

Investigating the Kinematics of Coronal Mass Ejections using the CORIMP Catalogue

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ABSTRACT

Subject headings: Sun: activity; Sun: corona; Sun: coronal mass ejections (CMEs)

1. Introduction

2. Numerical Differentiation Techniques (recap)

When presented with a moving object through a sequence of image frames such that it is possible to measure its position at each time step, the technique of numerical differentiation is often used to derive the velocity and acceleration of the object. In the standard 2-point approach, it should be possible to derive the time evolution of a system at time step $t + \delta t$ according to the system values at time step t . This may be applied through the technique of forward, reverse or centre differencing, resulting in an estimate of the speed of the object at a specific time step given its positional information. More commonly, a 3-point Lagrangian interpolation is applied to better approximate the kinematics of a moving object by solving for the Lagrange polynomials that best fit across three given datapoints (e.g. DERIV.PRO in IDL). Each of these schemes is based upon the Taylor series expansion of a real function $f(t)$:

$$f(t_0 + \delta t) = f(t_0) + f'(t_0)\delta t + \frac{f''(t_0)}{2!}(\delta t)^2 + \dots \quad (1)$$

but due to the approximation of an infinite series with a finite number of terms and iterations, an error must be associated with the result, based on its deviation from the true solution. Generally the

Euler method is employed, using the formula:

$$y_{n+1} = y_n + hf(t_n, y_n) \quad (2)$$

to solve the initial value problem $y' = f(t, y)$ given $y(t_0) = y_0$, where h is the stepsize such that $t_n = t_0 + nh$. The convergence of such an approximation to the actual solution is prone to two sources of error; truncation error (the difference between the true solution and the approximation) and round-off error (the limited precision of the approximation). Add to this fact that the data measurements themselves are subject to uncertainties in both the positional and temporal information, and the ability of the numerical differentiation techniques to derive kinematics becomes highly jeopardised, as shall be shown.

Given a function $x = f(u, v)$, the error propagation equation (based on the standard deviations σ of the variables) is written:

$$\sigma_x^2 = \sigma_u^2 \left(\frac{\partial x}{\partial u} \right)^2 + \sigma_v^2 \left(\frac{\partial x}{\partial v} \right)^2 + 2\sigma_{uv} \left(\frac{\partial x}{\partial u} \right) \left(\frac{\partial x}{\partial v} \right) + \dots \quad (3)$$

Specifically in the case of kinematic analyses, this is used to propagate the errors on the height-time data $r(t)$ into the velocity $v(t)$ and acceleration $a(t)$ profiles to determine the associated uncertainties. In the case of height-time data the covariance terms are zero because the quantities are uncorrelated.

In the case of relatively low sampling of the data, as in the case of coronagraph observations of CMEs, it is generally found that the simplest differentiation techniques are not applicable. The forward and/or reverse differencing techniques act to shift the kinematic profiles by one time-step, which is substantial enough to be of concern here (i.e., they derive a result at the current time-step, based on the pro-/preceeding time-step). Centre differencing employs the two neighbouring datapoints of the point under examination, and so is a better indication of the result at that time-step, but it fails at both endpoints. In any case these should not be employed when the spacing of the datapoints is unequal, i.e., when the cadence δt is not constant, and so the 3-point Lagrangian interpolation technique is used for such cases (and gives the same result as the centre-difference otherwise, but includes the endpoints). The Lagrangian interpolation polynomial is given by:

$$L(x) = \sum_{j=0}^2 y_j l_j(x) \quad (4)$$

$$\text{where } l_j(x) = \prod_{i=0, i \neq j}^2 \frac{x - x_i}{x_j - x_i} \quad (5)$$

The derivative at point x is given by $L' = \partial_x L(x)$. So for the case of height-time data being used to derive velocity (and similarly for acceleration) the associated error is given by:

$$\sigma_{v_1}^2 = \frac{\sigma_{r(t_2)}^2 + \sigma_{r(t_0)}^2}{(t_2 - t_0)^2} + v^2 \left(\frac{\sigma_{t_2}^2 + \sigma_{t_0}^2}{(t_2 - t_0)^2} \right) \quad (6)$$

with the endpoint errors derived from a weighting of the pro-/preceeding three datapoints, and therefore larger.

Although the 3-point Lagrangian is mathematically sound, its application to CME data has proved problematic due to the limitations of the data. The main drawback is that the noise level across the few, low-cadence measurements, often belies the actual trends of the data such that the numerical derivations of velocity and acceleration become untrustworthy and even misleading. As an example of this, we simulate a height-time profile of a CME that propagates with a constant acceleration of 2 m s^{-2} and initial velocity 300 km s^{-1} , and add random noise of various levels up to a maximum of $\sim 20\%$. An errorbar on each datapoint is determined by its distance from the true height-time profile, to represent the worst best-case scenario wherein all measurements lie on the

very edge of the known error. Various instances of datapoint scatters result in erroneous trends in the velocity and acceleration profiles - even with the proper error treatment ascribed by the 3-point Lagrangian interpolation technique. Two such cases are shown in Figure 2 where completely opposing acceleration trends are revealed, indicating that the nature of the scatter in the samples is not satisfactorily reflected in the derived kinematics and associated errors. The worry here is that for CMEs undergoing non-constant acceleration, the resulting trends in the derived profiles may be completely intractable via these methods. One possible assurance has been that instead of trusting the endpoints, even given their larger errorbars, they be removed. Figure 2 would then show three datapoints for CME velocity and one for acceleration, that would in the left case go somewhat toward removing the biased trend and implying a constant acceleration, while in the right case it would remain inconclusive. So qualitatively this might achieve a more accurate result and avoid assigning erroneous trends, especially to fast CMEs for which there are few datapoints.

Beyond the interest certainly in the initial acceleration from the forces that drive a CME to erupt, there is a real physical motivation to resolve the subsequent low or zero acceleration as precisely as possible. This is important for determining whether the predominant forces acting are such that the CME is, in general, undergoing continued driving (internal) forces, or positive or negative drag (external) forces, or most assuredly some interplay of both. Any changes to its acceleration that result from different phases of dominating force, and where or why this occurs, are of great interest. Yet this all remains somewhat elusive given the limitations of current CME data, especially as ? demonstrate that the errors in the acceleration values can be of the same order as the accelerations typically measured.

We investigate the output of the recently developed CORIMP catalogue with new techniques (and stringent error analyses) for deriving CME kinematics.

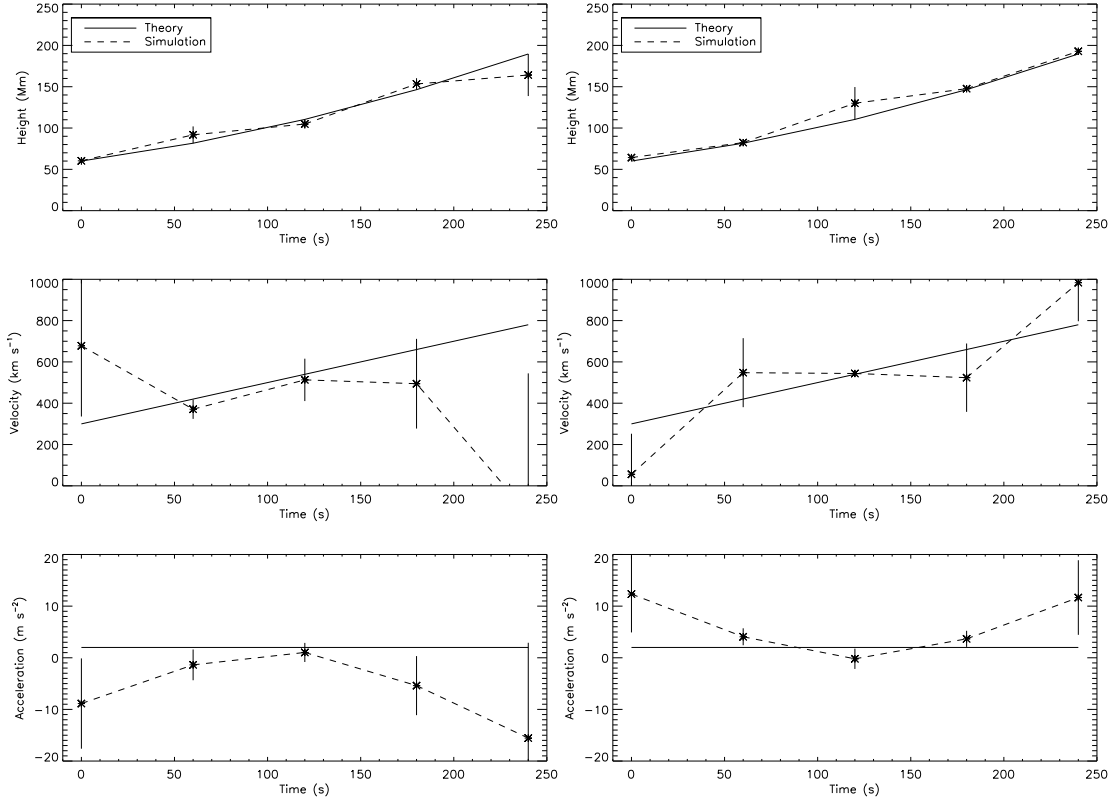


Fig. 1.— A theoretical model for a CME with constant acceleration 2 m s^{-2} and initial velocity 300 km s^{-1} , and two simulations of how the resulting profiles for a noisy sample of datapoints behave using 3-point Lagrangian interpolation.

3. Models

3.1. Testing for Constant Acceleration

3.2. Testing for Non-constant Acceleration

4. Conclusions

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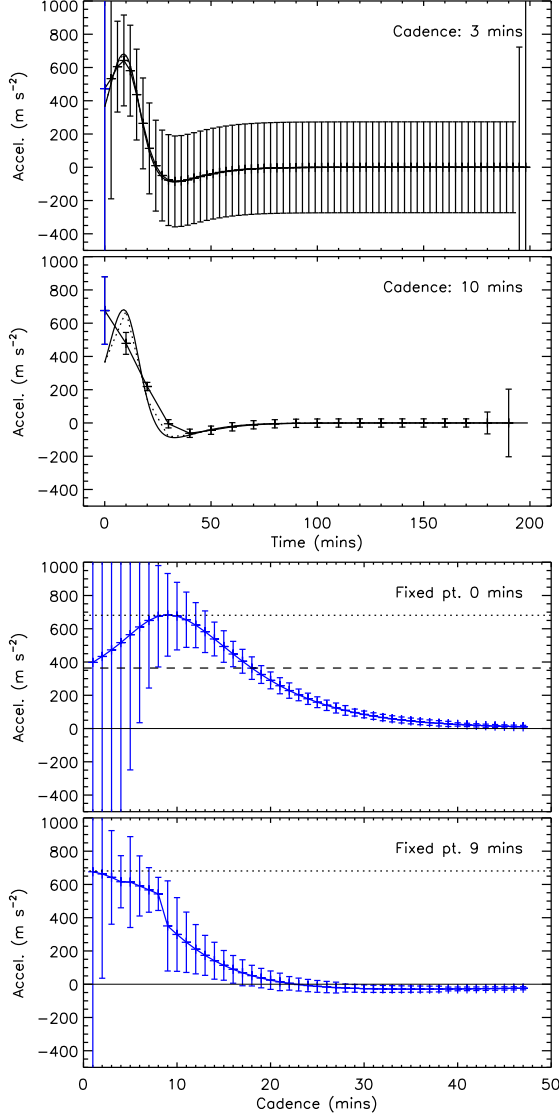


Fig. 2.— Demonstration of the effects of cadence (sampling rate) on the error propagation according to the 3-point Lagrangian interpolation. A theoretical model for a CME with non-constant acceleration peaking at 676 m s^{-2} is tested for varying cadences of 1 to 47 mins. The top two plots show the cases when the cadence is 3 mins. and 10 mins. while the bottom plot shows the derived acceleration of the first point for each cadence, and its associated errorbar.