A study on Ricci dark energy in bulk-brane interaction

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In this work we have investigated the effects of the interaction between a brane universe and the bulk in which it is embedded. Considering the effects of the interaction between a brane universe and the bulk we have obtained the equation of state for the interacting holographic Ricci dark energy density $\rho_{\Lambda} = 3c^2(\dot{H} + 2H^2)$ in the flat universe. We have investigated the impact of c^2 on the equation of state ω_{Λ} . Also, considering the power law for of the scale factor, we have observed that non-trivial contributions of dark energy which differ from the standard matter fields confined to the brane is increasing with the evolution of the universe.

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I. INTRODUCTION

The observed accelerated expansion of the universe is usually attributed to the presence of an exotic kind of energy, called "dark energy" (DE)[1–8]. The measurement of WMAP and the supernova analysis predicts that exactly 95.4915028% of the cosmos must be dark energy [10–12]. A great variety of DE models have been proposed so far. Models of dark energy are conveniently characterized by the equation-of-state parameter $\omega = p/\rho$, where ρ is the energy density and p is the pressure [9, 13–15]. The models are discussed at length in the references mentioned above. Examples of DE models are tachyon [16], phantom [17], Chaplygin gas [18], K-essence [19], hessence [20], dilaton [21], holographic dark energy [22] etc. Observational evidences in support of DE are SNIa, CMB and BAO [25–27]. The Type Ia supernova (SNIa) observations, cosmic microwave background (CMB) and baryon acoustic oscillations (BAO) have confirmed that about 73% of the energy density of the present universe consists of dark energy. The so called "holographic dark energy" (HDE), based on the holographic principle [23], was proposed by Li [28]. In holographic principle the black holes are regarded as the maximally entropic objects of a given region and this principle postulates that the maximum entropy inside this region behaves non-extensively, growing

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only as its surface area [29]. Therefore, the number of independent degrees of freedom is bounded by the surface area in Planck units. This implies that an effective field theory with UV cutoff Λ in a box of size L is not self consistent, if it does not satisfy the Bekenstein entropy bound [29]. It was suggested suggested in [30] that the total energy in a region of size L should not exceed the mass of a black hole of the same size. Based on this assumption, reference [28] proposed HDE density as

$$\rho_h = 3c^2 M_p^2 L^{-2} \tag{1}$$

where c^2 is a dimensionless constant. Hsu [31] took Hubble horizon is taken as the IR cut-off in HDE, and subsequently the HDE is written as $\rho_h = 3c^2M_p^2H^2$. This consideration by [31] failed to give the accelerated expansion of the universe. Gao et al [32] proposed a holographic dark energy model in which the future event horizon is replaced by the inverse of the Ricci scalar curvature, and dubbed this model the Ricci dark energy model (RDE). The Ricci curvature of FRW universe is given by

$$R = -6\left(\dot{H} + 2H^2 + \frac{k}{a^2}\right) \tag{2}$$

The density of the RDE is therefore [33]

$$\rho_{\Lambda} = 3c^2 M_p^2 R = 3c^2 M_p^2 \left(\dot{H} + 2H^2 + \frac{k}{a^2} \right)$$
 (3)

Cai et al. [34] pointed out that the Ricci dark energy can be viewed as originated from taking the causal connection scale as the IR cutoff in the holographic setting. Here we mention some significant works on RDE. Feng [35] investigated the statefinder diagnostics of the RDE. Feng [36] investigated the nature of RDE with respect to the holographic principle of quantum gravity theory. Feng [37] regarded the f(R) theory as an effective description for the acceleration of the universe and reconstruct the function f(R) from the RDE. Feng and Li [38] investigated the viscous RDE model by assuming that there is bulk viscosity in the linear barotropic fluid and the RDE. In another work Feng and Zhang [39] discussed the role of c^2 in the RDE density (equation 3) and revealed that if $c^2 < 0.5$ then equation of state of RDE will evolve across the cosmological constant boundary -1 i.e. RDE will behave like quintom. In the present work, we are going to discuss the RDE in bulk-brane interaction.

In recent years, the idea that the universe is a brane embedded in a higher-dimensional space, has got much attraction of the researchers [40–47]. Friedmann equation on the brane contains corrections to the usual four-dimensional equation [43]. Binetruy et al [42] found a term $H \propto \rho$,

which is observationally problematic. The model is consistent if a cosmological constant in the bulk and the tension on the brane are considered. This leads to a cosmological version of the RandallSundrum scenario of warped geometries [43]. Bruck et al [43] considered an interaction between the bulk and the brane, which is another non-trivial aspect of brane world theories. The purpose of the present work is to disclose the effect of the energy exchange between the brane and the bulk on the evolution of the universe by considering the flow of energy onto or away from the brane. Employing the RDE in a flat universe, we obtain the equation of state parameter for RDE density. This study is motivated by the works of [49, 51–53]. In an interaction between the bulk and the brane, [49] considered holographic model of dark energy in non-flat universe under the assumption that the CDM energy density on the brane is conserved, but the holographic dark energy density on the brane is not conserved due to branebulk energy exchange. Sheykhi [51] considered the agegraphic models of dark energy in a braneworld scenario with branebulk energy exchange under the assumption that the adiabatic equation for the dark matter is satisfied while it is violated for the agegraphic dark energy due to the energy exchange between the brane and the bulk. In the study of [51], the author revealed the the equation of state (EoS) parameter can evolve from the quintessence to phantom. Myung and Kim [52] introduced the brane-bulk interaction to discuss a limitation of the cosmological Cardy-Verlinde formula which is useful for the holographic description of brane cosmology and showed that the presence of the brane-bulk interaction, one cannot find the entropy representation of the first Friedmann equation. Saridakis [47] considered a generalized version of holographic dark energy arguing that it must be considered in the maximally subspace of a cosmological model and showed that in the context of brane cosmology it leads to a bulk holographic dark energy which transfers its holographic nature to the effective 4D dark energy. Saridakis [48] applied the bulk holographic dark energy in general 5Dtwo-brane models and extracted the Friedmann equation on the physical brane and showed that in the general moving-brane case the effective 4D holographic dark energy behaves as a quintom for a large parameter-space area of a simple solution subclass.

In the present study we consider an interaction between the bulk and the brane, which is non-trivial aspect of braneworld theories. We shall discuss the flow of energy onto or away from the brane-universe. Then, using the holographic RDE in flat universe obtained equation of state for interacting RDE. Firstly, we review the formalism of bulk-brane energy exchange. Then we apply this material to a brane-world cosmology under the assumption that the cold dark matter energy density on the brane is conserved, but the RDE density on the brane is not conserved due to branebulk energy exchange.

II. BULK-BRANE ENERGY EXCHANGE

The bulk-brane action is given by [49, 54]

$$S = \int d^5x \sqrt{-G} \left(\frac{R_5}{2\kappa_5^2 - \Lambda_5 + L_R^m} \right) + \int d^4x \sqrt{-g} (-\sigma + L_b^m)$$
 (4)

where R_5 is the curvature scalar of the five-dimensional metric, Λ_5 is the bulk cosmological constant and σ is the brane tension, L_B^m and L_b^m are the matter Lagrangian in the bulk and on the brane, respectively. We consider the cosmological solution with a metric of the form [49, 54]

$$ds^{2} = -n^{2}(t, y)dt^{2} + a^{2}(t, y)\gamma_{ij}dx^{i}dy^{j} + b^{2}(t, y)dy^{2}$$

$$(5)$$

The non-zero components of Einstein tensor can be written as [49, 54]

$$G_{00} = 3\left[\frac{\dot{a}}{a}\left(\dot{a}a + \dot{b}b\right) - \frac{n^2}{b^2}\left(\frac{a''}{a}\frac{a'}{a}\left(\frac{a'}{a} - \frac{b'}{b}\right)\right) + k\frac{n^2}{b^2}\right] , \tag{6}$$

$$G_{ij} = \frac{a^2}{b^2} \gamma_{ij} \left[\frac{a'}{a} \left(\frac{a'}{a} + 2 \frac{n'}{n} \right) - \frac{b'}{b} \left(\frac{n'}{n} + 2 \frac{a'}{a} \right) + 2 \frac{a''}{a} + \frac{n''}{n} \right] + \frac{a^2}{n^2} \gamma_{ij} \left[\frac{\dot{a}}{a} \left(-\frac{\dot{a}}{a} + 2 \frac{\dot{n}}{n} \right) - 2 \frac{\ddot{a}}{a} + \frac{\dot{b}}{b} \left(-2 \frac{\dot{a}}{a} + \frac{\dot{n}}{n} \right) - \frac{\ddot{b}}{b} \right] - k \gamma_{ij} ,$$

$$(7)$$

$$G_{05} = 3\left(\frac{n'}{n}\frac{\dot{a}}{a} + \frac{a'}{a}\frac{\dot{b}}{b} - \frac{\dot{a}'}{a}\right) , \qquad (8)$$

$$G_{55} = 3\left[\frac{a'}{a}\left(\frac{a'}{a} + \frac{n'}{n}\right) - \frac{b^2}{n^2}\left(\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a} - \frac{\dot{n}}{n}\right) + \frac{\ddot{a}}{a}\right) - k\frac{b^2}{a^2}\right],\tag{9}$$

where k denotes the curvature of space k=0,1,1 for flat, closed and open universe, respectively and γ_{ij} is the metric for the maximally symmetric three-dimensional space. The primes and dots denote the derivatives with respect to y and t respectively. The three-dimensional brane is assumed at y=0. The Einstein equations are $G_{\mu\nu}=\kappa_5^2T_{\mu\nu}$, where the stress-energy momentum tensor has bulk and brane components and can be written as [49, 54]

$$T^{\mu}_{\nu} = T^{\mu}_{\nu}|_{\sigma,b} + T^{\mu}_{\nu}|_{m,b} + T^{\mu}_{\nu}|_{\Lambda,B} + T^{\mu}_{\nu}|_{m,B} \tag{10}$$

where.

$$T^{\mu}_{\nu}|_{\sigma,b} = \frac{\delta(y)}{b} diag(-\sigma, -\sigma, -\sigma, -\sigma, 0), \tag{11}$$

$$T^{\mu}_{\nu}|_{\Lambda,B} = diag(-\Lambda_5, -\Lambda_5, -\Lambda_5, -\Lambda_5, -\Lambda_5), \tag{12}$$

$$T^{\mu}_{\nu}|_{m,b} = \frac{\delta(y)}{b} diag(-\rho, p, p, p, 0),$$
 (13)

where, p and ρ are the pressure and density on the brane, respectively. Integrating equations (6) and (7) with respect to y around y = 0 give the following jump conditions

$$a'_{+} = -a'_{-} = -\frac{\kappa_5^2}{6} a_0 b_0(\sigma + \rho),$$
 (14)

$$n'_{+} = -n'_{-} = \frac{\kappa_5^2}{6} b_0 n_0 (-\sigma + 2\rho + 3p), \tag{15}$$

Using equations (14) and (15), one can derive

$$\dot{\rho} + 3\frac{\dot{a}_0}{a_0}(\rho + p) = -\frac{2n_0^2}{b_0}T_5^0,\tag{16}$$

$$\frac{1}{n_0^2} \left[\frac{\ddot{a}_0}{a_0} + \left(\frac{\dot{a}_0}{a_0} \right)^2 - \frac{\dot{a}_0 \dot{n}_0}{a_0 n_0} \right] + \frac{k}{a_0^2} = \frac{\kappa_5^2}{3} \left(\Lambda_5 + \frac{\kappa_5^2 \sigma^2}{6} \right) - \frac{\kappa_5^4}{36} \left[\sigma(3p - \rho) + \rho(3p + \rho) \right] - \frac{\kappa_5^2}{3} T_5^5, \quad (17)$$

where, T_{05} and T_{55} are the 05 and 55 components of $T_{\mu\nu}|_{m,b}$ evaluated on brane. Assuming that the bulk matter relative to bulk vacuum energy is much less than the ratio of the brane matter to the brane vacuum energy, we can neglect the T_5^5 term and this can lead to the derivation of a solution that is largely independent of the bulk dynamics. Considering this approximation and concentrating on the low-energy region with $\rho/\sigma \ll 1$, equations (16) and (17) can be simplified into

$$\dot{\rho} + 3H(1+w)\rho = -2T_5^0 = T,\tag{18}$$

$$H^{2} = \frac{8\pi G_{4}}{3}(\rho + \chi) - \frac{k}{a^{2}} + \lambda, \tag{19}$$

$$\dot{\chi} + 4H\chi \approx 2T_5^0 = -T. \tag{20}$$

The auxiliary field χ incorporates non-trivial contributions of dark energy which differ from the standard matter fields confined to the brane. Hence, with the energy exchange T between the bulk and brane, the usual energy conservation is broken down.

III. RDE IN THE BULK-BRANE INTERACTION

In the present work, we are going to discuss Ricci dark energy (RDE) in bulk-brane interaction. The bulk-brane interaction has been studied for various aspects in the works of [49, 50, 54]. We shall denote the energy density of RDE by ρ_{Λ} . Since we shall consider two dark components in the universe, namely, dark matter and dark energy, we shall have $\rho = \rho_{\Lambda} + \rho_{m}$. We assume that the adiabatic equation for the dark matter is satisfied, while it is violated for the dark energy due to the energy exchange between the brane and the bulk [49, 54],

$$\dot{\rho}_m + 3H\rho_m = 0,\tag{21}$$

$$\dot{\rho}_{\lambda} + 3H(1 + \omega_{\Lambda})\rho_{\Lambda} = T. \tag{22}$$

The interaction between bulk and brane is given by the quantity $T = \Gamma \rho_{\Lambda}$, where Γ is the rate of interaction. Considering $u = \frac{\chi}{\rho_{\Lambda} + \rho_{m}}$, the above equations lead to [49]

$$\dot{u} = \left(\frac{3Hu\Omega_{\Lambda}}{\Omega_{\Lambda} + \Omega_{m}}\right) \left[\omega_{\Lambda} - \frac{1}{3}\left(\frac{\Omega_{m}}{\Omega_{\Lambda}} + 1\right) - \frac{1+u}{u}\frac{\Gamma}{3H}\right]$$
(23)

In our present work, we take $\lambda = 0$ and k = 0. Furthermore, following [49] we have chosen

$$\Gamma = 3b^2(1+u)H\tag{24}$$

In flat (k = 0) FRW universe, the density of the RDE is given by [55]

$$\rho_{\Lambda} = 3c^2 \left(\dot{H} + 2H^2 \right) \tag{25}$$

Differentiating equation (19) with respect to t and using equations (18) to (20) one has (considering $8\pi G_4 = 1$) for flat universe i.e. k = 0

$$\dot{H} = -\frac{1}{6} \left[3\rho_{\Lambda} (1 + \omega_{\Lambda}) + 3\rho_m + 4\chi \right] \tag{26}$$

Using equations (26) and (19) in equation (25) we get

$$\omega_{\Lambda} = \frac{1}{3} \left(1 + \frac{\Omega_m}{\Omega_{\Lambda}} \right) - \frac{2}{3c^2} \tag{27}$$

where, it is defined

$$\Omega_{\Lambda} = \frac{\rho_{\Lambda}}{3H^2}, \quad \Omega_m = \frac{\rho_m}{3H^2}, \quad \Omega_{\chi} = \frac{\chi}{3H^2}.$$
(28)

and subsequently

$$u = \frac{1 - \Omega_{\Lambda} - \Omega_{m}}{\Omega_{\Lambda}} \tag{29}$$

Using $\Omega_{m0} = 0.27$ and $\Omega_{\Lambda0} = 0.73$ we plot the ω_{Λ} in figure 1. In this figure we observe that the EoS parameter is evolving from < -1 to > -1 as we are changing the value of c^2 . This implies a transition from quintessence to phantom. Moreover, we find that for $c^2 < 0.5$ the EoS is staying below -1 and for $c^2 > 0.5$ the EoS is staying above -1. In our calculation, we have taken $\Gamma = 3b^2(1+u)H$ Furthermore, we have considered scale factor in power law form.

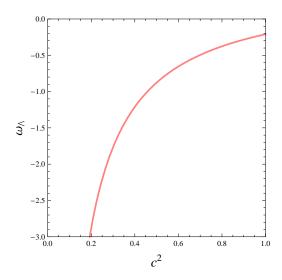


FIG. 1: Variation of ω_{Λ} with c^2 pertaining to RDE. We have taken $\Omega_{\Lambda 0}=0.73$ and $\Omega_{m0}=0.27$.

From equation (27) we can write

$$\dot{u} = -\left(\frac{3Hu\Omega_{\Lambda}}{\Omega_{\Lambda} + \Omega_{m}}\right) \left[\frac{2}{3c^{2}} + \frac{b^{2}(1+u)^{2}}{u}\right]$$
(30)

Since we have $\Omega_{\Lambda} + \Omega_{m} = (1+u)^{-1}$ the equation (30) can be rewritten as

$$\dot{u} = \frac{3Hu(1+u)}{\Omega_{\Lambda}} \left[\frac{2}{3c^2} + \frac{b^2(1+u)^2}{u} \right]$$
 (31)

Let us select the scale factor a in the power law form i.e. $a(t) = a_0 t^n$. Then

$$H = \frac{n}{t} \tag{32}$$

Subsequently, equation (31) becomes

$$\dot{u} = -\frac{3n^2u(1+u)}{tc^2(2n-1)} \left[\frac{2}{3c^2} + \frac{b^2(1+u)^2}{u} \right]$$
(33)

In figure 2 we have plotted u against t. We observe that u is increasing with time. This indicates that non-trivial contributions of dark energy which differ from the standard matter fields confined to the brane is increasing with the evolution of the universe.

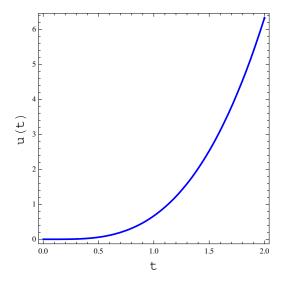


FIG. 2: Pattern of u with evolution of the universe under power law expansion. We have taken c = 0.7, n = 1/3 and b = 2.

IV. CONCLUSION

In this work we have investigated the effects of the interaction between a brane universe and the bulk in which it is embedded. Considering the effects of the interaction between a brane universe and the bulk we have obtained the equation of state for the interacting holographic Ricci dark energy density $\rho_{\Lambda} = 3c^2(\dot{H} + 2H^2)$ in the flat universe. We have seen that there is a crossing of the barrier $\omega_{\Lambda} = -1$ as c^2 shifts from < 0.5 to > 0.5. If $c^2 < 0.5$ the EoS behaves like phantom (≤ -1) and if $c^2 > 0.5$ the EoS behaves like quintessence (≥ -1). Next we have chosen a particular form of scale factor $a(t) = a_0 t^n$. We expressed \dot{u} as a function of t and solving numerically we plotted u against t and we found an increasing behavior in u. We observed that non-trivial contributions of dark energy which differ from the standard matter fields confined to the brane is increasing with the evolution of the universe.

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