Solar Physics
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Article preparation guidelines

Solar Physics

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Abstract The derivation of kinematic profiles for eruptive events is prominent in the field of solar physics. The details on the acceleration of coronal mass ejections (CMEs) and large-scale coronal disturbances ('EIT waves') are important for indicating the driving mechanisms at play. The techniques used for deriving the velocity and acceleration profiles of events based upon the height-time tracks .....

Keywords: CME, EIT Waves, Corona, Mathematical Techniques

1. Introduction

2. Numerical Differentiation Techniques

When presented with a moving object through a sequence of image frames such that it is possible to measure its position at each time step, the technique of numerical differentiation is often used to derive the velocity and acceleration of the object. In the standard 2-point approach, it should be possible to derive the time evolution of a system at time step $t + \Delta t$ according to the system values at time step t. This may be applied through the technique of forward, reverse or centre differencing, resulting in an estimate of the speed of the object at a specific time step given its positional information. More commonly, a 3-point Langrangian interpolation is applied to better approximate the kinematics of a moving object by solving for the Lagrange polynomials that best fit across 3 given datapoints (e.g. DERIV.PRO in IDL). Each of these schemes is based upon the Taylor series expansion of a real function $f(t)$ about the point $t = t_0$:

$$f(t) \ = \ f(t_0) + f'(t_0)(t - t_0) + \frac{f''(t_0)}{2!}(t - t_0)^2 + ... \tag{1}$$

An alternative form is obtained by letting $t - t_0 = \Delta t$ so that $t = t_0 + \Delta t$ to give:

$$f(t_0 + \Delta t) \ = \ f(t_0) + f'(t_0)\Delta t + \frac{f''(t_0)}{2!}(\Delta t)^2 + ... \tag{2}$$

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This expansion can be used to determine the numerical derivative of a function according to the choice of technique, detailed in Sections 2.1, 2.2, 2.3 and 2.4 below.

Due to the approximation of an infinite series with a finite number of terms and iterations, a truncation error must be associated with the result, based on its deviation from the true solution. In the case of polynomial (linear, quadratic, cubic, etc.) interpolation of a function $y' = f(t, y)$, the following sequence is computed:

$$y_{n+1} \ = \ y_n + hA\left(t_n, y_n, h, f\right) \tag{3}$$

and this has an associated truncation error of

$$\tau_{n+1} \ = \ \left|y\left(t_{n+1}\right) - \left(y\left(t_n\right) + hA\right)\right| \tag{4}$$

where we assume constant step size $h = t_{n+1} - t_n$ and A is the increment function which approximates the average slope $\left(y_{n+1} - y_n\right)/h$ between datapoints. It is useful to determine the absolute relative approximate error between the true value of the function and the approximation to it, or between successive iterations, until a sufficient degree of confidence is achieved:

$$\epsilon \ = \ \left|\frac{y'_{final} - y'_{initial}}{y'_{final}}\right| \times 100 \tag{5}$$

Given a function $x = f(u,\, v)$, the error propagation equation (based on the standard deviations σ of the variables) is written:

$$\sigma_x^2 \ = \ \sigma_u^2\left(\frac{\partial x}{\partial u}\right)^2 + \sigma_v^2\left(\frac{\partial x}{\partial v}\right)^2 + 2\sigma_{uv}^2\left(\frac{\partial x}{\partial u}\right)\left(\frac{\partial x}{\partial v}\right) + ... \tag{6}$$

Specifically in the case of kinematic analyses, this is used to propagate the errors on the height-time data $r(t)$ into the velocity $v(t)$ and acceleration $a(t)$ profiles to determine the associated uncertainties for each technique detailed below. In the case of height-time data the covariance terms are zero because the quantities are uncorrelated.

2.1. Forward Differencing

The forward differencing technique involves extrapolating forward from each time step t to derive the evolution of the system. Thus rewriting Equation 2 to express the distance measurement at time step $t + \Delta t$ gives:

$$r(t + \Delta t) \ = \ r(t) + r'(t)\Delta t + \frac{r''(t)}{2!}(\Delta t)^2 + ... \tag{7}$$

$$\Rightarrow \quad v(t) \ \equiv \ r'(t) \ = \ \frac{r(t + \Delta t) - r(t)}{\Delta t} + O(\Delta t) \tag{8}$$

where $O(\Delta t)$ is the truncation error term, proportional to the step size Δt between datapoints.

The forward difference technique inherently assumes that there is a straight-line gradient between points, and its application removes a point from the end of the dataset.

To propagate a one standard deviation uncertainty σ_r of the height data, and σ_t of the time data, into the velocity profile and obtain the associated uncertainty in velocity σ_v, the error propagation equation (Eqn. 6) is written:

$$\sigma_v^2 \; = \; \frac{\sigma^2_{r(t+\Delta t)} + \sigma^2_{r(t)}}{\Delta t^2} + v^2 \left(\frac{\sigma^2_{t+\Delta t} + \sigma^2_t}{\Delta t^2} \right) \tag{9}$$

It is clear that the inverse dependence on Δt^2 will be an important consideration when looking at the effects of measurement cadence for determining kinematics and their associated uncertainties. In effect, reducing the cadence increases the accuracy but decreases the precision (and vice versa) of the derived kinematics.

2.2. Reverse Differencing

The reverse difference technique works in the same manner as the forward difference but is applied at the point $t - \Delta t$ by extrapolating backwards from the point t. This results in:

$$v(t) \; \equiv \; r'(t) \; = \; \frac{r(t) - r(t - \Delta t)}{\Delta t} + O(\Delta t) \tag{10}$$

where $O(\Delta t)$ is the truncation error term, which, as with the forward difference method, is proportional to the step size Δt between datapoints.

Similar to forward differencing, the reverse difference technique inherently assumes that there is a straight-line gradient between points, and its application removes a point from the beginning of the dataset.

The error propagation equation for reverse differencing is written:

$$\sigma_v^2 \; = \; \frac{\sigma^2_{r(t)} + \sigma^2_{r(t-\Delta t)}}{\Delta t^2} + v^2 \left(\frac{\sigma^2_t + \sigma^2_{t-\Delta t}}{\Delta t^2} \right) \tag{11}$$

It may be noted that the trends in the derivatives produced by both forward and reverse differencing are identical, the only difference between them being where the resulting profile sits with respect to the time axis.

2.3. Centre Differencing

The centre difference technique uses the two neighbouring points to the point $r(t)$ under examination, i.e., $r(t - \Delta t)$ and $r(t + \Delta t)$, according to the equation:

$$v \; \equiv \; r'(t) \; = \; \frac{r(t + \Delta t) - r(t - \Delta t)}{2\Delta t} + O(\Delta t)^2 \tag{12}$$

The truncation error in this case is proportional to the square of the step size Δt between datapoints. The centre-difference method effectively smoothes the data set while differentiating it by using the two points either side of the point under examination. It is only applicable when the spacing between datapoints is equal, i.e., when Δt remains constant.

The error propagation equation for centre differencing is written:

$$\sigma_v^2 \;=\; \frac{\sigma^2_{r(t+\Delta t)} + \sigma^2_{r(t-\Delta t)}}{4\Delta t^2} + v^2 \left(\frac{\sigma^2_{t+\Delta t} + \sigma^2_{t-\Delta t}}{4\Delta t^2} \right) \tag{13}$$

2.4. 3-Point Lagrangian Interpolation

3-point Lagrangian interpolation is used in order to determine the first and second derivatives corresponding to the velocity and acceleration of an object in a more robust manner than simple forward, reverse or centre difference techniques, and specifically when the spacing between datapoints in not equal, i.e., when Δt is non-constant. Considering three data points, (x_0, y_0), (x_1, y_1), (x_2, y_2), the Lagrangian interpolation polynomial is given by:

$$L(x) \;=\; \sum_{j=0}^{2} y_j l_j(x) \quad \text{where} \quad l_j(x) \;=\; \prod_{i=0, i\neq j}^{2} \frac{x - x_i}{x_j - x_i} \tag{14}$$

The derivative at point x is given by $L' = \partial_x L(x)$. The error propagation equation is used to determine the errors on the resulting derivative points in $L' \equiv f(L(x), x)$:

$$\sigma^2_{L'} \;=\; \sigma_L^2 \left(\frac{\partial L'}{\partial L} \right)^2 + \sigma_x^2 \left(\frac{\partial L'}{\partial x} \right)^2 + ... \tag{15}$$

$$= \; \frac{\sigma_L^2}{\partial x^2} + \frac{\sigma_x^2}{\partial x^2} \left(\frac{\partial L}{\partial x} \right)^2 \tag{16}$$

Or more appropriately written in this context as:

$$\sigma_d^2 \;=\; \frac{\sigma^2_{y_{n+1}} + \sigma^2_{y_{n-1}}}{dx^2} + \frac{\sigma^2_{x_{n+1}} + \sigma^2_{x_{n-1}}}{dx^2} \left(\frac{dy}{dx} \right)^2 \tag{17}$$

such that the errors on the end points become:

$$\sigma^2_{d_0} \;=\; \frac{9\sigma^2_{y_0} + 16\sigma^2_{y_1} + \sigma^2_{y_2}}{(x_2 - x_0)^2} + \frac{\sigma^2_{x_2} + \sigma^2_{x_0}}{(x_2 - x_0)^2} \left(\frac{3y_0 - 4y_1 + y_2}{x_2 - x_0} \right)^2 \tag{18}$$

$$\sigma^2_{d_n} \;=\; \frac{9\sigma^2_{y_n} + 16\sigma^2_{y_{n-1}} + \sigma^2_{y_{n-2}}}{(x_n - x_{n-2})^2} + \frac{\sigma^2_{x_{n-2}} + \sigma^2_{x_n}}{(x_{n-2} - x_n)^2} \left(\frac{3y_n - 4y_{n-1} + y_{n-2}}{x_{n-2} - x_n} \right)^2 \tag{19}$$

Given three consecutive height-time data points $r(t_1)$, $r(t_2)$, $r(t_3)$, if they are equally spaced such that $\Delta t = |t_1 - t_2| = |t_2 - t_3|$ the resulting error propagation equation when deriving the velocity at t_2 by 3-point Lagrangian interpolation is the same as for centre differencing (Eqn. 13). Otherwise, for unevenly spaced data points it is written:

$$\sigma_v^2 \;=\; \frac{\sigma^2_{r(t_3)} + \sigma^2_{r(t_1)}}{(t_3 - t_1)^2} + v^2 \left(\frac{\sigma^2_{t_3} + \sigma^2_{t_1}}{(t_3 - t_1)^2} \right) \tag{20}$$

3. Models

3.1. Constant Acceleration Model

Given a model with constant acceleration, sampled at regular intervals (i.e., the cadence Δt is constant), then, on average, the chi-squared value of the velocity scatter using forward (and similarly reverse) differencing is larger than that of the centre difference technique due to the larger degree of scatter involved. If a model constant acceleration of 50 m s^{-2} is specified, with initial velocity 400 km s^{-1}, starting from an initial height of 100 Mm, then this scatter may be investigated by adding random Gaussian noise of varying magnitudes to the height-time profile. Figure 1 illustrates the effect for a 1% noise level on the height-time data, (at 50 second sampling cadence). Fitting a constant acceleration model to the resulting profiles of the forward and centre differencing schemes produces values very close to the model acceleration of 50 m s^{-2}, and indeed repeating this for many different random noise samplings (e.g. 10,000 iterations) produces a distribution of best-fits that average $\sim$50 m s^{-2}. Changing the noise affects the scatter, as does changing the sampling cadence (see Figures 2 and 3), which can affect individual fits since they are less constrained, but, again, iterating many times produces a distribution of fits that average $\sim$50 m s^{-2}.

3.2. Testing for Non-constant Acceleration

We investigate the scatter introduced by the differencing schemes for different magnitudes of error or sampling cadence on a non-constant acceleration model. Given a model of the form $h = h_0 + v_0 t + 0.5 a_0 t^{5/2}$ we observe that a reduced sampling cadence gives a more precise but less accurate approximation of the model acceleration, especially if the noise is increased, as seen when comparing Figures 4, 5 and 6. Indeed, reducing the sampling cadence to intervals of 1 s dramatically increases the derived uncertainty to propagate through the kinematics, as shown in Fig. 7.

4. Cadence

When looking at the kinematics of an event, too low a cadence makes it impossible to accurately determine possible acceleration profiles. Given a model acceleration profile, the effects of the cadence may be tested for how each numerical differencing scheme fares in determining the acceleration at a given time step, assuming no uncertainty in the measurements. For example, given the non-constant acceleration model of Figs. 4 and 5, the effects of different cadences are investigated with regards to the determined acceleration of each scheme; forward, centre or lagrangian. It shows that for a simple non-constant acceleration profile, the centre-differencing and 3-point Lagrangian interpolation are consistently good at determining the acceleration for varying cadence windows. The important issue with cadence is how the true profile may not be apparent if it is insufficiently sampled.

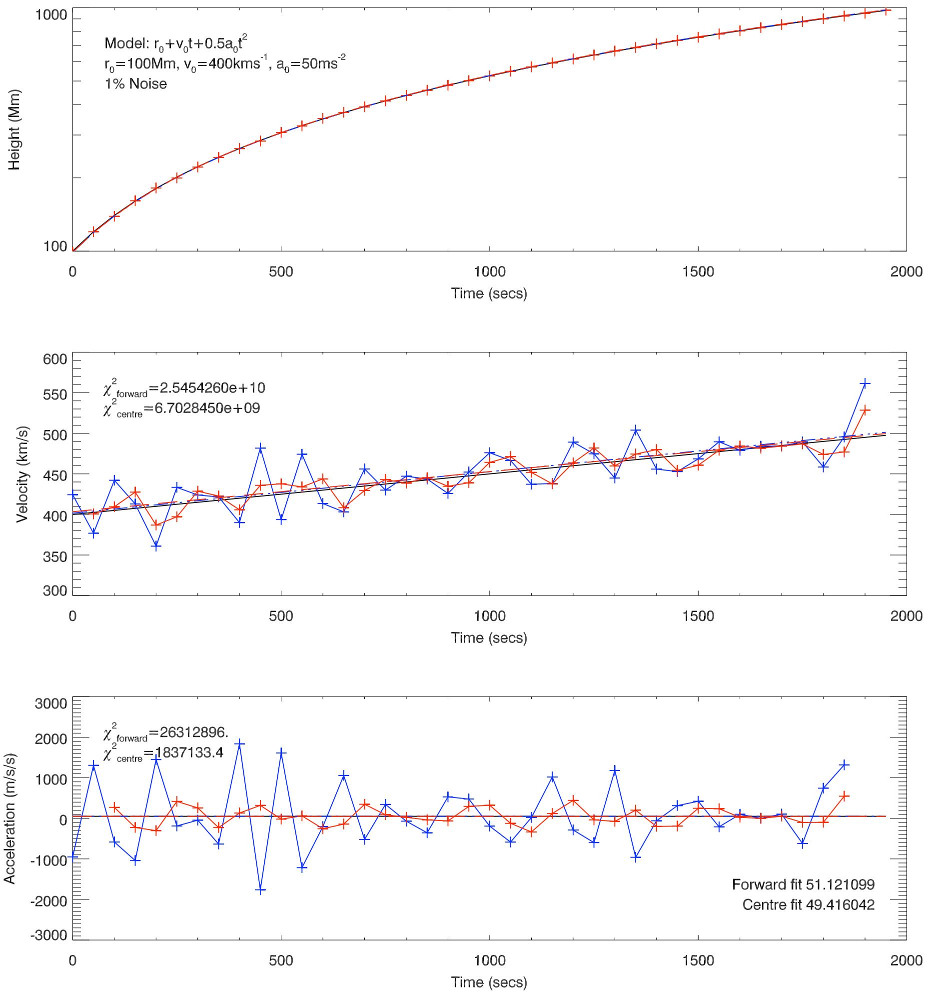

Figure 1. Constant acceleration model with 1% noise added, at 50 s sampling cadence, and the forward and centre differencing techniques applied. Black = Model / Blue = Forward / Red = Centre

5. Data

5.1. CMEs

5.2. EIT waves

6. Bootstrapping

6.1. Using BibTeX

The use of BibTeX simplifies the inclusion of references. Only the references cited and labeled in the text are included at compilation, and an error message

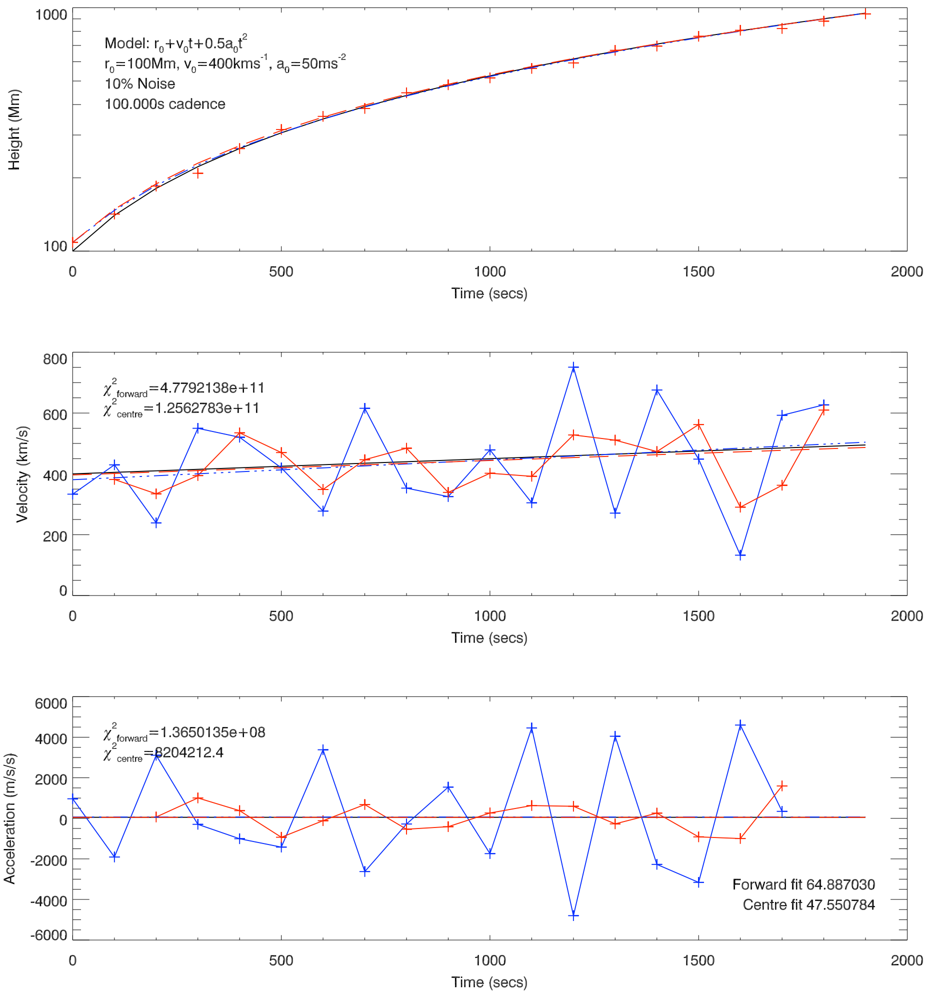

Figure 2. Constant acceleration model with 10% noise added, at 100 s sampling cadence, and the forward and centre differencing techniques applied. Black = Model / Blue = Forward / Red = Centre

appears if some references are missing. Any new reference will automatically be written at the correct location in the reference list after compilation. Moreover the references are stored, in any order, in a separate file (with the `.bib` extension) in the BIBTEX format, so independently of the journal format. Such a personal reference file can be re-used with any journal. The formatting of the references and their listing order are made automatically at compilation (using the information given in the `.bst` file).

The references in BIBTEX format can be downloaded from the Astrophysics Data System (ADS), then stored in `SOLA_bibliography_example.bib` (file name of the present example). The main extra work is to define a proper and easy label for each citation (a convenient one is simply first-author-name-year). Further-

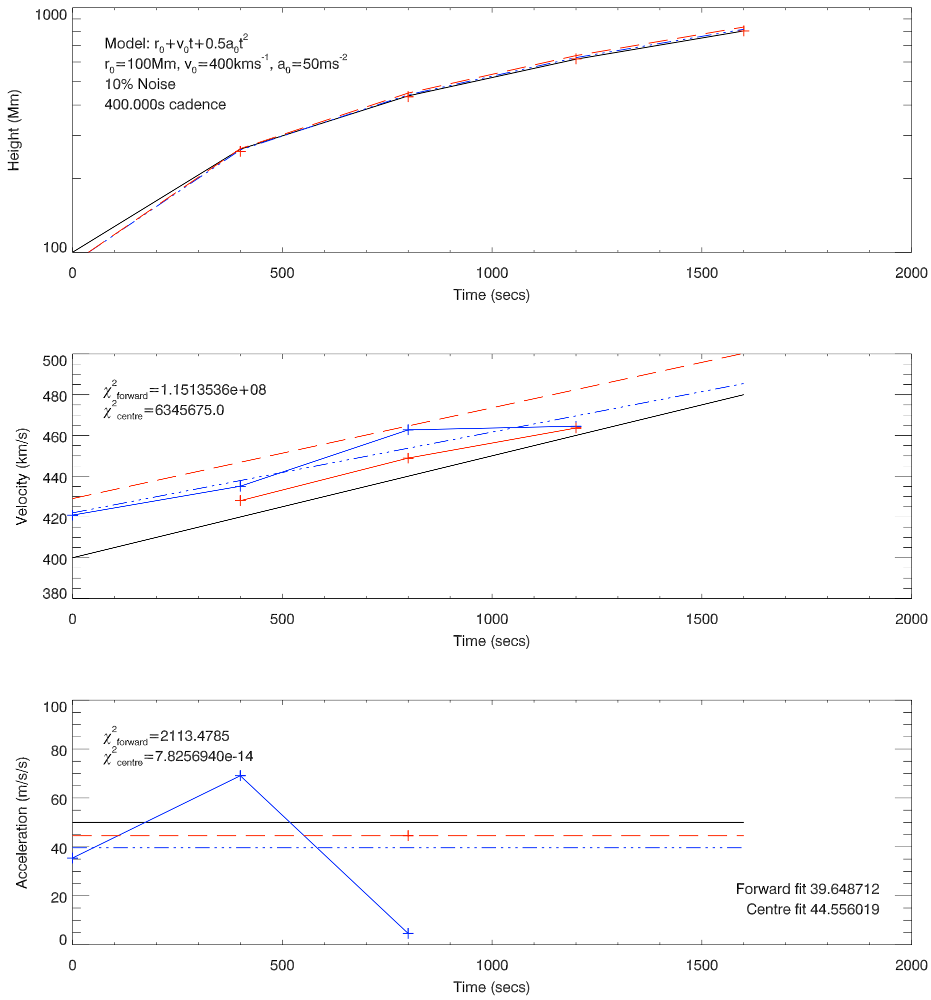

Figure 3. Constant acceleration model with 10% noise added, at 400 s sampling cadence, and the forward and centre differencing techniques applied. Black = Model / Blue = Forward / Red = Centre

more, it is better to have the journal names defined by commands (for example `\solphys`), as defined at the beginning of this `.tex` file. This provides an homogeneity in the reference list and permits flexibility when changing for journals. Some caution should be taken for some journals since ADS does not necessarily provide a uniform format for the journal names. This is the case for *J. Geophys. Res.* Moreover since *J. Geophys. Res.* has a new way to refer to an article (since 2002 it has no page number), then the ADS references need to be corrected. More generally, it is worth verifying each reference from the original publication (independently of BibTeX use).

The full LaTeX and BibTeX compilation is made in four steps:

1) `latex filename` (stores the labels in the `.aux` file)

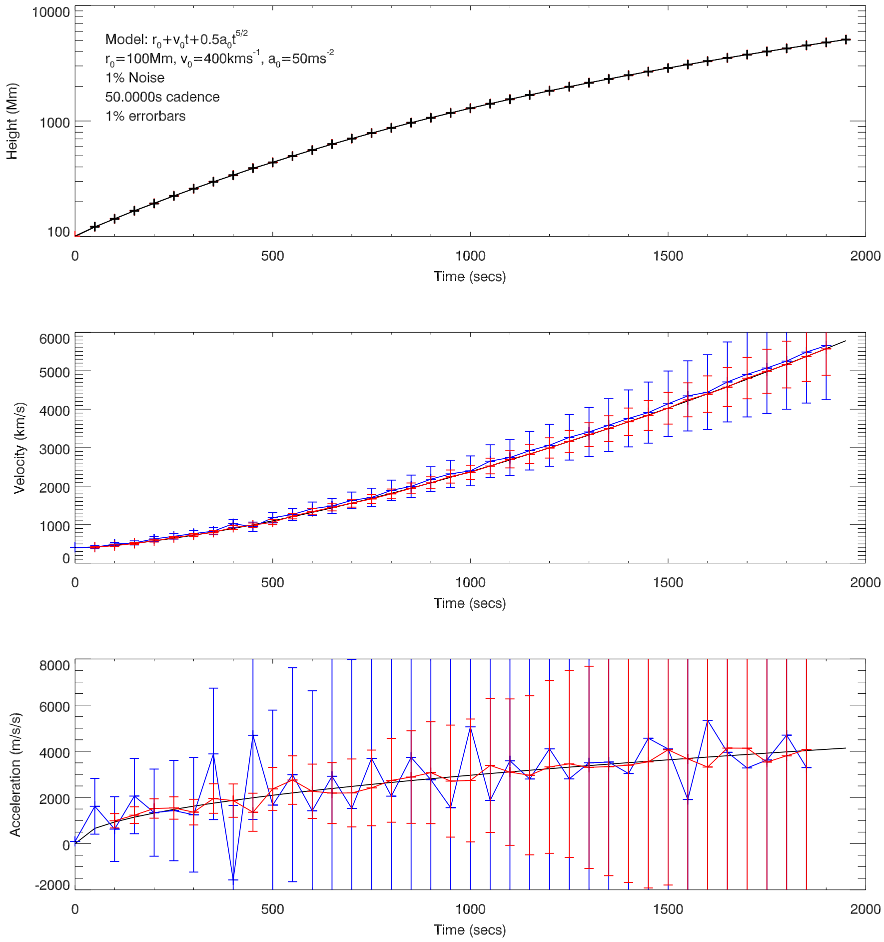

Figure 4. Non-constant acceleration model with 1% noise added, at 50 s sampling cadence, with errorbars at 1% of the data, and the forward and centre differencing techniques applied. Black = Model / Blue = Forward / Red = Centre

2) `bibtex filename`	(loads the bibliography in the `.bbl` file)
3) `latex filename`	(reads the .bbl, stores in the `.aux`)
4) `latex filename`	(replaces all labels)

where `filename` is the name of your LaTeX file (for example, the present file) **without** typing its `.tex` extension. If a `(?)` is still present in the output (at the place of a label), it means that this label has not been properly defined. (for example, LaTeX labels are case sensitive). Any undefined label has a warning written in the `console window` (it is better to have this window open by default, since LaTeX warning and error messages are very useful to localize the problem).

When the references are not changed, it is unnecessary to re-run BibTeX. When no new labels are added, running latex once is sufficient to refresh the

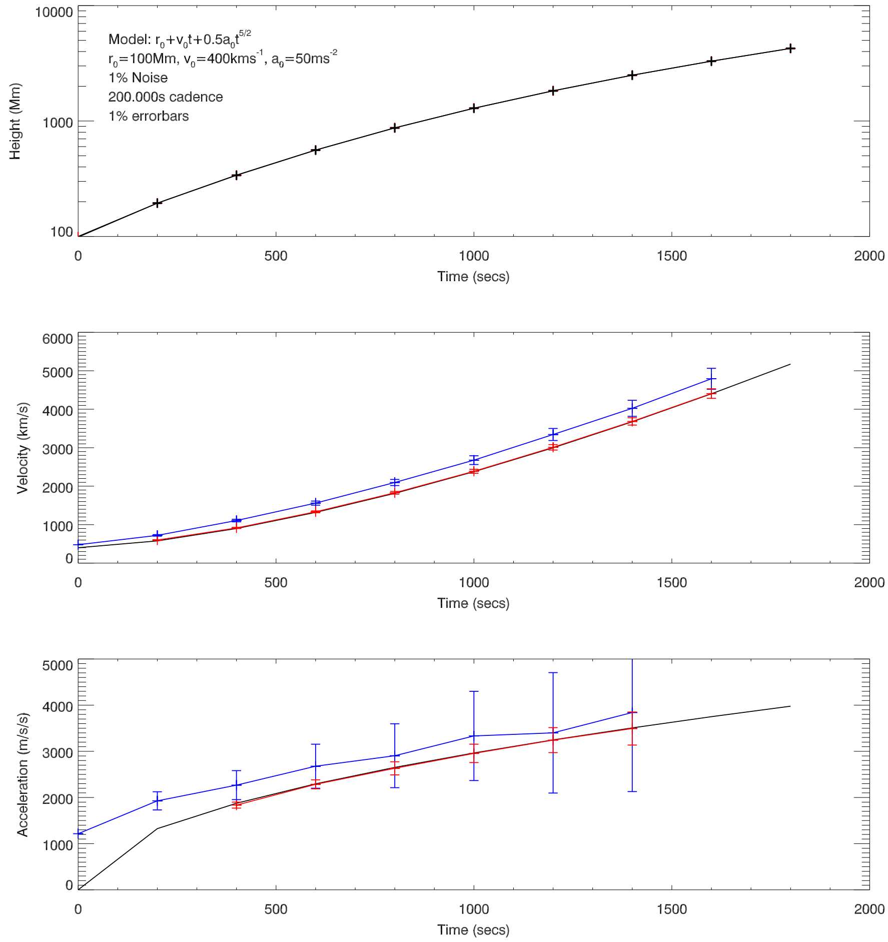

Figure 5. Non-constant acceleration model with 1% noise added, at 200 s sampling cadence, with errorbars at 1% of the data, and the forward and centre differencing techniques applied. Black = Model / Blue = Forward / Red = Centre

LaTeX output. So, except for the first, and the final time (safest), running LaTeX once is sufficient in most cases to update the LaTeX output, if the compilation files created are not erased! For example BibTeX keeps the bibliography in the usual environment,
`\begin{thebibliography}{} ... \end{thebibliography}`
in the file with the `.bbl` extension.

6.2. Miscellaneous Other Features

Long URL's can be quite messy when broken across lines `http://gong.nso.edu/data/magmap/` as normal text, however the `url` package does a nice job of this, *e.g.* http://gong.nso.edu/data/magmap/.

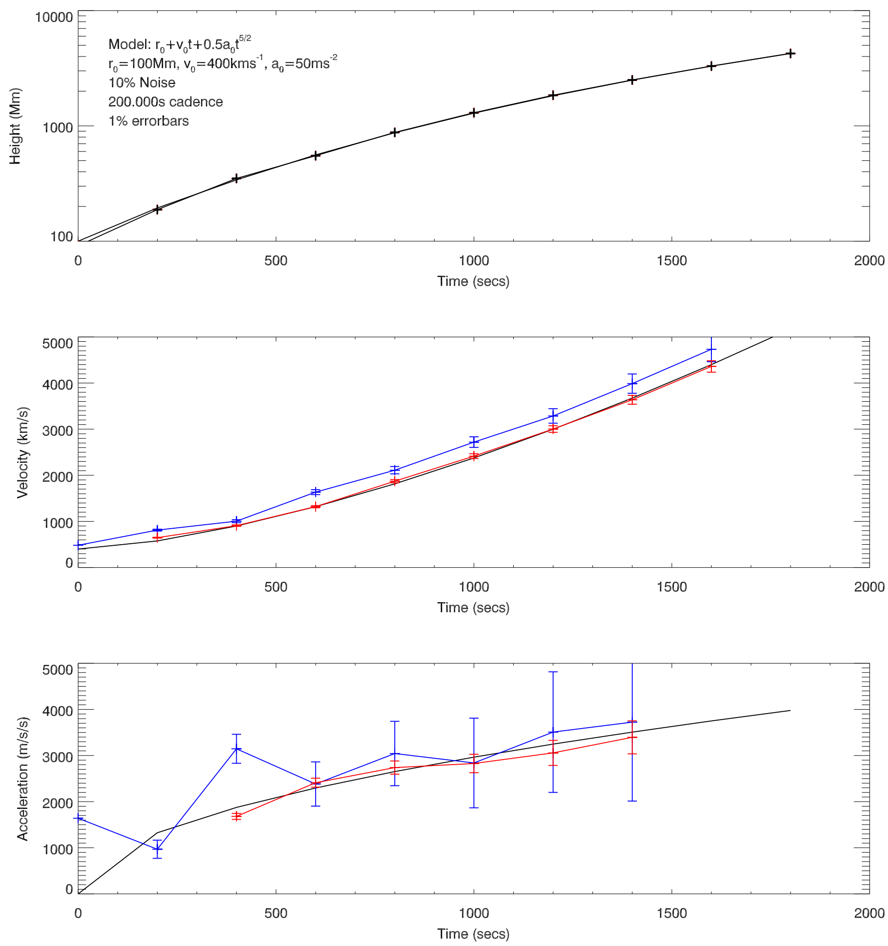

Figure 6. Non-constant acceleration model with 10% noise added, at 200 s sampling cadence, with errorbars at 1% of the data, and the forward and centre differencing techniques applied. Black = Model / Blue = Forward / Red = Centre

7. Conclusion

We hope authors of *Solar Physics* will find this guide useful. Please send us feedback on how to improve it.

LaTeX is very convenient to write a scientific text, in particular with the use of labels for figures, tables, and references. Moreover, the labels and list of references are checked by the software against one another, and, the formatting should be effortless with BibTeX.

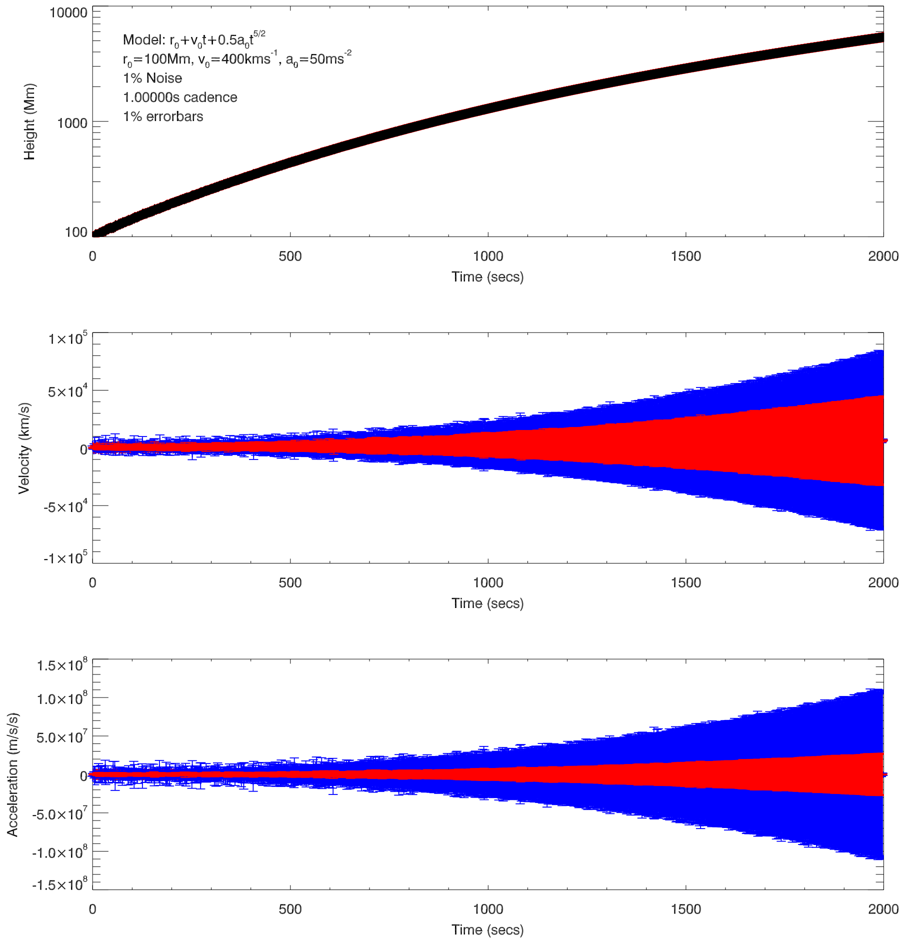

Figure 7. Non-constant acceleration model with 1% noise added, at 1 s sampling cadence, with errorbars at 1% of the data, and the forward and centre differencing techniques applied. Black = Model / Blue = Forward / Red = Centre

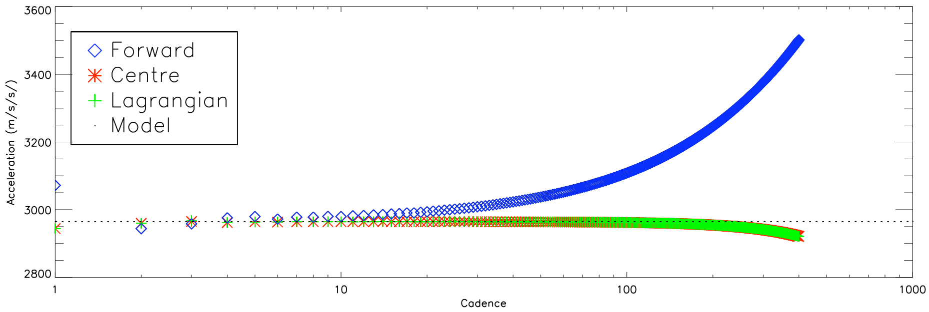

Figure 8. The acceleration determined at time step 1000 s by the numercal differencing schemes; forward, centre and lagrangian, as compared to the model value, for different cadences ranging from 1 s up to 400 s.

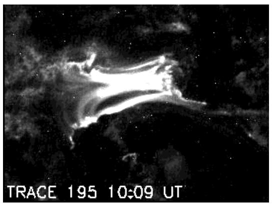

Figure 9. Example of a simple figure in an appendix.

Table 1. A simple table in an appendix.

Rot.	Date	CMEs obs.	CMEs cor.	α $10^{-2}\mathrm{Mm}^{-1}$
1	02–Nov–97	16	24.1	-1.26
2	29–Nov–97	–	2.53	0.94

Appendix

After the `\appendix` command, the sections are referenced with capital letters. The numbering of equations, figures and labels is is just the same as with classical sections.

A. Abbreviations of some Journal Names

Journal names are abbreviated in *Solar Physics* with the IAU convention (IAU Style Book published in Transactions of the IAU XXB, 1988, pp. Si-S3. www.iau.org/Abbreviations.235.0.html). Here are a few journals with their LaTeX commands (see the beginning of this `.tex` file).

`\aap`	*Astron. Astrophys.*
`\apj`	*Astrophys. J.*
`\jgr`	*J. Geophys. Res.*
`\mnras`	*Mon. Not. Roy. Astron. Soc.*
`\pasj`	*Pub. Astron. Soc. Japan*
`\pasp`	*Pub. Astron. Soc. Pac.*
`\solphys`	*Solar Phys.*

Acknowledgements The authors thank ... (*note the reduced point size*)

Bibliography Included with BibTeX

With BibTeX the formatting will be done automatically for all the references cited with one of the `\cite` commands (Section **??**). Besides the usual items, it includes the title of the article and the concluding page number.

Bibliography included manually

The articles can be entered, formatted, and ordered by the author with the command `\bibitem`. ADS provides references in the *Solar Physics* format by selecting the format `SoPh format` under the menu `Select short list format`. Including the article title and the concluding page number are optional; however, we require consistency in the author's choice. That is, all of the references should have the article title, or none, and similarly for ending page numbers.

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