Let's address the points listed in the image you uploaded:

1. **Influence of number of nodes on the solution**: The number of nodes (**Nx**) determines the spatial resolution of the simulation. Increasing **Nx** will provide a more detailed solution, capturing smaller features of the wave. However, as **Nx** increases, computational cost also increases. The influence of **Nx** on the solution can be analyzed by running simulations with different values of **Nx** and observing changes in the accuracy and stability of the solution.
2. **Speed of calculation**: The speed of calculation depends on the number of grid points (**Nx**), number of time steps (**Nt**), and the complexity of computations within the time loop. The use of vectorized operations and efficient memory management can speed up the calculations. Profiling tools can be used to measure the time taken by different parts of the code to identify bottlenecks.
3. **Time taken for the wave to leave the domain**: This can be determined by tracking the maximum value of the wave's amplitude as it propagates and noting the time step at which it leaves the predefined domain. For a more accurate assessment, you could implement a condition that checks when the amplitude at the domain boundaries falls below a certain threshold.
4. **Capture the locations of the wave at various time instances**: Your code already does this by plotting the wave at specific time instances defined in **times\_to\_plot**. To capture the exact locations, you could record the position of the wave's peak amplitude at each desired time instance. This can be done by finding the index of the maximum value of **u\_solution** for each time step that corresponds to a time in **times\_to\_plot**.

**1. Introduction**

This section should establish the context of your study:

* **Relevance to CFD**: Describe how wave propagation models are fundamental in various engineering applications, such as the design of aircraft, ships, and environmental engineering.
* **Importance of the Project**: Emphasize the project's significance in understanding fluid behavior under different conditions. This can be an introductory point to explain why modeling such phenomena computationally provides valuable insights, for example, in optimizing designs to reduce drag or improve stability.
* **Choice of Method**: Justify the use of upwind discretization. You might argue that for linear advection equations, this method, despite its simplicity, captures the essence of wave propagation and is particularly well-suited to handle problems with sharp gradients or discontinuities without introducing non-physical oscillations.

**2. Governing Equations and Discretization**

Elaborate on the mathematical foundations of your project:

* **Wave Equation**: Introduce the scalar convection equation, which models the transport of a scalar quantity by a flow field. You should detail the physical interpretation of each term in the equation.
* **Boundary Conditions**: The periodic boundary conditions used in your model imply that the domain is a closed loop. This can be visualized as a wave on a circular track where the wavefront exiting on one end immediately re-enters from the opposite end.
* **Stabilized FE Discretization**: Expand on the finite difference method, specifically the upwind scheme. Explain why it's called "upwind" and how it uses information from the direction of the wind (or wave propagation) to compute derivatives.
* **Stability Analysis (CFL Condition)**: Delve into the CFL condition, explaining that it's a necessary condition for convergence in the numerical solution of hyperbolic partial differential equations. Discuss how it's used to choose an appropriate time step size ��*dt* given the grid spacing ��*dx* and the wave speed �*u*.

**3. Code Structure**

Outline the architecture of your simulation code:

* **Pre-Processing**: Define the setup phase where parameters are initialized. This includes creating a spatial grid, initializing the wave profile (cosine wave as shown in the figure), and setting up velocity and time-stepping parameters.
* **Main-Processing**: Explain the computational heart of your simulation. This is where the discrete version of the wave equation is solved iteratively. Describe the algorithm for advancing the solution in time, including how the boundary conditions are applied at each time step.
* **Post-Processing**: Describe the analysis and visualization phase. Discuss how you plot the wave at different instances to visualize its propagation and compute its exit time from the domain. Mention any data extraction for further analysis, like tracking the wave peak over time.

**4. Results and Discussion**

Present and interpret the simulation results:

* **Simulated Wave Propagation**: Show plots of the wave at different times, similar to the initial cosine profile provided, and discuss how they change over time for different velocities.
* **Effect of Nodes**: Discuss the relationship between spatial resolution (number of nodes) and the accuracy of the simulation. Consider showing how results converge with increasing resolution.
* **Computation Speed**: Present the timing results of your simulations, discussing any observed trends related to computational workload and efficiency.
* **Wave Exit Time**: Provide the calculated exit times for the wave at different velocities and discuss any trends or notable observations.
* **Wave Locations**: Discuss the captured locations of the wave's peak at different times and how they relate to the physical expectations of wave propagation.

**5. Conclusions**

Wrap up your report with a summary and reflection:

* **Summary of Findings**: Review the key results from your simulations. This could include observations about the impact of velocity on wave shape and the influence of discretization parameters on the simulation outcome.
* **Lessons Learned**: Reflect on the broader lessons about numerical simulation, such as the trade-off between accuracy and computational cost, the importance of stability conditions, and the impact of boundary conditions on the solution.
* **Future Work**: Suggest directions for future research, such as incorporating more complex physics, exploring non-linear equations, or extending the simulation to two or three dimensions.

Incorporating these details will provide a comprehensive understanding of your project in the report. Each heading should guide the reader through your thought process, the analytical framework you've used, and the insights gained from the project.