Classification of differential equations

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Order

The *order* of a differential equation is the highest order of any differential contained in it.

Examples:
$\frac{\mathrm{d}y}{\mathrm{d}x} = ax$ is 1st order, $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} + \frac{y}{x} = b$ is 3rd order, and $\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial x} + z = 0$ is 2nd order.

Ordinary *vs.* partial

An *ordinary* differential equation (ODE) contains differentials with respect to only one variable, *partial* differential equations (PDE) contain differentials with respect to several independent variables.

Examples:
$\frac{\mathrm{d}y}{\mathrm{d}x} = ax$ and $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} + \frac{y}{x} = b$ are ODE, but $\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial x} + z = 0$ and $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y}$ are PDE.

The straight and curly \`d\`s give it away if used properly. The real test is whether the dependent variable depends on just one or on more independent variables. In the examples above, we have y(x) but z(x,y).

Linear *vs.* non-linear

Linear differential equations do not contain any higher powers of either the dependent variable (function) or any of its differentials, *non-linear* differential equations do.

Examples:
All of the examples above are linear, but $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = y$ isn't.

Note that $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 \neq \frac{\mathrm{d}^2y}{\mathrm{d}x^2}$!

Homogeneous *vs.* heterogeneous

A differential equation is *homogeneous* if it contains no non-differential terms and *heterogeneous* if it does.

Examples:
$\frac{\mathrm{d}y}{\mathrm{d}x} = ax$ and $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} + \frac{\mathrm{d}y}{\mathrm{d}x} = b$ are heterogeneous (unless the coefficients *a* and *b* are zero),
but $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y}$ is homogeneous.

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Sunset today: 21:06
1'35 later than y'day

A zero right-hand side is a sign of a tidied-up homogeneous differential equation, but beware of non-differential terms hidden on the left-hand side!

Solving heterogeneous differential equations usually involves finding a solution of the corresponding homogeneous equation as an intermediate step.

- ODE have been dealt with in Year-1 but we will have a brief review.
- Non-linear differential equations will not be discussed here. There are specialist maths lectures on this topic.
- This leaves us with
  - linear PDE of first order (homogeneous/heterogeneous) and
  - linear PDE of higher order (homogeneous/heterogeneous)

Blame me, not my employer etc. etc.
Content of this page last modified: 121030