NORTHWESTERN UNIVERSITY | SCHOOL OF PROFESSIONAL STUDIES

Midterm Exam I

Points possible: 100

Description: The midterm exam will cover topics from sessions 1-4.

Resources: The exam is completely open book. You may use course textbooks, materials provided on Canvas, graphing calculators (such as TI 83 or 84); *but any more advanced calculators, Excel Solver, Web calculators, Web-graphic calculators, or simplex method calculators are not allowed. Programming languages other than Python are also not permitted.*

For questions that require calculations, all calculations should be shown, not just the final answer. This will allow for partial credit for those answers that might be set up correctly but have calculation errors. For questions that specifically require Python, the code and output should be included with your answer. For questions that require graphs, only use Python.

Restrictions: All answers are to be your work only. You are not to receive assistance from any other person.

To complete the exam:

1. Answer all questions on the exam thoroughly. Create a Microsoft Word document, including the question number, the question, your typed answer, and graphs if required. You may use Word's equation editor to complete your answers.

2. Once you have completed your exam, return to the exam item where you downloaded the exam PDF, click View/Complete Assignment, and submit your document.

1. (a) Use Python to find the inverse of the following matrix:

A= $\begin{bmatrix} 1 & 0 & 1 \\ 4 & 1 & -2 \\ 3 & 1 & -1 \end{bmatrix}$

(b) Use Python and your answer to part (a) to find the solution to the following linear system:

$$\begin{aligned} x_1 + \quad & \quad x_3 = 4 \\ 4x_1 + x_2 - 2x_3 & = 0 \\ 3x_1 + x_2 - \ x_3 & = 2 \end{aligned}$$

2. An electronics company produces transistors, resistors, and computer chips. Each transistor requires 3 units of copper, 1 unit of zinc and 2 units of glass. Each resistor requires 3, 2 and 1 units of the three materials respectively, and each computer chip requires 2, 1, and 2 units of these materials respectively. How many of each product can be made with the following amount of material? 90000 units of copper, 50000 units of zinc, and 61000 units of glass?

3. Using Python, Plot the feasible region for the following set of constraints:

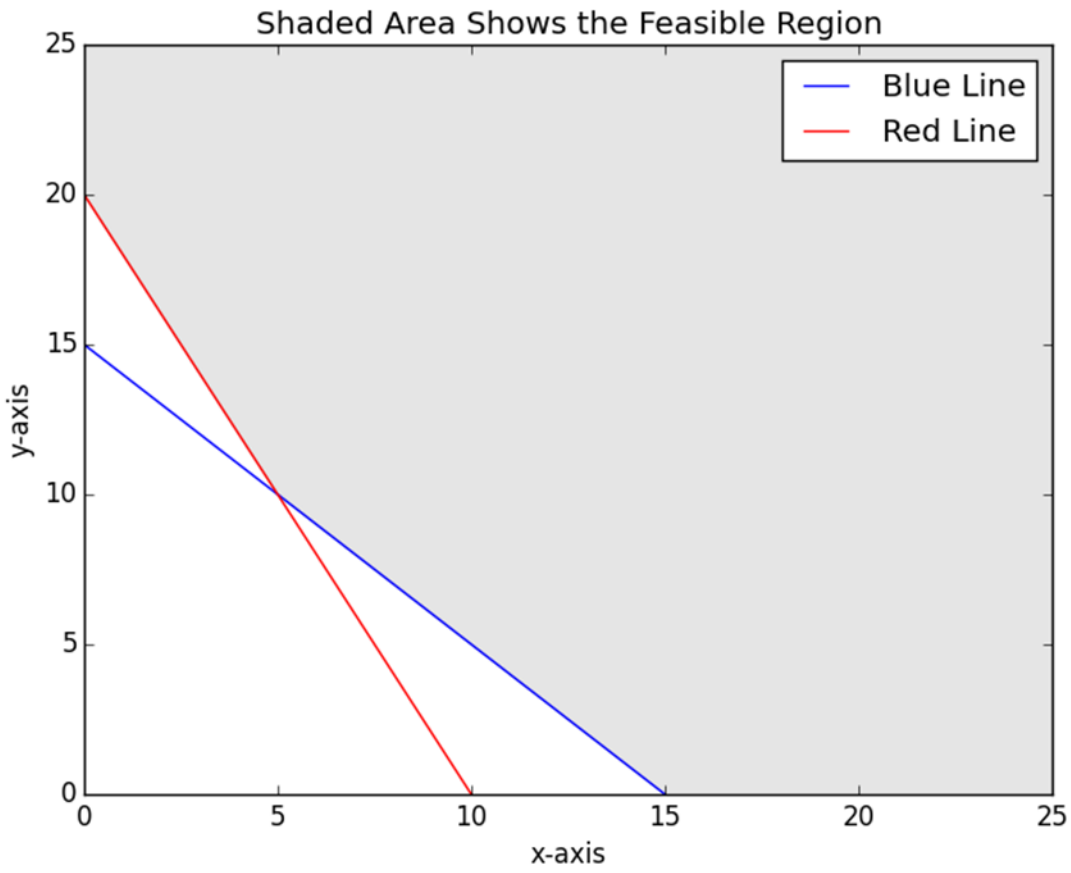
$$\begin{aligned} 2x_1 + x_2 &\leq 100 \\ x_1 + x_2 &\leq 80 \\ x_1 &\leq 40 \end{aligned}$$

$$x_1, x_2 \geq 0$$

4. Jayla is raising money for the homeless and discovers each church group requires 2 hours of letter writing and 1 hour of follow-up calls, while each labor union needs 2 hours of letter writing and 3 hours of follow-up. She can raise $100 from each church group and $175 from each union. She has a maximum of 20 hours of letter writing and 14 hours of follow-up available each month. Determine the most profitable mixture of groups she should contact and the most money she can raise in a month.

5. Debbi has decided to invest a $100,000 inheritance in securities that earn 7% per year, municipal bonds that earn 6% per year, and mutual funds that earn 10% per year. She will spend at least $40,000 on securities and wants at least half the inheritance to go to bonds and mutual funds, Securities have an initial fee of 2%, bonds have an initial fee of 1% and mutual funds have an initial fee of 3%. Debbi has 2400 to pay initial fees. How much should be invested each way to maximize interest yet meet the constraints?

6. Using Python, re-create the following graph. Be sure to show the equations of Red Line and Blue Line.



7. The chemistry department at a college decides to stock at least 900 small test tubes and 600 large test tubes. It wants to buy at least 2700 test tubes to take advantage of a special price. The department will order at least twice as many small tubes as large tubes.
   If the small test tubes cost 18 cents each and the large one's cost 15cents each, how many of each size should be ordered to minimize total cost?

8. Gary's exercise regimen includes running, swimming and playing drums. He has at most 10hrs per week for all three activities combined. He wants the time spent running and playing the drums to be at least twice as long as he swims. His neighbors will not tolerate him playing the drums more than 4 hours per week. Gary burns 388 calories per hour running, 518 calories per hour swimming, and 345 calories per hour playing the drums.
   How many hours per week should he spend on each activity to maximize the number of calories he will burn? What is the maximum calories burnt?

9. Among users of automated teller machines (ATMs), 93% use ATMs to withdraw cash and 32% use them to check their account balance. Suppose that 96% use ATMs to either withdraw cash or check their account balance (or both). Given someone who uses an ATM to check his or her balance, what is the probability that this person also uses an ATM to withdraw cash?

10. Medical Diagnosis: Assume that a patient's heart condition can be categorized according to the following table.

	State of health, H	**P(H)**
H_1	Patient has normal heart	0.8
H_2	Patient has minor heart problems	0.12
H_3	Patient has severe heart condition	0.08

Assume the examination for heart condition consists of stethoscope examination and cardiogram. The outcome of the exam can be one of the following:

$$C_1 = stethoscope\ shows\ normal\ heart\ and\ cardiogram\ shows\ normal\ heart$$

$$C_2 = stethoscope\ shows\ normal\ heart\ and\ cardiogram\ shows\ minor\ problems$$

$$C_3 = stethoscope\ shows\ normal\ heart\ and\ cardiogram\ shows\ severe\ problems$$

The following probabilities are given:

$$P(C_1\,|H_1\,) = 0.9$$

$$P(C_1|H_2\,) = 0.4$$

$$P(C_1\,|H_3) = 0.1$$

Find the probability that the patient has a normal heart given that the examination showed a normal stethoscope examination **and** a normal cardiogram.