

Find the second order differentiation

The second derivative of a function $f(x)$ is usually denoted $f''(x)$. That is:

$$F'' = (f')'$$

The second derivative of a dependent variable y with respect to an independent variable x is written

$$\frac{d^2y}{dx^2}$$

This notation is derived from the following formula:

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right).$$

Example:

Find the second order derivatives of the function.

$$x^2 + 3x + 2$$

Solution:

$$\text{Let } y = x^2 + 3x + 2$$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}(x^2) + \frac{d}{dx}(3x) + \frac{d}{dx}(2) = 2x + 3 + 0 = 2x + 3$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}(2x + 3) = \frac{d}{dx}(2x) + \frac{d}{dx}(3) = 2 + 0 = 2$$

Example:

Find the second order derivatives of the function.

$$e^{6x} \cos 3x$$

Solution :

$$\text{Let } y = e^{6x} \cos 3x$$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}(e^{6x} \cdot \cos 3x) = \cos 3x \cdot \frac{d}{dx}(e^{6x}) + e^{6x} \cdot \frac{d}{dx}(\cos 3x)$$

$$= \cos 3x \cdot e^{6x} \cdot \frac{d}{dx}(6x) + e^{6x} \cdot (-\sin 3x) \cdot \frac{d}{dx}(3x)$$

$$= 6e^{6x} \cos 3x - 3e^{6x} \sin 3x \quad \dots\dots\dots(1)$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}(6e^{6x} \cos 3x - 3e^{6x} \sin 3x) = 6 \cdot \frac{d}{dx}(e^{6x} \cos 3x) - 3 \cdot \frac{d}{dx}(e^{6x} \sin 3x)$$

$$= 6[6e^{6x} \cos 3x - 3e^{6x} \sin 3x] - 3 \left[\sin 3x \cdot \frac{d}{dx}(e^{6x}) + e^{6x} \cdot \frac{d}{dx}(\sin 3x) \right] \quad [\text{Using (1)}]$$

$$= 36e^{6x} \cos 3x - 18e^{6x} \sin 3x - 3[\sin 3x \cdot e^{6x} \cdot 6 + e^{6x} \cdot \cos 3x \cdot 3]$$

$$= 36e^{6x} \cos 3x - 18e^{6x} \sin 3x - 18e^{6x} \sin 3x - 9e^{6x} \cos 3x$$

$$= 27e^{6x} \cos 3x - 36e^{6x} \sin 3x$$

$$= 9e^{6x} (3\cos 3x - 4\sin 3x)$$