

## Solar Physics

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**Abstract** The derivation of kinematic profiles for eruptive events is prominent in the field of solar physics. The details on the acceleration of coronal mass ejections (CMEs) and large-scale coronal disturbances (‘EIT waves’) are important for indicating the driving mechanisms at play. The techniques used for deriving the velocity and acceleration profiles of events based upon the height-time tracks  
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**Keywords:** CME, EIT Waves, Corona, Mathematical Techniques

## 1. Introduction

## 2. Numerical Differentiation Techniques

When presented with a moving object through a sequence of image frames such that it is possible to measure its position at each time step, the technique of numerical differentiation is often used to derive the velocity and acceleration of the object. In the standard 2-point approach, it should be possible to derive the time evolution of a system at time step  $t + \Delta t$  according to the system values at time step  $t$ . This may be applied through the technique of forward, reverse or centre differencing, resulting in an estimate of the speed of the object at a specific time step given its positional information. More commonly, a 3-point Lagrangian interpolation is applied to better approximate the kinematics of a moving object by solving for the Lagrange polynomials that best cross 3 given datapoints (e.g. DERIV.PRO in IDL). Each of these schemes is based upon the Taylor series expansion of a real function  $f(t)$  about the point  $t = t_0$ :

$$f(t) = f(t_0) + f'(t_0)(t - t_0) + \frac{f''(t_0)}{2!}(t - t_0)^2 + \dots \quad (1)$$

An alternative form is obtained by letting  $t - t_0 = \Delta t$  so that  $t = t_0 + \Delta t$  to give:

$$f(t_0 + \Delta t) = f(t_0) + f'(t_0)\Delta t + \frac{f''(t_0)}{2!}(\Delta t)^2 + \dots \quad (2)$$

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This expansion can be used to determine the numerical derivative of a function according to the choice of technique, detailed in Sections 2.1, 2.2, 2.3 and 2.4 below.

Given a function  $x = f(u, v)$ , the error propagation equation (based on the standard deviations  $\sigma$  of the variables) is written:

$$\sigma_x^2 = \sigma_u^2 \left( \frac{\partial x}{\partial u} \right)^2 + \sigma_v^2 \left( \frac{\partial x}{\partial v} \right)^2 + 2\sigma_{uv} \left( \frac{\partial x}{\partial u} \right) \left( \frac{\partial x}{\partial v} \right) + \dots \quad (3)$$

Specifically in the case of kinematic analyses, this is used to propagate the errors on the height-time data  $r(t)$  into the velocity  $v(t)$  and acceleration  $a(t)$  profiles to determine the associated uncertainties for each technique detailed below. In the case of height-time data the covariance terms are zero because the quantities are uncorrelated.

### 2.1. Forward Differencing

The forward differencing technique involves extrapolating forward from each time step  $t$  to derive the evolution of the system. Thus rewriting Equation 2 to express the distance measurement at time step  $t + \Delta t$  gives:

$$r(t + \Delta t) = r(t) + r'(t)\Delta t + \frac{r''(t)}{2!}(\Delta t)^2 + \dots \quad (4)$$

$$\Rightarrow v(t) \equiv r'(t) = \frac{r(t + \Delta t) - r(t)}{\Delta t} + O(\Delta t) \quad (5)$$

where  $O(\Delta t)$  is the truncation error term, determined by the distance between neighbouring points  $\Delta t$ .

The forward difference technique inherently assumes that there is a straight-line gradient between points, and its application removes a point from the end of the dataset.

To propagate a one standard deviation uncertainty  $\sigma_r$  of the height data, and  $\sigma_t$  of the time data, into the velocity profile and obtain the associated uncertainty in velocity  $\sigma_v$ , the error propagation equation (Eqn. 3) is written:

$$\sigma_v^2 = \frac{\sigma_{r(t+\Delta t)}^2 + \sigma_{r(t)}^2}{\Delta t^2} + v^2 \left( \frac{\sigma_{t+\Delta t}^2 + \sigma_t^2}{\Delta t^2} \right) \quad (6)$$

It is clear that the inverse dependence on  $\Delta t^2$  will be an important consideration when looking at the effects of measurement cadence for determining kinematics and their associated uncertainties. In effect, reducing the cadence increases the accuracy but decreases the precision (and vice versa) of the derived kinematics.

### 2.2. Reverse Differencing

The reverse difference technique works in the same manner as the forward difference but is applied at the point  $t - \Delta t$  by extrapolating backwards from the

point  $t$ . This results in:

$$v(t) \equiv r'(t) = \frac{r(t) - r(t - \Delta t)}{\Delta t} + O(\Delta t) \quad (7)$$

where  $O(\Delta t)$  is the truncation error term which, as with the forward difference method, is determined by the distance between neighbouring points  $\Delta t$ , assuming a straight line gradient between consecutive points.

Similar to forward differencing, the reverse difference technique inherently assumes that there is a straight-line gradient between points, and its application removes a point from the beginning of the dataset.

The error propagation equation for reverse differencing is written:

$$\sigma_v^2 = \frac{\sigma_{r(t)}^2 + \sigma_{r(t-\Delta t)}^2}{\Delta t^2} + v^2 \left( \frac{\sigma_t^2 + \sigma_{t-\Delta t}^2}{\Delta t^2} \right) \quad (8)$$

It may be noted that the trends in the derivatives produced by both forward and reverse differencing are identical, the only difference between them being where the resulting profile sits with respect to the time axis.

### 2.3. Centre Differencing

The centre difference technique uses the two neighbouring points to the point  $r(t)$  under examination, i.e.,  $r(t - \Delta t)$  and  $r(t + \Delta t)$ , according to the equation:

$$v \equiv r'(t) = \frac{r(t + \Delta t) - r(t - \Delta t)}{2\Delta t} + O(\Delta t)^2 \quad (9)$$

The truncation error term in this case is determined by the square of the distance between neighbouring points. The centre-difference method effectively smoothes the data set while differentiating it by using the two points either side of the point under examination. It is only applicable when the spacing between datapoints is equal, i.e., when  $\Delta t$  remains constant.

The error propagation equation for centre differencing is written:

$$\sigma_v^2 = \frac{\sigma_{r(t+\Delta t)}^2 + \sigma_{r(t-\Delta t)}^2}{4\Delta t^2} + v^2 \left( \frac{\sigma_{t+\Delta t}^2 + \sigma_{t-\Delta t}^2}{4\Delta t^2} \right) \quad (10)$$

### 2.4. 3-Point Lagrangian Interpolation

3-point Lagrangian interpolation is used in order to determine the first and second derivatives corresponding to the velocity and acceleration of an object in a more robust manner than simple forward, reverse or centre difference techniques, and specifically when the spacing between datapoints is not equal, i.e., when  $\Delta t$  is non-constant. Considering three data points,  $(x_0, y_0)$ ,  $(x_1, y_1)$ ,  $(x_2, y_2)$ , the Lagrangian interpolation polynomial is given by:

$$L(x) = \sum_{j=0}^2 y_j l_j(x) \quad \text{where} \quad l_j(x) = \prod_{i=0, i \neq j}^2 \frac{x - x_i}{x_j - x_i} \quad (11)$$

The derivative at point  $x$  is given by  $L' = \partial_x L(x)$ . The error propagation equation is used to determine the errors on the resulting derivative points in  $L' \equiv f(L(x), x)$ :

$$\sigma_{L'}^2 = \sigma_L^2 \left( \frac{\partial L'}{\partial L} \right)^2 + \sigma_x^2 \left( \frac{\partial L'}{\partial x} \right)^2 + \dots \quad (12)$$

$$= \frac{\sigma_L^2}{\partial x^2} + \frac{\sigma_x^2}{\partial x^2} \left( \frac{\partial L}{\partial x} \right)^2 \quad (13)$$

Or more appropriately written in this context as:

$$\sigma_d^2 = \frac{\sigma_{y_{n+1}}^2 + \sigma_{y_{n-1}}^2}{dx^2} + \frac{\sigma_{x_{n+1}}^2 + \sigma_{x_{n-1}}^2}{dx^2} \left( \frac{dy}{dx} \right)^2 \quad (14)$$

which for the case of three height-time data points  $r(t - \Delta t)$ ,  $r(t)$ ,  $r(t + \Delta t)$ , is simply the error propagation equation again (Eqn. ??). So the errors on the end points become:

$$\sigma_{d_0}^2 = \frac{9\sigma_{y_0}^2 + 16\sigma_{y_1}^2 + \sigma_{y_2}^2}{(x_2 - x_0)^2} + \frac{\sigma_{x_2}^2 + \sigma_{x_0}^2}{(x_2 - x_0)^2} \left( \frac{3y_0 - 4y_1 + y_2}{x_2 - x_0} \right)^2 \quad (15)$$

$$\sigma_{d_n}^2 = \frac{9\sigma_{y_n}^2 + 16\sigma_{y_{n-1}}^2 + \sigma_{y_{n-2}}^2}{(x_n - x_{n-2})^2} + \frac{\sigma_{x_{n-2}}^2 + \sigma_{x_n}^2}{(x_{n-2} - x_n)^2} \left( \frac{3y_n - 4y_{n-1} + y_{n-2}}{x_{n-2} - x_n} \right)^2 \quad (16)$$

Given three height-time data points  $r(t - \Delta t)$ ,  $r(t)$ ,  $r(t + \Delta t)$  to derive the velocity profile by Lagrangian interpolation, the resulting error propagation equation is written:

$$\sigma_v^2 = \frac{\sigma_{r(t+\Delta t)}^2 + \sigma_{r(t-\Delta t)}^2}{4\Delta t^2} + v^2 \left( \frac{\sigma_{t+\Delta t}^2 + \sigma_{t-\Delta t}^2}{4\Delta t^2} \right) \quad (17)$$

### 3. Models

#### 3.1. const. vel.

Given a model with constant velocity, sampled at regular intervals (i.e., the cadence  $\Delta t$  is constant), then, on average, the chi-squared value of the velocity scatter using forward/reverse differencing is approximately 4 times higher than the velocity scatter using centre differencing (and  $\gtrsim 16$  times higher for acceleration).

#### 3.2. const. accel.

Given a model with constant acceleration, sampled at regular intervals (i.e., the cadence  $\Delta t$  is constant), then, on average, the chi-squared value of the velocity scatter using forward/reverse differencing depends on the steepness of the velocity profile; decreasing from approximately 4 times higher (for low acceleration) to 4 times lower (for high acceleration) than the velocity scatter using centre differencing (and  $\gtrsim 16$  times higher for acceleration).

3.3. non-const. accel.

3.4. Cadence

3.5. S/N

## 4. Data

4.1. CMEs

4.2. EIT waves

## 5. Bootstrapping

5.1. Using BIB<sub>T</sub>E<sub>X</sub>

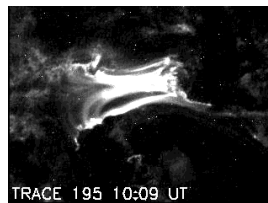
The use of BIB<sub>T</sub>E<sub>X</sub> simplifies the inclusion of references. Only the references cited and labeled in the text are included at compilation, and an error message appears if some references are missing. Any new reference will automatically be written at the correct location in the reference list after compilation. Moreover the references are stored, in any order, in a separate file (with the `.bib` extension) in the BIB<sub>T</sub>E<sub>X</sub> format, so independently of the journal format. Such a personal reference file can be re-used with any journal. The formatting of the references and their listing order are made automatically at compilation (using the information given in the `.bst` file).

The references in BIB<sub>T</sub>E<sub>X</sub> format can be downloaded from the Astrophysics Data System (ADS), then stored in `SOLA_bibliography_example.bib` (file name of the present example). The main extra work is to define a proper and easy label for each citation (a convenient one is simply first-author-name-year). Furthermore, it is better to have the journal names defined by commands (for example `\solphys`), as defined at the beginning of this `.tex` file. This provides an homogeneity in the reference list and permits flexibility when changing for journals. Some caution should be taken for some journals since ADS does not necessarily provide a uniform format for the journal names. This is the case for *J. Geophys. Res.* Moreover since *J. Geophys. Res.* has a new way to refer to an article (since 2002 it has no page number), then the ADS references need to be corrected. More generally, it is worth verifying each reference from the original publication (independently of BIB<sub>T</sub>E<sub>X</sub> use).

The full L<sup>A</sup>T<sub>E</sub>X and BIB<sub>T</sub>E<sub>X</sub> compilation is made in four steps:

- |                                 |  |
|---------------------------------|--|
| 1) <code>latex filename</code>  | (stores the labels in the <code>.aux</code> file)                |
| 2) <code>bibtex filename</code> | (loads the bibliography in the <code>.bbl</code> file)           |
| 3) <code>latex filename</code>  | (reads the <code>.bbl</code> , stores in the <code>.aux</code> ) |
| 4) <code>latex filename</code>  | (replaces all labels)  |

where `filename` is the name of your L<sup>A</sup>T<sub>E</sub>X file (for example, the present file) **without** typing its `.tex` extension. If a (?) is still present in the output (at the place of a label), it means that this label has not been properly defined. (for



**Figure 1.** Example of a simple figure in an appendix.

example,  $\text{\LaTeX}$  labels are case sensitive). Any undefined label has a warning written in the `console window` (it is better to have this window open by default, since  $\text{\LaTeX}$  warning and error messages are very useful to localize the problem).

When the references are not changed, it is unnecessary to re-run  $\text{\BibTeX}$ . When no new labels are added, running `latex` once is sufficient to refresh the  $\text{\LaTeX}$  output. So, except for the first, and the final time (safest), running  $\text{\LaTeX}$  once is sufficient in most cases to update the  $\text{\LaTeX}$  output, if the compilation files created are not erased! For example  $\text{\BibTeX}$  keeps the bibliography in the usual environment,

```
\begin{thebibliography}{} ... \end{thebibliography}
```

in the file with the `.bbl` extension.

## 5.2. Miscellaneous Other Features

Long URL's can be quite messy when broken across lines `http://gong.nso.edu/data/magmap/` as normal text, however the `url` package does a nice job of this, *e.g.* `http://gong.nso.edu/data/magmap/`.

## 6. Conclusion

We hope authors of *Solar Physics* will find this guide useful. Please send us feedback on how to improve it.

$\text{\LaTeX}$  is very convenient to write a scientific text, in particular with the use of labels for figures, tables, and references. Moreover, the labels and list of references are checked by the software against one another, and, the formatting should be effortless with  $\text{\BibTeX}$ .

## Appendix

After the `\appendix` command, the sections are referenced with capital letters. The numbering of equations, figures and labels is just the same as with classical sections.

### A. Abbreviations of some Journal Names

Journal names are abbreviated in *Solar Physics* with the IAU convention (IAU Style Book published in Transactions of the IAU XXB, 1988, pp. Si-S3. [www.iau.org](http://www.iau.org)).

**Table 1.** A simple table in an appendix.

Rot.	Date	CMEs obs.	CMEs cor.	$\alpha$ $10^{-2}\text{Mm}^{-1}$
1	02–Nov–97	16	24.1	-1.26
2	29–Nov–97	–	2.53	0.94

iau.org/Abbreviations.235.0.html). Here are a few journals with their L<sup>A</sup>T<sub>E</sub>X commands (see the beginning of this .tex file).

\aap      *Astron. Astrophys.*  
\apj      *Astrophys. J.*  
\jgr      *J. Geophys. Res.*  
\mnras    *Mon. Not. Roy. Astron. Soc.*  
\pasj      *Pub. Astron. Soc. Japan*  
\pasp      *Pub. Astron. Soc. Pac.*  
\solphys   *Solar Phys.*

**Acknowledgements**    The authors thank ... (*note the reduced point size*)

### Bibliography Included with BibT<sub>E</sub>X

With BibT<sub>E</sub>X the formatting will be done automatically for all the references cited with one of the \cite commands (Section ??). Besides the usual items, it includes the title of the article and the concluding page number.

### Bibliography included manually

The articles can be entered, formatted, and ordered by the author with the command \bibitem. ADS provides references in the *Solar Physics* format by selecting the format **SoPh format** under the menu **Select short list format**. Including the article title and the concluding page number are optional; however, we require consistency in the author's choice. That is, all of the references should have the article title, or none, and similarly for ending page numbers.

### References

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