b_i = number of basis functions (edges) in box i
L_λ = number of Legendre points in level λ
m = number of points in Lagrange polynomial interpolation
d_λ = diameter of every box in level λ

Data	Shape	Indexing			$\mathcal{I}_i^{\mathcal{S}}$	$\mathcal{I}_i^{\mathcal{F}}$	$\mathcal{I}_i^{\mathcal{S}}$	
Z_{ij}^{near}	$b_i \times b_j$	$\lambda_{max}, \mathcal{B}_i, \mathcal{B}_j : \mathcal{B}_j \in \mathcal{I}_i^{\mathcal{N}}$	r					
I_i	$b_i \times 1$	$\lambda_{max}, \mathcal{B}_i$	r	r				
V_i	$b_i \times 1$	$\lambda_{max}, \mathcal{B}_i$	w					w
$\hat{\kappa}$	$2(L_\lambda + 1) \times (L_\lambda + 1)$	λ						
F_{ij}	$2(L_\lambda + 1) \times (L_\lambda + 1)$	$\lambda_{max}, j \in \mathcal{B}_i$		r				
R_{ij}	$2(L_\lambda + 1) \times (L_\lambda + 1)$	$\lambda_{max}, j \in \mathcal{B}_i$						r
$\tilde{F}_i$	$2(L_\lambda + 1) \times (L_\lambda + 1)$	$\lambda, \mathcal{B}_i$		w	rw	r		
G_i	$2(L_\lambda + 1) \times (L_\lambda + 1)$	$\lambda, \mathcal{B}_i$				w	rw	r
$\mathcal{T}_i$	$2(L_\lambda + 1) \times (L_\lambda + 1)$	$\lambda, \mathcal{B}_i$				r		
$\mathcal{P}$	$m \times m$??	$\mathcal{P}_\theta, \mathcal{P}_\phi$			r		r	

Near Field

$$
\begin{aligned}
V_i^{b_i\times 1} = Z_{ij}^{b_i\times b_j} I_j^{b_j\times 1}, \quad \forall \mathcal{B}_j \text{ neighbor } \mathcal{B}_i \\
V = [\mathbf{V_x}\ \mathbf{V_y}\ \mathbf{V_z}\ ] \quad Z = [\mathbf{Z_x}\ \mathbf{Z_y}\ \mathbf{Z_z}] \quad I = [\mathbf{I_x}\ \mathbf{I_y}\ \mathbf{I_z}] \\
\mathbf{V_x} = \mathbf{Z_x}\mathbf{I_x} \quad \mathbf{V_y} = \mathbf{Z_y}\mathbf{I_y} \quad \mathbf{V_z} = \mathbf{Z_z}\mathbf{I_z}
\end{aligned} \tag{1}
$$

Box Field

$$
\tilde{F}_{m\lambda}^{K_\lambda\times 1} = F_m^{K_\lambda\times b_m} I_m^{b_m\times 1}, \quad \forall \mathcal{B}_m \in \lambda_{max} \wedge \lambda = \lambda_{max} \tag{2}
$$

C2P

$$
\begin{gathered}
\tilde{F}_{P,\lambda-1}^{K_{\lambda-1}\times 1} = \sum_{\mathcal{B}_C} E_{CP}^{K_\lambda\times 1} \odot A_{1,\lambda} \tilde{F}_{C\lambda}^{K_\lambda\times 1} A_{2,\lambda}\ , \quad \forall \mathcal{B}_C \text{ child of } \mathcal{B}_P \\
E_{mn} = e^{\imath\kappa_n \mathbf{d}_{\mathcal{B}_n,\mathcal{B}_m}}
\end{gathered} \tag{3}
$$

Far Field

$$
G_{m\lambda}^{K_\lambda\times 1} = \sum_{\mathcal{B}_n} T_{nm}^{K_\lambda\times 1} \odot \tilde{F}_{n\lambda}^{K_\lambda\times 1}, \quad \forall \mathcal{B}_n \in \mathcal{I}_{m\lambda}^{\mathcal{F}} \tag{4}
$$

P2C

$$
G_{C,\lambda+1}^{K_{\lambda+1}\times 1} = -E_{CP}^{K_\lambda\times 1} \odot B_{1,\lambda} G_{P\lambda}^{K_\lambda\times 1} B_{2,\lambda}\ , \quad \forall \mathcal{B}_C \text{ child of } \mathcal{B}_P \tag{5}
$$

Receiving

$$
V_m^{b_m\times 1} = R_m \odot G_{m,\lambda_{max}}^{K_{\lambda_{max}}\times 1}, \quad \mathcal{B}_m \in \lambda_{max} \tag{6}
$$

$$
V_i = Z_{ij} I_j \tag{1}
$$

$$
\tilde{F}_m = F_m I_m \tag{2}
$$

$$
\tilde{F}_P = \cdots \tilde{F}_C \cdots \tag{3}
$$

$$
G_m = \cdots \tilde{F}_n \cdots \tag{4}
$$

$$
G_C = \cdots G_P \cdots \tag{5}
$$

$$
V_m = \cdots G_m \cdots \tag{6}
$$

- Specific
  $D_{L_{max} \times g}$ is dummy data owned by every host

```
for L=L_max-1:-1:1
  for gi=1:g
    C2P_t D(L+1, gi  ),D(L,gi)
    XLT_t D(L  , gi-1),D(L,gj)    ; for gj=1:g
  end
end
for L=1:L_max
  for gi=1:g
    P2C_t D(L-1,gi),D(L,gi)
  end
end
for gi=1:g
  for gj=1:g
    NF_t D(L_max,gi),D(L_max,gj)
  end
end
C2P_t(A,B) Kernel
  A , B are partitioned hierarchically
  for a in A
    for b in  B
      c2p_t(a,b);
    end
  end
end
C2P_t(D1,D2) Kernel
  for p in D2
    for c in p.children
      c2p_t c,p
    end
  end
end
XLT_t(D1,D2) Kernel
  for x in D2
    for b in D1
      if b in x.far_field
        xlt_t b,x
      end
    end
  end
end
P2C_t(D1,D2) Kernel
  for p in D1
    for c in p.children
```

```
      if c in D2, p2c_t p,c
    end
  end
end
```

- General

```
type,D1,D2,D3,d3_box
```