NORTHWESTERN UNIVERSITY | SCHOOL OF PROFESSIONAL STUDIES

Final Exam MSPA Fall 2016

Points possible: 100

Description: The final exam will cover topics from sessions 1-9.

Resources: The exam is completely open book. You may use course textbooks, materials provided on Canvas, graphing calculators (such as TI 83 or 84); *but any more advanced calculators, Excel Solver, Web calculators, Web-graphic calculators, or simplex method calculators are not allowed. Programming languages other than Python are also not permitted*.

For questions that require calculations, all calculations should be shown, not just the final answer. This will allow for partial credit for those answers that might be set up correctly but have calculation errors. For questions that specifically require Python, the code and output should be included with your answer. For questions that require graphs, only use Python.

Restrictions: All answers are to be your work only. You are not to receive assistance from any other person.

To complete the exam:

1. Answer all questions on the exam thoroughly. Create a Microsoft Word document, including the question number, the question, your typed answer, and graphs if required. You may use Word's equation editor to complete your answers.

2. Once you have completed your exam, return to the exam item where you downloaded the exam PDF, click View/Complete Assignment, and submit your document.

1. The Apex Television Company has to decide on the number of 27- and 20-inch sets to be produced at one of its factories. Market research indicates that at most 40 of the 27-inch sets and 10 of the 20-inch sets can be sold per month. The maximum number of work-hours available is 500 per month. A 27-inch set requires 20 work-hours and a 20-inch set requires 10 work-hours. Each 27-inch set sold produces a profit of $120 and each 20-inch set produces a profit of $80. A wholesaler has agreed to purchase all the television sets produced if the numbers do not exceed the maxima indicated by the market research.

   (a) Formulate a linear programming model for this problem. (b) Use the graphical method to solve this model.

2. MARY has just been diagnosed as having a cancer at a fairly advanced stage. Specifically, she has a large malignant tumor in the bladder area (a "whole bladder lesion"). Mary is to receive the most advanced medical care available to give her every possible chance for survival. This care will include extensive radiation therapy.

Two radiation beams are to be used in combination: beam 1 and beam 2. The two decision variables x_1 and x_2 represent the dose (in kilorads) at the entry point for beam 1 and beam 2, respectively. Because the total dosage reaching the healthy anatomy is to be minimized, let Z denote this quantity. The model can be represented by the following LP:

Area	**Beam 1(ratio)**	**Beam 2(ratio)**	**Total Dosage(kilorads)**
Healthy Tissue	0.4	0.5	Minimize
Critical tissues	0.3	0.1	<=2.7
Tumor region	0.5	0.5	<=6
Tumor Center	0.6	0.4	>=6

2.(cont'd).

The model can be represented by the following Linear Program:

Minimize: $Z = 0.4x_1 + 0.5x_2$

Subject to:

$0.3x_1 + 0.1x_2 \leq 2.7$(critical tissues)

$0.5x_1 + 0.5x_2 \leq 6$(tumor region)

$0.6x_1 + 0.4x_2 \geq 6$(center of tumor)

$$x_1, x_2 \geq 0$$

(a) Using **Python**, graph the feasible region for these constraints.
(b) Solve the Linear program graphically and determine the values of x_1, x_2 which minimize Z as well as the minimum value of total dosage Z.

3.(a) The price of a product is given by P(x) = [(120/x) +900] where x represents the demand for the product. Find the rate of change of price when the demand is 2.

(b)Find the derivative of f(x) if:

f(x) = $\dfrac{5}{(2x-3)^4}$

4. The population of students at Northwestern is given by the formula $p(t) = (t^2 + 100) * \ln(t + 2)$, where t represents the time in years since 2000. Using **Python**, find the rate of change of the student population in both 2006 and 2016.

5. A hotel has 270 units. All rooms are occupied when the hotel charges $90 per day for a room. For every increase of ***x dollars*** in the daily room rate, there are ***x*** rooms vacant. Each occupied room costs $28 per day to service and maintain. Using **Python**, find what the hotel charge per day should be in order to maximize daily profit.

6. The rate of growth of the profit (in **millions**) from an invention is approximated by $P(x) = xe^{-x^2}$ where x represents time measured in years. The total profit in year 2 that the invention is in operation is $10,000. Find the total profit function. Round to three decimal places where appropriate.

7. The population of Asian Carp in Lake Michigan is described by the logistic equation $G(t) = \frac{12000}{1+19e^{-1.2t}}$ where G (t) is the population after t years. Find the point in time at which the **growth rate** of this population begins to decline.

8. The life in years of a certain species of bird is a random variable with probability density function given by: $f(x) = \frac{1}{9} * (1 + 2/\sqrt{x}) \quad for\ x\ in\ [4,9].$ Find the mean life of this species of bird.

9. Determine whether the following function is a probability density function on the specified interval:

$$f(x) = \frac{2}{109}x^2 - \frac{2}{109}x + \frac{1}{654}, \qquad on\ interval\ [0,6]$$

10. For the following function, determine the domain, critical points, intervals where the function is increasing or decreasing, inflection points, intervals of concavity, intercepts, and asymptotes where applicable. Use this information to graph the function.

$f(x) = -5x - (15/x)$