ELSEVIER

BioSystems 45 (1998) 1–10

BioSystems

A model for the influence of the greenhouse effect on insect and microorganism geographical distribution and population dynamics

Ioannis Karafyllidis *

Department of Electrical and Computer Engineering, Democritus University of Thrace, GR-671 00 Xanthi, Greece

Received 31 March 1997; accepted 16 June 1997

Abstract

A model for the influence of the greenhouse effect on insect and microorganism geographical distribution and population dynamics using cellular automata is presented. Based on this model, an algorithm has been developed and used to determine the geographical distribution and population dynamics of a hypothetical species in an scenario of global warming. The species' initial population distribution is assumed to be Gaussian. After the initiation of global warming, the population moves and after a few decades the population distribution is no longer Gaussian. Larger populations are found in the direction of population movement. © 1998 Elsevier Science Ireland Ltd.

Keywords: Greenhouse effect; Global warming; Geographical distribution; Population dynamics; Cellular automata; Modelling

1. Introduction

Most of the solar energy reaching the earth is visible electromagnetic radiation, to which the earth's atmosphere is almost transparent. A large part of the solar energy absorbed by the earth's surface is radiated back into space. This energy re-emitted by the earth is mainly infrared radiation which is absorbed effectively in the lower level of the atmosphere, trapping much of the heat close to earth's surface. This phenomenon is called the greenhouse effect because the atmosphere, like the glass of a greenhouse, transmits sunlight while trapping heat inside (Cunningham and Saigo, 1995). It is possible that increasing atmospheric carbon dioxide (CO_2) due to human activities will cause a global warming that could cause major climatic changes. About 8.5 billion metric tonnes of CO_2 are released annually, causing atmospheric levels to rise $\sim 0.4\%$ every year. If current trends continue, preindustrial concen-

* Tel.: +30 54179548; fax: +30 54129813; e-mail: ykar@demokritos.cc.duth.gr

PII S0303-2647(97)00061-0

trations will have doubled by the year 2075. Computer models predict that doubling atmospheric CO_2 could cause global mean temperatures to rise from 1.5 to 4.5°C (Cunningham and Saigo, 1995; Cubasch et al., 1995; Pittock, 1995). In addition to CO_2, methane, nitrous oxide and chlorofluorocarbons (CFCs) cause climate warming. These gases are known as greenhouse gases, and their effect on global climate change has been extensively studied (Washington et al., 1990; Washington and Bettge, 1990; Zapert et al., 1994). Many scientists are convinced that global climate change has already begun, based on observations showing that global tropospheric temperatures have been rising during the 20th century, with a sharp rise since the mid-1970s (Graham, 1995), and 4 of the 5 warmest years on record were in the late 1980s and early 1990s (Schuurmans, 1994).

Climatic changes have drastic effects on biotic assemblages. Climate change leads to change in the ecosystem's carrying capacity for species (deGroot, 1992). When climatic change is gradual, species may have time to adapt, to adjust their population or move to more suitable locations. When climatic change is relatively abrupt, organisms are less likely to respond before conditions exceed their tolerance limits (Cunningham and Saigo, 1995; deGroot, 1992). A population of a species is a number of members of the same species that occupy a specific geographical area at a given time (Cunningham and Saigo, 1995). The carrying capacity of the environment for a species is the maximum number of the individuals that can be supported on a long term basis. Populations tend to increase as far as their environment will allow (deGroot, 1992). Due to the interaction of a population with other populations and because of the limiting environmental factors, most populations are in a dynamic state of equilibrium (Klamka, 1994) or in a chaotic limit cycle (Peitgen et al., 1992). These environmental factors, which are also called regulatory factors, may be either biotic, if caused by biota, or abiotic otherwise. Regulatory factors act either in a density dependent or independent manner. A factor is density dependent if its effect depends on population density and density independent otherwise. Climate is one of the most important abiotic density-independent factors that affect natality and mortality of species (Cunningham and Saigo, 1995; Mayer et al., 1992; Lischke, 1992; Morse, 1993).

Many insects and microorganisms live or become active within a certain temperature range. At each time of the year, a mean temperature range corresponds roughly to a certain zone on earth, i.e. in November the temperature range from 10 to 0°C corresponds to a zone between 30 and 50° North latitude. The population of a species living in this temperature range is located in the aforementioned zone in November. Global warming will shift the temperature ranges in which certain species live to other than their usual zones on earth. This will force species to move to other geographical regions. It is also possible that certain temperature ranges, will become larger or smaller. This will force species to increase or decrease their populations. Species which have a 1-year life cycle live in a certain season, produce offspring (eggs, spores, etc.) and die. Global warming will increase or decrease the duration of the season in which these species live, resulting in larger or fewer offspring production. Global warming can also increase or decrease the carrying capacity of the environment.

Modelling the influence of global warming on geographical distribution and population dynamics of such species is a highly non-linear problem with many interacting factors, because mean temperature increment influences species movement, offspring production and carrying capacity of the environment. It is very difficult to describe the action of all these parameters using partial differential equations (PDEs). Such an attempt will probably lead to a system of PDEs which will be very difficult to interpret. Cellular automata (CAs) are an alternative to PDEs and have been used successfully in modelling physical systems and processes (Toffoli, 1984a; Omohundro, 1984). The aim of this work is to investigate the possibility of using CAs for modelling the influence of the greenhouse effect on insect and microorganism geographical distribution and population dynamics.

In the current framework, a model for the influence of global warming on species geographical distribution and population dynamics has been developed based on CAs. Based on this model, an algorithm has been developed and has been used to determine the geographical distribution and population dynamics of a hypothetical species in a scenario of global warming according to which the mean temperature increment is 0.1°C each year. The species lives in a temperature range from 25 to 32°C and the initial population distribution is assumed to be Gaussian. After the initiation of global warming the population moves and after a few decades the population distribution is no longer Gaussian. Larger populations are found in the direction of population movement. The model presented here is incomplete because important regulatory factors such as predator population, stress-related disease, and species competition were not included.

2. Cellular automata

CAs (von Neumann, 1966) are models of physical systems where space and time are discrete and interactions are local. They have been extensively used as models for complex systems (Wolfram, 1994). CAs have also been applied to several physical problems where local interactions are involved (Gerhard and Schuster, 1989; Gerhard et al., 1990; Weimar et al., 1992; Karafyllidis and Thanailakis, 1995). In spite of the simplicity of their structure, CAs exhibit complex dynamical behaviour and can describe many physical systems and processes. A CA consists of a regular uniform n-dimensional lattice (or array), usually of infinite extent. At each site of the lattice (cell) a physical quantity takes on values. This physical quantity is the global state of the CA, and the value of this quantity at each cell is its local state. Each cell is restricted to local neighbourhood interaction only, and as a result it is incapable of immediate global communication (von Neumann, 1966). The neighbourhood of a cell is taken to be the cell itself and some of (or all) the immediately adjacent cells. The states at each cell are updated simultaneously at discrete time steps, based on the states in their neighbourhood at the preceding time step. The algorithm used to compute the next cell state is referred to as the CA local rule. Usually the same local rule applies to all cells of the CA.

A CA is characterised by five properties:

1. the number of spatial dimensions (n);
2. the width of each side of the array (w). w_j is the width of the jth side of the array, where $j = 1, 2, 3,\dots$ n;.
3. the width of the neighbourhood of the cell (d). d_j is the width of the neighbourhood at the jth side of the array;
4. the state of the CA cells;
5. the CA rule, which is an arbitrary function F.

The state of a cell at time step $(t+1)$ is computed according to F. F is a function of the state of this cell at time step (t) and the states of the cells in its neighbourhood at time step (t).

CAs have sufficient expressive dynamics to represent phenomena of arbitrary complexity and at the same time can be simulated exactly by digital computers because of their intrinsic discreteness, i.e. the topology of the simulated object is reproduced in the simulating device (Vichniac, 1984). Mathematical tools for simulating physics, namely PDEs, contain much more information than is usually needed, because variables may take an infinite number of values in a continuous space. Moreover, the value of a physical quantity cannot be measured at a point but instead over a finite volume (Toffoli, 1984b). PDEs are used to compute values of physical quantities at points, whereas CAs are used to compute values of physical quantities over finite volumes (CA cells). Therefore, for the above reasons, algorithms that are based on CAs run quickly on digital computers (Toffoli, 1984a). As models for physical systems, CAs have many limitations. They are classical systems and therefore they cannot represent quantum mechanical systems. CAs should not be used to simulate systems where speeds are comparable to that of light because of the anisotropy induced by the discrete space. More about modelling physics with CAs may be found in (Vichniac, 1984; Minsky, 1982; Feynman, 1982; Zeigler, 1982).

Models based on CAs are leading to algorithms which are fast when implemented on serial computers because they exploit the inherent parallelism of the CA structure. These algorithms are also appropriate for implementation on a massively parallel computers, such as cellular automata machine (CAM) (Toffoli, 1984b). Moreover, due to the local interconnections and the discreteness in space and time, synchronous very large scale integration (VLSI) circuits have been used as an implementation medium of algorithms based on CAs (Andreadis et al., 1996; Karafyllidis et al., 1996). Such algorithms, when implemented on hardware, lead to application-specific integrated circuits (ASICs) with a maximum frequency of operation ranging between 25 and 50 MHz when 1.0 μm CMOS technology is used. Therefore, CAs are suitable for modelling and simulating large and complicated systems.

3. The model

Fig. 1 shows a schematic representation of earth. The grey zone in Fig. 1 is defined by two parallel circles which are perpendicular to the equatorial plane. The distances of the North pole from both planes on which these two circles lie are equal. This zone is divided into 900 equal rectangular cells and is represented by a CA by considering each zone cell as a CA cell. The left edge of cell 1 lies on the equatorial or on 0° North

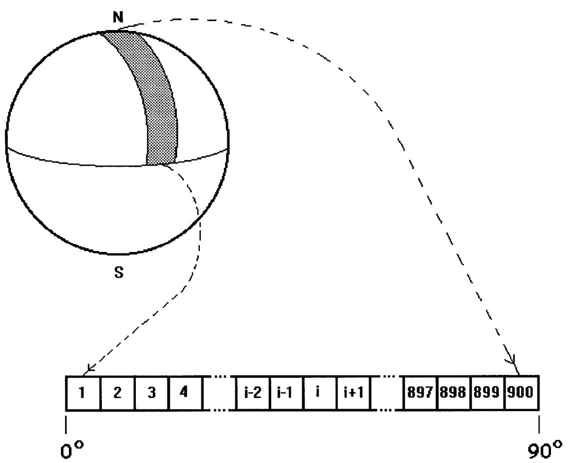

Fig. 1. A schematic representation of earth. The grey zone is divided into 900 equal rectangular cells and is represented by a CA by considering each zone cell as a CA cell.

latitude. The North pole is located in the right edge of cell 900 and thus this edge is located in 90° North latitude. Each North latitude degree is represented by ten CA cells. Each CA cell represents a geographical area. Time in this CA model advances at discrete and equal steps. Each step corresponds to 15 days, i.e. a year is represented by 24 time steps.

The local state of a CA cell at a time t is defined as the normalised number of individuals of a species, i.e. the ratio of the number of the individuals in the geographical area represented by the CA cell at time t, to the carrying capacity of the same area at the same time step:

$$C_i^t = \frac{P_i^t}{M_i^t} \quad (1)$$

C_i^t being the local state of the ith cell, at time step t, P_i^t is the number of individuals in this cell at time t and M_i^t is the carrying capacity of this cell at time t. C_i^t may take any value between 0 and 1. The state of a cell in which there exist no individuals of this species is zero, whereas the state of a cell is one if the number of individuals in this cell is equal to its carrying capacity.

Each cell of the CA is allocated a temperature T. This temperature is not constant, but varies with time. T_i^t is the temperature at the ith cell at time step t. This temperature is the mean temperature at the geographical area represented by the cell for the 15 days represented by the time step. If the first time step corresponds to the 1–15 May, then the second time step corresponds to the 16–31 May and so on. T_{407}^8 is the mean temperature of the 16–30 August at the geographical area represented by the 407th cell. The 16–30 August corresponds to the eighth time step. If the climate is stable then it is assumed that the mean temperature does not change over the years. This means that $T_{407}^8 = T_{407}^{32}$, i.e. the temperature on the 16–30 August next year ($32 = 24 + 8$) will be equal to the mean temperature on the 16–30 August this year. Generally, in the case of no global warming:

$$T_{407}^8 = T_{407}^{32} = T_{407}^{56} = \ldots = T_{407}^{24n+8} \quad (2)$$

Due to global warming, the mean temperature is assumed to increase by R°C per year. Global warming is modelled by increasing the temperature at each CA cell by R°C each year:

$$T_{407}^{8} = T_{407}^{32+R} = T_{407}^{56+2R} = \ldots = T_{407}^{24n+nR+8} \quad (3)$$

It is assumed that individuals of a species (organisms) live within a certain temperature range, defined by $T_{\min}$ and $T_{\max}$. Organisms in a CA cell live (i.e. get active or hatched) if the temperature T is:

$$T_{\min} < T < T_{\max}$$

and perish if:

$$T < T_{\min} \quad \text{or} \quad T_{\max} < T \quad (4)$$

Living organisms in a cell produce offspring (eggs, etc.) before they perish. If the number of living organisms in the ith cell at time t is equal to P_i^t, then the number of offspring O_i^t is determined according to:

$$O_i^t = pP_i^t \quad (5)$$

The user may define the fitting parameter p or enter another equation instead of Eq. (5). The number of offspring at time step $t+1$ equals:

$$O_i^{t+1} = O_i^t + pP_i^{t+1} \quad (6)$$

If at a time step $t+n$ no living organisms exist in the ith cell ($P_i^{t+n} = 0$) then the number of offspring will be constant but not zero ($O_i^{t+n+2} = O_i^{t+n+1}$, etc.) until temperature comes again into the range where this species lives. If this happens at time step $t+m$, the number of organisms is determined according to:

$$P_i^{t+m} = qO_i^{t+m} \quad (7)$$

where q is a user defined parameter. In the next time step $(t+m+1)$ the number of organisms is given by:

$$P_i^{t+m+1} = P_i^{t+m} + qO_i^{t+m+1} \quad (8)$$

The number of offspring at time step $(t+m+1)$ is given simply by:

$$O_i^{t+m+1} = (1-q)O_i^{t+m} \quad (9)$$

The cell carrying capacity is not constant but depends on temperature. The variation of carrying capacity with temperature is different for different species. A general non-linear form which relates the temperature at the ith cell, at time step t, to its carrying capacity is:

$$M_i^t = a\exp(bT_i^t) \quad (10)$$

Eq. (10) will be used throughout. The user may give any value to the fitting parameters a and b or may enter another equation to replace Eq. (10).

At each time step, organisms are only able to move from a cell to one of its two nearest cells, provided that the temperature at these cells is within the temperature range where these organisms live. In other words only north/south movement is allowed. At a time step t, organisms from the ith cell can migrate to $i-1$ and $i+1$ cells if:

$$\begin{aligned} T_{\min} &< T_{i-1}^t < T_{\max} \\ T_{\min} &< T_{i+1}^t < T_{\max} \end{aligned} \quad (11)$$

If at time step t the state of the $(i+1)$st cell is 0.7 (i.e. the population in this cell is at 70% of its carrying capacity) and the state of the ith cell is 0.5 (i.e. 50% of its carrying capacity), organisms will move from the $(i+1)$st cell to the ith cell. Suppose that at time step t in the ith cell P_i^t organisms exist and in the $(i-1)$st and $(i+1)$st cells P_{i-1}^t and P_{i+1}^t organisms exist, respectively. The corresponding carrying capacities of these cells are M_i^t, M_{i-1}^t and M_{i+1}^t. At time step t, organisms will move from the $(i+1)$st cell to the ith cell if:

$$\frac{P_{i+1}^t}{M_{i+1}^t} > \frac{P_i^t}{M_i^t} \quad \text{or} \quad C_{i+1}^t > C_i^t \quad (12)$$

The ith cell may accept organisms from both its neighbours, or send organisms to both its neighbours, or send organisms to one neighbour and accept organisms from the other. The driving force for organism movement from a cell to another is the difference of their states $(C_{i+1}^t - C_i^t)$. If this difference is positive, organisms will move from the $(i+1)$st cell to the ith, whereas, if the difference is negative, organisms will move from the ith cell to the $(i+1)$st cell. Organisms do not move instantly because the environment offers resistance to organism movement. This movement resistance acts as inertia to the movement driving force. Movement is supposed to take place during a time step, i.e. during the 15 days that correspond to the time step. In the beginning of the 16th day, or at time step $t+1$, the new state of the ith cell is calculated according to:

$$C_i^{t+1} = C_i^t + \frac{qO_i^t}{M_i^t} + c\,\exp(d(C_{i-1}^t - C_i^t)) + c\,\exp(d(C_{i+1}^t - C_i^t)) \quad (13)$$

The state of a CA cell at time step $t+1$ is a function of the state of the same cell at time step t and of the states of its two neighbouring cells at time step t. The parameters c and d are user defined and model the environmental resistance to population movement. The second term on the right-hand side of Eq. (13) represents the possible population increment by offspring. Replacing Eq. (1) and Eq. (10) in Eq. (13):

$$C_i^{t+1} = C_i^t + \frac{qO_i^t}{M_i^t} + c\,\exp\left(d\left(\frac{P_{i-1}^t}{a\,\exp(bT_{i-1}^t)} - \frac{P_i^t}{a\,\exp(bT_i^t)}\right)\right) + c\,\exp\left(d\left(\frac{P_{i+1}^t}{a\,\exp(bT_{i+1}^t)} - \frac{P_i^t}{a\,\exp(bT_i^t)}\right)\right) \quad (14)$$

The state of the ith CA cell at time step $t+1$ is a function of populations and temperatures at cells $i-1$, i, $i+1$ at time step t. Populations and temperatures are the mean populations and temperatures of the 15 days that correspond to each time step. At each time step, the population is calculated from Eq. (1), i.e. the population in a cell is equal to the product of the state and the carrying capacity of this cell.

The initial conditions of the CA are the temperatures, the populations, and the carrying capacities of all cells at time step 0. It is assumed that global warming will start at time step 25, i.e. 1 year after the initial conditions have been imposed. The initial conditions are entered by the user. The user-defined parameters and equations are:

- temperature at all cells at time step 0;
- population in all cells at time step 0;
- carrying capacity of all cells at time step 0;
- Eqs. (5) and (7) and Eq. (10);
- parameters p, q, a, b, c, d;
- the temperature increment per year R°C.

Cells 1 and 900 have only one neighbour. Their second neighbour is determined by the boundary conditions. If the grey zone in Fig. 1 is allowed to go around earth, then cell 1 and the cell on its left will have the same mean temperature at all times because they are symmetric to the equatorial plane and, therefore, cell 1 will be used as the left neighbour of cell 1. Similarly, the cell on the right of cell 900 will be used as the right neighbour of cell 900.

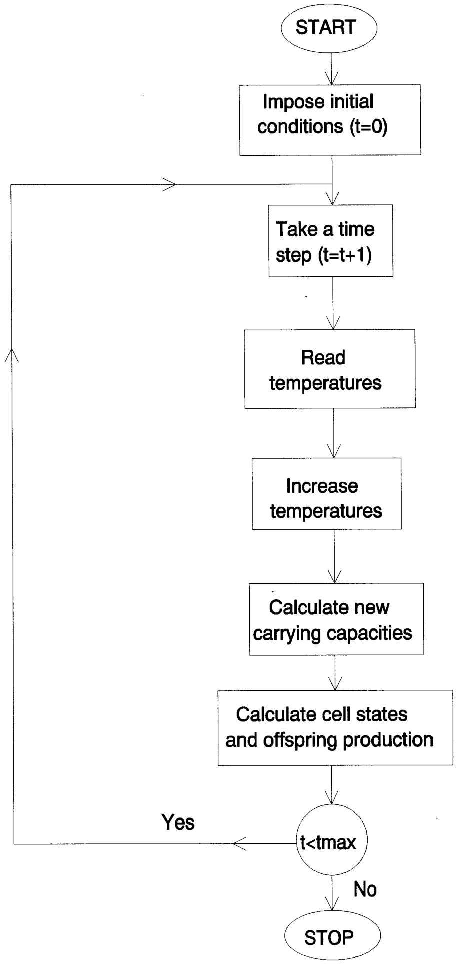

Fig. 2. The flow chart of the algorithm.

4. The algorithm

Fig. 2 shows the flow chart of the algorithm based on the previous section. The algorithm starts at time step 0. The initial conditions are imposed (i.e. temperatures, populations, and car-

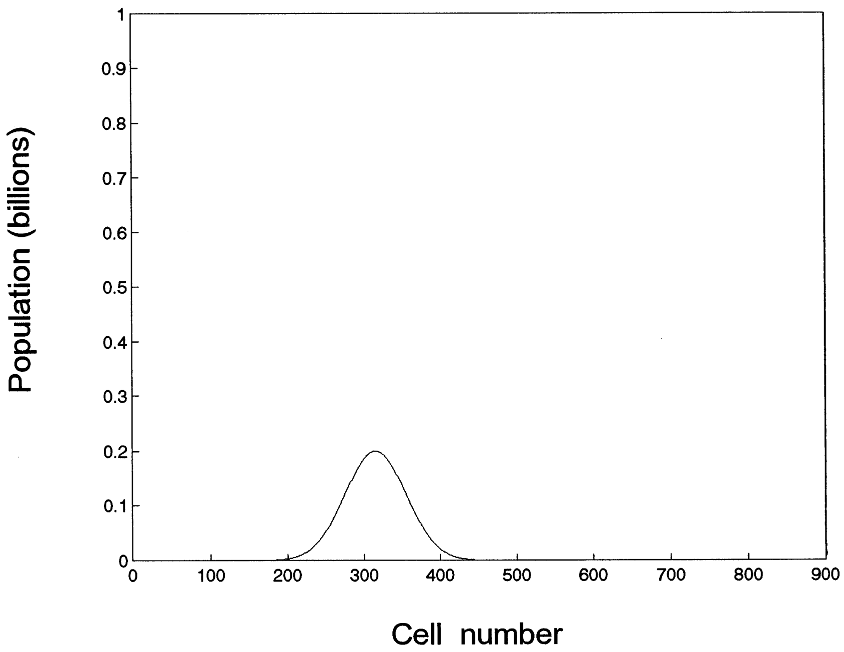

Fig. 3. The population initial conditions.

rying capacities of all cells) and these values are kept in three files in the form of [1×900] matrices. These three files and one additional file containing the values of the user-defined parameters are read. A time step is taken and the temperatures of the cells at this time step are read from a file. Mean temperatures for various latitudes for every day of the year are found in the literature. After that, temperatures are increased by R°C. In the 1st year the temperatures are not increased, in the 2nd year ($24 < t < 49$) temperatures are increased by R°C, in the 3rd year ($48 < t < 73$) by $2R$°C and so on. After temperature increment, the carrying capacities of the cells are calculated using Eq. (10). After that, the new cell states are calculated using Eq. (14), and the offspring production using Eq. (5).

The algorithm was implemented in an hypothetical scenario according to which the mean temperature increment is 0.1°C each year. The hypothetical species lives in a temperature range from 25 to 32°C. The population initial conditions ($t = 0$) that were imposed are shown in Fig. 3. It is supposed that $t = 0$ corresponds to the 1–15 April. At this time the species occupies a geographical area represented by the cells 180, 181,..., 440 and the population distribution in these cells is assumed to be Gaussian. The model parameters were taken to be: $p = 1.40$, $q = 0.75$, $a = 0.50$, $b = 1.10$, $c = 0.60$, $d = 0.90$.

The population distribution of the species 10, 20 and 30 years after is shown in Fig. 4. All three curves correspond to a period from 16 to 30 April. The dash-dotted curve represents the population distribution 10 years later ($t = 241$), the dashed curve the population distribution 20 years latter ($t = 481$) and the solid curve the population distribution 30 years later ($t = 721$). The species migrates from South to North and its mean population is increased. The population distribution is no longer Gaussian. Larger populations are found in the direction of population movement and it could be said that the peak of the initial Gaussian distribution has been displaced towards the direction of population movement.

Fig. 5 shows the same data as Fig. 4 but from 1 to 15 October. After 10 years no individuals of this species live this time of year, but after 20

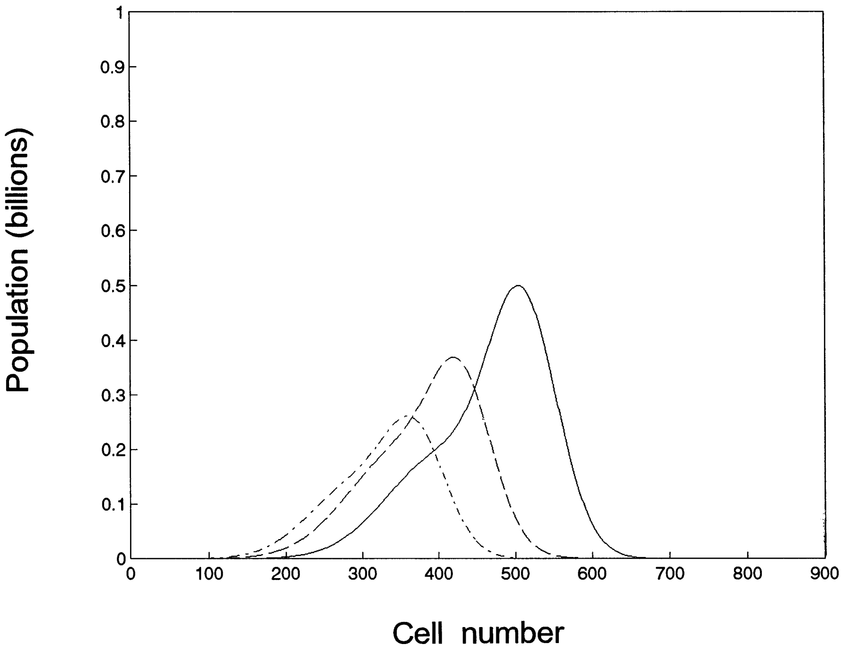

Fig. 4. The population distribution of the species 10 (dashed–dotted curve), 20 (dashed curve) and 30 (solid curve) years after. All three curves correspond to a period from 16 to 30 April.

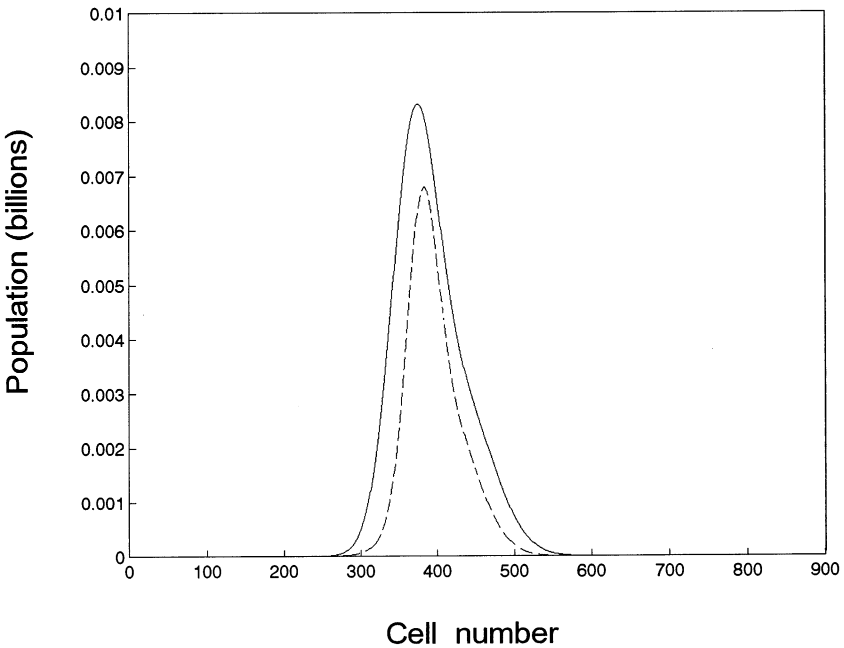

Fig. 5. The population distribution of the species 20 (dashed curve) and 30 (solid curve) years after. Both curves correspond to a period from 1 to 15 October.

years a number of individuals live and this number is increased after 30 years. In this time of year, larger populations are found in the direction opposite to that of the population movement.

5. Conclusions

A model for the influence of the greenhouse effect on insect and microorganism geographical distribution and population dynamics using cellular automata has been presented. An algorithm based on this model has been developed and used to determine the geographical distribution and population dynamics of an hypothetical species in an hypothetical scenario of global warming. Important regulatory factors such as predator population, stress-related diseases and species competition were not included. It has been shown, however, that cellular automata are suitable for modelling such a complicated non-linear problem. The problem is far more complicated in two dimensions where geographical areas such as seas, lakes, and mountains should be taken into account in order to simulate earth's surface. It will be practically impossible to use partial differential equations in the two dimensional problem because of the complex boundaries. Cellular automata can handle complex boundaries and could be used in the two dimensional problem.

Acknowledgements

The author would like to thank Dr J.W. Haefner for his help and encouragement and Dr D.B. Fogel and the anonymous reviewers for their constructive criticism which helped improve the presentation of this paper.

References

Andreadis, I., Karafyllidis, I., Tzionas, P., Thanailakis, A., Tsalides, Ph., 1996. A new hardware module for automated visual inspection based on a cellular automaton architecture. J. Intell. Robot. Sys. 16, 89–102.

Cubasch, U., Hegerl, G.C., Hellbach, A., Hoeck, H., 1995. A climate change simulation starting from 1935. Clim. Dyn. 11 (2), 71–84.

Cunningham, W.P., Saigo, B.W., 1995. Environmental Science. W.C. Brown, Dubuque, IA.

Feynman, R.P., 1982. Simulating physics with computers. Int. J. Theor. Phys. 21, 467–488.

Gerhard, M., Schuster, H., 1989. A cellular automaton describing the formation of spatially ordered structures in chemical systems. Physica D 36, 209–221.

Gerhard, M., Schuster, H., Tyson, J.J., 1990. A cellular automaton model of excitable media. Physica D 46, 392–415.

Graham, N.E., 1995. Simulation of recent global temperature trends. Science 267 (5198), 666–671.

deGroot, W.T., 1992. Environmental Science Theory. Elsevier, Amsterdam.

Karafyllidis, I., Thanailakis, A., 1995. Simulation of two-dimensional photoresist etching process in integrated circuit fabrication using cellular automata. Model. Simul. Mater. Sci. Eng. 3, 629–642.

Karafyllidis, I., Andreadis, I., Tzionas, P., Tsalides, Ph., Thanailakis, A., 1996. A cellular automaton for the determination of the mean velocity of moving objects and its VLSI implementation. Pattern Recognit. 29, 689–699.

Klamka, J., 1994. Controllability of non-linear models in modelling population dynamics. Appl. Math. Comput. Sci. 4 (2), 203–209.

Lischke, H., 1992. In: J.G. Buwalda (Ed.), A Model to Simulate the Population Dynamics of the Codling Moth (Cydia-pomonella): Parameter Estimation, Validation and Sensitivity Analysis. Int. Soc. Horticultural Sci., Wageningen, The Netherlands.

Mayer, D.G., Atzeni, M.G., Butler, D.G., 1992. Adaptation of CLIMEX for spatial screwworm fly population dynamics. Math. Comput. Simul. 33 (5/6), 439–444.

Minsky, M., 1982. Cellular vacuum. Int. J. Theor. Phys. 21, 537–551.

Morse, D.R., 1993. In: Kerridge E.L. (Ed.), Spatial Simulation Modelling of Insect Population Dynamics on a Transputer Network. IOS Press, Amsterdam.

von Neumann, J., 1966. Theory of Self-Reproducing Automata. University of Illinois, Urbana, IL.

Omohundro, S., 1984. Modelling cellular automata with partial differential equations. Physica D 10, 128–134.

Peitgen, H., Jurgens, H., Saupe, D., 1992. Chaos and Fractals, Springer-Verlag, Berlin.

Pittock, A.B., 1995. Comments on climate change scenario development. Math. Comput. Model. 21 (9), 1–4.

Schuurmans, C.J.E., 1994. Numerical simulation and prediction of climate. Dyn. Atmos. Oceans 20 (3), 271–284.

Toffoli, T., 1984a. Cellular automata as an alternative to (rather than an approximation of) differential equations in modeling physics. Physica D 10, 117–127.

Toffoli, T., 1984b. CAM: a high-performance cellular automaton machine. Physica D 10, 195–204.

Vichniac, G.Y., 1984. Simulating physics with cellular automata. Physica D 10, 96–116.

Washington, W.M., Bettge, T.W., 1990. Computer simulation of the greenhouse effect. Comput. Phys. 4 (3), 240–246.

Washington, W.M., Bettge, T.W., Meehl, G.A., Yost, J.B., 1990. Computer simulation of the global climatic effects of increased greenhouse gases. Int. J. Supercomput. Appl. 4 (2), 5–19.

Weimar, J.R., Tyson, J.J., Watson, L.T., 1992. Diffusion and wave propagation in cellular automaton models of excitable media. Physica D 55, 309–327.

Wolfram, S., 1994. Cellular Automata and Complexity. Addison-Wesley, Reading, MA.

Zapert, R., Filar, J.A., Gaertner, P.S., 1994. Uncertainty analysis of an integrated greenhouse effect model.' In: Zannetti, P. (Ed.), Environmental Systems. Computational Mechanics Publications, Southampton, UK.

Zeigler, B.P., 1982. Discrete event models for cell space simulation. Int. J. Theor. Phys. 21, 573–588.