A Survey of Game Theory Topics in Generative Adversarial Networks

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1 Background

As given by its name, game theory is the study and usage of mathematical models of strategic interaction for an outcome of rational decisions. Simply, we are trying to look at different ways in which the choices of agents produce different outcomes, with respect to the preferences of these agents. One of the major topics looked at in the study of game theory is what would be known as the minimax (or maximin) value. This point basically looks at minimizing the worst possible scenario in terms of loss, or in the case of maximin, maximizing the minimum gain. This was originally formulated for a two-player, zero-sum game, in which each player's loss or gain of utility is exactly balanced by the gains or losses of the other player (the total gain of both players will sum to zero). The minimax/maximin solution, zero-sum games and topics relating to them are some of the most important and fundamental ideas of game theory.

Meanwhile, generative adversarial networks, or GANs for short, are a relatively new form of machine learning framework that was introduced by Ian Goodfellow and colleagues in 2014 [1]. GANs make use of two neural networks, where one is pitted against the other, in order to generate new (but synthetic) instances of data that are so accurate that they pass as real data itself.. In general, machine learning models can be broken down into two main categories, generative and discriminative. The first of the two neural networks used by GANs, called the generator, learns to generate plausible data. These generated instances then become negative training examples for the other neural network within the GAN, called the discriminator. The discriminators job is to distinguish between real data and the synthetic data that the generator outputs; this is done by the discriminator penalizing the generator for producing the incorrect data. This process of creating synthetic data and then improving on it is done cyclically until the data produced by the generator appears so real that the discriminator can no longer distinguish between the two and can be seen in figure 1 below. The potential of GANs is massive as they essentially can learn to mimic almost any kind or distribution of data, making the applications of these form of machine learning numerous. In just six short years since their introduction, they are already widely used in the fields of image, video and even voice generation.

2 Objectives

While these two fields, game theory and generative adversarial networks, may not seem like they overlap too much from the outset, what is apparent after studying GANs is that the basis of how they operate depends on game theory at its core. Most standard GAN approaches have the main goal of finding a pure Nash Equilibrium, typically achieved by using traditional gradient-based techniques in order to minimize each player's cost (in an alternating fashion). Thus, our objective in this survey paper is to study and look at different techniques, strategies or even ideas related to the game theoretic side of generative adversarial networks relating to either increasing the performance or addressing some of the shortcomings of GANs. The focus will particularly be on the study of different alterations and innovations to the two-player, minimax game that the generator and discriminator, in traditional GANs, tend to play. Finally, we will look to propose a broad vision for future research in this field based on the current direction and potential shortcomings we feel still exist in regard to this topic. Section 3 will cover the two-player game model that traditional GANs are built off of, while also introducing ideas that will be covered in our survey of work. Section 4 looks at some of the research being done in this field while section 5 finishes off with an open call for future research.

3 System Model and Problem Formulation

Before understanding what advances are being made in this field, we must first understand how traditional generative adversarial networks work from a theoretic standpoint. As discussed earlier, the seminal paper on GANs authored by Ian Goodfellow and colleagues [1] introduced the two-player minimax game strategy that the majority of traditional GANs follow, which we will outline here. This general framework is easiest to apply and understand when both of the neural networks are multilayer perceptrons. To first learn the generator's distribution, p_g, over data x, there must be input noise variables $p_z(z)$ and a mapping function to the data space defined at $G(z;\theta_g)$ (the multilayer perceptrons neural network has parameters θ_g). The discriminator, which is probabilistic by nature, is defined as $D(x;\theta_d)$, which outputs a single scalar and essentially represents the probability that x came from the data rather then p_g. As discussed earlier, we will train D to maximize the chance that both the training data and samples from G are given the correct label. Meanwhile, G will be trained simultaneously to minimize the value of $\log(1 - D(G(z)))$, thus the objective for G and D, the generator and discriminator, is to either maximize or minimize their objectives. This results in the following two-player, zero-sum minimax game (with the value defined as $V(G, D)$:

$$\min_G \max_D V(D, G) = \mathbb{E}_{\boldsymbol{x}\sim p_{\text{data}}(\boldsymbol{x})}[\log D(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{z}\sim p_{\boldsymbol{z}}(\boldsymbol{z})}[\log(1 - D(G(\boldsymbol{z})))].$$

This equation provides the backbone for the process in which GANs operate and illustrates how the two-player game alternates between the generator and discriminator. However, there have been multiple problems identified with this original equation mostly revolving around the fact that attempting to find a pure Nash Equilibrium through a process like this often causes the gradient descent to not converge. This is one of the central issues in the field of GANs and is even addressed in the seminal paper discussed above as they suggest that the minimax game has a global optimum at $p_g = p_{data}$. Of course, this is not always guaranteed or even possible in some cases and can often lead to a situation where one players progress is repeatedly undone by the other players work, leading to a lack of convergence. This is the motivation for what we will be studying in this survey paper as we will look at various methods and research that has been done with a goal of minimizing this central issue, among others. In particular, the research covered in this paper tends to deal with changes to the traditional two-player game setting laid out above, and how they affect the results and performance of GANs.

4 Survey of Related Work

The first paper we will look at takes the approach of training GANs by finding a mixed strategy in a zero-sum two-player game [2]. The authors of this paper propose the idea that the minimax value of a pure strategy, in terms of the generator that is, is always higher than the minimax value of the equilibrium strategy. Of course, what they are focusing on is getting greater performance out of the generator in terms of the minimax objective, and they propose by finding a mixed strategy, instead of a pure one, that the minimax value of their algorithm will always be

$\epsilon - optimal$ with respect to the minimax value (that is less then or equal to). Without going too in-depth into the algorithm they propose, they essentially are assuming that the game is semi-concave and zero-sum and by utilizing a form of a follow-the-regularized-leader (FTRL) algorithm they are able to output two uniform mixed strategies. The game (M) is considered semi-concave if for any fixed $u_0 \in K_1$, the function $g(v) \coloneqq M(u_0, v)$ is concave in v. While using a basic example of rock-paper-scissors to illustrate how the minimax of mixed strategies is always less then the pure strategy, they also provide a full proof of this lemma, which formally is stated as:

$$\max_{\mathbf{v}\in\mathcal{K}_2} \mathbb{E}_{\mathbf{u}\sim\mathcal{D}_1}[M(\mathbf{u},\mathbf{v})] \leq \min_{\mathbf{u}\in\mathcal{K}_1} \max_{\mathbf{v}\in\mathcal{K}_2} M(\mathbf{u},\mathbf{v}) + \varepsilon$$

An analogous result holds for a mixed strategy with respect to the pure maximin objective. By using these mixed strategies, they are able to take advantage of advances made in supervised deep learning, which makes the reduction of their problem easier, hence making it fairly efficient to find an equilibrium for GANs. They were able to apply their algorithm to practical datasets and compared their performance to that of standard GANs and found that their improved implementation, called CHEKHOV GAN, performed much better in decreasing non-convergence along with decreasing mode dropping, another common problem in the field of GANs.

There has been more research done about the idea of using a mixed Nash Equilibrium to train GANs, including a paper that uses a classical lifting trick to show that essentially all existing GAN objectives can be relaxed into their mixed strategy forms [3]. They prove that the global optima can be solved via sampling, in contrast to the use of optimization frameworks that are typically used in standard implementations of GANs. The authors of this paper also propose a mean-approximation sampling scheme, which they claim allows them to exploit methods for bi-affine games (were affine transformations are applied twice), with the goal of delineating novel and practical training algorithms for GANs. The work done in this paper is similar to that of [2], both of which show through experimentation that their new algorithms outperform that of other GAN training algorithms.

Another innovative approach to solving some of the problems relating to GANs is covered in this next paper, which looks to add a third player to the game, hence resulting in a triple generative adversarial network (called Triple-GAN) [4]. The approach isn't vastly different with the authors proposing the addition of a classifier player in addition to the generator and discriminator. The classifier approximately characterizes the conditional distribution $p_c(y|x) \approx p(y|x)$ while a class-conditional generator approximately characterizes the conditional distribution in the other direction, i.e. $p_g(x|y) \approx p(x|y)$. The discriminator distinguishes whether a pair of data (x, y) comes from the true distribution $p(x, y)$, all of which are neural networks of course. A problem with their initial formulation is that they cannot guarantee that the true distribution (which is equal to the conditional distribution of the generator and the classifier respectively) is the unique global optimum. To address this problem, they propose a standard supervised loss to the classifier, $\mathcal{R}_{\mathcal{L}} = E_{(x,y)\sim p(x,y)}[-\log p_c(y|x)]$, which is essentially equal to the KL-divergence between $p_c(x, y)$ and $p_g(x, y)$. Thus, they define their game as the following, which they prove, given it has utilities $\widetilde{U}$, has the unique global optimum for the classifier and generator.

$$\min_{C,G}\max_{D}\tilde{U}(C,G,D) = E_{(x,y)\sim p(x,y)}[\log D(x,y)] + \alpha E_{(x,y)\sim p_c(x,y)}[\log(1-D(x,y))]$$
$$+(1-\alpha)E_{(x,y)\sim p_g(x,y)}[\log(1-D(G(y,z),y))] + \mathcal{R}_{\mathcal{L}}.$$

More theoretical analysis, theorems and proofs are given to back up their proposal, which can be found in the paper in more detail. Most importantly they theorize that if and only if $p(x,y) = p_c(x,y) = p_g(x,y)$, equilibrium can be achieved. This is the main motivation behind their design of the Triple-GAN, ensuring that both the classifier and generator will converge to the true data distribution if the model has been trained to achieve the optimum.

In addition to the research outlined above, more intriguing work has been done in terms of modifying the two-player game scenario involved with GANs including exploring the idea of using generalized zero-sum games, or even multi-player zero-sum games, for training GANs [5]. It appears that a multi-player setup resulted in much less mode dropping and overall improved performance in terms of generating synthetic images. Another unique approach to expanding on the general two-player adversarial game was done by a group of authors that proposed a new multi-player objective called Discriminator Discrepancy Loss (DDL) [6]. Their DDL-GAN algorithm adds competitions besides the original between the generator and discriminator including one that minimizes the original GAN loss and maximizes DDL and one that while training the generator, minimizes DDL to encourage the generator to confuse the discriminators.

5 Open Call for Future Research

As seen throughout this survey of different variations to the GAN model, in particular with respect to many game theoretic ideas, there is plenty of room for more innovation and advancement. This paper focuses mainly on improving issues with the traditional GAN architecture including reducing non-convergence, mode dropping and vanishing gradients, all of which seem to be topics of active research being done in this field. Despite that fact, these issues still need to remain at the forefront of future research of GANs, as especially non-convergence and mode dropping seem to be the main contributors of weakened performance in terms of GANs. While the papers reviewed in this survey attempt to remedy these issues, none tend to completely remove both issues, as often decreasing the amount of one is done at the cost of the other. Hence, more research needs to be done in this area. Particularly, I would like to see more work done on topics that don't involve adding more players to the two-player adversarial game, as this tends to result in increased computation times and complexity that often cancel out some of the gains achieved by their new implementation.

Another point of research that seems to be something that needs more attention is regarding the hardware side of GANs. As it stands already, the dueling neural network approach puts a heavy strain on computer systems and CPUs in particular due to the large size of the models used and complexity of the algorithms. More research and development must be done with the goal of reducing the complexity and computational strain that generative adversarial networks put on computer systems. GANs are a relatively young and new type of machine learning framework that will require years of future research in order to both increase the performance and decrease the computational horsepower required to run them. Improving on the game theory side of GANs seem to be one of the more efficient and effective ways of achieving these goals.

6 References

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