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1. Misc

1.1. Contest

1.1.1. Makefile

```
1 .PRECIOUS: ./p%
2
3 %: p%
4     ulimit -s unlimited && ./$<
5 p%: p%.cpp
6     g++ -O $@ <-std=c++17 -Wall -Wextra -Wshadow \
7     -fsanitize=address,undefined
```

1.2. How Did We Get Here?

1.2.1. Macros

Use vectorizations and math optimizations at your own peril.
For gcc \geq 9, there are `[[likely]]` and `[[unlikely]]` attributes.
Call gcc with `-fopt-info-optimized-missed-optall` for optimization info.

```
1 #define _GLIBCXX_DEBUG 1 // for debug mode
2 #define _GLIBCXX_SANITIZE_VECTOR 1 // for asan on vectors
3 #pragma GCC optimize("O3", "unroll-loops")
4 #pragma GCC optimize("fast-math")
5 #pragma GCC target("avx,avx2,abm,bmi,bmi2") // tip:
6 // before a loop
7 #pragma GCC unroll 16 // 0 or 1 -> no unrolling
# pragma GCC ivdep
```

1.2.2. Fast I/O

```
1 struct scanner {
2     static constexpr size_t LEN = 32 << 20;
3     char *buf, *buf_ptr, *buf_end;
4     scanner()
5         : buf(new char[LEN]), buf_ptr(buf + LEN),
6           buf_end(buf + LEN) {}
7     ~scanner() { delete[] buf; }
8     char getc() {
9         if (buf_ptr == buf_end) [[unlikely]]
10            buf_end = buf + fread_unlocked(buf, 1, LEN, stdin);
11            buf_ptr = buf;
12        return *(buf_ptr++);
13    }
14    char seek(char del) {
15        char c;
16        while ((c = getc()) < del) {}
17        return c;
18    }
19    void read(int &t) {
20        bool neg = false;
21        char c = seek('-');
22        if (c == '-') neg = true, t = 0;
23        else t = c ^ '0';
24        while ((c = getc()) >= '0') t = t * 10 + (c ^ '0');
25        if (neg) t = -t;
26    }
27}; struct printer {
28    static constexpr size_t CPI = 21, LEN = 32 << 20;
29    char *buf, *buf_ptr, *buf_end, *tbuf;
30    char *int_buf, *int_buf_end;
31    printer()
32        : buf(new char[LEN]), buf_ptr(buf),
33          buf_end(buf + LEN), int_buf(new char[CPI + 1]),
34          int_buf_end(int_buf + CPI - 1) {}
35    ~printer() {
36        flush();
37        delete[] buf, delete[] int_buf;
38    }
39    void flush() {
40        fwrite_unlocked(buf, 1, buf_ptr - buf, stdout);
41        buf_ptr = buf;
```

```
43 }
44 void write_(const char &c) {
45     *buf_ptr = c;
46     if (++buf_ptr == buf_end) [[unlikely]]
47         flush();
48 }
49 void write(const char *s) {
50     for (; *s != '\0'; ++s) write_(*s);
51 }
52 void write(int x) {
53     if (x < 0) write_('-'), x = -x;
54     if (x == 0) [[unlikely]]
55         return write_('\0');
56     for (tbuf = int_buf_end; x != 0; --tbuf, x /= 10)
57         *tbuf = '0' + char(x % 10);
58     write_(++tbuf);
59 }
```

Kotlin

```
import java.io.*
import java.util.*

@JvmField val cin = System.`in`.bufferedReader()
@JvmField val cout = PrintWriter(System.out, false)
@JvmField var tokenizer: StringTokenizer = StringTokenizer("")
fun nextLine() = cin.readLine()!!
fun read(): String {
    while (!tokenizer.hasMoreTokens())
        tokenizer = StringTokenizer(nextLine())
    return tokenizer.nextToken()
}

// example
fun main() {
    val n = read().toInt()
    val a = DoubleArray(n) { read().toDouble() }
    cout.println("omg hi")
    cout.flush()
}
```

1.2.3. constexpr

Some default limits in gcc (7.x - trunk):

- constexpr recursion depth: 512
- constexpr loop iteration per function: 262 144
- constexpr operation count per function: 33 554 432
- template recursion depth: 900 (gcc *might* segfault first)

```
01 constexpr array<int, 10> fibonacci[] {
02     array<int, 10> a{};
03     a[0] = a[1] = 1;
04     for (int i = 2; i < 10; i++) a[i] = a[i - 1] + a[i - 2];
05     return a;
06 }();
07 static_assert(fibonacci[9] == 55, "CE");
08
09 template <typename F, typename INT, INT... S>
10 constexpr void for_constexpr(integer_sequence<INT, S...>, F &&func) {
11     int _[] = {{func(integral_constant<INT, S>{}, 0)...}};
12 }
13 // example
14 template <typename... T> void print_tuple(tuple<T...> t) {
15     for_constexpr(make_index_sequence<sizeof...(T)>{},
16                 [&](auto i) { cout << get<i>(t) << '\n'; });
17 }
```

1.2.4. Bump Allocator

```

1
3
// global bump allocator
5 char mem[256 << 20]; // 256 MB
size_t rsp = sizeof mem;
7 void*operator new(size_t s) {
    assert(s < rsp); // MLE
9     return (void*)&mem[rsp -= s];
}
11 void operator delete(void *) {}

13 // bump allocator for STL / pbds containers
char mem[256 << 20];
size_t rsp = sizeof mem;
template <typename T> struct bump {
17     typedef T value_type;
     bump() {}
19     template <typename U> bump(U, ...) {}
T *allocate(size_t n) {
21         rsp -= n * sizeof(T);
        rsp &= 0 - alignof(T);
        return (T*)(mem + rsp);
}
25     void deallocate(T *, size_t n) {}
};

```

1.3. Tools

1.3.1. Floating Point Binary Search

```

1 union di {
2     double d;
3     ull i;
4 };
5 bool check(double);
// binary search in [L, R) with relative error 2^-eps
7 double binary_search(double L, double R, int eps) {
8     di l = {L}, r = {R}, m;
9     while (r.i - l.i > 1LL << (52 - eps)) {
10         m.i = (l.i + r.i) >> 1;
11         if (check(m.d)) r = m;
12         else l = m;
13     }
14     return l.d;
15 }

```

1.3.2. SplitMix64

```

1 using ull = unsigned long long;
2 inline ull splitmix64(ull x) {
3     // change to `static ull x = SEED;` for DRBG
4     ull z = (x += 0x9E3779B97F4A7C15);
5     z = (z ^ (z >> 30)) * 0xBF58476D1CE4E5B9;
6     z = (z ^ (z >> 27)) * 0x94D049BB133111EB;
7     return z ^ (z >> 31);
}

```

1.3.3. <random>

```

1 #ifdef __unix__
random_device rd;
3 mt19937_64 RNG(rd());
#else
5 const auto SEED = chrono::high_resolution_clock::now()
                    .time_since_epoch()
                    .count();
7 mt19937_64 RNG(SEED);
9 #endif
// random uint_fast64_t: RNG();
// uniform random of type T (int, double, ...) in [l,
11 // uniform_int_distribution<T> dist(l, r); dist(RNG);
13

```

1.3.4. x86 Stack Hack

```

1 constexpr size_t size = 200 << 20; // 200MiB
2 int main() {
3     register long rsp asm("rsp");
4     char *buf = new char[size];
5     asm("movq %0, %%rsp\n" :: "r"(buf + size));
6     // do stuff
7     asm("movq %0, %%rsp\n" :: "r"(rsp));
8     delete[] buf;
9 }

```

1.4. Algorithms

1.4.1. Bit Hacks

```

1 // next permutation of x as a bit sequence
2 ull next_bits_permutation(ull x) {
3     ull c = __builtin_ctzll(x), r = x + (1ULL << c);
4     return (r ^ x) >> (c + 2) | r;
5 }
// iterate over all (proper) subsets of bitset s
7 void subsets(ull s) {
8     for (ull x = s; x;) { --x &= s; /* do stuff */ }
9 }

```

1.4.2. Infinite Grid Knight Distance

```

1 ll get_dist(ll dx, ll dy) {
2     if (++(dx = abs(dx)) > ++(dy = abs(dy))) swap(dx, dy);
3     if (dx == 1 && dy == 2) return 3;
4     if (dx == 3 && dy == 3) return 4;
5     ll lb = max(dy / 2, (dx + dy) / 3);
6     return ((dx ^ dy ^ lb) & 1) ? ++lb : lb;
7 }

```

1.4.3. Longest Increasing Subsequence

```

1
2 template <class I> vi lis(const vector<I> &S) {
3     if (S.empty()) return {};
4     vi prev(sz(S));
5     typedef pair<I, int> p;
6     vector<p> res;
7     rep(i, 0, sz(S)) {
8         // change 0 -> i for longest non-decreasing subsequence
9         auto it = lower_bound(all(res), p{S[i], 0});
10        if (it == res.end())
11            res.emplace_back(), it = res.end() - 1;
12        *it = {S[i], i};
13        prev[i] = it == res.begin() ? 0 : (it - 1)->second;
14    }
15    int L = sz(res), cur = res.back().second;
16    vi ans(L);
17    while (L--) ans[L] = cur, cur = prev[cur];
18    return ans;
}

```

1.4.4. Mo's Algorithm on Tree

```

1 void MoAlgoOnTree() {
2     Dfs(0, -1);
3     vector<int> euler(tk);
4     for (int i = 0; i < n; ++i) {
5         euler[tin[i]] = i;
6         euler[tout[i]] = i;
7     }
8     vector<int> l(q), r(q), qr(q), sp(q, -1);
9     for (int i = 0; i < q; ++i) {
10        if (tin[u[i]] > tin[v[i]]) swap(u[i], v[i]);
11        int z = GetLCA(u[i], v[i]);
12        sp[i] = z[i];
13        if (z == u) l[i] = tin[u[i]], r[i] = tin[v[i]];
14        else l[i] = tout[u[i]], r[i] = tin[v[i]];
15        qr[i] = i;
}

```

```
17     }
18     sort(qr.begin(), qr.end(), [&](int i, int j) {
19         if (l[i] / kB == l[j] / kB) return r[i] < r[j];
20         return l[i] / kB < l[j] / kB;
21     });
22     vector<bool> used(n);
23     // Add(v): add/remove v to/from the path based on used[v]
24     for (int i = 0, tl = 0, tr = -1; i < q; ++i) {
25         while (tl < l[qr[i]]) Add(euler[tl++]);
26         while (tl > l[qr[i]]) Add(euler[-tl]);
27         while (tr > r[qr[i]]) Add(euler[tr--]);
28         while (tr < r[qr[i]]) Add(euler[++tr]);
29     }
}
```

2. Data Structures

2.1. GNU PBDS

```
1 #include <ext/pb_ds/assoc_container.hpp>
2 #include <ext/pb_ds/priority_queue.hpp>
3 #include <ext/pb_ds/tree_policy.hpp>
4 using namespace __gnu_pbds;
5
6 // most std::map + order_of_key, find_by_order, sp
7 template <typename T, typename U = null_type>
8 using ordered_map = tree<T, U, std::less<>, rb_tree
9                               tree_order_statistics_node
10 // useful tags: rb_tree_tag, splay_tree_tag
11
12 template <typename T> struct myhash {
13     size_t operator()(T x) const; // splitmix, bswap
14 };
15 // most of std::unordered_map, but faster (needs g
16 template <typename T, typename U = null_type>
17 using hash_table = gp_hash_table<T, U, myhash<T>>;
18
19 // most std::priority_queue + modify, erase, split
20 using heap = priority_queue<int, std::less<>>;
21 // useful tags: pairing_heap_tag, binary_heap_tag,
22 //                  (rc_)binomial_heap_tag, thin_heap
```

2.2. 2D Partial Sums

```

1 using vvi = vector<vector<int>>;
2 using vvll = vector<vector<ll>>;
3 using vll = vector<ll>;
4
5 struct PrefixSum2D {
6     vvll pref; // 0-based 2-D prefix sum
7     void build(const vvll &v) { // creates a copy
8         int n = v.size(), m = v[0].size();
9         pref.assign(n, vll(m, 0));
10        for (int i = 0; i < n; i++) {
11            for (int j = 0; j < m; j++) {
12                pref[i][j] = v[i][j] + (i ? pref[i - 1][j]
13                                         : (j ? pref[i][j - 1] : 0)) -
14                                         (i && j ? pref[i - 1][j - 1]
15                                         : 0);
16            }
17        }
18    }
19    ll query(int ulx, int uly, int brx, int bry) const {
20        ll ans = pref[brx][bry];
21        if (ulx) ans -= pref[ulx - 1][bry];
22        if (uly) ans -= pref[brx][uly - 1];
23        if (ulx && uly) ans += pref[ulx - 1][uly - 1];
24        return ans;
25    }
26    ll query(int ulx, int uly, int size) const {
27        return query(ulx, uly, ulx + size - 1, uly + size - 1);
28    }
29 };
30 struct PartialSum2D : PrefixSum2D {
31     vvll diff; // 0 based
32     int n, m;
33     PartialSum2D(int _n, int _m) : n(_n), m(_m) {
34         diff.assign(n + 1, vll(m + 1, 0));
35     }
36     // add c from [ulx,uly] to [brx,bry]
37     void update(int ulx, int uly, int brx, int bry,
38                 diff[ulx][uly] += c;
39                 diff[ulx][bry + 1] -= c;
40                 diff[brx + 1][uly] -= c;
41                 diff[brx + 1][bry + 1] += c;
42             }
43             void update(int ulx, int uly, int size, ll c) {
44                 int brx = ulx + size - 1;
45                 int bry = uly + size - 1;
46                 update(ulx, uly, brx, bry, c);
47             }
48             // process the grid using prefix sum

```

```
49     void process() { this->build(diff); }
50 };
51 // usage
52 PrefixSum2D pref;
53 pref.build(v); // takes 2d 0-based vector as input
54 pref.query(x1, y1, x2, y2); // sum of region
55
56 PartialSum2D part(n, m);           // dimension of grid 0 based
57 part.update(x1, y1, x2, y2, 1); // add 1 in region
58 // must run after all updates
59 part.process(); // prefix sum on diff array
60 // only exists after processing
61 update(y1, &grid = part.pref); // processed diff array
62 part.query(x1, y1, x2, y2); // gives sum of region
```

2.3. Heavy-Light Decomposition

3. Graph

3.1. Modeling

- Maximum/Minimum flow with lower bound / Circulation problem
 1. Construct super source S and sink T .
 2. For each edge (x, y, l, u) , connect $x \rightarrow y$ with capacity $u - l$.
 3. For each vertex v , denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
 4. If $in(v) > 0$, connect $S \rightarrow v$ with capacity $in(v)$, otherwise, connect $v \rightarrow T$ with capacity $-in(v)$.
 - To maximize, connect $t \rightarrow s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T . If $f \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
 - To minimize, let f be the maximum flow from S to T . Connect $t \rightarrow s$ with capacity ∞ and let the flow from S to T be f' . If $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, f' is the answer.
 5. The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
 1. Redirect every edge: $y \rightarrow x$ if $(x, y) \in M$, $x \rightarrow y$ otherwise.
 2. DFS from unmatched vertices in X .
 3. $x \in X$ is chosen iff x is unvisited.
 4. $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
 1. Construct super source S and sink T
 2. For each edge (x, y, c) , connect $x \rightarrow y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \rightarrow x$ with $(cost, cap) = (-c, 1)$
 3. For each edge with $c < 0$, sum these cost as K , then increase $d(y)$ by 1, decrease $d(x)$ by 1
 4. For each vertex v with $d(v) > 0$, connect $S \rightarrow v$ with $(cost, cap) = (0, d(v))$
 5. For each vertex v with $d(v) < 0$, connect $v \rightarrow T$ with $(cost, cap) = (0, -d(v))$
 6. Flow from S to T , the answer is the cost of the flow $C + K$
- Maximum density induced subgraph
 1. Binary search on answer, suppose we're checking answer T
 2. Construct a max flow model, let K be the sum of all weights
 3. Connect source $s \rightarrow v, v \in G$ with capacity K
 4. For each edge (u, v, w) in G , connect $u \rightarrow v$ and $v \rightarrow u$ with capacity w
 5. For $v \in G$, connect it with sink $v \rightarrow t$ with capacity $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
 6. T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
 1. For each $v \in V$ create a copy v' , and connect $u' \rightarrow v'$ with weight $w(u, v)$.
 2. Connect $v \rightarrow v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v .
 3. Find the minimum weight perfect matching on G' .
- Project selection problem
 1. If $p_v > 0$, create edge (s, v) with capacity p_v ; otherwise, create edge (v, t) with capacity $-p_v$.
 2. Create edge (u, v) with capacity w with w being the cost of choosing u without choosing v .
 3. The mincut is equivalent to the maximum profit of a subset of projects.
- 0/1 quadratic programming

$$\sum_x c_x x + \sum_y c_y \bar{y} + \sum_{xy} c_{xy} x \bar{y} + \sum_{xyx'y'} c_{xyx'y'} (x \bar{y} + x' \bar{y'})$$

can be minimized by the mincut of the following graph:

1. Create edge (x, t) with capacity c_x and create edge (s, y) with capacity c_y .
2. Create edge (x, y) with capacity c_{xy} .
3. Create edge (x, y) and edge (x', y') with capacity $c_{xyx'y'}$.

3.2. Matching/Flows

3.2.1. Kuhn-Munkres algorithm

```

1 // Maximum Weight Perfect Bipartite Matching
2 // Detect non-perfect-matching:
3 // 1. set all edge[i][j] as INF
4 // 2. if solve() >= INF, it is not perfect matching.
5
6 typedef long long ll;
7 struct KM {
8     static const int MAXN = 1050;
9     static const ll INF = 1LL << 60;
10    int n, match[MAXN], vx[MAXN], vy[MAXN];
11    ll edge[MAXN][MAXN], lx[MAXN], ly[MAXN], slack[MAXN];
12    void init(int _n) {
13        n = _n;
14        for (int i = 0; i < n; i++)
15            for (int j = 0; j < n; j++) edge[i][j] = 0;
16    }
17    bool add_edge(int x, int y, ll w) { edge[x][y] = w; }
18    bool DFS(int x) {
19        vx[x] = 1;
20        for (int y = 0; y < n; y++) {
21            if (vy[y]) continue;
22            if (lx[x] + ly[y] > edge[x][y]) {
23                slack[y] =
24                    min(slack[y], lx[x] + ly[y] - edge[x][y]);
25            } else {
26                vy[y] = 1;
27                if (match[y] == -1 || DFS(match[y])) {
28                    match[y] = x;
29                    return true;
30                }
31            }
32        }
33        return false;
34    }
35    ll solve() {
36        fill(match, match + n, -1);
37        fill(lx, lx + n, -INF);
38        fill(ly, ly + n, 0);
39        for (int i = 0; i < n; i++)
40            for (int j = 0; j < n; j++)
41                lx[i] = max(lx[i], edge[i][j]);
42        for (int i = 0; i < n; i++) {
43            fill(slack, slack + n, INF);
44            while (true) {
45                fill(vx, vx + n, 0);
46                fill(vy, vy + n, 0);
47                if (DFS(i)) break;
48                ll d = INF;
49                for (int j = 0; j < n; j++)
50                    if (!vy[j]) d = min(d, slack[j]);
51                for (int j = 0; j < n; j++) {
52                    if (vx[j]) lx[j] -= d;
53                    if (vy[j]) ly[j] += d;
54                    else slack[j] -= d;
55                }
56            }
57            ll res = 0;
58            for (int i = 0; i < n; i++) {
59                res += edge[match[i]][i];
60            }
61        }
62    }
63 } graph;

```

3.3. Shortest Path Faster Algorithm

```

1 struct SPFA {
2     static const int maxn = 1010, INF = 1e9;
3     int dis[maxn];
4     bitset<maxn> inq, inneg;
5     queue<int> q, tq;
6     vector<pii> v[maxn];
7 
```

```

7 void make_edge(int s, int t, int w) {
8     v[s].emplace_back(t, w);
9 }
10 void dfs(int a) {
11     inneg[a] = 1;
12     for (pii i : v[a])
13         if (!inneg[i.F]) dfs(i.F);
14 }
15 bool solve(int n, int s) { // true if have neg-cycle
16     for (int i = 0; i <= n; i++) dis[i] = INF;
17     dis[s] = 0, q.push(s);
18     for (int i = 0; i < n; i++) {
19         inq.reset();
20         int now;
21         while (!q.empty()) {
22             now = q.front(), q.pop();
23             for (pii &i : v[now]) {
24                 if (dis[i.F] > dis[now] + i.S) {
25                     dis[i.F] = dis[now] + i.S;
26                     if (!inq[i.F]) tq.push(i.F), inq[i.F] = 1;
27                 }
28             }
29         }
30         q.swap(tq);
31     }
32     bool re = !q.empty();
33     inneg.reset();
34     while (!q.empty()) {
35         if (!inneg[q.front()]) dfs(q.front());
36         q.pop();
37     }
38     return re;
39 }
40 void reset(int n) {
41     for (int i = 0; i <= n; i++) v[i].clear();
42 }
43 }
```

3.4. Strongly Connected Components

```

1 struct TarjanScc {
2     int n, step;
3     vector<int> time, low, instk, stk;
4     vector<vector<int>> e, scc;
5     TarjanScc(int n)
6         : n(n_), step(0), time(n), low(n), instk(n),
7           void add_edge(int u, int v) { e[u].push_back(v); }
8     void dfs(int x) {
9         time[x] = low[x] = ++step;
10        stk.push_back(x);
11        instk[x] = 1;
12        for (int y : e[x])
13            if (!time[y]) {
14                dfs(y);
15                low[x] = min(low[x], low[y]);
16            } else if (instk[y]) {
17                low[x] = min(low[x], time[y]);
18            }
19        if (time[x] == low[x]) {
20            scc.emplace_back();
21            for (int y = -1; y != x;) {
22                y = stk.back();
23                stk.pop_back();
24                instk[y] = 0;
25                scc.back().push_back(y);
26            }
27        }
28    }
29    void solve() {
30        for (int i = 0; i < n; i++)
31            if (!time[i]) dfs(i);
32        reverse(scc.begin(), scc.end());
33        // scc in topological order
34    }
35 }
```

3.5. Biconnected Components

3.5.1. Articulation Points

```

1 void dfs(int x, int p) {
2     tin[x] = low[x] = ++t;
3     int ch = 0;
4     for (auto u : g[x])
5         if (u.first != p) {
6             if (!ins[u.second])
7                 st.push(u.second), ins[u.second] = true;
8             if (tin[u.first]) {
9                 low[x] = min(low[x], tin[u.first]);
10                continue;
11            }
12            ++ch;
13            dfs(u.first, x);
14            low[x] = min(low[x], low[u.first]);
15            if (low[u.first] >= tin[x]) {
16                cut[x] = true;
17                ++sz;
18                while (true) {
19                    int e = st.top();
20                    st.pop();
21                    bcc[e] = sz;
22                    if (e == u.second) break;
23                }
24            }
25        }
26    if (ch == 1 && p == -1) cut[x] = false;
27 }
```

3.5.2. Bridges

```

1 // if there are multi-edges, then they are not bridges
2 void dfs(int x, int p) {
3     tin[x] = low[x] = ++t;
4     st.push(x);
5     for (auto u : g[x])
6         if (u.first != p) {
7             if (tin[u.first]) {
8                 low[x] = min(low[x], tin[u.first]);
9                 continue;
10            }
11            dfs(u.first, x);
12            low[x] = min(low[x], low[u.first]);
13            if (low[u.first] == tin[u.first]) br[u.second] = true;
14        }
15    if (tin[x] == low[x]) {
16        ++sz;
17        while (st.size()) {
18            int u = st.top();
19            st.pop();
20            bcc[u] = sz;
21            if (u == x) break;
22        }
23    }
24 }
```

3.6. Centroid Decomposition

```

1 void get_center(int now) {
2     v[now] = true;
3     vtx.push_back(now);
4     sz[now] = 1;
5     mx[now] = 0;
6     for (int u : G[now])
7         if (!v[u])
8             get_center(u);
9             mx[now] = max(mx[now], sz[u]);
10            sz[now] += sz[u];
11        }
12    }
13    void get_dis(int now, int d, int len) {
14        dis[d][now] = cnt;
15        v[now] = true;
```

```
17   for (auto u : G[now])
18     if (!v[u.first]) { get_dis(u, d, len + u.second); }
19 void dfs(int now, int fa, int d) {
20   get_center(now);
21   int c = -1;
22   for (int i : vtx) {
23     if (max(mx[i], (int)vtx.size() - sz[i]) <=
24         (int)vtx.size() / 2)
25       c = i;
26     v[i] = false;
27   }
28   get_dis(c, d, 0);
29   for (int i : vtx) v[i] = false;
30   v[c] = true;
31   vtx.clear();
32   dep[c] = d;
33   p[c] = fa;
34   for (auto u : G[c])
35     if (u.first != fa && !v[u.first]) {
36       dfs(u.first, c, d + 1);
37     }
}
```

4. Math

4.1. Number Theory

4.1.1. Mod Struct

A list of safe primes: 26003, 27767, 28319, 28979, 29243, 29759, 30467, 910927547, 919012223, 947326223, 990669467, 1007939579, 1019126695, 929760389146037459, 975500632317046523, 989312547895528379

NTT prime p	$p - 1$	primitive root	
65537	$1 \ll 16$	3	
998244353	$119 \ll 23$	3	
2748779069441	$5 \ll 39$	3	
1945555039024054273	$27 \ll 56$	5	

Requires: Extended GCD

```

1
3 template <typename T> struct M {
4     static T MOD; // change to constexpr if already known
5     T v;
6     M(T x = 0) {
7         v = (-MOD <= x && x < MOD) ? x : x % MOD;
8         if (v < 0) v += MOD;
9     }
10    explicit operator T() const { return v; }
11    bool operator==(const M &b) const { return v == b.v; }
12    bool operator!=(const M &b) const { return v != b.v; }
13    M operator-() { return M(-v); }
14    M operator+(M b) { return M(v + b.v); }
15    M operator-(M b) { return M(v - b.v); }
16    M operator*(M b) { return M((__int128)v * b.v % MOD); }
17    M operator/(M b) { return *this * (b^(MOD - 2)); }
18    // change above implementation to this if MOD is not prime
19    M inv() {
20        auto [p, _, g] = extgcd(v, MOD);
21        return assert(g == 1), p;
22    }
23    friend M operator^(M a, ll b) {
24        M ans(1);
25        for (; b; b >= 1, a *= a)
26            if (b & 1) ans *= a;
27        return ans;
28    }
29    friend M &operator+=(M &a, M b) { return a = a + b; }
30    friend M &operator-=(M &a, M b) { return a = a - b; }
31    friend M &operator*=(M &a, M b) { return a = a * b; }
32    friend M &operator/=(M &a, M b) { return a = a / b; }
33};
34 using Mod = M<int>;
35 template <> int Mod::MOD = 1'000'000'007;
36 int &MOD = Mod::MOD;

```

4.1.2. Miller-Rabin

Requires: Mod Struct

```

1
3 // checks if Mod::MOD is prime
4 bool is_prime() {
5     if (MOD < 2 || MOD % 2 == 0) return MOD == 2;
6     Mod A[] = {2, 7, 61}; // for int values (< 2^31)
7     // ll: 2, 325, 9375, 28178, 450775, 9780504, 179526509
8     int s = __builtin_ctzll(MOD - 1), i;
9     for (Mod a : A) {
10         Mod x = a ^ (MOD >> s);
11         for (i = 0; i < s && (x + 1).v > 2; i++) x *= x;
12         if (i && x != -1) return 0;
13     }
14     return 1;
15 }

```

4.1.3. Linear Sieve

```

1 constexpr ll MAXN = 1000000;
2 bitset<MAXN> is_prime;
3 vector<ll> primes;
4 ll mpf[MAXN], phi[MAXN], mu[MAXN];
5
6 void sieve() {
7     is_prime.set();
8     is_prime[1] = 0;
9     mu[1] = phi[1] = 1;
10    for (ll i = 2; i < MAXN; i++) {
11        if (is_prime[i]) {
12            mpf[i] = i;
13            primes.push_back(i);
14            phi[i] = i - 1;
15            mu[i] = -1;
16        }
17        for (ll p : primes) {
18            if (p > mpf[i] || i * p >= MAXN) break;
19            is_prime[i * p] = 0;
20            mpf[i * p] = p;
21            mu[i * p] = -mu[i];
22            if (i % p == 0)
23                phi[i * p] = phi[i] * p, mu[i * p] = 0;
24            else phi[i * p] = phi[i] * (p - 1);
25        }
26    }
27 }

```

4.1.4. Get Factors

Requires: Linear Sieve

```

1
2 vector<ll> all_factors(ll n) {
3     vector<ll> fac = {1};
4     while (n > 1) {
5         const ll p = mpf[n];
6         vector<ll> cur = {1};
7         while (n % p == 0) {
8             n /= p;
9             cur.push_back(cur.back() * p);
10        }
11        vector<ll> tmp;
12        for (auto x : cur)
13            tmp.push_back(x * p);
14        cur.swap(tmp);
15    }
16    return fac;
17 }

```

4.1.5. Binary GCD

```

1 // returns the gcd of non-negative a, b
2 ull bin_gcd(ull a, ull b) {
3     if (!a || !b) return a + b;
4     int s = __builtin_ctzll(a | b);
5     a >>= __builtin_ctzll(a);
6     while (b) {
7         if ((b >>= __builtin_ctzll(b)) < a) swap(a, b);
8         b -= a;
9     }
10    return a << s;
11 }

```

4.1.6. Extended GCD

```

1 // returns (p, q, g): p * a + q * b == g == gcd(a, b)
2 // g is not guaranteed to be positive when a < 0 or b < 0
3 tuple<ll, ll, ll> extgcd(ll a, ll b) {
4     ll s = 1, t = 0, u = 0, v = 1;
5     while (b) {
6         ll q = a / b;
7         swap(a -= q * b, b);
8         swap(s -= q * t, t);
9     }
10    return {a, s, g};
11 }

```

```

9     swap(u -= q * v, v);
11    return {s, u, a};
}

```

4.1.7. Chinese Remainder Theorem

Requires: Extended GCD

```

1 // for 0 <= a < m, 0 <= b < n, returns the smallest
3 // such that x % m == a and x % n == b
ll crt(ll a, ll m, ll b, ll n) {
5   if (n > m) swap(a, b), swap(m, n);
  auto [x, y, g] = extgcd(m, n);
7   assert((a - b) % g == 0); // no solution
  x = ((b - a) / g * x) % (n / g) * m + a;
9   return x < 0 ? x + m / g * n : x;
}

```

4.1.8. Pollard's Rho

```

1 ll f(ll x, ll mod) { return (x * x + 1) % mod; }
// n should be composite
3 ll pollard_rho(ll n) {
  if (!(n & 1)) return 2;
  while (1) {
    ll y = 2, x = RNG() % (n - 1) + 1, res = 1;
    for (int sz = 2; res == 1; sz *= 2) {
      for (int i = 0; i < sz && res <= 1; i++) {
        x = f(x, n);
        res = __gcd(abs(x - y), n);
      }
      y = x;
    }
    if (res != 0 && res != n) return res;
  }
}

```

4.1.9. Rational Number Binary Search

```

1 struct QQ {
  ll p, q;
3  QQ go(QQ b, ll d) { return {p + b.p * d, q + b.q * d}; }
};
5 bool pred(QQ);
// returns smallest p/q in [lo, hi] such that
// pred(p/q) is true, and 0 <= p, q <= N
7 QQ frac_bs(ll N) {
9  QQ lo{0, 1}, hi{1, 0};
  if (pred(lo)) return lo;
11  assert(pred(hi));
  bool dir = 1, L = 1, H = 1;
13  for (; L || H; dir = !dir) {
    ll len = 0, step = 1;
15  for (int t = 0; t < 2 && (t ? step /= 2 : step *= 2);)
      if (QQ mid = hi.go(lo, len + step);
          mid.p > N || mid.q > N || dir ^ pred(mid))
        t++;
19    else len += step;
    swap(lo, hi = hi.go(lo, len));
21    (dir ? L : H) = !!len;
  }
23  return dir ? hi : lo;
}

```

4.2. Combinatorics

4.2.1. De Bruijn Sequence

```

1 int res[kN], aux[kN], a[kN], sz;
void Rec(int t, int p, int n, int k) {
3  if (t > n) {
    if (n % p == 0)
      for (int i = 1; i <= p; ++i) res[sz++] = aux[i];
    } else {
}

```

```

7   aux[t] = aux[t - p];
9   Rec(t + 1, p, n, k);
11  for (aux[t] = aux[t - p] + 1; aux[t] < k; ++aux[t])
    Rec(t + 1, t, n, k);
}
13 int DeBruijn(int k, int n) {
// return cyclic string of length k^n such that every
// string of length n using k character appears as a
// substring.
15 >=if (k == 1) return res[0] = 0, 1;
  fill(aux, aux + k * n, 0);
19  return sz = 0, Rec(1, 1, n, k), sz;
}

```

5. Numeric

5.1. Long Long Multiplication

```
1 using ull = unsigned long long;
2 using ll = long long;
3 using ld = long double;
4 // returns a * b % M where a, b < M < 2**63
5 ull mult(ull a, ull b, ull M) {
6     ll ret = a * b - M * ull(ld(a) * ld(b) / ld(M));
7     return ret + M * (ret < 0) - M * (ret >= (ll)M);
}
```

6. Geometry

6.1. Point

```

1 template <typename T> struct P {
2     T x, y;
3     P(T x = 0, T y = 0) : x(x), y(y) {}
4     bool operator<(const P &p) const {
5         return tie(x, y) < tie(p.x, p.y);
6     }
7     bool operator==(const P &p) const {
8         return tie(x, y) == tie(p.x, p.y);
9     }
P operator-() const { return {-x, -y}; }
P operator+(P p) const { return {x + p.x, y + p.y}; }
P operator-(P p) const { return {x - p.x, y - p.y}; }
P operator*(T d) const { return {x * d, y * d}; }
P operator/(T d) const { return {x / d, y / d}; }
T dist2() const { return x * x + y * y; }
double len() const { return sqrt(dist2()); }
P unit() const { return *this / len(); }
friend T dot(P a, P b) { return a.x * b.x + a.y * b.y; }
friend T cross(P a, P b) { return a.x * b.y - a.y * b.x; }
friend T cross(P a, P b, P o) {
    return cross(a - o, b - o);
}
};  

using pt = P<ll>;

```

6.1.1. Quaternion

```

1 constexpr double PI = 3.141592653589793;
2 constexpr double EPS = 1e-7;
3 struct Q {
4     using T = double;
5     T x, y, z, r;
6     Q(T r = 0) : x(0), y(0), z(0), r(r) {}
7     Q(T x, T y, T z, T r = 0) : x(x), y(y), z(z), r(r) {}
8     friend bool operator==(const Q &a, const Q &b) {
9         return (a - b).abs2() <= EPS;
10    }
11    friend bool operator!=(const Q &a, const Q &b) {
12        return !(a == b);
13    }
14    Q operator-() { return Q(-x, -y, -z, -r); }
15    Q operator+(const Q &b) const {
16        return Q(x + b.x, y + b.y, z + b.z, r + b.r);
17    }
18    Q operator-(const Q &b) const {
19        return Q(x - b.x, y - b.y, z - b.z, r - b.r);
20    }
21    Q operator*(const T &t) const {
22        return Q(x * t, y * t, z * t, r * t);
23    }
24    Q operator*(const Q &b) const {
25        return Q(r * b.x + x * b.r + y * b.z - z * b.y,
26                  r * b.y - x * b.z + y * b.r + z * b.x,
27                  r * b.z + x * b.y - y * b.x + z * b.r,
28                  r * b.r - x * b.x - y * b.y - z * b.z);
29    }
30    Q operator/(const Q &b) const { return *this * b.inv(); }
31    T abs2() const { return r * r + x * x + y * y + z * z; }
32    T len() const { return sqrt(abs2()); }
33    Q conj() const { return Q(-x, -y, -z, r); }
34    Q unit() const { return *this * (1.0 / len()); }
35    Q inv() const { return conj() * (1.0 / abs2()); }
36    friend T dot(Q a, Q b) {
37        return a.x * b.x + a.y * b.y + a.z * b.z;
38    }
39    friend Q cross(Q a, Q b) {
40        return Q(a.y * b.z - a.z * b.y, a.z * b.x - a.x * b.z,
41                  a.x * b.y - a.y * b.x);
42    }
43    friend Q rotation_around(Q axis, T angle) {
44        return axis.unit() * sin(angle / 2) + cos(angle) * this;
45    }

```

```

47     Q rotated_around(Q axis, T angle) {
48         Q u = rotation_around(axis, angle);
49         return u * *this / u;
50     }
51     friend Q rotation_between(Q a, Q b) {
52         a = a.unit(), b = b.unit();
53         if (a == -b) {
54             // degenerate case
55             Q ortho = abs(a.y) > EPS ? cross(a, Q(1, 0, 0))
56                                         : cross(a, Q(0, 1, 0));
57             return rotation_around(ortho, PI);
58         }
59         return (a * (a + b)).conj();
60     }
61 }

```

6.1.2. Spherical Coordinates

```

1 struct car_p {
2     double x, y, z;
3 };
4 struct sph_p {
5     double r, theta, phi;
6 };
7 sph_p conv(car_p p) {
8     double r = sqrt(p.x * p.x + p.y * p.y + p.z * p.z);
9     double theta = asin(p.y / r);
10    double phi = atan2(p.y, p.x);
11    return {r, theta, phi};
12 }
13 car_p conv(sph_p p) {
14     double x = p.r * cos(p.theta) * sin(p.phi);
15     double y = p.r * cos(p.theta) * cos(p.phi);
16     double z = p.r * sin(p.theta);
17     return {x, y, z};
18 }

```

6.2. Segments

```

1 // for non-collinear ABCD, if segments AB and CD intersect
2 bool intersects(pt a, pt b, pt c, pt d) {
3     if (cross(b, c, a) * cross(b, d, a) > 0) return false;
4     if (cross(d, a, c) * cross(d, b, c) > 0) return false;
5     return true;
6 }
7 // the intersection point of lines AB and CD
8 pt intersect(pt a, pt b, pt c, pt d) {
9     auto x = cross(b, c, a), y = cross(b, d, a);
10    if (x == y) {
11        // if(abs(x, y) < 1e-8) {
12            // is parallel
13        } else {
14            return d * (x / (x - y)) - c * (y / (x - y));
15        }
16    }

```

6.3. Convex Hull

```

1 // returns a convex hull in counterclockwise order
2 // for a non-strict one, change cross >= to >
3 vector<pt> convex_hull(vector<pt> p) {
4     sort(ALL(p));
5     if (p[0] == p.back()) return {p[0]};
6     int n = p.size(), t = 0;
7     vector<pt> h(n + 1);
8     for (int i = 2, s = 0; _ --; s = --t, reverse(ALL(p)))
9         for (pt i : p) {
10             while (t > s + 1 && cross(i, h[t - 1], h[t - 2]) >= 0)
11                 t--;
12             h[t++] = i;
13         }
14     return h.resize(t), h;
15 }

```

6.3.1. 3D Hull

```

1
3 typedef Point3D<double> P3;
5
5 struct PR {
6     void ins(int x) { (a == -1 ? a : b) = x; }
7     void rem(int x) { (a == x ? a : b) = -1; }
8     int cnt() { return (a != -1) + (b != -1); }
9     int a, b;
10 };
11
12 struct F {
13     P3 q;
14     int a, b, c;
15 };
16
17 vector<F> hull3d(const vector<P3> &A) {
18     assert(sz(A) >= 4);
19     vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {
20         #define E(x, y) E[f.x][f.y]
21         vector<F> FS;
22         auto mf = [&](int i, int j, int k, int l) {
23             P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
24             if (q.dot(A[l]) > q.dot(A[i])) q = q * -1;
25             F f{q, i, j, k};
26             E(a, b).ins(k);
27             E(a, c).ins(j);
28             E(b, c).ins(i);
29             FS.push_back(f);
30         };
31         rep(i, 0, 4) rep(j, i + 1, 4) rep(k, j + 1, 4)
32             mf(i, j, k, 6 - i - j - k);
33
34         rep(i, 4, sz(A)) {
35             rep(j, 0, sz(FS)) {
36                 F f = FS[j];
37                 if (f.q.dot(A[i]) > f.q.dot(A[f.a])) {
38                     E(a, b).rem(f.c);
39                     E(a, c).rem(f.b);
40                     E(b, c).rem(f.a);
41                     swap(FS[j--], FS.back());
42                     FS.pop_back();
43                 }
44                 int nw = sz(FS);
45                 rep(j, 0, nw) {
46                     F f = FS[j];
47                     #define C(a, b, c)
48                     if (E(a, b).cnt() != 2) mf(f.a, f.b, i, f.c);
49                     C(a, b, c);
50                     C(a, c, b);
51                     C(b, c, a);
52                 }
53             }
54             for (F &it : FS)
55                 if ((A[it.b] - A[it.a])
56                     .cross(A[it.c] - A[it.a])
57                     .dot(it.q) <= 0)
58                     swap(it.c, it.b);
59             return FS;
60     };
61 }
```

6.4. Angular Sort

```

1 auto angle_cmp = [](const pt &a, const pt &b) {
2     auto btm = [](const pt &a) {
3         return a.y < 0 || (a.y == 0 && a.x < 0);
4     };
5     return make_tuple(btm(a), a.y * b.x, abs2(a)) <
6         make_tuple(btm(b), a.x * b.y, abs2(b));
7 };
8 void angular_sort(vector<pt> &p) {
9     sort(p.begin(), p.end(), angle_cmp);
10 }
```

6.5. Convex Polygon Minkowski Sum

```

1 // O(n) convex polygon minkowski sum
2 // must be sorted and counterclockwise
3 vector<pt> minkowski_sum(vector<pt> p, vector<pt> q) {
4     auto diff = [](vector<pt> &c) {
5         auto rcmp = [](pt a, pt b) {
6             return pt{a.y, a.x} < pt{b.y, b.x};
7         };
8         rotate(c.begin(), min_element(ALL(c), rcmp), c.end());
9         c.push_back(c[0]);
10        vector<pt> ret;
11        for (int i = 1; i < c.size(); i++)
12            ret.push_back(c[i] - c[i - 1]);
13        return ret;
14    };
15    auto dp = diff(p), dq = diff(q);
16    pt cur = p[0] + q[0];
17    vector<pt> d(dp.size() + dq.size()), ret = {cur};
18    // include angle_cmp from angular-sort.cpp
19    merge(ALL(dp), ALL(dq), d.begin(), angle_cmp);
20    // optional: make ret strictly convex (UB if degenerate)
21    int now = 0;
22    for (int i = 1; i < d.size(); i++) {
23        if (cross(d[i], d[now]) == 0) d[now] = d[now] + d[i];
24        else d[++now] = d[i];
25    }
26    d.resize(now + 1);
27    // end optional part
28    for (pt v : d) ret.push_back(cur = cur + v);
29    return ret.pop_back(), ret;
30 }
```

6.6. Point In Polygon

```

1 bool on_segment(pt a, pt b, pt p) {
2     return cross(a, b, p) == 0 && dot((p - a), (p - b)) <= 0;
3 }
4 // p can be any polygon, but this is O(n)
5 bool inside(const vector<pt> &p, pt a) {
6     int cnt = 0, n = p.size();
7     for (int i = 0; i < n; i++) {
8         pt l = p[i], r = p[(i + 1) % n];
9         // change to return 0; for strict version
10        if (on_segment(l, r, a)) return 1;
11        cnt += ((a.y < l.y) - (a.y < r.y)) * cross(l, r, a) > 0;
12    }
13    return cnt;
14 }
```

6.6.1. Convex Version

```

1 // no preprocessing version
2 // p must be a strict convex hull, counterclockwise
3 // if point is inside or on border
4 bool is_inside(const vector<pt> &c, pt p) {
5     int n = c.size(), l = 1, r = n - 1;
6     if (cross(c[0], c[1], p) < 0) return false;
7     if (cross(c[n - 1], c[0], p) < 0) return false;
8     while (l < r - 1) {
9         int m = (l + r) / 2;
10        T a = cross(c[0], c[m], p);
11        if (a > 0) l = m;
12        else if (a < 0) r = m;
13        else return dot(c[0] - p, c[m] - p) <= 0;
14    }
15    if (l == r) return dot(c[0] - p, c[l] - p) <= 0;
16    else return cross(c[l], c[r], p) >= 0;
17 }
18
19 // with preprocessing version
20 vector<pt> vecs;
21 pt center;
22 // p must be a strict convex hull, counterclockwise
23 // BEWARE OF OVERFLOWS!!
24 void preprocess(vector<pt> p) {
```

```

25   for (auto &v : p) v = v * 3;
26   center = p[0] + p[1] + p[2];
27   center.x /= 3, center.y /= 3;
28   for (auto &v : p) v = v - center;
29   vecs = (angular_sort(p), p);
30 }
31 bool intersect_strict(pt a, pt b, pt c, pt d) {
32   if (cross(b, c, a) * cross(b, d, a) > 0) return false;
33   if (cross(d, a, c) * cross(d, b, c) >= 0) return false;
34   return true;
35 }
36 // if point is inside or on border
37 bool query(pt p) {
38   p = p * 3 - center;
39   auto pr = upper_bound(ALL(vecs), p, angle_cmp);
40   if (pr == vecs.end()) pr = vecs.begin();
41   auto pl = (pr == vecs.begin()) ? vecs.back() : *(pr - 1);
42   return !intersect_strict({0, 0}, p, pl, *pr);
43 }
```

6.6.2. Offline Multiple Points Version

Requires: Point, GNU PBDS

```

1
3
5
using Double = __float128;
7 using Point = pt<Double, Double>;
9
int n, m;
vector<Point> poly;
vector<Point> query;
vector<int> ans;
13
struct Segment {
15   Point a, b;
16   int id;
17 };
18 vector<Segment> segs;
19
Double Xnow;
21 inline Double get_y(const Segment &u, Double xnow) {
22   const Point &a = u.a;
23   const Point &b = u.b;
24   return (a.y * (b.x - xnow) + b.y * (xnow - a.x)) / (b.x - a.x);
25 }
26
bool operator<(Segment u, Segment v) {
27   Double yu = get_y(u);
28   Double yv = get_y(v);
29   if (yu != yv) return yu < yv;
30   return u.id < v.id;
31 }
32
ordered_map<Segment> st;
33
struct Event {
34   int type; // +1 insert seg, -1 remove seg, 0 query
35   Double x, y;
36   int id;
37 };
38
bool operator<(Event a, Event b) {
39   if (a.x != b.x) return a.x < b.x;
40   if (a.type != b.type) return a.type < b.type;
41   return a.y < b.y;
42 }
43
vector<Event> events;
44
void solve() {
45   set<Double> xs;
46   set<Point> ps;
47   for (int i = 0; i < n; i++) {
48     xs.insert(poly[i].x);
49     ps.insert(poly[i]);
50   }
51 }
```

```

55   for (int i = 0; i < n; i++) {
56     Segment s{poly[i], poly[(i + 1) % n], i};
57     if (s.a.x > s.b.x || (s.a.x == s.b.x && s.a.y > s.b.y)) {
58       swap(s.a, s.b);
59     }
60     segs.push_back(s);
61   }
62   for (int i = 0; i < m; i++) {
63     events.push_back({0, query[i].x, query[i].y, i});
64   }
65
66   sort(events.begin(), events.end());
67   int cnt = 0;
68   for (Event e : events) {
69     int i = e.id;
70     Xnow = e.x;
71     if (e.type == 0) {
72       Double x = e.x;
73       Double y = e.y;
74       Segment tmp = {{x - 1, y}, {x + 1, y}, -1};
75       auto it = st.lower_bound(tmp);
76
77       if (ps.count(query[i]) > 0) {
78         ans[i] = 0;
79       } else if (xs.count(x) > 0) {
80         ans[i] = -2;
81       } else if (it != st.end() && get_y(*it) == get_y(tmp)) {
82         ans[i] = 0;
83       } else if (it != st.begin() && get_y(*prev(it)) == get_y(tmp)) {
84         ans[i] = 0;
85       } else {
86         int rk = st.order_of_key(tmp);
87         if (rk % 2 == 1) {
88           ans[i] = 1;
89         } else {
90           ans[i] = -1;
91         }
92       }
93     } else if (e.type == 1) {
94       st.insert(segs[i]);
95       assert((int)st.size() == ++cnt);
96     } else if (e.type == -1) {
97       st.erase(segs[i]);
98       assert((int)st.size() == --cnt);
99     }
100   }
101 }
```

6.7. Closest Pair

```

1 vector<pll> p; // sort by x first!
2 bool cmpy(const pll &a, const pll &b) const {
3   return a.y < b.y;
4 }
5 ll sq(ll x) { return x * x; }
6 // returns (minimum dist)^2 in [l, r)
7 ll solve(int l, int r) {
8   if (r - l <= 1) return 1e18;
9   int m = (l + r) / 2;
10  ll mid = p[m].x, d = min(solve(l, m), solve(m, r));
11  auto pb = p.begin();
12  inplace_merge(pb + l, pb + m, pb + r, cmpy);
13  vector<pll> s;
14  for (int i = l; i < r; i++)
15    if (sq(p[i].x - mid) < d) s.push_back(p[i]);
16  for (int i = 0; i < s.size(); i++) {
17    for (int j = i + 1;
18         j < s.size() && sq(s[j].y - s[i].y) < d; j++)
19      d = min(d, dis(s[i], s[j]));
20 }
```

```
21 }  
return d;
```

6.8. Minimum Enclosing Circle

```
1  
3 typedef Point<double> P;  
4 double ccRadius(const P &A, const P &B, const P &C) {  
5     return (B - A).dist() * (C - B).dist() * (A - C).dist() /  
6         abs((B - A).cross(C - A)) / 2;  
7 }  
8 P ccCenter(const P &A, const P &B, const P &C) {  
9     P b = C - A, c = B - A;  
10    return A + (b * c.dist2() - c * b.dist2()).perp() /  
11        b.cross(c) / 2;  
12 }  
13 pair<P, doublevector<P> ps) {  
14     shuffle(all(ps), mt19937(time(0)));  
15     P o = ps[0];  
16     double r = 0, EPS = 1 + 1e-8;  
17     rep(i, 0, sz(ps)) if ((o - ps[i]).dist() > r * EPS) {  
18         o = ps[i], r = 0;  
19         rep(j, 0, i) if ((o - ps[j]).dist() > r * EPS) {  
20             o = (ps[i] + ps[j]) / 2;  
21             r = (o - ps[i]).dist();  
22             rep(k, 0, j) if ((o - ps[k]).dist() > r * EPS) {  
23                 o = ccCenter(ps[i], ps[j], ps[k]);  
24                 r = (o - ps[i]).dist();  
25             }  
26         }  
27     }  
28     return {o, r};  
29 }
```

7. Strings

7.1. Knuth-Morris-Pratt Algorithm

```

1 vector<int> pi(const string &s) {
2     vector<int> p(s.size());
3     for (int i = 1; i < s.size(); i++) {
4         int g = p[i - 1];
5         while (g && s[i] != s[g]) g = p[g - 1];
6         p[i] = g + (s[i] == s[g]);
7     }
8     return p;
9 }
10 vector<int> match(const string &s, const string &pat) {
11     vector<int> p = pi(pat + '\0' + s), res;
12     for (int i = p.size() - s.size(); i < p.size(); i++)
13         if (p[i] == pat.size())
14             res.push_back(i - 2 * pat.size());
15     return res;
16 }
```

7.2. Z Value

```

1 int z[n];
2 void zval(string s) {
3     // z[i] => longest common prefix of s and s[i:],
4     int n = s.size();
5     z[0] = 0;
6     for (int b = 0, i = 1; i < n; i++) {
7         if (z[b] + b <= i) z[i] = 0;
8         else z[i] = min(z[i - b], z[b] + b - i);
9         while (s[i + z[i]] == s[z[i]]) z[i]++;
10    }
11 }
```

7.3. Manacher's Algorithm

```

1 int z[n];
2 void manacher(string s) {
3     // z[i] => longest odd palindrome centered at i
4     //      s[i - z[i] ... i + z[i]]
5     // to get all palindromes (including even length)
6     // insert a '#' between each s[i] and s[i + 1]
7     int n = s.size();
8     z[0] = 0;
9     for (int b = 0, i = 1; i < n; i++) {
10        if (z[b] + b >= i)
11            z[i] = min(z[2 * b - i], b + z[b] - i);
12        else z[i] = 0;
13        while (i + z[i] + 1 < n && i - z[i] - 1 >= 0 &&
14              s[i + z[i] + 1] == s[i - z[i] - 1])
15            z[i]++;
16        if (z[i] + i > z[b] + b) b = i;
17    }
18 }
```

7.4. Minimum Rotation

```

1 int min_rotation(string s) {
2     int a = 0, n = s.size();
3     s += s;
4     for (int b = 0; b < n; b++) {
5         for (int k = 0; k < n; k++) {
6             if (a + k == b || s[a + k] < s[b + k]) {
7                 b += max(0, k - 1);
8                 break;
9             }
10            if (s[a + k] > s[b + k]) {
11                a = b;
12                break;
13            }
14        }
15    }
16    return a;
17 }
```

7.5. Aho-Corasick Automaton

```

1 struct Aho_Corasick {
2     static const int maxc = 26, maxn = 4e5;
3     struct NODES {
4         int Next[maxc], fail, ans;
5     };
6     NODES T[maxn];
7     int top, qtop, q[maxn];
8     int get_node(const int &fail) {
9         fill_n(T[top].Next, maxc, 0);
10        T[top].fail = fail;
11        T[top].ans = 0;
12        return top++;
13    }
14    int insert(const string &s) {
15        int ptr = 1;
16        for (char c : s) { // change char id
17            c -= 'a';
18            if (!T[ptr].Next[c]) T[ptr].Next[c] = get_node(ptr);
19            ptr = T[ptr].Next[c];
20        }
21        return ptr;
22    }
23    } // return ans_last_place
24    void build_fail(int ptr) {
25        int tmp;
26        for (int i = 0; i < maxc; i++) {
27            if (T[ptr].Next[i]) {
28                tmp = T[ptr].fail;
29                while (tmp != 1 && !T[tmp].Next[i])
30                    tmp = T[tmp].fail;
31                if (T[tmp].Next[i] != T[ptr].Next[i])
32                    if (T[tmp].Next[i]) tmp = T[tmp].Next[i];
33                T[T[ptr].Next[i]].fail = tmp;
34                q[qtop++] = T[ptr].Next[i];
35            }
36        }
37    void AC_auto(const string &s) {
38        int ptr = 1;
39        for (char c : s) {
40            while (ptr != 1 && !T[ptr].Next[c]) ptr = T[ptr].fail;
41            if (T[ptr].Next[c]) {
42                ptr = T[ptr].Next[c];
43                T[ptr].ans++;
44            }
45        }
46    void Solve(string &s) {
47        for (char &c : s) // change char id
48            c -= 'a';
49        for (int i = 0; i < qtop; i++) build_fail(q[i]);
50        AC_auto(s);
51        for (int i = qtop - 1; i > -1; i--)
52            T[T[q[i]].fail].ans += T[q[i]].ans;
53    }
54    void reset() {
55        qtop = top = q[0] = 1;
56        get_node(1);
57    }
58 } AC;
59 // usage example
60 string s, S;
61 int n, t, ans_place[50000];
62 int main() {
63     Tie cin >> t;
64     while (t--) {
65         AC.reset();
66         cin >> S >> n;
67         for (int i = 0; i < n; i++) {
68             cin >> s;
69             ans_place[i] = AC.insert(s);
70         }
71         AC.Solve(S);
72         for (int i = 0; i < n; i++)
73             cout << AC.T[ans_place[i]].ans << '\n';
74     }
75 }
```

75 }]

8. Debug List

- ```
1 - Pre-submit:
 - Did you make a typo when copying a template?
 - Test more cases if unsure.
 - Write a naive solution and check small cases.
 - Submit the correct file.

7 - General Debugging:
 - Read the whole problem again.
 - Have a teammate read the problem.
 - Have a teammate read your code.
 - Explain your solution to them (or a rubber duck).
 - Print the code and its output / debug output.
 - Go to the toilet.

15 - Wrong Answer:
 - Any possible overflows?
 - > `__int128` ?
 - Try `#pragma GCC optimize("trapv")`
 - Floating point errors?
 - > `long double` ?
 - turn off math optimizations
 - check for `==`, `>=`, `acos(1.000000001)`, etc.
 - Did you forget to sort or unique?
 - Generate large and worst "corner" cases.
 - Check your `m` / `n`, `i` / `j` and `x` / `y`.
 - Are everything initialized or reset properly?
 - Are you sure about the STL thing you are using?
 - Read cppreference (should be available).
 - Print everything and run it on pen and paper.

31 - Time Limit Exceeded:
 - Calculate your time complexity again.
 - Does the program actually end?
 - Check for `while(q.size())` etc.
 - Test the largest cases locally.
 - Did you do unnecessary stuff?
 - e.g. pass vectors by value
 - e.g. `memset` for every test case
 - Is your constant factor reasonable?

41 - Runtime Error:
 - Check memory usage.
 - Forget to clear or destroy stuff?
 - > `vector::shrink_to_fit()`
 - Stack overflow?
 - Bad pointer / array access?
 - Try `fsanitize=address`
 - Division by zero? NaN's?
```