SUPPLEMENT TO

CHAPTER 8

The Transportation Model

SUPPLEMENT OUTLINE

Transportation Problem, Model, and Solution, 2

Other Related Applications, 6

OM in Action: Some Applications of the Transportation Model, 9

Problems, 10

Mini-Case: NewPage's Wickliffe Kentucky Paper Mill, 15

LEARNING OBJECTIVES

After completing this supplement, you should be able to:

- LO1 Describe the transportation problem, set it up as a transportation model, and solve it using Excel's Solver.
- LO2 Describe Assignment and Transshipment problems, and solve them using Excel's Solver.

LO 1 TRANSPORTATION PROBLEM, MODEL, AND SOLUTION

The transportation problem involves finding the lowest-cost plan for distributing goods from multiple sources that supply the goods to multiple destinations that demand the goods. For instance, a company might have three factories (supply), all of which are capable of producing identical units of the same product, and four warehouses (demand) that stock those products, as depicted in Figure 8S-1. The *transportation model* can be used to represent the structure and data of transportation problem to determine how to allocate the supplies available from the factories to the warehouses in such a way that total transportation cost is minimized. Usually, analysis of the problem will produce a shipping plan that pertains to a certain period of time (day, week, month), although once the plan is established it will generally not change unless one or more of the parameters of the problem (supply, demand, unit transportation cost) changes.

Although Figure 8S-1 illustrates the nature of the transportation problem, in real life managers must often deal with allocation problems that are considerably larger in scope. A beer maker may have four or five breweries and hundreds or even thousands of distributors, and an automobile manufacturer may have eight assembly plants scattered throughout North America and thousands of dealers that must be supplied with those cars. In such cases, the ability to identify the optimal distribution plan makes the transportation model very important.

Transportation Model

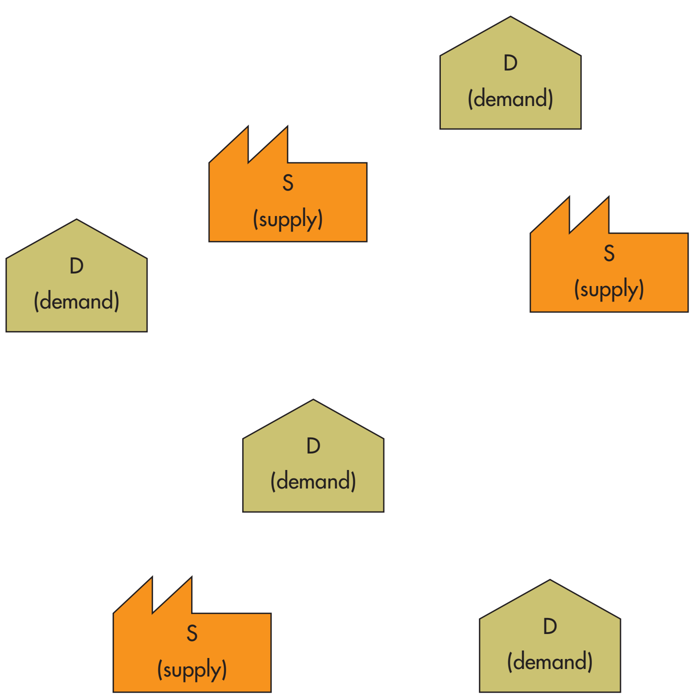
The supply points can be factories, warehouses, or any other place from which goods are sent. Destinations can be warehouses, retail stores, or other points that receive goods. The information needed to use the model consists of the following:

1. A list of the sources and each one's capacity or supply quantity per period.
2. A list of the destinations and each one's demand per period.
3. The unit cost of transporting items from each source to each destination.

This information is arranged in a *transportation table* (see Table 8S-1), where S_i = supply of Source i, $i = 1, \ldots, 3$, D_j = demand of Destination j, $j = A, B, C, D$, $\sum S_i$ = total

Figure 8S-1

The transportation problem involves determining a minimum-cost plan for transporting a product from multiple sources to multiple destinations



	Warehouse				
	A	B	C	D	Supply
Factory 1	C_{1A}	C_{1B}	C_{1C}	C_{1D}	S_1
Factory 2	C_{2A}	C_{2B}	C_{2C}	C_{2D}	S_2
Factory 3	C_{3A}	C_{3B}	C_{3C}	C_{3D}	S_3
Demand	D_1	D_2	D_3	D_4	ΣD_j / ΣS_i

Table 8S-1

A transportation table (symbols represent the parameters)

supply, $\sum D_j$ = total demand, and C_{ij} = cost to transport 1 unit of the product from Source i to Destination j.

The first thing to do in transportation modelling is to collect or estimate values of the input parameters.

Example 1: A company makes a product in three factories and distributes them to four warehouses. The transportation cost (dollar per unit), and supply and demand (in units per week) are shown in Table 8S-2. Note that total supply = total demand. If this was not the case, an extra ("dummy") origin or destination with zero unit transportation costs should be added to the table. If a route (cell) is not possible, a large unit transportation cost, e.g., \$999, can be used for it.

	Warehouse				
	A	B	C	D	Supply
Factory 1	4	7	7	1	100
Factory 2	12	3	8	8	200
Factory 3	8	10	16	5	150
Demand	80	90	120	160	450 / 450

Table 8S-2

The transportation table for Example 1

	Warehouse				
	A	B	C	D	Supply
Factory 1	4 x_{1A}	7 x_{1B}	7 x_{1C}	1 x_{1D}	100
Factory 2	12 x_{2A}	3 x_{2B}	8 x_{2C}	8 x_{2D}	200
Factory 3	8 x_{3A}	10 x_{3B}	16 x_{3C}	5 x_{3D}	150
Demand	80	90	120	160	450 / 450

Table 8S-3

The transportation table for Example 1 with quantity symbols added

The output from the transportation model is the optimal quantity of product to be transported on each source–destination route and may be displayed inside the cells of the transportation table (see Table 8S-3), where x_{ij} = units to transport from Factory i, $i = 1, \ldots, 3$, to Warehouse j, j = A, B, C, D.

Use of the transportation model implies that certain assumptions are satisfied. The major ones are:

1. The items to be transported are homogeneous (i.e., they are the same regardless of their source or destination).
2. Transportation cost per unit C_{ij} is the same regardless of the number of units transported.
3. There is only one route or mode of transportation being used between each source and each destination.

Linear Programming Formulation The transportation model is a special case of linear programming model. The decision variables for the transportation model are the quantities to be transported x_{ij}. The objective function consists of the sum of cell costs C_{ij} times decision variables x_{ij}. In most cases, the capacity of each source cannot be exceeded, but the demand at each destination must be met. For example, the transportation problem of Example 1 can be formulated as a linear programming model as follows:

Minimize $4x_{1A} + 7x_{1B} + 7x_{1C} + 1x_{1D} + 12x_{2A} + 3x_{2B} + 8x_{2C} + 8x_{2D} + 8x_{3A} + 10x_{3B} + 16x_{3C} + 5x_{3D}$

Subject to:

Supply (rows)

$$x_{1A} + x_{1B} + x_{1C} + x_{1D} \le 100$$
$$x_{2A} + x_{2B} + x_{2C} + x_{2D} \le 200$$
$$x_{3A} + x_{3B} + x_{3C} + x_{3D} \le 150$$

Demand (columns)

$$x_{1A} + x_{2A} + x_{3A} = 80$$
$$x_{1B} + x_{2B} + x_{3B} = 90$$
$$x_{1C} + x_{2C} + x_{3C} = 120$$
$$x_{1D} + x_{2D} + x_{3D} = 160$$

If total supply is less than total demand, the problem will be infeasible. To avoid this, add an extra (dummy) row (source) with the necessary supply and unit transportation costs of zero to restore feasibility.

Compact Formulation and Solution We can represent the linear programming model of a transportation problem in a more compact form in Excel. Figure 8S-2 illustrates the compact Excel formulation of Example 1. The Input Table at the top shows the unit transportation costs, supplies, and demands, whereas the Solutions Table at the bottom shows the decision variables—the quantities to transport (in yellow), Row Totals (i.e., sum of units sent out of each source), Column Totals (i.e., sum of units received in each destination, and Total Cost (i.e., sum product of the unit transportation costs and decision variables).

Solver is shown on the top right if Data menu item is clicked. Then, clicking on Solver will open the Solver Parameters window. Enter the coordinates of Total Cost in front of Set Objective, click in the radio button before Min, enter the range of decision variables after "By Changing Variable Cells," and enter the constraints (Column Total = Demand and Row Total ≤ Supply). See Figure 8S-3. Note that there is no need to enter each supply or each demand constraint individually. Also note that "Simplex LP" should be chosen as the Solving Method.

Optimal Solution to the problem (Example 1) is given in Figure 8S-4.

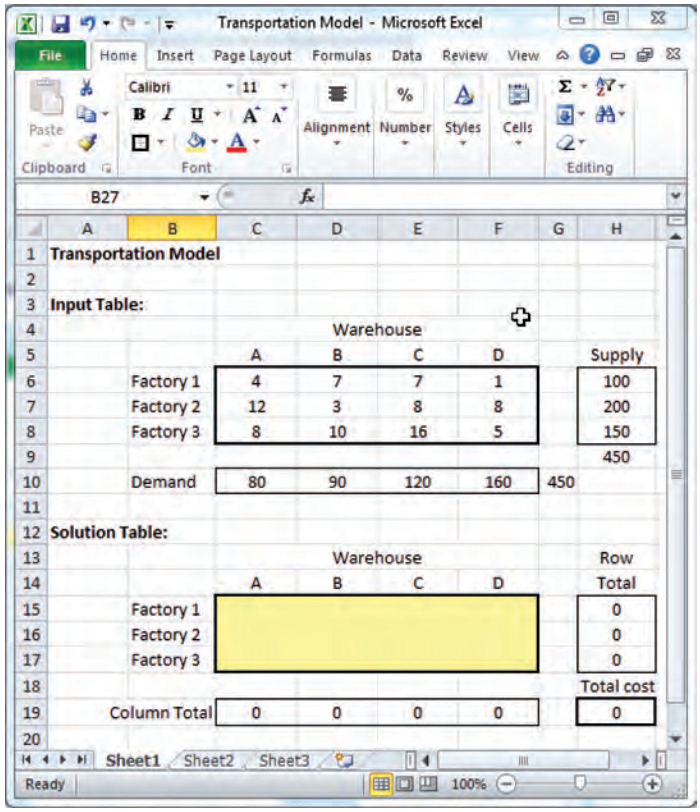


Figure 8S-2

Compact Excel Formulation of Example 1

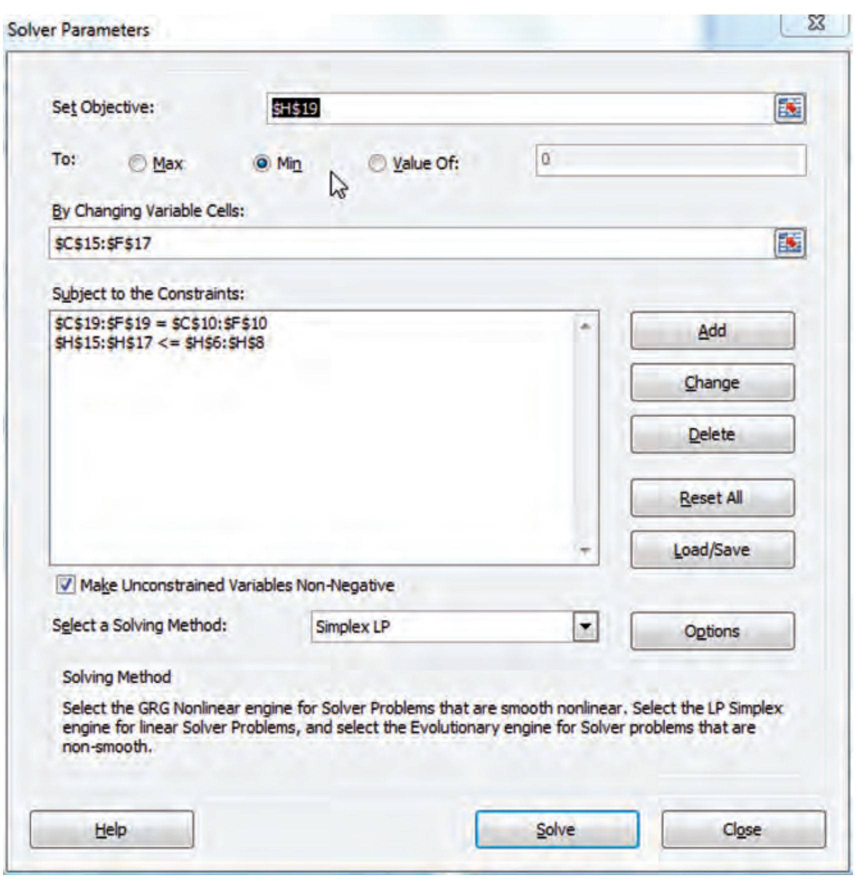
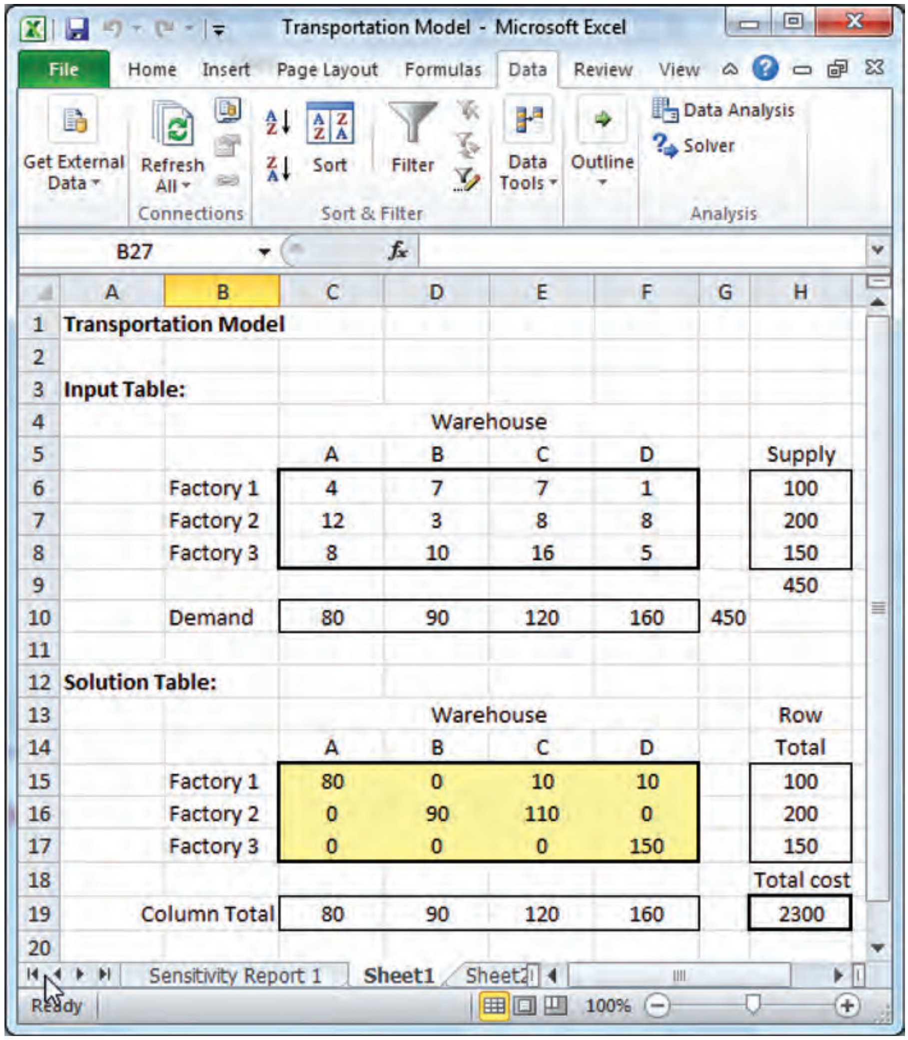


Figure 8S-3

Solver parameters for the problem given in Figure 8S-2

Figure 8S-4

Optimal solution to the problem given in Figure 8S-2



	A	B	C	D	E	F	G	H
1	**Transportation Model**							
2								
3	**Input Table:**							
4				Warehouse				
5			A	B	C	D		Supply
6		Factory 1	4	7	7	1		100
7		Factory 2	12	3	8	8		200
8		Factory 3	8	10	16	5		150
9								450
10		Demand	80	90	120	160	450	
11								
12	**Solution Table:**							
13				Warehouse				Row
14			A	B	C	D		Total
15		Factory 1	80	0	10	10		100
16		Factory 2	0	90	110	0		200
17		Factory 3	0	0	0	150		150
18								Total cost
19		Column Total	80	90	120	160		2300
20								

LO2 OTHER RELATED APPLICATIONS

Some of the other uses of the transportation model include aggregate operations planning (illustrated in Chapter 13), assignment of resources to jobs, transshipment problems, and location decisions.

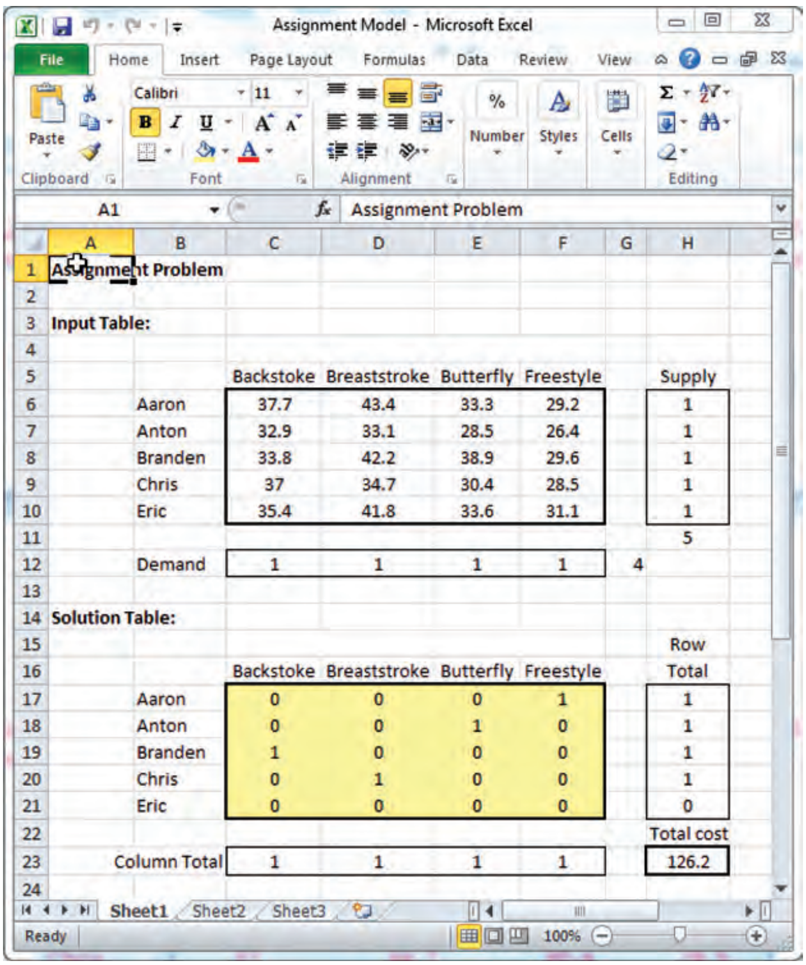
Assignment Problem

When there are multiple units of a resource that are slightly different in terms of capability and jobs requiring one unit of the resource, the assignment of units to jobs can be done by the transportation method. The only change is to use 1 for each demand and supply quantity.

Example 2: The coach of a university swimming team needs to choose swimmers for the 200-metre medley men's relay team (four swimmers, each swimming 50 meters of one of the four strokes). Since most of the best swimmers are very fast in more than one stroke, it is not clear which swimmer should be assigned to each of the four strokes. The five fastest swimmers and their best times (in seconds) in each of the strokes (for 50 meters) are:

	Backstroke	Breaststroke	Butterfly	Freestyle
Aaron	37.7	43.4	33.3	29.2
Anton	32.9	33.1	28.5	26.4
Branden	33.8	42.2	38.9	29.6
Chris	37	34.7	30.4	28.5
Eric	35.4	41.8	33.6	31.1

Solution:



	A	B	C	D	E	F	G	H
1	Assignment Problem							
2								
3	Input Table:							
4								
5			Backstoke	Breaststroke	Butterfly	Freestyle		Supply
6		Aaron	37.7	43.4	33.3	29.2		1
7		Anton	32.9	33.1	28.5	26.4		1
8		Branden	33.8	42.2	38.9	29.6		1
9		Chris	37	34.7	30.4	28.5		1
10		Eric	35.4	41.8	33.6	31.1		1
11								5
12		Demand	1	1	1	1	4	
13								
14	Solution Table:							
15								Row
16			Backstoke	Breaststroke	Butterfly	Freestyle		Total
17		Aaron	0	0	0	1		1
18		Anton	0	0	1	0		1
19		Branden	1	0	0	0		1
20		Chris	0	1	0	0		1
21		Eric	0	0	0	0		0
22								Total cost
23		Column Total	1	1	1	1		126.2
24								

Transshipment Problem

Transshipment problems, in addition to sources, have major distribution centres (transshipment points) that in turn redistribute to smaller market destinations. A transshipment problem can be formulated as a linear program in a manner similar to the transportation model. However, a transshipment point does not have a fixed supply or demand quantity—instead, we should equate (balance) the total input into it to total output out of it.

Example 3: An agricultural cooperative that buys wheat from the surrounding farms is faced with deciding to which of two mills to send its recently purchased 50,000 bushels.[1] From the mills, the ground flour is to be sent to three local markets. The Cooperative, as

[1]R. D. Shapiro, *Optimization Models for Planning and Allocation: Text and Cases in Mathematical Programming*, New York: Wiley, 1984, p. 25.

well as deciding how much wheat to send to each mill, must instruct each mill as to which market(s) to deliver the flour to. The transportation and milling costs are shown below along with demand for flour in each of the markets. Note: A bushel of flour is defined as the amount of flour resulting from milling a bushel of wheat. Formulate this transshipment problem as a linear program and solve it using Solver.

Transportation costs ($/bushel):

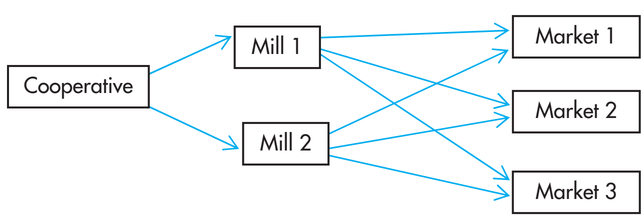
	To:	Mill 1	Mill 2
From:	Cooperative	.62	.66

	To:	Market 1	Market 2	Market 3
From:	Mill 1	.36	.44	.54
	Mill 2	.50	.46	.38

Milling cost ($/bushel):

Mill 1	Mill 2
.26	.32

Demand (bushels):	Market 1	Market 2	Market 3
	10,000	15,000	25,000



Solution:

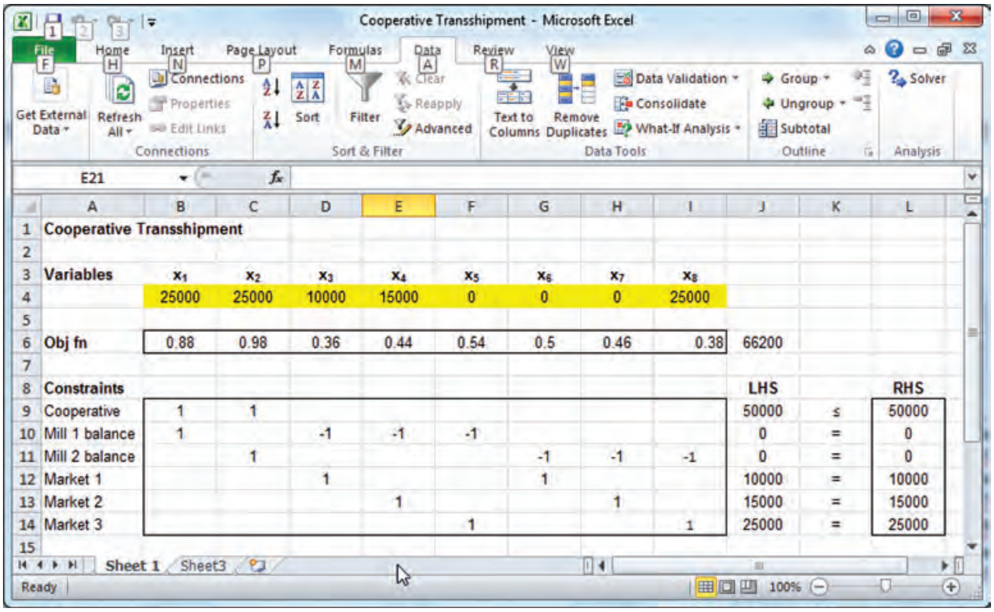
x_1 = amount of wheat to send from the Cooperative to Mill 1 (in bushels)
x_2 = amount of wheat to send from the Cooperative to Mill 2 (in bushels)
x_3 = amount of flour to send from Mill 1 to Market 1 (in bushels)
x_4 = amount of flour to send from Mill 1 to Market 2 (in bushels)
x_5 = amount of flour to send from Mill 1 to Market 3 (in bushels)
x_6 = amount of flour to send from Mill 2 to Market 1 (in bushels)
x_7 = amount of flour to send from Mill 2 to Market 2 (in bushels)
x_8 = amount of flour to send from Mill 2 to Market 3 (in bushels)

Minimize $(.62 + .26)\, x_1 + (.66 + .32)\, x_2 + .36\, x_3 + .44\, x_4 + .54\, x_5 + .50\, x_6 + .46\, x_7 + .38\, x_8$

S.t.

Cooperative	$x_1 + x_2$	$\leq 50{,}000$
Mill 1 balance	$x_1 - x_3 - x_4 - x_5$	$= 0$
Mill 2 balance	$x_2 - x_6 - x_7 - x_8$	$= 0$
Market 1	$x_3 + x_6$	$= 10{,}000$
Market 2	$x_4 + x_7$	$= 15{,}000$
Market 3	$x_5 + x_8$	$= 25{,}000$
	$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \geq 0$	

Note that the milling cost was added to the transportation cost from the Cooperative to each mill. The optimal solution obtained by Solver is as follows:



	A	B	C	D	E	F	G	H	I	J	K	L
1	Cooperative Transshipment											
2												
3	Variables	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8			
4		25000	25000	10000	15000	0	0	0	25000			
5												
6	Obj fn	0.88	0.98	0.36	0.44	0.54	0.5	0.46	0.38	66200		
7												
8	Constraints									LHS		RHS
9	Cooperative	1	1							50000	≤	50000
10	Mill 1 balance	1		-1	-1	-1				0	=	0
11	Mill 2 balance		1				-1	-1	-1	0	=	0
12	Market 1			1			1			10000	=	10000
13	Market 2				1			1		15000	=	15000
14	Market 3					1			1	25000	=	25000

Location Decisions

The transportation model can be used to compare location alternatives in terms of their impact on the total distribution cost. The procedure involves working through a separate problem for each location being considered and then comparing the resulting total costs.

If other costs, such as production costs, differ among locations, these can easily be included in the analysis, provided they can be determined on a per-unit basis. For each factory, just add the unit production cost to the unit transportation cost of factory-demand point pairs.

The above procedure can be automated by formulating the problem as a variation of linear programing called mixed integer linear programming. The resulting formulation is called location-allocation model.

OM in *ACTION*

Some Applications of the Transportation Model

TNT Express

TNT Express is a European parcel delivery company. In 2006 TNT Express streamlined its transportation network in the UK using MapMechanic's OptiSite.[2] Optisite is an optimization software that uses mixed-integer linear programming to solve location-allocation problems. OptiSite helped evaluate some alternative scenarios for TNT Express's network. OptiSite made it easier for TNT Express analysts to examine the implications of adding, say, an extra depot, and comparing that with the cost of changing existing depot boundaries.

Armstrong World Industries

Armstrong is a major producer of flooring and ceiling material.[3] Armstrong operates a complex supply chain: 35 plants across

[2]http://www.mapmechanics.com/mapmechanicspressroom/archive/press_tntexpress2006.htm.
[3]https://www-01.ibm.com/software/success/cssdb.nsf/CS/CPOR-8BP7F8?OpenDocument&Site=default&cty=en_us.

9 countries supplying multiple distribution centres and many customers. Armstrong's modelling team helped management improve their decision making in the supply chain with IBM ILOG LogicNet Plus XE, a network design software. Management has taken significant transportation, warehousing, and manufacturing costs out of the supply chain, added capacity in growing markets in a timely manner, removed capacity in shrinking markets, and responded when major customers restructured their supply chain.

Horoz Lojistik

Horoz Lojistik is a logistics services provider in Turkey.[4] Horoz used AIMMS (a linear programming model building software) to optimize the distribution (transportation and storage) of fertilizer in Turkey for one of its customers. Fertilizers are either manufactured locally or imported into Turkish harbours, where they are then destined for the final client. There are 3 manufacturing facilities, 17 possible harbour locations for imports, 14 stocking locations (local warehouses), 22 commodity types, and 80 major client destinations. A mathematical model was developed using client's data (production, demand, etc.) and Horoz's data (capacities, costs, and route and harbour options). The solution was analyzed, and sensitivity analysis was carried out on it. Multiple scenarios were tested for potential changes in the distribution network. Addition of new facilities or routes was assessed. The result was annual cost savings of US$400,000.

HeidelbergCement

HeidelbergCement needed to make more intelligent decisions about its supply chain investments in more than 40 countries.[5] Using JDA's Supply Chain Strategist software, HeidelbergCement has been able to analyze decisions such as the construction of new plants, acquisitions, and profitability of routes. An example of how HeidelbergCement analysts and management are using JDA's Supply Chain Strategist occurred in 2007. Company management studied whether they should build a new green field plant in order to serve growing customer demand. They modelled three of the existing plants in that region, and then added the new plant to the model. The JDA model actually suggested that they sequentially upgrade the capacity of the existing plants, instead of investing in a new facility—a decision that saved about $200 million euros.

Royal Mail

Royal Mail operates the UK's largest road vehicle fleet of 40,000 vehicles as well as overnight rail and air networks, moving some 81 million items a day. The road fleet includes vehicles for national distribution network (4,100 large vehicles and trailers) and fleet for local delivery and collection.[6] In 2004, Royal Mail selected PlanOp, Jeppesen's logistics optimization software, to assist in reviewing and rationalizing its postal transport network. PlanOp helped develop a new distribution network for Royal Mail that reduced the operating costs by £41 million per annum.

Problems

1. Formulate the following linear program as a compact transportation model in Excel and solve it using Excel's Solver. (LO1)

$$\begin{aligned}
&\text{Minimize } 8x_{11} + 2x_{12} + 5x_{13} + 2x_{21} + x_{22} + 3x_{23} + 7x_{31} + 2x_{32} + 6x_{33} \\
&\text{Subject to } x_{11} + x_{12} + x_{13} \le 90 \\
&\quad x_{21} + x_{22} + x_{23} \le 105 \\
&\quad x_{31} + x_{32} + x_{33} \le 105 \\
&\quad x_{11} + x_{21} + x_{31} = 150 \\
&\quad x_{12} + x_{22} + x_{32} = 75 \\
&\quad x_{13} + x_{23} + x_{33} = 75 \\
&\quad \text{All variable} \ge 0
\end{aligned}$$

2. A toy manufacturer wants to open a third shop/warehouse that will supply three retail outlets. The new shop/warehouse can supply 500 units of backyard play-sets per week. Two locations are being studied, N1 and N2. Unit transportation costs for location N1 to stores A, B, and C are $6, $8, and $7, respectively; for location N2, the unit transportation costs are $10, $6, and $4, respectively. The existing network's data are shown in the following table. Which location would result in lower total transportation cost? Assume that unit production costs are the same in N1 and N2. (LO2)

	Store A	Store B	Store C	Capacity/wk
Shop/Warehouse 1	8	3	7	500
Shop/Warehouse 2	5	10	9	400
Demand/wk	400	600	350	

[4] http://www.aimms.com/references/case-studies/horoz-lojistik.
[5] http://chinese.jda.com/file_bin/casestudies/HeidelbergCement-AG_case-study.pdf.
[6] http://www.jeppesen.com/documents/land/Royal-Mail-network-redesign.pdf.

3. A large firm is contemplating construction of a new manufacturing facility. The two leading locations are Hamilton and Thunder Bay. The new factory would have a supply capacity of 160 units per week. Unit transportation cost from each potential location and existing manufacturing facilities locations 1, 2, and 3 to markets A, B, and C are shown in the following tables. Determine which new location would provide lower total transportation cost. (LO2)

From Hamilton To	Transport Cost per Unit	From Thunder Bay to	Transport Cost per Unit
A …………..	$18	A ………..	$7
B …………..	8	B ………..	17
C …………..	13	C ………..	13

	A	B	C	Supply/wk
1	10	14	10	210
2	12	17	20	140
3	11	11	12	150
				500
Demand/wk	220	220	220	660

4. A large retailer is planning to open a new store (the retailer currently has stores A and B). Three locations are currently under consideration: South Coast Plaza (SCP), Fashion Island (FI), and Laguna Hills (LH). Unit transportation costs from the warehouses 1, 2, and 3 to the new locations and unit transportation costs, demands, and supplies from the warehouses to the existing stores are shown below. Each of the new store locations has a demand potential of 300 units per week. Which new store location would yield the lowest total transportation cost for the distribution network? (LO2)

From Warehouse	To SCP	FI	LH
1	$4	$7	$5
2	11	6	5
3	5	5	6

Warehouse	Store A	B	Supply/wk
1	15	9	660
2	10	7	340
3	14	18	200
			1200
Demand/wk	400	500	900

5. A petroleum company has two refineries in NE and SW United States from which it supplies gasoline to three markets.[7] The supplies and demands of premium gasoline and unit transportation cost from each refinery (S_1 and S_2) to each market (D_1, D_2, and D_3) are given below. Formulate this problem as a transportation model in Excel and solve it using Excel's Solver. (LO1)

	Proposed Volumes (in 1,000 barrels/day)
NE refinery (S_1)	30.70
SW refinery (S_2)	28.06
NE market (D_1)	29.38
SE market (D_2)	8.81
SW market (D_3)	20.57

[7] J. S. Aronofsky et al, *Managerial Planning with Linear Programming in Process Industry Operations*, New York: Wiley, 1978, p. 168.

	Unit Transportation Cost ($/barrel)
$S_1 \rightarrow D_1$	1.4
$S_1 \rightarrow D_2$	2.8
$S_1 \rightarrow D_3$	3.5
$S_2 \rightarrow D_1$	3.2
$S_2 \rightarrow D_2$	2.4
$S_2 \rightarrow D_3$	1.2

6. An oil company has three oil fields and five refineries.[8] The availabilities and requirements (in million barrels per week), and unit transportation costs (in $ per barrel) are given below: (LO1)

	Refineries					
Oil field	R1	R2	R3	R4	R5	Availability
OF1	5	3	3	3	7	4
OF2	5	4	4	2	1	6
OF3	5	4	2	6	2	7
Requirements	2	2	3	4	4	

Formulate this problem as a transportation model in Excel and solve it using Excel's Solver.

7. A construction company has to move 4 large cranes from 4 old construction sites to 4 new construction sites.[9] The distances (in km) between the old and new sites are: (LO2)

	N1	N2	N3	N4
O1	25	30	17	43
O2	20	23	45	30
O3	42	32	18	26
O4	17	21	40	50

Determine a plan for moving the cranes that will minimize the total distance involved in the move.

8. A rent-a-car company has to fulfill the excess demand for cars in 4 different cities: 2 cars in City A, 3 in City B, 5 in City C, and 7 in City D. The company has excess supply of cars in 3 cities: 6 cars in City E, 1 in City F, and 10 in City G. The distances between the cities (in 100 km) are: (LO2)

	A	B	C	D
E	7	11	3	2
F	7	6	10	1
G	9	15	8	5

What is the optimal distribution policy that minimizes total travelling distance?

9. The association responsible for central power transmission in Norway would like to determine the usage of each independent electric power generator (subscriber) of the common power network.[10] Each subscriber may have power generators in various locations and may sell its power to various customers. Suppose a subscriber has three sources (power generators in three locations) and four major customers. The supply and demand, and the distances of sources and destinations are given below. Determine the minimum total km-megawatt needed to supply

[8] S. P. Bradley et al., *Applied Mathematical Programming*, Massachusetts: Addison-Wesley, 1977, p. 359.
[9] A. J. Hughes and D. E. Grawiog, *Linear Programming: An Emphasis on Decision Making*, Massachusetts: Addison-Wesley, 1973, p. 350.
[10] O. Aarvik and P. Randolph, "The Application of Linear Programming to the Determination of Transmission Line Fees in an Electric Power Network," *Interfaces*, 6(1), Nov 1975, pp. 47–49.

the demands. The association agrees that this minimum is an acceptable measure of the usage of the network by this subscriber. The usage of the common network is then multiplied by a fee for each km-megawatt to determine the charge to the subscriber. (LO1)

Supply (1000 megawatt):	Source 1	Source 2		Source 3
	35	50		40
Demand (1000 megawatt):	Cust 1	Cust 2	Cust 3	Cust 4
	45	20	30	30

Distances (100 km):

	Cust 1	Cust 2	Cust 3	Cust 4
Source 1	8	6	10	9
Source 2	9	12	13	7
Source 3	14	9	16	5

10. The school board in Monroe County of Indiana must choose the bus companies to drive their students during next academic year.[11] Six companies have bid on eight bus routes (some bid on only some bus routes). The bids (in $1000) are as follows: (LO2)

Company	Route 1	Route 2	Route 3	Route 4	Route 5	Route 6	Route 7	Route 8
1		8.2	7.8	5.4		3.9		
2	7.8	8.2		6.3		3.3	4.9	
3		4.8				4.4	5.6	3.6
4			8	5	6.8		6.7	4.2
5	7.2	6.4		3.9	6.4	2.8		3
6	7	5.8	7.5	4.5	5.6		6	4.2

a. If the school board wishes to limit the number of bus routes awarded to any company to 2, determine the awards.

b. If there is no limit on the maximum number of bus routes a company can receive, what would the awards be? Hint: You can determine this visually from the bids.

***11.** The Austin-based out-of-state Auditors of State of Texas were to be relocated out of Texas.[12] This move was because of the high cost of frequent trips that the auditors took in order to audit the out-of-state companies that did business in Texas. The audits were to make sure that the companies paid sales, franchise, and fuel taxes. Five locations were considered. The fixed annual cost of having an office in each location is given below. Also, the companies to be visited have been grouped into five regions, with the demand for audit represented as number of trips to the companies in that region per year (each trip lasted approximately two weeks); see below. Finally, the data for typical airfare from each potential office location to a city in each region is given below. The maximum number of audit trips out of each office is limited to 375 (in order to limit the number of auditors in each office to 15). (LO2)

a. Determine the optimal locations of offices by using the transportation model for each possible set of potential office locations.

b. Determine the optimal locations of offices by using the location-allocation formulation of the transportation model as follows: Define a binary variable for each potential location, for each row calculate the Row Total × Binary variables – 999 × Binary variable, add sum of Fixed cost × Binary variable to the objective function cell; in the Solver Parameters window, include binary variables as variables, add constraints defining binary variables,

[11] W. L. Winston and S. C. Albright, *Practical Management Science*, 2nd Ed, California: Duxbury, 2001, p. 225.
[12] J. A. Fitzsimmons and L. A. Allen, “A Warehouse Location Model Helps Texas Comptroller Select Out-of-State Audit Offices,” *Interfaces*, 13(5), 1983, pp. 40–46.

and add constraints Row Total × Binary variable − 999 × Binary variable ≤ 0. A binary variable is a variable that can assume only values 0 or 1. The last set of constraints will force the binary variable for a location to be 1 if any quantities are sent out from it.

Fixed annual cost (in $1000):

LA	Tulsa	Chicago	Atlanta	New York
160	80	140	70	200

Demand for audits/year:

West(W)	South-central(SC)	Midwest(MW)	South(S)	Northeast(NE)
460	230	230	270	310

Typical air fare between potential office locations and a city in each region:

	W (Portland)	SC (Denver)	MW (Minot)	S (Miami)	NE (Pittsburgh)
LA	$360	210	435	373	349
Tulsa	406	290	584	412	361
Chicago	441	313	518	234	224
Atlanta	537	293	537	477	278
New York	518	349	530	275	125

***12.** The crude oil to be exploited from under Elk Hills Naval field in California must be awarded to the highest bidders.[13] A smaller version of the problem is as follows. There are two shipping points, A and B, each with an expected production of 10,000 barrels of crude oil per day. There are two bidders: Company C's bid for crude oil at shipping point A offers a bonus of $1/barrel over the highest price offered for similar grade crude oil produced from non-government fields in the area. Company D's bid for shipping point A crude oil offers a $0.90/barrel bonus. Company C's bid for shipping point B crude oil has a $2/barrel bonus and Company D's bid for shipping point B crude oil has a $1.5/barrel bonus. According to legislation, no bidder should receive more than 75 percent of the total crude oil exploited. In addition, Company D cannot handle more than 12,000 barrels per day. (LO1 & 2)

a. Formulate this problem as a Transportation model but *maximize* the total bonus received by the government.

b. Because of fixed costs involved, both companies have requested a minimum quantity of 5,000 barrels per day for any shipping point sales awarded to them (i.e., if $x_{ij} > 0$ then it must be ≥ 5,000). Represent this in your formulation of Part a, and re-solve the problem. Hint: you need to introduce a binary variable for each shipment quantity and compute the (Shipment quantity – 10,000 × Binary variable) and (Shipment quantity – 5,000 × Binary variable), and in the Solver Parameters window, include the binary variables as variables, and add binary variables, (Shipment quantity – 10,000 × Binary variable ≤ 0) and (Shipment quantity – 5,000 × Binary variable ≥ 0) as constraints.

[13] B. L. Jackson and J. M. Brown, "Using LP for Crude Oil Sales at Elk Hills: A Case Study," *Interfaces*, 10(3), 1980, pp. 65–69.

MINI-CASE

NewPage's Wickliffe Kentucky Paper Mill

Every day large rolls of paper have to be distributed from NewPage's Wickliffe paper mill to its North American customers.[14] First the load planner consolidates less-than truckloads and splits larger-than truckloads into two or more truckloads. Then, the transportation planner assigns the truckloads to available carriers (some contracted). A small version of the problem is as follows. There are 32 truckloads that have to be delivered to 12 destinations (some trucks need to stop on the way and drop off 1 to 3 loads). There are 33 trucks available (one will not be used) with specific number of trucks from each of the 6 carriers. All carriers have a minimum charge per truckload and stop-off charge (if any). Some carriers should receive a minimum number of truckloads according to their contract. These data, including the distance to the destinations and transport cost ($ per truckload per mile) for each destination/carrier pair are given in the following table. What is the least-cost assignment of truckloads to carriers that will meet the above requirements?

					Carrier					
Destination	**State**	**Truckloads**	**Stops**	**Miles**	**A**	**B**	**C**	**D**	**E**	**F**
Atlanta	GA	4	0	612	–	.88	1.15	.87	.95	1.05
Everett	MA	1	3	612	–	1.18	1.27	1.39	1.35	1.28
Ephrata	PA	3	0	190	–	3.42	1.73	1.71	1.82	2
Riverview	MI	5	0	383	.79	1.01	1.25	.96	.95	1.11
Carson	CA	1	2	3063	–	.8	.87	–	1	–
Chamblee	GA	1	0	429	–	1.23	1.1	1.22	1.33	1.47
Roseville	MN	1	3	600	1.24	1.13	1.89	1.32	1.41	1.41
Hanover	PA	1	0	136	–	4.78	2.23	2.39	2.26	2.57
Sparks	NV	2	0	2439	–	1.45	–	1.2	–	–
Parsippany	NJ	1	1	355	–	1.62	1.36	1.39	1.03	1.76
Effingham	IL	5	0	570	.87	.87	1.25	.87	.9	1.31
Kearny	NJ	7	0	324	–	2.01	1.54	1.53	1.28	1.95
		Minimum charge per truckload			350	400	350	300	350	300
		Stop-off charge			50	75	50	35	50	50
		Available trucks			4	8	7	7	3	4
		Commitment (per contract)			1	7	6	0	0	4

[14] W. L. Winston and S. C. Albright, *Practical Management Science*, 2nd Ed, California: Duxbury, 2001, pp. 271–273.