1. Lower triangular matrix

Implement the type of the lower triangular matrix. Elements above the main diagonal are zero. It is enough to store the main diagonal and the lower items in a sequence. Implement the following operations: getting an item in the *i*th row and *j*th column, adding and multiplying two matrices.

7x7 lower triangular matrix

	1	2	3	4	5	6	7
1	1						
2	2	3					
3	4	5	6				
4	7	8	9	10			
5	11	12	13	14	15		
6	16	17	18	19	20	21	
7	22	23	24	25	26	27	28

The elements' index in the array.

The items' order in the array.

1	[1,1]
2	[2,1]
3	[2,2]
4	[3,1]
5	[3,2]
6	[3,3]
27	[7,6]
28	[7,7]
29	0

Zero can be stored once.

$$ind(i,j) = j + \sum_{k=1}^{i-1} k = j + \frac{i(i-1)}{2}, \text{ if } 1 \le j \le i \le n$$

ind(1,1)=1
ind(2,1)=2
...
ind(4,2)=8
...
ind(7,7)=28

Type specification:

Type values:	**Type operations:**
$LTM(\mathbb{R}^{n\times n})$	**Sum:** $A=(a, b, c{:}LTM(\mathbb{R}^{n\times n}))$ $c := a+b$
	Product: $A=(a, b, c{:}\ LTM(\mathbb{R}^{n\times n}))$ $c := a*b$
	GetElement: $A=(a{:}\ LTM(\mathbb{R}^{n\times n}), i{:}\mathbb{N}, j{:}\mathbb{N}, e{:}\mathbb{R})$ $e := a[i,j]$

Representation: n: $\mathbb{N}$ v:$\mathbb{R}^{n(n+1)/2}$	**Sum:** $\forall i \in [1..n(n+1)/2]$: $c.v[i] := a.v[i]+b.v[i]$
	Product: $\forall i,j \in [1..n]$: if $i \geq j$ then $c.v[ind(i,j)] = \sum_{k=j}^{i} a.v[ind(i,k)]b.v[ind(k,j)]$
	GetElement: if $i \geq j$ then $e := a.v[\text{ind}(i,j)]$ else $e := 0$

Class: