Chapter 1

Introduction

1.1 Motivation

In automated fruit grading, the size of fruit has to be determined in realtime, as they travel along a conveyor. Depending on their size, the fruit are dropped at different locations, resulting in automated sorting. Such a fruit grading system is shown in figure 1.1.

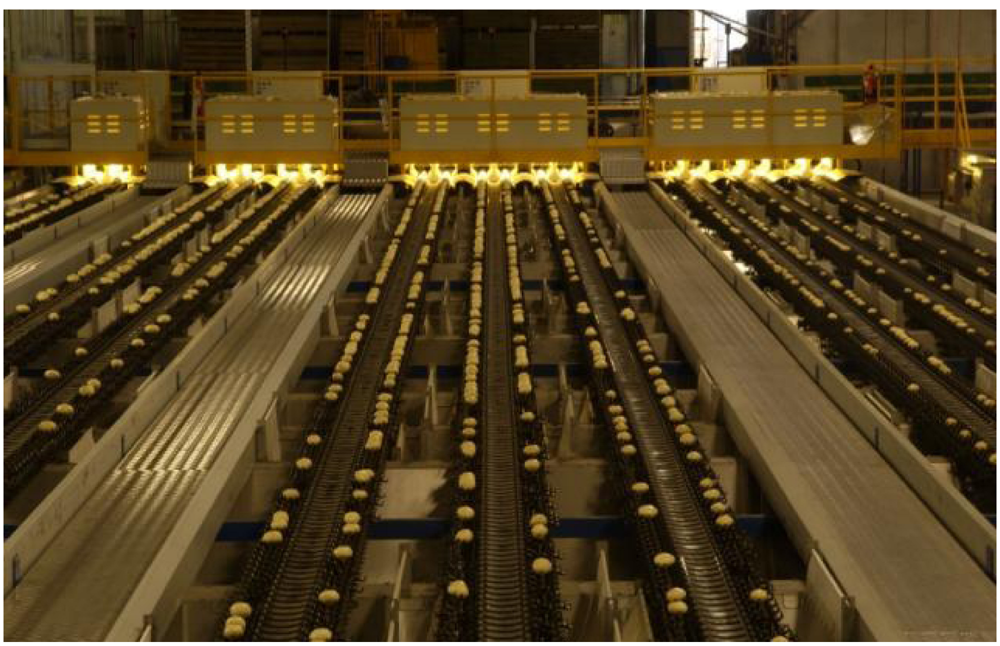

Figure 1.1: A Machine used for Automated Grading of Fruit

Two cameras are situated on either side of the conveyor, which image the fruit simultaneously as they pass by. Images thus obtained are the input for determining the size of the fruit for the purpose of sorting.

There are two ways to solve the problem of size determination: by using 2D image data directly, or by reconstructing the 3D shape from the images. The former is not reliable, as different views of the same fruit give different projections, while the latter is more accurate but is slower. Further, reconstruction of the actual shape requires several views or texture information to be used in methods such as shape from shading, or stereo matching. However, only two views are available in the target application, and fruit usually possess smooth surfaces with uniform colouring. This renders such methods difficult, if not impossible to be used in the problem under consideration.

Therefore, a method of reconstruction which uses the only reliable source of information available: the silhouettes of the images, has to be employed. Modelling of the fruit as quadric surfaces is one such solution that overcomes the problems associated with 2D size detection and actual 3D reconstruction.

The basic theory behind multiple view quadric reconstruction from silhouettes is set forth in Cross [20]. There, it is proven that two silhouettes are not sufficient to fully reconstruct a proper quadric, and can only be done so up to a one-parameter ambiguity. To remove this ambiguity, the use of a point on the surface or a tangent plane is suggested in Cross [20]. However, in the application under consideration, obtaining a point is not reliable due to the uniform texture of the fruit, and finding a tangent plane is not practical as they do not rest on a planar surface. Therefore, the final constraint to achieve full reconstruction has to take a different form: a known surface tangent to the quadric. The rollers that the fruit rest on satisfy this requirement and are used to provide the remaining constraint.

As no method is available, to the author's knowledge, for the solution of two view reconstruction with tangency information, the main objective of the thesis is to address this issue. Further, since the proposed solution is aimed at a realtime application, a practical objective is to achieve higher speed without compromising accuracy. In view of these requirements, not only the reconstruction, but also issues that arise in practical situations are investigated to provide a complete solution. The reconstruction process is thus divided into sub-problems, and efficient algorithms are introduced at each step to ensure the realization of the above objectives.

1.2 Related Work

As the problem addressed here is quadric reconstruction given a tangent surface, the first sub-problem addressed is determining tangency between two quadrics. It is related to the issue of obtaining the intersection of quadrics, which is therefore investigated as well. The same problem in 2D (the intersection of conics) is also explored.

The calculation of the intersection of two quadrics require the solution of two quadratic equations in three variables. To avoid the complexity of their direct solution, the *pencil method* can be used as first introduced by Levin [53]. There, the pencil formed by the two quadrics is searched to obtain a ruled quadric, which is then parameterized and used in the calculation of the intersection curves. The use of a ruled quadric simplifies the calculation, and therefore, has been the basis of many methods such as Sarraga [76], Farouki *et al.* [27], Wang *et al.* [89], and Dupont *et al.* [23, 24, 25]. However, as these methods are not primarily concerned with determining tangency, it has to be obtained from the properties of the intersection curves, or from a separate classification based on Segre characteristics (see appendix B). It is sought in the thesis to overcome this problem and introduce a method of determining tangency as a precursor to the calculation of intersection curves. Similar focus is also given to the 2D version of the same problem.

As conics are the projections of quadrics (to be reconstructed), the next sub problem considered here is the fitting of conics to images of the object to be reconstructed. This is a well researched field of study, and therefore, many methods have been developed to deal with it. The most popular of these can be classified into two: clustering techniques [7, 43, 45] and least squares fitting [4, 5, 28, 32]. As clustering techniques are computationally intensive, least squares methods have gained more attention in recent times. A broad categorization of the latter results in algebraic and orthogonal distance techniques, based on the distance measure they minimize. Algebraic methods are simpler and require less computation, but are less accurate than orthogonal fitting methods [5].

The existing orthogonal distance methods are either conic specific (Gander *et al.* [31], Späth *et al.* [85]) or iterative in the calculation of orthogonal distances (Ahn *et al.* [5]). Therefore, a compromise has to be reached as to the speed and type independence of the algorithms. Overcoming this weakness of the existing methods also gains focus in the thesis.

However, for realtime applications, the speed achieved by orthogonal distance methods is not sufficient. Algebraic methods such as Bookstein [12], Rosin [73], and Fitzgibbon *et al.* [29] are more suitable in such instances. In an effort to further reduce the time, and to take advantage of application specific details (such as ability of simple segmentation, and elliptic nature of the images), another conic fitting method is also devised.

A practical aspect of the overall problem of reconstruction lies in the non-adherence of the fitted conics to the geometry of the two-camera system (or epipolar geometry [39]). To proceed with the next step, these conics have to be adjusted so that they satisfy epipolar geometry. Cross [20] introduces a method of adjustment that involves a nested non-linear optimization. However, this method is not suitable for realtime applications due to speed limitations and dependency on initializations. Therefore, removing these issues from the conic adjustment is the next objective.

The final stage of the process is the actual reconstruction of quadric surfaces. The specific problem addressed here requires the use of silhouettes from two calibrated cameras. Karl *et al.* [51], Ma and Li [57], Kahl and Heyden [48], and Cross [20] all discuss methods of reconstructing quadrics from silhouettes, while Cross [20] formally introduce the constraints that tie a quadric to its projections.

As discussed in Cross [20], using two views, a quadric can only be reconstructed up to a one-parameter ambiguity. An additional constraint should be used to fully reconstruct it, and in the particular problem addressed here, this takes the form of a surface tangent to the quadric to be reconstructed. To the author's knowledge, there are no methods in literature that deal with this specific problem, and therefore, the final objective of the thesis is to devise solutions for it which are suitable for realtime applications.

1.3 Contributions

To improve the speed and accuracy, novel algorithms are introduced at each stage of the reconstruction process. First, the issue of finding the relationship (intersection, tangency, or disjointedness) of conic/quadrics is considered, and the contribution there lies in two algorithms that use the pencil method for conics/quadrics. The first is for conics, and is similar to the method of calculating intersections given in Miller [64]. However, a new theoretical perspective to the problem is given here, as well as a complete algorithm which takes into consideration arbitrary conics of any type. As a preliminary step to calculating intersections, the type of relationship of the conics is also obtained. The novel algorithm thus discussed is used to calculate orthogonal distances in the general conic fitting method introduced in Wijewickrema *et al.* [96].

This method is then extended to 3D, to calculate the intersection curves of quadrics. As opposed to other methods that use properties of quadric pencils, such as Levin [53] which search for ruled quadrics, the algorithm introduced here selects any degenerate quadric from the pencil. As degenerate quadrics are a subset of ruled quadrics, they also can be parameterized easily. To achieve this purpose, novel parameterizations are introduced for cones and cylinders. Finding the relationship of quadrics is also reduced to a 2D problem. This algorithm is later used in the ellipsoid reconstruction algorithm, and resulted in the publication of Wijewickrema *et al.* [94].

Next, two algorithms are introduced for the purpose of conic fitting. The first of these is an orthogonal distance least squares fitting algorithm for general conics. This novel algorithm simultaneously removes the problems of nested iteration, and type specific calculations. This is achieved by introducing a direct way of calculating the orthogonal distance for a general conic, and resulted in Wijewickrema *et al.* [96].

Although the above mentioned method is faster than the nested iterative schemes of orthogonal distance fitting, it is still not sufficiently fast to be used in applications such as automated fruit grading. Taking into consideration that only ellipses need to be fitted in this application, an ellipse specific method based on principal component analysis [42] is also introduced. The resulting publications of this work are Wijewickrema and Papliński [92, 93].

The novel conic correction algorithm introduced next is faster, similarly accurate, and more robust than the nested non-linear optimization of Cross [20]. It is a direct method, and only uses information available from the silhouettes of two views, and known camera matrices, to deduce the geometry of the two-camera system. This algorithm was published as part of the work discussed in Wijewickrema *et al.* [97].

The main problem that this thesis focuses on: quadric reconstruction, is the next stage of the process. Three novel algorithms are introduced to achieve this purpose. The first of these deals with the derivation of a unique quadric, given two views and a tangent surface. Although theoretically robust, this method is sensitive to slight deviations from the ideal. Hence, a more practical method is required for real applications such as fruit grading. To this end, a novel ellipsoid specific algorithm is also introduced, which resulted in the publication of Wijewickrema *et al.* [97].

The third and final contribution is the reconstruction of spheres. The novelty lies in the fact that the proposed method only requires the silhouettes of two views to fully reconstruct the sphere in primal space. This work was published in Wijewickrema *et al.* [98], and was used in the texture extraction algorithm of Wijewickrema *et al.* [95].

A key feature of the contributions is that they aim at achieving higher speed by focusing on simplicity of computation, while ensuring that accuracy is preserved.

1.4 Thesis Outline

The rest of the thesis is organized as follows. First, chapter 2 discusses general concepts that are used throughout the thesis. Since the solution of the particular problem considered here spans the range of several research areas, only the general theoretical concepts used, are introduced there. More comprehensive literature surveys are given in each of the content chapters that follow.

Chapter 3 derives two algorithms from the properties of conic/quadric pencils, to obtain the relationship between two arbitrary conics/quadrics in space. These algorithms also deal with the calculation of intersection points/curves in instances where they exist. The contributions of this chapter are mainly theoretical, but experimental proof is also provided, to illustrate their validity.

Next, in chapter 4, fitting of conics to a set of points is discussed. This chapter is divided into two parts, the first of which deals with a general conic fitting algorithm, while the second introduces an ellipse specific method. Experiments are also carried out, to test their performance with respect to well known methods of their kind.

Chapter 5 addresses the purely practical issue of adjusting the conics thus fitted, to adhere to epipolar geometry. Here too, the method is derived from mathematical concepts, but experiments are also done to observe its practical performance.

The final content chapter of the thesis is chapter 6, which discusses the actual reconstruction of quadrics. It is divided into three main sections, the first of which introduces a method of general quadric reconstruction. The second section provides a more practical method for the special case of ellipsoids. The third is for reconstructing spheres, where the number of constraints is less than that required for other proper quadrics. Each section includes theoretical, as well as experimental validations.

Finally the thesis concludes with a discussion of the work presented, and possible paths for future research.