Solar Physics
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Article preparation guidelines

Solar Physics

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Abstract The derivation of kinematic profiles for eruptive events is prominent in the field of solar physics. The details on the acceleration of coronal mass ejections (CMEs) and large-scale coronal disturbances ('EIT waves') are important for indicating the driving mechanisms at play. The techniques used for deriving the velocity and acceleration profiles of events based upon the height-time tracks .....

Keywords: CME, EIT Waves, Corona, Mathematical Techniques

1. Introduction

2. Numerical Differentiation Techniques

When presented with a moving object through a sequence of image frames such that it is possible to measure it's position at each time step, the technique of numerical differentiation is often used to derive the velocity and acceleration of the object. In the standard 2-point approach, it should be possible to derive the time evolution of a system at time step $t + \Delta t$ according to the system values at time step t. This may be applied through the technique of forward, reverse or centre differencing, resulting in an estimate of the speed of the object at a specific time step given its positional information. More commonly, a 3-point Langrangian interpolation is applied to better approximate the kinematics of a moving object by solving for the Lagrange polynomials that bestacross 3 given datapoints (e.g. DERIV.PRO in IDL). Each of these schemes is based upon the Taylor series expansion of a real function $f(t)$ about the point $t = t_0$:

$$f(t) \ = \ f(t_0) + f'(t_0)(t - t_0) + \frac{f''(t_0)}{2!}(t - t_0)^2 + ... \qquad (1)$$

An alternative form is obtained by letting $t - t_0 = \Delta t$ so that $t = t_0 + \Delta t$ to give:

$$f(t_0 + \Delta t) \ = \ f(t_0) + f'(t_0)\Delta t + \frac{f''(t_0)}{2!}(\Delta t)^2 + ... \qquad (2)$$

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This expansion can be used to determine the numerical derivative of a function according to the choice of technique, detailed in Sections 2.1, 2.2, 2.3 and 2.4 below.

Given a function $x = f(u, v)$, the error propagation equation (based on the standard deviations σ of the variables) is written:

$$\sigma_x^2 \;=\; \sigma_u^2 \left(\frac{\partial x}{\partial u}\right)^2 + \sigma_v^2 \left(\frac{\partial x}{\partial v}\right)^2 + 2\sigma_{uv}^2 \left(\frac{\partial x}{\partial u}\right)\left(\frac{\partial x}{\partial v}\right) + ... \qquad (3)$$

Specifically in the case of kinematic analyses, this is used to propagate the errors on the height-time data $r(t)$ into the velocity $v(t)$ and acceleration $a(t)$ profiles to determine the associated uncertainties for each technique detailed below.

2.1. Forward Differencing

The forward differencing technique involves extrapolating forward from each time step t to derive the evolution of the system. Thus rewriting Equation 2 to express the distance measurement at time step $t + \Delta t$ gives:

$$r(t + \Delta t) \;=\; r(t) + r'(t)\Delta t + \frac{r''(t)}{2!}(\Delta t)^2 + ... \qquad (4)$$

$$\Rightarrow \quad v(t) \;\equiv\; r'(t) \;=\; \frac{r(t + \Delta t) - r(t)}{\Delta t} + O(\Delta t) \qquad (5)$$

where $O(\Delta t)$ is the truncation error term, determined by the distance between neighbouring points (Δt). It is possible to derive the value of the truncation error term in terms of the original $r(t)$ values. The truncation error is given as

$$O(\Delta t) = \frac{r''(t)}{2!}(\Delta t) \qquad (6)$$

The $r''(t)$ term may be decomposed using the original forward-difference definition:

$$r''(t) = \frac{r'(t + \Delta t) - r'(t)}{\Delta t} \qquad (7)$$

Rewriting each term using the original functional forms produces

$$O(\Delta t) = \frac{r(t + 2\Delta t) - 2r(t + \Delta t) + r(t)}{2!\Delta t} \qquad (8)$$

The error term associated with the velocity estimate using the forward-difference technique is therefore dependent on the value of the function $r(t)$ at the points t, $t + \Delta t$ and $t + 2\Delta t$.

In the case of estimating the acceleration error term, we obtain:

$$O(\Delta t) = \frac{v(t + 2\Delta t) - 2v(t + \Delta t) + v(t)}{2!\Delta t} \qquad (9)$$

Here, the velocity function $v(t)$ is treated as the base function, rather than the distance function $r(t)$ as above.

The forward difference technique inherently assumes that there is a straight-line gradient between points, and it's application removes a point from the end of the dataset.

2.2. Reverse Differencing

The reverse difference technique works in the same manner as the forward difference but is applied at the point $t - \Delta t$ by extrapolating backwards from the point t. This results in:

$$v(t) \;\equiv\; r'(t) \;=\; \frac{r(t) - r(t-\Delta t)}{\Delta t} + O(\Delta t) \tag{10}$$

where $O(\Delta t)$ is the truncation error term which, as with the forward difference method, is determined by the distance between neighbouring points (Δt), assuming a straight line gradient between points.

Once again, the truncation error can be estimated in terms of the original distance function $r(t)$, given by:

$$O(\Delta t) \;=\; \frac{r(t) - 2r(t-\Delta t) + r(t-2\Delta t)}{2!\Delta t} \tag{11}$$

and the truncation error associated with the acceleration term is:

$$O(\Delta t) \;=\; \frac{v(t) - 2v(t-\Delta t) + v(t-2\Delta t)}{2!\Delta t} \tag{12}$$

Similar to forward differencing, the reverse difference technique inherently assumes that there is a straight-line gradient between points, and it's application removes a point from the beginning of the dataset.

2.3. Centre Differencing

The centre difference technique uses the two neighbouring points to the point $r(t)$ under examination, i.e., $r(t-\Delta t)$ and $r(t+\Delta t)$, according to the equation:

$$v \;\equiv\; r'(t) \;=\; \frac{r(t+\Delta t) - r(t-\Delta t)}{2\Delta t} + O(\Delta t)^2 \tag{13}$$

where $O(\Delta t)^2$ is the truncation error term. The truncation error term in this case is determined by the square of the distance between neighbouring points, and thus has greater precision than the forward and reverse difference methods.

The centre-difference method effectively smoothes the data set while differentiating it by using the two points either side of the point under examination. The truncation error term may be written as:

$$O(\Delta t)^2 \;=\; \frac{r(t+3\Delta t) - 3r(t+\Delta t) + 3r(t-\Delta t) - r(t-3\Delta t)}{(3!)(8)\Delta t} \tag{14}$$

Similarly the truncation error term for the acceleration is given by:

$$O(\Delta t)^2 \; = \; \frac{v(t+3\Delta t) - 3v(t+\Delta t) + 3v(t-\Delta t) - v(t-3\Delta t)}{(3!)(8)\Delta t} \tag{15}$$

The centre-difference technique produces a smaller mean truncation error and less scatter in the data than the forward and reverse difference techniques. The centre difference technique is therefore determined to be more precise.

2.4. 3-Point Lagrangian Interpolation

3-point Lagrangian interpolation is used on a discrete set of data points in order to determine the first and second derivatives corresponding to the velocity and acceleration of an object in a more robust manner than simple forward, reverse or centre difference techniques. Considering three data points, (x_0, y_0), (x_1, y_1), (x_2, y_2), the Lagrangian interpolation polynomial is given by:

$$L(x) \; = \; \sum_{j=0}^{2} y_j l_j(x) \quad \text{where} \quad l_j(x) \; = \prod_{i=0, i\neq j}^{2} \frac{x - x_i}{x_j - x_i} \tag{16}$$

The error propagation equation is used to determine the errors on the resulting derivative points in $L' \equiv f(L(x), x)$:

$$\sigma^2_{L'} \; = \sigma^2_L \left(\frac{\partial L'}{\partial L}\right)^2 + \sigma^2_x \left(\frac{\partial L'}{\partial x}\right)^2 + ... \tag{17}$$

$$= \frac{\sigma^2_L}{\partial x^2} + \frac{\sigma^2_x}{\partial x^2} \left(\frac{\partial L}{\partial x}\right)^2 \tag{18}$$

Or more appropriately written in this context as:

$$\sigma^2_d \; = \; \frac{\sigma^2_{y_{n+1}} + \sigma^2_{y_{n-1}}}{dx^2} + \frac{\sigma^2_{x_{n+1}} + \sigma^2_{x_{n-1}}}{dx^2} \left(\frac{dy}{dx}\right)^2 \tag{19}$$

So the errors on the end points become:

$$\sigma^2_{d_0} \; = \frac{9\sigma^2_{y_0} + 16\sigma^2_{y_1} + \sigma^2_{y_2}}{(x_2 - x_0)^2} + \frac{\sigma^2_{x_2} + \sigma^2_{x_0}}{(x_2 - x_0)^2} \left(\frac{3y_0 - 4y_1 + y_2}{x_2 - x_0}\right)^2 \tag{20}$$

$$\sigma^2_{d_n} \; = \frac{9\sigma^2_{y_n} + 16\sigma^2_{y_{n-1}} + \sigma^2_{y_{n-2}}}{(x_n - x_{n-2})^2} + \frac{\sigma^2_{x_{n-2}} + \sigma^2_{x_n}}{(x_{n-2} - x_n)^2} \left(\frac{3y_n - 4y_{n-1} + y_{n-2}}{x_{n-2} - x_n}\right)^2 \tag{21}$$

This effect is reflected in the larger errorbars on the end points of the derived kinematics of Section **??**.

The errors on the heights are used to constrain the best fit to a constant acceleration model of the form:

$$h(t) \; = \; at^2 + v_0 t + h_0 \tag{22}$$

where t is time and a, v_0 and h_0 are the acceleration, initial velocity and initial height respectively. This provides a linear fit to the derived velocity points and a constant fit to the acceleration. An important point to note is the small time error (taken to be the image exposure time of the coronagraph data) since the analysis is performed upon the observed data frames individually. Previous methods of temporal-differencing would increase this time error. With these more accurate measurements we are better able to determine the velocity and acceleration errors, leading to improved constraints upon the data and providing greater confidence in comparing to theoretical models.

A more advanced numerical differentiation method is the three-point Lagrangian method used by the DERIV routine in IDL. This method uses three adjacent points ($t-\Delta t$, t and $t+\Delta t$) to fit a Lagrange polynomial function of the form

$$\begin{aligned} P(x) &= \frac{(x-t)(x-(t+\Delta t))}{((t-\Delta t)-t)((t-\Delta t)-(t+\Delta t))}y_1 \\ &+ \frac{(x-(t-\Delta t))(x-(t+\Delta t))}{(t-(t-\Delta t))(t-(t+\Delta t))}y_2 \\ &+ \frac{(x-(t-\Delta t))(x-t)}{((t+\Delta t)-(t-\Delta t))((t+\Delta t)-t))}y_3 \end{aligned} \tag{23}$$

to the three points, and hence find the derived value at a point numerically.

This method gives a more accurate indication of the numerically differentiated data as it does not assume a straight line in between points. In addition, the routine compensates for edge points, increasing the errors of the points. It also uses three points to find the numerical derivative, rather than two (c.f. centre, forward and reverse-difference techniques). The Lagrangian technique also smoothes the data-set, removing the spiky appearance that arises from use of the different Taylor series techniques.

The problems encountered in this work arise from the small data-sets available in the case of these disturbances. The small data-sets mean that there are only four or five points available of which three are needed to calculate the numerical derivative of each point. This makes the Lagrangian technique unsuitable for estimating the kinematics of these disturbances.

The three-point Lagrangian interpolation method used by IDL uses the standard deviation of the derivative calculated by the Lagrangian polynomial as the truncation error of the numerical derivative. This produces a consistent estimate of the mean truncation error that only varies with the end-points due to the increased uncertainty of these points.

Table no. shows the different estimates for the truncation error term for each numerical differencing technique. These truncation error estimates are given in terms of the original base function, the exception being for the Lagrangian technique, which takes the standard deviation of the Lagrange derivative at a point as the truncation error estimate at that point.

3. Models

3.1. const. vel.

3.2. const. accel.

3.3. non-const. accel.

3.4. Cadence

3.5. S/N

4. Data

4.1. CMEs

4.2. EIT waves

5. Bootstrapping

5.1. Using BibTeX

The use of BibTeX simplifies the inclusion of references. Only the references cited and labeled in the text are included at compilation, and an error message appears if some references are missing. Any new reference will automatically be written at the correct location in the reference list after compilation. Moreover the references are stored, in any order, in a separate file (with the `.bib` extension) in the BibTeX format, so independently of the journal format. Such a personal reference file can be re-used with any journal. The formatting of the references and their listing order are made automatically at compilation (using the information given in the `.bst` file).

The references in BibTeX format can be downloaded from the Astrophysics Data System (ADS), then stored in `SOLA_bibliography_example.bib` (file name of the present example). The main extra work is to define a proper and easy label for each citation (a convenient one is simply first-author-name-year). Furthermore, it is better to have the journal names defined by commands (for example `\solphys`), as defined at the beginning of this `.tex` file. This provides an homogeneity in the reference list and permits flexibility when changing for journals. Some caution should be taken for some journals since ADS does not necessarily provide a uniform format for the journal names. This is the case for *J. Geophys. Res.* Moreover since *J. Geophys. Res.* has a new way to refer to an article (since 2002 it has no page number), then the ADS references need to be corrected. More generally, it is worth verifying each reference from the original publication (independently of BibTeX use).

The full LaTeX and BibTeX compilation is made in four steps:

1) `latex filename`	(stores the labels in the `.aux` file)
2) `bibtex filename`	(loads the bibliography in the `.bbl` file)
3) `latex filename`	(reads the .bbl, stores in the `.aux`)
4) `latex filename`	(replaces all labels)

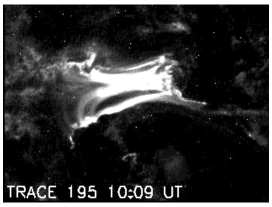

Figure 1. Example of a simple figure in an appendix.

where `filename` is the name of your LaTeX file (for example, the present file) **without** typing its `.tex` extension. If a `(?)` is still present in the output (at the place of a label), it means that this label has not been properly defined. (for example, LaTeX labels are case sensitive). Any undefined label has a warning written in the `console window` (it is better to have this window open by default, since LaTeX warning and error messages are very useful to localize the problem).

When the references are not changed, it is unnecessary to re-run BibTeX. When no new labels are added, running latex once is sufficient to refresh the LaTeX output. So, except for the first, and the final time (safest), running LaTeX once is sufficient in most cases to update the LaTeX output, if the compilation files created are not erased! For example BibTeX keeps the bibliography in the usual environment,

```
\begin{thebibliography}{} ... \end{thebibliography}
```

in the file with the `.bbl` extension.

5.2. Miscellaneous Other Features

Long URL's can be quite messy when broken across lines `http://gong.nso.edu/data/magmap/` as normal text, however the `url` package does a nice job of this, *e.g.* http://gong.nso.edu/data/magmap/.

6. Conclusion

We hope authors of *Solar Physics* will find this guide useful. Please send us feedback on how to improve it.

LaTeX is very convenient to write a scientific text, in particular with the use of labels for figures, tables, and references. Moreover, the labels and list of references are checked by the software against one another, and, the formatting should be effortless with BibTeX.

Appendix

After the `\appendix` command, the sections are referenced with capital letters. The numbering of equations, figures and labels is is just the same as with classical sections.

Table 1. A simple table in an appendix.

Rot.	Date	CMEs obs.	CMEs cor.	α $10^{-2}\mathrm{Mm}^{-1}$
1	02–Nov–97	16	24.1	-1.26
2	29–Nov–97	–	2.53	0.94

A. Abbreviations of some Journal Names

Journal names are abbreviated in *Solar Physics* with the IAU convention (IAU Style Book published in Transactions of the IAU XXB, 1988, pp. Si-S3. www.iau.org/Abbreviations.235.0.html). Here are a few journals with their LaTeX commands (see the beginning of this `.tex` file).

`\aap`	*Astron. Astrophys.*
`\apj`	*Astrophys. J.*
`\jgr`	*J. Geophys. Res.*
`\mnras`	*Mon. Not. Roy. Astron. Soc.*
`\pasj`	*Pub. Astron. Soc. Japan*
`\pasp`	*Pub. Astron. Soc. Pac.*
`\solphys`	*Solar Phys.*

Acknowledgements The authors thank ... (*note the reduced point size*)

Bibliography Included with BibTeX

With BibTeX the formatting will be done automatically for all the references cited with one of the `\cite` commands (Section **??**). Besides the usual items, it includes the title of the article and the concluding page number.

Bibliography included manually

The articles can be entered, formatted, and ordered by the author with the command `\bibitem`. ADS provides references in the *Solar Physics* format by selecting the format `SoPh format` under the menu `Select short list format`. Including the article title and the concluding page number are optional; however, we require consistency in the author's choice. That is, all of the references should have the article title, or none, and similarly for ending page numbers.

References

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