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## Order

The *order* of a differential equation is the highest order of any differential contained in it.

Examples:

$\frac{dy}{dx} = ax$  is 1st order,  $\frac{d^3y}{dx^3} + \frac{y}{x} = b$  is 3rd order, and  $\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial x} + z = 0$  is 2nd order.

## Ordinary vs. partial

An *ordinary* differential equation (ODE) contains differentials with respect to only one variable, *partial* differential equations (PDE) contain differentials with respect to several independent variables.

Examples:

$\frac{dy}{dx} = ax$  and  $\frac{d^3y}{dx^3} + \frac{y}{x} = b$  are ODE, but  $\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial x} + z = 0$  and  $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y}$  are PDE.

The straight and curly `d`s give it away if used properly. The real test is whether the dependent variable depends on just one or on more independent variables. In the examples above, we have  $y(x)$  but  $z(x,y)$ .

## Linear vs. non-linear

*Linear* differential equations do not contain any higher powers of either the dependent variable (function) or any of its differentials, *non-linear* differential equations do.

Examples:

All of the examples above are linear, but  $\left(\frac{dy}{dx}\right)^2 = y$  isn't.

Note that  $\left(\frac{dy}{dx}\right)^2 \neq \frac{d^2y}{dx^2}$  !

## Homogeneous vs. heterogeneous

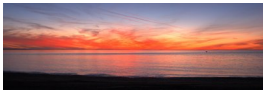
A differential equation is *homogeneous* if it contains no non-differential terms and *heterogeneous* if it does.

Examples:

$\frac{dy}{dx} = ax$  and  $\frac{d^3y}{dx^3} + \frac{dy}{dx} = b$  are heterogeneous (unless the coefficients  $a$  and  $b$  are zero), but  $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y}$  is homogeneous.

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Sunset today: 21:06  
1'35 later than y'day

A zero right-hand side is a sign of a tidied-up homogeneous differential equation, but beware of non-differential terms hidden on the left-hand side!

Solving heterogeneous differential equations usually involves finding a solution of the corresponding homogeneous equation as an intermediate step.

- ODE have been dealt with in Year-1 but we will have a [brief review](#).
- Non-linear differential equations will not be discussed here. There are specialist maths lectures on this topic.
- This leaves us with
  - linear PDE of first order (homogeneous/heterogeneous) and
  - linear PDE of higher order (homogeneous/heterogeneous)

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*Blame me, not my employer etc. etc.*

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