Problem Statement:

In the case, a student named Kelly would like to build the next semester's schedule. Given data is the list of acceptable courses with scheduled times and Kelly's preference ratings. She would like to ***determine a schedule of classes that will maximize the total rating.*** Kelly's requirements are as follows:

1. Do not take more than one section of each course.
2. Do not take more than one course during any time slot.
3. Take MGT 490 and FIN 358.
4. Take exactly two courses out of FIN 325, 352, 356, and 359.
5. Take one of CIS 102T and CIS 102W.
6. Take exactly five courses.

A key step is to organize the given data in a table format that lends itself to a linear program.

2.1. Input Data:

Given the list of courses in the case, the ratings can be placed in a table where rows represent the distinct meeting patterns (or "time slots") and columns represent the distinct courses. The table is shown in Figure 1. Notice the time slots 6, 7, and 9 represent twice-a-week meeting times. It is possible to have some time slots overlap as long as we remember not to choose more than one course in the overlapping time slots. Using the term "course section" to represent time slot-course pair, this table includes ratings of only the course sections under consideration. The empty cells in the table represent the course sections that are unavailable or unacceptable.

	Available Courses, Time slots, and Ratings								
		Courses							
		1	2	3	4	5	6	7	8
	Time Slots	MGT 490	FIN 358	CIS 102T	CIS 102W	FIN 325	FIN 352	FIN 356	FIN 359
1	M Eve	4.3					**3.6**		3
2	T Eve	3.8			**3.7**			3.2	
3	W Eve	3.5	**3.5**						3.5
4	Th Eve					3			
5	F Eve	3.5							
6	M 1:25-3:15 W 1:25-2:20						3.9		
7	M 1:25-2:20 W 1:25-3:15	**4.6**				3.7			
8	W 2:30-5:15			4.4	3.5				
9	T 1:25-3:15 Th 1:25-2:20	2.7	3.3					**3.4**	
10	Th 2:30-5:15			3.1					

Figure 1. Time Slot and Ratings for each class

Heuristic Solution

In the following, let cell (i,j) represent the cell in row i and column j (i=1,2,…,10; j=1,2,…,8). Repeat the step below until all rows and columns are crossed out: Select a cell (i,j) with the highest rating. Add this course section to the schedule. Cross out row i to indicate time slot i has been taken. Cross out column j to indicate course j is taken. Also, cross out any other row corresponding to a time slot that overlaps with i and any other column that corresponds to a course that is no longer is required because course j is taken. Note: When there is a tie, i.e., more than one cell with the highest rating, break the tie with some rule. A simple rule is the "northwest-first" rule, choosing the cell in the uppermost row and the leftmost column. The steps for the given data in Figure 1 are as follows:

1. Choose cell (7,1), MGT 490 at MW 1:25 pm as it has the highest rating of 4.6. Cross out row 7 and column 1. In addition, since row 7 overlaps with rows 6 and 8, cross them out as well.

2. Among the remaining cells, the highest rating is 3.7 at cell (2,4), CIS 102W on Tuesday evening. Cross out row 2 and columns 3 and 4 (since only one of the two CIS courses is needed).

3. Next, choose cell (1,6), FIN 352 on Monday evening with the rating of 3.6. Cross out row 1 and column 6.

4. There are two cells with the highest rating of 3.5: (3,2) and (3,8). Choose (3,2) using the northwest-first tie-breaking rule. This means taking FIN 358 on Wednesday evening. Cross out row 3 and column 2.

5. There are two remaining cells with ratings: 3 at cell (4,5) and 3.4 at cell (9,7). Choose (9,7), FIN 356 at TTh 1:25 pm. Cross out row 9 and column 7. This is the second of the two finance electives required, so cross out all the remaining finance elective columns: 5 and 8. All of the cells are now crossed out.

The resulting schedule is a 4-day schedule, shown in Table 2, with total rating of 18.8. At the end, all of the cells will be highlighted. This is helpful as a visual aid. The discussion of the heuristic is followed by the linear program formulation.

Table 2: Heuristic Solution

Period	M	T	W	Th	F
1:25-2:20	MGT 490	FIN 356	MGT 490	FIN 356	
2:30-3:15		FIN 356	MGT 490		
3:30-4:25					
4:30-5:25					
Eve	FIN 352	CIS 102 W	FIN 358		

Linear Programming - Algebraic Model The model we use is a variant of the assignment model with binary decision variables. We explain that for each available course section, we either take it or not take it, so we can use "yes-or-no" variables. Let

$$x_{ij}=\left\{\begin{matrix} 1 & if\ course\ j\ is\ taken\ at\ time\ slot\ i \\ 0 & otherwise \\ & i = 1-10, j = 1-8 \end{matrix}\right\}$$

As the problem size is small, it is not necessary to limit the decision variables to only the existing course sections. In the Excel model, it is easier to use all of the 80 course sections (all the cells in the data in Figure 1) instead of only the 20 existing course sections (the ones with positive ratings in Figure 1). Here, it should be clarified that because the ratings for non-existing course sections are 0, including non-existing course sections in the Excel model does not affect the solution. ***The objective is to maximize the total rating from the courses in the schedule:***

$$\sum_{i=1}^{10}\sum_{j=1}^{8} c_{ij}x_{ij}$$

Where c_{ij} = rating of the course j at time slot i shown in Figure 1. In our course, we do not use the sigma notation, but **write out the objective function using the given ratings.**

The constraints are in the form of "number of course sections taken (i.e., sum of relevant variables) is 'equal to' or 'less than or equal to' some value." The time slots are similar to supply points in a transportation model. For each time slot, no more than one course can be taken:

For each time slot, no more than one course can be taken:

$$\sum_{j=1}^{8} x_{ij} \leq 1 \ (i = 1,,,10)$$

Note time slots 6 and 7 overlap and 7 and 8 overlap. This means only one course can be taken across each pair of overlapping time slots. Hence, the following additional constraints are needed

$$\sum_{j=1}^{8} x_{6j} + \sum_{j=1}^{8} x_{7j} \leq \ 1$$

Write a Similar equation for time 7 and 8.

As for the courses, the required courses and elective courses call for different types of constraints. For each required course, MGT 490 and FIN 358, exactly one section must be taken (regardless of how much Kelly loves the subject):

$$\sum_{i=1}^{10} x_{ij} = 1 \ (j = 1,2)$$

For each elective course, at most one section must be taken:

$$\sum_{i=1}^{10} x_{ij} \leq 1 \ (j = 3,4,5,6,7,8)$$

Questions: (15 pts)

1. Write out the objective function to maximize the total rating
2. Write ALL constraints for all courses AND time slots
3. Combine step 1 and 2 to formulate the Linear Program

Extra Credit: 5pts

Solve your Linear Program formulated above to get the optimal course schedule. You can use Python

Or MS Excel Solver(text)