# Bernoulli equations

Sometimes a first-order differential equation $y' = f(t, y)$ can be transformed into a linear equation by an appropriate substitution, One important example of this phenomenon is the class of *Bernoulli equations*:

$$y' + py = qy^n. \tag{1}$$

If $n \neq 0, 1$ then the substitution $v = y^{1-n}$ transforms this into a linear equation in $v$.

Let's work thru an example:

$$y' + (t-2)y = \sin(t)y^3 \tag{2}$$

First we divide both sides of the equation by $y^3$:

$$y^{-3}y' + (t-2)y^{-2} = \sin(t). \tag{3}$$

If we set $v = y^{-2}$ then $v' = -2y^{-3}y'$. Hence

$$-\frac{1}{2}v' + (t-2)v = \sin(t). \tag{4}$$

We can now solve this by Leibniz' method. (Finish!)

## Further reading

Bernoulli equations are covered in exercises 27–31, page 77.

## Reading quiz

1. What is a Bernoulli equation?
2. Outline the method for solving a Bernoulli equation.
3. True or false: Every Bernoulli equation is linear. Explain!
4. True or false: Every Bernoulli equation is separable. Explain!
5. True or false: Every Bernoulli equation is homogeneous. Explain!

## Extra credit 2: due Monday, 13 February

For each of the following, draw a slope field, solve the equation, and then graph several integral curves on your slope field. Summarize the behavior of the solutions, paying particular attention to the domain of validity and the asymptotic behavior. Is the domain of validity the same for each integral curve?

1. $y' = \sqrt{t}y - t\sqrt{y}$.
2. $y' = ry - ky^2$, where $r$, and $k$ are positive constants.
3. $y' = \epsilon y - \sigma y^3$, where $\epsilon$, and $\sigma$ are positive constants.