	0.0578	0.0578	0.003341	14.4500	14.4500					
6	0.71	0.3466	0.3634	0.3634	0.132060	51.1831	51.1831					
7	0.43	0.3509	0.0791	0.0791	0.006257	18.3953	18.3953					
8	0.41	0.3552	0.0548	0.0548	0.003003	13.3659	13.3659					
9	0.56	0.3596	0.2004	0.2004	0.040160	35.7857	35.7857					
10	0.70	0.3639	0.3361	0.3361	0.112963	48.0143	48.0143					
11	0.66	0.3683	0.2917	0.2917	0.085089	44.1970	44.1970					
12	0.60	0.3726	0.2274	0.2274	0.051711	37.9000	37.9000					

Table C. MINITAB® 17 Calculation of Forecast Errors for Holdout Data.

The first two columns of Table C consist of the actual values of the holdout data set and the SAS (AFS)® forecasted values, based on training data. The remaining columns give forecast errors, the absolute forecast errors, the squared forecast errors, the relative forecast errors and absolute forecast percentage errors using the MINITAB® calculator function.

Obs	DATE	ACTUAL	PREDICT	UPPER	LOWER	ERROR	STD	NERROR	_LEVEL_	_TREND_
114	1993	.	0.3249	0.5847	0.0651	.	0.1325	.	0.3249	0.004338
115	1994	.	0.3292	0.5977	0.0607	.	0.1370	.	0.3292	0.004338
116	1995	.	0.3336	0.6106	0.0566	.	0.1413	.	0.3336	0.004338
117	1996	.	0.3379	0.6231	0.0526	.	0.1455	.	0.3379	0.004338
118	1997	.	0.3422	0.6355	0.0489	.	0.1496	.	0.3422	0.004338
119	1998	.	0.3466	0.6477	0.0454	.	0.1536	.	0.3466	0.004338
120	1999	.	0.3509	0.6597	0.0421	.	0.1575	.	0.3509	0.004338
121	2000	.	0.3552	0.6715	0.0390	.	0.1614	.	0.3552	0.004338
122	2001	.	0.3596	0.6832	0.0360	.	0.1651	.	0.3596	0.004338
123	2002	.	0.3639	0.6947	0.0332	.	0.1688	.	0.3639	0.004338
124	2003	.	0.3683	0.7061	0.0305	.	0.1723	.	0.3683	0.004338
125	2004	.	0.3726	0.7173	0.0279	.	0.1759	.	0.3726	0.004338

Table D. Forecasted values for the last 12 years of Holdout Data.

Table D contains forecasted values for the final twelve years for the Anomaly variable, and the corresponding lower and upper 95% prediction limits. The Root Mean Square Prediction Error, Level and Trend smoothing components are also shown..

Model Parameter	Estimate	Std. Error	T	Prob>|T|
LEVEL Smoothing Weight	0.26140	0.0438	5.9660	<.0001
TREND Smoothing Weight	0.00100	0.0077	0.1301	0.8967
Residual Variance (sigma squared)	0.01757	.	.	.
Smoothed Level	0.32054	.	.	.
Smoothed Trend	0.00434	.	.	.

Table E. Parameter Estimates for Training Data using the Linear (Holt) Exponential Smoothing model suggested by SAS (AFS)®.

Results

Figure 3b reveals the SAS(AFS) could not forecast values with acceptably small prediction error. In addition, few testing values are outside of the confidence bands of the forecast values. There are no spikes in the ACF function, which means forecast errors are uncorrelated. The MPE and MAPE are very big in all cases.

TABLE B.7 (WHOLE FOODS MARKET STOCK PRICE, DAILY CLOSING ADJUSTED FOR SPLITS) ANALYSIS

Table B.7 comprises data on the Whole Foods market stock price, daily closing adjusted for Splits from January 2001 through December 2001, where the dollar variable is collected daily. The data set excludes last month values as a holdout set in order to analyze the accuracy of SAS(AFS), which suggested a log linear (Holt) exponential smoothing model as the "best model" for this training, or initialization, data set. SAS(AFS) also produced the predicted values for the next month. I have calculated different types of forecast errors based on the hold out data and predicted data.

The model equation is

$$w_t = \ln(y_t) = \mu_t + \beta_t t + \varepsilon_t$$

where,

Y_t = observed value i.e. reading at time t

μ_t = smoothed level and $\mu_t = 3.74951$

β_t = smoothed trend and $\beta_t = 0.00211$

Now, the k-step prediction equation is

$$\hat{y}_t(k) = \exp(\hat{w}_t(k))$$

where, $\hat{w}_t(k) = L_t + K T_t$

and the smoothing equations are

$$L_t = \alpha Y_t + (1-\alpha)(L_{t-1} + T_{t-1})$$

and $T_t = \gamma (L_t - L_{t-1}) + (1-\gamma) T_{t-1}$

where,

L_t = Level estimate and $L_0 = \mu_t = 3.74951$

T_t = Trend estimate and $T_0 = \beta_t = 0.00211$

α = level smoothing and $\alpha = 0.999$

γ = trend smoothing and $\gamma = 0.00100$

Table B.7: Whole Foods Market Stock Price, Daily Closing Adjusted for Splits

Date	Dollars	Date	Dollars	Date	Dollars	Date	Dollars	Date	Dollars
1/2/01	28.05	3/15/01	22.01	5/25/01	27.88	8/7/01	32.24	10/23/01	35.20
1/3/01	28.23	3/16/01	22.26	5/29/01	27.78	8/8/01	31.60	10/24/01	35.30
1/4/01	26.25	3/19/01	22.35	5/30/01	28.03	8/9/01	31.78	10/25/01	35.65
1/5/01	25.41	3/20/01	23.06	5/31/01	28.36	8/10/01	32.99	10/26/01	35.96
1/8/01	26.25	3/21/01	22.78	6/1/01	28.31	8/13/01	32.69	10/29/01	35.86
1/9/01	26.03	3/22/01	22.19	6/4/01	27.58	8/14/01	33.31	10/30/01	35.61
1/10/01	26.09	3/23/01	22.19	6/5/01	27.43	8/15/01	32.78	10/31/01	34.42
1/11/01	26.28	3/26/01	22.66	6/6/01	27.16	8/16/01	32.78	11/1/01	34.55
1/12/01	26.00	3/27/01	22.50	6/7/01	27.92	8/17/01	32.82	11/2/01	35.43
1/16/01	25.63	3/28/01	21.36	6/8/01	27.36	8/20/01	33.04	11/5/01	34.92
1/17/01	25.57	3/29/01	20.71	6/11/01	27.17	8/21/01	33.79	11/6/01	35.56
1/18/01	25.57	3/30/01	20.86	6/12/01	27.39	8/22/01	32.69	11/7/01	35.85
1/19/01	25.16	4/2/01	20.95	6/13/01	27.58	8/23/01	32.40	11/8/01	36.89

Date	Dollars	Date	Dollars	Date	Dollars	Date	Dollars	Date	Dollars
1/22/01	26.52	4/3/01	20.12	6/14/01	27.55	8/24/01	32.91	11/9/01	37.24
1/23/01	27.18	4/4/01	19.50	6/15/01	27.49	8/27/01	33.38	11/12/01	37.01
1/24/01	26.93	4/5/01	20.30	6/18/01	27.70	8/28/01	34.72	11/13/01	37.52
1/25/01	26.50	4/6/01	20.09	6/19/01	27.19	8/29/01	35.22	11/14/01	37.24
1/26/01	26.50	4/9/01	20.38	6/20/01	26.76	8/30/01	34.77	11/15/01	40.36
1/29/01	27.27	4/10/01	21.13	6/21/01	26.53	8/31/01	34.85	11/16/01	39.42
1/30/01	27.70	4/11/01	20.63	6/22/01	26.45	9/4/01	33.91	11/19/01	40.16
1/31/01	28.17	4/12/01	20.35	6/25/01	25.97	9/5/01	34.39	11/20/01	42.64
2/1/01	28.26	4/16/01	20.39	6/26/01	26.11	9/6/01	34.49	11/21/01	41.86
2/2/01	28.29	4/17/01	20.95	6/27/01	26.50	9/7/01	34.37	11/23/01	42.58
2/5/01	28.23	4/18/01	21.94	6/28/01	26.98	9/10/01	33.44	11/26/01	42.63
2/6/01	28.54	4/19/01	21.43	6/29/01	26.84	9/17/01	33.24	11/27/01	42.14
2/7/01	28.94	4/20/01	21.37	7/2/01	28.03	9/18/01	33.18	11/28/01	41.62
2/8/01	28.51	4/23/01	21.24	7/3/01	28.00	9/19/01	31.26	11/29/01	42.59
2/9/01	27.55	4/24/01	21.13	7/5/01	28.01	9/20/01	31.04	11/30/01	42.50
2/12/01	28.05	4/25/01	22.36	7/6/01	27.20	9/21/01	30.33	12/3/01	42.38
2/13/01	27.98	4/26/01	22.93	7/9/01	27.92	9/24/01	30.69	12/4/01	42.77
2/14/01	23.55	4/27/01	23.26	7/10/01	27.10	9/25/01	30.84	12/5/01	43.80
2/15/01	24.21	4/30/01	24.07	7/11/01	27.15	9/26/01	29.95	12/6/01	45.13
2/16/01	23.92	5/1/01	23.79	7/12/01	27.19	9/27/01	29.22	12/7/01	45.40
2/20/01	23.77	5/2/01	24.56	7/13/01	26.69	9/28/01	31.11	12/10/01	43.81
2/21/01	23.74	5/3/01	24.43	7/16/01	26.79	10/1/01	30.93	12/11/01	42.16
2/22/01	23.55	5/4/01	24.29	7/17/01	27.17	10/2/01	30.98	12/12/01	41.24
2/23/01	23.34	5/7/01	23.33	7/18/01	26.72	10/3/01	32.59	12/13/01	40.91
2/26/01	23.22	5/8/01	25.20	7/19/01	26.33	10/4/01	32.50	12/14/01	41.05
2/27/01	22.87	5/9/01	24.94	7/20/01	26.23	10/5/01	32.12	12/17/01	41.13
2/28/01	21.36	5/10/01	24.95	7/23/01	26.59	10/8/01	32.09	12/18/01	41.55
3/1/01	21.30	5/11/01	25.25	7/24/01	26.82	10/9/01	32.85	12/19/01	41.35
3/2/01	21.51	5/14/01	25.70	7/25/01	27.24	10/10/01	33.44	12/20/01	41.27
3/5/01	21.32	5/15/01	26.33	7/26/01	28.49	10/11/01	32.68	12/21/01	42.46
3/6/01	21.67	5/16/01	27.81	7/27/01	31.65	10/12/01	32.54	12/24/01	42.96
3/7/01	21.48	5/17/01	28.04	7/30/01	34.47	10/15/01	32.07	12/26/01	43.63
3/8/01	21.85	5/18/01	28.75	7/31/01	33.63	10/16/01	33.18	12/27/01	43.63
3/9/01	21.49	5/21/01	28.72	8/1/01	32.58	10/17/01	33.45	12/28/01	43.59
3/12/01	21.48	5/22/01	28.33	8/2/01	32.62	10/18/01	34.35	12/31/01	43.14
3/13/01	22.10	5/23/01	27.61	8/3/01	32.09	10/19/01	33.95		
3/14/01	21.79	5/24/01	27.98	8/6/01	32.41	10/22/01	34.42		

The collection of Figures and Tables that follow are individually identified below.

List of Figures:

List of Tables:

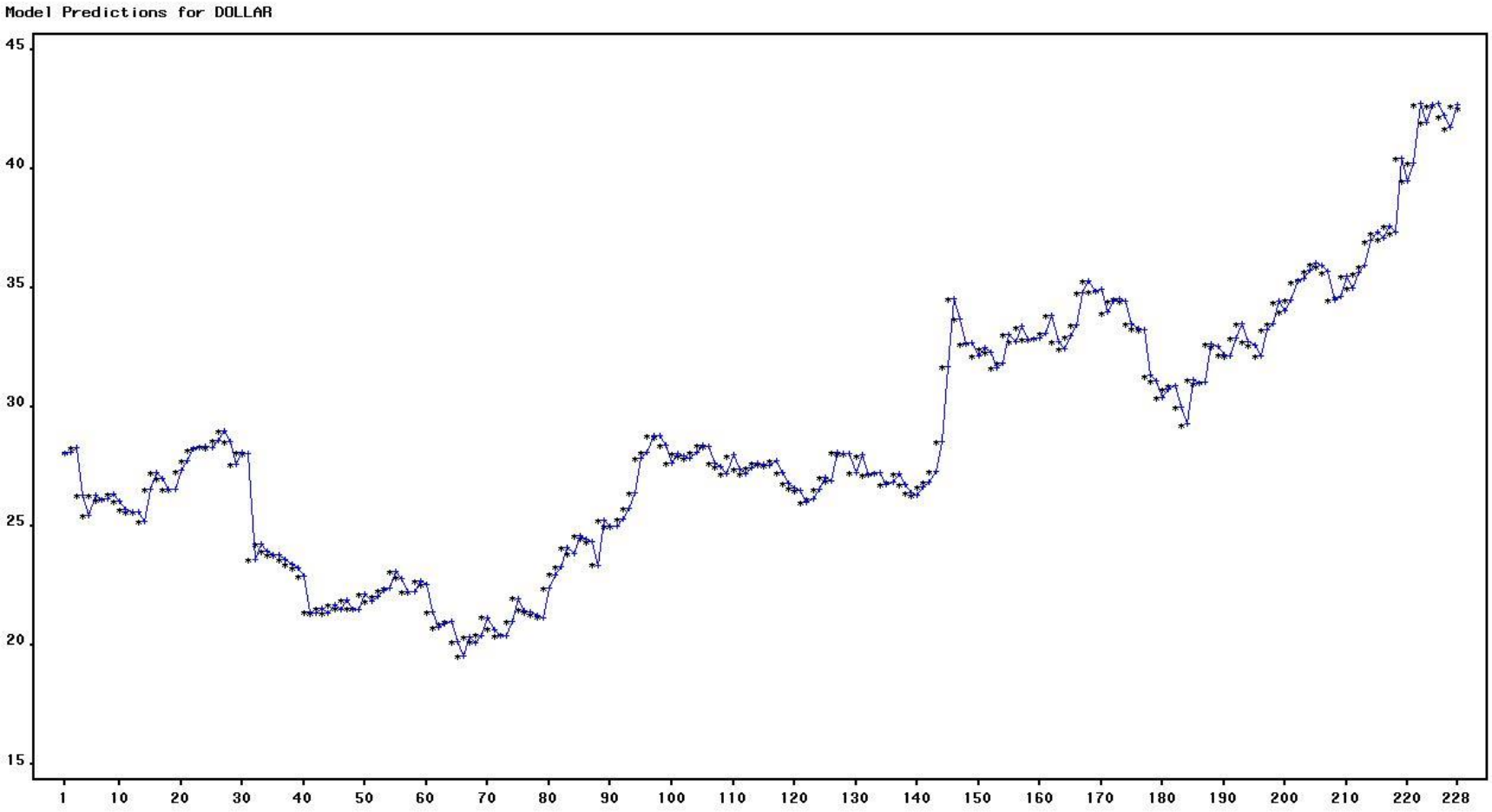

Figure 1 depicts a time series graph for predicted values ((+) signs) using the training data ((*) signs) set for Dollar variable, as reported by SAS(AFS).

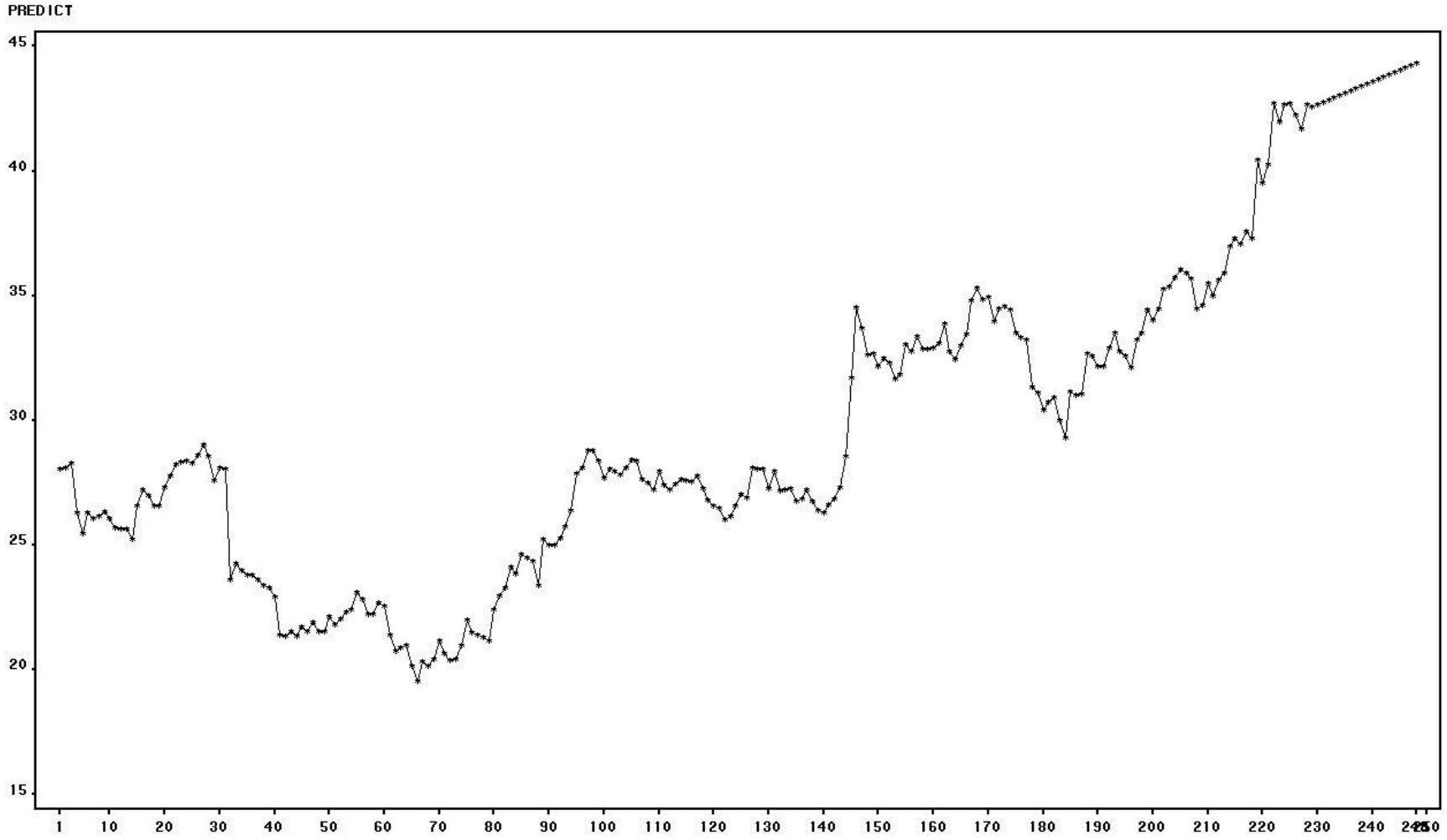

Figure 2 shows a time series graph for predicted values ((*) signs) for Dollar variable as reported by SAS(AFS), along with 12-month forecasted values for the Dollar variable. Here, predicted and forecast values are plotted using ((*) signs).

Figure 3a depicts a time series graph for predicted values ((+) signs) and the training data ((*) signs) for the Dollar variable, as reported by SAS(AFS), along with next month forecasted values and prediction limits.

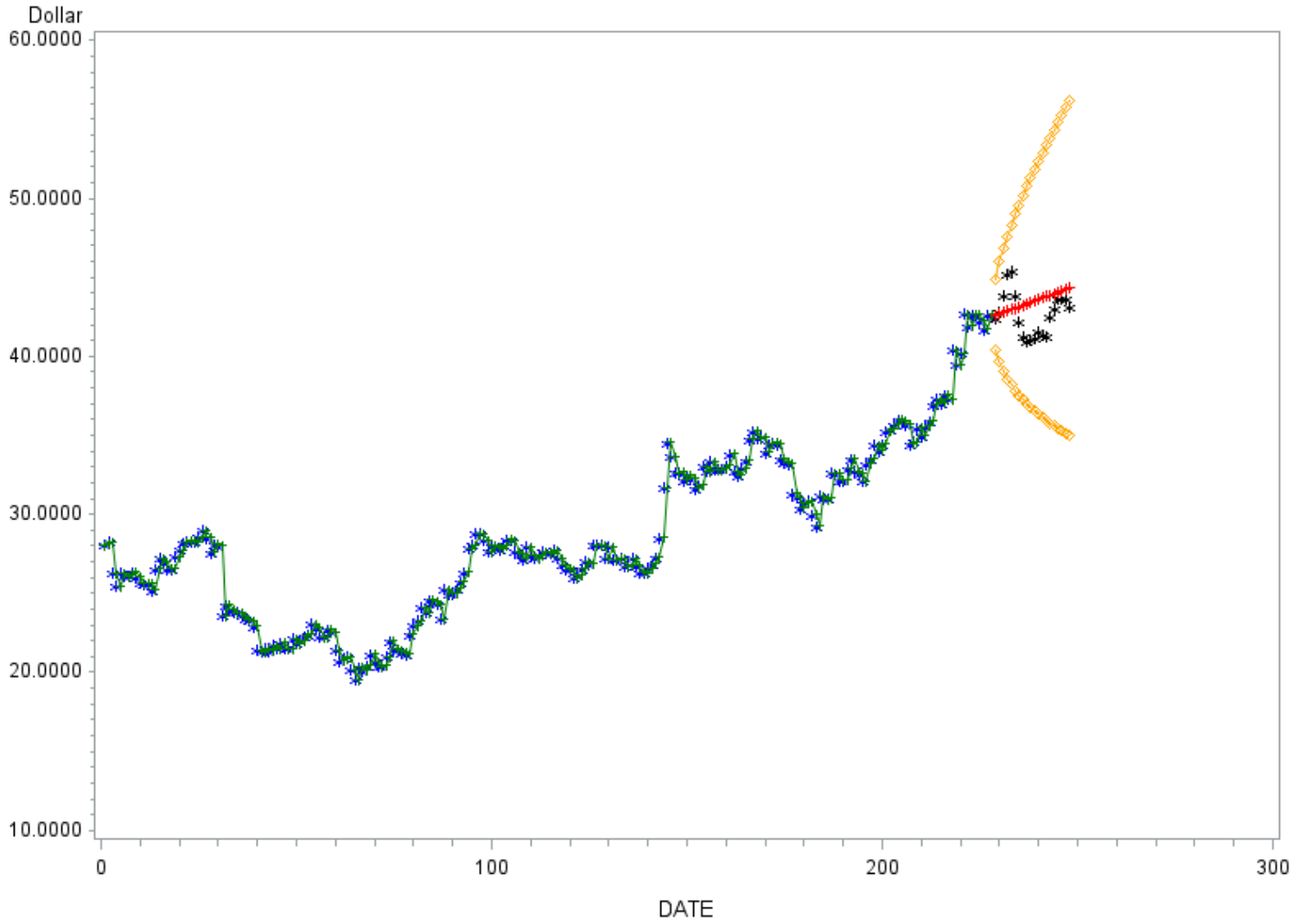

Figure 3b shows a time series graph for predicted values (green-colored (+) signs) for the Dollar variable as reported by SAS(AFS), along with next month forecasted values, which are plotted using red-colored plus (+) signs. The training data are plotted using blue-colored star (*) signs, and testing data are plotted using black-colored star (*) signs, while the next month confidence limits are yellow-colored.

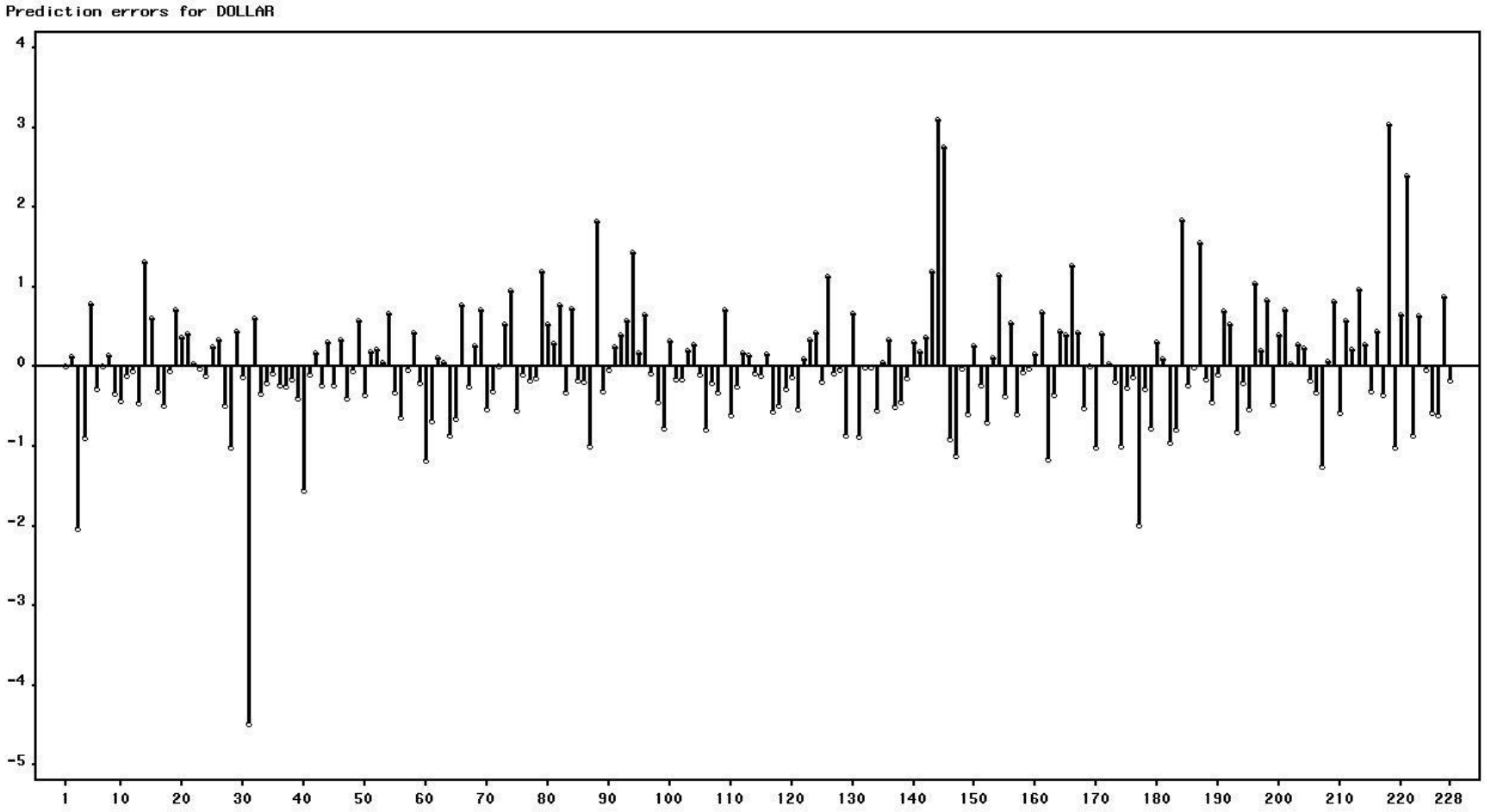

Figure 4. Prediction errors by time series values for the training data set.

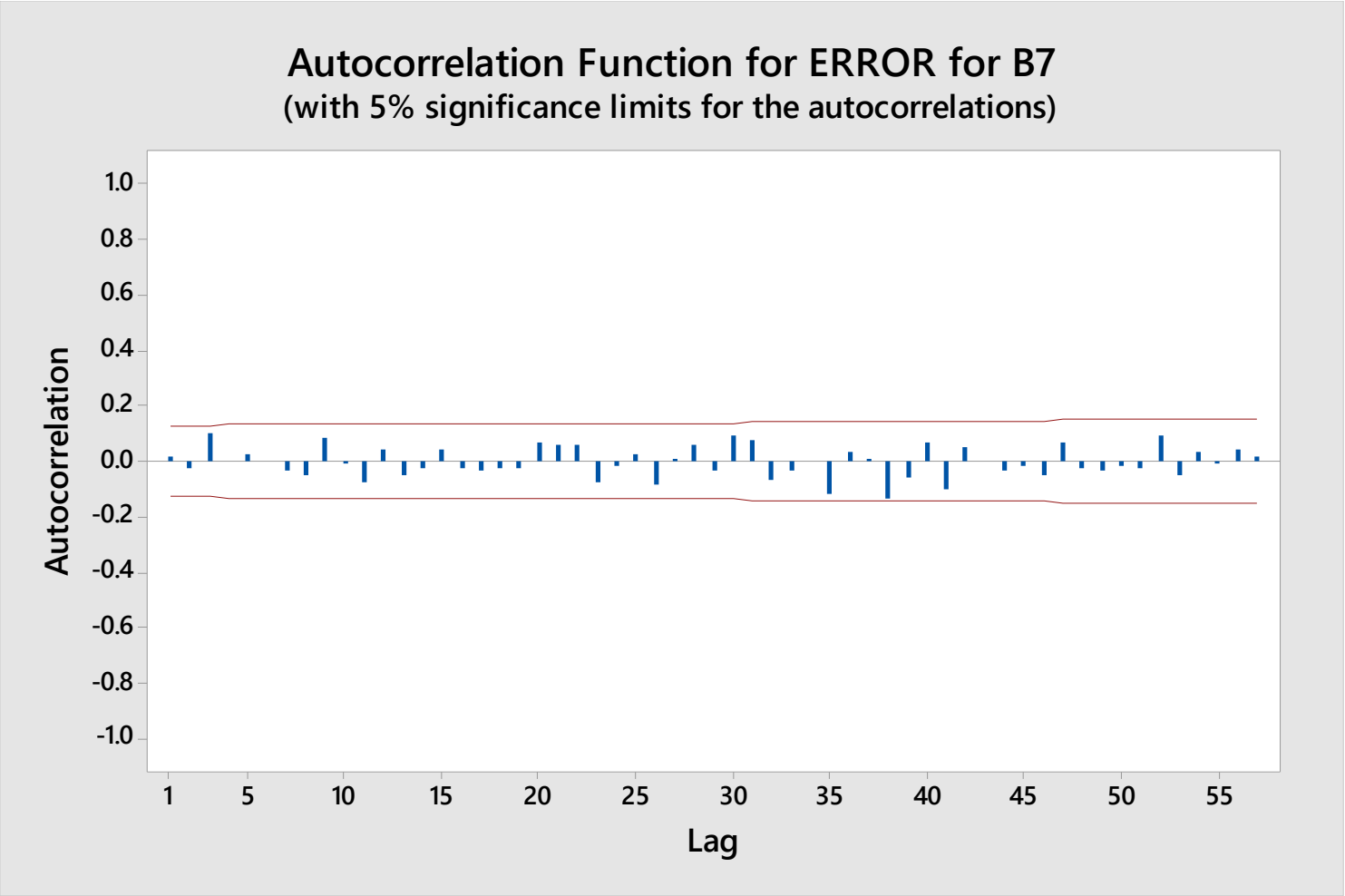

Figure 5. Autocorrelation Function (ACF) for the training data set.

Accuracy Measure for Table B.7 data.

Actual y_t	Predict $\widehat{y_t}$(t-1)	Forecast error $e_t(1) = y_t - \widehat{y_t}(t-1)$	Absolute error $\lvert e_t(1)\rvert$	Square error $[e_t(1)]^2$	Relative error $re_t(1) = 100(e_t(1)/y_t)$	Absolute percentage error $\lvert re_t(1)\rvert$
42.38	42.5901	-0.2101	0.2101	0.04414	-0.49575	0.49575
42.77	42.6803	0.0897	0.0897	0.00805	0.20973	0.20973
43.80	42.7707	1.0293	1.0293	1.05946	2.35000	2.35000
45.13	42.8612	2.2688	2.2688	5.14745	5.02725	5.02725
45.40	42.9519	2.4481	2.4481	5.99319	5.39229	5.39229
43.81	43.0429	0.7671	0.7671	0.58844	1.75097	1.75097
42.16	43.1340	-0.9740	0.9740	0.94868	-2.31025	2.31025
41.24	43.2253	-1.9853	1.9853	3.94142	-4.81402	4.81402
40.91	43.3168	-2.4068	2.4068	5.79269	-5.88316	5.88316
41.05	43.4085	-2.3585	2.3585	5.56252	-5.74543	5.74543
41.13	43.5004	-2.3704	2.3704	5.61880	-5.76319	5.76319
41.55	43.5925	-2.0425	2.0425	4.17181	-4.91576	4.91576
41.35	43.6847	-2.3347	2.3347	5.45082	-5.64619	5.64619
41.27	43.7772	-2.5072	2.5072	6.28605	-6.07512	6.07512
42.46	43.8699	-1.4099	1.4099	1.98782	-3.32054	3.32054
42.96	43.9628	-1.0028	1.0028	1.00561	-2.33426	2.33426
43.63	44.0558	-0.4258	0.4258	0.18131	-0.97593	0.97593
43.63	44.1491	-0.5191	0.5191	0.26946	-1.18978	1.18978
43.59	44.2425	-0.6525	0.6525	0.42576	-1.49690	1.49690
43.14	44.3362	-1.1962	1.1962	1.43089	-2.77283	2.77283

Table A. Error Measures table B7 time series data.

Data set	**MSE**	$\widehat{\sigma_{e(1)}}$	**MAD**	**MPE**	**MAPE**
Full data (248 data)	0.58602	0.76552	0.52204	-0.06791	1.78272
Training data (228 data)	0.58501	0.76486	0.51722	-0.04489	1.82035
Holdout data (20 data)	2.79572	1.67204	1.44994	-1.95044	3.42347

Table B. Accuracy Measures table B7 time series data.

Table B provides SAS (AFS)® forecast errors for the full data set, the training data set, and the holdout (or test) data set. The log linear (Holt) exponential smoothing model was suggested as the "best model" for this data set. SAS (AFS)® and MINITAB® produced the accuracy measures shown in Table B, which comprise the Mean Square Error (MSE), the Standard Deviation of the Forecast Errors (SDFE), the Mean Absolute Deviation (MAD), the Mean Percent Forecast Error (MPE), and Mean Absolute Percent Error (MAPE).

Here, the accuracy statistics are computed as follows:

Mean square error:

$$\textbf{MSE} = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = 2.79572$$

Standard Deviation of Forecast Error (Root Mean Square Error):

$$\widehat{\sigma_{e(1)}} = \sqrt{MSE} = \sqrt{2.79572} = 1.67204$$

Mean Absolute Deviation (Mean Absolute Error):

$$\textbf{MAD} = \frac{1}{n}\sum_{t=1}^{n}|e_t(1)| = 1.44994$$

Mean Percent Forecast Error:

$$\textbf{MPE} = \frac{1}{n}\sum_{t=1}^{n}re_t(1) = -1.95044$$

Mean Absolute Percent Error:

$$\textbf{MAPE} = \frac{1}{n}\sum_{t=1}^{n}|re_t(1)| = 3.42347$$

Worksheet 1 ***

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	Actual	Predict	Forecast_Error	Abs_Error	Sq_Error	Relative_Error	Abs_Percentage_Error	MSE	Root_MSE	MAD	MPE	MAPE
1	42.38	42.5901	-0.2101	0.2101	0.04414	-0.49575	0.49575	2.79572	1.67204	1.44994	-1.95044	3.42347
2	42.77	42.6803	0.0897	0.0897	0.00805	0.20973	0.20973					
3	43.80	42.7707	1.0293	1.0293	1.05946	2.35000	2.35000					
4	45.13	42.8612	2.2688	2.2688	5.14745	5.02725	5.02725					
5	45.40	42.9519	2.4481	2.4481	5.99319	5.39229	5.39229					
6	43.81	43.0429	0.7671	0.7671	0.58844	1.75097	1.75097					
7	42.16	43.1340	-0.9740	0.9740	0.94868	-2.31025	2.31025					
8	41.24	43.2253	-1.9853	1.9853	3.94142	-4.81402	4.81402					
9	40.91	43.3168	-2.4068	2.4068	5.79269	-5.88316	5.88316					
10	41.05	43.4085	-2.3585	2.3585	5.56252	-5.74543	5.74543					
11	41.13	43.5004	-2.3704	2.3704	5.61880	-5.76319	5.76319					
12	41.55	43.5925	-2.0425	2.0425	4.17181	-4.91576	4.91576					
13	41.35	43.6847	-2.3347	2.3347	5.45082	-5.64619	5.64619					
14	41.27	43.7772	-2.5072	2.5072	6.28605	-6.07512	6.07512					
15	42.46	43.8699	-1.4099	1.4099	1.98782	-3.32054	3.32054					
16	42.96	43.9628	-1.0028	1.0028	1.00561	-2.33426	2.33426					
17	43.63	44.0558	-0.4258	0.4258	0.18131	-0.97593	0.97593					
18	43.63	44.1491	-0.5191	0.5191	0.26946	-1.18978	1.18978					
19	43.59	44.2425	-0.6525	0.6525	0.42576	-1.49690	1.49690					
20	43.14	44.3362	-1.1962	1.1962	1.43089	-2.77283	2.77283					

Table C. MINITAB® 17 Calculation of Forecast Errors for Holdout Data.

The first two columns of Table C consist of the actual values of the holdout data set and the SAS (AFS)® forecasted values, based on training data. The remaining columns give forecast errors, the absolute forecast errors, the squared forecast errors, the relative forecast errors and absolute forecast percentage errors using the MINITAB® calculator function.

Obs	DATE	ACTUAL	PREDICT	UPPER	LOWER	ERROR	STD	NERROR	RESIDUAL	RESSTD	NRESID	_LEVEL_	_TREND_
229	229	.	42.5901	44.8888	40.4092	.	1.1430	.	.	.	.	3.7516	0.002115
230	230	.	42.6803	45.9740	39.6225	.	1.6209	.	.	.	.	3.7537	0.002115
231	231	.	42.7707	46.8490	39.0474	.	1.9913	.	.	.	.	3.7559	0.002115
232	232	.	42.8612	47.6164	38.5809	.	2.3066	.	.	.	.	3.7580	0.002115
233	233	.	42.9519	48.3160	38.1834	.	2.5871	.	.	.	.	3.7601	0.002115
234	234	.	43.0429	48.9682	37.8345	.	2.8431	.	.	.	.	3.7622	0.002115
235	235	.	43.1340	49.5848	37.5224	.	3.0809	.	.	.	.	3.7643	0.002115
236	236	.	43.2253	50.1739	37.2390	.	3.3043	.	.	.	.	3.7664	0.002115
237	237	.	43.3168	50.7407	36.9791	.	3.5160	.	.	.	.	3.7685	0.002115
238	238	.	43.4085	51.2892	36.7386	.	3.7183	.	.	.	.	3.7707	0.002115
239	239	.	43.5004	51.8225	36.5147	.	3.9124	.	.	.	.	3.7728	0.002115
240	240	.	43.5925	52.3427	36.3050	.	4.0997	.	.	.	.	3.7749	0.002115
241	241	.	43.6847	52.8517	36.1078	.	4.2810	.	.	.	.	3.7770	0.002115
242	242	.	43.7772	53.3510	35.9214	.	4.4570	.	.	.	.	3.7791	0.002115
243	243	.	43.8699	53.8418	35.7448	.	4.6285	.	.	.	.	3.7812	0.002115
244	244	.	43.9628	54.3251	35.5770	.	4.7959	.	.	.	.	3.7833	0.002115
245	245	.	44.0558	54.8018	35.4170	.	4.9596	.	.	.	.	3.7855	0.002115
246	246	.	44.1491	55.2725	35.2642	.	5.1200	.	.	.	.	3.7876	0.002115
247	247	.	44.2425	55.7380	35.1179	.	5.2775	.	.	.	.	3.7897	0.002115
248	248	.	44.3362	56.1988	34.9776	.	5.4322	.	.	.	.	3.7918	0.002115

Table D. Forecasted values for the last 12 month of Holdout Data.

Table D contains forecasted values for the final month for the Dollar variable, and the corresponding lower and upper 95% prediction limits. The Root Mean Square Prediction Error, Level and Trend smoothing components are also shown..

Model Parameter	Estimate	Std. Error	T	Prob>|T|
LEVEL Smoothing Weight	0.99900	0.0470	21.2373	<.0001
TREND Smoothing Weight	0.00100	0.0035	0.2821	0.7782
Residual Variance (sigma squared)	0.0007193	.	.	.
Smoothed Level	3.74951	.	.	.
Smoothed Trend	0.00211	.	.	.

Table E. Parameter Estimates for Training Data using the Log Linear (Holt) Exponential Smoothing model suggested by SAS (AFS)®.

Results

In Figure 3b, it appears the model fits the data appropriately, however, the SAS(AFS) could not forecast values with acceptably small prediction error. There are no spikes in the ACF function, which means forecast errors are uncorrelated. The MSE, RMSE, MAD and MAPE are little bit higher in hold out data compare to training data.

TABLE B.8 (UNEMPLOYMENT RATE-FULL-TIME LABOR FORCE, NOT SEASONALLY ADJUSTED) ANALYSIS

Table B.8 comprises data on the Unemployment Rate–full-time labor force, not seasonally adjusted from January 1963 through December 2004, where the percentage rate variable is collected monthly. The data set excludes last month values as a holdout set in order to analyze the accuracy of SAS(AFS), which suggested a ARIMA (2,1,2)(0,1,1)s NOINT model as the "best model" for this training, or initialization, data set. SAS(AFS) also produced the predicted values for the next month. I have calculated different types of forecast errors based on the hold out data and predicted data.

Table B.8: Unemployment Rate-Full-Time Labor Force, Not Seasonally Adjusted

Month	Rate (%)	Month	Rate (%)	Month	Rate (%)	Month	Rate (%)	Month	Rate (%)	Month	Rate (%)
Jan-1963	6.8	Jan-1970	3.8	Jan-1977	7.9	Jan-1984	8.9	Jan-1991	7.0	Jan-1998	5.0
Feb-1963	6.8	Feb-1970	4.3	Feb-1977	8.2	Feb-1984	8.5	Feb-1991	7.4	Feb-1998	4.8
Mar-1963	6.2	Mar-1970	4.2	Mar-1977	7.6	Mar-1984	8.3	Mar-1991	7.1	Mar-1998	4.8
Apr-1963	5.6	Apr-1970	4.1	Apr-1977	6.7	Apr-1984	7.8	Apr-1991	6.6	Apr-1998	4.0
May-1963	5.4	May-1970	4.2	May-1977	6.4	May-1984	7.4	May-1991	6.7	May-1998	4.2
Jun-1963	6.0	Jun-1970	5.5	Jun-1977	7.3	Jun-1984	7.4	Jun-1991	7.0	Jun-1998	4.6
Jul-1963	5.4	Jul-1970	5.1	Jul-1977	6.9	Jul-1984	7.6	Jul-1991	6.9	Jul-1998	4.6
Aug-1963	4.9	Aug-1970	4.7	Aug-1977	6.6	Aug-1984	7.2	Aug-1991	6.5	Aug-1998	4.3
Sep-1963	4.3	Sep-1970	4.5	Sep-1977	6.0	Sep-1984	6.8	Sep-1991	6.3	Sep-1998	4.1
Oct-1963	4.4	Oct-1970	4.5	Oct-1977	5.9	Oct-1984	6.9	Oct-1991	6.3	Oct-1998	3.9
Nov-1963	4.9	Nov-1970	4.9	Nov-1977	5.9	Nov-1984	6.8	Nov-1991	6.6	Nov-1998	3.8
Dec-1963	5.1	Dec-1970	5.2	Dec-1977	5.7	Dec-1984	7.1	Dec-1991	7.0	Dec-1998	3.9
Jan-1964	6.2	Jan-1971	6.1	Jan-1978	6.7	Jan-1985	8.0	Jan-1992	8.1	Jan-1999	4.6
Feb-1964	6.1	Feb-1971	6.2	Feb-1978	6.6	Feb-1985	7.9	Feb-1992	8.3	Feb-1999	4.6
Mar-1964	5.7	Mar-1971	5.9	Mar-1978	6.2	Mar-1985	7.5	Mar-1992	7.9	Mar-1999	4.3
Apr-1964	5.1	Apr-1971	5.4	Apr-1978	5.5	Apr-1985	7.1	Apr-1992	7.4	Apr-1999	4.0
May-1964	4.7	May-1971	5.2	May-1978	5.5	May-1985	7.0	May-1992	7.4	May-1999	3.9
Jun-1964	5.7	Jun-1971	6.4	Jun-1978	6.1	Jun-1985	7.5	Jun-1992	8.0	Jun-1999	4.3
Jul-1964	4.7	Jul-1971	6.0	Jul-1978	6.1	Jul-1985	7.4	Jul-1992	7.8	Jul-1999	4.4
Aug-1964	4.5	Aug-1971	5.6	Aug-1978	5.5	Aug-1985	6.8	Aug-1992	7.3	Aug-1999	4.1
Sep-1964	4.0	Sep-1971	5.1	Sep-1978	5.1	Sep-1985	6.6	Sep-1992	7.0	Sep-1999	3.8
Oct-1964	4.0	Oct-1971	4.8	Oct-1978	4.8	Oct-1985	6.5	Oct-1992	6.7	Oct-1999	3.7
Nov-1964	4.0	Nov-1971	5.1	Nov-1978	5.0	Nov-1985	6.6	Nov-1992	7.0	Nov-1999	3.6
Dec-1964	4.3	Dec-1971	5.2	Dec-1978	5.2	Dec-1985	6.6	Dec-1992	7.1	Dec-1999	3.7
Jan-1965	5.3	Jan-1972	6.1	Jan-1979	5.9	Jan-1986	7.3	Jan-1993	7.9	Jan-2000	4.3
Feb-1965	5.5	Feb-1972	6.0	Feb-1979	6.1	Feb-1986	7.8	Feb-1993	7.9	Feb-2000	4.2
Mar-1965	4.9	Mar-1972	5.8	Mar-1979	5.7	Mar-1986	7.5	Mar-1993	7.5	Mar-2000	4.1
Apr-1965	4.5	Apr-1972	5.2	Apr-1979	5.3	Apr-1986	7.0	Apr-1993	6.9	Apr-2000	3.5
May-1965	4.1	May-1972	5.1	May-1979	5.0	May-1986	7.1	May-1993	6.9	May-2000	3.7
Jun-1965	5.1	Jun-1972	6.0	Jun-1979	5.9	Jun-1986	7.3	Jun-1993	7.2	Jun-2000	4.0
Jul-1965	4.2	Jul-1972	5.7	Jul-1979	5.7	Jul-1986	7.0	Jul-1993	7.1	Jul-2000	4.0
Aug-1965	3.9	Aug-1972	5.2	Aug-1979	5.5	Aug-1986	6.4	Aug-1993	6.5	Aug-2000	3.9
Sep-1965	3.4	Sep-1972	4.6	Sep-1979	5.1	Sep-1986	6.5	Sep-1993	6.2	Sep-2000	3.5
Oct-1965	3.2	Oct-1972	4.5	Oct-1979	5.1	Oct-1986	6.3	Oct-1993	6.1	Oct-2000	3.5
Nov-1965	3.3	Nov-1972	4.2	Nov-1979	5.2	Nov-1986	6.4	Nov-1993	6.0	Nov-2000	3.5
Dec-1965	3.4	Dec-1972	4.2	Dec-1979	5.3	Dec-1986	6.3	Dec-1993	6.2	Dec-2000	3.6
Jan-1966	4.1	Jan-1973	5.1	Jan-1980	6.5	Jan-1987	7.2	Jan-1994	7.5	Jan-2001	4.5
Feb-1966	4.0	Feb-1973	5.2	Feb-1980	6.5	Feb-1987	7.1	Feb-1994	7.4	Feb-2001	4.4
Mar-1966	3.8	Mar-1973	4.9	Mar-1980	6.4	Mar-1987	6.7	Mar-1994	7.0	Mar-2001	4.4

Month	Rate (%)	Month	Rate (%)	Month	Rate (%)	Month	Rate (%)	Month	Rate (%)	Month	Rate (%)
Apr-1966	3.5	Apr-1973	4.4	Apr-1980	6.6	Apr-1987	6.1	Apr-1994	6.3	Apr-2001	4.0
May-1966	3.4	May-1973	4.2	May-1980	7.2	May-1987	6.1	May-1994	5.9	May-2001	4.1
Jun-1966	4.3	Jun-1973	5.1	Jun-1980	8.0	Jun-1987	6.4	Jun-1994	6.3	Jun-2001	4.6
Jul-1966	3.7	Jul-1973	4.7	Jul-1980	8.1	Jul-1987	6.1	Jul-1994	6.4	Jul-2001	4.6
Aug-1966	3.2	Aug-1973	4.3	Aug-1980	7.6	Aug-1987	5.6	Aug-1994	5.8	Aug-2001	4.7
Sep-1966	2.9	Sep-1973	3.9	Sep-1980	6.9	Sep-1987	5.3	Sep-1994	5.5	Sep-2001	4.7
Oct-1966	2.8	Oct-1973	3.6	Oct-1980	6.8	Oct-1987	5.3	Oct-1994	5.3	Oct-2001	4.9
Nov-1966	3.0	Nov-1973	4.0	Nov-1980	7.0	Nov-1987	5.4	Nov-1994	5.2	Nov-2001	5.2
Dec-1966	3.1	Dec-1973	4.1	Dec-1980	7.0	Dec-1987	5.3	Dec-1994	5.0	Dec-2001	5.5
Jan-1967	3.8	Jan-1974	5.2	Jan-1981	8.0	Jan-1988	6.1	Jan-1995	6.1	Jan-2002	6.5
Feb-1967	3.6	Feb-1974	5.3	Feb-1981	8.0	Feb-1988	6.1	Feb-1995	5.8	Feb-2002	6.3
Mar-1967	3.5	Mar-1974	5.0	Mar-1981	7.6	Mar-1988	5.8	Mar-1995	5.7	Mar-2002	6.2
Apr-1967	3.2	Apr-1974	4.6	Apr-1981	7.0	Apr-1988	5.2	Apr-1995	5.5	Apr-2002	5.9
May-1967	3.0	May-1974	4.5	May-1981	7.2	May-1988	5.4	May-1995	5.4	May-2002	5.7
Jun-1967	4.3	Jun-1974	5.6	Jun-1981	7.8	Jun-1988	5.4	Jun-1995	5.7	Jun-2002	6.2
Jul-1967	3.7	Jul-1974	5.4	Jul-1981	7.4	Jul-1988	5.4	Jul-1995	5.8	Jul-2002	6.0
Aug-1967	3.4	Aug-1974	4.9	Aug-1981	7.0	Aug-1988	5.2	Aug-1995	5.5	Aug-2002	5.6
Sep-1967	3.1	Sep-1974	4.9	Sep-1981	6.9	Sep-1988	4.8	Sep-1995	5.2	Sep-2002	5.3
Oct-1967	3.1	Oct-1974	5.0	Oct-1981	7.3	Oct-1988	4.7	Oct-1995	5.0	Oct-2002	5.4
Nov-1967	3.0	Nov-1974	5.7	Nov-1981	7.8	Nov-1988	4.9	Nov-1995	5.1	Nov-2002	5.7
Dec-1967	3.0	Dec-1974	6.3	Dec-1981	8.5	Dec-1988	5.0	Dec-1995	5.2	Dec-2002	5.9
Jan-1968	3.7	Jan-1975	8.7	Jan-1982	9.5	Jan-1989	5.7	Jan-1996	6.2	Jan-2003	6.6
Feb-1968	3.8	Feb-1975	9.0	Feb-1982	9.6	Feb-1989	5.5	Feb-1996	5.9	Feb-2003	6.5
Mar-1968	3.4	Mar-1975	9.1	Mar-1982	9.7	Mar-1989	5.2	Mar-1996	5.8	Mar-2003	6.3
Apr-1968	2.9	Apr-1975	8.7	Apr-1982	9.4	Apr-1989	5.0	Apr-1996	5.3	Apr-2003	6.0
May-1968	2.7	May-1975	8.6	May-1982	9.5	May-1989	5.0	May-1996	5.3	May-2003	6.0
Jun-1968	4.2	Jun-1975	9.3	Jun-1982	10.3	Jun-1989	5.3	Jun-1996	5.4	Jun-2003	6.6
Jul-1968	3.7	Jul-1975	8.7	Jul-1982	10.1	Jul-1989	5.3	Jul-1996	5.6	Jul-2003	6.4
Aug-1968	3.1	Aug-1975	7.9	Aug-1982	9.8	Aug-1989	4.9	Aug-1996	4.9	Aug-2003	6.1
Sep-1968	2.7	Sep-1975	7.6	Sep-1982	9.7	Sep-1989	4.7	Sep-1996	4.8	Sep-2003	5.7
Oct-1968	2.7	Oct-1975	7.4	Oct-1982	10.1	Oct-1989	4.6	Oct-1996	4.7	Oct-2003	5.6
Nov-1968	2.6	Nov-1975	7.5	Nov-1982	10.6	Nov-1989	4.9	Nov-1996	4.9	Nov-2003	5.7
Dec-1968	2.5	Dec-1975	7.5	Dec-1982	11.0	Dec-1989	4.9	Dec-1996	4.9	Dec-2003	5.6
Jan-1969	3.3	Jan-1976	8.6	Jan-1983	11.9	Jan-1990	5.8	Jan-1997	5.8	Jan-2004	6.3
Feb-1969	3.3	Feb-1976	8.4	Feb-1983	11.9	Feb-1990	5.6	Feb-1997	5.5	Feb-2004	6.1
Mar-1969	3.1	Mar-1976	8.0	Mar-1983	11.4	Mar-1990	5.4	Mar-1997	5.4	Mar-2004	6.2
Apr-1969	2.9	Apr-1976	7.2	Apr-1983	10.6	Apr-1990	5.2	Apr-1997	4.7	Apr-2004	5.4
May-1969	2.7	May-1976	6.8	May-1983	10.3	May-1990	5.1	May-1997	4.7	May-2004	5.5
Jun-1969	4.0	Jun-1976	8.1	Jun-1983	10.6	Jun-1990	5.3	Jun-1997	5.1	Jun-2004	5.8
Jul-1969	3.6	Jul-1976	7.6	Jul-1983	9.8	Jul-1990	5.4	Jul-1997	5.0	Jul-2004	5.8
Aug-1969	3.1	Aug-1976	7.3	Aug-1983	9.4	Aug-1990	5.2	Aug-1997	4.6	Aug-2004	5.3
Sep-1969	3.0	Sep-1976	6.9	Sep-1983	8.8	Sep-1990	5.3	Sep-1997	4.5	Sep-2004	5.1
Oct-1969	2.8	Oct-1976	6.7	Oct-1983	8.4	Oct-1990	5.2	Oct-1997	4.2	Oct-2004	5.0
Nov-1969	2.7	Nov-1976	7.0	Nov-1983	8.2	Nov-1990	5.7	Nov-1997	4.1	Nov-2004	5.1
Dec-1969	2.8	Dec-1976	7.2	Dec-1983	8.2	Dec-1990	6.0	Dec-1997	4.3	Dec-2004	5.2

The collection of Figures and Tables that follow are individually identified below.

List of Figures:

List of Tables:

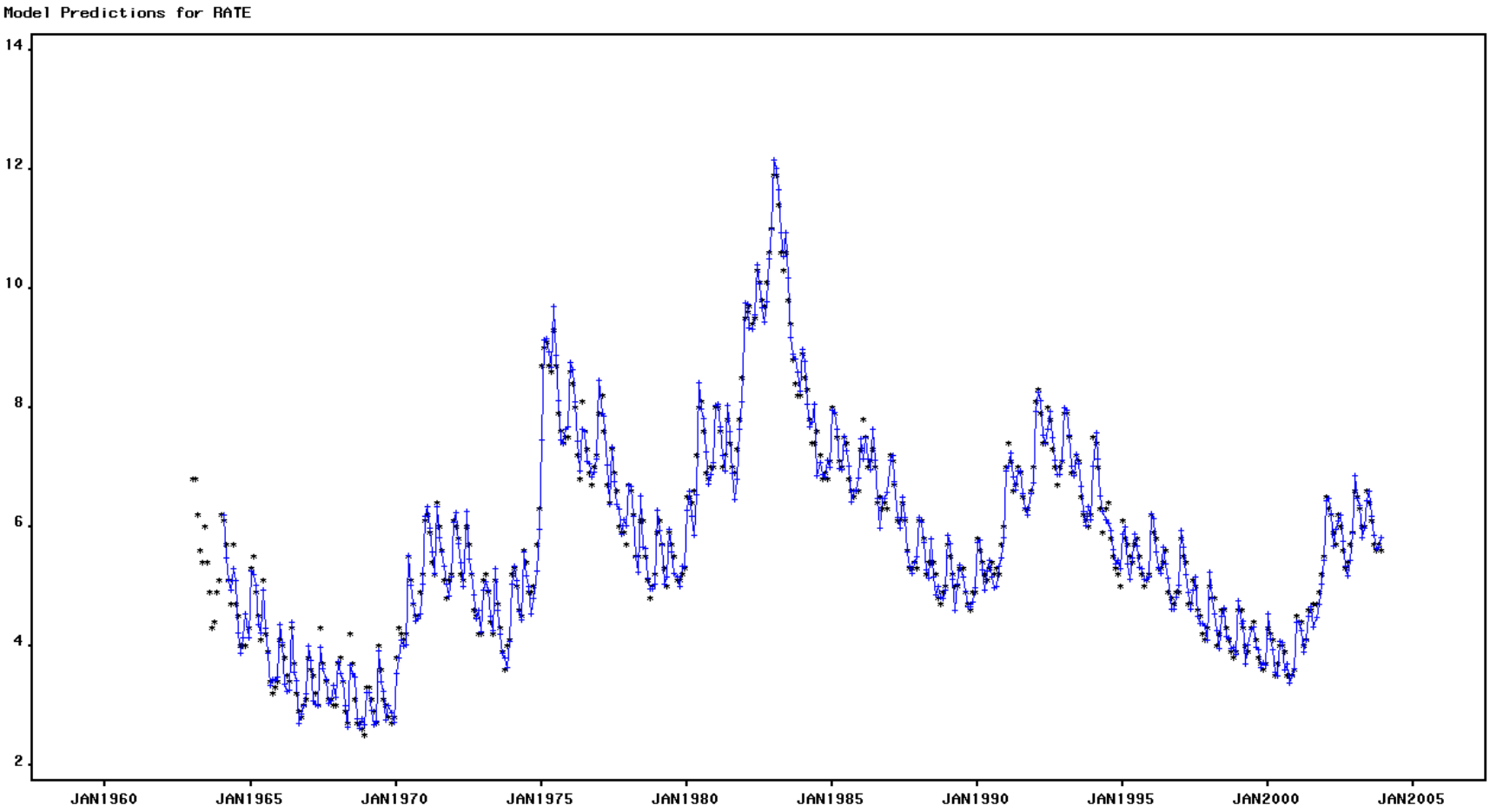

Figure 1 depicts a time series graph for predicted values ((+) signs) using the training data ((*) signs) set for Rate variable, as reported by SAS(AFS).

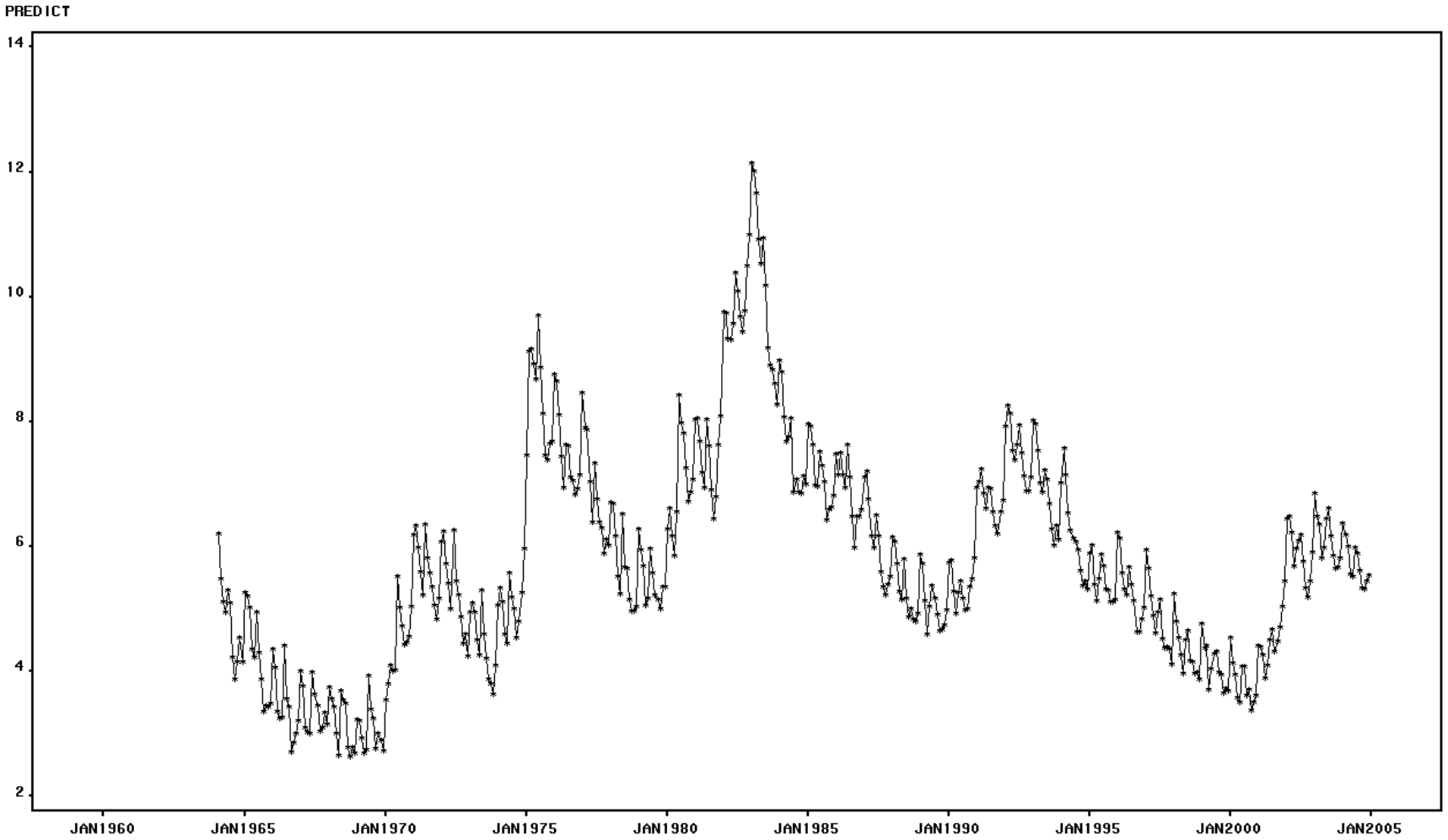

Figure 2 shows a time series graph for predicted values ((*) signs) for Rate variable as reported by SAS(AFS), along with 12-month forecasted values for the Rate variable. Here, predicted and forecast values are plotted using ((*) signs).

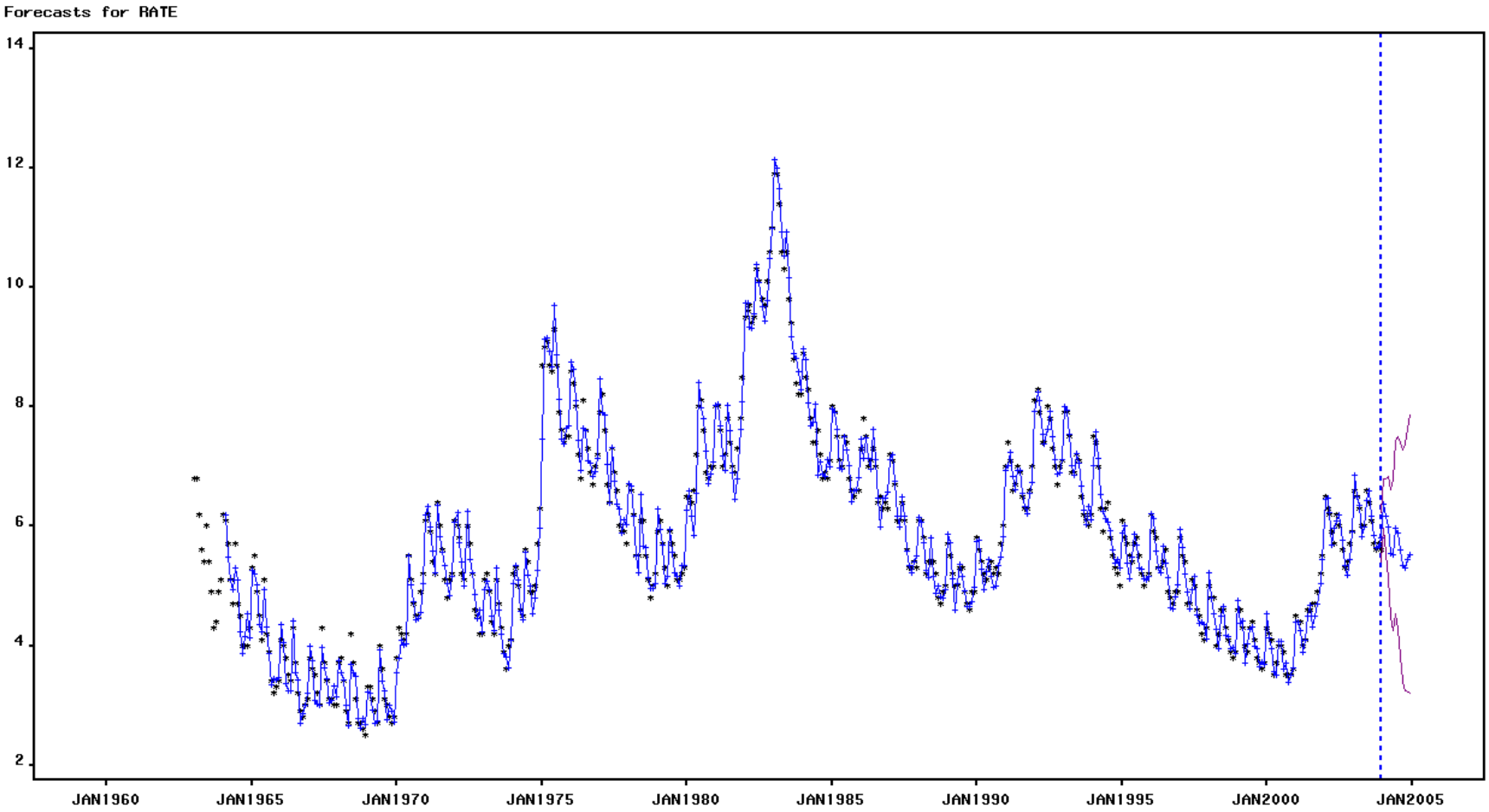

Figure 3a depicts a time series graph for predicted values ((+) signs) and the training data ((*) signs) for the Rate variable, as reported by SAS(AFS), along with 12-month forecasted values and prediction limits.

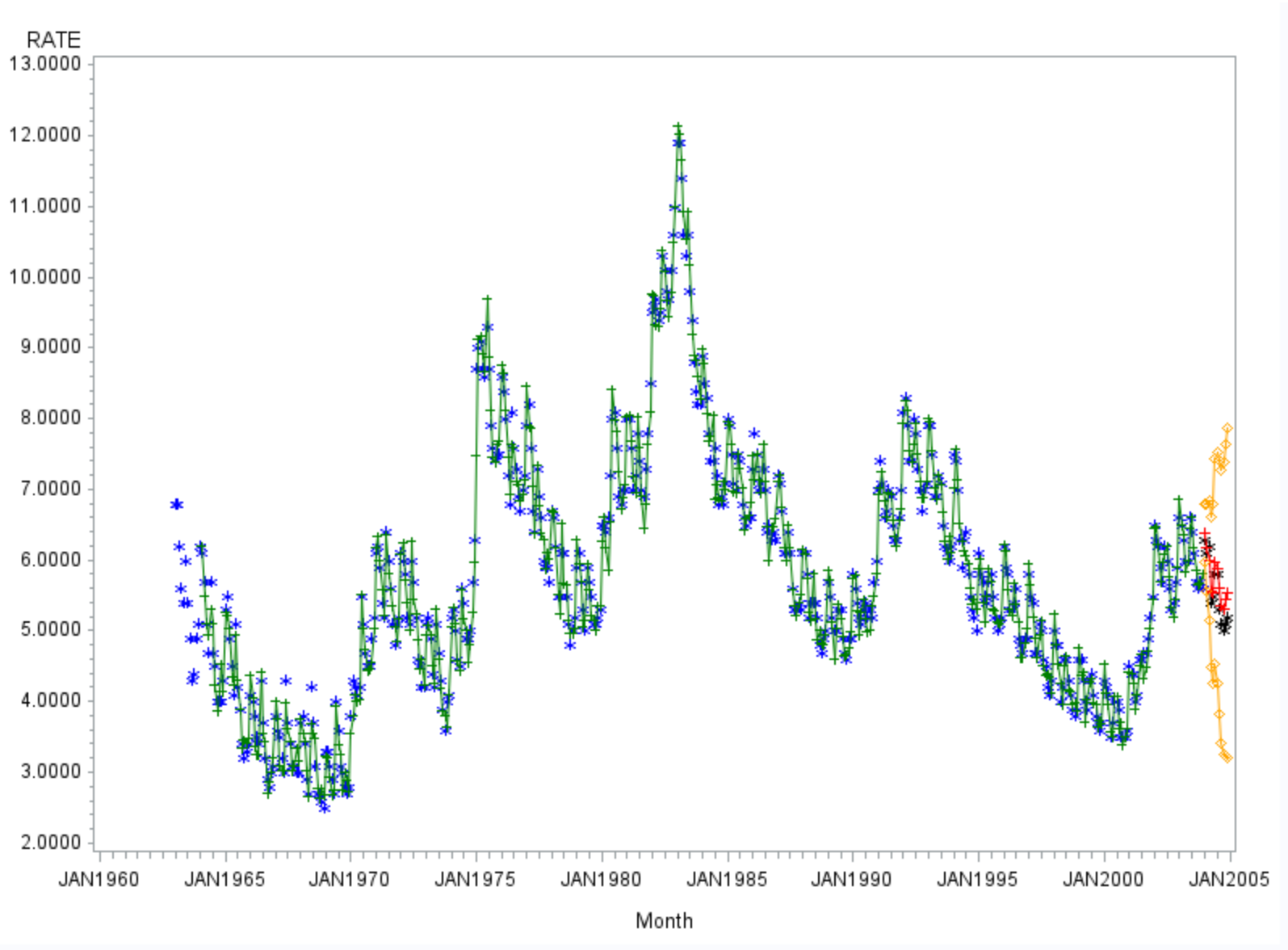

Figure 3b shows a time series graph for predicted values (green-colored (+) signs) for the Rate variable as reported by SAS(AFS), along with 12-month forecasted values, which are plotted using red-colored plus (+) signs. The training data are plotted using blue-colored star (*) signs, and testing data are plotted using black-colored star (*) signs, while the 12-month confidence limits are yellow-colored.

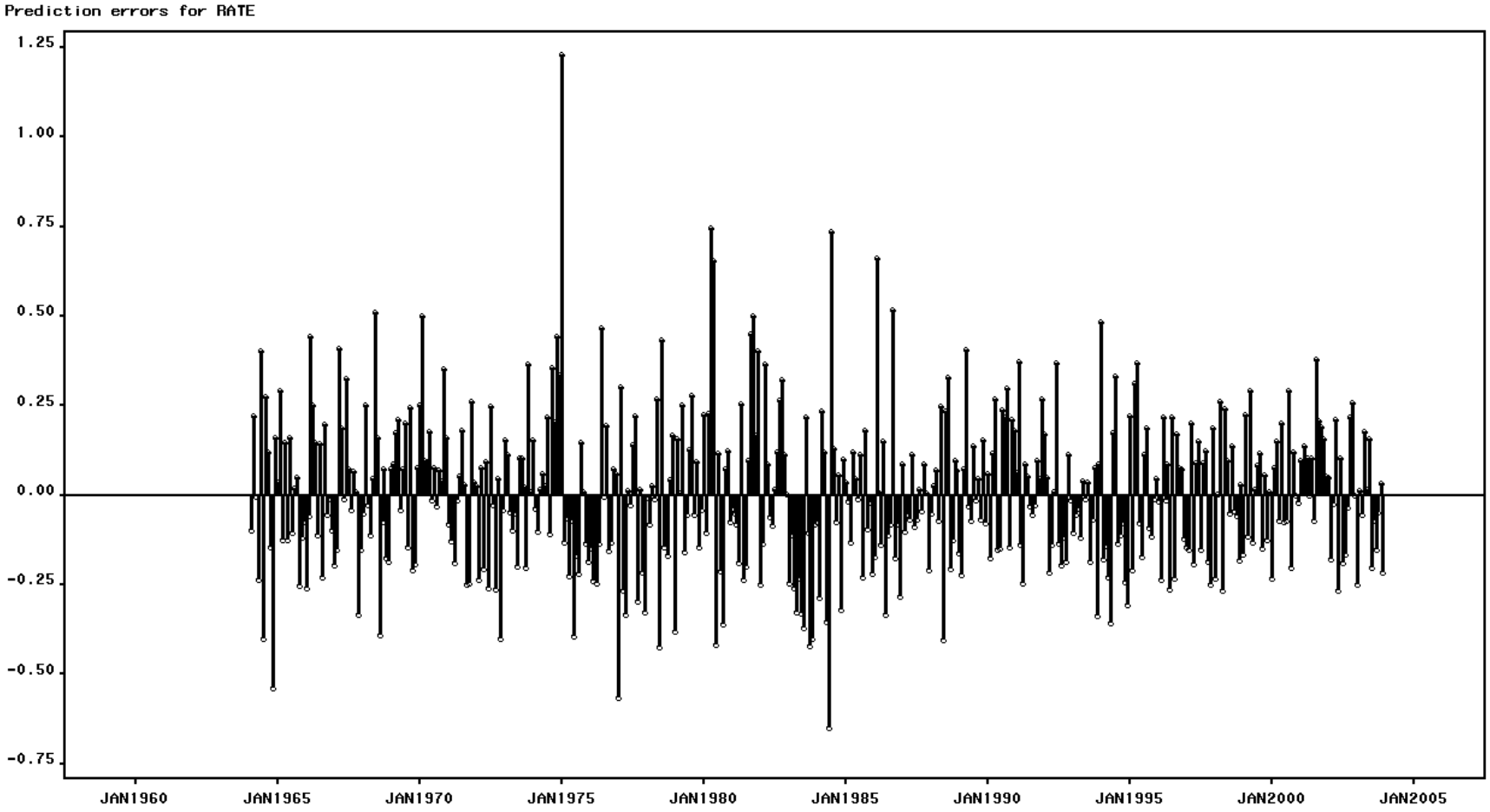

Figure 4. Prediction errors by time series values for the training data set.

Accuracy Measure for Table B.8 data.

Actual y_t	Predict $\widehat{y_t}$(t-1)	Forecast error $e_t(1) = y_t - \widehat{y_t}$(t-1)	Absolute error $\lvert e_t(1)\rvert$	Square error $[e_t(1)]^2$	Relative error $re_t(1) = 100(e_t(1)/y_t)$	Absolute percentage error $\lvert re_t(1)\rvert$
6.3	6.3741	-0.0741	0.0741	0.005491	-1.17619	1.17619
6.1	6.1758	-0.0758	0.0758	0.005746	-1.24262	1.24262
6.2	5.9911	0.2089	0.2089	0.043639	3.36935	3.36935
5.4	5.5516	-0.1516	0.1516	0.022983	-2.80741	2.80741
5.5	5.5264	-0.0264	0.0264	0.000697	-0.48000	0.48000
5.8	5.9767	-0.1767	0.1767	0.031223	-3.04655	3.04655
5.8	5.8887	-0.0887	0.0887	0.007868	-1.52931	1.52931
5.3	5.6142	-0.3142	0.3142	0.098722	-5.92830	5.92830
5.1	5.3422	-0.2422	0.2422	0.058661	-4.74902	4.74902
5.0	5.3134	-0.3134	0.3134	0.098220	-6.26800	6.26800
5.1	5.4370	-0.3370	0.3370	0.113569	-6.60784	6.60784
5.2	5.5318	-0.3318	0.3318	0.110091	-6.38077	6.38077

Table A. Error Measures table B8 time series data.

Data set	**MSE**	$\widehat{\sigma_{e(1)}}$	**MAD**	**MPE**	**MAPE**
Full data (504 data)	0.04465	0.21130	0.16267	0.08215	2.97999
Training data (492 data)	0.04509	0.21234	0.16388	0.09151	3.00348
Holdout data (12 data)	0.0497423	0.223030	0.195067	-3.07056	3.63211

Table B. Accuracy Measures table B8 time series data.

Table B provides SAS (AFS)® forecast errors for the full data set, the training data set, and the holdout (or test) data set. The ARIMA (2,1,2)(0,1,1)s NOINT model was suggested as the "best model" for this data set. SAS (AFS)® and MINITAB® produced the accuracy measures shown in Table B, which comprise the Mean Square Error (MSE), the Standard Deviation of the Forecast Errors (SDFE), the Mean Absolute Deviation (MAD), the Mean Percent Forecast Error (MPE), and Mean Absolute Percent Error (MAPE).

Here, the accuracy statistics are computed as follows:

Mean square error:

$$\textbf{MSE} = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = 0.0497423$$

Standard Deviation of Forecast Error (Root Mean Square Error):

$$\widehat{\sigma_{e(1)}} = \sqrt{MSE} = \sqrt{0.0497423} = 0.223030$$

Mean Absolute Deviation (Mean Absolute Error):

$$\textbf{MAD} = \frac{1}{n}\sum_{t=1}^{n} |e_t(1)| = 0.195067$$

Mean Percent Forecast Error:

$$\textbf{MPE} = \frac{1}{n}\sum_{t=1}^{n} re_t(1) = -3.07056$$

Mean Absolute Percent Error:

$$\textbf{MAPE} = \frac{1}{n}\sum_{t=1}^{n} |re_t(1)| = 3.63211$$

Worksheet 1 ***

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	Actual	Predict	Forecast_Error	Abs_Error	Sq_Error	Relative_Error	Abs_Percentage_Error	MSE	Root_MSE	MAD	MPE	MAPE
1	6.3	6.3741	-0.0741	0.0741	0.005491	-1.17619	1.17619	0.0497423	0.223030	0.195067	-3.07056	3.63211
2	6.1	6.1758	-0.0758	0.0758	0.005746	-1.24262	1.24262					
3	6.2	5.9911	0.2089	0.2089	0.043639	3.36935	3.36935					
4	5.4	5.5516	-0.1516	0.1516	0.022983	-2.80741	2.80741					
5	5.5	5.5264	-0.0264	0.0264	0.000697	-0.48000	0.48000					
6	5.8	5.9767	-0.1767	0.1767	0.031223	-3.04655	3.04655					
7	5.8	5.8887	-0.0887	0.0887	0.007868	-1.52931	1.52931					
8	5.3	5.6142	-0.3142	0.3142	0.098722	-5.92830	5.92830					
9	5.1	5.3422	-0.2422	0.2422	0.058661	-4.74902	4.74902					
10	5.0	5.3134	-0.3134	0.3134	0.098220	-6.26800	6.26800					
11	5.1	5.4370	-0.3370	0.3370	0.113569	-6.60784	6.60784					
12	5.2	5.5318	-0.3318	0.3318	0.110091	-6.38077	6.38077					

Table C. MINITAB® 17 Calculation of Forecast Errors for Holdout Data.

The first two columns of Table C consist of the actual values of the holdout data set and the SAS (AFS)® forecasted values, based on training data. The remaining columns give forecast errors, the absolute forecast errors, the squared forecast errors, the relative forecast errors and absolute forecast percentage errors using the MINITAB® calculator function.

Obs	DATE	ACTUAL	PREDICT	UPPER	LOWER	ERROR	STD	NERROR
493	JAN2004	.	6.3741	6.7881	5.9601	.	0.2112	.
494	FEB2004	.	6.1758	6.7964	5.5552	.	0.3166	.
495	MAR2004	.	5.9911	6.8333	5.1489	.	0.4297	.
496	APR2004	.	5.5516	6.6106	4.4925	.	0.5403	.
497	MAY2004	.	5.5264	6.7885	4.2643	.	0.6439	.
498	JUN2004	.	5.9767	7.4264	4.5269	.	0.7397	.
499	JUL2004	.	5.8887	7.5116	4.2659	.	0.8280	.
500	AUG2004	.	5.6142	7.3974	3.8310	.	0.9098	.
501	SEP2004	.	5.3422	7.2746	3.4097	.	0.9859	.
502	OCT2004	.	5.3134	7.3856	3.2413	.	1.0573	.
503	NOV2004	.	5.4370	7.6408	3.2332	.	1.1244	.
504	DEC2004	.	5.5318	7.8602	3.2034	.	1.1880	.

Table D. Forecasted values for the last 12 month of Holdout Data.

Table D contains forecasted values for the final twelve month for the Rate variable, and the corresponding lower and upper 95% prediction limits. The Root Mean Square Prediction Error, Level and Trend smoothing components are also shown..

Model Parameter	Estimate	Std. Error	T	Prob>|T|
Moving Average, Lag 1	0.59193	0.2698	2.1938	0.0287
Moving Average, Lag 2	-0.24163	0.1929	-1.2527	0.2109
Seasonal Moving Average, Lag 12	0.73998	0.0329	22.4847	<.0001
Autoregressive, Lag 1	0.70859	0.2755	2.5720	0.0104
Autoregressive, Lag 2	-0.06568	0.2390	-0.2748	0.7836
Model Variance (sigma squared)	0.04462	.	.	.

Table E. Parameter Estimates for Training Data using the ARIMA (2,1,2)(0,1,1)s NOINT model suggested by SAS (AFS)®.

Results

In Figure 3b, it seems the model fits the data appropriately, in addition, the SAS(AFS) forecast values with acceptably small prediction error. The MSE, RMSE and MAD are similar for both the training and holdout data sets; that is, the variability in forecast errors are approximately similar.

TABLE B.9 (INTERNATIONAL SUNSPOT NUMBERS) ANALYSIS

Table B.9 comprises data on the International Sunspot numbers from 1700 through 2004, where the sunspot number variable is collected yearly. The data set excludes last ten year values as a holdout set in order to analyze the accuracy of SAS(AFS), which suggested a Simple Exponential Smoothing model as the "best model" for this training, or initialization, data set. SAS(AFS) also produced the predicted values for the next 10 year. I have calculated different types of forecast errors based on the hold out data and predicted data.

The model equation is

$$Y_t = \mu_t + \varepsilon_t$$

where,

Y_t = observed value i.e. sunspot number at time t

μ_t = smoothed level and $\mu_t = 30.02474$

Now, the k-step prediction equation is

$$\hat{Y}_t(k) = L_t$$

and the smoothing equations are

$$L_t = \alpha Y_t + (1-\alpha) L_{t-1}$$

where,

L_t = Level estimate and $L_0 = \mu_t = 30.02474$

α = level smoothing and $\alpha = 0.999$

Table B.9: International Sunspot Numbers

Year	Sunspot Number	Year	Sunspot Number	Year	Sunspot Number	Year	Sunspot Number	Year	Sunspot Number
1700	5.1	1761	86	1822	4.1	1883	63.8	1944	9.7
1701	11.1	1762	61.3	1823	1.9	1884	63.6	1945	33.3
1702	16.1	1763	45.2	1824	8.6	1885	52.3	1946	92.7
1703	23.1	1764	36.5	1825	16.7	1886	25.5	1947	151.7
1704	36.1	1765	21	1826	36.4	1887	13.2	1948	136.4
1705	58.1	1766	11.5	1827	49.7	1888	6.9	1949	134.8
1706	29.1	1767	37.9	1828	64.3	1889	6.4	1950	84
1707	20.1	1768	69.9	1829	67.1	1890	7.2	1951	69.5
1708	10.1	1769	106.2	1830	71	1891	35.7	1952	31.6
1709	8.1	1770	100.9	1831	47.9	1892	73.1	1953	14
1710	3.1	1771	81.7	1832	27.6	1893	85.2	1954	4.5
1711	0.1	1772	66.6	1833	8.6	1894	78.1	1955	38.1
1712	0.1	1773	34.9	1834	13.3	1895	64.1	1956	141.8
1713	2.1	1774	30.7	1835	57	1896	41.9	1957	190.3
1714	11.1	1775	7.1	1836	121.6	1897	26.3	1958	184.9
1715	27.1	1776	19.9	1837	138.4	1898	26.8	1959	159.1
1716	47.1	1777	92.6	1838	103.3	1899	12.2	1960	112.4
1717	63.1	1778	154.5	1839	85.8	1900	9.6	1961	54
1718	60.1	1779	126	1840	64.7	1901	2.8	1962	37.7
1719	39.1	1780	84.9	1841	36.8	1902	5.1	1963	28
1720	28.1	1781	68.2	1842	24.3	1903	24.5	1964	10.3

Year	Sunspot Number	Year	Sunspot Number	Year	Sunspot Number	Year	Sunspot Number	Year	Sunspot Number
1721	26.1	1782	38.6	1843	10.8	1904	42.1	1965	15.2
1722	22.1	1783	22.9	1844	15.1	1905	63.6	1966	47.1
1723	11.1	1784	10.3	1845	40.2	1906	53.9	1967	93.8
1724	21.1	1785	24.2	1846	61.6	1907	62.1	1968	106
1725	40.1	1786	83	1847	98.6	1908	48.6	1969	105.6
1726	78.1	1787	132.1	1848	124.8	1909	44	1970	104.6
1727	122.1	1788	131	1849	96.4	1910	18.7	1971	66.7
1728	103.1	1789	118.2	1850	66.7	1911	5.8	1972	69
1729	73.1	1790	90	1851	64.6	1912	3.7	1973	38.1
1730	47.1	1791	66.7	1852	54.2	1913	1.5	1974	34.6
1731	35.1	1792	60.1	1853	39.1	1914	9.7	1975	15.6
1732	11.1	1793	47	1854	20.7	1915	47.5	1976	12.7
1733	5.1	1794	41.1	1855	6.8	1916	57.2	1977	27.6
1734	16.1	1795	21.4	1856	4.4	1917	104	1978	92.6
1735	34.1	1796	16.1	1857	22.8	1918	80.7	1979	155.5
1736	70.1	1797	6.5	1858	54.9	1919	63.7	1980	154.7
1737	81.1	1798	4.2	1859	93.9	1920	37.7	1981	140.6
1738	111.1	1799	6.9	1860	95.9	1921	26.2	1982	116
1739	101.1	1800	14.6	1861	77.3	1922	14.3	1983	66.7
1740	73.1	1801	34.1	1862	59.2	1923	5.9	1984	46
1741	40.1	1802	45.1	1863	44.1	1924	16.8	1985	18
1742	20.1	1803	43.2	1864	47.1	1925	44.4	1986	13.5
1743	16.1	1804	47.6	1865	30.6	1926	64	1987	29.3
1744	5.1	1805	42.3	1866	16.4	1927	69.1	1988	100.3
1745	11.1	1806	28.2	1867	7.4	1928	77.9	1989	157.7
1746	22.1	1807	10.2	1868	37.7	1929	65	1990	142.7
1747	40.1	1808	8.2	1869	74.1	1930	35.8	1991	145.8
1748	60.1	1809	2.6	1870	139.1	1931	21.3	1992	94.4
1749	81	1810	0.1	1871	111.3	1932	11.2	1993	54.7
1750	83.5	1811	1.5	1872	101.7	1933	5.8	1994	30
1751	47.8	1812	5.1	1873	66.3	1934	8.8	1995	17.6
1752	47.9	1813	12.3	1874	44.8	1935	36.2	1996	8.7
1753	30.8	1814	14	1875	17.1	1936	79.8	1997	21.6
1754	12.3	1815	35.5	1876	11.4	1937	114.5	1998	64.4
1755	9.7	1816	45.9	1877	12.5	1938	109.7	1999	93.4
1756	10.3	1817	41.1	1878	3.5	1939	88.9	2000	119.7
1757	32.5	1818	30.2	1879	6.1	1940	67.9	2001	111.1
1758	47.7	1819	24	1880	32.4	1941	47.6	2002	104.1
1759	54.1	1820	15.7	1881	54.4	1942	30.7	2003	63.8
1760	63	1821	6.7	1882	59.8	1943	16.4	2004	40.5
1713	2.1	1774	30.7	1835	57	1896	41.9	1957	190.3
1714	11.1	1775	7.1	1836	121.6	1897	26.3	1958	184.9
1715	27.1	1776	19.9	1837	138.4	1898	26.8	1959	159.1
1716	47.1	1777	92.6	1838	103.3	1899	12.2	1960	112.4
1717	63.1	1778	154.5	1839	85.8	1900	9.6	1961	54
1718	60.1	1779	126	1840	64.7	1901	2.8	1962	37.7
1719	39.1	1780	84.9	1841	36.8	1902	5.1	1963	28
1720	28.1	1781	68.2	1842	24.3	1903	24.5	1964	10.3
1721	26.1	1782	38.6	1843	10.8	1904	42.1	1965	15.2
1722	22.1	1783	22.9	1844	15.1	1905	63.6	1966	47.1

The collection of Figures and Tables that follow are individually identified below.

List of Figures:

List of Tables:

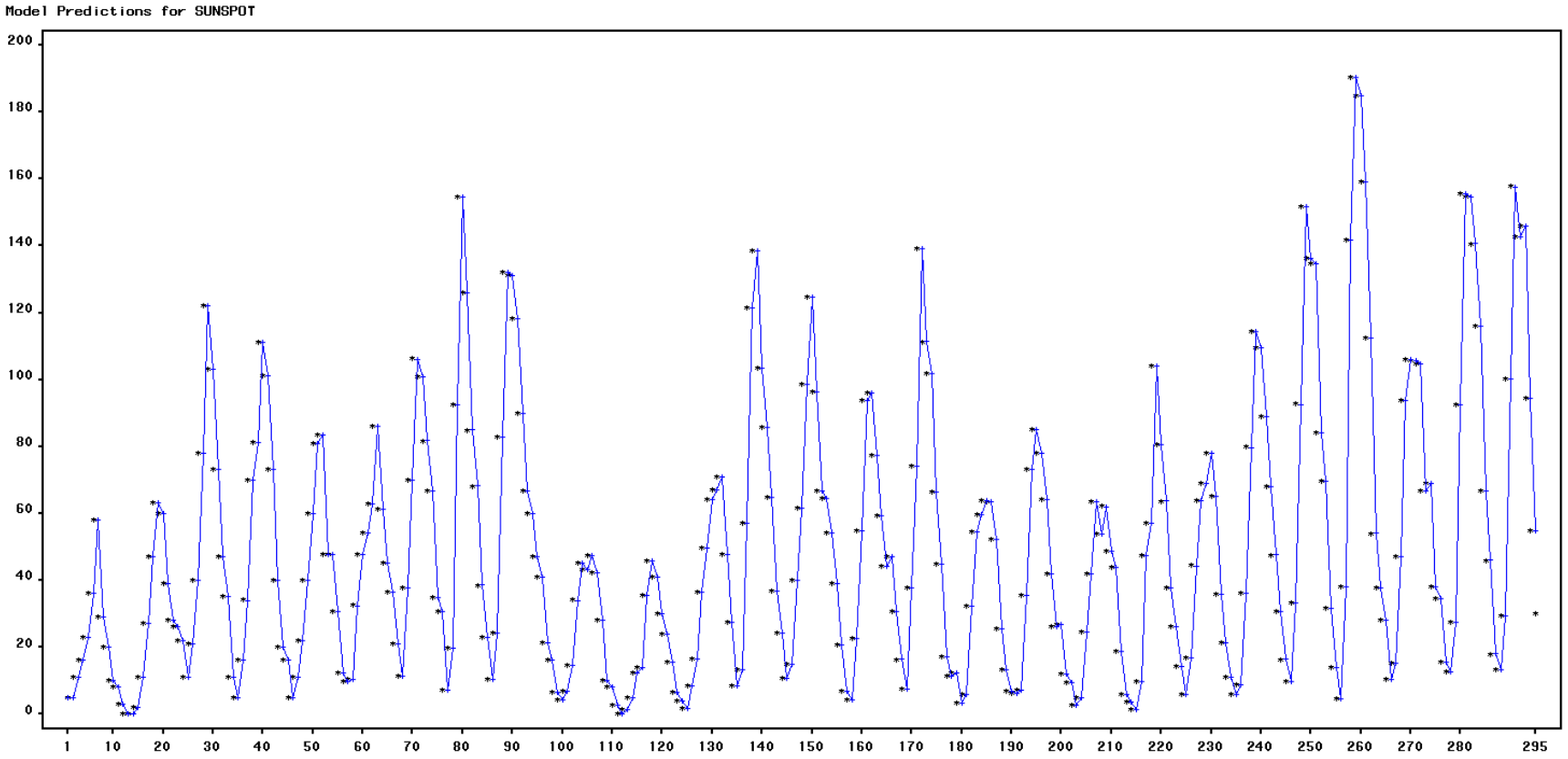

Figure 1 depicts a time series graph for predicted values ((+) signs) using the training data ((*) signs) set for Sunspot Number variable, as reported by SAS(AFS).

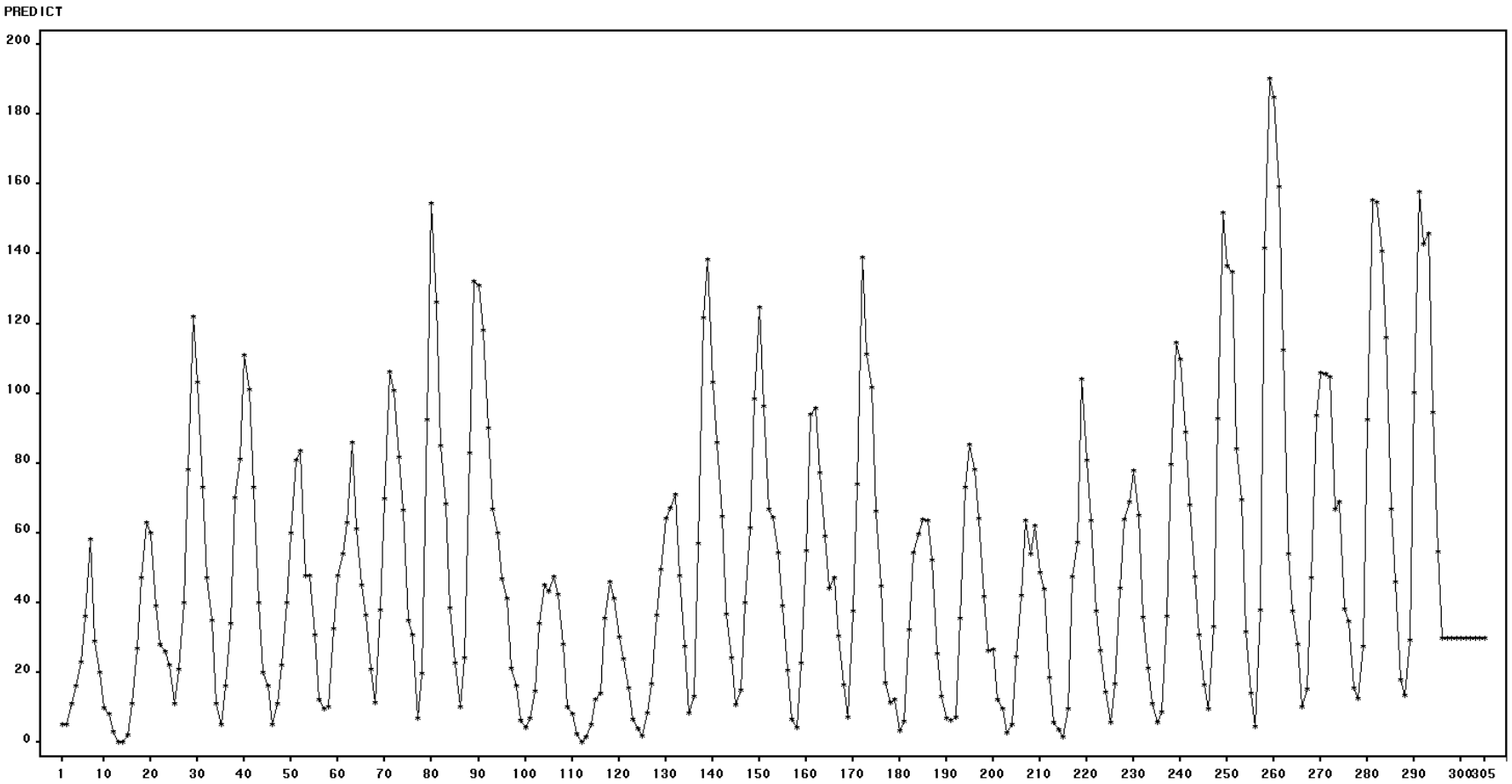

Figure 2 shows a time series graph for predicted values ((*) signs) for Sunspot Number variable as reported by SAS(AFS), along with 10-year forecasted values for the sunspot number variable. Here, predicted and forecast values are plotted using ((*) signs).

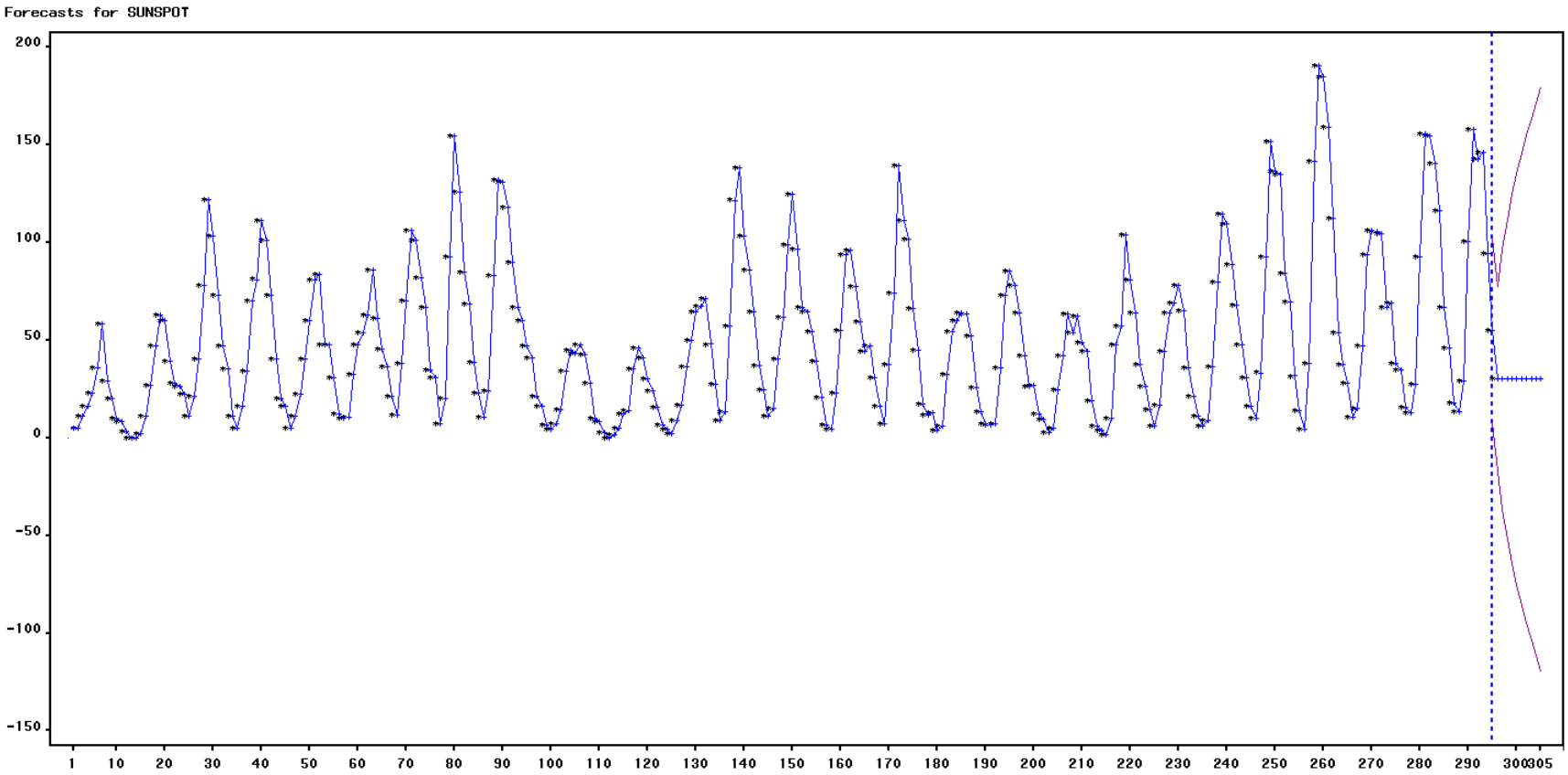

Figure 3a depicts a time series graph for predicted values ((+) signs) and the training data ((*) signs) for the Sunspot Number variable, as reported by SAS(AFS), along with 10-year forecasted values and prediction limits.

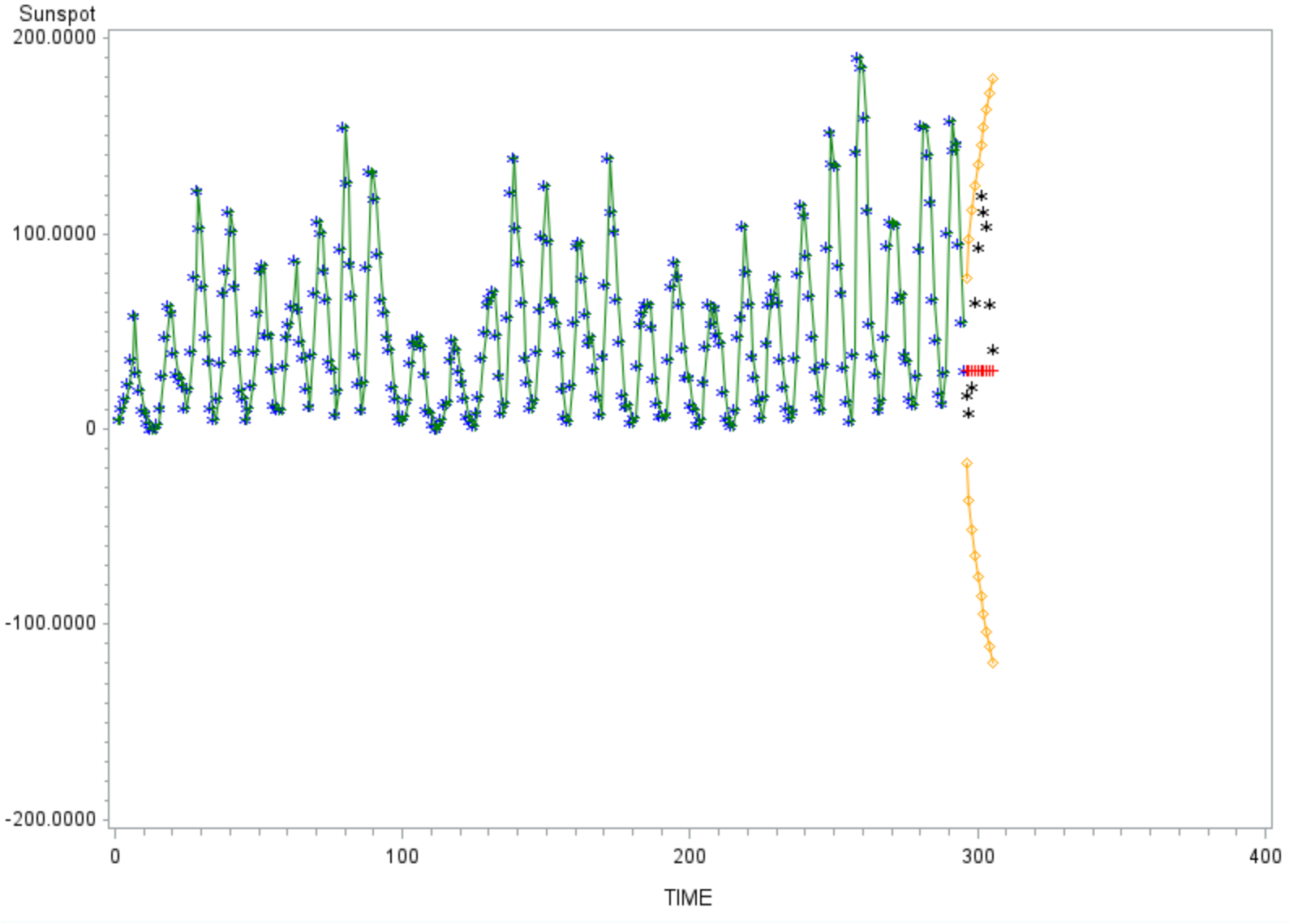

Figure 3b shows a time series graph for predicted values (green-colored (+) signs) for the Sunspot number variable as reported by SAS(AFS), along with 10-time point forecasted values, which are plotted using red-colored plus (+) signs. The training data are plotted using blue-colored star (*) signs, and testing data are plotted using black-colored star (*) signs, while the 10-time point confidence limits are yellow-colored.

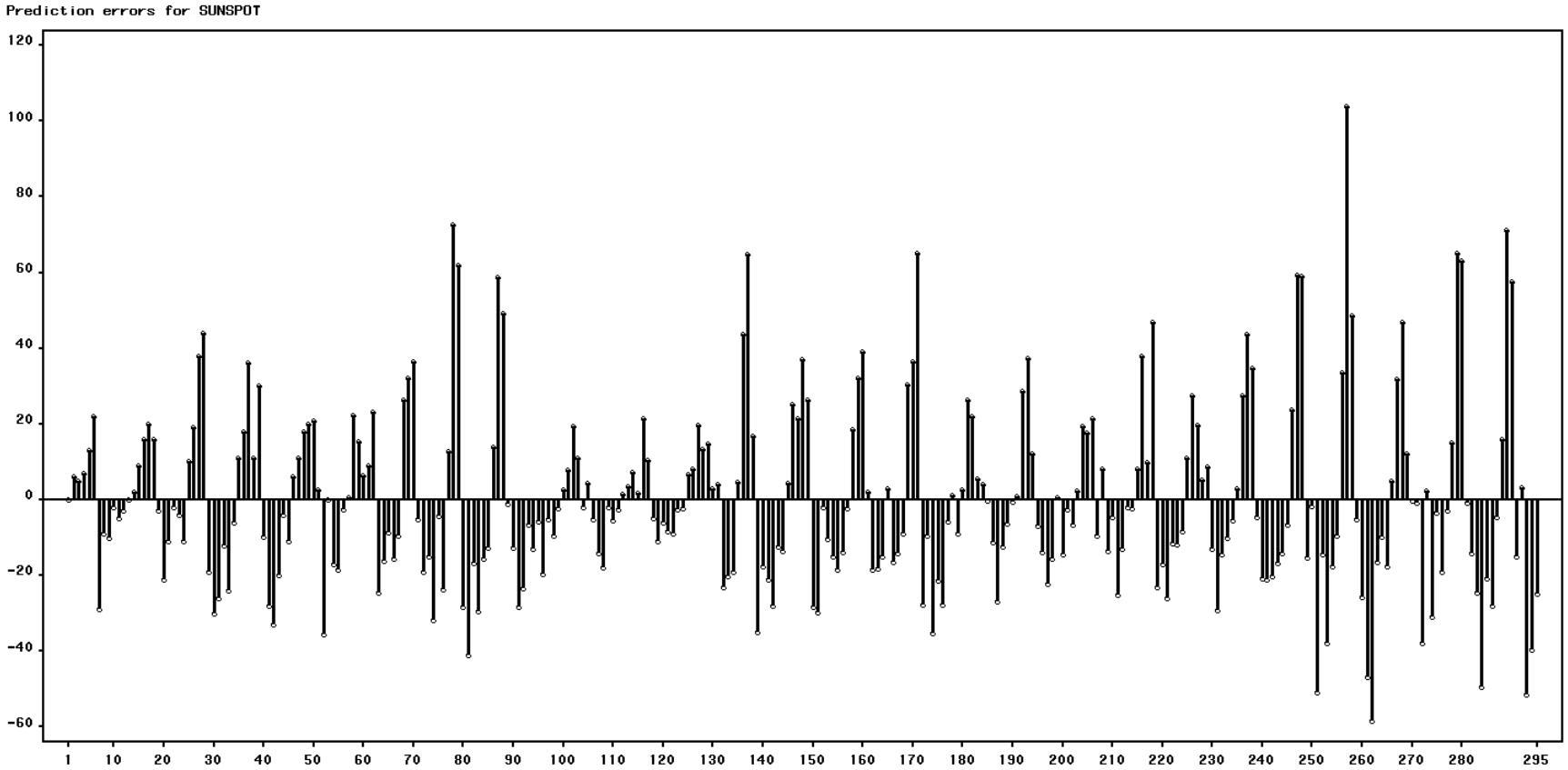

Figure 4. Prediction errors by time series values for the training data set.

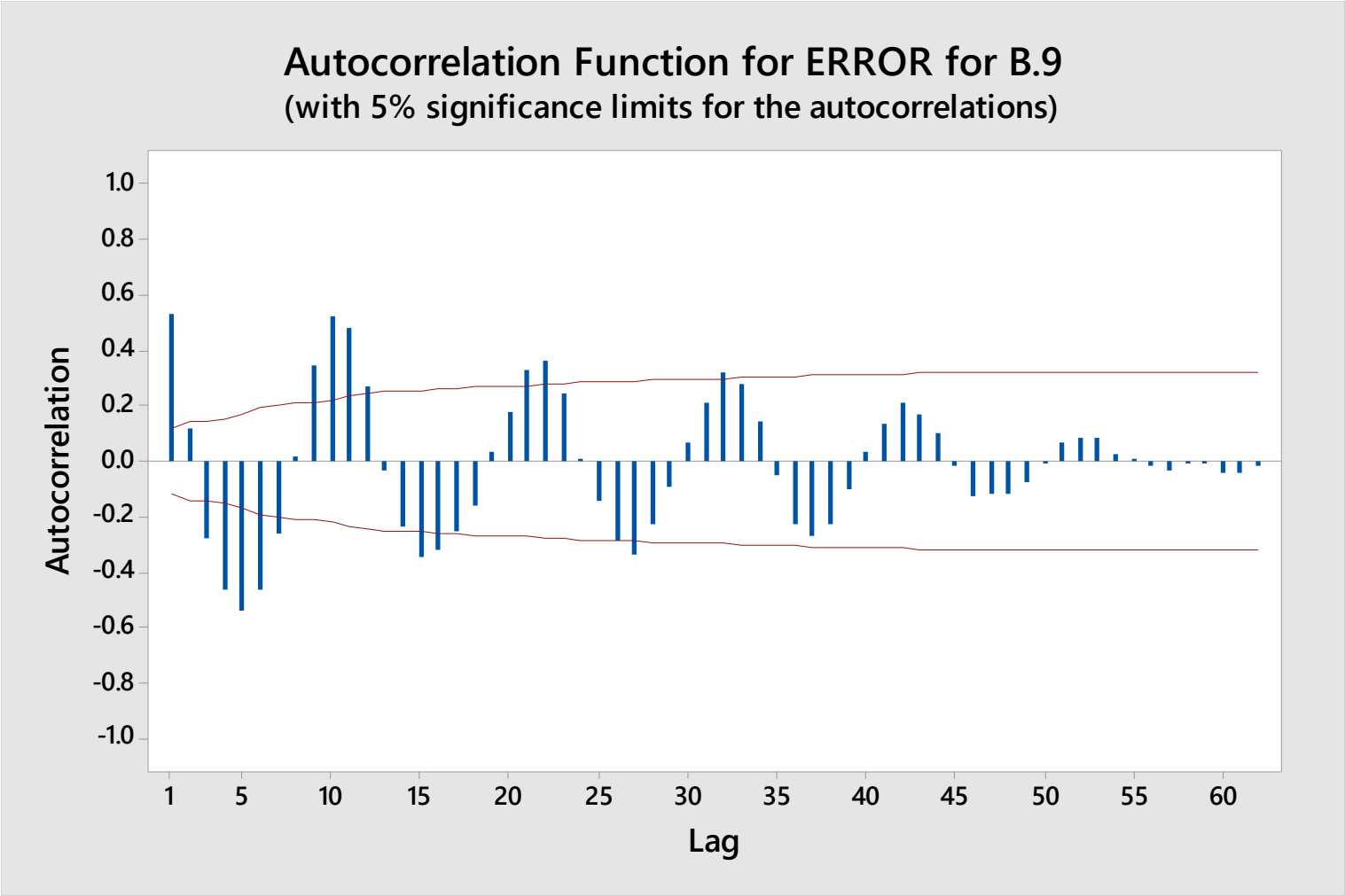

Figure 5. Autocorrelation Function (ACF) for the training data set.

Accuracy Measure for Table B.9 data.

Actual y_t	Predict $\widehat{y_t}$(t-1)	Forecast error $e_t(1) = y_t - \widehat{y_t}$(t-1)	Absolute error $\lvert e_t(1)\rvert$	Square error $[e_t(1)]^2$	Relative error $re_t(1) = 100(e_t(1)/y_t)$	Absolute percentage error $\lvert re_t(1)\rvert$
17.6	30.0247	-12.4247	12.4247	154.37	-70.595	70.595
8.7	30.0247	-21.3247	21.3247	454.74	-245.111	245.111
21.6	30.0247	-8.4247	8.4247	70.98	-39.003	39.003
64.4	30.0247	34.3753	34.3753	1181.66	53.378	53.378
93.4	30.0247	63.3753	63.3753	4016.43	67.854	67.854
119.7	30.0247	89.6753	89.6753	8041.66	74.917	74.917
111.1	30.0247	81.0753	81.0753	6573.20	72.975	72.975
104.1	30.0247	74.0753	74.0753	5487.15	71.158	71.158
63.8	30.0247	33.7753	33.7753	1140.77	52.939	52.939
40.5	30.0247	10.4753	10.4753	109.73	25.865	25.865

Table A. Error Measures table B9 time series data.

Data set	**MSE**	$\widehat{\sigma_{e(1)}}$	**MAD**	**MPE**	**MAPE**
Full data (305 data)	579.82075	24.07947	18.26618	-37.04412	72.51572
Training data (295 data)	578.92761	24.06091	18.16794	-37.86242	73.32118
Holdout data (10 data)	2723.07	52.1830	42.9001	6.43757	77.3795

Table B. Accuracy Measures table B9 time series data.

Table B provides SAS (AFS)® forecast errors for the full data set, the training data set, and the holdout (or test) data set. The Simple Exponential Smoothing model was suggested as the "best model" for this data set. SAS (AFS)® and MINITAB® produced the accuracy measures shown in Table B, which comprise the Mean Square Error (MSE), the Standard Deviation of the Forecast Errors (SDFE), the Mean Absolute Deviation (MAD), the Mean Percent Forecast Error (MPE), and Mean Absolute Percent Error (MAPE).

Here, the accuracy statistics are computed as follows:

Mean square error:

$$\textbf{MSE} = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = 2723.07$$

Standard Deviation of Forecast Error (Root Mean Square Error):

$$\widehat{\sigma_{e(1)}} = \sqrt{MSE} = \sqrt{2723.07} = 52.1830$$

Mean Absolute Deviation (Mean Absolute Error):

$$\textbf{MAD} = \frac{1}{n}\sum_{t=1}^{n} |e_t(1)| = 42.9001$$

Mean Percent Forecast Error:

$$\textbf{MPE} = \frac{1}{n}\sum_{t=1}^{n} re_t(1) = 6.43757$$

Mean Absolute Percent Error:

$$\textbf{MAPE} = \frac{1}{n}\sum_{t=1}^{n} |re_t(1)| = 77.3795$$

Worksheet 1 ***

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	Actual	Predict	Forecast_Error	Abs_Error	Sq_Error	Relative_Error	Abs_Percentage_Error	MSE	Root_MSE	MAD	MPE	MAPE
1	17.6	30.0247	-12.4247	12.4247	154.37	-70.595	70.595	2723.07	52.1830	42.9001	6.43757	77.3795
2	8.7	30.0247	-21.3247	21.3247	454.74	-245.111	245.111					
3	21.6	30.0247	-8.4247	8.4247	70.98	-39.003	39.003					
4	64.4	30.0247	34.3753	34.3753	1181.66	53.378	53.378					
5	93.4	30.0247	63.3753	63.3753	4016.43	67.854	67.854					
6	119.7	30.0247	89.6753	89.6753	8041.66	74.917	74.917					
7	111.1	30.0247	81.0753	81.0753	6573.20	72.975	72.975					
8	104.1	30.0247	74.0753	74.0753	5487.15	71.158	71.158					
9	63.8	30.0247	33.7753	33.7753	1140.77	52.939	52.939					
10	40.5	30.0247	10.4753	10.4753	109.73	25.865	25.865					

Table C. MINITAB® 17 Calculation of Forecast Errors for Holdout Data.

The first two columns of Table C consist of the actual values of the holdout data set and the SAS (AFS)® forecasted values, based on training data. The remaining columns give forecast errors, the absolute forecast errors, the squared forecast errors, the relative forecast errors and absolute forecast percentage errors using the MINITAB® calculator function.

Obs	DATE	ACTUAL	PREDICT	UPPER	LOWER	ERROR	STD	NERROR	_LEVEL_
296	296	.	30.0247	77.2634	-17.2139	.	24.1018	.	30.0247
297	297	.	30.0247	96.7969	-36.7474	.	34.0681	.	30.0247
298	298	.	30.0247	111.7900	-51.7405	.	41.7177	.	30.0247
299	299	.	30.0247	124.4312	-64.3817	.	48.1675	.	30.0247
300	300	.	30.0247	135.5691	-75.5196	.	53.8502	.	30.0247
301	301	.	30.0247	145.6389	-85.5895	.	58.9879	.	30.0247
302	302	.	30.0247	154.8994	-94.8499	.	63.7127	.	30.0247
303	303	.	30.0247	163.5189	-103.4695	.	68.1105	.	30.0247
304	304	.	30.0247	171.6148	-111.5653	.	72.2411	.	30.0247
305	305	.	30.0247	179.2721	-119.2226	.	76.1480	.	30.0247

Table D. Forecasted values for the last 10 year of Holdout Data.

Table D contains forecasted values for the final ten year for the Sunspot Number variable, and the corresponding lower and upper 95% prediction limits. The Root Mean Square Prediction Error, Level and Trend smoothing components are also shown..

Model Parameter	Estimate	Std. Error	T	Prob>|T|
LEVEL Smoothing Weight	0.99900	0.0413	24.1965	<.0001
Residual Variance (sigma squared)	580.89675	.	.	.
Smoothed Level	30.02474	.	.	.

Table E. Parameter Estimates for Training Data using the Simple Exponential Smoothing model suggested by SAS (AFS)®.

Results

Figure 3b clearly shows that the model suggested by SAS(AFS) is of little use in forecasting future values appropriately. Additionally, there are many spikes in the ACF function of residuals, which suggests that forecast errors have structure and the original model must be improved. Moreover, MSE, RMSE, MAD and MAPE are also very large. SAS(AFS) does not do well for this data set.

TABLE B.10 (UNITED KINGDOM AIRLINE MILES FLOWN) ANALYSIS

Table B.10 comprises data on the United Kingdom Airline Miles Flown from January 1964 through December 1970, where the miles (in millions) variable is collected monthly. The data set excludes last twelve month values as a holdout set in order to analyze the accuracy of SAS(AFS), which suggested a Log Seasonal Exponential Smoothing model as the "best model" for this training, or initialization, data set. SAS(AFS) also produced the predicted values for the next 12 month. I have calculated different types of forecast errors based on the hold out data and predicted data.

Table B.10: United Kingdom Airline Miles Flown

Month	Miles, in Millions	Month	Miles, in Millions	Month	Miles, in Millions
Jan-1964	7.269	Jun-1966	12.282	Nov-1968	9.290
Feb-1964	6.775	Jul-1966	11.637	Dec-1968	10.925
Mar-1964	7.819	Aug-1966	11.577	Jan-1969	9.491
Apr-1964	8.371	Sep-1966	12.417	Feb-1969	8.919
May-1964	9.069	Oct-1966	9.637	Mar-1969	11.607
Jun-1964	10.248	Nov-1966	8.094	Apr-1969	8.852
Jul-1964	11.030	Dec-1966	9.280	May-1969	12.537
Aug-1964	10.882	Jan-1967	8.334	Jun-1969	14.759
Sep-1964	10.333	Feb-1967	7.899	Jul-1969	13.667
Oct-1964	9.109	Mar-1967	9.994	Aug-1969	13.731
Nov-1964	7.685	Apr-1967	10.078	Sep-1969	15.110
Dec-1964	7.682	May-1967	10.801	Oct-1969	12.185
Jan-1965	8.350	Jun-1967	12.953	Nov-1969	10.645
Feb-1965	7.829	Jul-1967	12.222	Dec-1969	12.161
Mar-1965	8.829	Aug-1967	12.246	Jan-1970	10.840
Apr-1965	9.948	Sep-1967	13.281	Feb-1970	10.436
May-1965	10.638	Oct-1967	10.366	Mar-1970	13.589
Jun-1965	11.253	Nov-1967	8.730	Apr-1970	13.402
Jul-1965	11.424	Dec-1967	9.614	May-1970	13.103
Aug-1965	11.391	Jan-1968	8.639	Jun-1970	14.933
Sep-1965	10.665	Feb-1968	8.772	Jul-1970	14.147
Oct-1965	9.396	Mar-1968	10.894	Aug-1970	14.057
Nov-1965	7.775	Apr-1968	10.455	Sep-1970	16.234
Dec-1965	7.933	May-1968	11.179	Oct-1970	12.389
Jan-1966	8.186	Jun-1968	10.588	Nov-1970	11.594
Feb-1966	7.444	Jul-1968	10.794	Dec-1970	12.772
Mar-1966	8.484	Aug-1968	12.770		
Apr-1966	9.864	Sep-1968	13.812		
May-1966	10.252	Oct-1968	10.857		

The collection of Figures and Tables that follow are individually identified below.

List of Figures:

List of Tables:

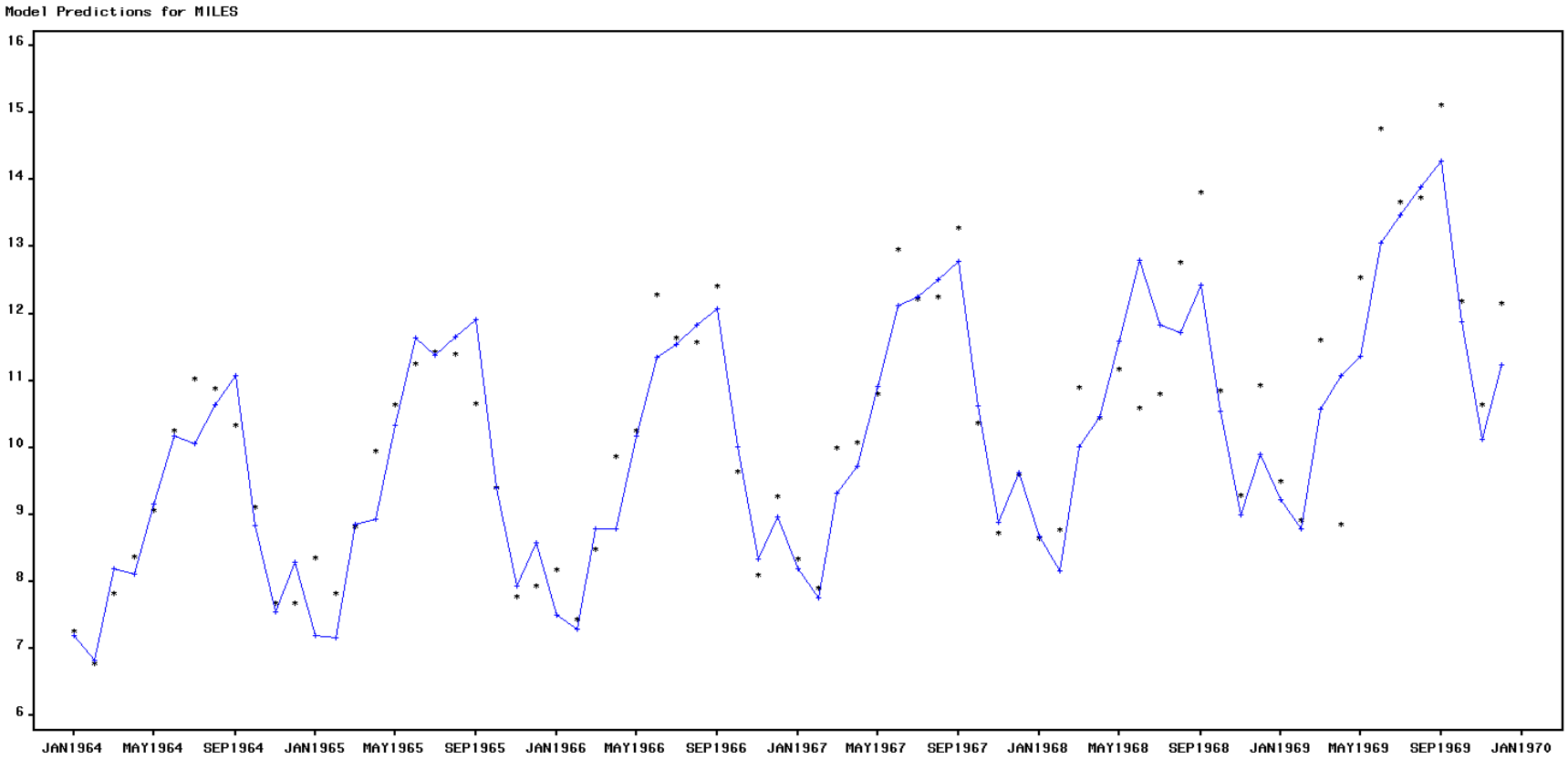

Figure 1 depicts a time series graph for predicted values ((+) signs) using the training data ((*) signs) set for Miles variable, as reported by SAS(AFS).

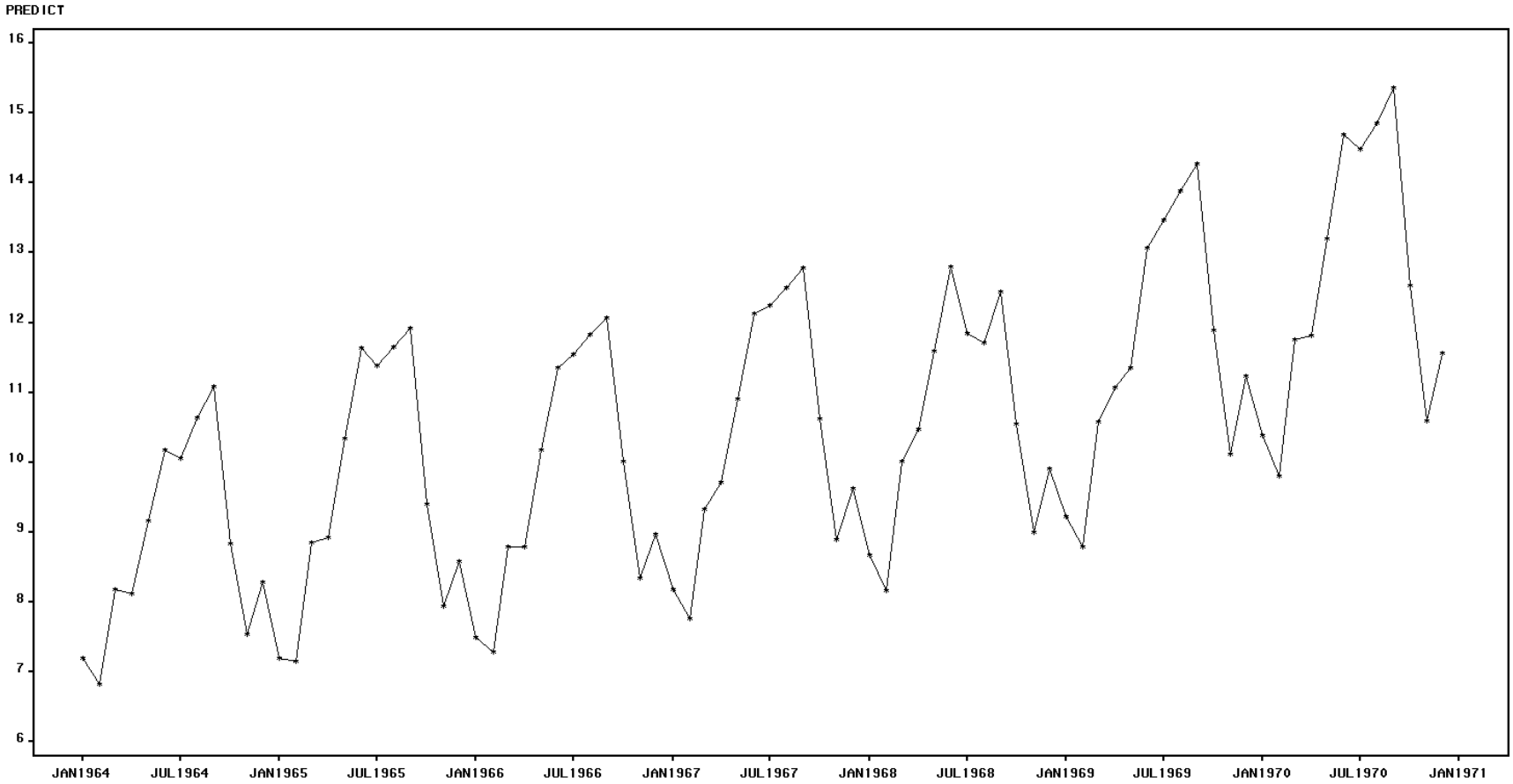

Figure 2 shows a time series graph for predicted values ((+) signs) for Miles variable as reported by SAS(AFS), along with 12-month forecasted values for the miles variable. Here, predicted and forecast values are plotted using ((*) signs).

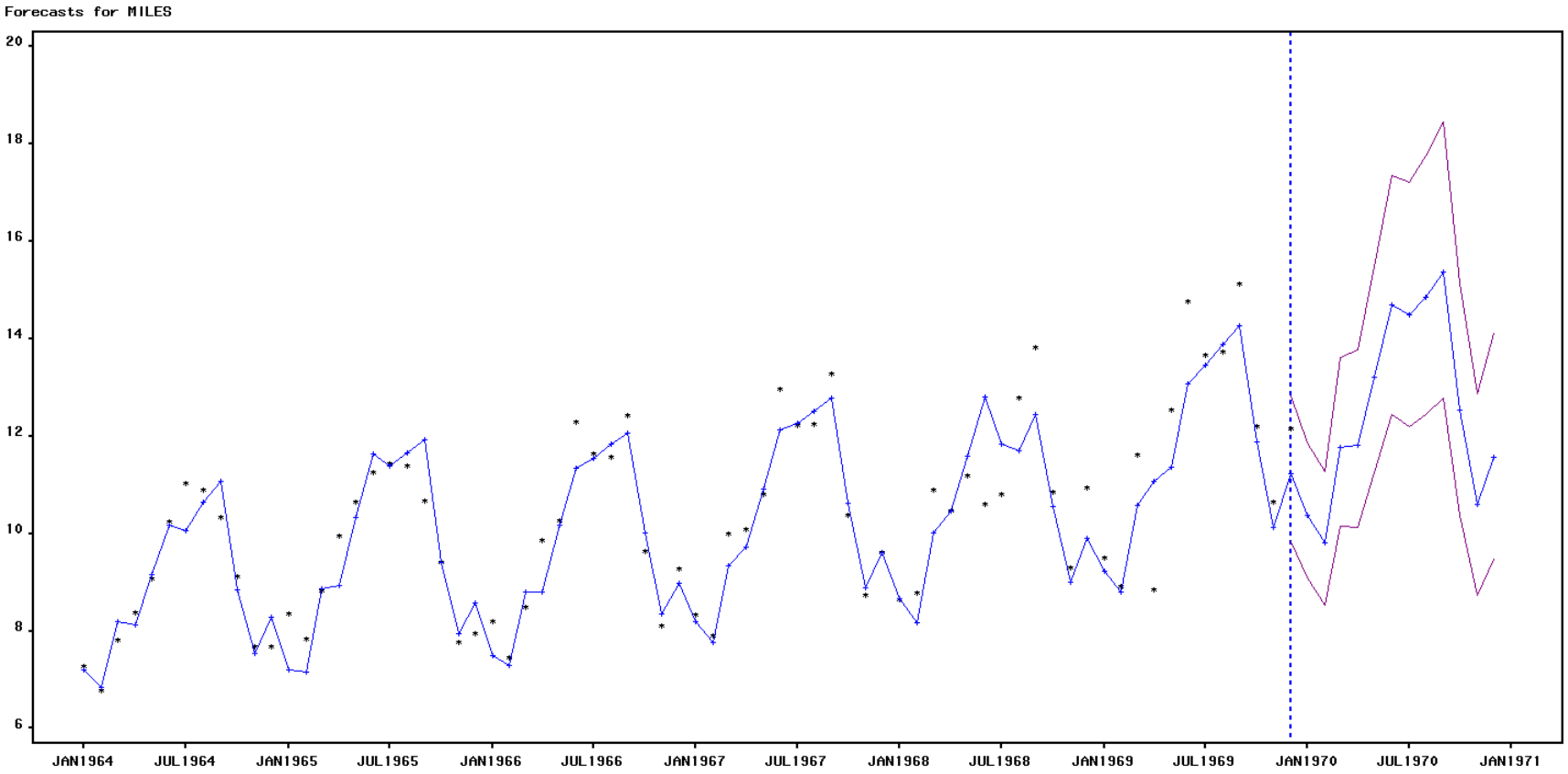

Figure 3a depicts a time series graph for predicted values ((+) signs) and the training data ((*) signs) for the Miles variable, as reported by SAS(AFS), along with 12-month forecasted values and prediction limits.

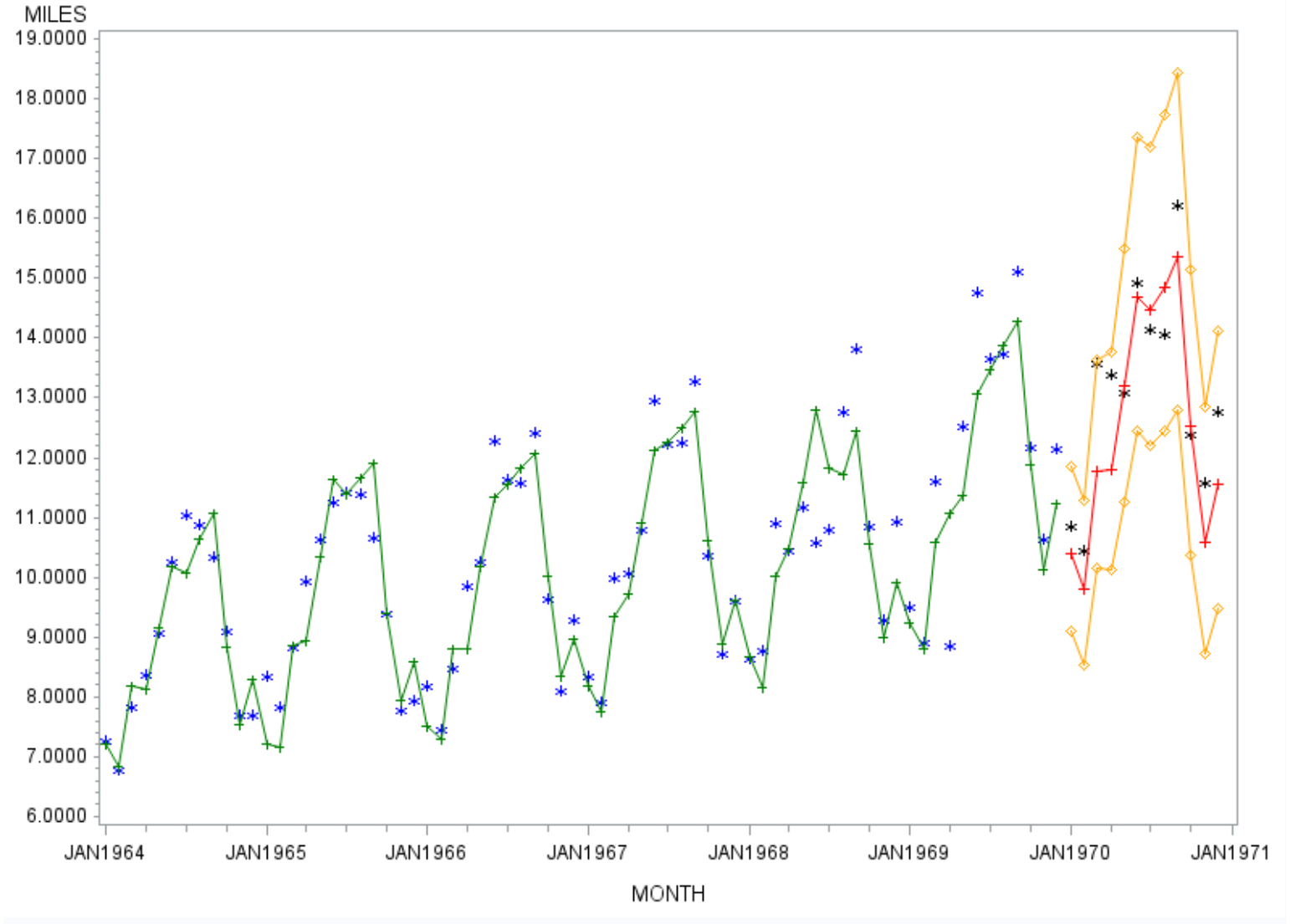

Figure 3b shows a time series graph for predicted values (green-colored (+) signs) for the Miles variable as reported by SAS(AFS), along with 12-month forecasted values, which are plotted using red-colored plus (+) signs. The training data are plotted using blue-colored star (*) signs, and testing data are plotted using black-colored star (*) signs, while the 12-month confidence limits are yellow-colored.

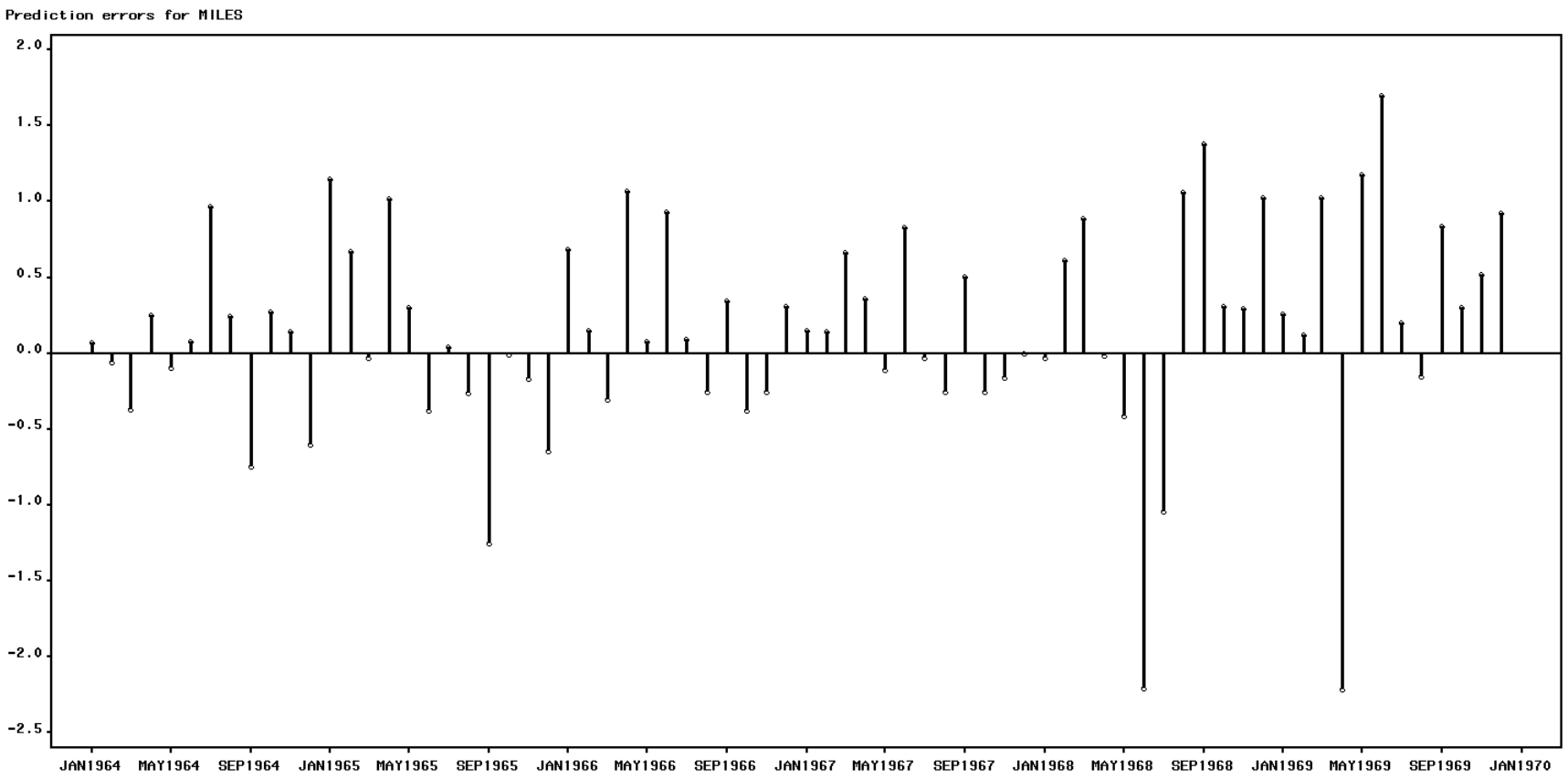

Figure 4. Prediction errors by time series values for the training data set.

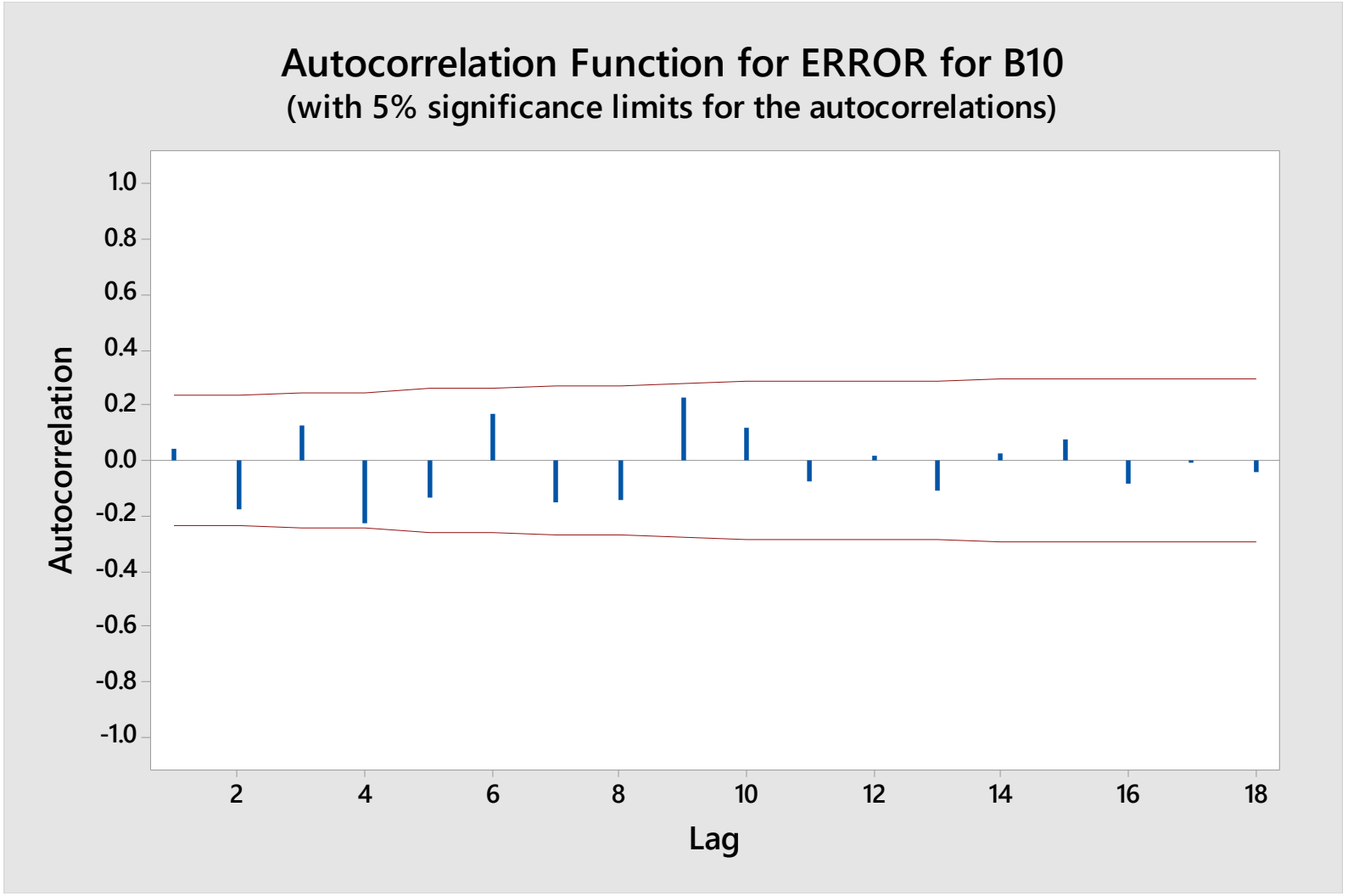

Figure 5. Autocorrelation Function (ACF) for the training data set.

Accuracy Measure for Table B.10 data.

Actual y_t	Predict $\widehat{y_t}$(t-1)	Forecast error $e_t(1) = y_t - \widehat{y_t}(t-1)$	Absolute error $\lvert e_t(1)\rvert$	Square error $[e_t(1)]^2$	Relative error $re_t(1) = 100(e_t(1)/y_t)$	Absolute percentage error $\lvert re_t(1)\rvert$
10.840	10.3829	0.4571	0.4571	0.20894	4.2168	4.2168
10.436	9.8000	0.6360	0.6360	0.40450	6.0943	6.0943
13.589	11.7624	1.8266	1.8266	3.33647	13.4418	13.4418
13.402	11.8094	1.5926	1.5926	2.53637	11.8833	11.8833
13.103	13.1983	-0.0953	0.0953	0.00908	-0.7273	0.7273
14.933	14.6909	0.2421	0.2421	0.05861	1.6212	1.6212
14.147	14.4839	-0.3369	0.3369	0.11350	-2.3814	2.3814
14.057	14.8523	-0.7953	0.7953	0.63250	-5.6577	5.6577
16.234	15.3569	0.8771	0.8771	0.76930	5.4029	5.4029
12.389	12.5299	-0.1409	0.1409	0.01985	-1.1373	1.1373
11.594	10.5925	1.0015	1.0015	1.00300	8.6381	8.6381

Table A. Error Measures table B10 time series data.

Data set	**MSE**	$\widehat{\sigma_{e(1)}}$	**MAD**	**MPE**	**MAPE**
Full data (84 data) – Seasonal Exponential Smoothing	0.51118	0.71497	0.53520	1.24912	5.07377
Training data (72 data) – Log Seasonal Exponential Smoothing	0.50050	0.70746	0.51191	1.24737	4.89930
Holdout data (12 data)	0.878339	0.937197	0.767058	4.23467	5.88529

Table B. Accuracy Measures table B10 time series data.

Table B provides SAS (AFS)® forecast errors for the full data set, the training data set, and the holdout (or test) data set. The Log Seasonal Exponential Smoothing model was suggested as the "best model" for this data set. SAS (AFS)® and MINITAB® produced the accuracy measures shown in Table B, which comprise the Mean Square Error (MSE), the Standard Deviation of the Forecast Errors (SDFE), the Mean Absolute Deviation (MAD), the Mean Percent Forecast Error (MPE), and Mean Absolute Percent Error (MAPE).

Here, the accuracy statistics are computed as follows:

Mean square error:

$$\textbf{MSE} = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = 0.878339$$

Standard Deviation of Forecast Error (Root Mean Square Error):

$$\widehat{\sigma_{e(1)}} = \sqrt{MSE} = \sqrt{0.878339} = 0.937197$$

Mean Absolute Deviation (Mean Absolute Error):

$$\textbf{MAD} = \frac{1}{n}\sum_{t=1}^{n} |e_t(1)| = 0.767058$$

Mean Percent Forecast Error:

$$\textbf{MPE} = \frac{1}{n}\sum_{t=1}^{n} re_t(1) = 4.23467$$

Mean Absolute Percent Error:

$$\textbf{MAPE} = \frac{1}{n}\sum_{t=1}^{n} |re_t(1)| = 5.88529$$

Worksheet 1 ***

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	Actual	Predict	Forecast_Error	Abs_Error	Sq_Error	Relative_Error	Abs_Percentage_Error	MSE	Root_MSE	MAD	MPE	MAPE
1	10.840	10.3829	0.4571	0.4571	0.20894	4.2168	4.2168	0.878339	0.937197	0.767058	4.23467	5.88529
2	10.436	9.8000	0.6360	0.6360	0.40450	6.0943	6.0943					
3	13.589	11.7624	1.8266	1.8266	3.33647	13.4418	13.4418					
4	13.402	11.8094	1.5926	1.5926	2.53637	11.8833	11.8833					
5	13.103	13.1983	-0.0953	0.0953	0.00908	-0.7273	0.7273					
6	14.933	14.6909	0.2421	0.2421	0.05861	1.6212	1.6212					
7	14.147	14.4839	-0.3369	0.3369	0.11350	-2.3814	2.3814					
8	14.057	14.8523	-0.7953	0.7953	0.63250	-5.6577	5.6577					
9	16.234	15.3569	0.8771	0.8771	0.76930	5.4029	5.4029					
10	12.389	12.5299	-0.1409	0.1409	0.01985	-1.1373	1.1373					
11	11.594	10.5925	1.0015	1.0015	1.00300	8.6381	8.6381					
12	12.772	11.5687	1.2033	1.2033	1.44793	9.4214	9.4214					

Table C. MINITAB® 17 Calculation of Forecast Errors for Holdout Data.

The first two columns of Table C consist of the actual values of the holdout data set and the SAS (AFS)® forecasted values, based on training data. The remaining columns give forecast errors, the absolute forecast errors, the squared forecast errors, the relative forecast errors and absolute forecast percentage errors using the MINITAB® calculator function.

Obs	DATE	ACTUAL	PREDICT	UPPER	LOWER	ERROR	STD	NERROR	RESIDUAL	RESSTD	NRESID	_LEVEL_	_SFACTOR
73	JAN1970	.	10.3829	11.8550	9.0936	.	0.7052	.	.	.	.	2.5192	-0.1790
74	FEB1970	.	9.8000	11.2721	8.5201	.	0.7029	.	.	.	.	2.5192	-0.2368
75	MAR1970	.	11.7624	13.6242	10.1550	.	0.8862	.	.	.	.	2.5192	-0.0543
76	APR1970	.	11.8094	13.7702	10.1277	.	0.9306	.	.	.	.	2.5192	-0.0503
77	MAY1970	.	13.1983	15.4885	11.2467	.	1.0838	.	.	.	.	2.5192	0.0609
78	JUN1970	.	14.6909	17.3464	12.4418	.	1.2533	.	.	.	.	2.5192	0.1680
79	JUL1970	.	14.4839	17.2038	12.1941	.	1.2803	.	.	.	.	2.5192	0.1539
80	AUG1970	.	14.8523	17.7427	12.4327	.	1.3573	.	.	.	.	2.5192	0.1790
81	SEP1970	.	15.3569	18.4477	12.7839	.	1.4479	.	.	.	.	2.5192	0.2124
82	OCT1970	.	12.5299	15.1331	10.3745	.	1.2166	.	.	.	.	2.5192	0.008933
83	NOV1970	.	10.5925	12.8604	8.7245	.	1.0575	.	.	.	.	2.5192	-0.1590
84	DEC1970	.	11.5687	14.1175	9.4801	.	1.1859	.	.	.	.	2.5192	-0.0709

Table D. Forecasted values for the last 12 month of Holdout Data.

Table D contains forecasted values for the final twelve month for the Miles variable, and the corresponding lower and upper 95% prediction limits. The Root Mean Square Prediction Error, Level and Trend smoothing components are also shown..

Model Parameter	Estimate	Std. Error	T	Prob>|T|
LEVEL Smoothing Weight	0.33780	0.0656	5.1467	<.0001
SEASONAL Smoothing Weight	0.04675	0.0475	0.9846	0.3282
Residual Variance (sigma squared)	0.00458	.	.	.
Smoothed Level	2.51918	.	.	.
Smoothed Seasonal Factor 1	-0.17902	.	.	.
Smoothed Seasonal Factor 2	-0.23680	.	.	.
Smoothed Seasonal Factor 3	-0.05427	.	.	.
Smoothed Seasonal Factor 4	-0.05029	.	.	.
Smoothed Seasonal Factor 5	0.06090	.	.	.
Smoothed Seasonal Factor 6	0.16804	.	.	.
Smoothed Seasonal Factor 7	0.15386	.	.	.
Smoothed Seasonal Factor 8	0.17897	.	.	.
Smoothed Seasonal Factor 9	0.21238	.	.	.
Smoothed Seasonal Factor 10	0.00893	.	.	.
Smoothed Seasonal Factor 11	-0.15904	.	.	.
Smoothed Seasonal Factor 12	-0.07088	.	.	.

Table E. Parameter Estimates for Training Data using the Log Seasonal Exponential Smoothing model suggested by SAS (AFS)®.

Results

In Figure 3b, it seems the model fits the data appropriately, however, the SAS(AFS) could not forecast values with acceptably small prediction error. There are no spikes in the ACF function, which means forecast errors uncorrelated. The MSE, RMSE and MAD are similar for both the training and holdout data sets; that is, the variability in forecast errors are approximately similar. However, MAPE is little bit big in all cases.

TABLE B.11 (CHAMPAGNE SALES) ANALYSIS

Table B.11 comprises data on the Champagne Sales from January 1962 through December 1969, where the sales (in thousand of bottles) variable is collected monthly. The data set excludes last twelve month values as a holdout set in order to analyze the accuracy of SAS(AFS), which suggested a Log Seasonal Exponential Smoothing model as the "best model" for this training, or initialization, data set. SAS(AFS) also produced the predicted values for the next 12 month. I have calculated different types of forecast errors based on the hold out data and predicted data.

Table B.11: Champagne Sales

Month	Sales, in Thousand of Bottles	Month	Sales, in Thousand of Bottles	Month	Sales, in Thousand of Bottles
Jan-1962	2.851	Sep-1964	3.528	May-1967	4.968
Feb-1962	2.672	Oct-1964	5.211	Jun-1967	4.677
Mar-1962	2.755	Nov-1964	7.614	Jul-1967	3.523
Apr-1962	2.721	Dec-1964	9.254	Aug-1967	1.821
May-1962	2.946	Jan-1965	5.375	Sep-1967	5.222
Jun-1962	3.036	Feb-1965	3.088	Oct-1967	6.873
Jul-1962	2.282	Mar-1965	3.718	Nov-1967	10.803
Aug-1962	2.212	Apr-1965	4.514	Dec-1967	13.916
Sep-1962	2.922	May-1965	4.520	Jan-1968	2.639
Oct-1962	4.301	Jun-1965	4.539	Feb-1968	2.899
Nov-1962	5.764	Jul-1965	3.663	Mar-1968	3.370
Dec-1962	7.132	Aug-1965	1.643	Apr-1968	3.740
Jan-1963	2.541	Sep-1965	4.739	May-1968	2.927
Feb-1963	2.475	Oct-1965	5.428	Jun-1968	3.986
Mar-1963	3.031	Nov-1965	8.314	Jul-1968	4.217
Apr-1963	3.266	Dec-1965	10.651	Aug-1968	1.738
May-1963	3.776	Jan-1966	3.633	Sep-1968	5.221
Jun-1963	3.230	Feb-1966	4.292	Oct-1968	6.424
Jul-1963	3.028	Mar-1966	4.154	Nov-1968	9.842
Aug-1963	1.759	Apr-1966	4.121	Dec-1968	13.076
Sep-1963	3.595	May-1966	4.647	Jan-1969	3.934
Oct-1963	4.474	Jun-1966	4.753	Feb-1969	3.162
Nov-1963	6.838	Jul-1966	3.965	Mar-1969	4.286
Dec-1963	8.357	Aug-1966	1.723	Apr-1969	4.676
Jan-1964	3.113	Sep-1966	5.048	May-1969	5.010
Feb-1964	3.006	Oct-1966	6.922	Jun-1969	4.874
Mar-1964	4.047	Nov-1966	9.858	Jul-1969	4.633
Apr-1964	3.523	Dec-1966	11.331	Aug-1969	1.659
May-1964	3.937	Jan-1967	4.016	Sep-1969	5.951
Jun-1964	3.986	Feb-1967	3.957	Oct-1969	6.981
Jul-1964	3.260	Mar-1967	4.510	Nov-1969	9.851
Aug-1964	1.573	Apr-1967	4.276	Dec-1969	12.670

The collection of Figures and Tables that follow are individually identified below.

List of Figures:

List of Tables:

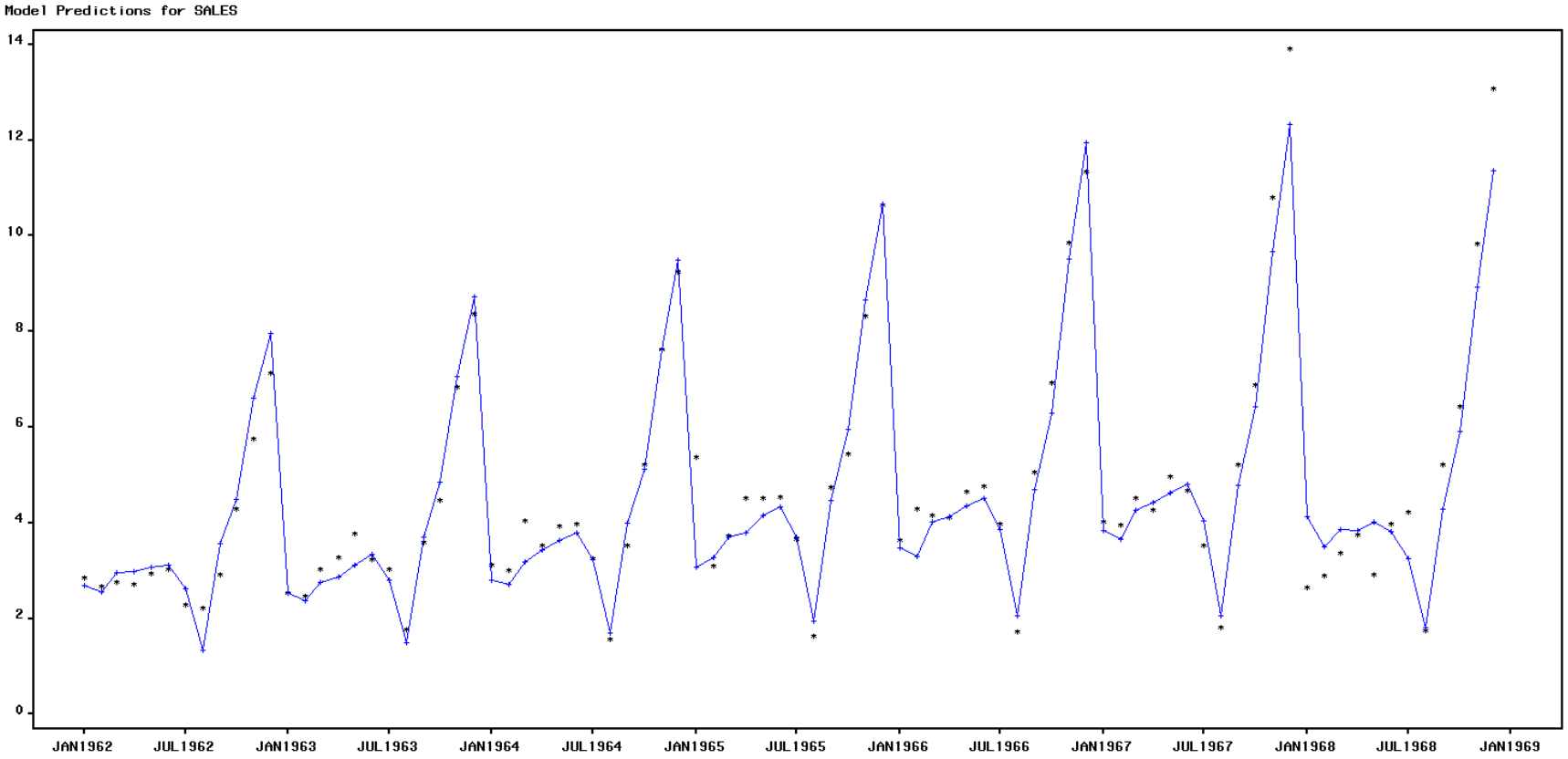

Figure 1 depicts a time series graph for predicted values ((+) signs) using the training data ((*) signs) set for Sales variable, as reported by SAS(AFS).

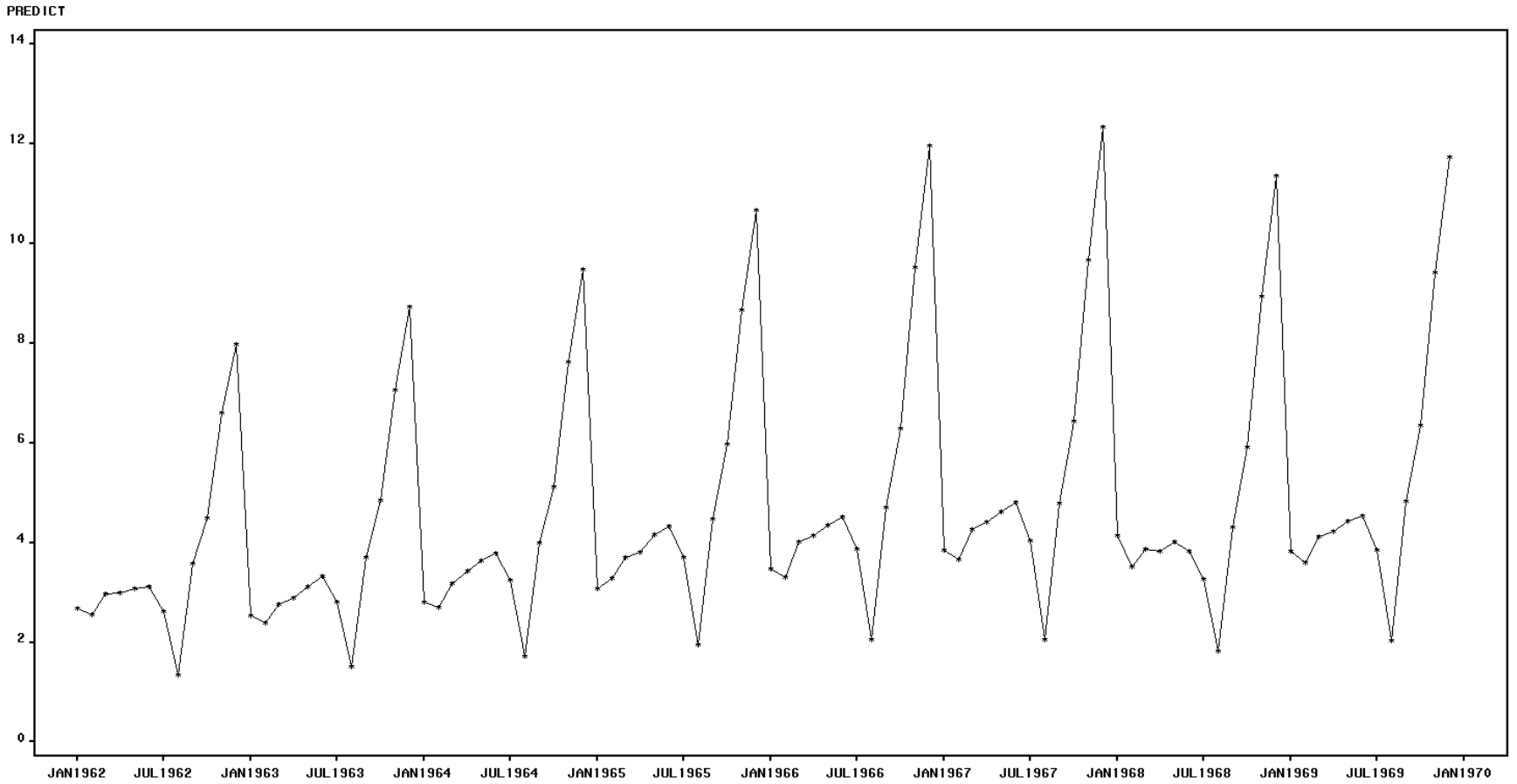

Figure 2 shows a time series graph for predicted values ((+) signs) for Sales variable as reported by SAS(AFS), along with 12-month forecasted values for the sales variable. Here, predicted and forecast values are plotted using ((*) signs).

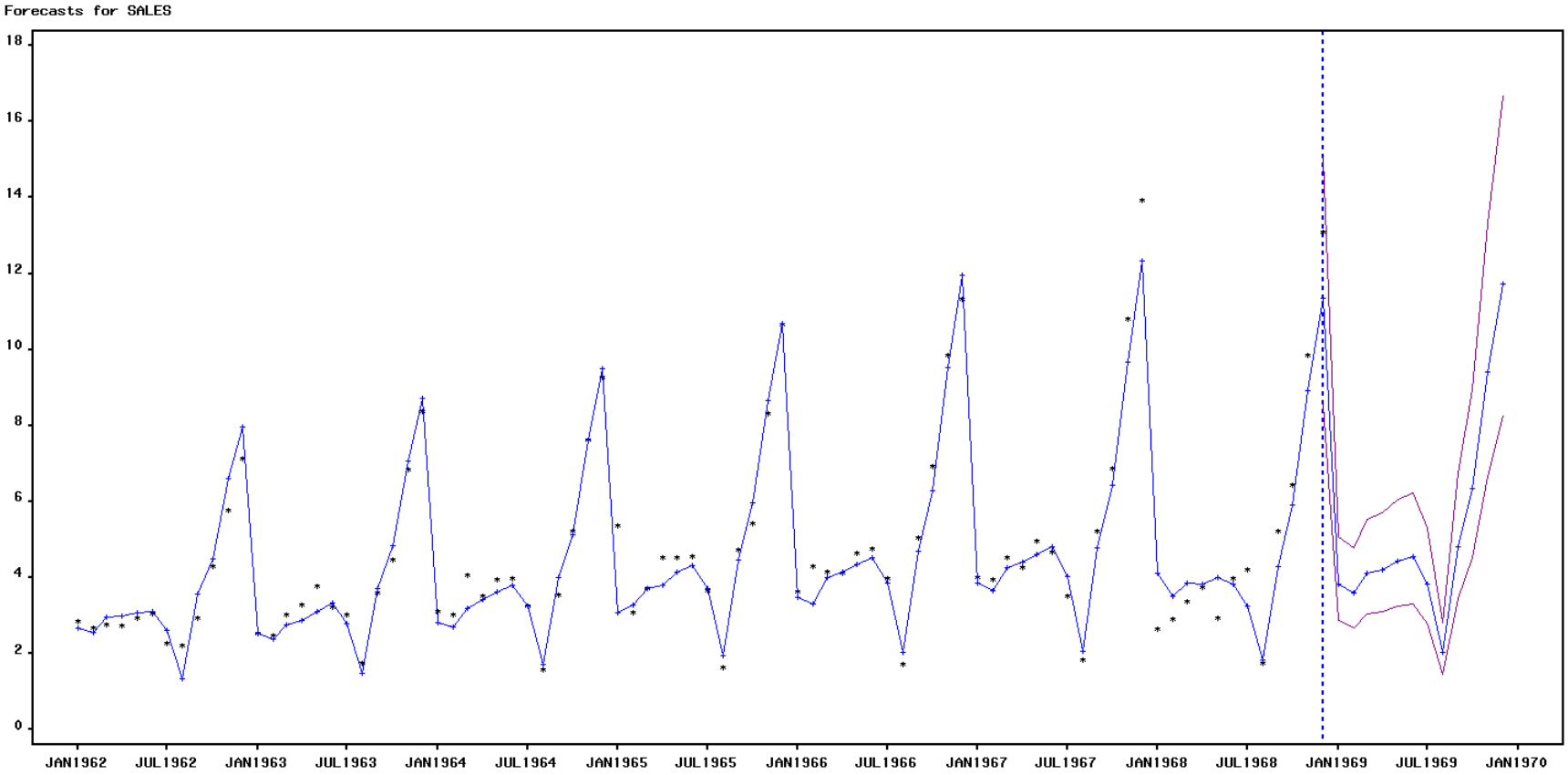

Figure 3a depicts a time series graph for predicted values ((+) signs) and the training data ((*) signs) for the Sales variable, as reported by SAS(AFS), along with 12-month forecasted values and prediction limits.

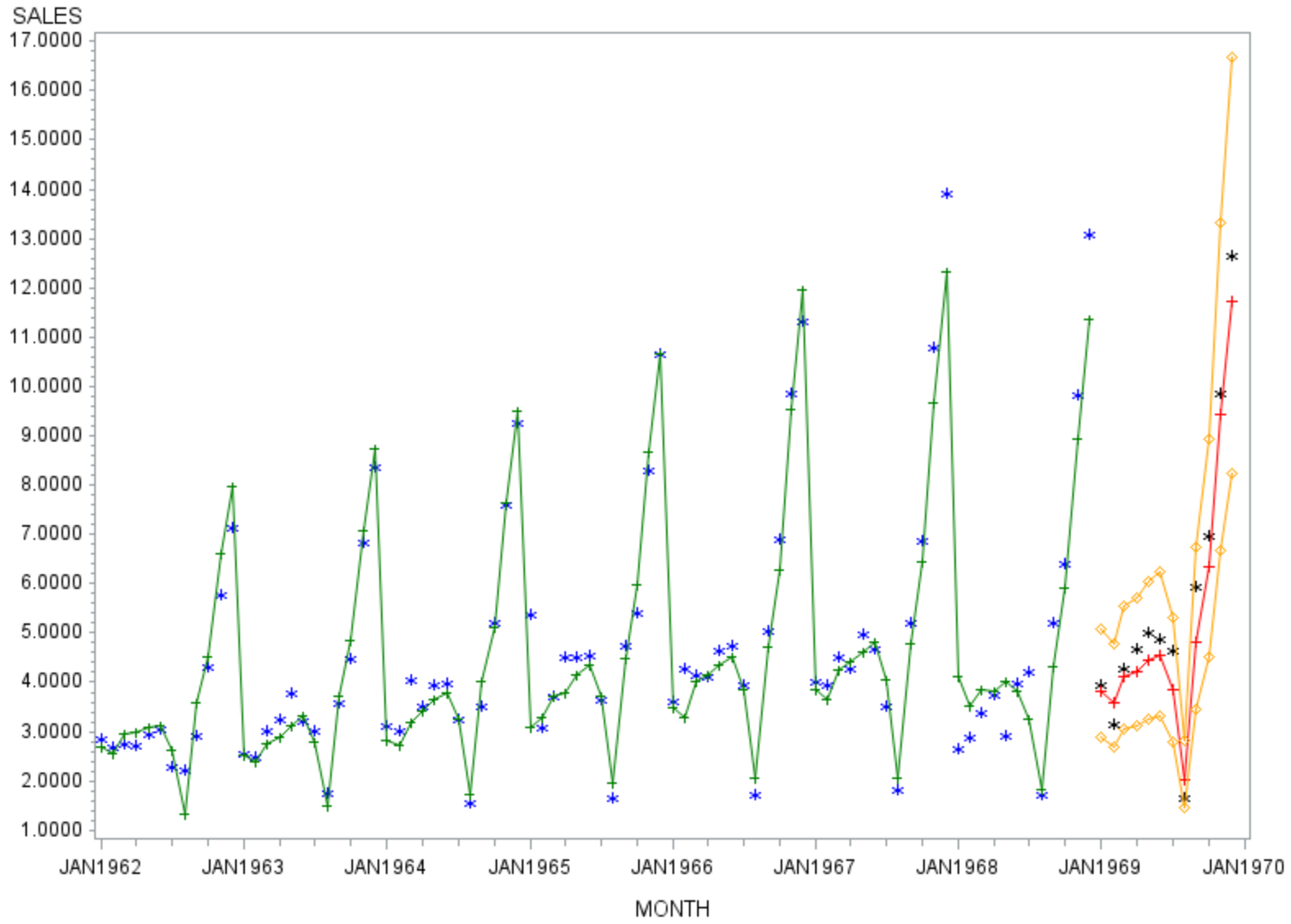

Figure 3b shows a time series graph for predicted values (green-colored (+) signs) for the Sales variable as reported by SAS(AFS), along with 12-month forecasted values, which are plotted using red-colored plus (+) signs. The training data are plotted using blue-colored star (*) signs, and testing data are plotted using black-colored star (*) signs, while the 12-month confidence limits are yellow-colored.

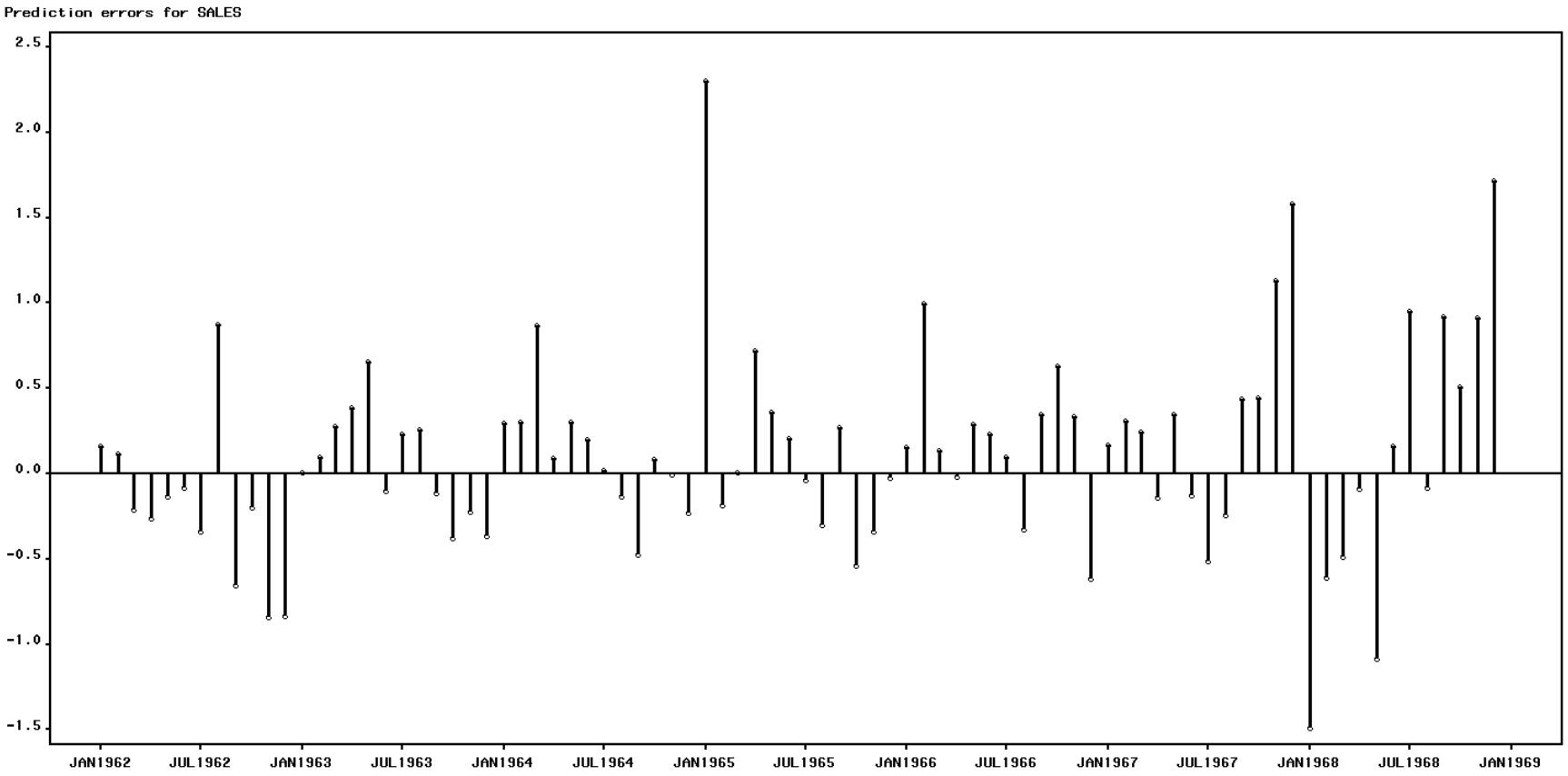

Figure 4. Prediction errors by time series values for the training data set.

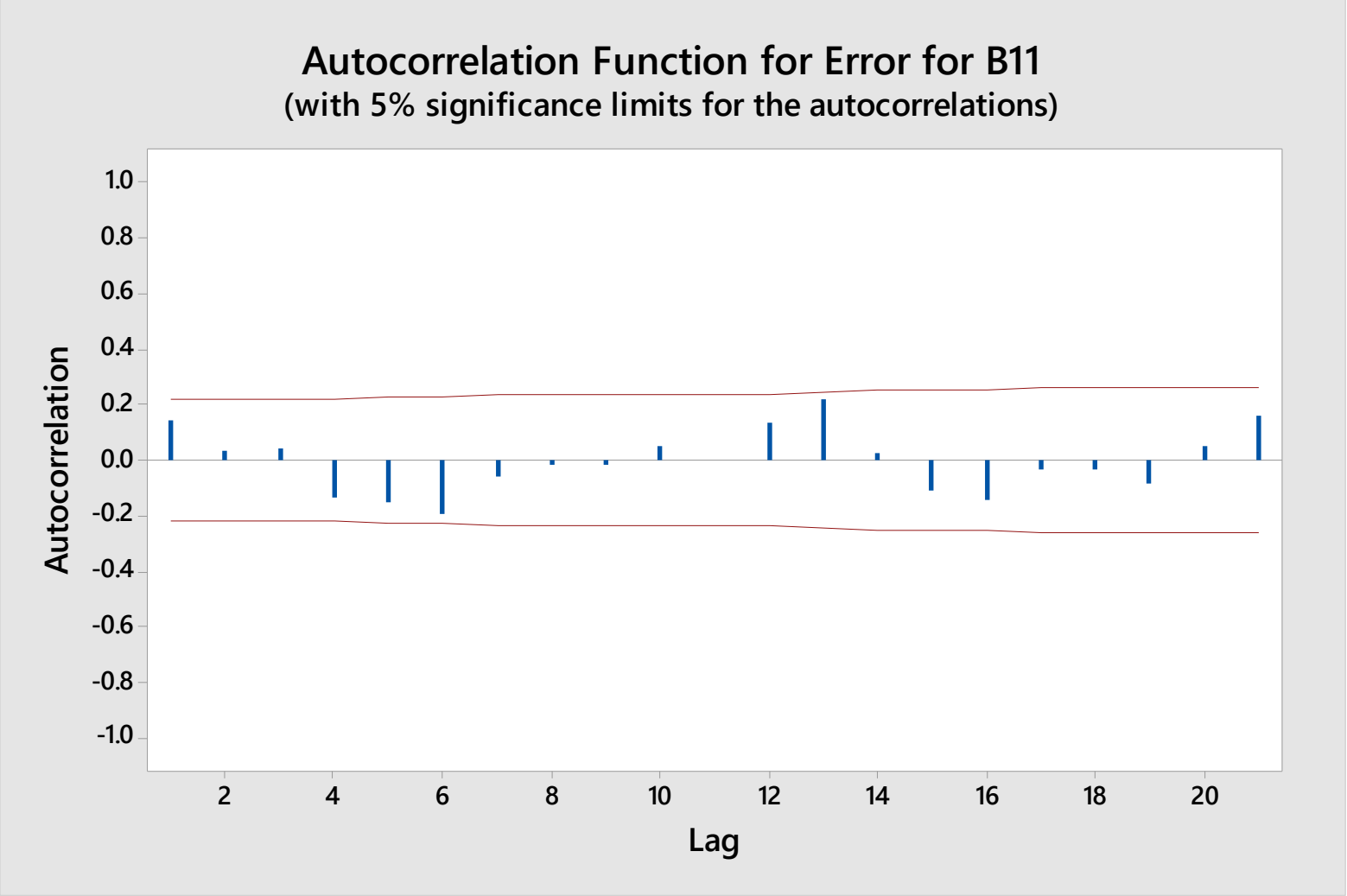

Figure 5. Autocorrelation Function (ACF) for the training data set.

Accuracy Measure for Table B.11 data.

Actual y_t	Predict $\widehat{y_t}$(t-1)	Forecast error $e_t(1) = y_t - \widehat{y_t}$(t-1)	Absolute error $\lvert e_t(1)\rvert$	Square error $[e_t(1)]^2$	Relative error $re_t(1) = 100(e_t(1)/y_t)$	Absolute percentage error $\lvert re_t(1)\rvert$
3.934	3.8191	0.1149	0.1149	0.01320	2.9207	2.9207
3.162	3.5890	-0.4270	0.4270	0.18233	-13.5041	13.5041
4.286	4.1192	0.1668	0.1668	0.02782	3.8917	3.8917
4.676	4.2144	0.4616	0.4616	0.21307	9.8717	9.8717
5.010	4.4378	0.5722	0.5722	0.32741	11.4212	11.4212
4.874	4.5410	0.3330	0.3330	0.11089	6.8322	6.8322
4.633	3.8396	0.7934	0.7934	0.62948	17.1250	17.1250
1.659	2.0231	-0.3641	0.3641	0.13257	-21.9470	21.9470
5.951	4.8273	1.1237	1.1237	1.26270	18.8825	18.8825
6.981	6.3578	0.6232	0.6232	0.38838	8.9271	8.9271
9.851	9.4220	0.4290	0.4290	0.18404	4.3549	4.3549
12.670	11.7258	0.9442	0.9442	0.89151	7.4522	7.4522

Table A. Error Measures table B11 time series data.

Data set	**MSE**	$\widehat{\sigma_{e(1)}}$	**MAD**	**MPE**	**MAPE**
Full data (96 data)	0.32552	0.57054	0.40840	1.02193	9.59394
Training data (84 data)	0.34529	0.58761	0.41672	0.88245	9.81278
Holdout data (12 data)	0.363618	0.603008	0.529425	4.68568	10.5942

Table B. Accuracy Measures table B11 time series data.

Table B provides SAS (AFS)® forecast errors for the full data set, the training data set, and the holdout (or test) data set. The Log Seasonal Exponential Smoothing model was suggested as the "best model" for this data set. SAS (AFS)® and MINITAB® produced the accuracy measures shown in Table B, which comprise the Mean Square Error (MSE), the Standard Deviation of the Forecast Errors (SDFE), the Mean Absolute Deviation (MAD), the Mean Percent Forecast Error (MPE), and Mean Absolute Percent Error (MAPE).

Here, the accuracy statistics are computed as follows:

Mean square error:

$$\textbf{MSE} = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = 0.363618$$

Standard Deviation of Forecast Error (Root Mean Square Error):

$$\widehat{\sigma_{e(1)}} = \sqrt{MSE} = \sqrt{0.363618} = 0.603008$$

Mean Absolute Deviation (Mean Absolute Error):

$$\textbf{MAD} = \frac{1}{n}\sum_{t=1}^{n} |e_t(1)| = 0.529425$$

Mean Percent Forecast Error:

$$\textbf{MPE} = \frac{1}{n}\sum_{t=1}^{n} re_t(1) = 4.68568$$

Mean Absolute Percent Error:

$$\textbf{MAPE} = \frac{1}{n}\sum_{t=1}^{n} |re_t(1)| = 10.5942$$

Worksheet 1 ***

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	Actual	Predict	Forecast_Error	Abs_Error	Sq_Error	Relative_Error	Abs_Percentage_Error	MSE	Root_MSE	MAD	MPE	MAPE
1	3.934	3.8191	0.1149	0.1149	0.01320	2.9207	2.9207	0.363618	0.603008	0.529425	4.68568	10.5942
2	3.162	3.5890	-0.4270	0.4270	0.18233	-13.5041	13.5041					
3	4.286	4.1192	0.1668	0.1668	0.02782	3.8917	3.8917					
4	4.676	4.2144	0.4616	0.4616	0.21307	9.8717	9.8717					
5	5.010	4.4378	0.5722	0.5722	0.32741	11.4212	11.4212					
6	4.874	4.5410	0.3330	0.3330	0.11089	6.8322	6.8322					
7	4.633	3.8396	0.7934	0.7934	0.62948	17.1250	17.1250					
8	1.659	2.0231	-0.3641	0.3641	0.13257	-21.9470	21.9470					
9	5.951	4.8273	1.1237	1.1237	1.26270	18.8825	18.8825					
10	6.981	6.3578	0.6232	0.6232	0.38838	8.9271	8.9271					
11	9.851	9.4220	0.4290	0.4290	0.18404	4.3549	4.3549					
12	12.670	11.7258	0.9442	0.9442	0.89151	7.4522	7.4522					

Table C. MINITAB® 17 Calculation of Forecast Errors for Holdout Data.

The first two columns of Table C consist of the actual values of the holdout data set and the SAS (AFS)® forecasted values, based on training data. The remaining columns give forecast errors, the absolute forecast errors, the squared forecast errors, the relative forecast errors and absolute forecast percentage errors using the MINITAB® calculator function.

Obs	DATE	ACTUAL	PREDICT	UPPER	LOWER	ERROR	STD	NERROR	RESIDUAL	RESSTD	NRESID	_LEVEL_	_SFACTOR
85	JAN1969	.	3.8191	5.0654	2.8795	.	0.5604	.	.	.	.	1.5544	-0.2144
86	FEB1969	.	3.5890	4.7938	2.6870	.	0.5402	.	.	.	.	1.5544	-0.2765
87	MAR1969	.	4.1192	5.5398	3.0628	.	0.6353	.	.	.	.	1.5544	-0.1388
88	APR1969	.	4.2144	5.7061	3.1127	.	0.6653	.	.	.	.	1.5544	-0.1159
89	MAY1969	.	4.4378	6.0481	3.2562	.	0.7164	.	.	.	.	1.5544	-0.0643
90	JUN1969	.	4.5410	6.2287	3.3105	.	0.7490	.	.	.	.	1.5544	-0.0413
91	JUL1969	.	3.8396	5.3000	2.7816	.	0.6465	.	.	.	.	1.5544	-0.2090
92	AUG1969	.	2.0231	2.8098	1.4566	.	0.3475	.	.	.	.	1.5544	-0.8498
93	SEP1969	.	4.8273	6.7455	3.4545	.	0.8453	.	.	.	.	1.5544	0.0199
94	OCT1969	.	6.3578	8.9373	4.5228	.	1.1342	.	.	.	.	1.5544	0.2953
95	NOV1969	.	9.4220	13.3226	6.6634	.	1.7113	.	.	.	.	1.5544	0.6886
96	DEC1969	.	11.7258	16.6759	8.2450	.	2.1670	.	.	.	.	1.5544	0.9074

Table D. Forecasted values for the last 12 month of Holdout Data.

Table D contains forecasted values for the final twelve month for the Miles variable, and the corresponding lower and upper 95% prediction limits. The Root Mean Square Prediction Error, Level and Trend smoothing components are also shown..

Model Parameter	Estimate	Std. Error	T	Prob>|T|
LEVEL Smoothing Weight	0.22465	0.0508	4.4198	<.0001
SEASONAL Smoothing Weight	0.00100	0.0237	0.0422	0.9665
Residual Variance (sigma squared)	0.02076	.	.	.
Smoothed Level	1.55441	.	.	.
Smoothed Seasonal Factor 1	-0.21438	.	.	.
Smoothed Seasonal Factor 2	-0.27654	.	.	.
Smoothed Seasonal Factor 3	-0.13876	.	.	.
Smoothed Seasonal Factor 4	-0.11590	.	.	.
Smoothed Seasonal Factor 5	-0.06426	.	.	.
Smoothed Seasonal Factor 6	-0.04127	.	.	.
Smoothed Seasonal Factor 7	-0.20905	.	.	.
Smoothed Seasonal Factor 8	-0.84980	.	.	.
Smoothed Seasonal Factor 9	0.01987	.	.	.
Smoothed Seasonal Factor 10	0.29527	.	.	.
Smoothed Seasonal Factor 11	0.68864	.	.	.
Smoothed Seasonal Factor 12	0.90738	.	.	.

Table E. Parameter Estimates for Training Data using the Log Seasonal Exponential Smoothing model suggested by SAS (AFS)®.

Results

In Figure 3b, it seems the model fits the data appropriately, however, the SAS(AFS) could not forecast values with acceptably small prediction error. The MSE, RMSE and MAD are similar for both the training and holdout data sets; that is, the variability in forecast errors are approximately similar. Howver, the Mean Absolute Percent Error (MAPE) are really big in all cases.

TABLE B.12 (CHEMICAL PROCESS YIELD) ANALYSIS

Table B.12 comprises data on the Chemical Process Yield, with Operating Temperature (Uncontrolled) for 50 hours, where the yield (in percent) variable is collected hourly. The data set excludes last twelve hour values as a holdout set in order to analyze the accuracy of SAS(AFS), which suggested a Damped Trend Exponential Smoothing model as the "best model" for this training, or initialization, data set. SAS(AFS) also produced the predicted values for the next 12 hour. I have calculated different types of forecast errors based on the hold out data and predicted data.

The model equation is

$$Y_t = \mu_t + \beta_t\, t + \varepsilon_t$$

where,

Y_t = observed value i.e. securities rate at time t

μ_t = smoothed level and $\mu_t = 99.20141$

β_t = smoothed trend and $\beta_t = 0.10409$

Now, the k-step prediction equation is

$$\hat{Y}_t(k) = L_t + \sum_{i=1}^{k} \phi^t\, T_t$$

and the smoothing equations are

$$L_t = \alpha\, Y_t + (1-\alpha)\,(L_{t-1} + \phi\, T_{t-1})$$

and $T_t = \gamma\,(L_t - L_{t-1}) + (1-\gamma)\,\phi\, T_{t-1}$

where,

L_t = Level estimate and $L_0 = \mu_t = 99.20141$

T_t = Trend estimate and $T_0 = \beta_t = 0.10409$

α = level smoothing and $\alpha = 0.96592$

γ = trend smoothing and $\gamma = 0.999000$ and

ϕ = damping smoothing and $\phi = 0.46766$

The ARIMA model equivalency to damped-trend linear exponential smoothing is the ARIMA (1, 1, 2) model,

$(1-\phi B)(1-B)\, Y_t = (1-\theta_1 B - \theta_2 B^2)\, \varepsilon_t$

$\theta_1 = 1 + \phi - \alpha - \alpha\,\gamma\,\phi$

$\theta_2 = (\alpha - 1)\,\phi$

Table B.11: Chemical Process Yield, with Operating Temperature (Uncontrolled)

Hour	Yield, %	Temperature, deg F	Hour	Yield, %	Temperature, deg F
1	89.0	153	26	99.4	152
2	90.5	152	27	99.6	153
3	91.5	153	28	99.8	153
4	93.2	153	29	98.8	154
5	93.9	154	30	99.9	154
6	94.6	151	31	98.2	153

Hour	Yield, %	Temperature, deg F	Hour	Yield, %	Temperature, deg F
7	94.7	153	32	98.7	153
8	93.5	152	33	97.5	154
9	91.2	151	34	97.9	152
10	89.3	150	35	98.3	152
11	85.6	150	36	98.8	151
12	80.3	149	37	99.1	150
13	75.9	149	38	99.2	149
14	75.3	147	39	98.6	148
15	78.3	146	40	95.3	147
16	89.1	143	41	94.2	146
17	88.3	148	42	91.3	148
18	89.2	151	43	90.6	145
19	90.1	152	44	91.2	143
20	94.3	153	45	88.3	145
21	97.7	154	46	84.1	150
22	98.6	152	47	86.5	147
23	98.7	153	48	88.2	150
24	98.9	152	49	89.5	151
25	99.2	152	50	89.5	152

The collection of Figures and Tables that follow are individually identified below.

List of Figures:

List of Tables:

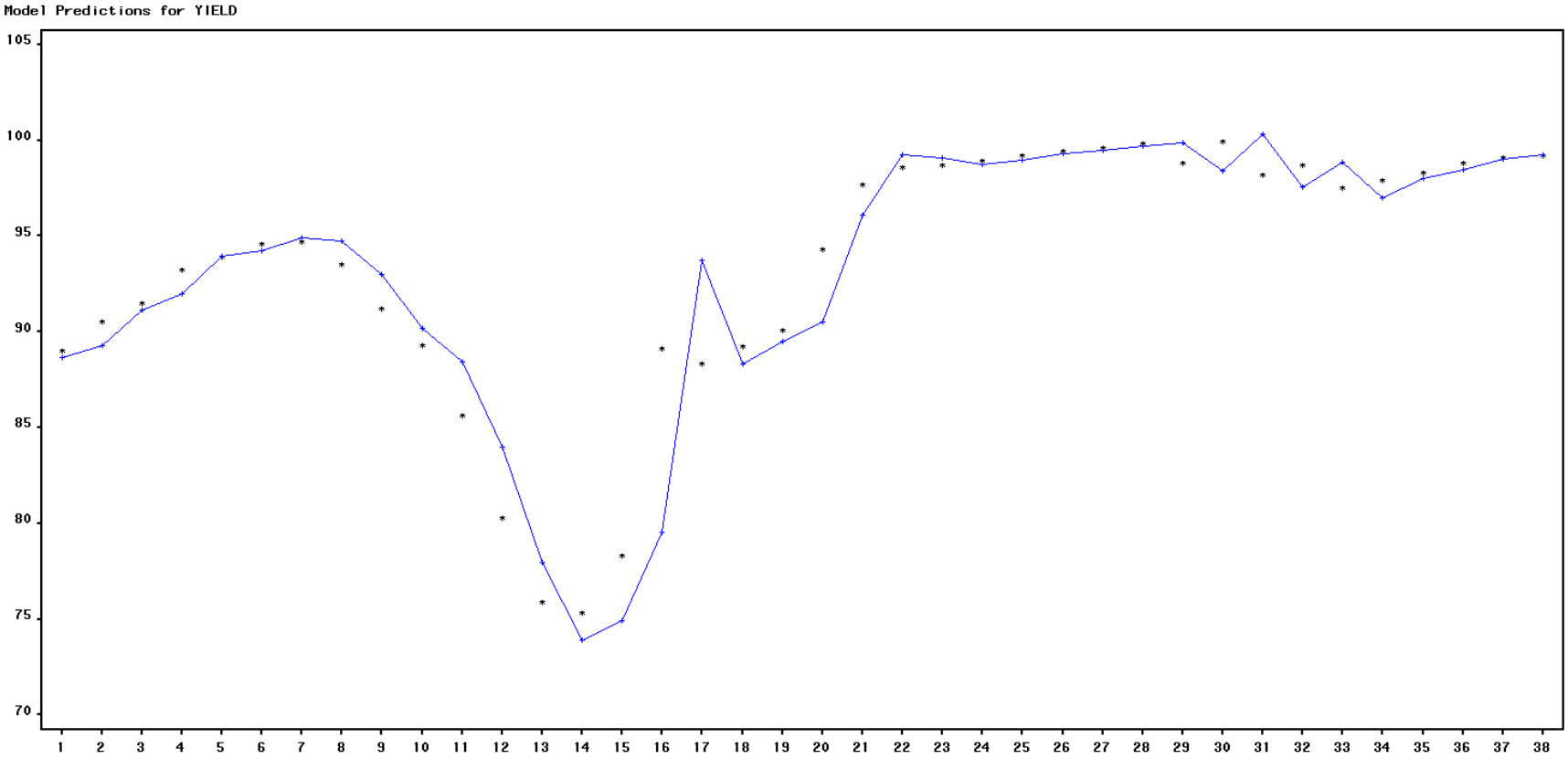

Figure 1 depicts a time series graph for predicted values ((+) signs) using the training data ((*) signs) set for Yield variable, as reported by SAS(AFS).

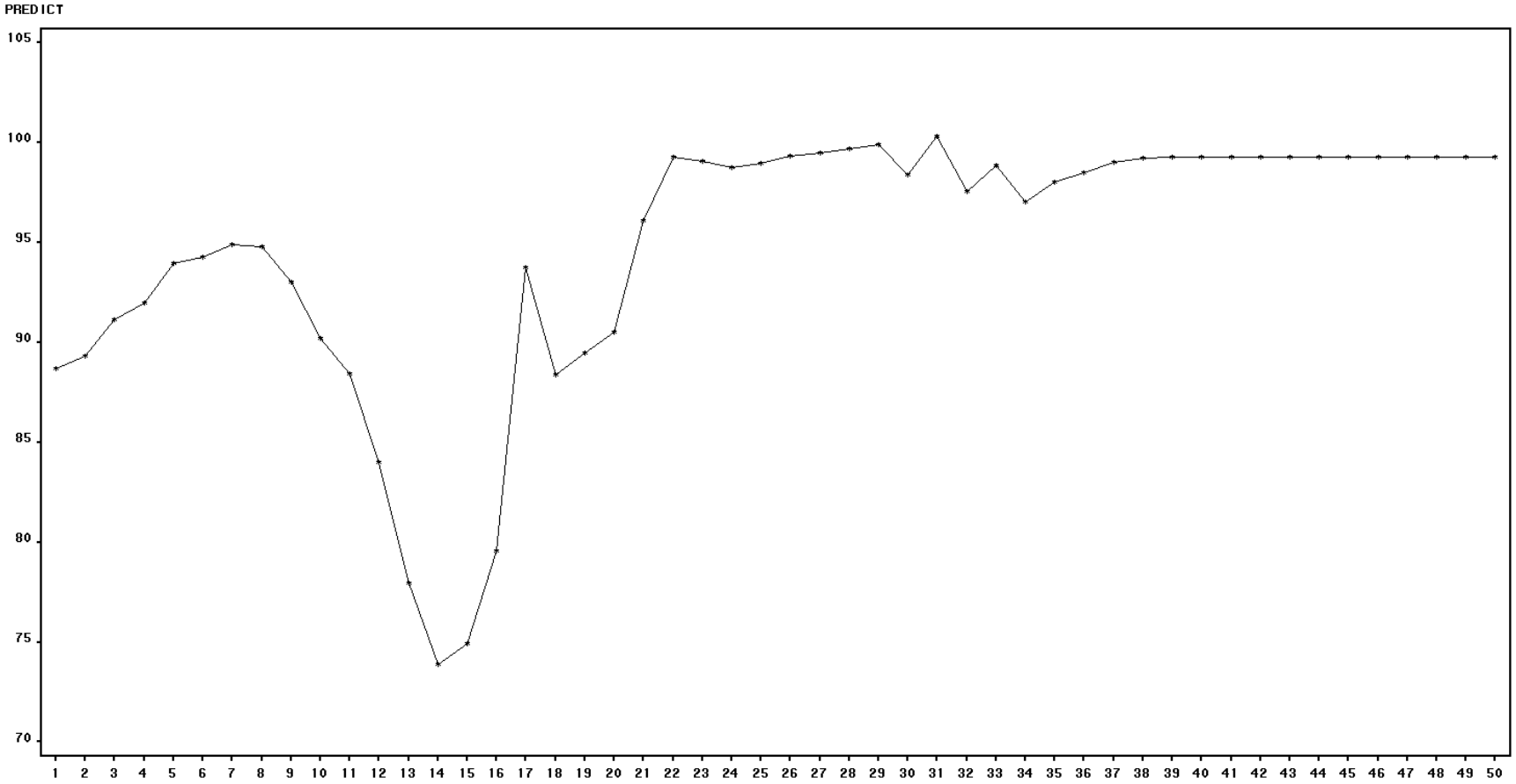

Figure 2 shows a time series graph for predicted values ((+) signs) for Yield variable as reported by SAS(AFS), along with 12-hour forecasted values for the yield variable. Here, predicted and forecast values are plotted using ((*) signs).

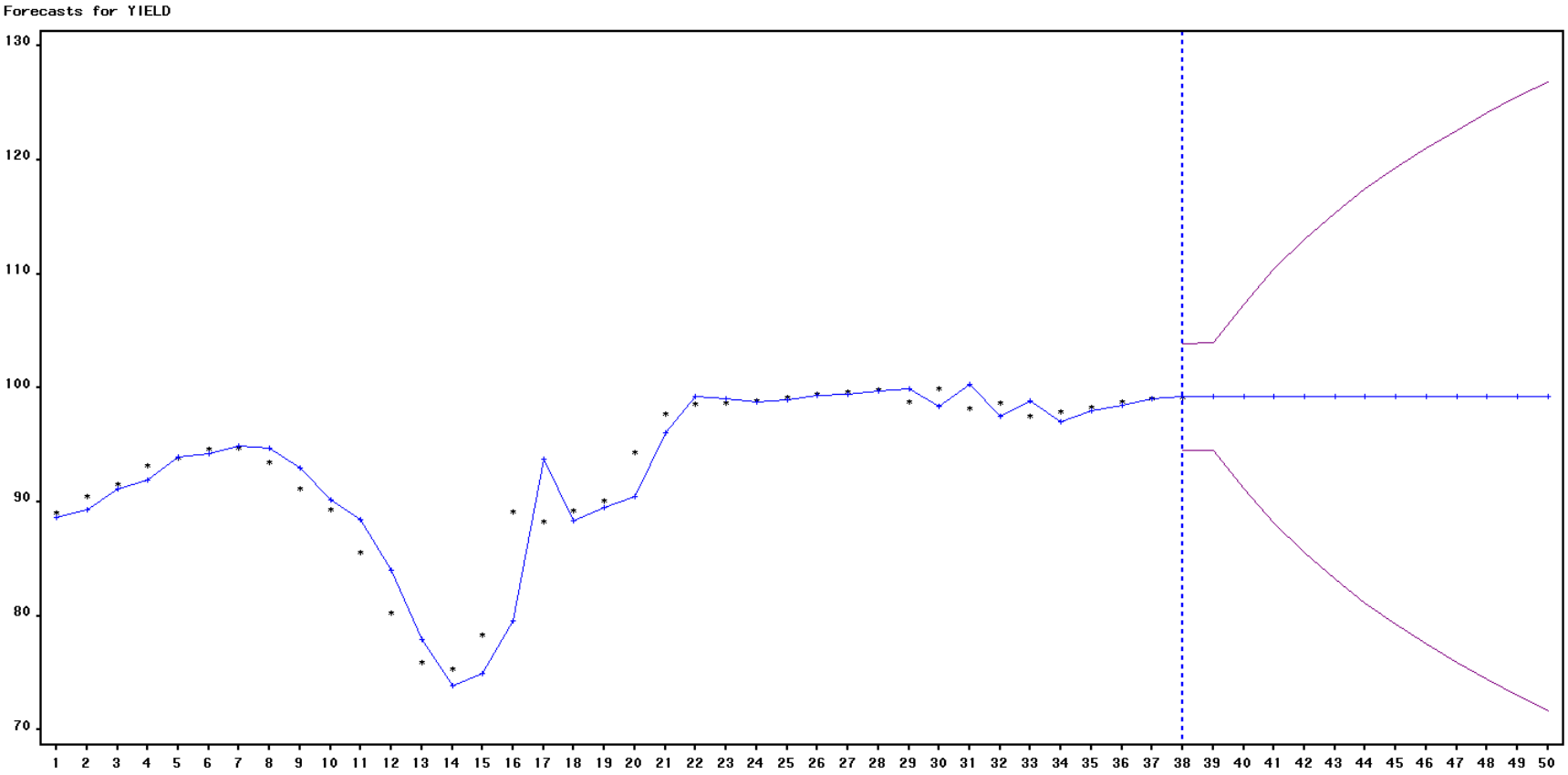

Figure 3a depicts a time series graph for predicted values ((+) signs) and the training data ((*) signs) for the Yield variable, as reported by SAS(AFS), along with 12-hour forecasted values and prediction limits.

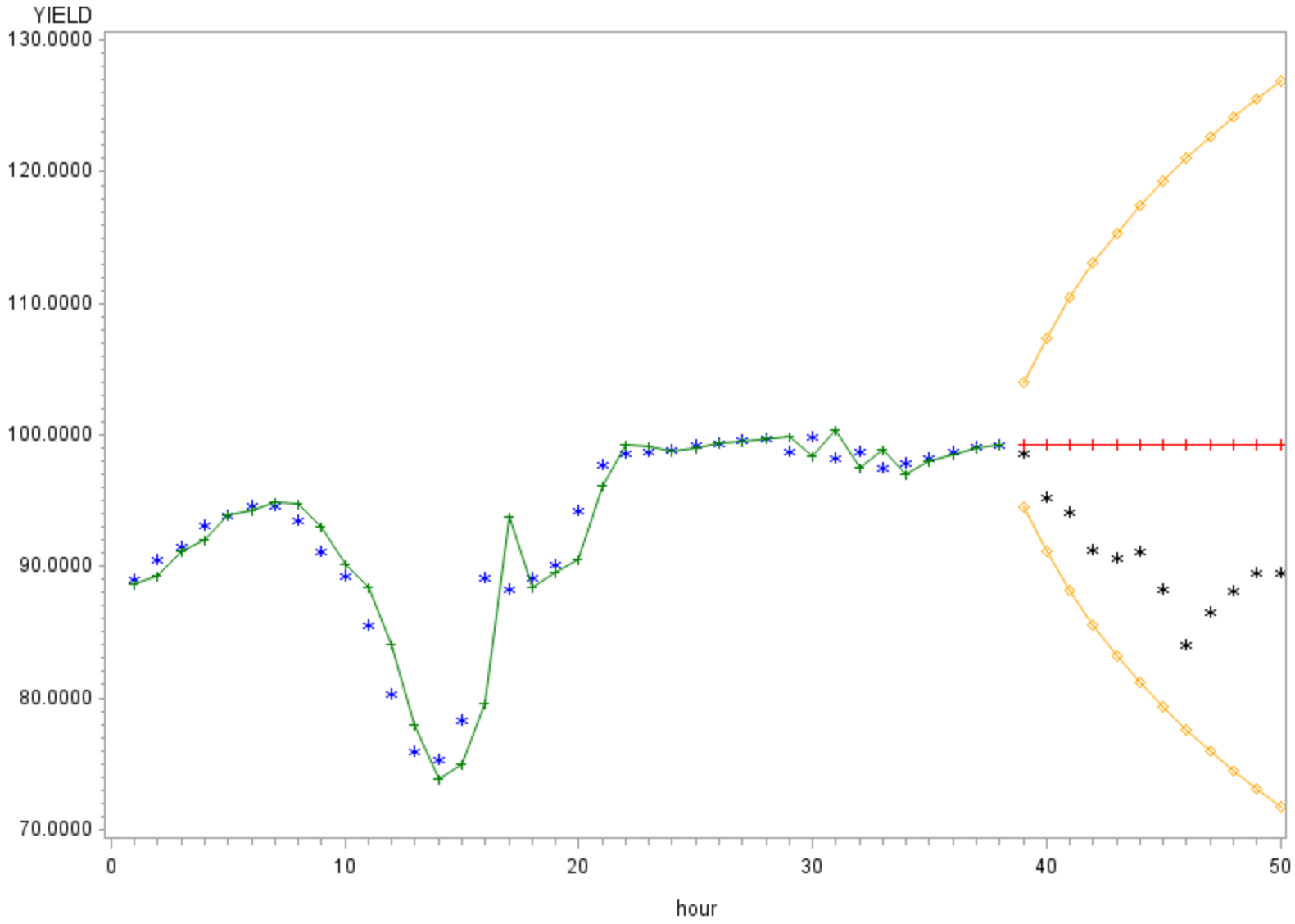

Figure 3b shows a time series graph for predicted values (green-colored (+) signs) for the Yield variable as reported by SAS(AFS), along with 12-hour forecasted values, which are plotted using red-colored plus (+) signs. The training data are plotted using blue-colored star (*) signs, and testing data are plotted using black-colored star (*) signs, while the 12-hour confidence limits are yellow-colored.

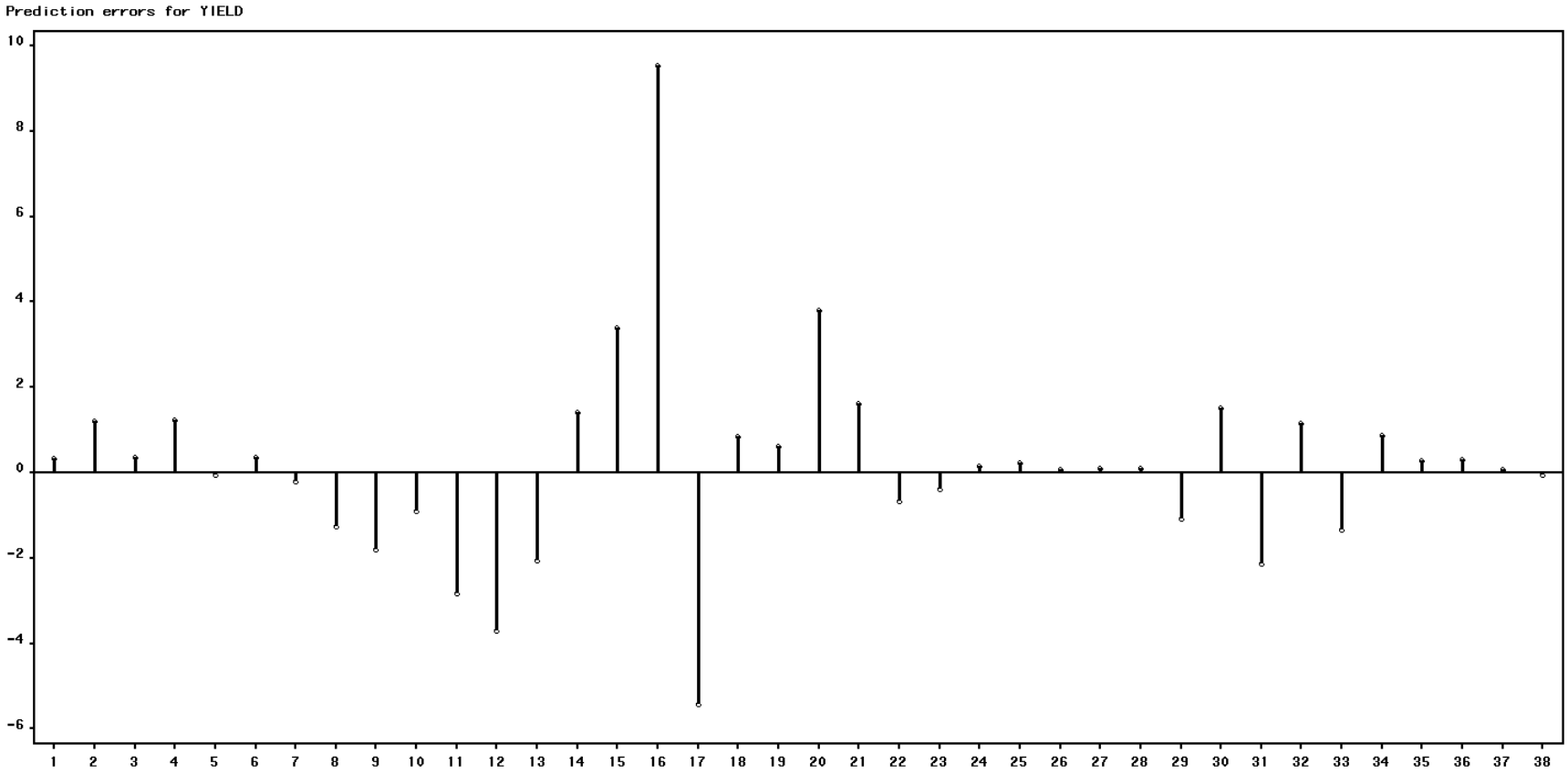

Figure 4. Prediction errors by time series values for the training data set.

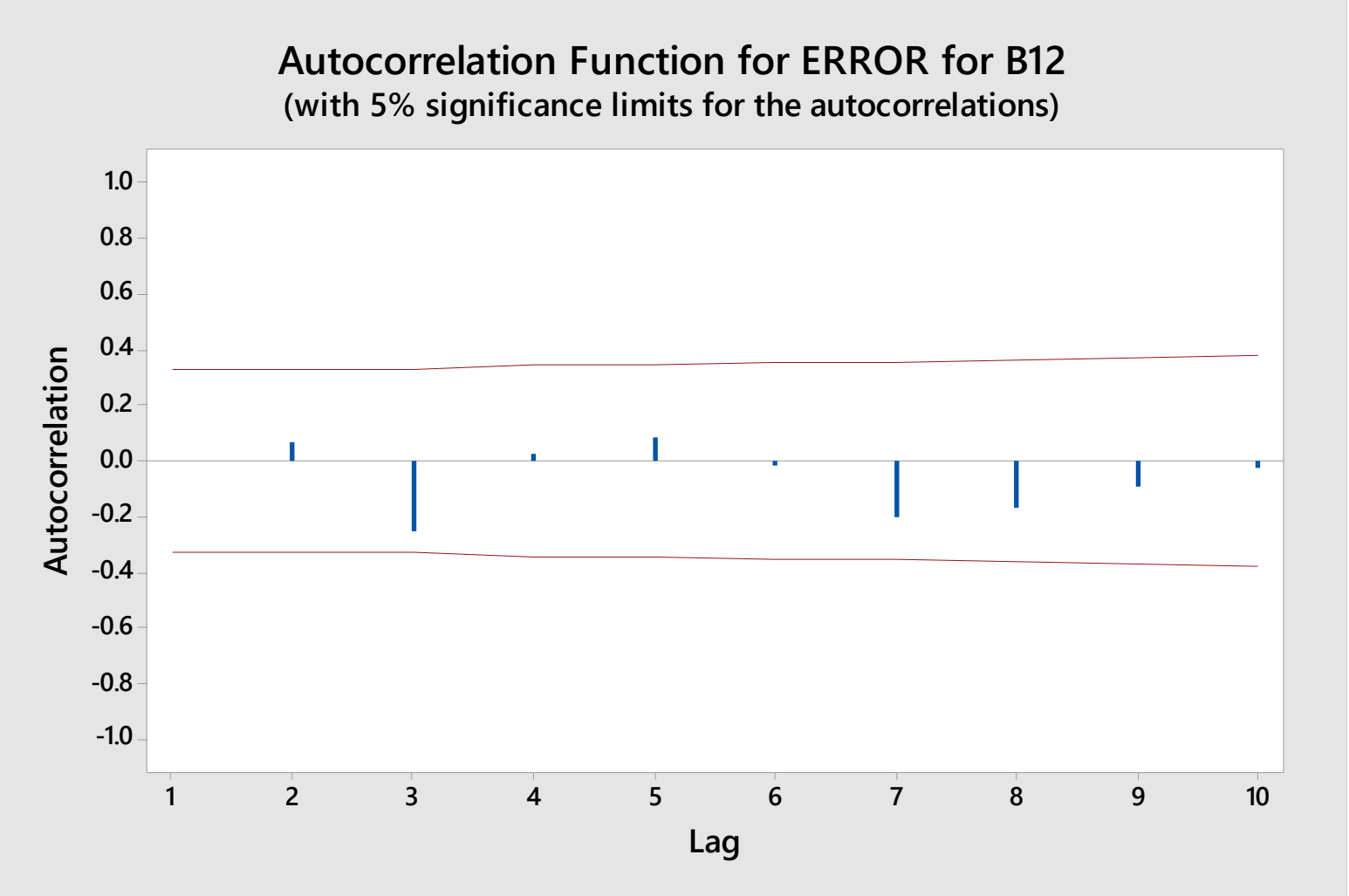

Figure 5. Autocorrelation Function (ACF) for the training data set.

Accuracy Measure for Table B.12 data.

Actual y_t	Predict $\widehat{y_t}$(t-1)	Forecast error $e_t(1) = y_t - \widehat{y_t}(t-1)$	Absolute error $\lvert e_t(1)\rvert$	Square error $[e_t(1)]^2$	Relative error $re_t(1) = 100(e_t(1)/y_t)$	Absolute percentage error $\lvert re_t(1)\rvert$
98.6	99.2501	-0.6501	0.6501	0.423	-0.6593	0.6593
95.3	99.2729	-3.9729	3.9729	15.784	-4.1688	4.1688
94.2	99.2835	-5.0835	5.0835	25.842	-5.3965	5.3965
91.3	99.2885	-7.9885	7.9885	63.816	-8.7497	8.7497
90.6	99.2908	-8.6908	8.6908	75.530	-9.5925	9.5925
91.2	99.2919	-8.0919	8.0919	65.479	-8.8727	8.8727
88.3	99.2924	-10.9924	10.9924	120.833	-12.4489	12.4489
84.1	99.2926	-15.1926	15.1926	230.815	-18.0649	18.0649
86.5	99.2928	-12.7928	12.7928	163.656	-14.7894	14.7894
88.2	99.2928	-11.0928	11.0928	123.050	-12.5769	12.5769
89.5	99.2928	-9.7928	9.7928	95.899	-10.9417	10.9417
89.5	99.2928	-9.7928	9.7928	95.899	-10.9417	10.9417

Table A. Error Measures table B12 time series data.

Data set	**MSE**	$\widehat{\sigma_{e(1)}}$	**MAD**	**MPE**	**MAPE**
Full data (50 data)	5.08408	2.25479	1.47190	-0.0012371	1.65275
Training data (38 data)	5.25790	2.29301	1.40580	0.15337	1.57969
Holdout data (12 data)	89.7521	9.47376	8.67783	-9.76692	9.76692

Table B. Accuracy Measures table B12 time series data.

Table B provides SAS (AFS)® forecast errors for the full data set, the training data set, and the holdout (or test) data set. The Damped Trend Exponential Smoothing model was suggested as the "best model" for this data set. SAS (AFS)® and MINITAB® produced the accuracy measures shown in Table B, which comprise the Mean Square Error (MSE), the Standard Deviation of the Forecast Errors (SDFE), the Mean Absolute Deviation (MAD), the Mean Percent Forecast Error (MPE), and Mean Absolute Percent Error (MAPE).

Here, the accuracy statistics are computed as follows:

Mean square error:

$$\textbf{MSE} = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = 89.7521$$

Standard Deviation of Forecast Error (Root Mean Square Error):

$$\widehat{\sigma_{e(1)}} = \sqrt{MSE} = \sqrt{89.7521} = 9.47376$$

Mean Absolute Deviation (Mean Absolute Error):

$$\textbf{MAD} = \frac{1}{n}\sum_{t=1}^{n} |e_t(1)| = 8.67783$$

Mean Percent Forecast Error:

$$\textbf{MPE} = \frac{1}{n}\sum_{t=1}^{n} re_t(1) = -9.76692$$

Mean Absolute Percent Error:

$$\textbf{MAPE} = \frac{1}{n}\sum_{t=1}^{n} |re_t(1)| = 9.76692$$

Worksheet 1 ***

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	Actual	Predict	Forecast_Error	Abs_Error	Sq_Error	Relative_Error	Abs_Percentage_Error	MSE	Root_MSE	MAD	MPE	MAPE
1	98.6	99.2501	-0.6501	0.6501	0.423	-0.6593	0.6593	89.7521	9.47376	8.67783	-9.76692	9.76692
2	95.3	99.2729	-3.9729	3.9729	15.784	-4.1688	4.1688					
3	94.2	99.2835	-5.0835	5.0835	25.842	-5.3965	5.3965					
4	91.3	99.2885	-7.9885	7.9885	63.816	-8.7497	8.7497					
5	90.6	99.2908	-8.6908	8.6908	75.530	-9.5925	9.5925					
6	91.2	99.2919	-8.0919	8.0919	65.479	-8.8727	8.8727					
7	88.3	99.2924	-10.9924	10.9924	120.833	-12.4489	12.4489					
8	84.1	99.2926	-15.1926	15.1926	230.815	-18.0649	18.0649					
9	86.5	99.2928	-12.7928	12.7928	163.656	-14.7894	14.7894					
10	88.2	99.2928	-11.0928	11.0928	123.050	-12.5769	12.5769					
11	89.5	99.2928	-9.7928	9.7928	95.899	-10.9417	10.9417					
12	89.5	99.2928	-9.7928	9.7928	95.899	-10.9417	10.9417					

Table C. MINITAB® 17 Calculation of Forecast Errors for Holdout Data.

The first two columns of Table C consist of the actual values of the holdout data set and the SAS (AFS)® forecasted values, based on training data. The remaining columns give forecast errors, the absolute forecast errors, the squared forecast errors, the relative forecast errors and absolute forecast percentage errors using the MINITAB® calculator function.

Obs	DATE	ACTUAL	PREDICT	UPPER	LOWER	ERROR	STD	NERROR	_LEVEL_	_TREND_
39	39	.	99.2501	103.9330	94.5672	.	2.3893	.	99.2501	0.0487
40	40	.	99.2729	107.3952	91.1505	.	4.1441	.	99.2729	0.0228
41	41	.	99.2835	110.4239	88.1431	.	5.6840	.	99.2835	0.0106
42	42	.	99.2885	113.0547	85.5223	.	7.0237	.	99.2885	0.004979
43	43	.	99.2908	115.3672	83.2145	.	8.2024	.	99.2908	0.002328
44	44	.	99.2919	117.4324	81.1514	.	9.2555	.	99.2919	0.001089
45	45	.	99.2924	119.3050	79.2798	.	10.2107	.	99.2924	0.000509
46	46	.	99.2926	121.0253	77.5600	.	11.0883	.	99.2926	0.000238
47	47	.	99.2928	122.6226	75.9629	.	11.9032	.	99.2928	0.000111
48	48	.	99.2928	124.1190	74.4666	.	12.6667	.	99.2928	0.0000521
49	49	.	99.2928	125.5309	73.0548	.	13.3870	.	99.2928	0.0000244
50	50	.	99.2928	126.8709	71.7147	.	14.0707	.	99.2928	0.0000114

Table D. Forecasted values for the last 12 hour of Holdout Data.

Table D contains forecasted values for the final twelve month for the Yield variable, and the corresponding lower and upper 95% prediction limits. The Root Mean Square Prediction Error, Level and Trend smoothing components are also shown..

Model Parameter	Estimate	Std. Error	T	Prob>|T|
LEVEL Smoothing Weight	0.96592	0.5431	1.7786	0.0840
TREND Smoothing Weight	0.99900	2.7212	0.3671	0.7157
DAMPING Smoothing Weight	0.46766	0.4863	0.9617	0.3428
Residual Variance (sigma squared)	5.70857	.	.	.
Smoothed Level	99.20141	.	.	.
Smoothed Trend	0.10409	.	.	.

Table E. Parameter Estimates for Training Data using the Damped Trend Exponential Smoothing model suggested by SAS (AFS)®.

Results

Figure 3b reveals the model fits the data appropriately, however, the SAS(AFS) could not forecast values with acceptably small prediction error. Forecast values are too far from testing values. There are no spikes in the ACF function, which means forecast errors are uncorrelated. The MSE, RMSE, MAD and MAPE, all measure of accuracy are very large for holdout out compare to training data; that is, the variability in forecast errors are not similar.

TABLE B.13 (US PRODCUTION OF ICE CREAM AND FROZEN YOGURT) ANALYSIS

Table B.13 comprises data on the US Production of Ice Cream and Frozen Yogurt from 1950 to 2000, where the Ice Cream (in 10^3 gal) variable is collected yearly. The data set excludes last twelve year values as a holdout set in order to analyze the accuracy of SAS(AFS), which suggested a Linear (Holt) Exponential Smoothing model as the "best model" for this training, or initialization, data set. SAS(AFS) also produced the predicted values for the next 12 year. I have calculated different types of forecast errors based on the hold out data and predicted data.

The model equation is

$$Y_t = \mu_t + \beta_t\, t + \varepsilon_t$$

where,

Y_t = observed value i.e. temperature anomaly at time t

μ_t = smoothed level and $\mu_t = 928601$

β_t = smoothed trend and $\beta_t = 8978$

Now, the k-step prediction equation is

$$\hat{Y}_t(k) = L_t + K\, T_t$$

and the smoothing equations are

$$L_t = \alpha\, Y_t + (1-\alpha)\,(L_{t-1} + T_{t-1})$$

and $T_t = \gamma\,(L_t - L_{t-1}) + (1-\gamma)\, T_{t-1}$

where,

L_t = Level estimate and $L_0 = \mu_t = 928601$

T_t = Trend estimate and $T_0 = \beta_t = 8978$

α = level smoothing and $\alpha = 0.75804$

γ = trend smoothing and $\gamma = 0.00192$

Table B.13: US Production of Ice Cream and Frozen Yogurt

Year	Ice Cream, 1000 gal	Frozen Yogurt, 1000 gal	Year	Ice Cream, 1000 gal	Frozen Yogurt, 1000 gal
1950	554,351	-	1975	836,552	-
1951	568,849	-	1976	818,241	-
1952	592,705	-	1977	809,849	-
1953	605,051	-	1978	815,360	-
1954	596,821	-	1979	811,079	-
1955	628,525	-	1980	829,798	-
1956	641,333	-	1981	832,450	-
1957	650,583	-	1982	852,072	-
1958	657,175	-	1983	881,543	-
1959	698,931	-	1984	894,468	-
1960	697,552	-	1985	901,449	-
1961	697,151	-	1986	923,597	-
1962	704,428	-	1987	928,356	-

Year	Ice Cream, 1000 gal	Frozen Yogurt, 1000 gal	Year	Ice Cream, 1000 gal	Frozen Yogurt, 1000 gal
1963	717,597	-	1988	882,079	-
1964	738,743	-	1989	831,159	82,454
1965	757,000	-	1990	823,610	117,577
1966	751,159	-	1991	862,638	147,137
1967	745,409	-	1992	866,110	134,067
1968	773,207	-	1993	866,248	149,933
1969	765,501	-	1994	876,097	150,565
1970	761,732	-	1995	862,232	152,097
1971	765,843	-	1996	878,572	114,168
1972	767,750	-	1997	913,770	92,167
1973	773,674	-	1998	937,485	87,777
1974	781,971	-	2000	979,645	94,478

The collection of Figures and Tables that follow are individually identified below.

List of Figures:

List of Tables:

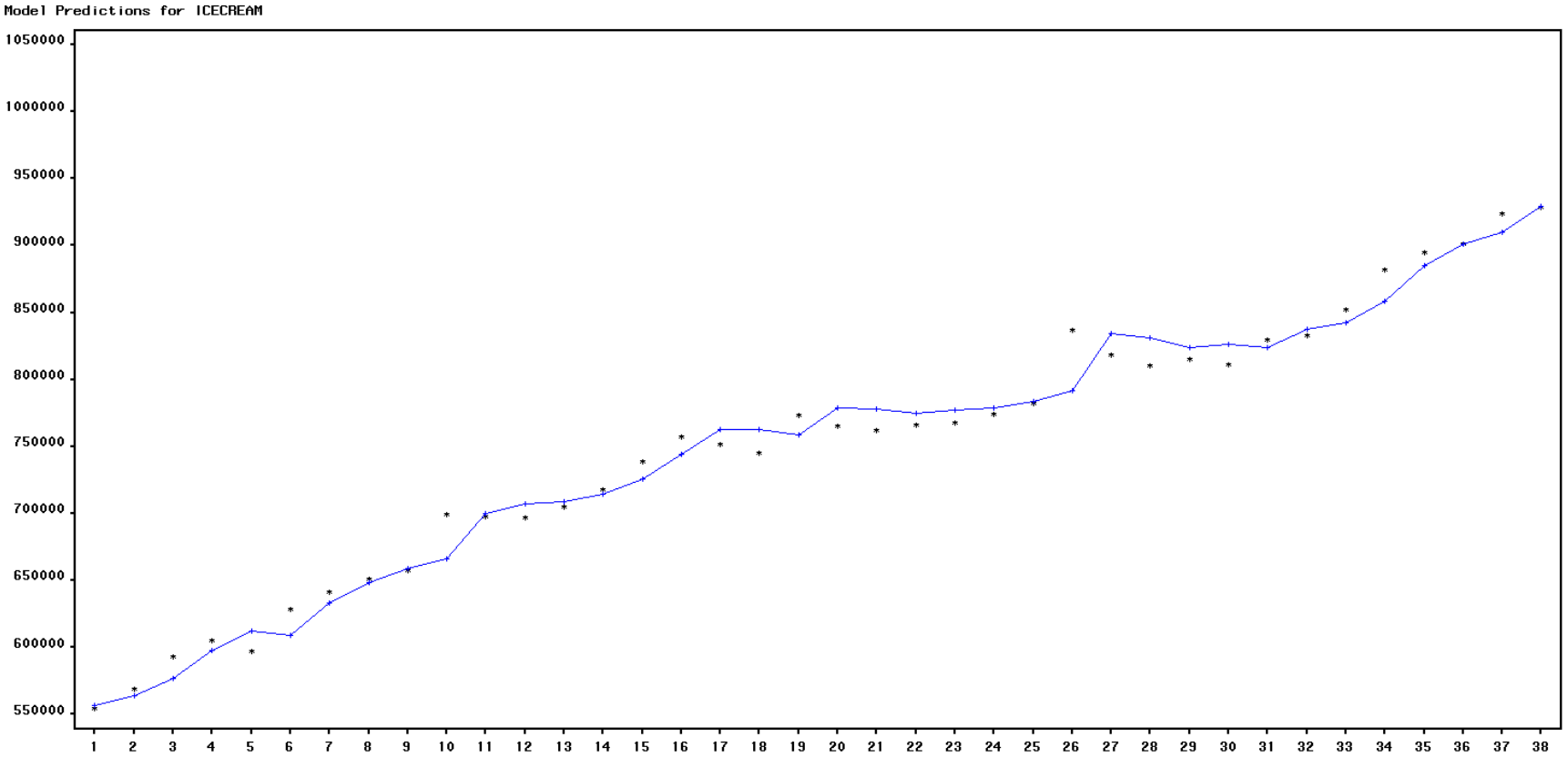

Figure 1 depicts a time series graph for predicted values ((+) signs) using the training data ((*) signs) set for Ice Cream variable, as reported by SAS(AFS).

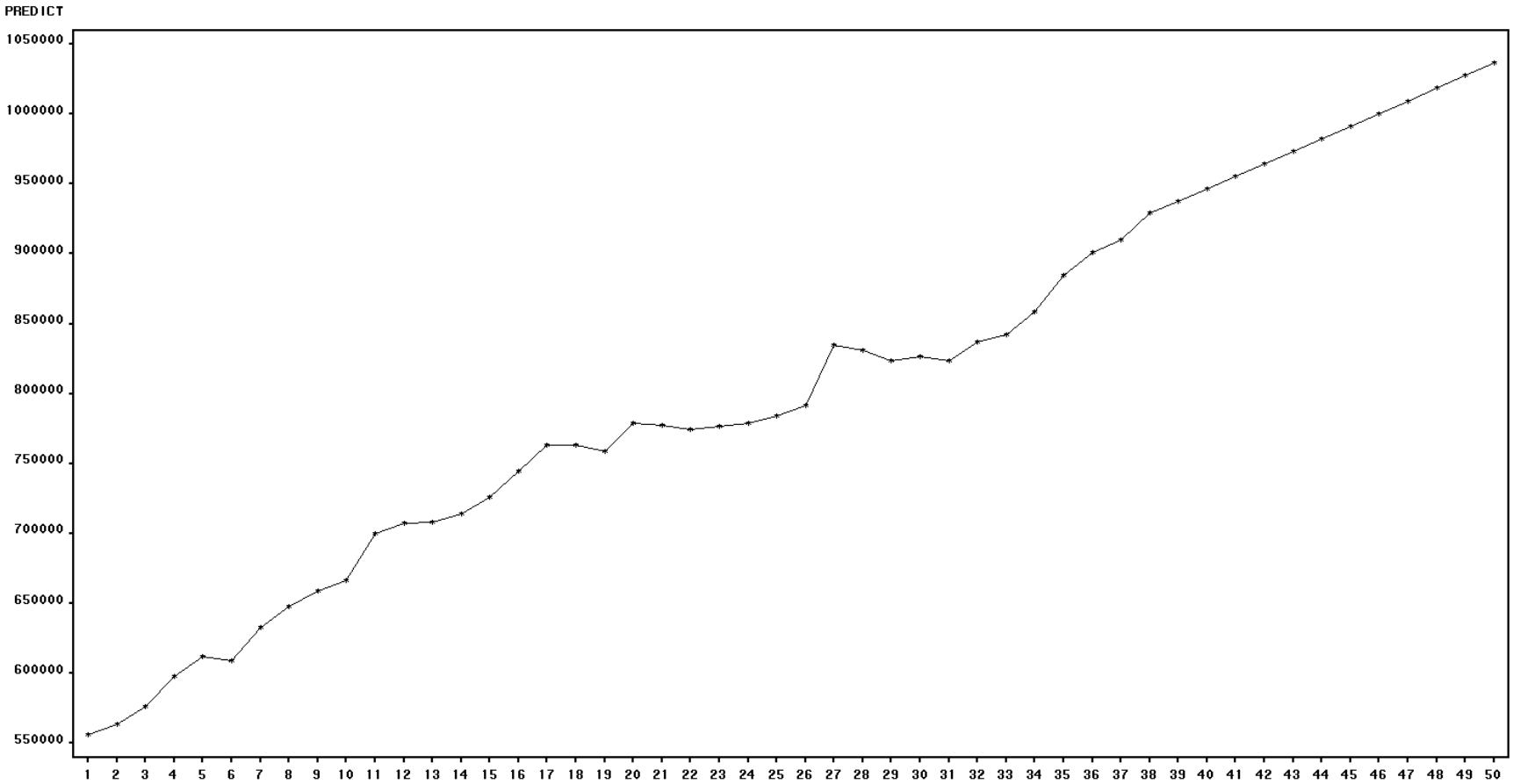

Figure 2 shows a time series graph for predicted values ((+) signs) for Ice Cream variable as reported by SAS(AFS), along with 12-year forecasted values for the yield variable. Here, predicted and forecast values are plotted using ((*) signs).

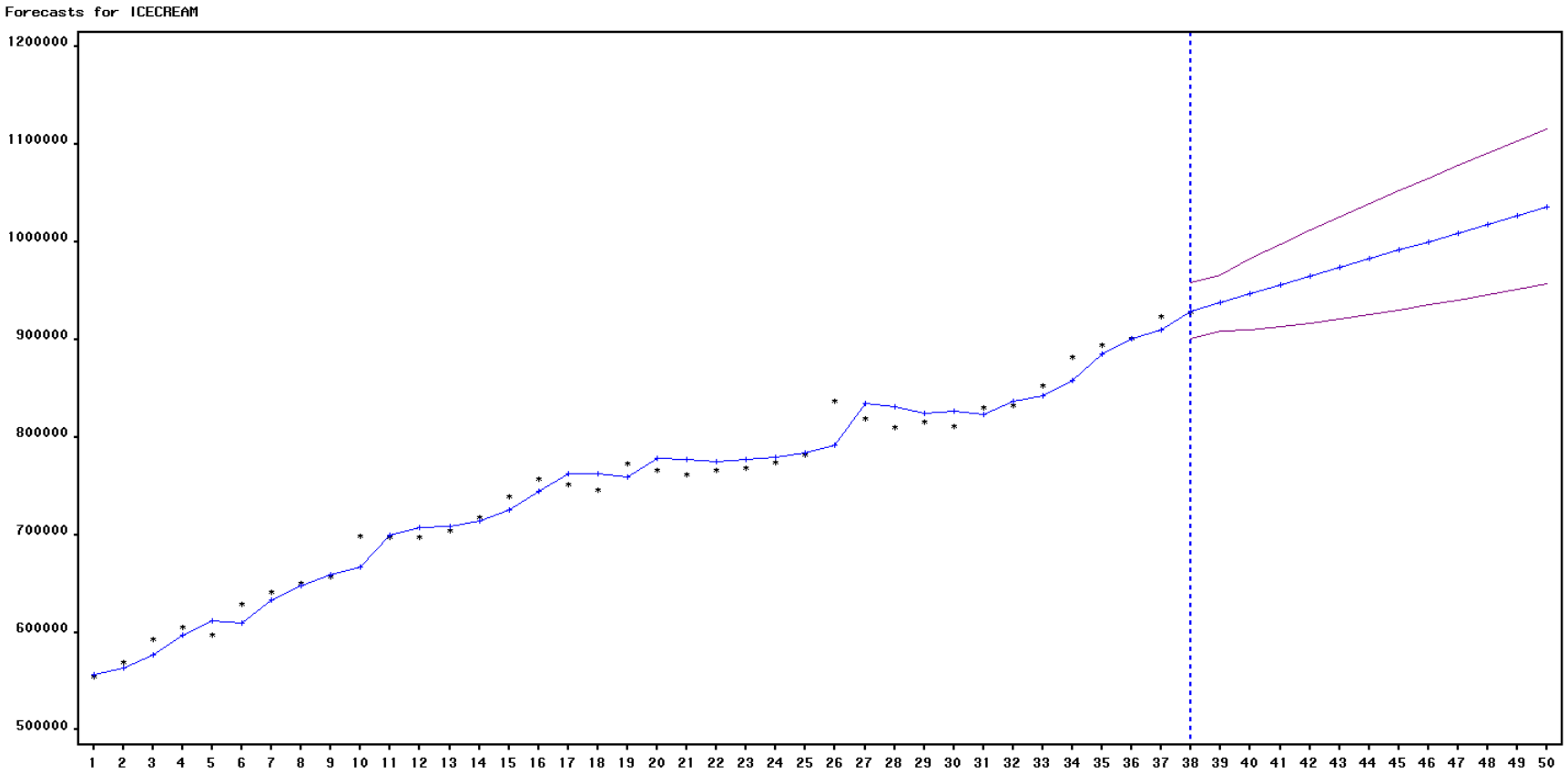

Figure 3a depicts a time series graph for predicted values ((+) signs) and the training data ((*) signs) for the Ice Cream variable, as reported by SAS(AFS), along with 12-year forecasted values and prediction limits.

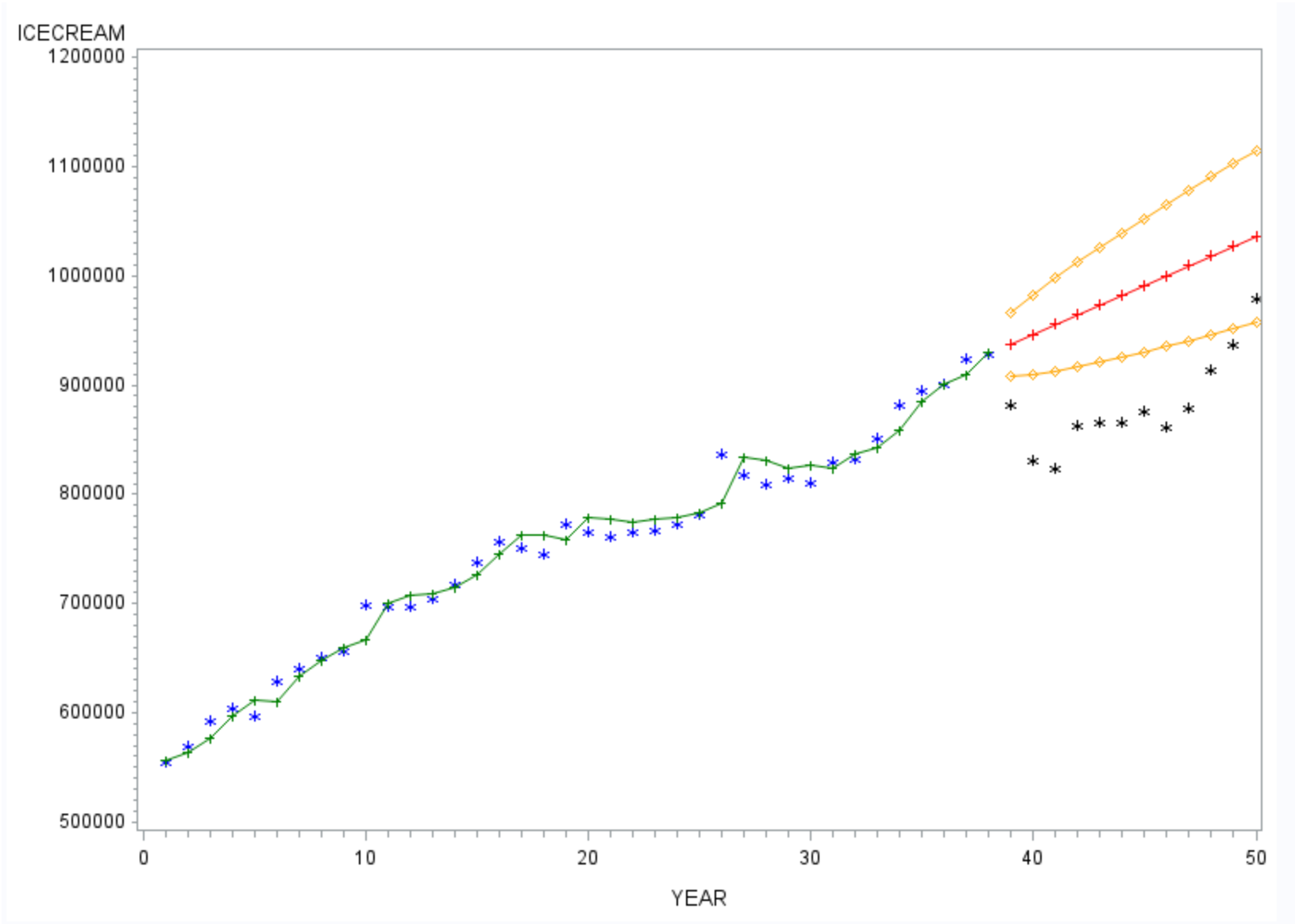

Figure 3b shows a time series graph for predicted values (green-colored (+) signs) for the Ice cream variable as reported by SAS(AFS), along with 12-year forecasted values, which are plotted using red-colored plus (+) signs. The training data are plotted using blue-colored star (*) signs, and testing data are plotted using black-colored star (*) signs, while the 12-year confidence limits are yellow-colored.

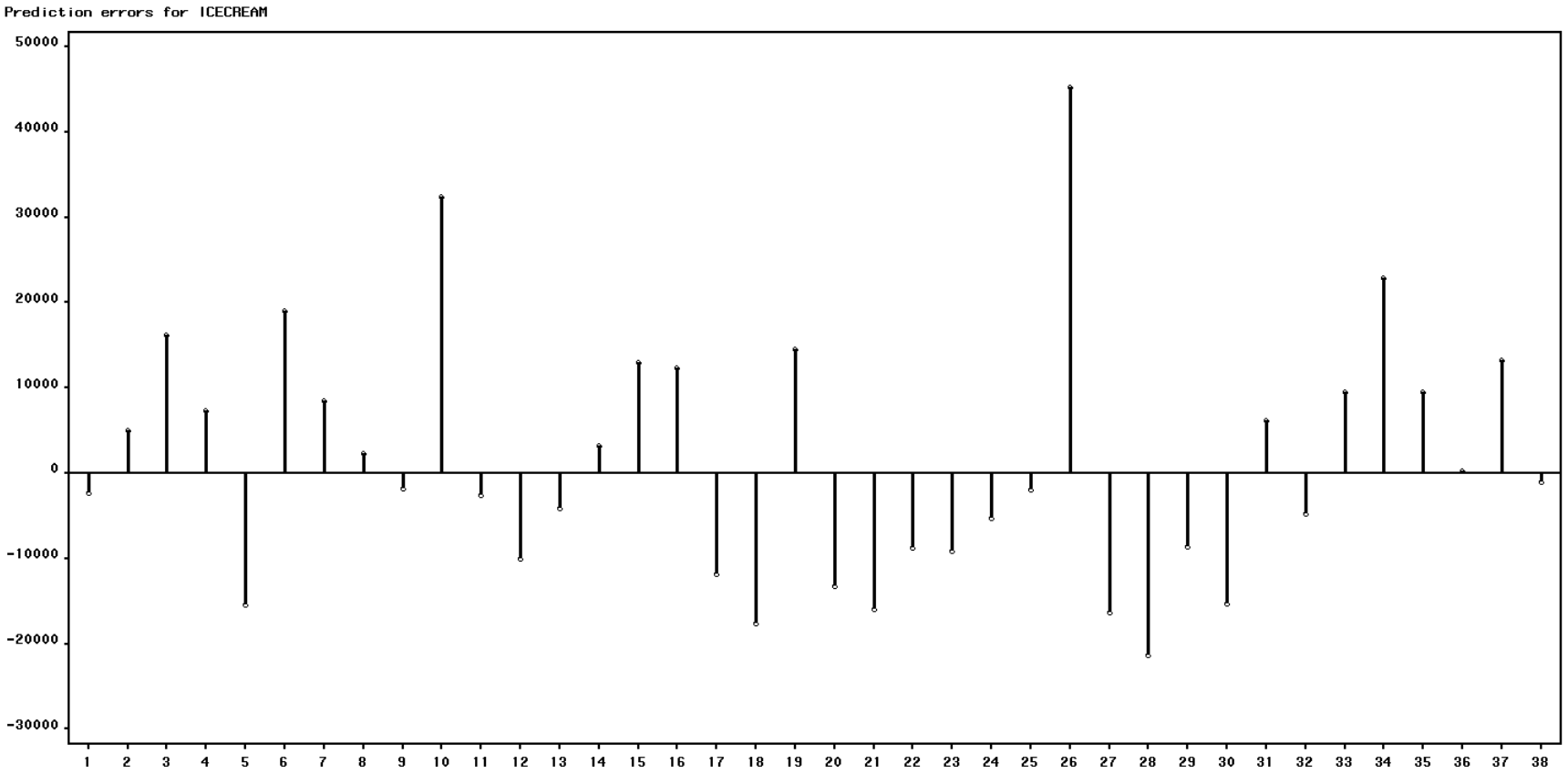

Figure 4. Prediction errors by time series values for the training data set.

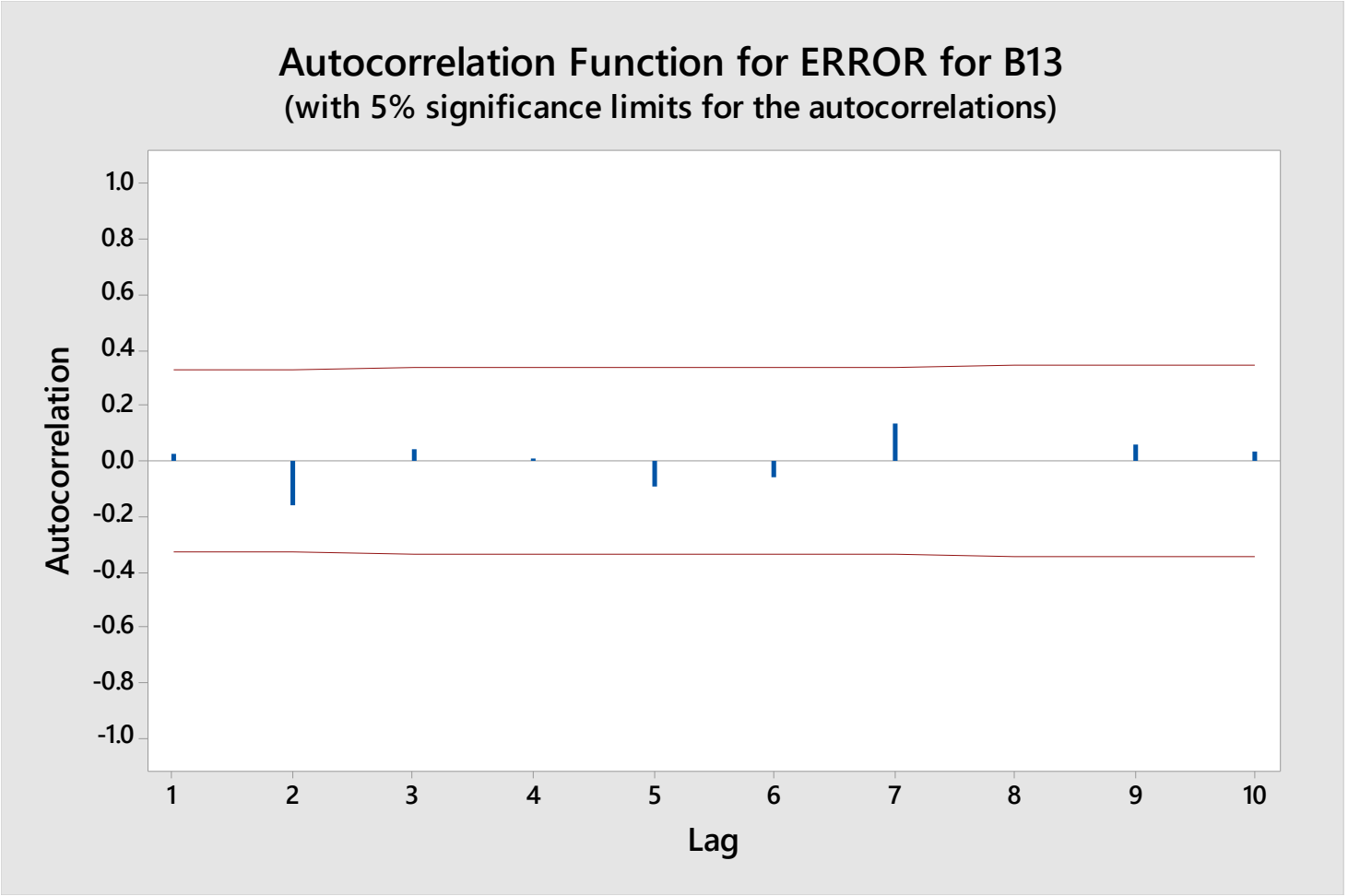

Figure 5. Autocorrelation Function (ACF) for the training data set.

Accuracy Measure for Table B.13 data.

Actual y_t	Predict $\widehat{y_t}$(t-1)	Forecast error $e_t(1) = y_t - \widehat{y_t}$(t-1)	Absolute error $\lvert e_t(1)\rvert$	Square error $[e_t(1)]^2$	Relative error $re_t(1) = 100(e_t(1)/y_t)$	Absolute percentage error $\lvert re_t(1)\rvert$
882079	937579	-55500	55500	3.08025E+09	-6.2920	6.2920
831159	946557	-115398	115398	1.33167E+10	-13.8840	13.8840
823610	955535	-131925	131925	1.74042E+10	-16.0179	16.0179
862638	964514	-101876	101876	1.03787E+10	-11.8098	11.8098
866110	973492	-107382	107382	1.15309E+10	-12.3982	12.3982
866248	982470	-116222	116222	1.35076E+10	-13.4167	13.4167
876097	991448	-115351	115351	1.33059E+10	-13.1665	13.1665
862232	1000427	-138195	138195	1.90979E+10	-16.0276	16.0276
878572	1009405	-130833	130833	1.71173E+10	-14.8916	14.8916
913770	1018383	-104613	104613	1.09439E+10	-11.4485	11.4485
937485	1027361	-89876	89876	8.07770E+09	-9.5869	9.5869
979645	1036340	-56695	56695	3.21432E+09	-5.7873	5.7873

Table A. Error Measures table B13 time series data.

Data set	**MSE**	$\widehat{\sigma_{e(1)}}$	**MAD**	**MPE**	**MAPE**
Full data (50 data)	375933968	19389.0	14006.0	0.21061	1.75970
Training data (38 data)	205928998	14350.2	11257.1	0.19435	1.50483
Holdout data (12 data)	1.17479E+10	108388	105322	-12.0606	12.0606

Table B. Accuracy Measures table B13 time series data.

Table B provides SAS (AFS)® forecast errors for the full data set, the training data set, and the holdout (or test) data set. The Linear (Holt) Exponential Smoothing model was suggested as the "best model" for this data set. SAS (AFS)® and MINITAB® produced the accuracy measures shown in Table B, which comprise the Mean Square Error (MSE), the Standard Deviation of the Forecast Errors (SDFE), the Mean Absolute Deviation (MAD), the Mean Percent Forecast Error (MPE), and Mean Absolute Percent Error (MAPE).

Here, the accuracy statistics are computed as follows:

Mean square error:

$$\textbf{MSE} = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = 1.17479E+10$$

Standard Deviation of Forecast Error (Root Mean Square Error):

$$\widehat{\sigma_{e(1)}} = \sqrt{MSE} = \sqrt{1.17479\text{E} + 10} = 108388$$

Mean Absolute Deviation (Mean Absolute Error):

$$\textbf{MAD} = \frac{1}{n}\sum_{t=1}^{n} |e_t(1)| = 105322$$

Mean Percent Forecast Error:

$$\textbf{MPE} = \frac{1}{n}\sum_{t=1}^{n} re_t(1) = -12.0606$$

Mean Absolute Percent Error:

$$\textbf{MAPE} = \frac{1}{n}\sum_{t=1}^{n} |re_t(1)| = 12.0606$$

Worksheet 1 ***

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	Actual	Predict	Forecast_Error	Abs_Error	Sq_Error	Relative_Error	Abs_Percentage_Error	MSE	Root_MSE	MAD	MPE	MAPE
1	882079	937579	-55500	55500	3.08025E+09	-6.2920	6.2920	1.17479E+10	108388	105322	-12.0606	12.0606
2	831159	946557	-115398	115398	1.33167E+10	-13.8840	13.8840					
3	823610	955535	-131925	131925	1.74042E+10	-16.0179	16.0179					
4	862638	964514	-101876	101876	1.03787E+10	-11.8098	11.8098					
5	866110	973492	-107382	107382	1.15309E+10	-12.3982	12.3982					
6	866248	982470	-116222	116222	1.35076E+10	-13.4167	13.4167					
7	876097	991448	-115351	115351	1.33059E+10	-13.1665	13.1665					
8	862232	1000427	-138195	138195	1.90979E+10	-16.0276	16.0276					
9	878572	1009405	-130833	130833	1.71173E+10	-14.8916	14.8916					
10	913770	1018383	-104613	104613	1.09439E+10	-11.4485	11.4485					
11	937485	1027361	-89876	89876	8.07770E+09	-9.5869	9.5869					
12	979645	1036340	-56695	56695	3.21432E+09	-5.7873	5.7873					

Table C. MINITAB® 17 Calculation of Forecast Errors for Holdout Data.

The first two columns of Table C consist of the actual values of the holdout data set and the SAS (AFS)® forecasted values, based on training data. The remaining columns give forecast errors, the absolute forecast errors, the squared forecast errors, the relative forecast errors and absolute forecast percentage errors using the MINITAB® calculator function.

Obs	DATE	ACTUAL	PREDICT	UPPER	LOWER	ERROR	STD	NERROR	_LEVEL_	_TREND_
39	39	.	937579	966476	908682	.	14743	.	937579	8978
40	40	.	946557	982843	910271	.	18514	.	946557	8978
41	41	.	955535	997964	913107	.	21648	.	955535	8978
42	42	.	964514	1012321	916706	.	24392	.	964514	8978
43	43	.	973492	1026149	920835	.	26866	.	973492	8978
44	44	.	982470	1039583	925357	.	29140	.	982470	8978
45	45	.	991448	1052709	930188	.	31256	.	991448	8978
46	46	.	1000427	1065585	935268	.	33245	.	1000427	8978
47	47	.	1009405	1078255	940555	.	35128	.	1009405	8978
48	48	.	1018383	1090749	946017	.	36922	.	1018383	8978
49	49	.	1027361	1103093	951629	.	38639	.	1027361	8978
50	50	.	1036340	1115306	957374	.	40290	.	1036340	8978

Table D. Forecasted values for the last 12 hour of Holdout Data.

Table D contains forecasted values for the final twelve month for the Ice Cream variable, and the corresponding lower and upper 95% prediction limits. The Root Mean Square Prediction Error, Level and Trend smoothing components are also shown..

Model Parameter	Estimate	Std. Error	T	Prob>|T|
LEVEL Smoothing Weight	0.75804	0.1147	6.6107	<.0001
TREND Smoothing Weight	0.00192	0.0208	0.0922	0.9271
Residual Variance (sigma squared)	217369498	.	.	.
Smoothed Level	928601	.	.	.
Smoothed Trend	8978	.	.	.

Table E. Parameter Estimates for Training Data using the Linear (Holt) Exponential Smoothing model suggested by SAS (AFS)®.

Results

In Figure 3b, it seems the model fits the data appropriately, however, the SAS(AFS) forecast values very inaccurately. The testing data values are totally outside of the confidence limit of the forecast values. There are no problem with ACF function. All measure of accuracy i.e., MSE, RMSE, MAD, MPE and MAPE are very big for holdout data than training data set.

TABLE B.14 (ATMOSPHERIC CO_2 CONCENTRATIONS AT MAUNA LOA OBSERVATORY) ANALYSIS

Table B.14 comprises data on the Atmospheric CO_2 Concentration at Mauna Loa Observatory from 1959 to 2003, where the Average CO_2 Concentration (in ppmv) variable is collected yearly. The data set excludes last twelve year values as a holdout set in order to analyze the accuracy of SAS(AFS), which suggested a Linear (Holt) Exponential Smoothing model as the "best model" for this training, or initialization, data set. SAS(AFS) also produced the predicted values for the next 12 year. I have calculated different types of forecast errors based on the hold out data and predicted data.

The model equation is

$$Y_t = \mu_t + \beta_t\, t + \varepsilon_t$$

where,

Y_t = observed value i.e. temperature anomaly at time t

μ_t = smoothed level and $\mu_t = 355.62015$

β_t = smoothed trend and $\beta_t = 1.54583$

Now, the k-step prediction equation is

$$\hat{Y}_t(k) = L_t + K\, T_t$$

and the smoothing equations are

$$L_t = \alpha\, Y_t + (1-\alpha)\,(L_{t-1} + T_{t-1})$$

and $T_t = \gamma\,(L_t - L_{t-1}) + (1-\gamma)\, T_{t-1}$

where,

L_t = Level estimate and $L_0 = \mu_t = 355.62015$

T_t = Trend estimate and $T_0 = \beta_t = 1.54583$

α = level smoothing and $\alpha = 0.99900$

γ = trend smoothing and $\gamma = 0.23151$

Table B.14: Atmospheric CO_2 Concentrations at Mauna Loa Observatory

Year	Average CO2 Concentration, ppmv	Year	Average CO2 Concentration, ppmv
1959	316.00	1982	341.09
1960	316.91	1983	342.75
1961	317.63	1984	344.44
1962	318.46	1985	345.86
1963	319.02	1986	347.14
1964	319.52	1987	348.99
1965	320.09	1988	351.44
1966	321.34	1989	352.94
1967	322.13	1990	354.19
1968	323.11	1991	355.62
1969	324.60	1992	356.36
1970	325.65	1993	357.10

Year	Average CO2 Concentration, ppmv	Year	Average CO2 Concentration, ppmv
1971	326.32	1994	358.86
1972	327.52	1995	360.90
1973	329.61	1996	362.58
1974	330.29	1997	363.84
1975	331.16	1998	366.58
1976	332.18	1999	368.30
1977	333.88	2000	369.47
1978	335.52	2001	371.03
1979	336.89	2002	373.07
1980	338.67	2003	375.61
1981	339.95		

The collection of Figures and Tables that follow are individually identified below.

List of Figures:

List of Tables:

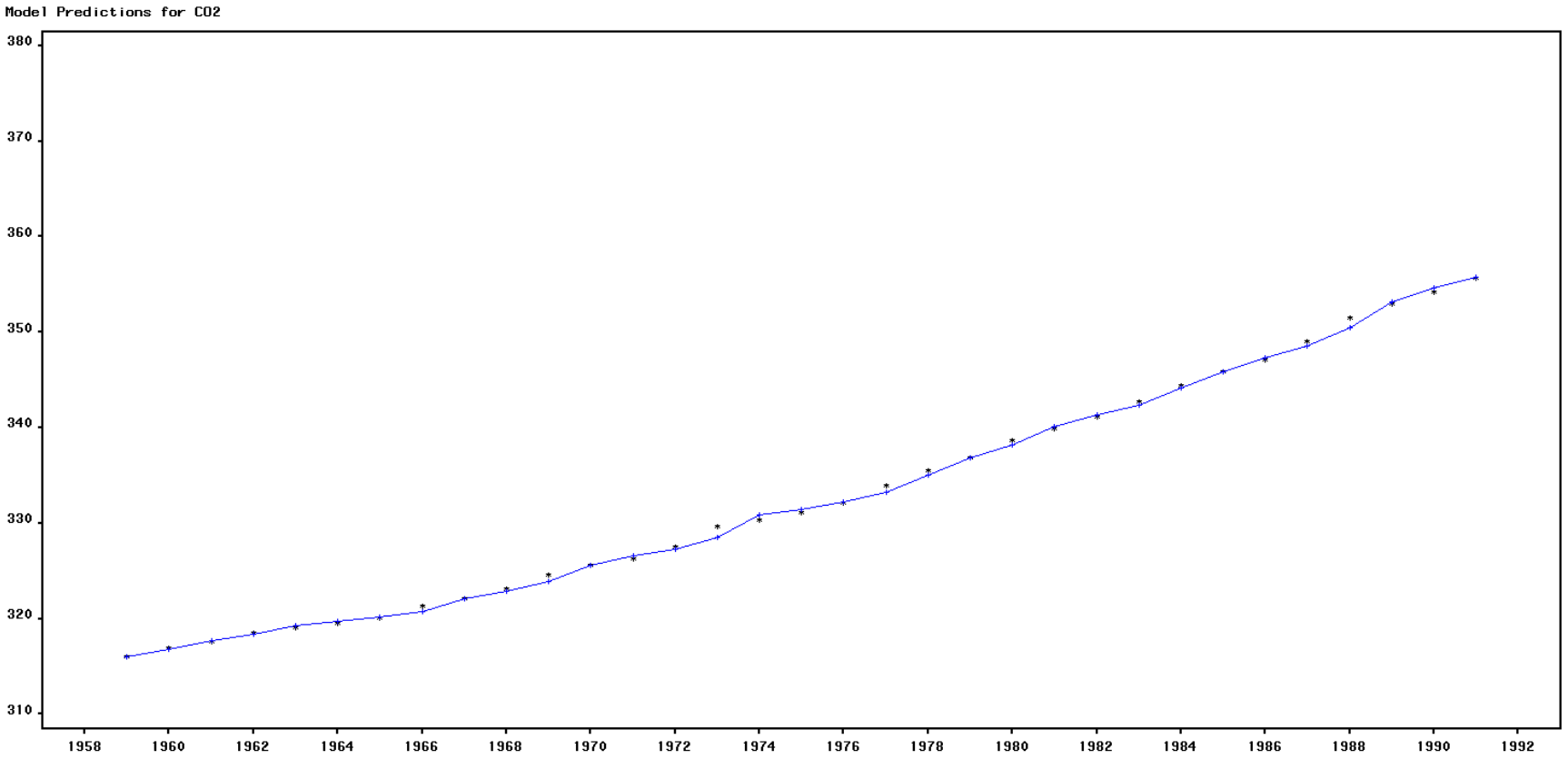

Figure 1 depicts a time series graph for predicted values ((+) signs) using the training data ((*) signs) set for Average CO_2 variable, as reported by SAS(AFS).

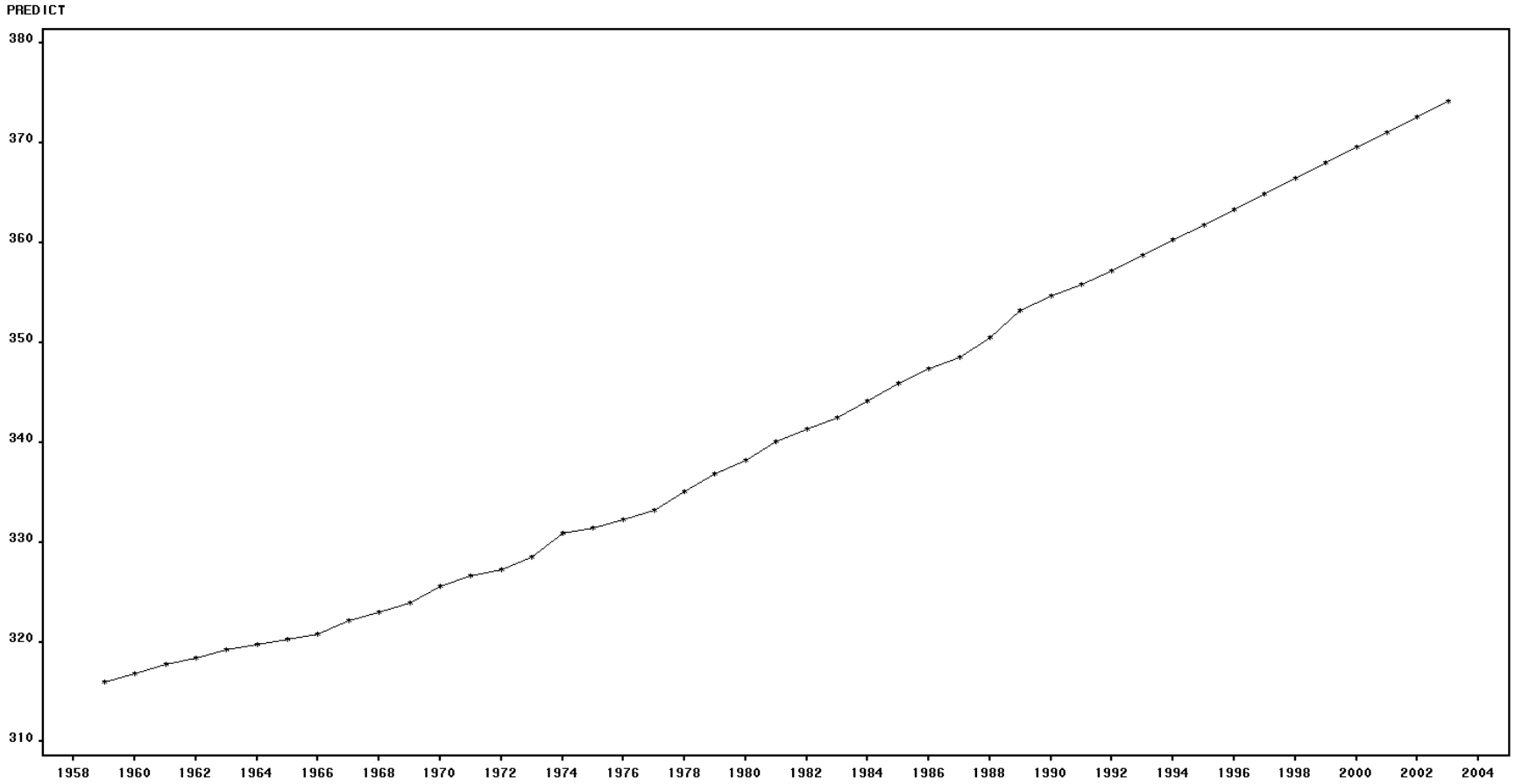

Figure 2 shows a time series graph for predicted values ((+) signs) for Ice Cream variable as reported by SAS(AFS), along with 12-year forecasted values for the average CO_2 variable. Here, predicted and forecast values are plotted using ((*) signs).

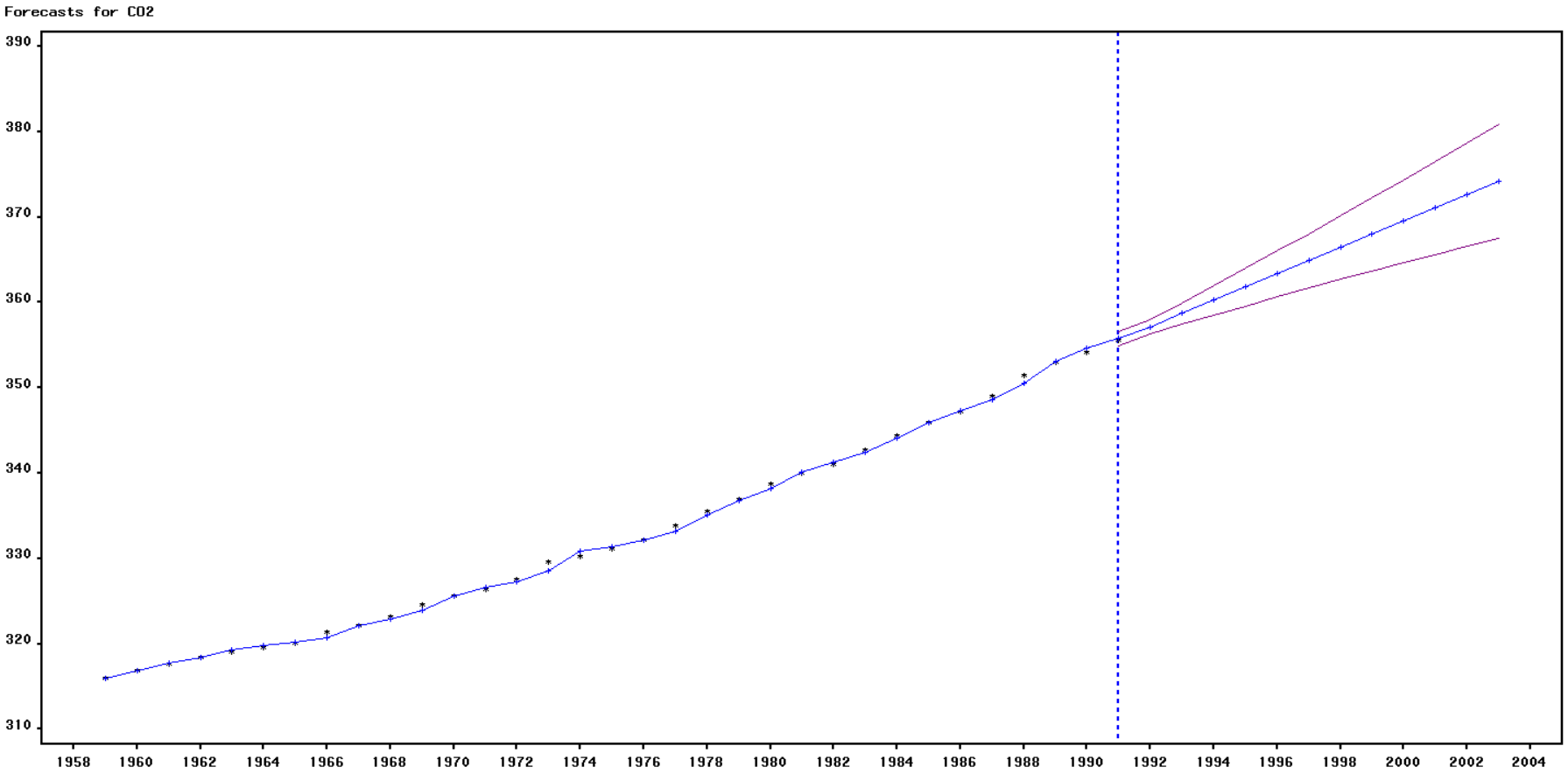

Figure 3a depicts a time series graph for predicted values ((+) signs) and the training data ((*) signs) for the Average CO_2 variable, as reported by SAS(AFS), along with 12-year forecasted values and prediction limits.

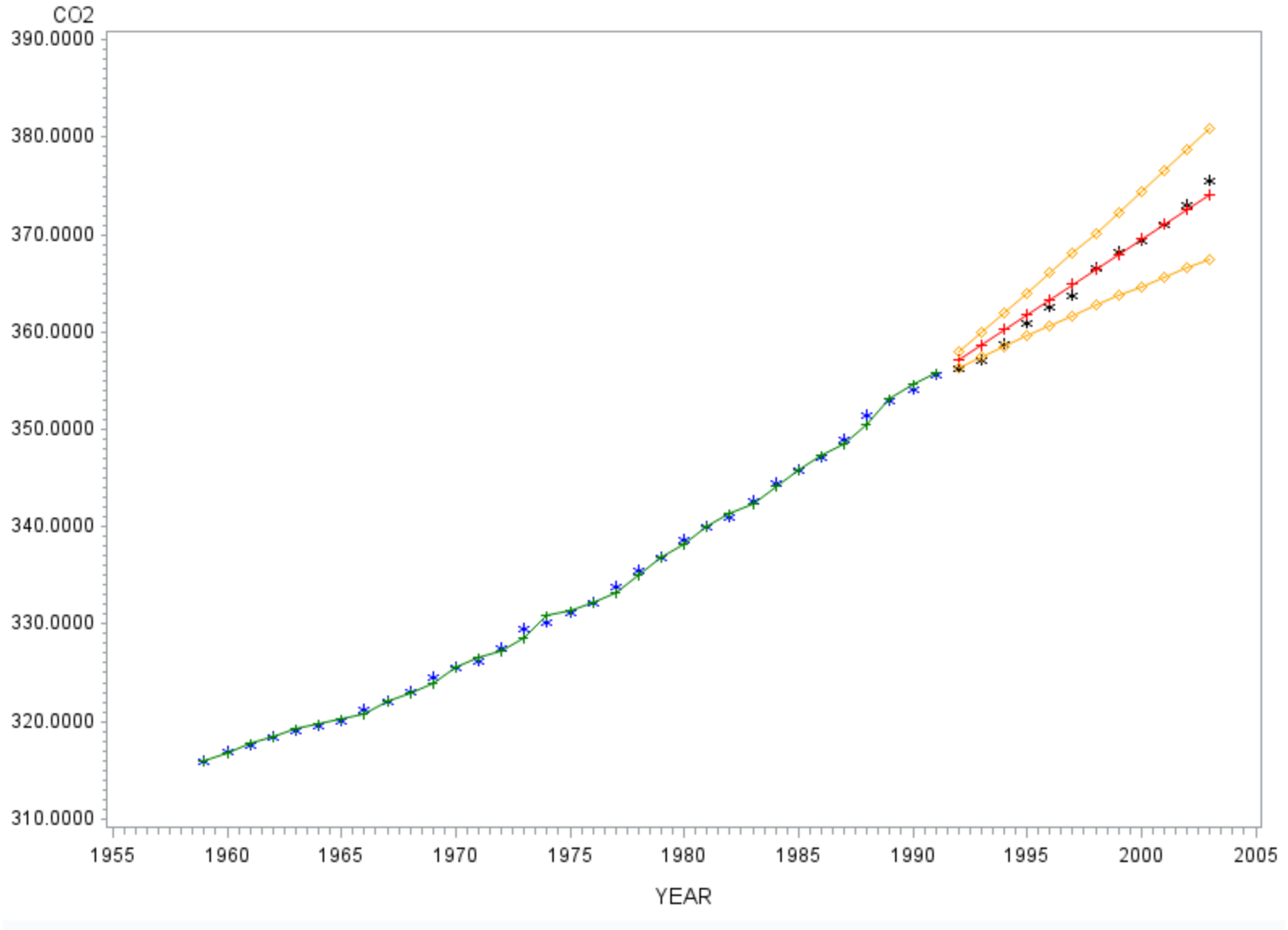

Figure 3b shows a time series graph for predicted values (green-colored (+) signs) for the CO_2 variable as reported by SAS(AFS), along with 12-year forecasted values, which are plotted using red-colored plus (+) signs. The training data are plotted using blue-colored star (*) signs, and testing data are plotted using black-colored star (*) signs, while the 12-year confidence limits are yellow-colored.

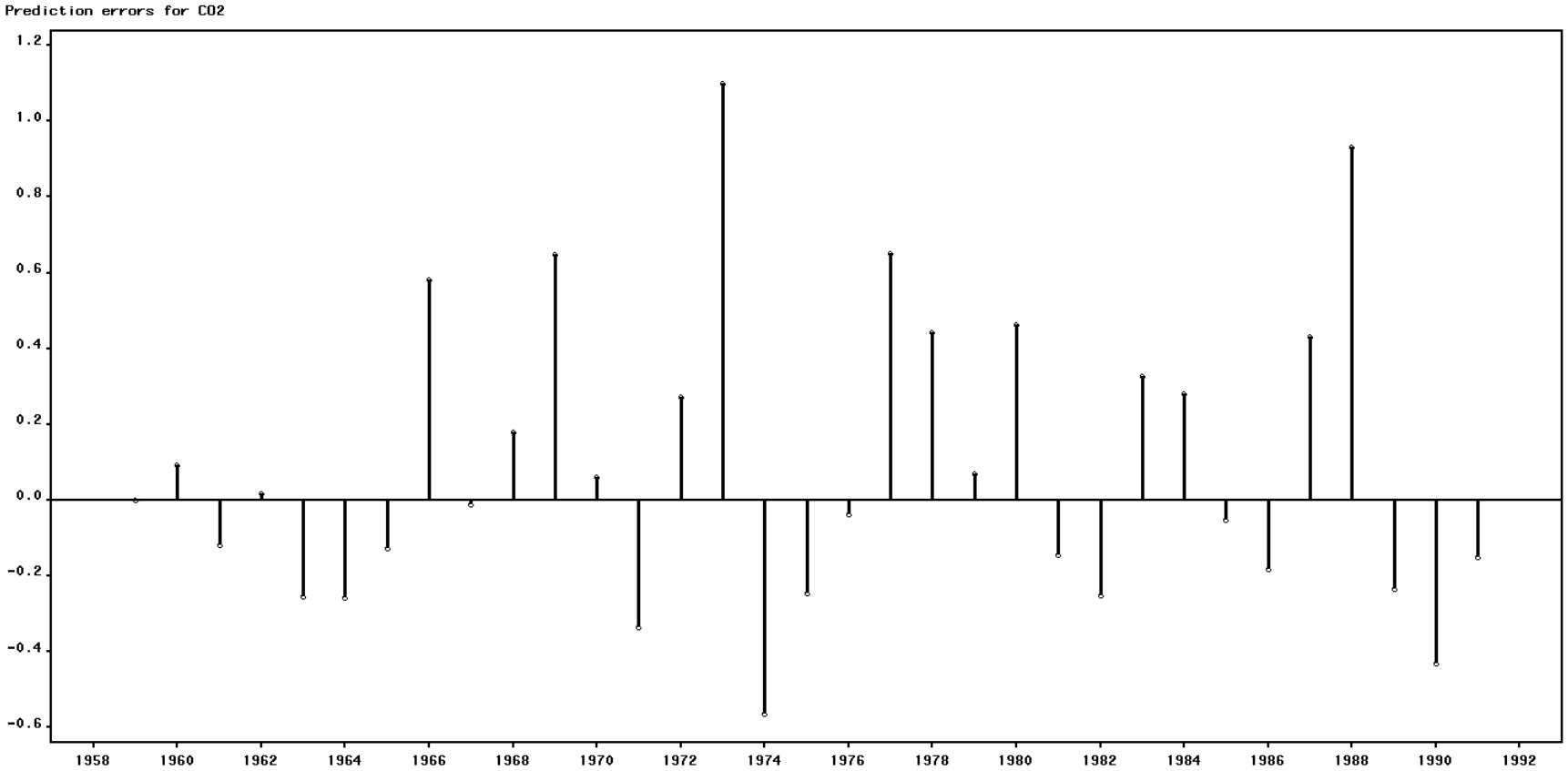

Figure 4. Prediction errors by time series values for the training data set.

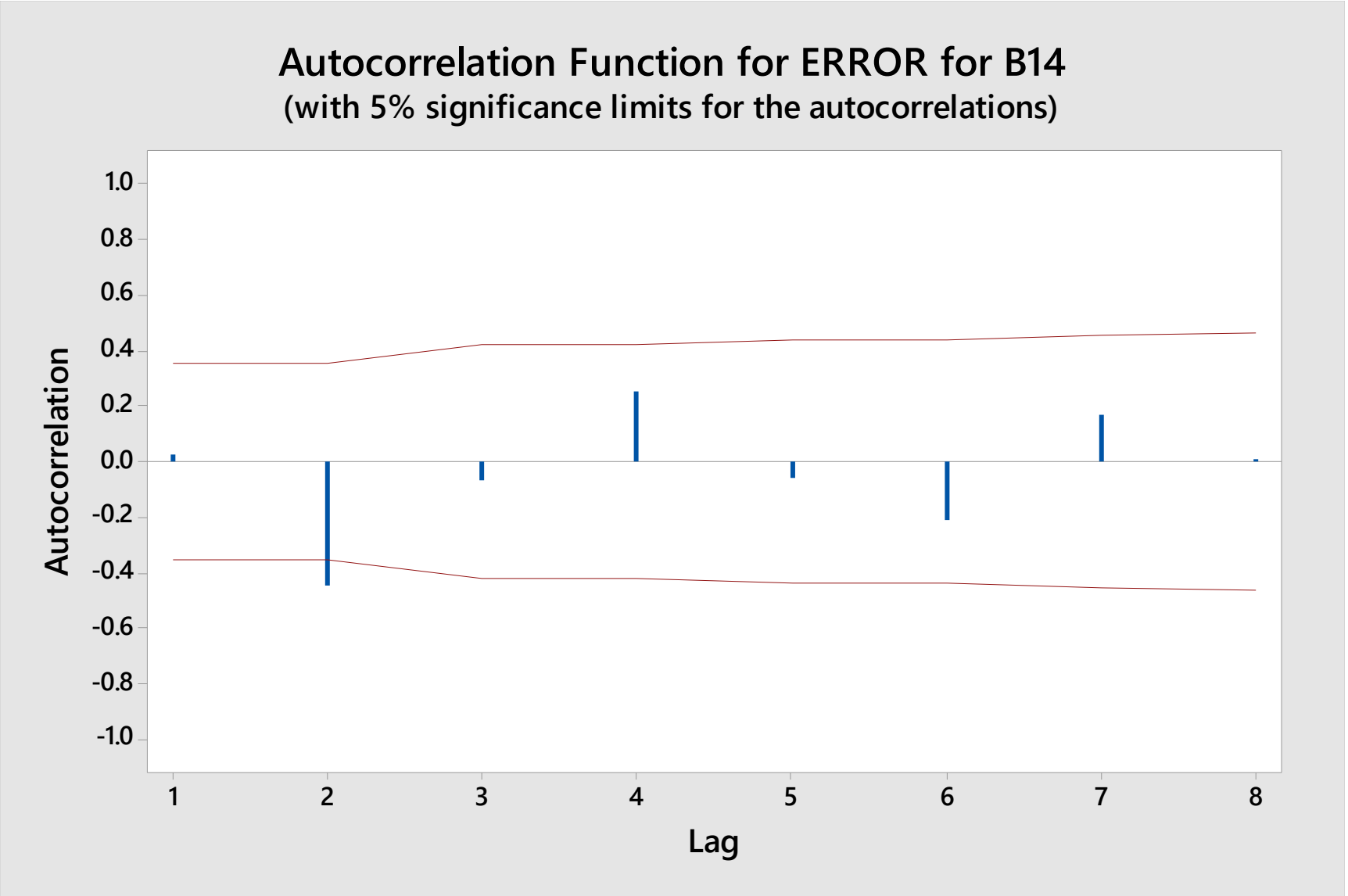

Figure 5. Autocorrelation Function (ACF) for the training data set.

Accuracy Measure for Table B.14 data.

Actual y_t	Predict $\widehat{y_t}$(t-1)	Forecast error $e_t(1) = y_t - \widehat{y_t}(t-1)$	Absolute error $\lvert e_t(1)\rvert$	Square error $[e_t(1)]^2$	Relative error $re_t(1) = 100(e_t(1)/y_t)$	Absolute percentage error $\lvert re_t(1)\rvert$
356.36	357.166	-0.8060	0.8060	0.64964	-0.226176	0.226176
357.10	358.712	-1.6118	1.6118	2.59790	-0.451358	0.451358
358.86	360.258	-1.3976	1.3976	1.95329	-0.389455	0.389455
360.90	361.803	-0.9035	0.9035	0.81631	-0.250346	0.250346
362.58	363.349	-0.7693	0.7693	0.59182	-0.212174	0.212174
363.84	364.895	-1.0550	1.0550	1.11303	-0.289963	0.289963
366.58	366.441	0.1391	0.1391	0.01935	0.037945	0.037945
368.30	367.987	0.3132	0.3132	0.09809	0.085039	0.085039
369.47	369.533	-0.0626	0.0626	0.00392	-0.016943	0.016943
371.03	371.078	-0.0484	0.0484	0.00234	-0.013045	0.013045
373.07	372.624	0.4458	0.4458	0.19874	0.119495	0.119495
375.61	374.170	1.4399	1.4399	2.07331	0.383350	0.383350

Table A. Error Measures table B14 time series data.

Data set	**MSE**	$\widehat{\sigma_{e(1)}}$	**MAD**	**MPE**	**MAPE**
Full data (45 data)	0.22078	0.46988	0.35919	0.03231	0.10388
Training data (33 data)	0.15758	0.39697	0.30137	0.02852	0.09000
Holdout data (12 data)	0.843145	0.918229	0.74935	-0.101969	0.206274

Table B. Accuracy Measures table B14 time series data.

Table B provides SAS (AFS)® forecast errors for the full data set, the training data set, and the holdout (or test) data set. The Linear (Holt) Exponential Smoothing model was suggested as the "best model" for this data set. SAS (AFS)® and MINITAB® produced the accuracy measures shown in Table B, which comprise the Mean Square Error (MSE), the Standard Deviation of the Forecast Errors (SDFE), the Mean Absolute Deviation (MAD), the Mean Percent Forecast Error (MPE), and Mean Absolute Percent Error (MAPE).

Here, the accuracy statistics are computed as follows:

Mean square error:

$$\textbf{MSE} = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = 0.843145$$

Standard Deviation of Forecast Error (Root Mean Square Error):

$$\widehat{\sigma_{e(1)}} = \sqrt{MSE} = \sqrt{0.843145} = 0.918229$$

Mean Absolute Deviation (Mean Absolute Error):

$$\textbf{MAD} = \frac{1}{n}\sum_{t=1}^{n} |e_t(1)| = 0.74935$$

Mean Percent Forecast Error:

$$\textbf{MPE} = \frac{1}{n}\sum_{t=1}^{n} re_t(1) = -0.101969$$

Mean Absolute Percent Error:

$$\textbf{MAPE} = \frac{1}{n}\sum_{t=1}^{n} |re_t(1)| = 0.206274$$

Worksheet 1 ***

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	Actual	Predict	Forecast_Error	Abs_Error	Sq_Error	Relative_Error	Abs_Percentage_Error	MSE	Root_MSE	MAD	MPE	MAPE
1	356.36	357.166	-0.8060	0.8060	0.64964	-0.226176	0.226176	0.843145	0.918229	0.74935	-0.101969	0.206274
2	357.10	358.712	-1.6118	1.6118	2.59790	-0.451358	0.451358					
3	358.86	360.258	-1.3976	1.3976	1.95329	-0.389455	0.389455					
4	360.90	361.803	-0.9035	0.9035	0.81631	-0.250346	0.250346					
5	362.58	363.349	-0.7693	0.7693	0.59182	-0.212174	0.212174					
6	363.84	364.895	-1.0550	1.0550	1.11303	-0.289963	0.289963					
7	366.58	366.441	0.1391	0.1391	0.01935	0.037945	0.037945					
8	368.30	367.987	0.3132	0.3132	0.09809	0.085039	0.085039					
9	369.47	369.533	-0.0626	0.0626	0.00392	-0.016943	0.016943					
10	371.03	371.078	-0.0484	0.0484	0.00234	-0.013045	0.013045					
11	373.07	372.624	0.4458	0.4458	0.19874	0.119495	0.119495					
12	375.61	374.170	1.4399	1.4399	2.07331	0.383350	0.383350					

Table C. MINITAB® 17 Calculation of Forecast Errors for Holdout Data.

The first two columns of Table C consist of the actual values of the holdout data set and the SAS (AFS)® forecasted values, based on training data. The remaining columns give forecast errors, the absolute forecast errors, the squared forecast errors, the relative forecast errors and absolute forecast percentage errors using the MINITAB® calculator function.

Obs	DATE	ACTUAL	PREDICT	UPPER	LOWER	ERROR	STD	NERROR	_LEVEL_	_TREND_
34	1992	.	357.1660	357.9687	356.3632	.	0.4096	.	357.1660	1.5458
35	1993	.	358.7118	359.9845	357.4391	.	0.6493	.	358.7118	1.5458
36	1994	.	360.2576	361.9886	358.5267	.	0.8832	.	360.2576	1.5458
37	1995	.	361.8035	364.0041	359.6028	.	1.1228	.	361.8035	1.5458
38	1996	.	363.3493	366.0379	360.6607	.	1.3718	.	363.3493	1.5458
39	1997	.	364.8951	368.0923	361.6979	.	1.6313	.	364.8951	1.5458
40	1998	.	366.4409	370.1682	362.7136	.	1.9017	.	366.4409	1.5458
41	1999	.	367.9868	372.2657	363.7078	.	2.1832	.	367.9868	1.5458
42	2000	.	369.5326	374.3844	364.6807	.	2.4755	.	369.5326	1.5458
43	2001	.	371.0784	376.5241	365.6327	.	2.7785	.	371.0784	1.5458
44	2002	.	372.6242	378.6842	366.5643	.	3.0919	.	372.6242	1.5458
45	2003	.	374.1701	380.8643	367.4759	.	3.4155	.	374.1701	1.5458

Table D. Forecasted values for the last 12 year of Holdout Data.

Table D contains forecasted values for the final twelve month for the Average CO_2 variable, and the corresponding lower and upper 95% prediction limits. The Root Mean Square Prediction Error, Level and Trend smoothing components are also shown..

Model Parameter	Estimate	Std. Error	T	Prob>|T|
LEVEL Smoothing Weight	0.99900	0.1245	8.0263	<.0001
TREND Smoothing Weight	0.23151	0.0962	2.4067	0.0222
Residual Variance (sigma squared)	0.16775	.	.	.
Smoothed Level	355.62015	.	.	.
Smoothed Trend	1.54583	.	.	.

Table E. Parameter Estimates for Training Data using the Linear (Holt) Exponential Smoothing model suggested by SAS (AFS)®.

Results

In Figure 3b, it seems the model fits the data appropriately as well as the SAS(AFS) forecast values with acceptably small prediction error. There are a few spikes in the ACF function, which means forecast errors may have additional structure not captured by the model. The MSE, RMSE, MAD and MAPE are not similar among the training and holdout data sets; that is, the variability in forecast errors are not similar.

TABLE B.15 (US NATIONAL VIOLENT CRIME RATE) ANALYSIS

Table B.15 comprises data on the US National Violent Crime Rate from 1984 to 2005, where the Violent Crime Rate (in per 100,000 inhabitants) variable is collected yearly. The data set excludes last six year values as a holdout set in order to analyze the accuracy of SAS(AFS), which suggested a Damped Trend Exponential Smoothing model as the "best model" for this training, or initialization, data set. SAS(AFS) also produced the predicted values for the next 6 year. I have calculated different types of forecast errors based on the hold out data and predicted data.

The model equation is

$$Y_t = \mu_t + \beta_t\, t + \varepsilon_t$$

where,

Y_t = observed value i.e. securities rate at time t

μ_t = smoothed level and μ_t = 526.00128

β_t = smoothed trend and β_t = -44.52805

Now, the k-step prediction equation is

$$\hat{Y}_t(k) = L_t + \sum_{i=1}^{k} \phi^t\, T_t$$

and the smoothing equations are

$$L_t = \alpha\, Y_t + (1-\alpha)\,(L_{t-1} + \phi\, T_{t-1})$$

and $T_t = \gamma\,(L_t - L_{t-1}) + (1-\gamma)\,\phi\, T_{t-1}$

where,

L_t = Level estimate and $L_0 = \mu_t$ = 526.00128

T_t = Trend estimate and $T_0 = \beta_t$ = -44.52805

α = level smoothing and α = 0.78722

γ = trend smoothing and γ = 0.99900 and

ϕ = damping smoothing and ϕ = 0.86809

The ARIMA model equivalency to damped-trend linear exponential smoothing is the ARIMA (1, 1, 2) model,

$(1-\phi B)(1-B)\, Y_t = (1-\theta_1 B - \theta_2 B^2)\, \varepsilon_t$

$\theta_1 = 1 + \phi - \alpha - \alpha\,\gamma\,\phi$

$\theta_2 = (\alpha - 1)\,\phi$

Table B.15: UN National Violent Crime Rate

Year	Violent Crime Rate, per 100,000 inhabitants	Year	Violent Crime Rate, per 100,000 inhabitants
1984	539.9	1995	684.5
1985	558.1	1996	636.6
1986	620.1	1997	611.0
1987	612.5	1998	567.6
1988	640.6	1999	523.0
1989	666.9	2000	506.5

Year	Violent Crime Rate, per 100,000 inhabitants	Year	Violent Crime Rate, per 100,000 inhabitants
1990	729.6	2001	504.5
1991	758.2	2002	494.4
1992	757.7	2003	475.8
1993	747.1	2004	463.2
1994	713.6	2005	469.2

The collection of Figures and Tables that follow are individually identified below.

List of Figures:

List of Tables:

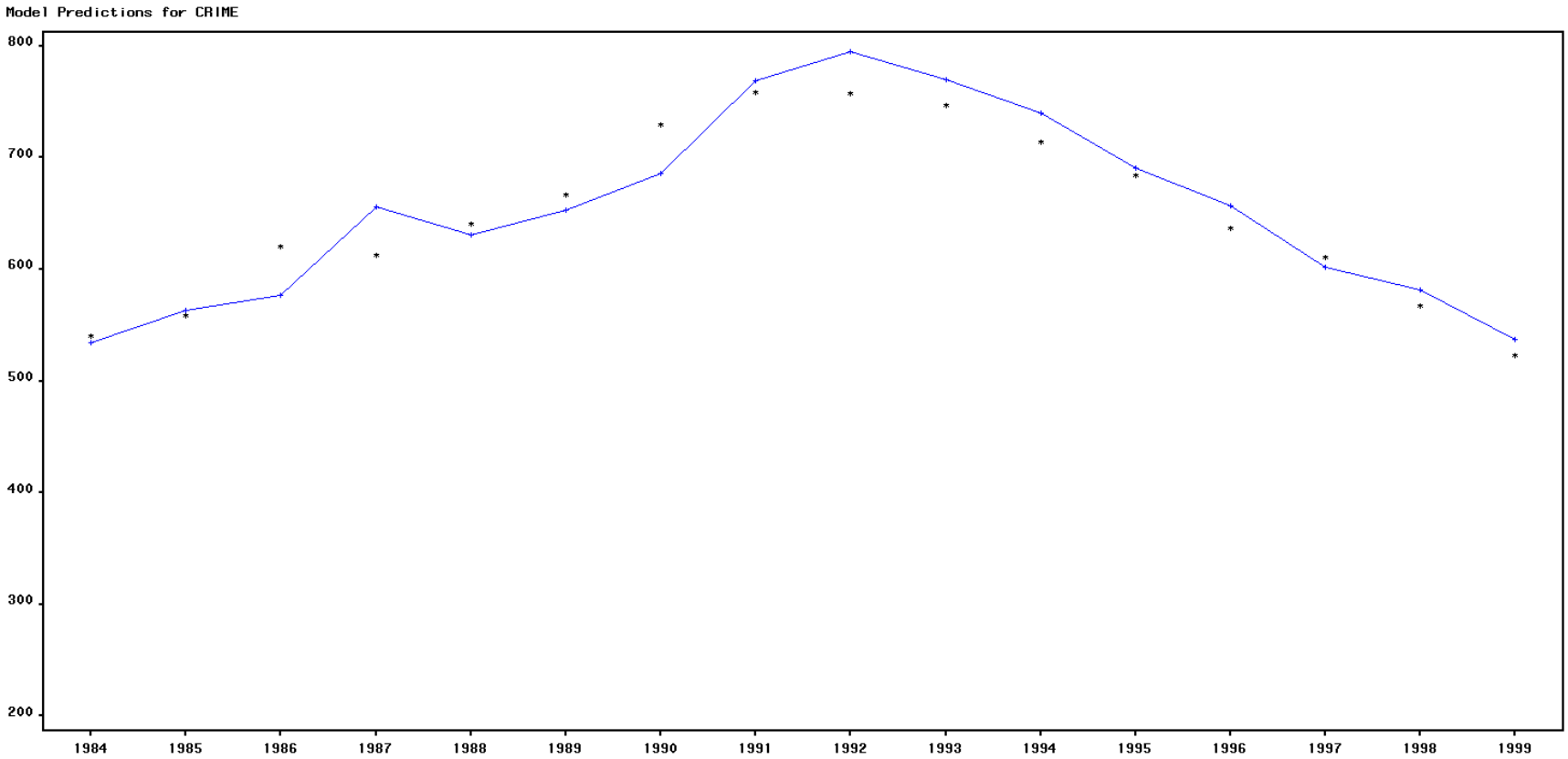

Figure 1 depicts a time series graph for predicted values ((+) signs) using the training data ((*) signs) set for Crime Rate variable, as reported by SAS(AFS).

Figure 2 shows a time series graph for predicted values ((+) signs) for Ice Cream variable as reported by SAS(AFS), along with 6-year forecasted values for the Crime Rate variable. Here, predicted and forecast values are plotted using ((*) signs).

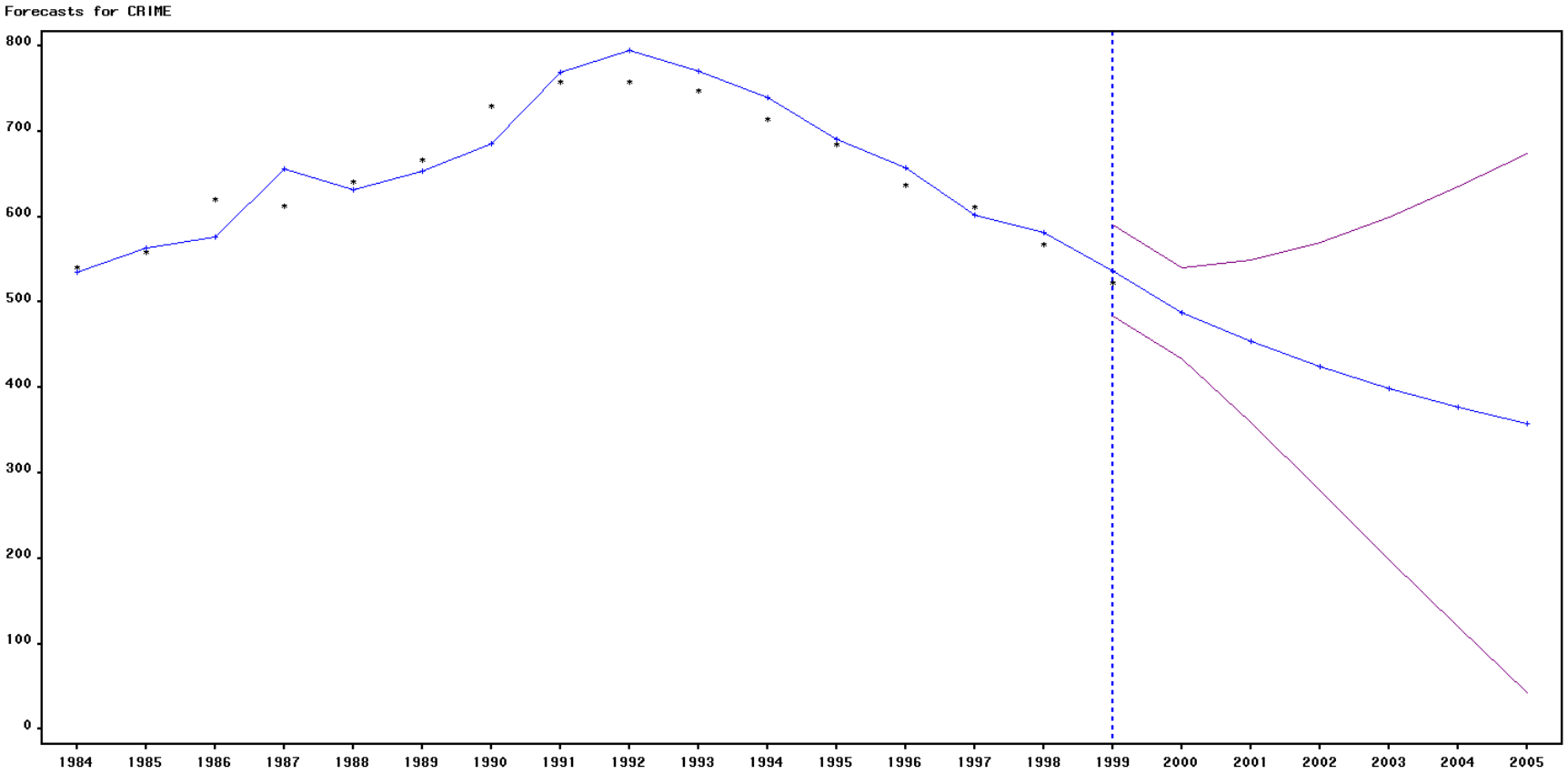

Figure 3a depicts a time series graph for predicted values ((+) signs) and the training data ((*) signs) for the Crime Rate variable, as reported by SAS(AFS), along with 6-year forecasted values and prediction limits.

Figure 3b shows a time series graph for predicted values (green-colored (+) signs) for the Crime variable as reported by SAS(AFS), along with 6-year forecasted values, which are plotted using red-colored plus (+) signs. The training data are plotted using blue-colored star (*) signs, and testing data are plotted using black-colored star (*) signs, while the 6-year confidence limits are yellow-colored.

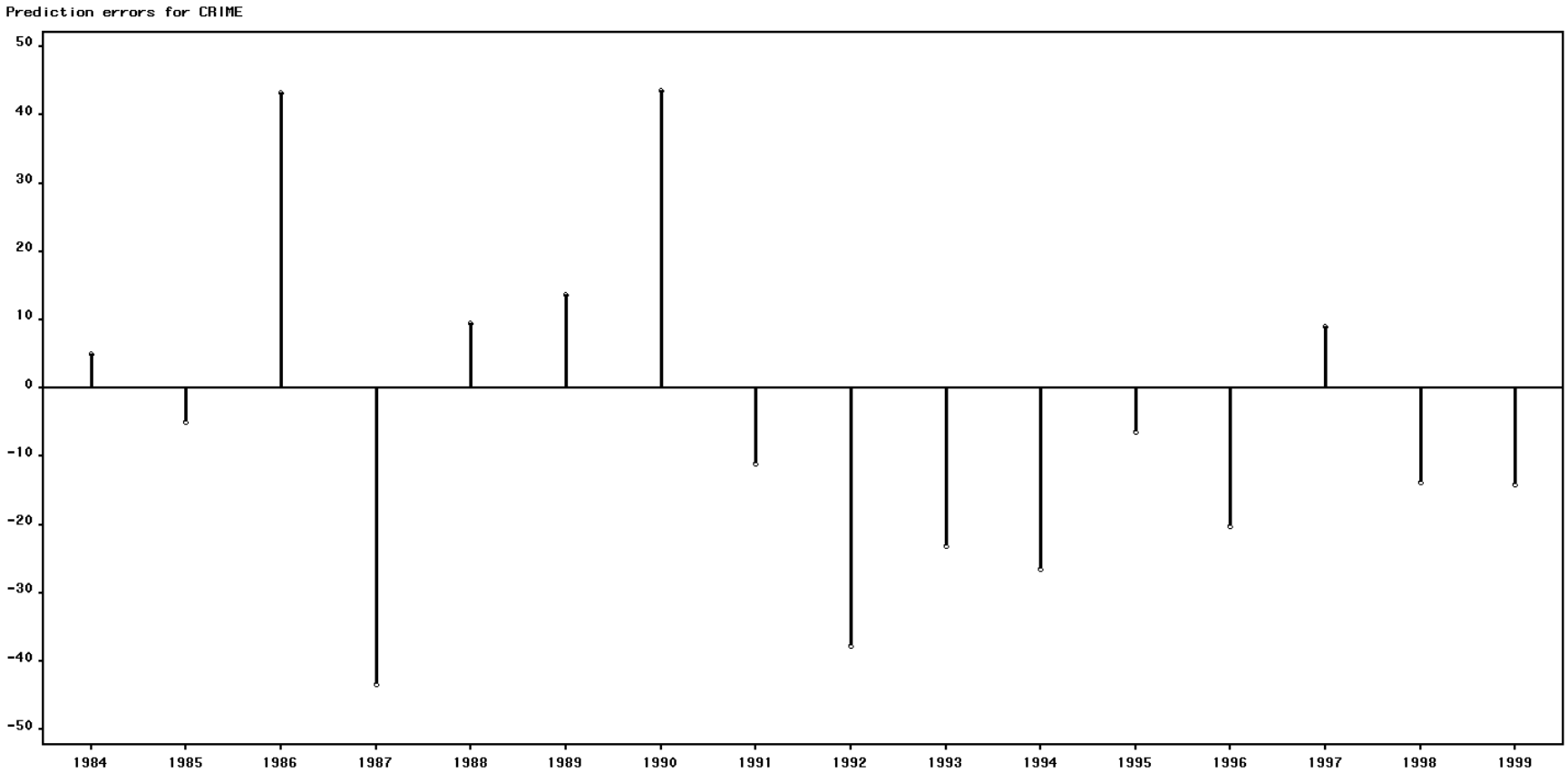

Figure 4. Prediction errors by time series values for the training data set.

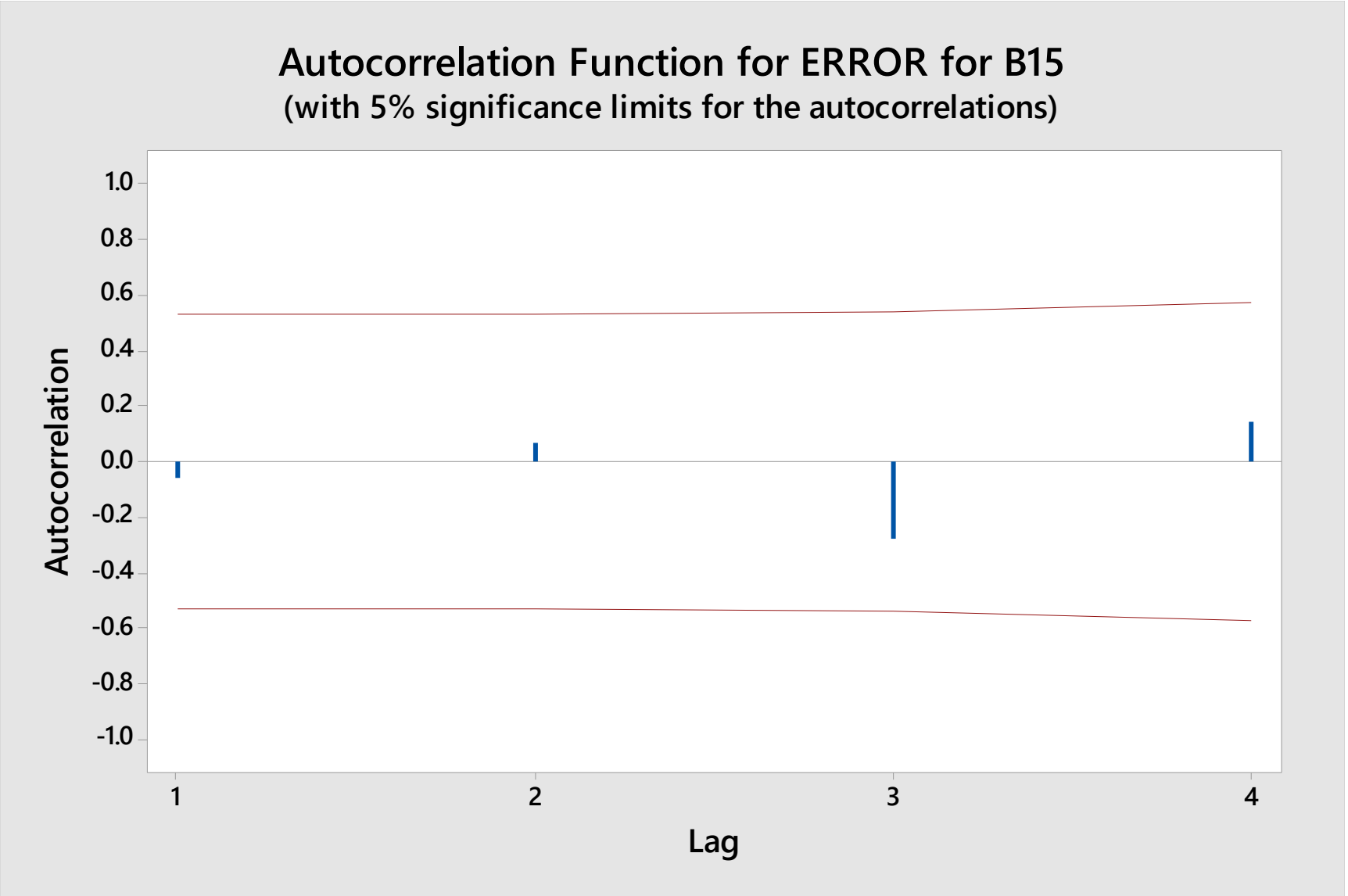

Figure 5. Autocorrelation Function (ACF) for the training data set.

Accuracy Measure for Table B.15 data.

Actual y_t	Predict $\widehat{y_t}$(t-1)	Forecast error $e_t(1) = y_t - \widehat{y_t}(t-1)$	Absolute error $\lvert e_t(1)\rvert$	Square error $[e_t(1)]^2$	Relative error $re_t(1) = 100(e_t(1)/y_t)$	Absolute percentage error $\lvert re_t(1)\rvert$
506.5	487.347	19.153	19.153	366.8	3.7815	3.7815
504.5	453.791	50.709	50.709	2571.4	10.0514	10.0514
494.4	424.661	69.739	69.739	4863.5	14.1057	14.1057
475.8	399.374	76.426	76.426	5840.9	16.0626	16.0626
463.2	377.422	85.778	85.778	7357.8	18.5185	18.5185
469.2	358.366	110.834	110.834	12284.2	23.6219	23.6219

Table A. Error Measures table B15 time series data.

Data set	**MSE**	$\widehat{\sigma_{e(1)}}$	**MAD**	**MPE**	**MAPE**
Full data (22 data)	499.41898	22.34768	18.24361	-0.16767	2.95898
Training data (16 data)	605.02941	24.59735	20.33607	-0.72154	3.08476
Holdout data (6 data)	5547.44	74.4811	68.7731	14.3569	14.3569

Table B. Accuracy Measures table B15 time series data.

Table B provides SAS (AFS)® forecast errors for the full data set, the training data set, and the holdout (or test) data set. The Damped Trend Exponential Smoothing model was suggested as the "best model" for this data set. SAS (AFS)® and MINITAB® produced the accuracy measures shown in Table B, which comprise the Mean Square Error (MSE), the Standard Deviation of the Forecast Errors (SDFE), the Mean Absolute Deviation (MAD), the Mean Percent Forecast Error (MPE), and Mean Absolute Percent Error (MAPE).

Here, the accuracy statistics are computed as follows:

Mean square error:

$$\textbf{MSE} = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = 5547.44$$

Standard Deviation of Forecast Error (Root Mean Square Error):

$$\widehat{\sigma_{e(1)}} = \sqrt{MSE} = \sqrt{5547.44} = 74.4811$$

Mean Absolute Deviation (Mean Absolute Error):

$$\textbf{MAD} = \frac{1}{n}\sum_{t=1}^{n} |e_t(1)| = 68.7731$$

Mean Percent Forecast Error:

$$\textbf{MPE} = \frac{1}{n}\sum_{t=1}^{n} re_t(1) = 14.3569$$

Mean Absolute Percent Error:

$$\textbf{MAPE} = \frac{1}{n}\sum_{t=1}^{n} |re_t(1)| = 14.3569$$

Worksheet 1 ***

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	Actual	Predict	Forecast_Error	Abs_Error	Sq_Error	Relative_Error	Abs_Percentage_Error	MSE	Root_MSE	MAD	MPE	MAPE
1	506.5	487.347	19.153	19.153	366.8	3.7815	3.7815	5547.44	74.4811	68.7731	14.3569	14.3569
2	504.5	453.791	50.709	50.709	2571.4	10.0514	10.0514					
3	494.4	424.661	69.739	69.739	4863.5	14.1057	14.1057					
4	475.8	399.374	76.426	76.426	5840.9	16.0626	16.0626					
5	463.2	377.422	85.778	85.778	7357.8	18.5185	18.5185					
6	469.2	358.366	110.834	110.834	12284.2	23.6219	23.6219					

Table C. MINITAB® 17 Calculation of Forecast Errors for Holdout Data.

The first two columns of Table C consist of the actual values of the holdout data set and the SAS (AFS)® forecasted values, based on training data. The remaining columns give forecast errors, the absolute forecast errors, the squared forecast errors, the relative forecast errors and absolute forecast percentage errors using the MINITAB® calculator function.

Obs	DATE	ACTUAL	PREDICT	UPPER	LOWER	ERROR	STD	NERROR	_LEVEL_	_TREND_
17	2000	.	487.3467	540.8308	433.8626	.	27.2883	.	487.3467	-38.6546
18	2001	.	453.7909	548.8765	358.7053	.	48.5140	.	453.7909	-33.5558
19	2002	.	424.6613	570.3001	279.0225	.	74.3069	.	424.6613	-29.1296
20	2003	.	399.3740	599.8936	198.8545	.	102.3078	.	399.3740	-25.2873
21	2004	.	377.4223	635.0281	119.8164	.	131.4340	.	377.4223	-21.9517
22	2005	.	358.3661	674.0402	42.6919	.	161.0612	.	358.3661	-19.0562

Table D. Forecasted values for the last 6 year of Holdout Data.

Table D contains forecasted values for the final twelve month for the Crime Rate variable, and the corresponding lower and upper 95% prediction limits. The Root Mean Square Prediction Error, Level and Trend smoothing components are also shown..

Model Parameter	Estimate	Std. Error	T	Prob>|T|
LEVEL Smoothing Weight	0.78722	0.2678	2.9399	0.0115
TREND Smoothing Weight	0.99900	0.7880	1.2677	0.2271
DAMPING Smoothing Weight	0.86809	0.1579	5.4991	0.0001
Residual Variance (sigma squared)	744.65158	.	.	.
Smoothed Level	526.00128	.	.	.
Smoothed Trend	-44.52805	.	.	.

Table E. Parameter Estimates for Training Data using the Damped Trend Exponential Smoothing model suggested by SAS (AFS)®.

Results

In Figure 3b, it seems the model fits the data appropriately, however, the SAS(AFS) could not forecast values with acceptably small prediction error. There are no problem in the ACF function. The MSE, RMSE, MAD and MAPE are not similar among the training and holdout data sets; that is, the variability in forecast errors are not similar.

TABLE B.16 (US GROSS DOMESTIC PRODUCT) ANALYSIS

Table B.16 comprises data on the US Gross Domestic Product from 1976 to 2002, where the GDP, Current Dollars (in Billions) variable is collected yearly. The data set excludes last six year values as a holdout set in order to analyze the accuracy of SAS(AFS), which suggested a Linear (Holt) Exponential Smoothing model as the "best model" for this training, or initialization, data set. SAS(AFS) also produced the predicted values for the next 6 year. I have calculated different types of forecast errors based on the hold out data and predicted data.

The model equation is

$$Y_t = \mu_t + \beta_t\, t + \varepsilon_t$$

where,

Y_t = observed value i.e. temperature anomaly at time t
μ_t = smoothed level and $\mu_t = 7809$
β_t = smoothed trend and $\beta_t = 346.90197$

Now, the k-step prediction equation is

$$\hat{Y}_t(k) = L_t + K\, T_t$$

and the smoothing equations are

$$L_t = \alpha\, Y_t + (1-\alpha)\,(L_{t-1} + T_{t-1})$$

and $T_t = \gamma\,(L_t - L_{t-1}) + (1-\gamma)\, T_{t-1}$

where,

L_t = Level estimate and $L_0 = \mu_t = 7809$
T_t = Trend estimate and $T_0 = \beta_t = 346.90197$
α = level smoothing and $\alpha = 0.94595$
γ = trend smoothing and $\gamma = 0.21476$

Table B.16: US Gross Domestic Product

Year	GDP, Current Dollars, billions	GDP, Real (1996) Dollars, billions	Year	GDP, Current Dollars, billions	GDP, Real (1996) Dollars, billions
1976	1,823.9	4,311.7	1990	5,803.2	6,707.9
1977	2,031.4	4,511.8	1991	5,986.2	6,676.4
1978	2,295.9	4,760.6	1992	6,318.9	6,880.0
1979	2,566.4	4,912.1	1993	6,642.3	7,062.6
1980	2,795.6	4,900.9	1994	7,054.3	7,347.7
1981	3,131.3	5,021.0	1995	7,400.5	7,543.8
1982	3,259.2	4,919.3	1996	7,813.2	7,813.2
1983	3,534.9	5,132.3	1997	8,318.4	8,159.5
1984	3,932.7	5,505.2	1998	8,781.5	8,508.9
1985	4,213.0	5,717.1	1999	9,274.3	8,859.0
1986	4,452.9	5,912.4	2000	9,824.6	9,191.4
1987	4,742.5	6,113.3	2001	10,082.2	9,214.5
1988	5,108.3	6,368.4	2002	10,446.2	9,439.9
1989	5,489.1	6,591.8			

The collection of Figures and Tables that follow are individually identified below.

List of Figures:

List of Tables:

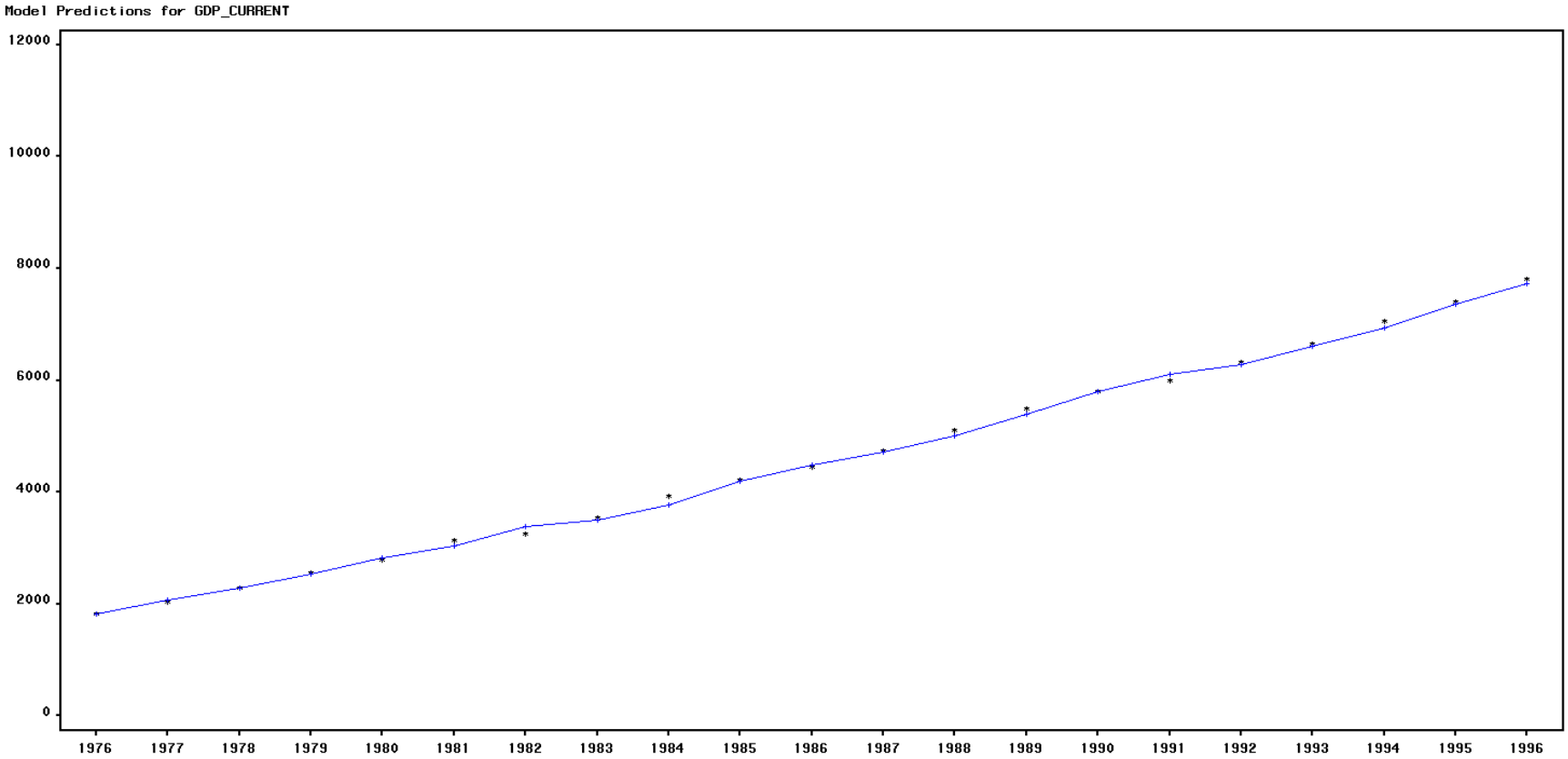

Figure 1 depicts a time series graph for predicted values ((+) signs) using the training data ((*) signs) set for GDP, Current Dollars variable, as reported by SAS(AFS).

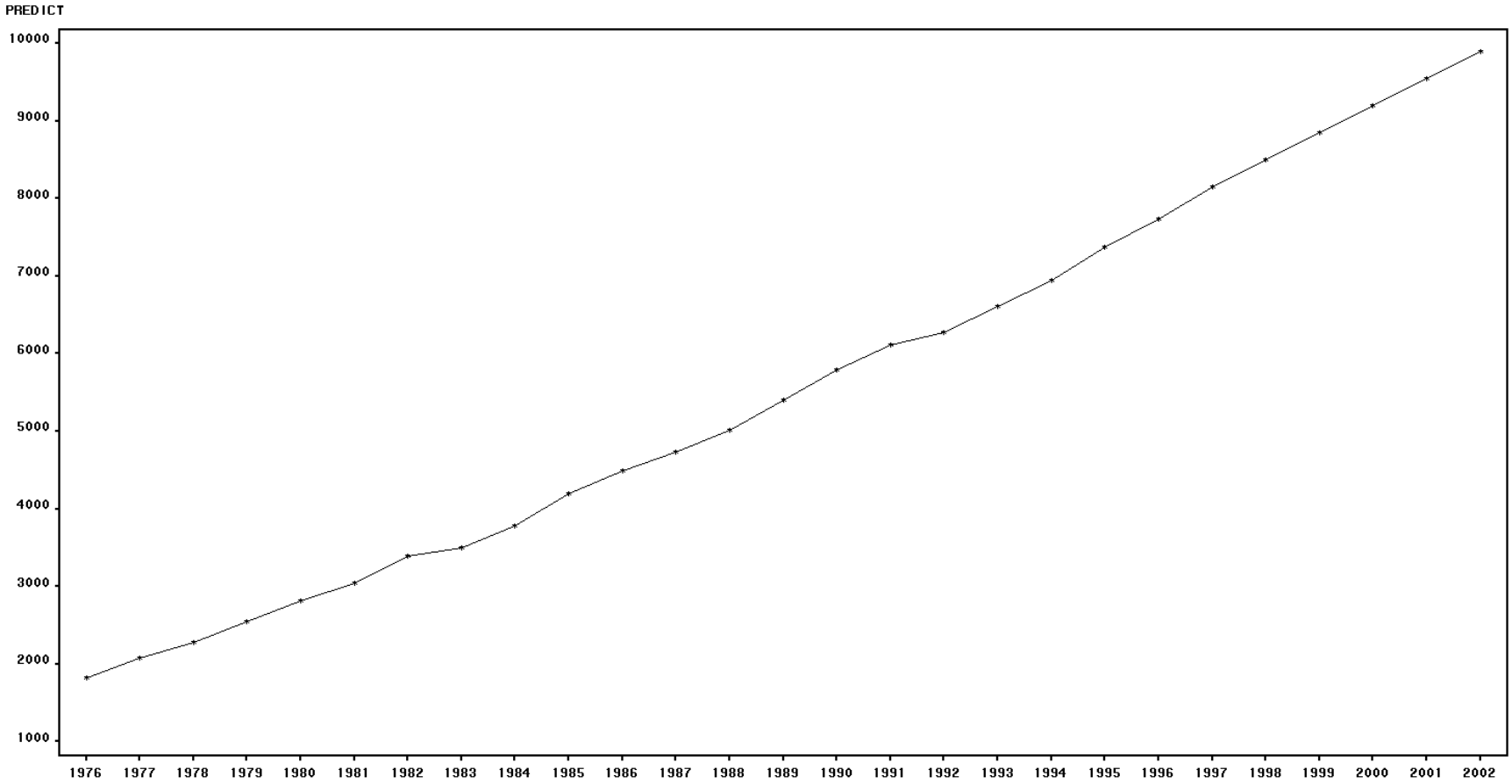

Figure 2 shows a time series graph for predicted values ((+) signs) for GDP, Current Dollars variable as reported by SAS(AFS), along with 6-year forecasted values for the average CO_2 variable. Here, predicted and forecast values are plotted using ((*) signs).

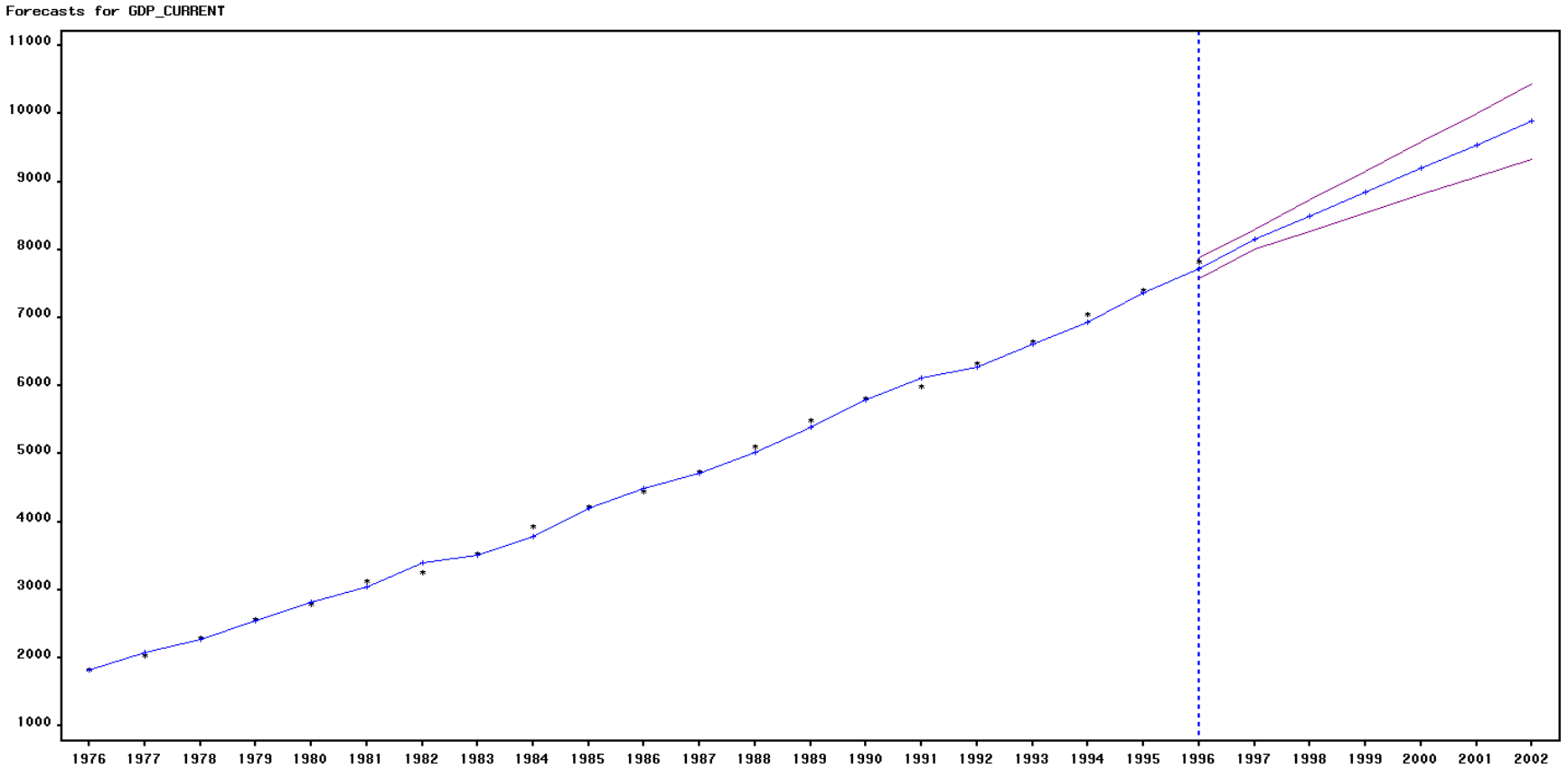

Figure 3a depicts a time series graph for predicted values ((+) signs) and the training data ((*) signs) for the GDP, Current Dollars variable, as reported by SAS(AFS), along with 6-year forecasted values and prediction limits.

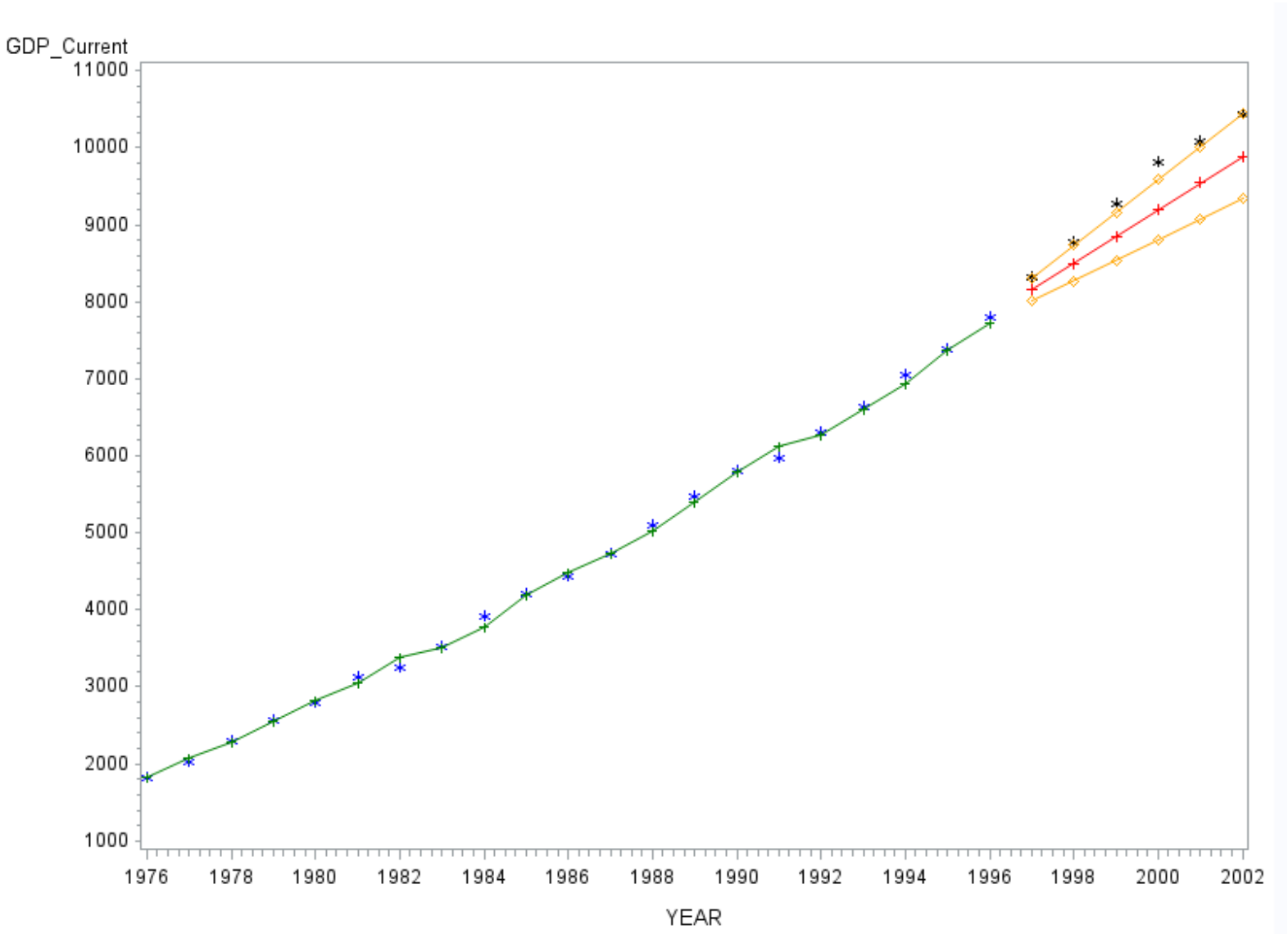

Figure 3b shows a time series graph for predicted values (green-colored (+) signs) for the GDP, Current Dollars variable as reported by SAS(AFS), along with 6-year forecasted values, which are plotted using red-colored plus (+) signs. The training data are plotted using blue-colored star (*) signs, and testing data are plotted using black-colored star (*) signs, while the 6-year confidence limits are yellow-colored.

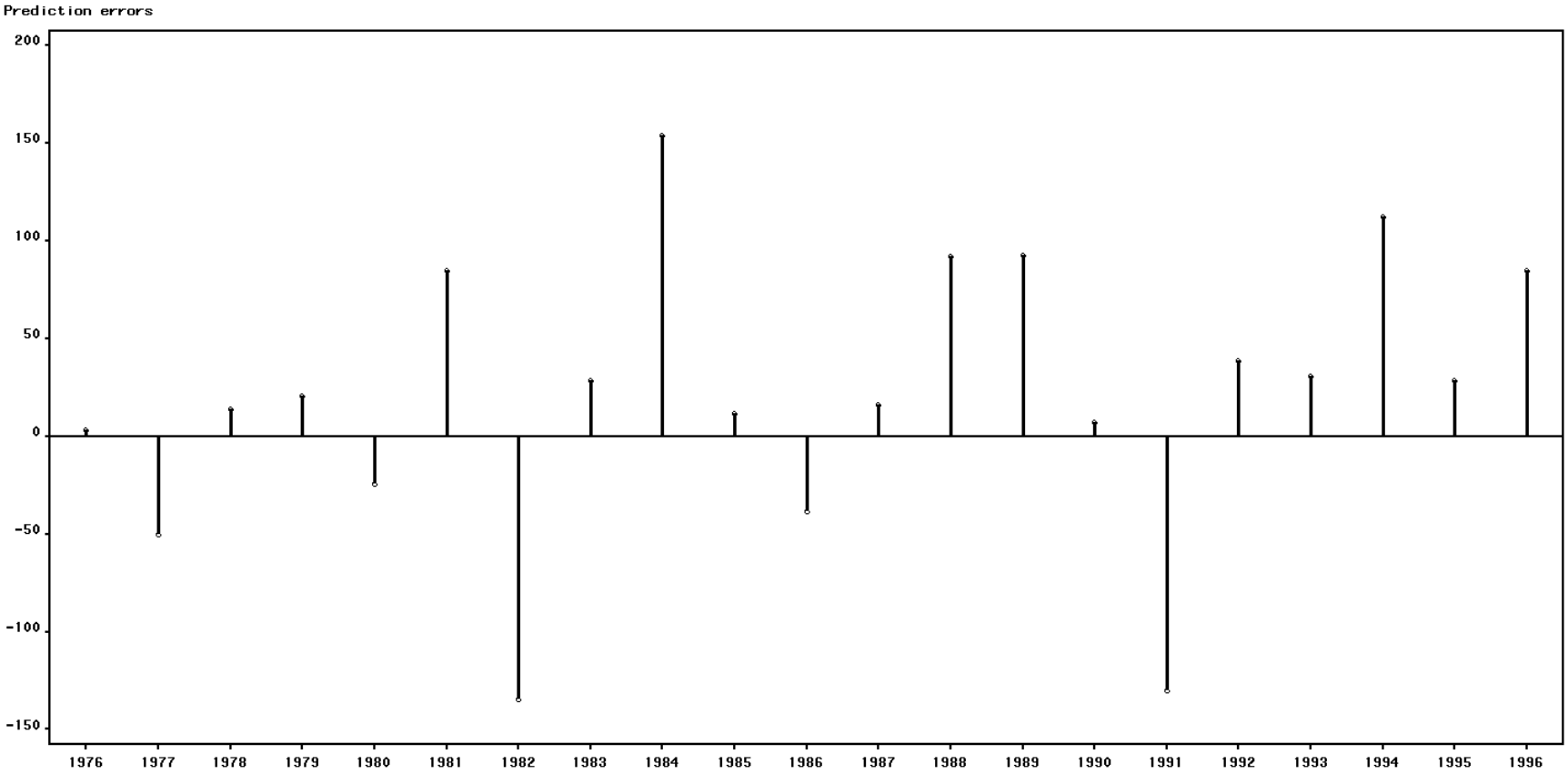

Figure 4. Prediction errors by time series values for the training data set.

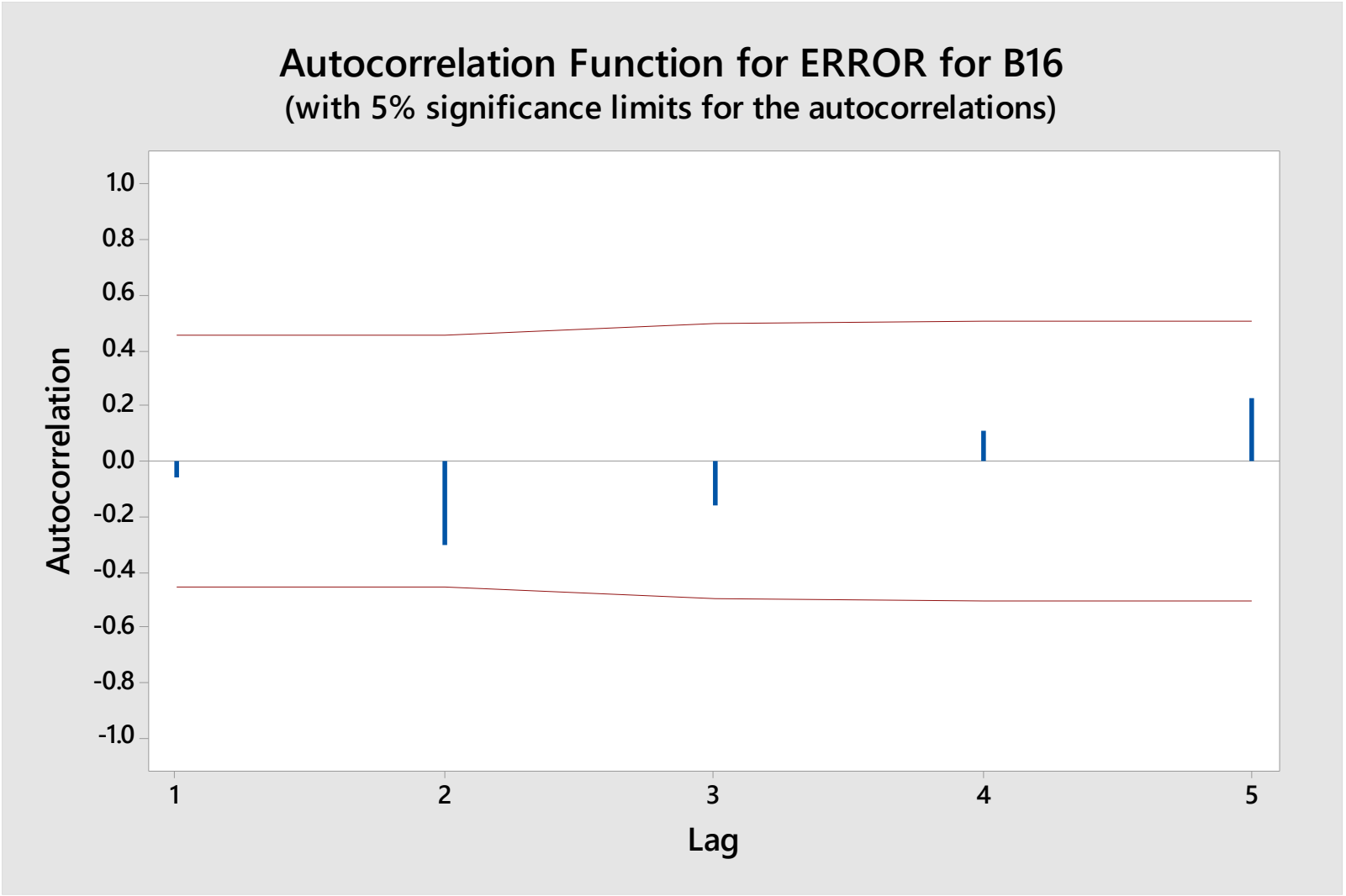

Figure 5. Autocorrelation Function (ACF) for the training data set.

Accuracy Measure for Table B.16 data.

Actual y_t	Predict $\widehat{y_t}$(t-1)	Forecast error $e_t(1) = y_t - \widehat{y_t}(t-1)$	Absolute error $\lvert e_t(1)\rvert$	Square error $[e_t(1)]^2$	Relative error $re_t(1) = 100(e_t(1)/y_t)$	Absolute percentage error $\lvert re_t(1)\rvert$
8159.5	8156	3.5	3.5	12	0.04289	0.04289
8508.9	8502	6.9	6.9	48	0.08109	0.08109
8859.0	8849	10.0	10.0	100	0.11288	0.11288
9191.4	9196	-4.6	4.6	21	-0.05005	0.05005
9214.5	9543	-328.5	328.5	107912	-3.56503	3.56503
9439.9	9890	-450.1	450.1	202590	-4.76806	4.76806

Table A. Error Measures table B16 time series data.

Data set	**MSE**	$\widehat{\sigma_{e(1)}}$	**MAD**	**MPE**	**MAPE**
Full data (27 data)	7600.0	87.17801	68.87009	0.30940	1.31653
Training data (21 data)	5353.9	73.17061	57.01024	0.32590	1.32886
Holdout data (6 data)	51780.5	227.553	133.933	-1.35771	1.43667

Table B. Accuracy Measures table B16 time series data.

Table B provides SAS (AFS)® forecast errors for the full data set, the training data set, and the holdout (or test) data set. The Linear (Holt) Exponential Smoothing model was suggested as the "best model" for this data set. SAS (AFS)® and MINITAB® produced the accuracy measures shown in Table B, which comprise the Mean Square Error (MSE), the Standard Deviation of the Forecast Errors (SDFE), the Mean Absolute Deviation (MAD), the Mean Percent Forecast Error (MPE), and Mean Absolute Percent Error (MAPE).

Here, the accuracy statistics are computed as follows:

Mean square error:

$$\textbf{MSE} = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = 51780.5$$

Standard Deviation of Forecast Error (Root Mean Square Error):

$$\widehat{\sigma_{e(1)}} = \sqrt{MSE} = \sqrt{51780.5} = 227.553$$

Mean Absolute Deviation (Mean Absolute Error):

$$\textbf{MAD} = \frac{1}{n}\sum_{t=1}^{n} |e_t(1)| = 133.933$$

Mean Percent Forecast Error:

$$\textbf{MPE} = \frac{1}{n}\sum_{t=1}^{n} re_t(1) = -1.35771$$

Mean Absolute Percent Error:

$$\textbf{MAPE} = \frac{1}{n}\sum_{t=1}^{n} |re_t(1)| = 1.43667$$

Worksheet 1 ***

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	Actual	Predict	Forecast_Error	Abs_Error	Sq_Error	Relative_Error	Abs_Percentage_Error	MSE	Root_MSE	MAD	MPE	MAPE
1	8159.5	8156	3.5	3.5	12	0.04289	0.04289	51780.5	227.553	133.933	-1.35771	1.43667
2	8508.9	8502	6.9	6.9	48	0.08109	0.08109					
3	8859.0	8849	10.0	10.0	100	0.11288	0.11288					
4	9191.4	9196	-4.6	4.6	21	-0.05005	0.05005					
5	9214.5	9543	-328.5	328.5	107912	-3.56503	3.56503					
6	9439.9	9890	-450.1	450.1	202590	-4.76806	4.76806					

Table C. MINITAB® 17 Calculation of Forecast Errors for Holdout Data.

The first two columns of Table C consist of the actual values of the holdout data set and the SAS (AFS)® forecasted values, based on training data. The remaining columns give forecast errors, the absolute forecast errors, the squared forecast errors, the relative forecast errors and absolute forecast percentage errors using the MINITAB® calculator function.

Obs	DATE	ACTUAL	PREDICT	UPPER	LOWER	ERROR	STD	NERROR	_LEVEL_	_TREND_
22	1997	.	8156	8306	8005	.	76.9254	.	8156	346.9020
23	1998	.	8502	8732	8273	.	117.1801	.	8502	346.9020
24	1999	.	8849	9156	8542	.	156.6901	.	8849	346.9020
25	2000	.	9196	9583	8810	.	197.1492	.	9196	346.9020
26	2001	.	9543	10012	9075	.	239.0976	.	9543	346.9020
27	2002	.	9890	10444	9336	.	282.7361	.	9890	346.9020

Table D. Forecasted values for the last 6 year of Holdout Data.

Table D contains forecasted values for the final twelve month for the GDP, Current Dollars variable, and the corresponding lower and upper 95% prediction limits. The Root Mean Square Prediction Error, Level and Trend smoothing components are also shown..

Model Parameter	Estimate	Std. Error	T	Prob>|T|
LEVEL Smoothing Weight	0.94595	0.1707	5.5402	<.0001
TREND Smoothing Weight	0.21476	0.1453	1.4776	0.1559
Residual Variance (sigma squared)	5918	.	.	.
Smoothed Level	7809	.	.	.
Smoothed Trend	346.90197	.	.	.

Table E. Parameter Estimates for Training Data using the Linear (Holt) Exponential Smoothing model suggested by SAS (AFS)®.

Results

Figure 3b reveal the model fits the data appropriately, however, the SAS(AFS) could not forecast values with acceptably small prediction error. The testing data values are outside of the confidence limit of the forecast values. There are no problem in the ACF function. The MSE, RMSE and MAD are not similar for both the training and holdout data sets; that is, the variability in forecast errors are not similar.

TABLE B.17 (TOTAL ANNUAL US ENERGY CONSUMPTION) ANALYSIS

Table B.17 comprises data on the Total Annual US Energy Consumption from 1949 to 2005, where the BTUs (in Billions) variable is collected yearly. The data set excludes last twelve year values as a holdout set in order to analyze the accuracy of SAS(AFS), which suggested a Damped Trend Exponential Smoothing model as the "best model" for this training, or initialization, data set. SAS(AFS) also produced the predicted values for the next 12 year. I have calculated different types of forecast errors based on the hold out data and predicted data.

The model equation is

$$Y_t = \mu_t + \beta_t\, t + \varepsilon_t$$

where,

Y_t = observed value i.e. securities rate at time t

μ_t = smoothed level and μ_t = 87651754

β_t = smoothed trend and β_t = 1196583

Now, the k-step prediction equation is

$$\hat{Y}_t(k) = L_t + \sum_{i=1}^{k} \phi^t\, T_t$$

and the smoothing equations are

$$L_t = \alpha\, Y_t + (1-\alpha)\,(L_{t-1} + \phi\, T_{t-1})$$

and $T_t = \gamma\,(L_t - L_{t-1}) + (1-\gamma)\,\phi\, T_{t-1}$

where,

L_t = Level estimate and $L_0 = \mu_t$ = 87651754

T_t = Trend estimate and $T_0 = \beta_t$ = 1196583

α = level smoothing and α = 0.99900

γ = trend smoothing and γ = 0.00100 and

ϕ = damping smoothing and ϕ = 0.99900

The ARIMA model equivalency to damped-trend linear exponential smoothing is the ARIMA (1, 1, 2) model,

$(1-\phi B)(1-B)\, Y_t = (1-\theta_1 B - \theta_2 B^2)\, \varepsilon_t$

$\theta_1 = 1 + \phi - \alpha - \alpha\gamma\phi$

$\theta_2 = (\alpha - 1)\,\phi$

Table B.17: Total Annual US Energy Consumption

Year	BTUs, billions	Year	BTUs, billions	Year	BTUs, billions	Year	BTUs, billions
1949	31,981,503	1964	51,817,177	1979	80,903,214	1994	89,291,713
1950	34,615,768	1965	54,017,221	1980	78,280,238	1995	91,199,841
1951	36,974,030	1966	57,016,544	1981	76,342,955	1996	94,225,791
1952	36,747,825	1967	58,908,107	1982	73,286,151	1997	94,800,047
1953	37,664,468	1968	62,419,392	1983	73,145,527	1998	95,200,433
1954	36,639,382	1969	65,620,879	1984	76,792,960	1999	96,836,647
1955	40,207,971	1970	67,844,161	1985	76,579,965	2000	98,976,371
1956	41,754,252	1971	69,288,965	1986	76,825,812	2001	96,497,865

Year	BTUs, billions	Year	BTUs, billions	Year	BTUs, billions	Year	BTUs, billions
1957	41,787,186	1972	72,704,267	1987	79,223,446	2002	97,966,872
1958	41,645,028	1973	75,708,364	1988	82,869,321	2003	98,273,323
1959	43,465,722	1974	73,990,880	1989	84,999,308	2004	100,414,461
1960	45,086,870	1975	71,999,191	1990	84,729,945	2005	99,894,296
1961	45,739,017	1976	76,012,373	1991	84,667,227		
1962	47,827,707	1977	77,999,554	1992	86,014,860		
1963	49,646,160	1978	79,986,371	1993	87,652,195		

The collection of Figures and Tables that follow are individually identified below.

List of Figures:

List of Tables:

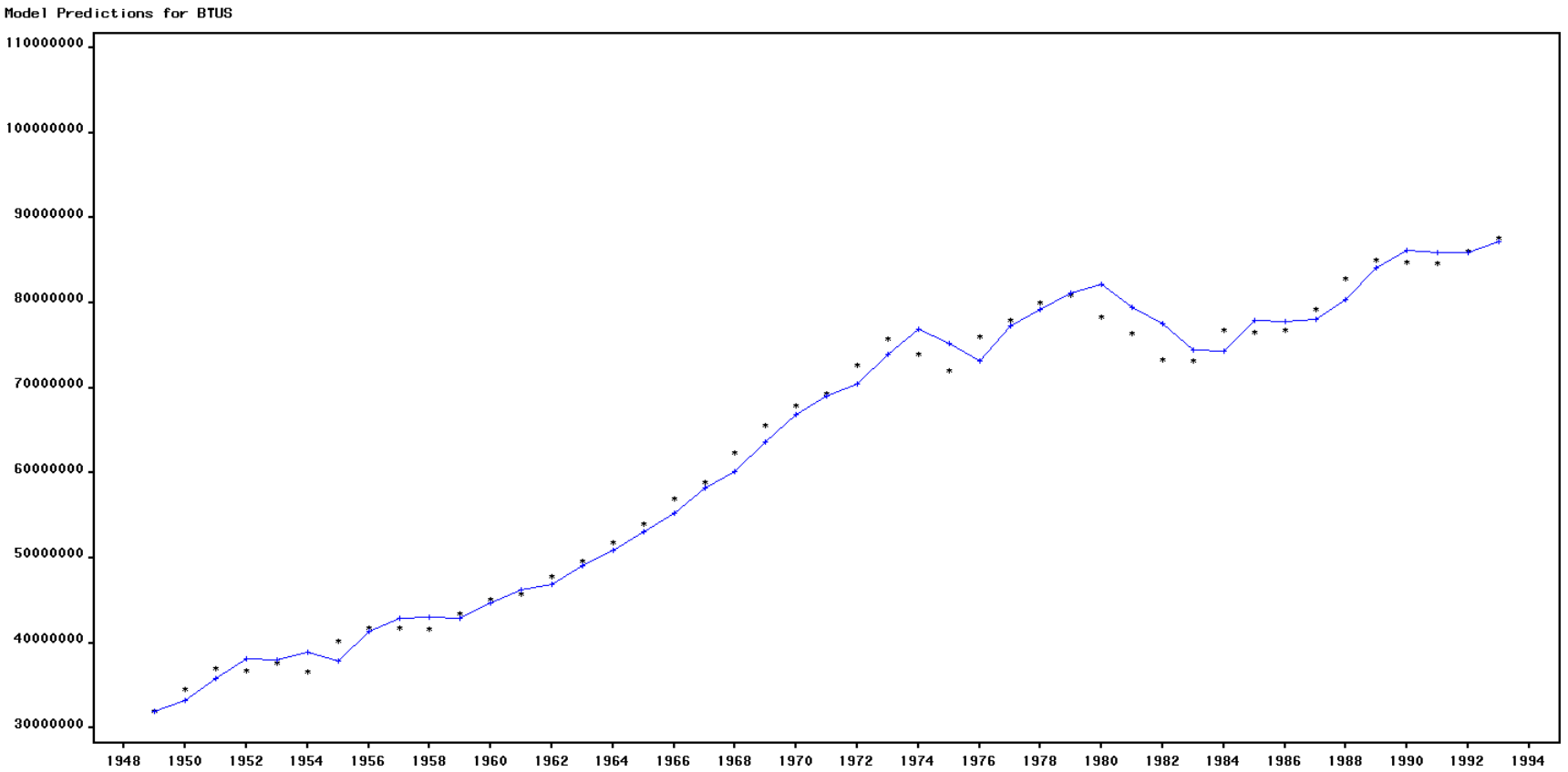

Figure 1 depicts a time series graph for predicted values ((+) signs) using the training data ((*) signs) set for BTUs variable, as reported by SAS(AFS).

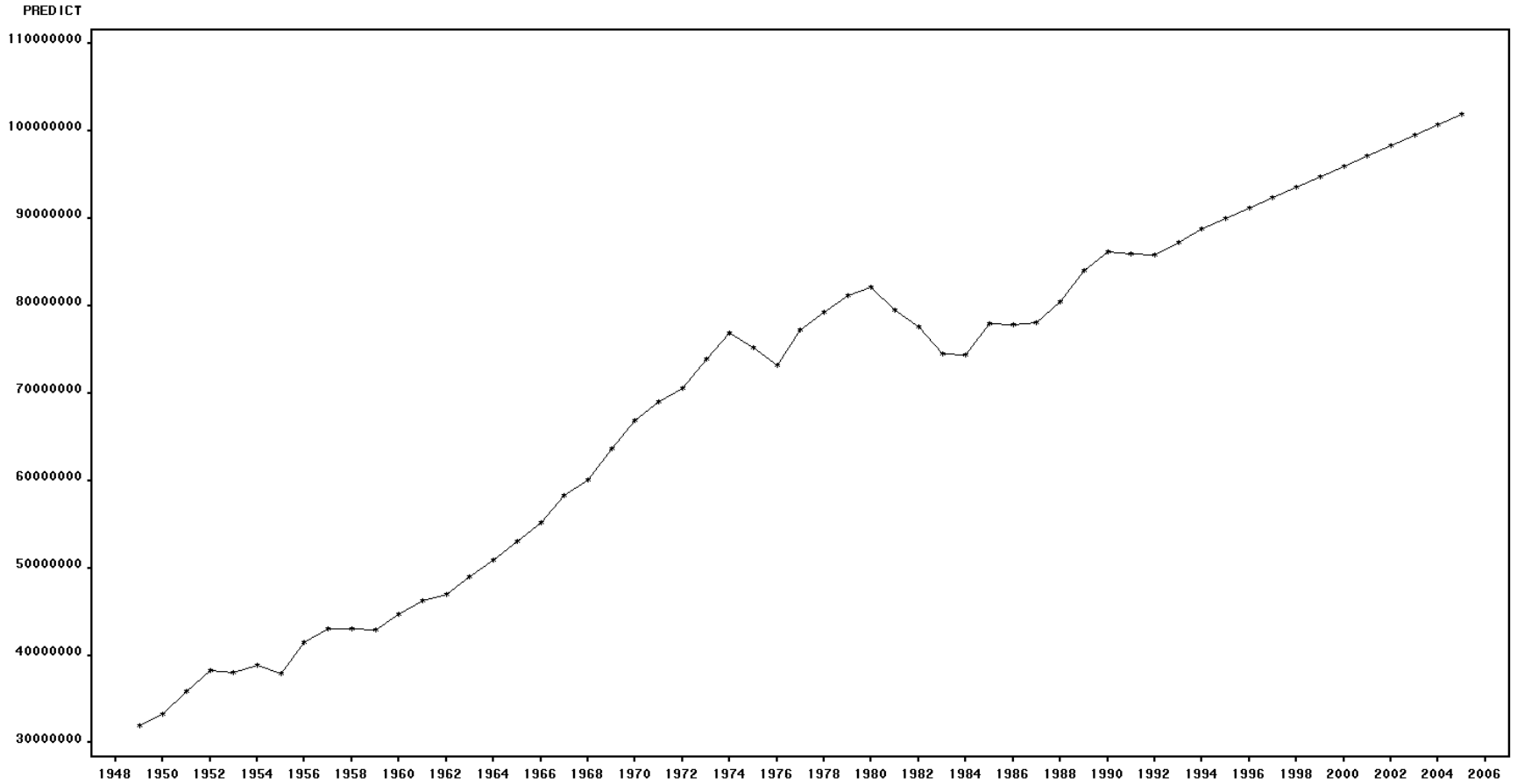

Figure 2 shows a time series graph for predicted values ((+) signs) for BTUs variable as reported by SAS(AFS), along with 12-year forecasted values for the average CO_2 variable. Here, predicted and forecast values are plotted using ((*) signs).

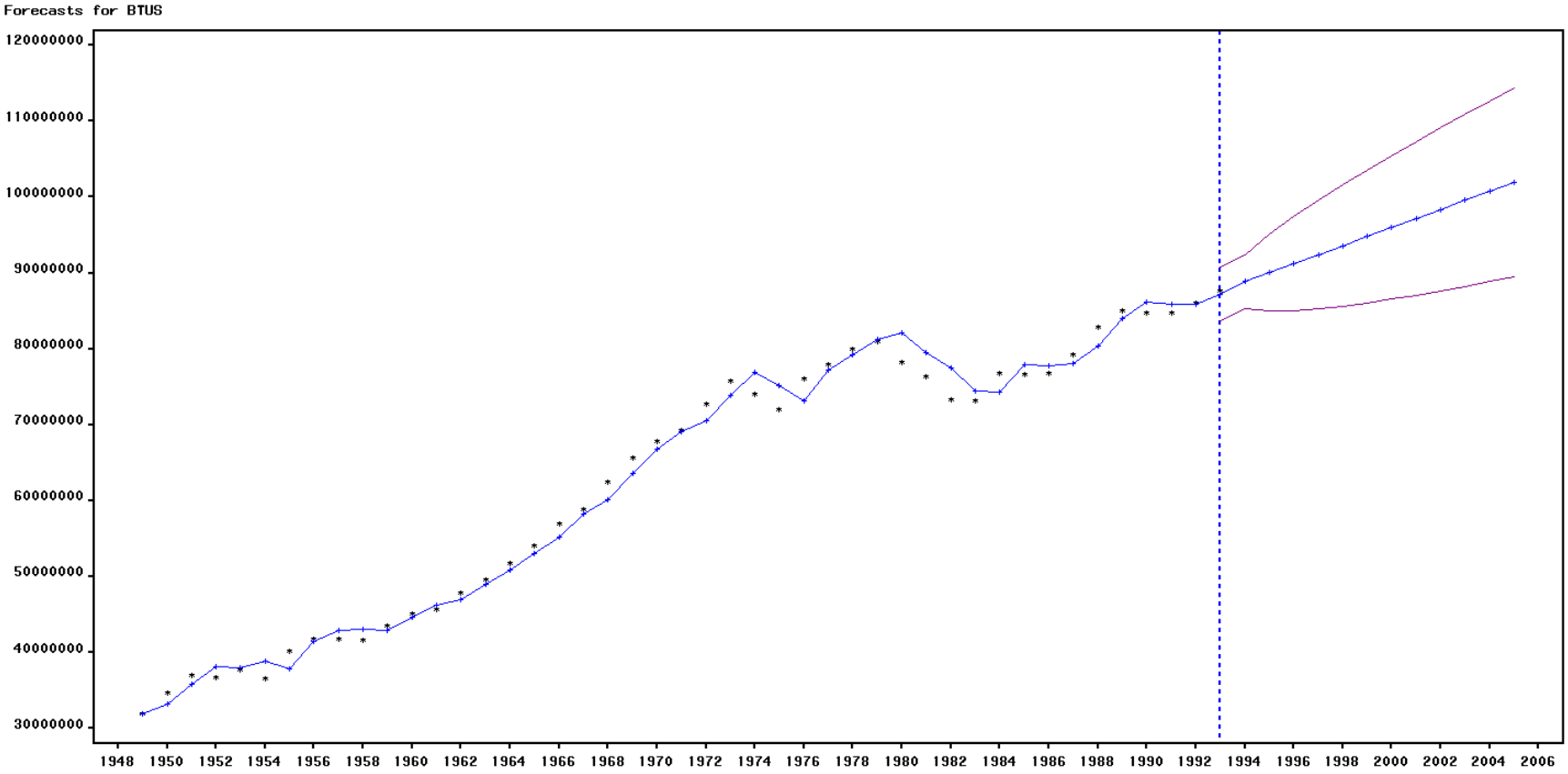

Figure 3a depicts a time series graph for predicted values ((+) signs) and the training data ((*) signs) for the BTUs variable, as reported by SAS(AFS), along with 12-year forecasted values and prediction limits.

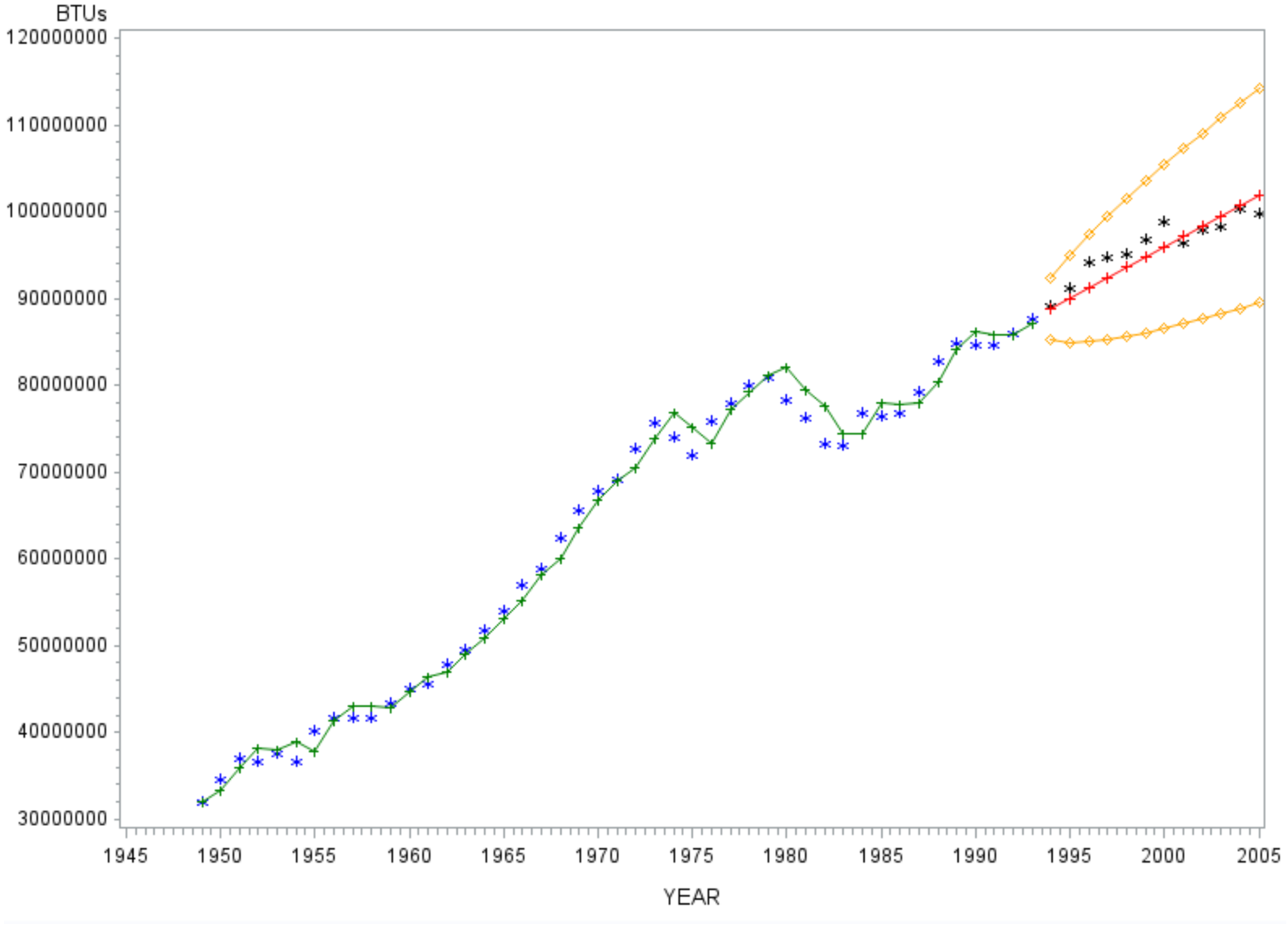

Figure 3b shows a time series graph for predicted values (green-colored (+) signs) for the BTUs variable as reported by SAS(AFS), along with 12-year forecasted values, which are plotted using red-colored plus (+) signs. The training data are plotted using blue-colored star (*) signs, and testing data are plotted using black-colored star (*) signs, while the 12-year confidence limits are yellow-colored.

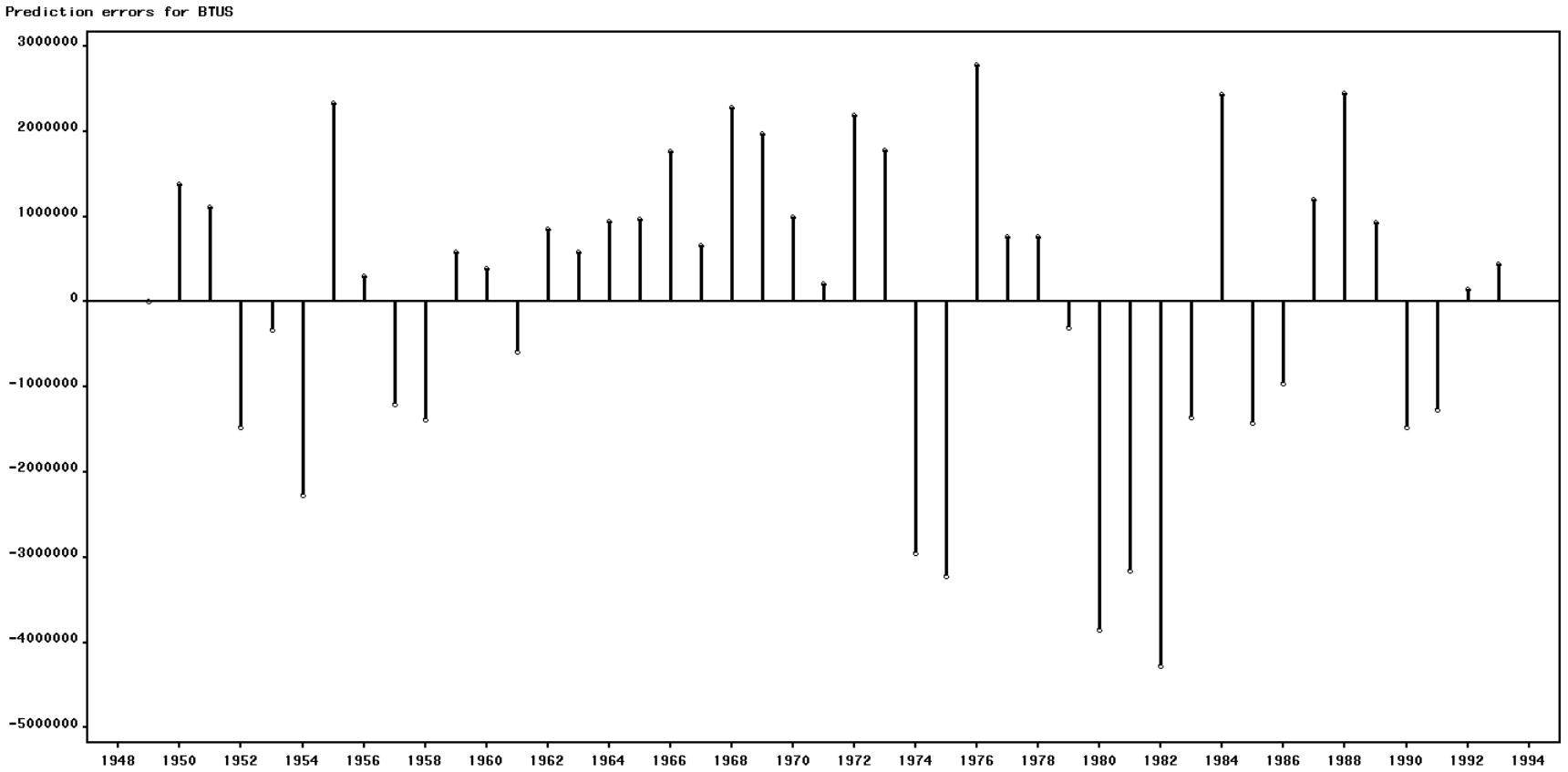

Figure 4. Prediction errors by time series values for the training data set.

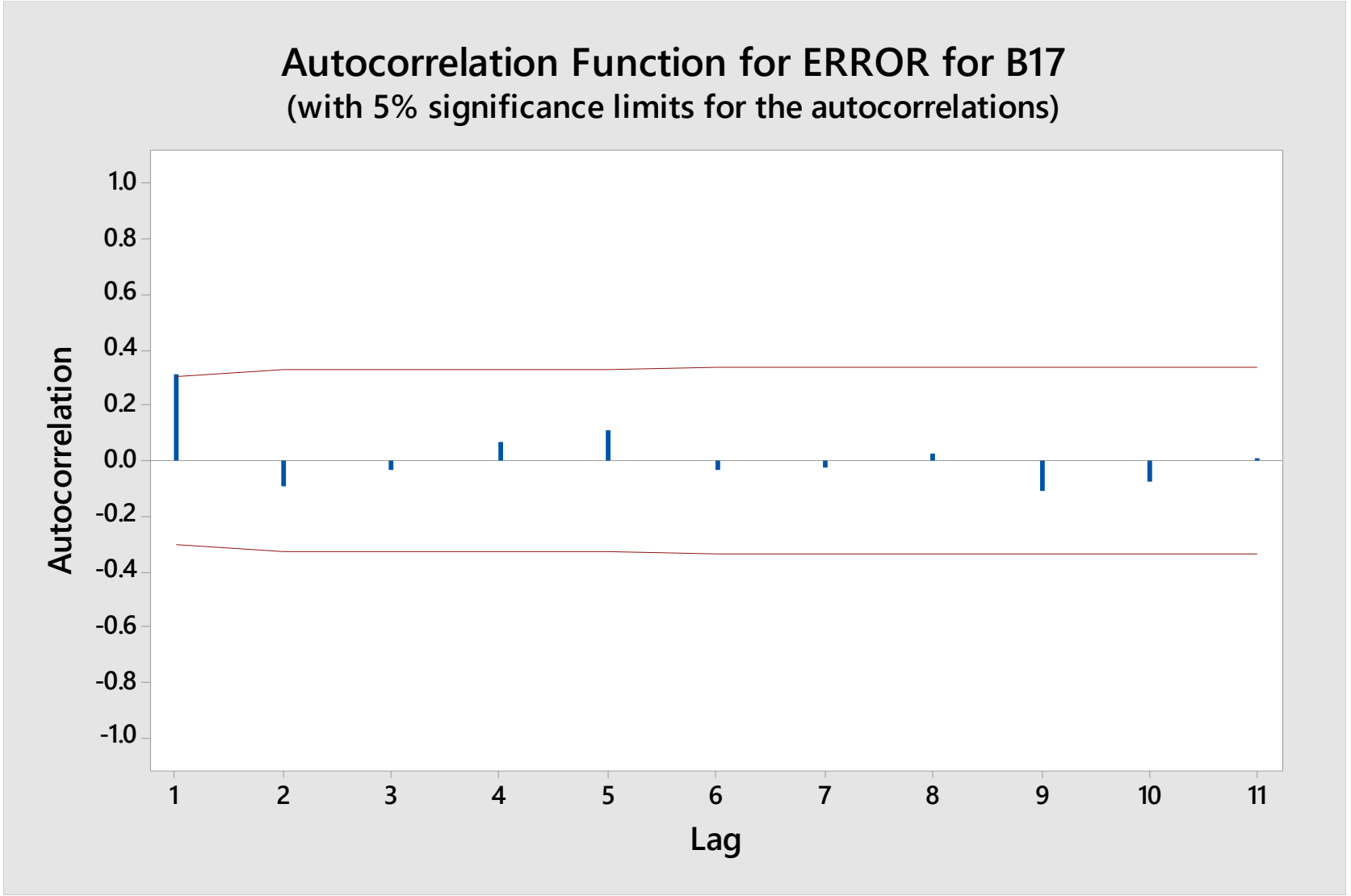

Figure 5. Autocorrelation Function (ACF) for the training data set.

Accuracy Measure for Table B.17 data.

Actual y_t	Predict $\hat{y}_t$(t-1)	Forecast error $e_t(1) = y_t - \hat{y}_t$(t-1)	Absolute error $\lvert e_t(1)\rvert$	Square error $[e_t(1)]^2$	Relative error $re_t(1) = 100(e_t(1)/y_t)$	Absolute percentage error $\lvert re_t(1)\rvert$
89291713	88847140	444573	444573	1.97645E+11	0.49789	0.49789
91199841	90041330	1158511	1158511	1.34215E+12	1.27030	1.27030
94225791	91234327	2991464	2991464	8.94886E+12	3.17478	3.17478
94800047	92426130	2373917	2373917	5.63548E+12	2.50413	2.50413
95200433	93616742	1583691	1583691	2.50808E+12	1.66353	1.66353
96836647	94806163	2030484	2030484	4.12287E+12	2.09681	2.09681
98976371	95994395	2981976	2981976	8.89218E+12	3.01282	3.01282
96497865	97181438	-683573	683573	4.67272E+11	-0.70838	0.70838
97966872	98367294	-400422	400422	1.60338E+11	-0.40873	0.40873
98273323	99551965	-1278642	1278642	1.63493E+12	-1.30111	1.30111
100414461	100735451	-320990	320990	1.03035E+11	-0.31967	0.31967
99894296	101917753	-2023457	2023457	4.09438E+12	-2.02560	2.02560

Table A. Error Measures table B17 time series data.

Data set	**MSE**	$\widehat{\sigma_{e(1)}}$	**MAD**	**MPE**	**MAPE**
Full data (45 data)	2.86994E12	1694090.5	1385045.7	0.17630	2.12488
Training data (33 data)	3.08694E12	1756969.9	1437579.6	0.10928	2.35112
Holdout data (12 data)	3.17560E+12	1782021	1522642	0.788065	1.58198

Table B. Accuracy Measures table B17 time series data.

Table B provides SAS (AFS)® forecast errors for the full data set, the training data set, and the holdout (or test) data set. The Damped Trend Exponential Smoothing model was suggested as the "best model" for this data set. SAS (AFS)® and MINITAB® produced the accuracy measures shown in Table B, which comprise the Mean Square Error (MSE), the Standard Deviation of the Forecast Errors (SDFE), the Mean Absolute Deviation (MAD), the Mean Percent Forecast Error (MPE), and Mean Absolute Percent Error (MAPE).

Here, the accuracy statistics are computed as follows:

Mean square error:

$$\textbf{MSE} = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = 3.17560\text{E+}12$$

Standard Deviation of Forecast Error (Root Mean Square Error):

$$\widehat{\sigma_{e(1)}} = \sqrt{MSE} = \sqrt{3.17560\text{E}+12} = 1782021$$

Mean Absolute Deviation (Mean Absolute Error):

$$\textbf{MAD} = \frac{1}{n}\sum_{t=1}^{n} |e_t(1)| = 1522642$$

Mean Percent Forecast Error:

$$\textbf{MPE} = \frac{1}{n}\sum_{t=1}^{n} re_t(1) = 0.788065$$

Mean Absolute Percent Error:

$$\textbf{MAPE} = \frac{1}{n}\sum_{t=1}^{n} |re_t(1)| = 1.58198$$

Worksheet 1 ***

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	Actual	Predict	Forecast_Error	Abs_Error	Sq_Error	Relative_Error	Abs_Percentage_Error	MSE	Root_MSE	MAD	MPE	MAPE
1	89291713	88847140	444573	444573	1.97645E+11	0.49789	0.49789	3.17560E+12	1782021	1522642	0.788065	1.58198
2	91199841	90041330	1158511	1158511	1.34215E+12	1.27030	1.27030					
3	94225791	91234327	2991464	2991464	8.94886E+12	3.17478	3.17478					
4	94800047	92426130	2373917	2373917	5.63548E+12	2.50413	2.50413					
5	95200433	93616742	1583691	1583691	2.50808E+12	1.66353	1.66353					
6	96836647	94806163	2030484	2030484	4.12287E+12	2.09681	2.09681					
7	98976371	95994395	2981976	2981976	8.89218E+12	3.01282	3.01282					
8	96497865	97181438	-683573	683573	4.67272E+11	-0.70838	0.70838					
9	97966872	98367294	-400422	400422	1.60338E+11	-0.40873	0.40873					
10	98273323	99551965	-1278642	1278642	1.63493E+12	-1.30111	1.30111					
11	100414461	100735451	-320990	320990	1.03035E+11	-0.31967	0.31967					
12	99894296	101917753	-2023457	2023457	4.09438E+12	-2.02560	2.02560					

Table C. MINITAB® 17 Calculation of Forecast Errors for Holdout Data.

The first two columns of Table C consist of the actual values of the holdout data set and the SAS (AFS)® forecasted values, based on training data. The remaining columns give forecast errors, the absolute forecast errors, the squared forecast errors, the relative forecast errors and absolute forecast percentage errors using the MINITAB® calculator function.

Obs	DATE	ACTUAL	PREDICT	UPPER	LOWER	ERROR	STD	NERROR	_LEVEL_	_TREND_
46	1994	.	88847140	92411602	85282677	.	1818637	.	88847140	1195386
47	1995	.	90041330	95082236	85000424	.	2571938	.	90041330	1194191
48	1996	.	91234327	97410201	85058453	.	3151014	.	91234327	1192996
49	1997	.	92426130	99560375	85291885	.	3639988	.	92426130	1191803
50	1998	.	93616742	101596644	85636840	.	4071453	.	93616742	1190612
51	1999	.	94806163	103551765	86060560	.	4462124	.	94806163	1189421
52	2000	.	95994395	105445196	86543594	.	4821926	.	95994395	1188232
53	2001	.	97181438	107289606	87073270	.	5157323	.	97181438	1187043
54	2002	.	98367294	109093795	87640793	.	5472805	.	98367294	1185856
55	2003	.	99551965	110864168	88239762	.	5771638	.	99551965	1184670
56	2004	.	100735451	112605556	88865346	.	6056287	.	100735451	1183486
57	2005	.	101917753	114321712	89513794	.	6328667	.	101917753	1182302

Table D. Forecasted values for the last 12 year of Holdout Data.

Table D contains forecasted values for the final twelve month for the BTUs variable, and the corresponding lower and upper 95% prediction limits. The Root Mean Square Prediction Error, Level and Trend smoothing components are also shown..

Model Parameter	Estimate	Std. Error	T	Prob>|T|
LEVEL Smoothing Weight	0.99900	0.1106	9.0363	<.0001
TREND Smoothing Weight	0.00100	0.0436	0.0230	0.9818
DAMPING Smoothing Weight	0.99900	0.0027	370.9814	<.0001
Residual Variance (sigma squared)	3.30744E12	.	.	.
Smoothed Level	87651754	.	.	.
Smoothed Trend	1196583	.	.	.

Table E. Parameter Estimates for Training Data using the Damped Trend Exponential Smoothing model suggested by SAS (AFS)®.

Results

In Figure 3b, it appears the model fits the data appropriately as well as the SAS(AFS) forecast values with acceptably small prediction error. There are no problem in the ACF function. In addition, The MSE, RMSE and MAD are similar for both the training and holdout data sets; that is, the variability in forecast errors are approximately similar. It seems SAS(AFS) model works well for this data set.

TABLE B.18 (ANNUAL US COAL PRODUCTION) ANALYSIS

Table B.18 comprises data on the Annual US Coal Production from 1949 to 2005, where the Coal Production (in 10^3 Short Tons) variable is collected yearly. The data set excludes last twelve year values as a holdout set in order to analyze the accuracy of SAS(AFS), which suggested a Damped Trend Exponential Smoothing model as the "best model" for this training, or initialization, data set. SAS(AFS) also produced the predicted values for the next 12 year. I have calculated different types of forecast errors based on the hold out data and predicted data.

The model equation is

$$Y_t = \mu_t + \beta_t\, t + \varepsilon_t$$

where,

Y_t = observed value i.e. securities rate at time t

μ_t = smoothed level and μ_t = 953971

β_t = smoothed trend and β_t = 10914

Now, the k-step prediction equation is

$$\hat{Y}_t(k) = L_t + \sum_{i=1}^{k} \phi^t\, T_t$$

and the smoothing equations are

$$L_t = \alpha\, Y_t + (1-\alpha)\,(L_{t-1} + \phi\, T_{t-1})$$

and $T_t = \gamma\,(L_t - L_{t-1}) + (1-\gamma)\,\phi\, T_{t-1}$

where,

L_t = Level estimate and $L_0 = \mu_t$ = 953971

T_t = Trend estimate and $T_0 = \beta_t$ = 10914 α = level smoothing and α = 0.86851

γ = trend smoothing and γ = 0.00100 and

ϕ = damping smoothing and ϕ = 0.99795

The ARIMA model equivalency to damped-trend linear exponential smoothing is the ARIMA (1, 1, 2) model,

$(1-\phi B)(1-B)\, Y_t = (1-\theta_1 B - \theta_2 B^2)\, \varepsilon_t$

$\theta_1 = 1 + \phi - \alpha - \alpha\gamma\phi$

$\theta_2 = (\alpha - 1)\phi$

Table B.18: Annual US Coal Production

Year	Coal Production, Short Tons in Thousands	Year	Coal Production, Short Tons in Thousands	Year	Coal Production, Short Tons in Thousands
1949	480,570	1968	556,706	1987	918,762
1950	560,388	1969	570,978	1988	950,265
1951	576,335	1970	612,661	1989	980,729
1952	507,424	1971	560,919	1990	1,029,076
1953	488,239	1972	602,492	1991	995,984
1954	420,789	1973	598,568	1992	997,545

Year	Coal Production, Short Tons in Thousands	Year	Coal Production, Short Tons in Thousands	Year	Coal Production, Short Tons in Thousands
1955	490,838	1974	610,023	1993	945,424
1956	529,774	1975	654,641	1994	1,033,504
1957	518,042	1976	684,913	1995	1,032,974
1958	431,617	1977	697,205	1996	1,063,856
1959	432,677	1978	670,164	1997	1,089,932
1960	434,329	1979	781,134	1998	1,117,535
1961	420,423	1980	829,700	1999	1,100,431
1962	439,043	1981	823,775	2000	1,073,612
1963	477,195	1982	838,112	2001	1,127,689
1964	504,182	1983	782,091	2002	1,094,283
1965	526,954	1984	895,921	2003	1,071,753
1966	546,822	1985	883,638	2004	1,112,099
1967	564,882	1986	890,315	2005	1,133,253

The collection of Figures and Tables that follow are individually identified below.

List of Figures:

List of Tables:

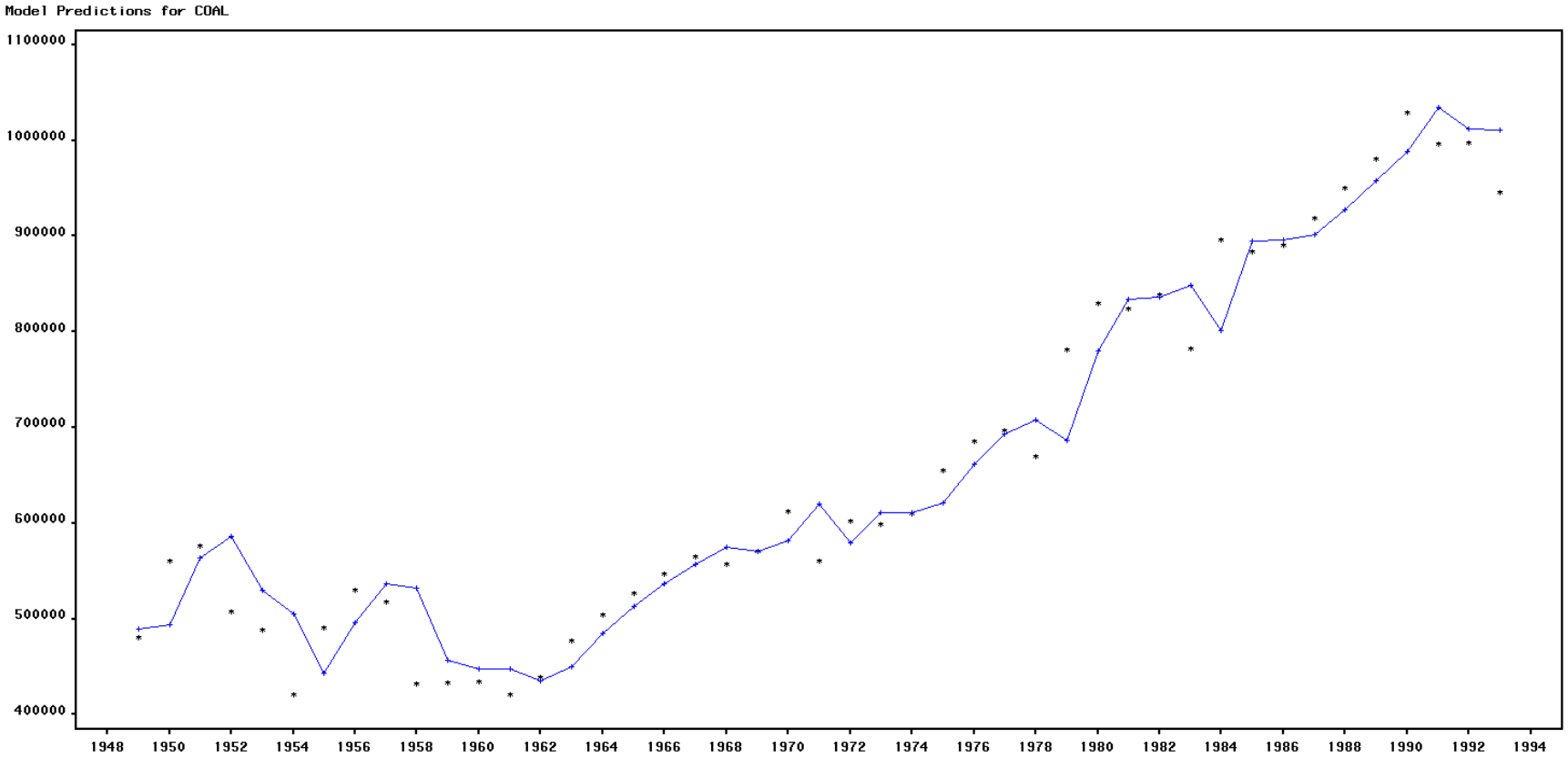

Figure 1 depicts a time series graph for predicted values ((+) signs) using the training data ((*) signs) set for Coal Production variable, as reported by SAS(AFS).

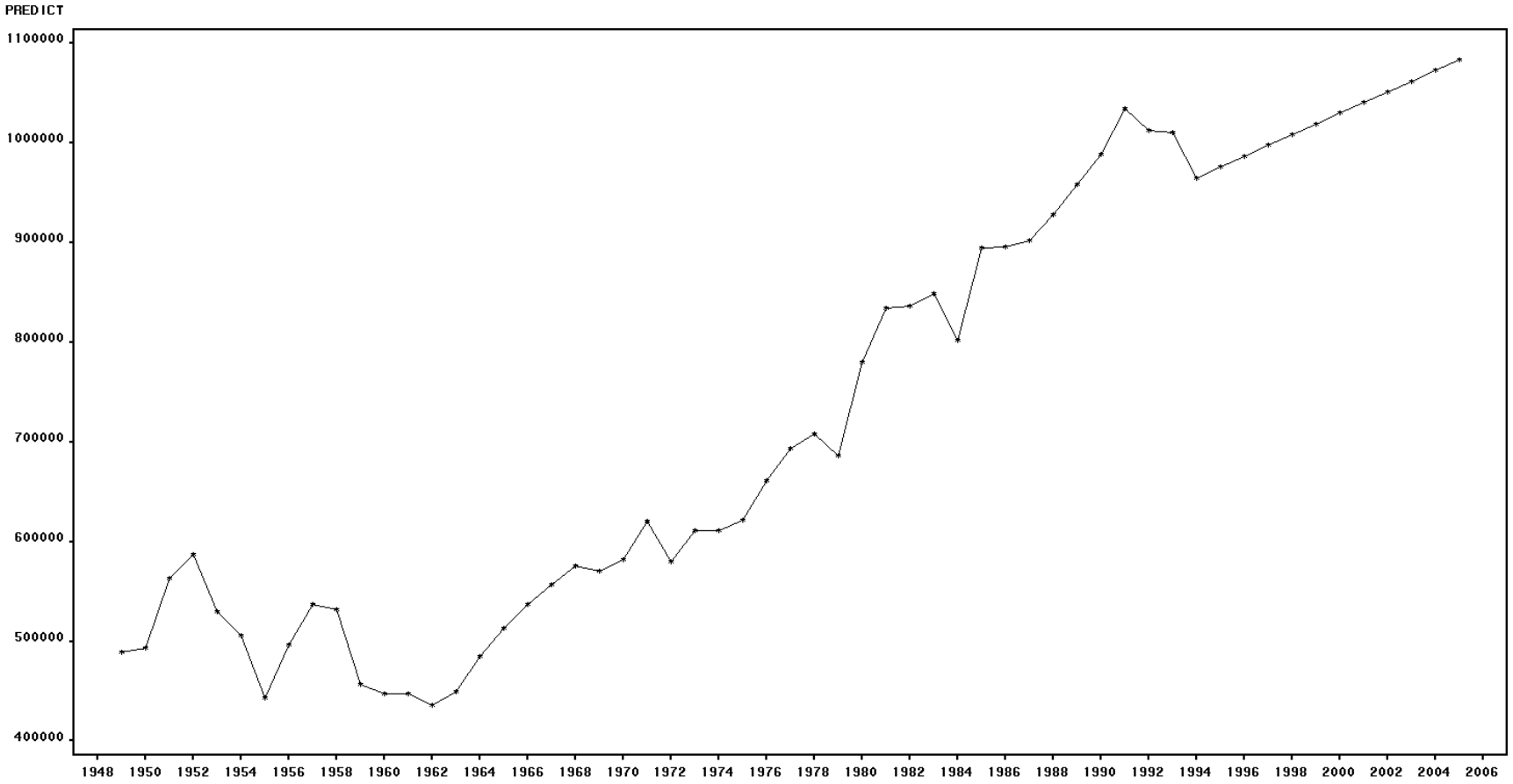

Figure 2 shows a time series graph for predicted values ((+) signs) for Coal Production variable as reported by SAS(AFS), along with 12-year forecasted values for the average CO_2 variable. Here, predicted and forecast values are plotted using ((*) signs).

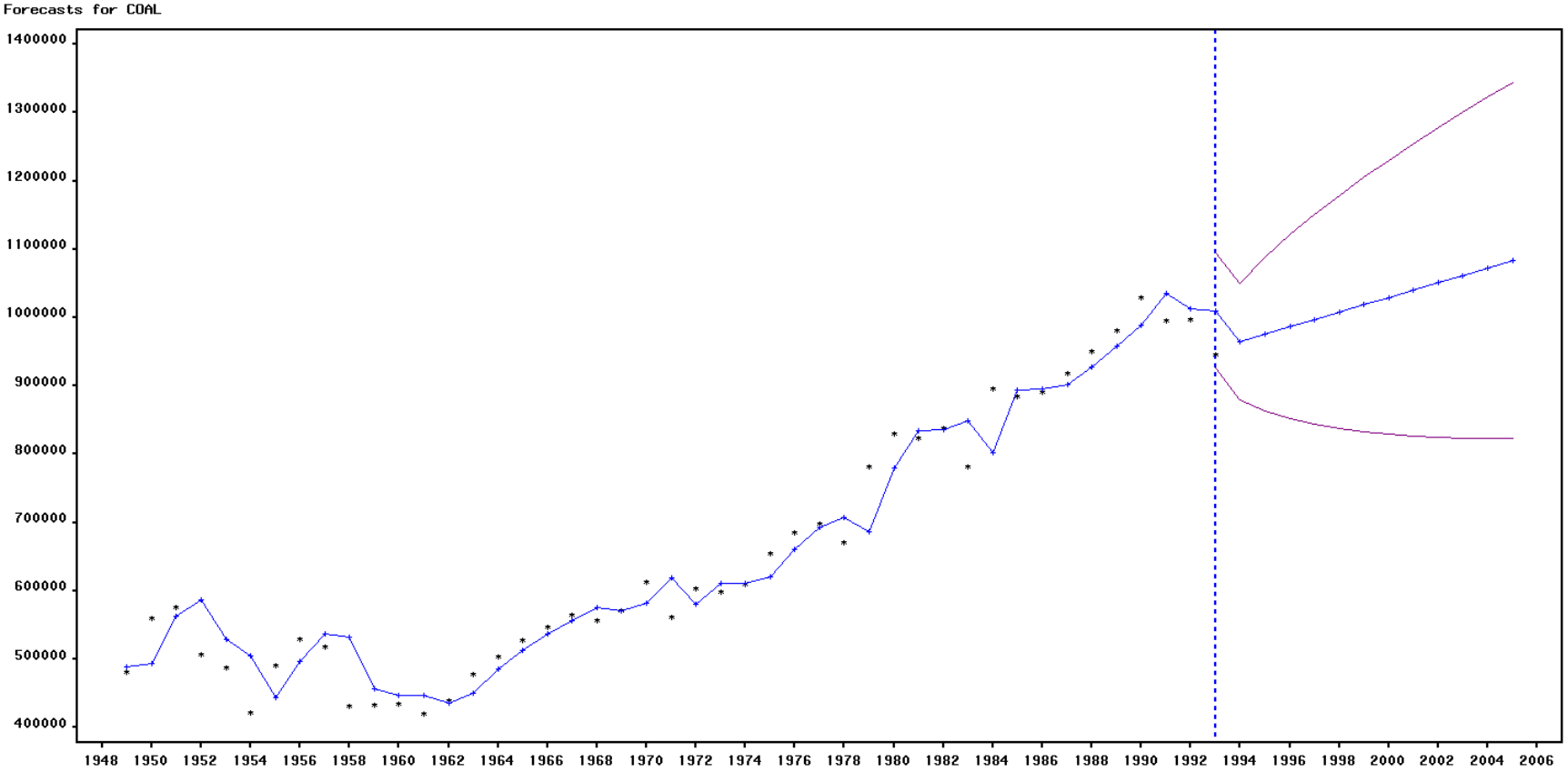

Figure 3a depicts a time series graph for predicted values ((+) signs) and the training data ((*) signs) for the Coal Production variable, as reported by SAS(AFS), along with 12-year forecasted values and prediction limits.

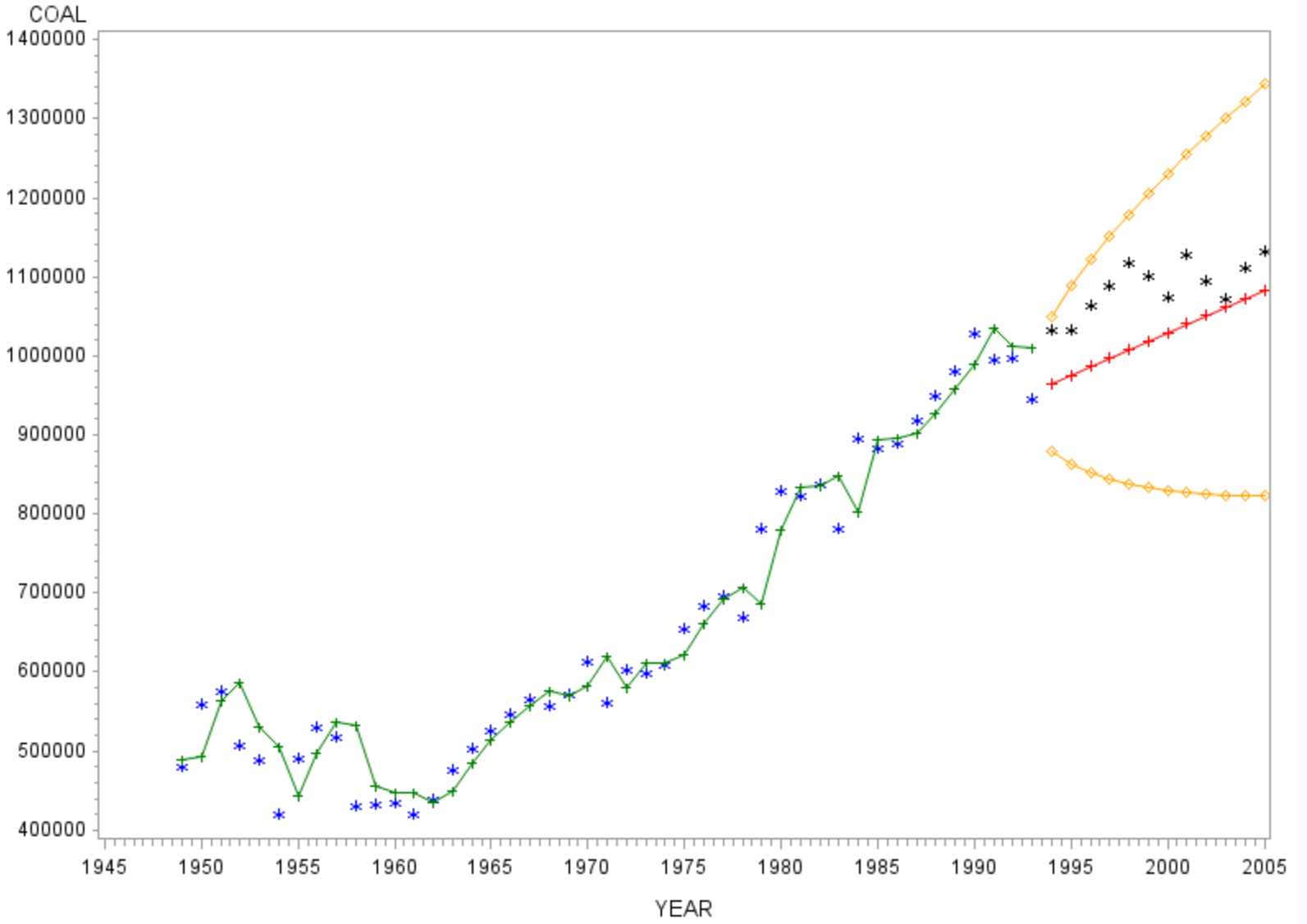

Figure 3b shows a time series graph for predicted values (green-colored (+) signs) for the Coal variable as reported by SAS(AFS), along with 12-year forecasted values, which are plotted using red-colored plus (+) signs. The training data are plotted using blue-colored star (*) signs, and testing data are plotted using black-colored star (*) signs, while the 12-year confidence limits are yellow-colored.

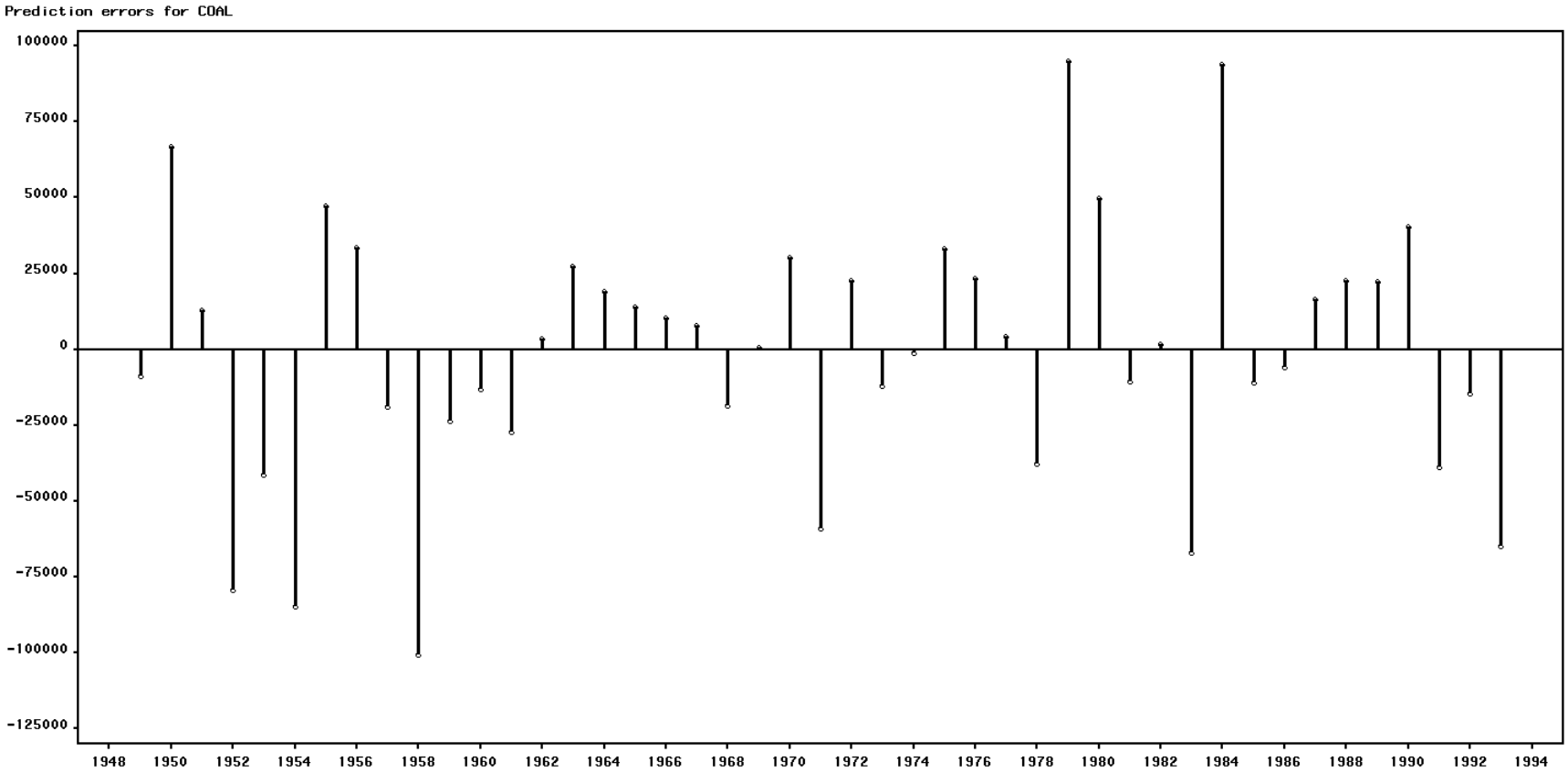

Figure 4. Prediction errors by time series values for the training data set.

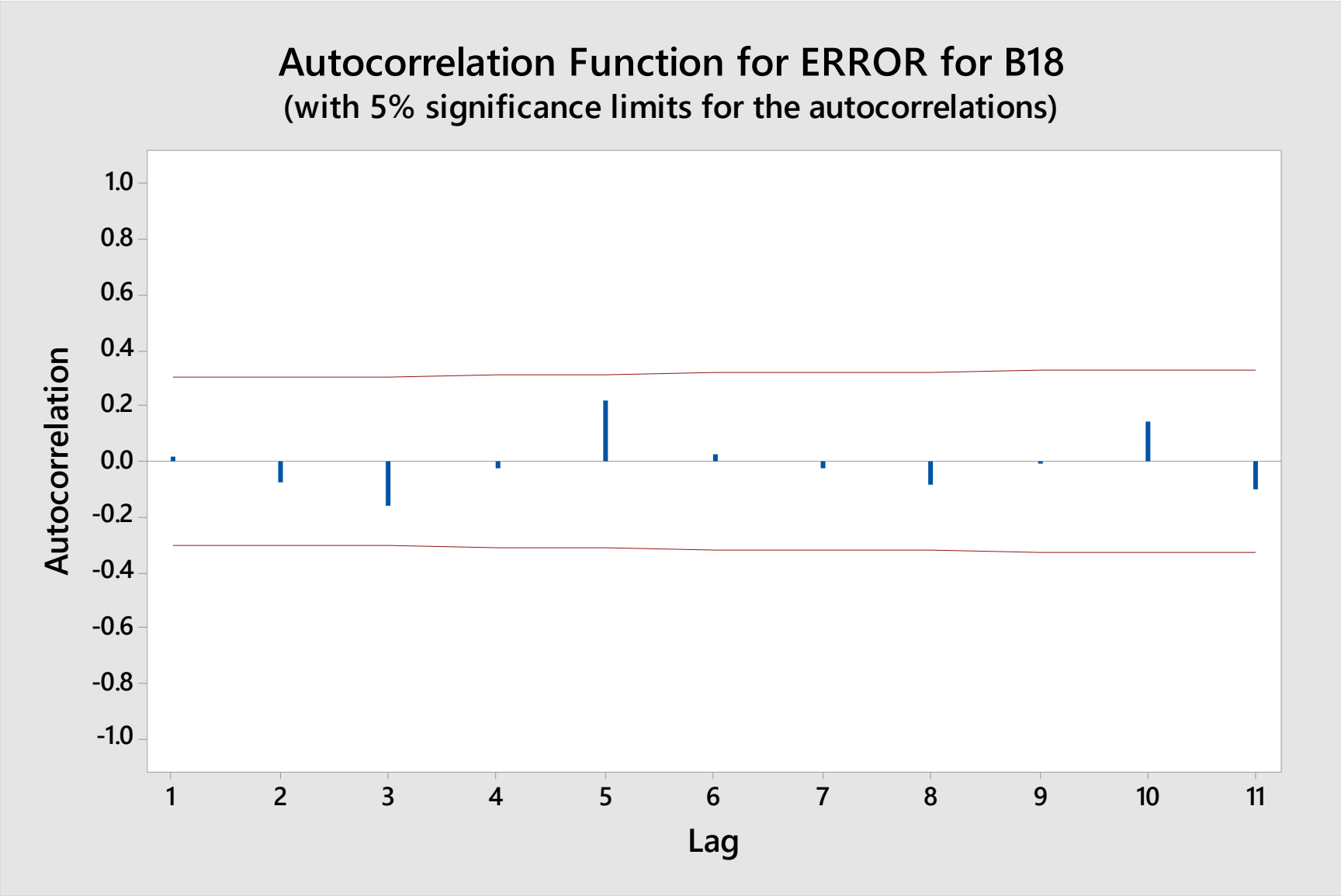

Figure 5. Autocorrelation Function (ACF) for the training data set.

Accuracy Measure for Table B.18 data.

Actual y_t	Predict $\widehat{y_t}$(t-1)	Forecast error $e_t(1) = y_t - \widehat{y_t}(t-1)$	Absolute error $\lvert e_t(1)\rvert$	Square error $[e_t(1)]^2$	Relative error $re_t(1) = 100(e_t(1)/y_t)$	Absolute percentage error $\lvert re_t(1)\rvert$
1033504	964863	68641	68641	4.71163E+09	6.64161	6.64161
1032974	975732	57242	57242	3.27662E+09	5.54145	5.54145
1063856	986579	77277	77277	5.97166E+09	7.26382	7.26382
1089932	997403	92529	92529	8.56158E+09	8.48941	8.48941
1117535	1008205	109330	109330	1.19531E+10	9.78315	9.78315
1100431	1018986	81445	81445	6.63336E+09	7.40123	7.40123
1073612	1029744	43868	43868	1.92436E+09	4.08598	4.08598
1127689	1040480	87209	87209	7.60538E+09	7.73341	7.73341
1094283	1051194	43089	43089	1.85667E+09	3.93765	3.93765
1071753	1061886	9867	9867	9.73493E+07	0.92060	0.92060
1112099	1072556	39543	39543	1.56364E+09	3.55570	3.55570
1133253	1083205	50048	50048	2.50485E+09	4.41635	4.41635

Table A. Error Measures table B18 time series data.

Data set	**MSE**	$\widehat{\sigma_{e(1)}}$	**MAD**	**MPE**	**MAPE**
Full data (45 data)	1612240138	40152.7	31382.9	-0.65115	4.78334
Training data (33 data)	1757307607	41920.3	31960.9	- 0.67330	5.26960
Holdout data (12 data)	4721680859	68714.5	63340.6	5.81420	5.81420

Table B. Accuracy Measures table B18 time series data.

Table B provides SAS (AFS)® forecast errors for the full data set, the training data set, and the holdout (or test) data set. The Damped Trend Exponential Smoothing model was suggested as the "best model" for this data set. SAS (AFS)® and MINITAB® produced the accuracy measures shown in Table B, which comprise the Mean Square Error (MSE), the Standard Deviation of the Forecast Errors (SDFE), the Mean Absolute Deviation (MAD), the Mean Percent Forecast Error (MPE), and Mean Absolute Percent Error (MAPE).

Here, the accuracy statistics are computed as follows:

Mean square error:

$$\textbf{MSE} = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = 4721680859$$

Standard Deviation of Forecast Error (Root Mean Square Error):

$$\widehat{\sigma_{e(1)}} = \sqrt{MSE} = \sqrt{4721680859} = 68714.5$$

Mean Absolute Deviation (Mean Absolute Error):

$$\textbf{MAD} = \frac{1}{n}\sum_{t=1}^{n}|e_t(1)| = 63340.6$$

Mean Percent Forecast Error:

$$\textbf{MPE} = \frac{1}{n}\sum_{t=1}^{n} re_t(1) = 5.81420$$

Mean Absolute Percent Error:

$$\textbf{MAPE} = \frac{1}{n}\sum_{t=1}^{n}|re_t(1)| = 5.81420$$

Worksheet 1 ***

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	Actual	Predict	Forecast_Error	Abs_Error	Sq_Error	Relative_Error	Abs_Percentage_Error	MSE	Root_MSE	MAD	MPE	MAPE
1	1033504	964863	68641	68641	4.71163E+09	6.64161	6.64161	4721680859	68714.5	63340.6	5.81420	5.81420
2	1032974	975732	57242	57242	3.27662E+09	5.54145	5.54145					
3	1063856	986579	77277	77277	5.97166E+09	7.26382	7.26382					
4	1089932	997403	92529	92529	8.56158E+09	8.48941	8.48941					
5	1117535	1008205	109330	109330	1.19531E+10	9.78315	9.78315					
6	1100431	1018986	81445	81445	6.63336E+09	7.40123	7.40123					
7	1073612	1029744	43868	43868	1.92436E+09	4.08598	4.08598					
8	1127689	1040480	87209	87209	7.60538E+09	7.73341	7.73341					
9	1094283	1051194	43089	43089	1.85667E+09	3.93765	3.93765					
10	1071753	1061886	9867	9867	9.73493E+07	0.92060	0.92060					
11	1112099	1072556	39543	39543	1.56364E+09	3.55570	3.55570					
12	1133253	1083205	50048	50048	2.50485E+09	4.41635	4.41635					

Table C. MINITAB® 17 Calculation of Forecast Errors for Holdout Data.

The first two columns of Table C consist of the actual values of the holdout data set and the SAS (AFS)® forecasted values, based on training data. The remaining columns give forecast errors, the absolute forecast errors, the squared forecast errors, the relative forecast errors and absolute forecast percentage errors using the MINITAB® calculator function.

Obs	DATE	ACTUAL	PREDICT	UPPER	LOWER	ERROR	STD	NERROR	_LEVEL_	_TREND_
46	1994	.	964863	1049909	879817	.	43392	.	964863	10891
47	1995	.	975732	1088424	863040	.	57497	.	975732	10869
48	1996	.	986579	1121401	851756	.	68788	.	986579	10847
49	1997	.	997403	1151239	843567	.	78489	.	997403	10825
50	1998	.	1008205	1178982	837428	.	87133	.	1008205	10802
51	1999	.	1018986	1205198	832774	.	95008	.	1018986	10780
52	2000	.	1029744	1230233	829255	.	102292	.	1029744	10758
53	2001	.	1040480	1254320	826640	.	109104	.	1040480	10736
54	2002	.	1051194	1277624	824764	.	115527	.	1051194	10714
55	2003	.	1061886	1300264	823509	.	121623	.	1061886	10692
56	2004	.	1072556	1322332	822781	.	127439	.	1072556	10670
57	2005	.	1083205	1343902	822508	.	133011	.	1083205	10648

Table D. Forecasted values for the last 12 year of Holdout Data.

Table D contains forecasted values for the final twelve month for the Coal Production variable, and the corresponding lower and upper 95% prediction limits. The Root Mean Square Prediction Error, Level and Trend smoothing components are also shown..

Model Parameter	Estimate	Std. Error	T	Prob>|T|
LEVEL Smoothing Weight	0.86851	0.1124	7.7257	<.0001
TREND Smoothing Weight	0.00100	0.0442	0.0226	0.9821
DAMPING Smoothing Weight	0.99795	0.0114	87.3748	<.0001
Residual Variance (sigma squared)	1882829579	.	.	.
Smoothed Level	953971	.	.	.
Smoothed Trend	10914	.	.	.

Table E. Parameter Estimates for Training Data using the Damped Trend Exponential Smoothing model suggested by SAS (AFS)®.

Results

In Figure 3b, it seems the model fits the data appropriately, however, the SAS(AFS) could not forecast values with acceptably small prediction error. There are no spikes in the ACF function, which means forecast errors are uncorrelated. The MSE, RMSE and MAD are not similar for both the training and holdout data sets; that is, the variability in forecast errors are not similar.

TABLE B.19 (ARIZONA DROWNING RATE, CHILDREN 1-4 YEARS OLD) ANALYSIS

Table B.19 comprises data on the Arizona Drowning Rate, Children 1-4 Years Old from 1970 to 2004, where the Drowing Rate (in per 100,000) variable is collected yearly. The data set excludes last six year values as a holdout set in order to analyze the accuracy of SAS(AFS), which suggested a Linear Trend model as the "best model" for this training, or initialization, data set. SAS(AFS) also produced the predicted values for the next 6 year. I have calculated different types of forecast errors based on the hold out data and predicted data.

Table B.19: Arizona Drowning Rate, Children 1-4 Years Old

Year	Drowning Rate per 100,000 Children 1-4 years old	Year	Drowning Rate per 100,000 Children 1-4 years old
1970	19.9	1988	9.2
1971	16.1	1989	11.9
1972	19.5	1990	5.8
1973	19.8	1991	8.5
1974	21.3	1992	7.1
1975	15.0	1993	7.9
1976	15.5	1994	8.0
1977	16.4	1995	9.9
1978	18.2	1996	8.5
1979	15.3	1997	9.1
1980	15.6	1998	9.7
1981	19.5	1999	6.2
1982	14.0	2000	7.2
1983	13.1	2001	8.7
1984	10.5	2002	5.8
1985	11.5	2003	5.7
1986	12.9	2004	5.2
1987	8.4		

The collection of Figures and Tables that follow are individually identified below.

List of Figures:

List of Tables:

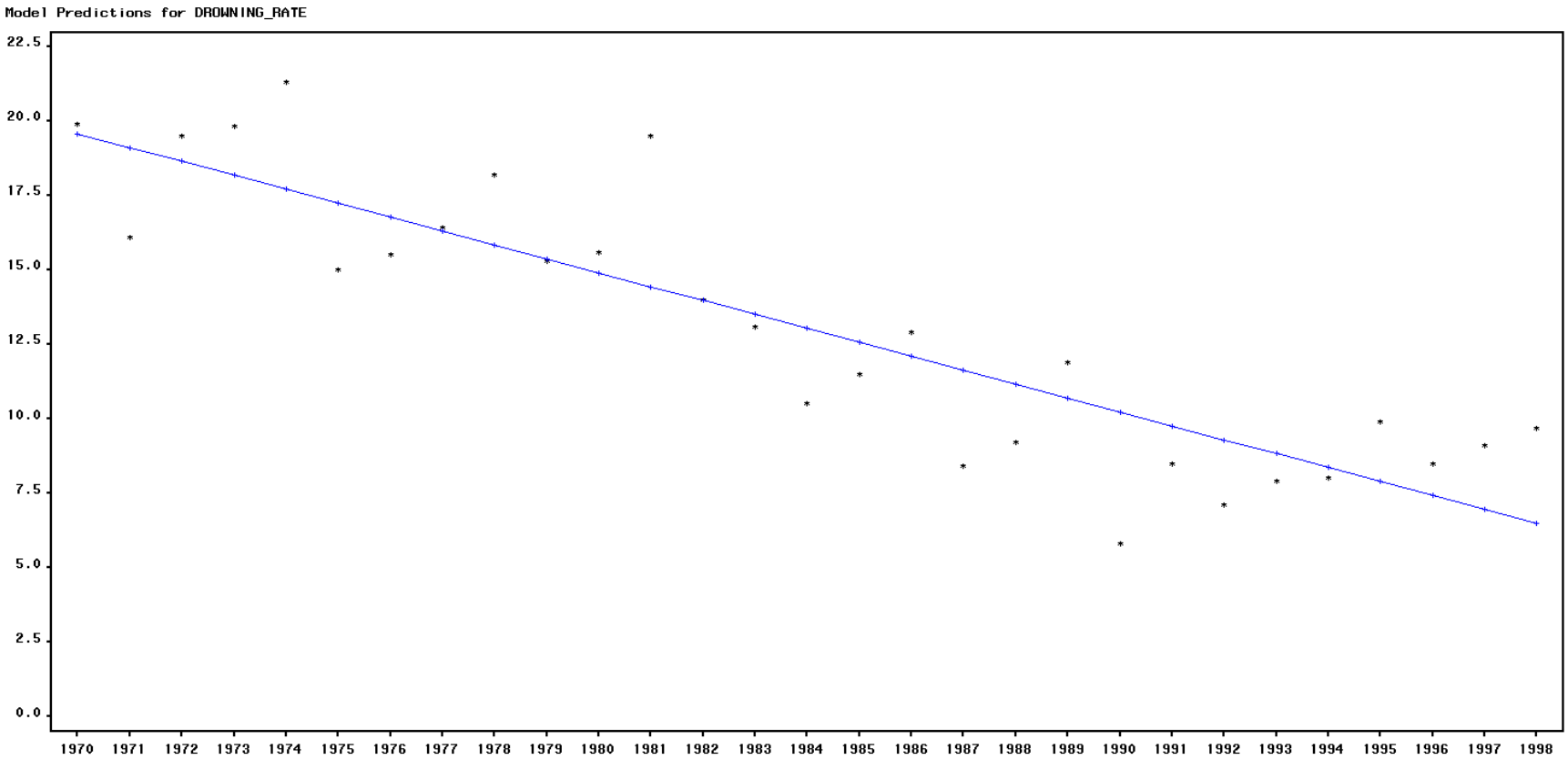

Figure 1 depicts a time series graph for predicted values ((+) signs) using the training data ((*) signs) set for Drowning Rate variable, as reported by SAS(AFS).

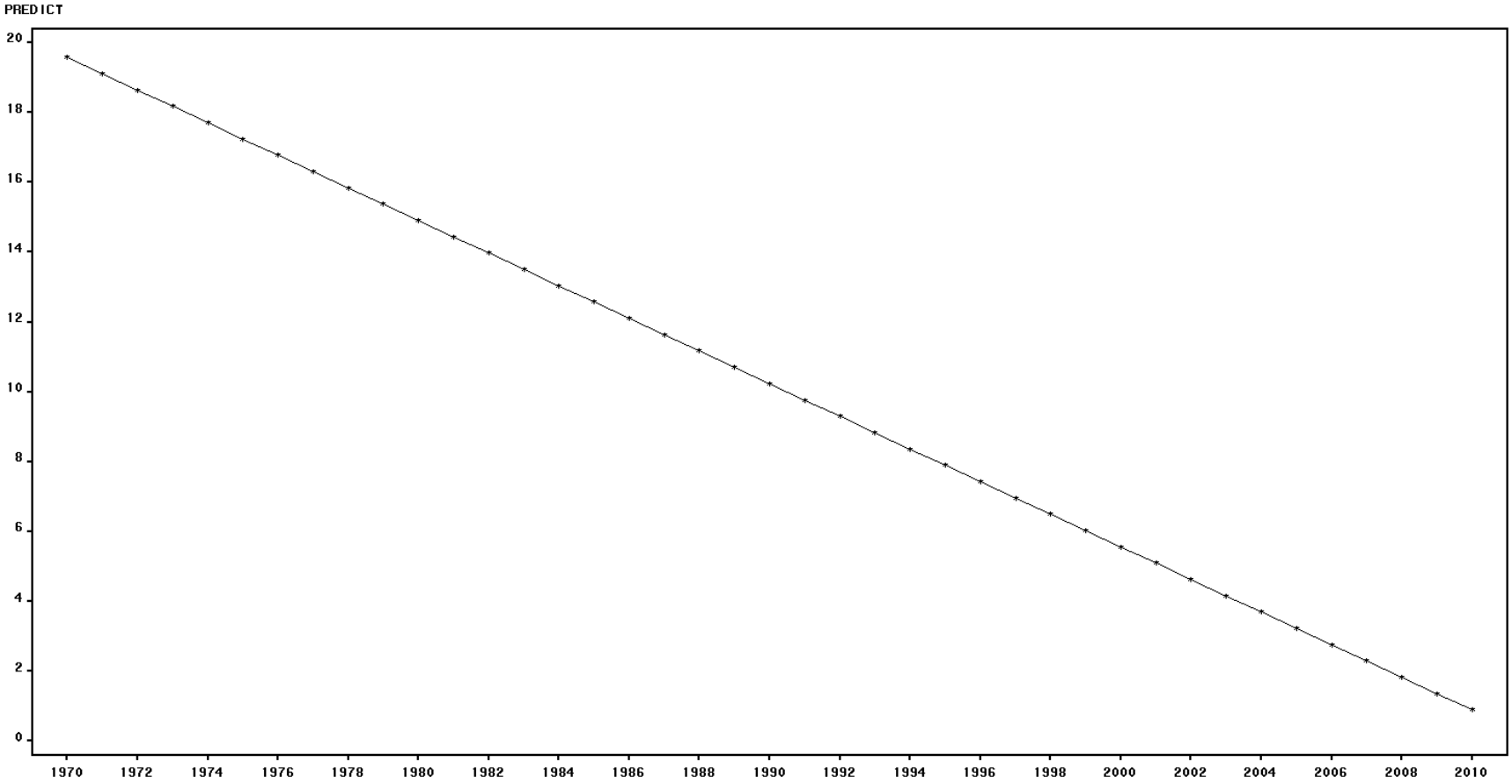

Figure 2 shows a time series graph for predicted values ((+) signs) for Drowning Rate variable as reported by SAS(AFS), along with 6-year forecasted values for the Drowning Rate variable. Here, predicted and forecast values are plotted using ((*) signs).

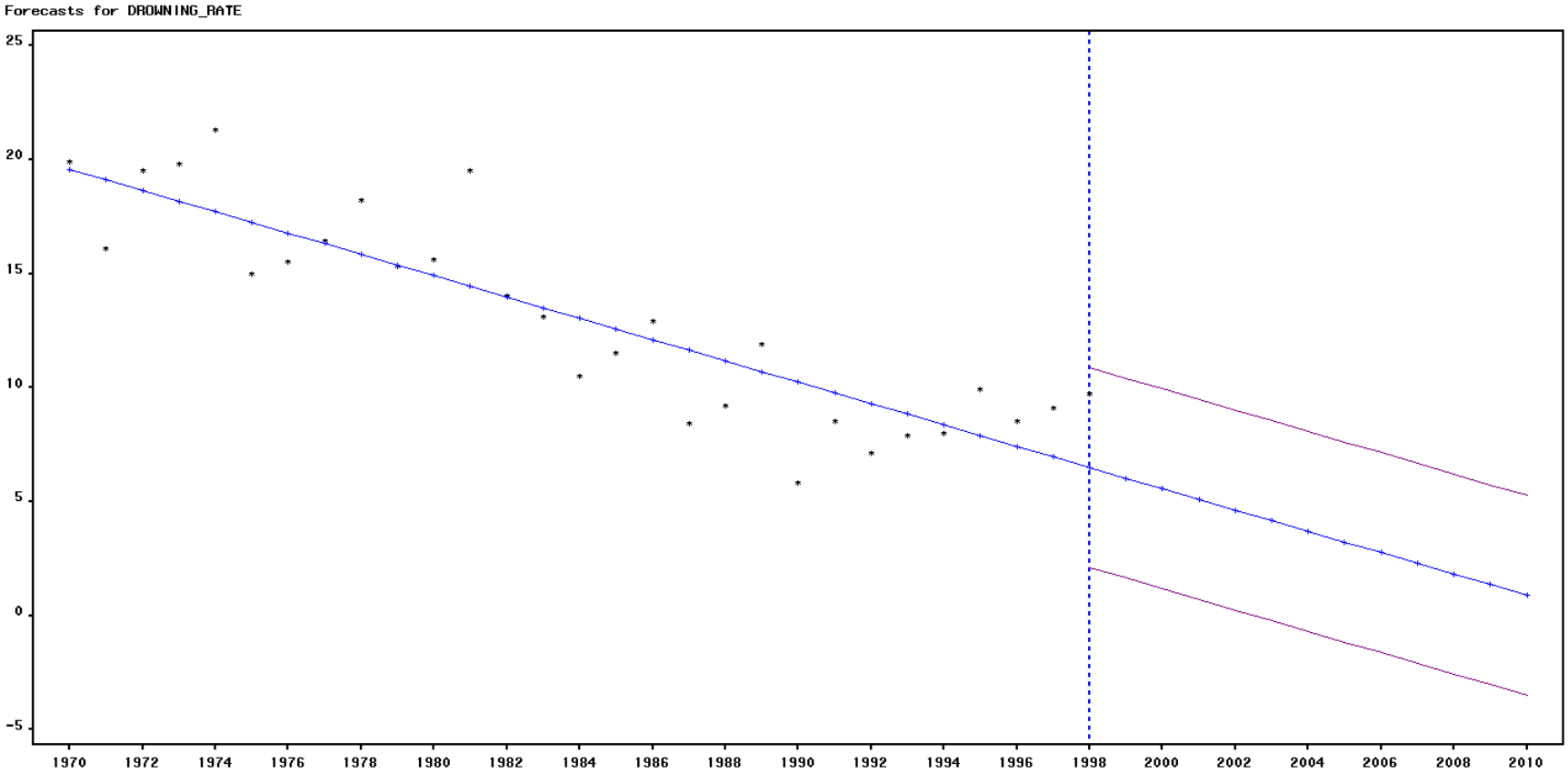

Figure 3a depicts a time series graph for predicted values ((+) signs) and the training data ((*) signs) for the Drowning Rate variable, as reported by SAS(AFS), along with 6-year forecasted values and prediction limits.

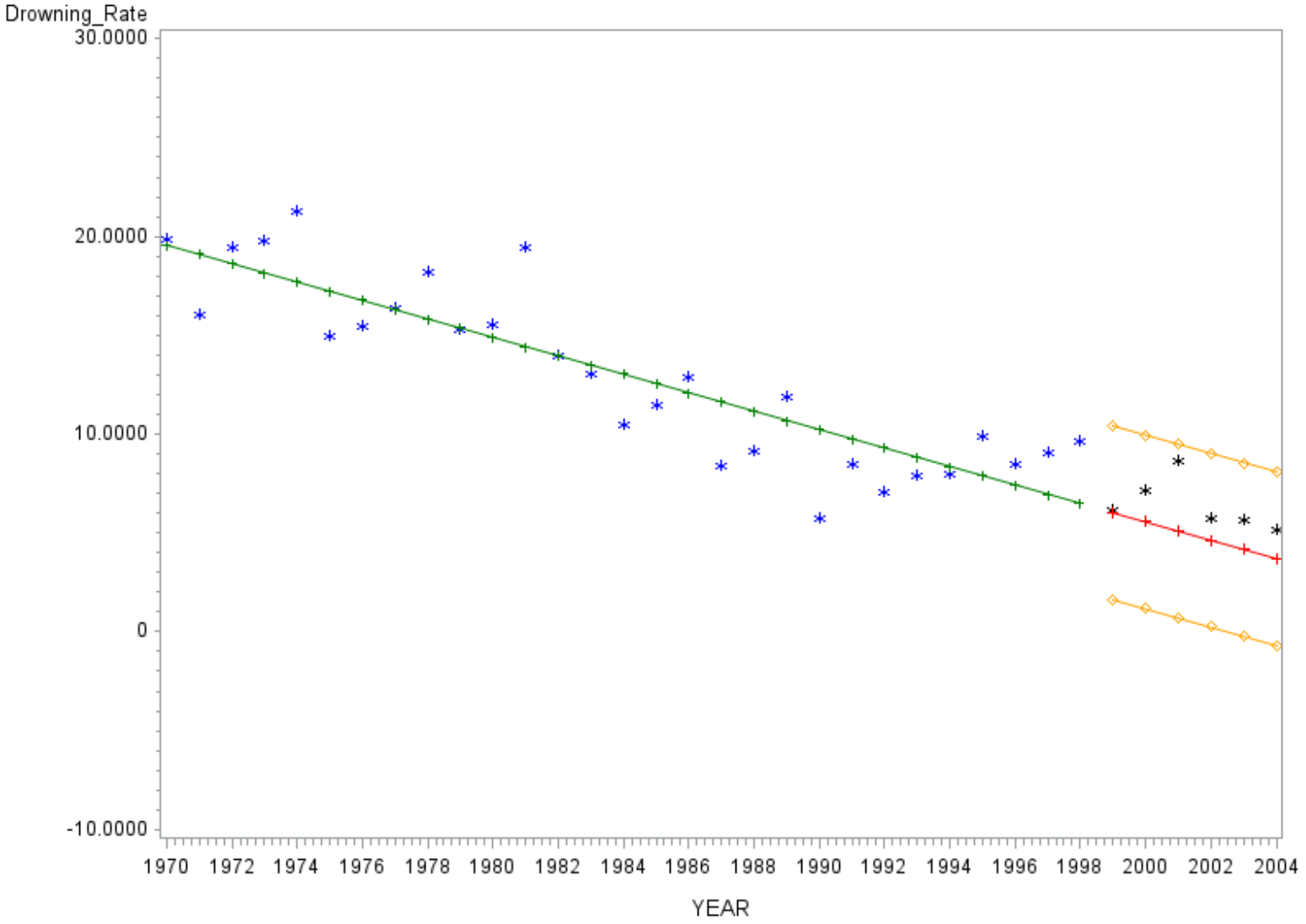

Figure 3b shows a time series graph for predicted values (green-colored (+) signs) for the Drowning Rate variable as reported by SAS(AFS), along with 6-year forecasted values, which are plotted using red-colored plus (+) signs. The training data are plotted using blue-colored star (*) signs, and testing data are plotted using black-colored star (*) signs, while the 6-year confidence limits are yellow-colored.

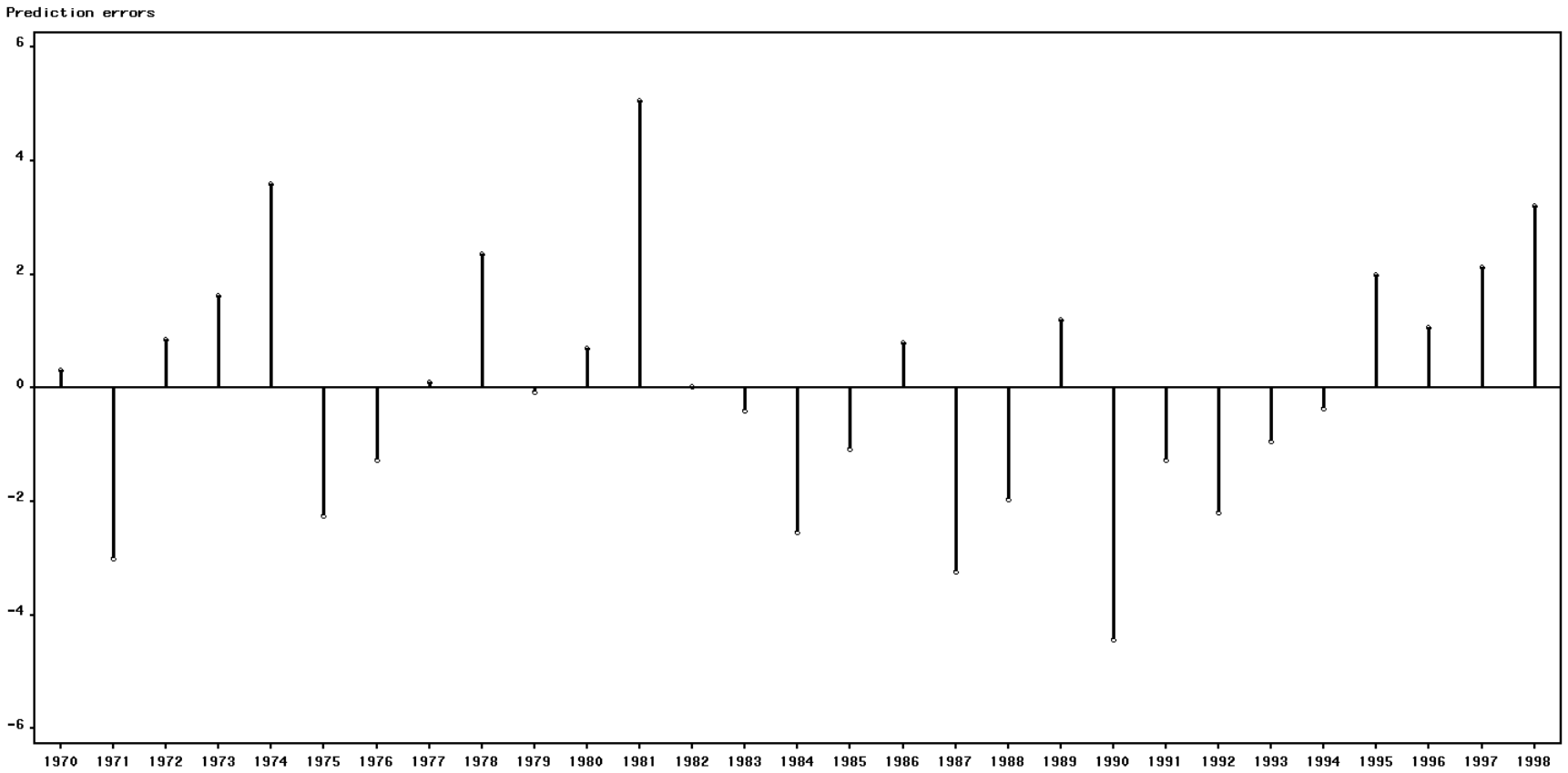

Figure 4. Prediction errors by time series values for the training data set.

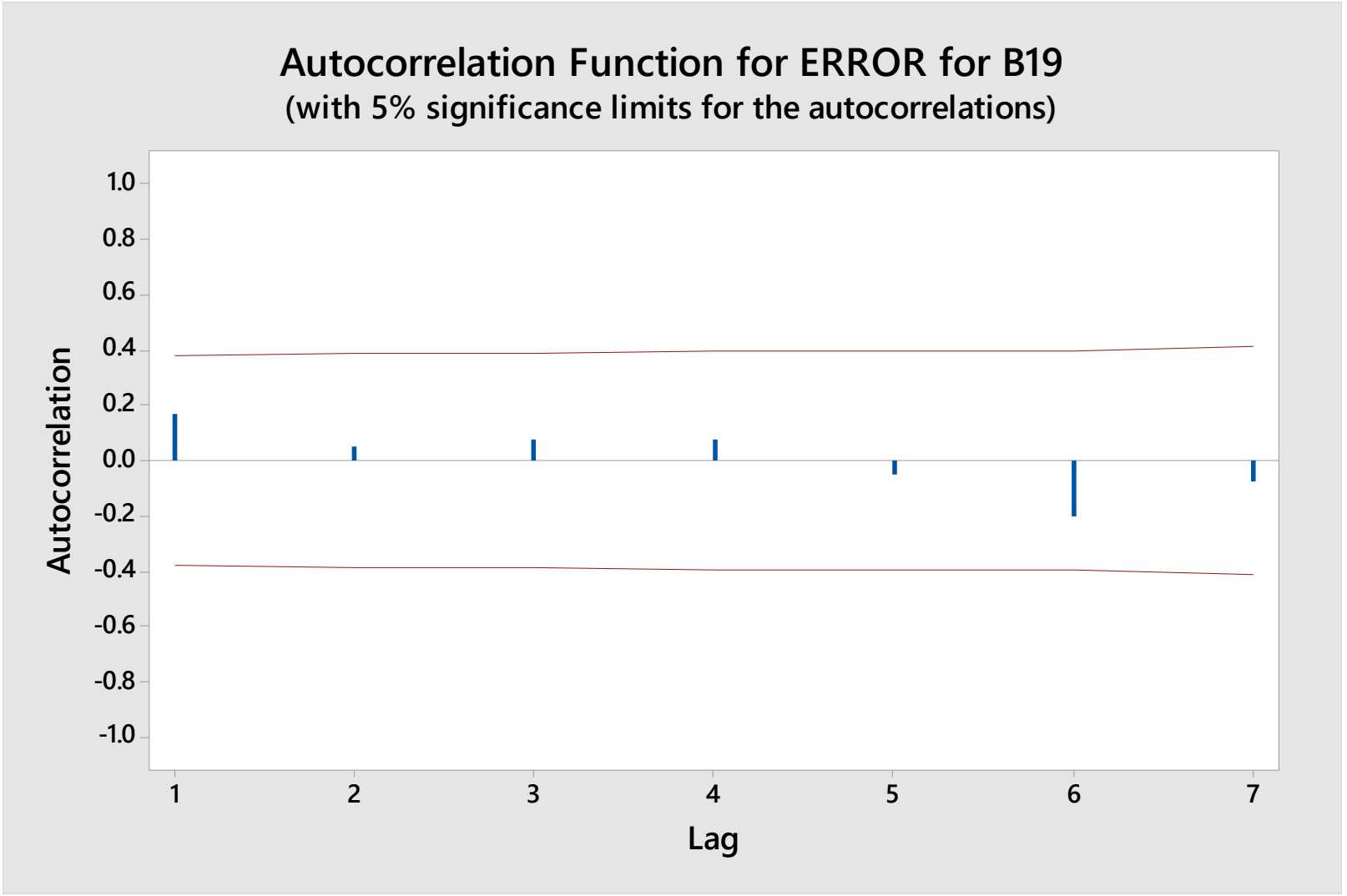

Figure 5. Autocorrelation Function (ACF) for the training data set.

Accuracy Measure for Table B.19 data.

Actual y_t	Predict $\widehat{y_t}$(t-1)	Forecast error $e_t(1) = y_t - \widehat{y_t}$(t-1)	Absolute error $\lvert e_t(1)\rvert$	Square error $[e_t(1)]^2$	Relative error $re_t(1) = 100(e_t(1)/y_t)$	Absolute percentage error $\lvert re_t(1)\rvert$
6.2	6.0315	0.1685	0.1685	0.0284	2.7177	2.7177
7.2	5.5644	1.6356	1.6356	2.6752	22.7167	22.7167
8.7	5.0973	3.6027	3.6027	12.9794	41.4103	41.4103
5.8	4.6302	1.1698	1.1698	1.3684	20.1690	20.1690
5.7	4.1632	1.5368	1.5368	2.3618	26.9614	26.9614
5.2	3.6961	1.5039	1.5039	2.2617	28.9212	28.9212

Table A. Error Measures table B19 time series data.

Data set	**MSE**	$\widehat{\sigma_{e(1)}}$	**MAD**	**MPE**	**MAPE**
Full data (35 data)	3.90882	1.97707	1.54160	-1.59385	14.19523
Training data (29 data)	4.65552	2.15767	1.72592	-3.34514	15.80423
Holdout data (6 data)	3.61249	1.90065	1.60288	23.8160	23.8160

Table B. Accuracy Measures table B19 time series data.

Table B provides SAS (AFS)® forecast errors for the full data set, the training data set, and the holdout (or test) data set. The Linear Trend model was suggested as the "best model" for this data set. SAS (AFS)® and MINITAB® produced the accuracy measures shown in Table B, which comprise the Mean Square Error (MSE), the Standard Deviation of the Forecast Errors (SDFE), the Mean Absolute Deviation (MAD), the Mean Percent Forecast Error (MPE), and Mean Absolute Percent Error (MAPE).

Here, the accuracy statistics are computed as follows:

Mean square error:

$$\textbf{MSE} = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = 3.61249$$

Standard Deviation of Forecast Error (Root Mean Square Error):

$$\widehat{\sigma_{e(1)}} = \sqrt{MSE} = \sqrt{3.61249} = 1.90065$$

Mean Absolute Deviation (Mean Absolute Error):

$$\textbf{MAD} = \frac{1}{n}\sum_{t=1}^{n} |e_t(1)| = 1.60288$$

Mean Percent Forecast Error:

$$\textbf{MPE} = \frac{1}{n}\sum_{t=1}^{n} re_t(1) = 23.8160$$

Mean Absolute Percent Error:

$$\textbf{MAPE} = \frac{1}{n}\sum_{t=1}^{n} |re_t(1)| = 23.8160$$

Worksheet 1 ***

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	Actual	Predict	Forecast_Error	Abs_Error	Sq_Error	Relative_Error	Abs_Percentage_Error	MSE	Root_MSE	MAD	MPE	MAPE
1	6.2	6.0315	0.1685	0.1685	0.0284	2.7177	2.7177	3.61249	1.90065	1.60288	23.8160	23.8160
2	7.2	5.5644	1.6356	1.6356	2.6752	22.7167	22.7167					
3	8.7	5.0973	3.6027	3.6027	12.9794	41.4103	41.4103					
4	5.8	4.6302	1.1698	1.1698	1.3684	20.1690	20.1690					
5	5.7	4.1632	1.5368	1.5368	2.3618	26.9614	26.9614					
6	5.2	3.6961	1.5039	1.5039	2.2617	28.9212	28.9212					

Table C. MINITAB® 17 Calculation of Forecast Errors for Holdout Data.

The first two columns of Table C consist of the actual values of the holdout data set and the SAS (AFS)® forecasted values, based on training data. The remaining columns give forecast errors, the absolute forecast errors, the squared forecast errors, the relative forecast errors and absolute forecast percentage errors using the MINITAB® calculator function.

Obs	DATE	ACTUAL	PREDICT	UPPER	LOWER	ERROR	STD	NERROR	_LINEAR_
30	1999	.	6.0315	10.4143	1.6487	.	2.2362	.	30
31	2000	.	5.5644	9.9472	1.1817	.	2.2362	.	31
32	2001	.	5.0973	9.4801	0.7146	.	2.2362	.	32
33	2002	.	4.6302	9.0130	0.2475	.	2.2362	.	33
34	2003	.	4.1632	8.5459	-0.2196	.	2.2362	.	34
35	2004	.	3.6961	8.0788	-0.6867	.	2.2362	.	35

Table D. Forecasted values for the last 6 year of Holdout Data.

Table D contains forecasted values for the final twelve month for the GDP, Current Dollars variable, and the corresponding lower and upper 95% prediction limits. The Root Mean Square Prediction Error, Level and Trend smoothing components are also shown..

Model Parameter	Estimate	Std. Error	T	Prob>|T|
Intercept	20.04433	0.8524	23.5140	<.0001
Linear Trend	-0.46709	0.0496	-9.4113	<.0001
Model Variance (sigma squared)	5.00038	.	.	.

Table E. Parameter Estimates for Training Data using the Linear Trend model suggested by SAS (AFS)®.

Results

In Figure 3b, it seems the model fits the data appropriately, however, the SAS(AFS) could not forecast values with acceptably small prediction error. There are no few spikes in the ACF function, which means forecast errors are uncorrelated. The MSE, RMSE and MAD are similar for both the training and holdout data sets; that is, the variability in forecast errors are approximately similar. However, the MAPE are too big in all cases.

TABLE B.20 (US INTERNAL REVENUE TAX REFUNDS) ANALYSIS

Table B.20 comprises data on the US INTERNAL REVENUE TAX REFUNDS from 1987 to 2006, where the Amount Refunded (in Millions Dollars) variable is collected yearly. The data set excludes last six year values as a holdout set in order to analyze the accuracy of SAS(AFS), which suggested a log Linear (Holt) Exponential Smoothing model as the "best model" for this training, or initialization, data set. SAS(AFS) also produced the predicted values for the next 6 year. I have calculated different types of forecast errors based on the hold out data and predicted data.

The model equation is

$$w_t = \ln(y_t) = \mu_t + \beta_t\, t + \varepsilon_t$$

where,

Y_t = observed value i.e. reading at time t

μ_t = smoothed level and $\mu_t = 12.18460$

β_t = smoothed trend and $\beta_t = 0.05405$

Now, the k-step prediction equation is

$$\hat{y}_t(k) = \exp(\hat{w}_t(k))$$

where, $\hat{w}_t(k) = L_t + K\, T_t$

and the smoothing equations are

$$L_t = \alpha\, Y_t + (1-\alpha)(L_{t-1} + T_{t-1})$$

and $T_t = \gamma (L_t - L_{t-1}) + (1-\gamma)\, T_{t-1}$

where,

L_t = Level estimate and $L_0 = \mu_t = 12.18460$

T_t = Trend estimate and $T_0 = \beta_t = 0.05405$

α = level smoothing and $\alpha = 0.99900$

γ = trend smoothing and $\gamma = 0.00100$

Table B.20: US Internal Revenue Tax Refunds

Fiscal Year	Amount Refunded, millions dollars	National Population, thousands	Fiscal Year	Amount Refunded, millions dollars	National Population, thousands
1987	96,969	242,289	1997	142,599	267,784
1988	94,480	244,499	1998	153,828	270,248
1989	93,613	246,819	1999	185,282	272,691
1990	99,656	249,464	2000	195,751	282,193
1991	104,380	252,153	2001	252,787	285,108
1992	113,108	255,030	2002	257,644	287,985
1993	93,580	257,783	2003	296,064	290,850
1994	96,980	260,327	2004	270,893	293,657
1995	108,035	262,803	2005	255,439	296,410
1996	132,710	265,229	2006	263,501	299,103

The collection of Figures and Tables that follow are individually identified below.

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List of Tables:

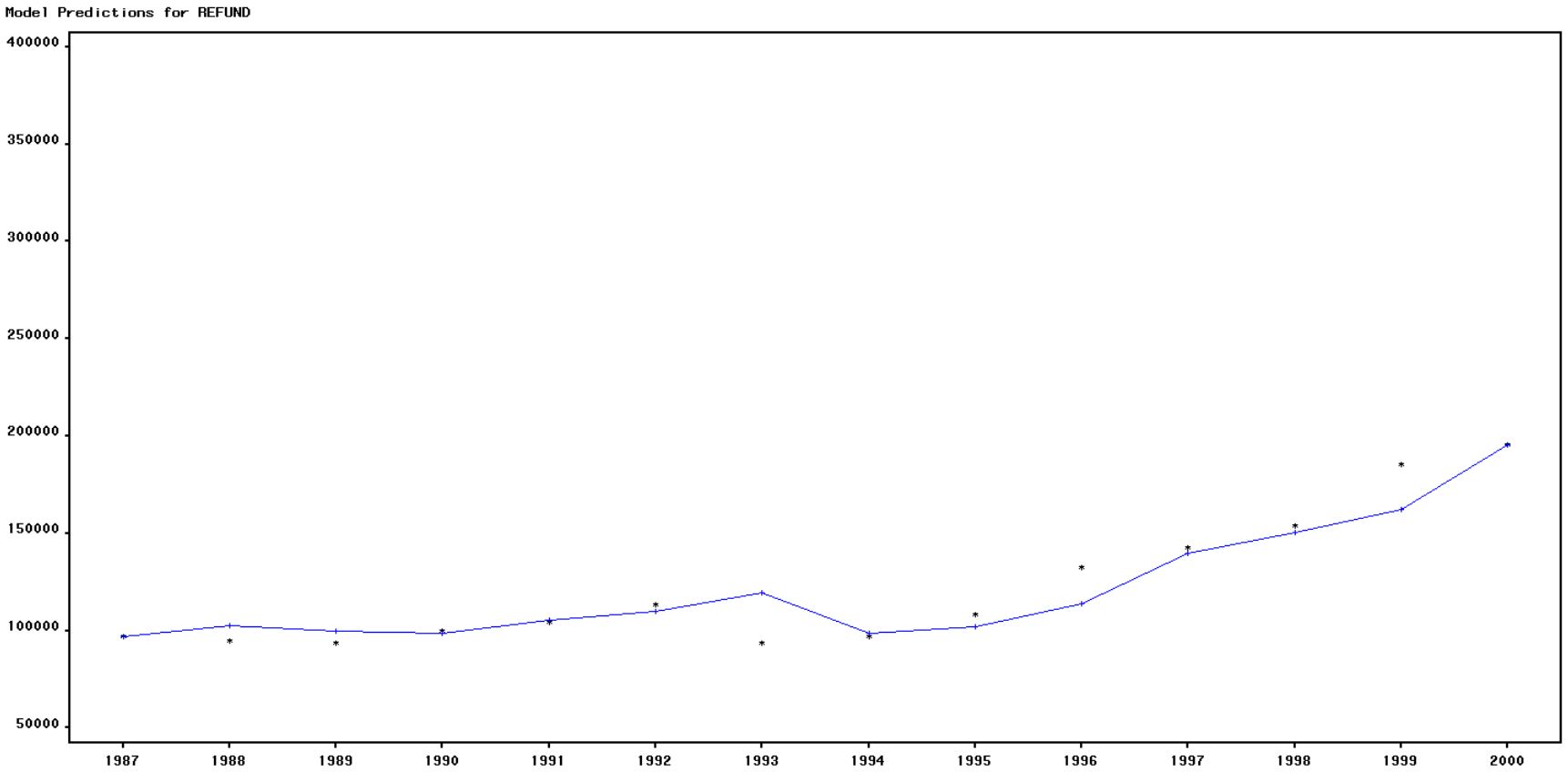

Figure 1 depicts a time series graph for predicted values ((+) signs) using the training data ((*) signs) set for Amount Refunded variable, as reported by SAS(AFS).

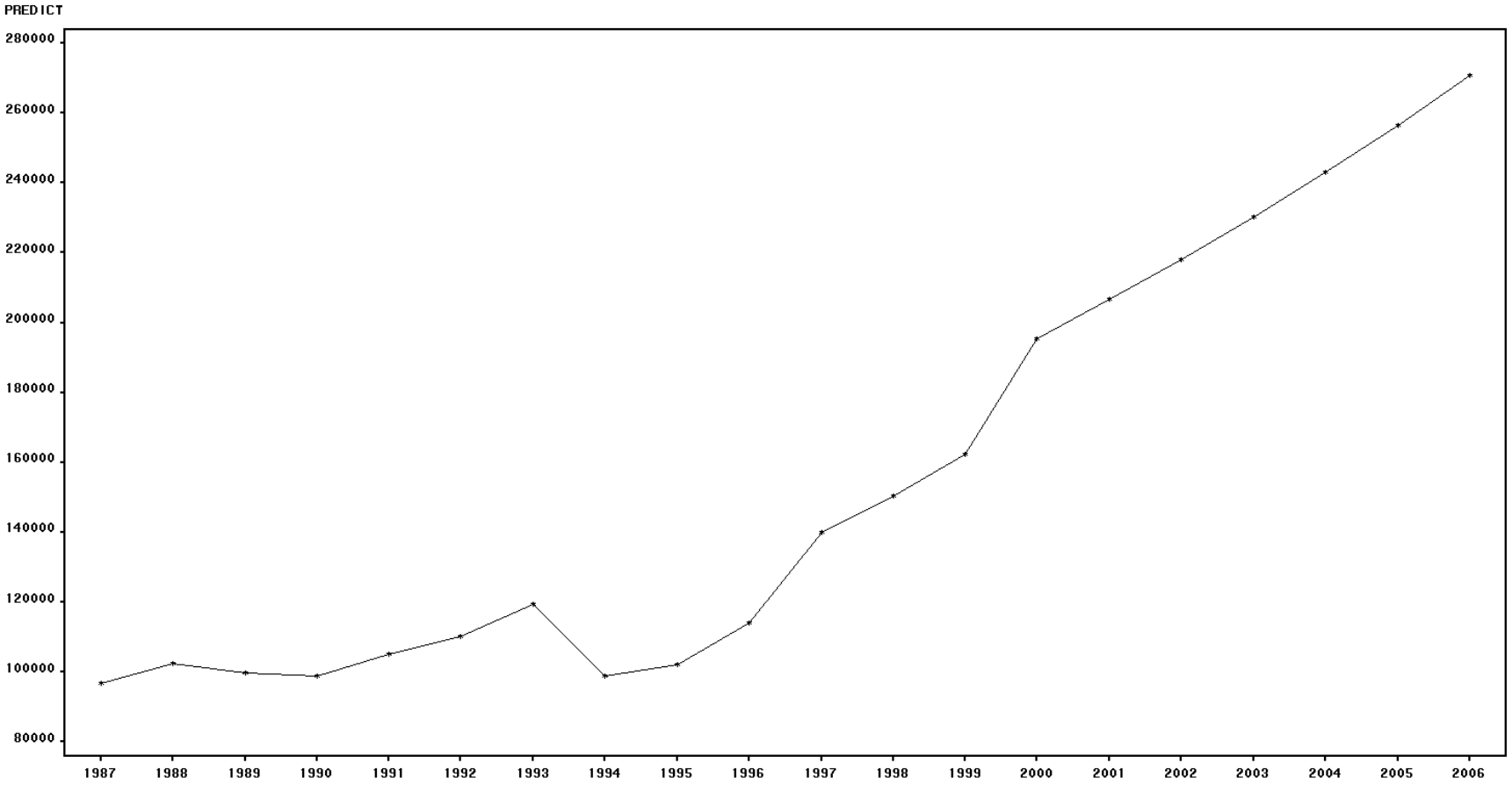

Figure 2 shows a time series graph for predicted values ((+) signs) for Amount Refunded variable as reported by SAS(AFS), along with 6-year forecasted values for the Amount Refunded variable. Here, predicted and forecast values are plotted using ((*) signs).

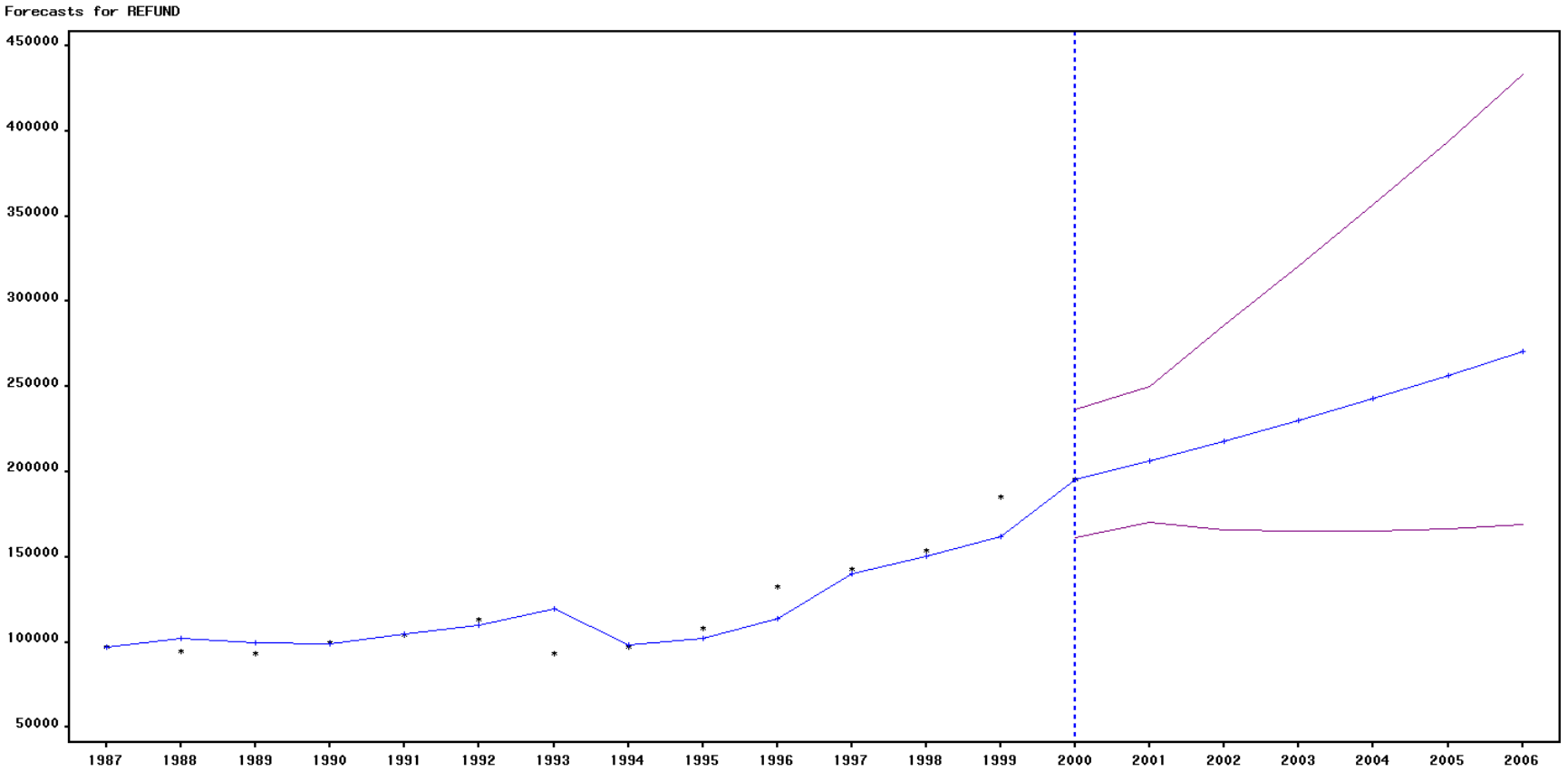

Figure 3a depicts a time series graph for predicted values ((+) signs) and the training data ((*) signs) for the Amount Refunded variable, as reported by SAS(AFS), along with 6-year forecasted values and prediction limits.

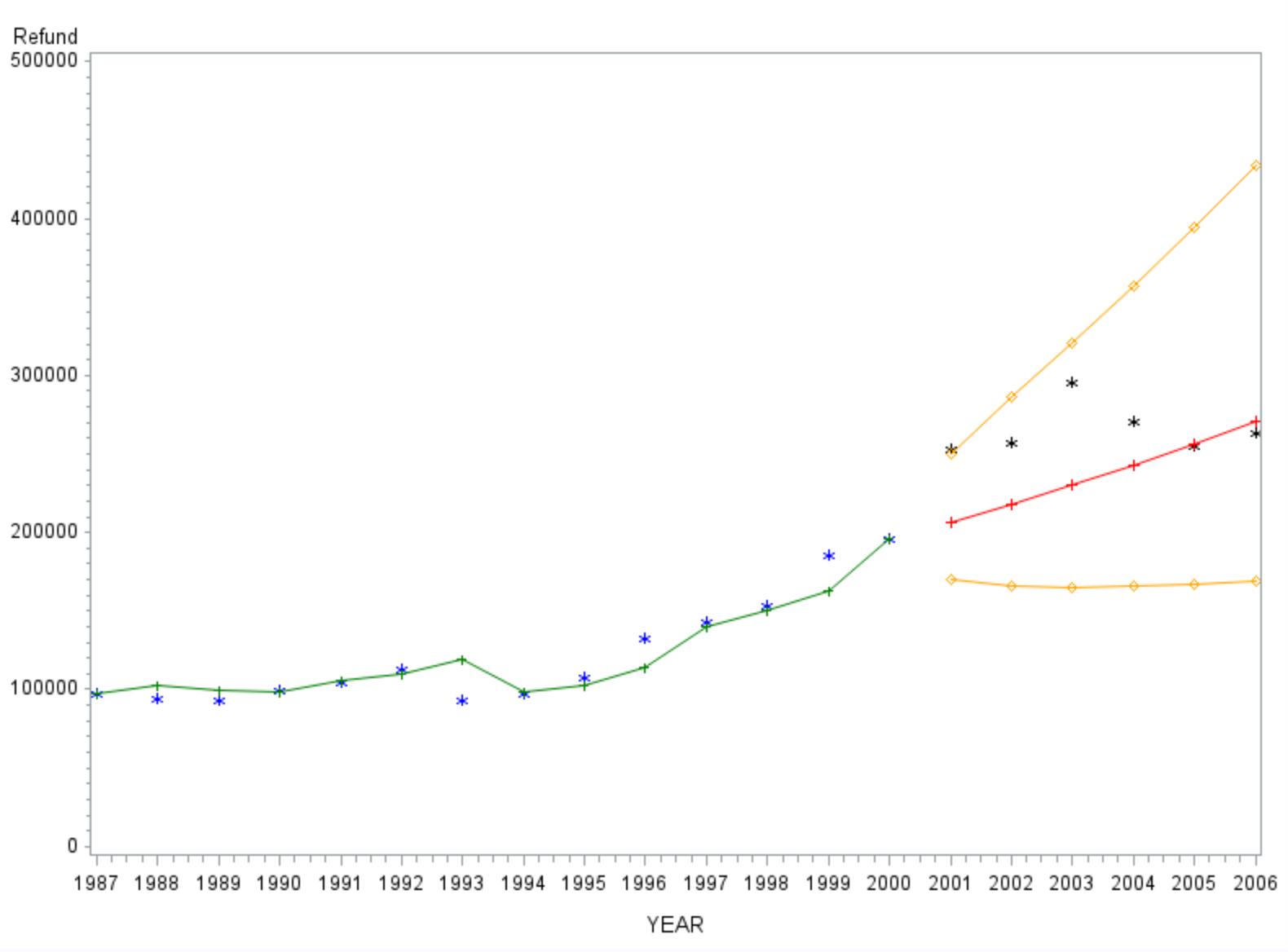

Figure 3b shows a time series graph for predicted values (green-colored (+) signs) for the Refunded variable as reported by SAS(AFS), along with 6-year forecasted values, which are plotted using red-colored plus (+) signs. The training data are plotted using blue-colored star (*) signs, and testing data are plotted using black-colored star (*) signs, while the 6-year confidence limits are yellow-colored.

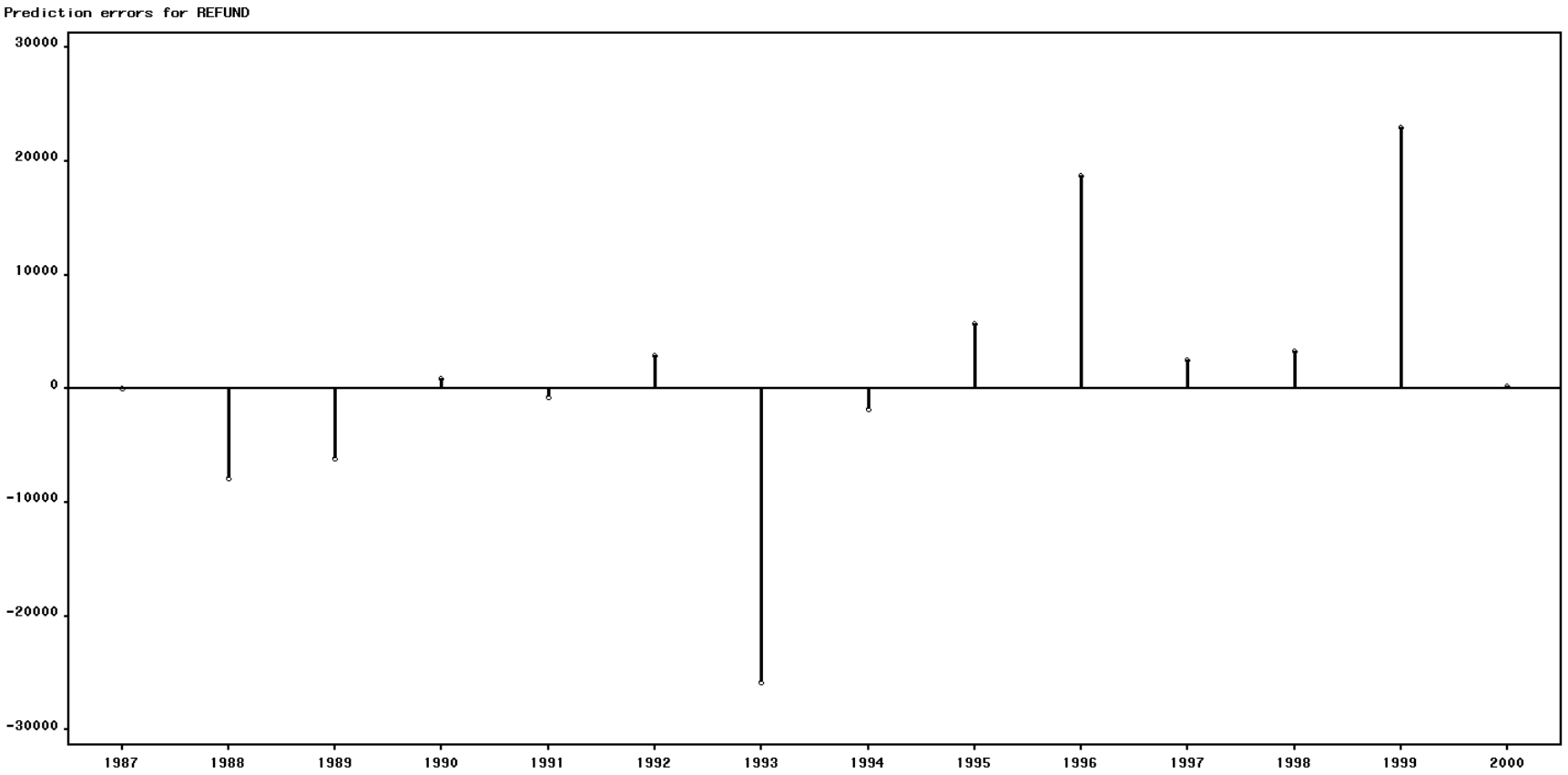

Figure 4. Prediction errors by time series values for the training data set.

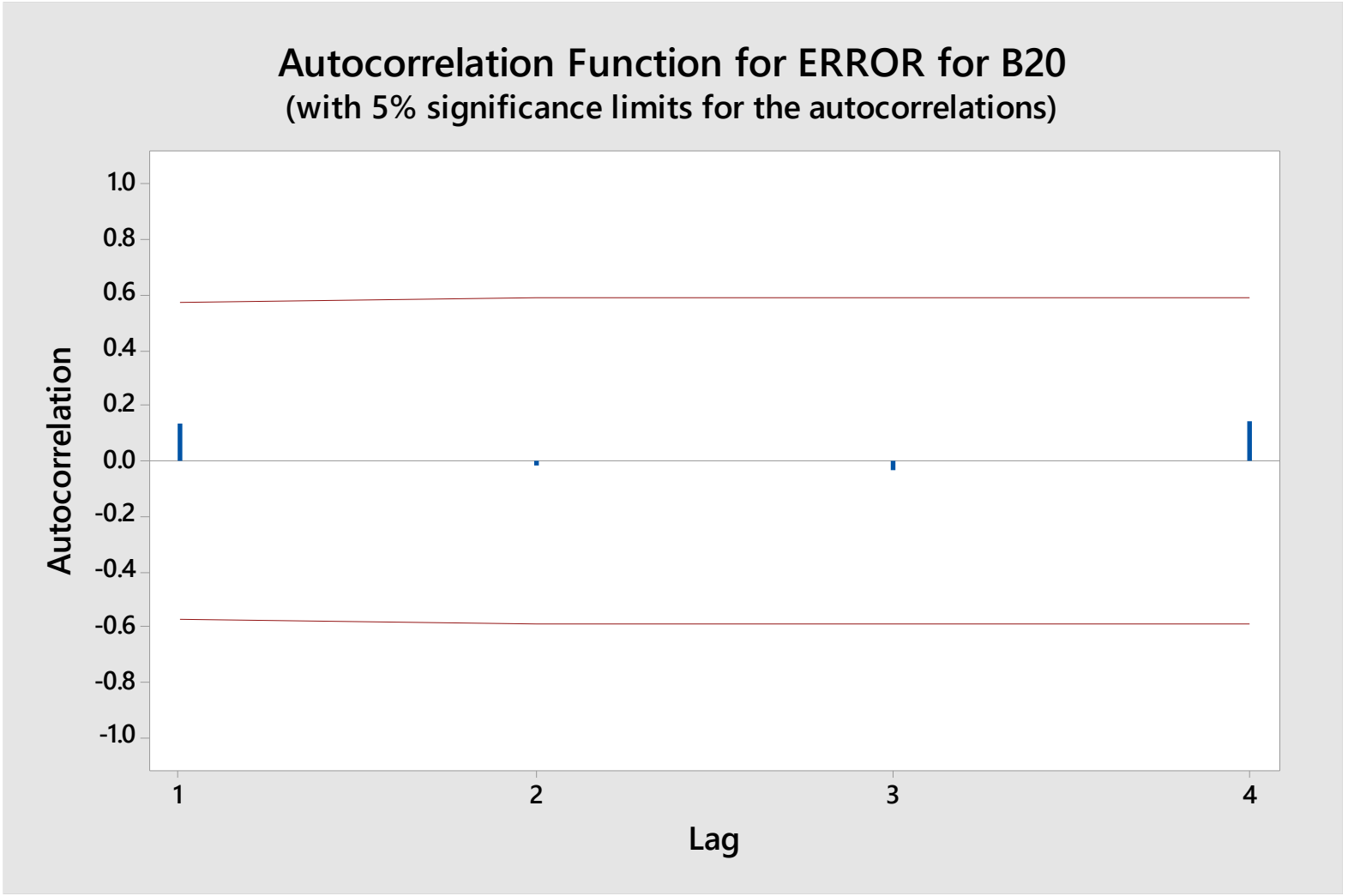

Figure 5. Autocorrelation Function (ACF) for the training data set.

Accuracy Measure for Table B.20 data.

Actual y_t	Predict $\widehat{y_t}$(t-1)	Forecast error $e_t(1) = y_t - \widehat{y_t}(t-1)$	Absolute error $\lvert e_t(1)\rvert$	Square error $[e_t(1)]^2$	Relative error $re_t(1) = 100(e_t(1)/y_t)$	Absolute percentage error $\lvert re_t(1)\rvert$
252787	206622	46165	46165	2131207225	18.2624	18.2624
257644	218097	39547	39547	1563965209	15.3495	15.3495
296064	230210	65854	65854	4336749316	22.2432	22.2432
270893	242995	27898	27898	778298404	10.2985	10.2985
255439	256490	-1051	1051	1104601	-0.4114	0.4114
263501	270734	-7233	7233	52316289	-2.7450	2.7450

Table A. Error Measures table B20 time series data.

Data set	**MSE**	$\widehat{\sigma_{e(1)}}$	**MAD**	**MPE**	**MAPE**
Full data (20 data)	343512813	18534.1	13404.9	2.32863	7.73958
Training data (14 data)	121757101	11034.4	7116.6	- 0.40897	6.02391
Holdout data (6 data)	1477273507	38435.3	31291.3	10.4995	11.5517

Table B. Accuracy Measures table B20 time series data.

Table B provides SAS (AFS)® forecast errors for the full data set, the training data set, and the holdout (or test) data set. The log Linear (Holt) Exponential Smoothing model was suggested as the "best model" for this data set. SAS (AFS)® and MINITAB® produced the accuracy measures shown in Table B, which comprise the Mean Square Error (MSE), the Standard Deviation of the Forecast Errors (SDFE), the Mean Absolute Deviation (MAD), the Mean Percent Forecast Error (MPE), and Mean Absolute Percent Error (MAPE).

Here, the accuracy statistics are computed as follows:

Mean square error:

$$\textbf{MSE} = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = 1477273507$$

Standard Deviation of Forecast Error (Root Mean Square Error):

$$\widehat{\sigma_{e(1)}} = \sqrt{MSE} = \sqrt{1477273507} = 38435.3$$

Mean Absolute Deviation (Mean Absolute Error):

$$\mathbf{MAD} = \frac{1}{n}\sum_{t=1}^{n} |e_t(1)| \ = 31291.3$$

Mean Percent Forecast Error:

$$\mathbf{MPE} = \ \frac{1}{n}\sum_{t=1}^{n} re_t(1) \ = 10.4995$$

Mean Absolute Percent Error:

$$\mathbf{MAPE} = \frac{1}{n}\sum_{t=1}^{n} |re_t(1)| \ = 11.5517$$

Worksheet 1 ***

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	Actual	Predict	Forecast_Error	Abs_Error	Sq_Error	Relative_Error	Abs_Percentage_Error	MSE	Root_MSE	MAD	MPE	MAPE
1	252787	206622	46165	46165	2131207225	18.2624	18.2624	1477273507	38435.3	31291.3	10.4995	11.5517
2	257644	218097	39547	39547	1563965209	15.3495	15.3495					
3	296064	230210	65854	65854	4336749316	22.2432	22.2432					
4	270893	242995	27898	27898	778298404	10.2985	10.2985					
5	255439	256490	-1051	1051	1104601	-0.4114	0.4114					
6	263501	270734	-7233	7233	52316289	-2.7450	2.7450					

Table C. MINITAB® 17 Calculation of Forecast Errors for Holdout Data.

The first two columns of Table C consist of the actual values of the holdout data set and the SAS (AFS)® forecasted values, based on training data. The remaining columns give forecast errors, the absolute forecast errors, the squared forecast errors, the relative forecast errors and absolute forecast percentage errors using the MINITAB® calculator function.

Obs	DATE	ACTUAL	PREDICT	UPPER	LOWER	ERROR	STD	NERROR	RESIDUAL	RESSTD	NRESID	_LEVEL_	_TREND_
15	2001	.	206622	250360	170525	.	20412	.	.	.	.	12.2386	0.0540
16	2002	.	218097	286140	166235	.	30726	.	.	.	.	12.2927	0.0540
17	2003	.	230210	321073	165061	.	40069	.	.	.	.	12.3467	0.0540
18	2004	.	242995	356860	165461	.	49267	.	.	.	.	12.4008	0.0540
19	2005	.	256490	394235	166873	.	58656	.	.	.	.	12.4548	0.0540
20	2006	.	270734	433652	169023	.	68423	.	.	.	.	12.5089	0.0540

Table D. Forecasted values for the last 6 year of Holdout Data.

Table D contains forecasted values for the final twelve month for the GDP, Current Dollars variable, and the corresponding lower and upper 95% prediction limits. The Root Mean Square Prediction Error, Level and Trend smoothing components are also shown..

Model Parameter	Estimate	Std. Error	T	Prob>|T|
LEVEL Smoothing Weight	0.99900	0.2020	4.9457	0.0003
TREND Smoothing Weight	0.00100	0.0785	0.0127	0.9900
Residual Variance (sigma squared)	0.00960	.	.	.
Smoothed Level	12.18460	.	.	.
Smoothed Trend	0.05405	.	.	.

Table E. Parameter Estimates for Training Data using the log Linear (Holt) Exponential Smoothing model suggested by SAS (AFS)®.

Results

In Figure 3b, it seems the model fits the data appropriately, however, the SAS(AFS) could not forecast values with acceptably small prediction error. There are no spikes in the ACF function, which means forecast errors are uncorrelated. The MSE, RMSE and MAD are not similar for both the training and holdout data sets; that is, the variability in forecast errors are not similar.

Conclusions

The SAS Automatic Forecasting System (AFS) for identifying the "best model" in time series applications is sometimes quite useful. However, in some instances that system is quite limited and of little value. In the end, it cannot be concluded that SAS(AFS) should be exclusively relied upon; the skill and acumen of the savvy data analyst cannot be replaced in all scenarios. Nonetheless, SAS (AFS) can at least be used as a "first pass" tool for large volumes of individual time series data that requires forecasts within a short period of time. Beyond that, the data analyst must be called upon to review the output and affect the necessary model-building whenever warranted.

References

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