Modern Physics Letters A, Vol. 17, No. 32 (2002) 2089–2094

THE CARDY–VERLINDE FORMULA AND ENTROPY OF TOPOLOGICAL REISSNER–NORDSTRÖM BLACK HOLES IN DE SITTER SPACES

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Received 20 September 2002
Revised 16 October 2002

In this paper we discuss the question of whether the entropy of cosmological horizon in topological Reissner–Nordström–de Sitter spaces can be described by the Cardy–Verlinde formula, which is supposed to be an entropy formula of conformal field theory in any dimension. Furthermore, we find that the entropy of black hole horizon can also be rewritten in terms of the Cardy–Verlinde formula for these black holes in de Sitter spaces, if we use the definition due to Abbott and Deser for conserved charges in asymptotically de Sitter spaces. Our result is in favour of the dS/CFT correspondence.

Keywords: The Cardy–Verlinde formula; cosmological horizon; black hole horizon; ds/CFT correspondence; asymptotically de Sitter spaces.

1. Introduction

Holography is believed to be one of the fundamental principles of the true quantum theory of gravity.[1,2] An explicitly calculable example of holography is the much-studied AdS/CFT correspondence. Unfortunately, it seems that we live in a universe with a positive cosmological constant which will look like de Sitter space–time in the future. Therefore, we should try to understand quantum gravity or string theory in de Sitter space preferably in a holographic way. Of course, physics in de Sitter space is interesting even without its connection to the real world; de Sitter entropy and temperature have always been mysterious aspects of quantum gravity.[3]

While string theory has successfully addressed the problem of entropy for black holes, dS entropy remains a mystery. One reason is that the finite entropy seems to suggest that the Hilbert space of quantum gravity for asymptotically de Sitter space is finite dimensional.[4,5] Another related reason is that the horizon and entropy in de Sitter space have an obvious observer dependence. For a black hole in flat space (or even in AdS) we can take the point of view of an outside observer who can assign a unique entropy to the black hole. The problem of what an observer venturing inside the black hole experiences, is much more tricky and has not been given a satisfactory answer within the string theory. While the idea of black hole complementarity provides useful clues,[6] rigorous calculations are still limited to the perspective of the outside observer. In de Sitter space there is no way to escape the problem of the observer dependent entropy. This contributes to the difficulty of de Sitter space.

More recently, it has been proposed that in a manner defined analogous to the AdS/CFT correspondence, quantum gravity in a de Sitter (dS) space is dual to a certain Euclidean CFT living on a spacelike boundary of the dS space[7] (see also earlier works[8–10]). Following the proposal, some investigations on the dS space have been carried out recently.[9–27] According to the dS/CFT correspondence, it might be expected that as the case of AdS black holes,[28] the thermodynamics of cosmological horizon in asymptotically dS spaces can be identified with that of a certain Euclidean CFT residing on a spacelike boundary of the asymptotically dS spaces.

In this paper, we will show that the entropy of cosmological horizon in the topological Reissner–Nordström–de Sitter spaces (TRNdS) can also be rewritten in the form of Cardy–Verlinde formula. We then show that if one uses the Abbott and Deser (AD) prescription,[29] the entropy of black hole horizons in dS spaces can also be expressed by the Cardy–Verlinde formula.

2. Topological Reissner–Nordström–de Sitter Black Holes

We start with an $(n+2)$-dimensional TRNdS black hole solution, whose metric is

$$ds^2 = -f(r)dt^2 + f(r)^{-1}\,dr^2 + r^2\gamma_{ij}\,dx^i\,dx^j\,,$$
$$f(r) = k - \frac{\omega_n M}{r^{n-1}} + \frac{n\omega_n^2 Q^2}{8(n-1)r^{2n-2}} - \frac{r^2}{l^2}\,, \tag{1}$$

where

$$\omega_n = \frac{16\pi G_{n+2}}{n\ \mathrm{Vol}(\Sigma)}\,, \tag{2}$$

where γ_{ij} denotes the line element of an n-dimensional hypersurface Σ with constant curvature $n(n-1)k$ and volume $\mathrm{Vol}(\Sigma)$, G_{n+2} is the $(n+2)$-dimensional Newtonian gravity constant, M is an integration constant, Q is the electric/magnetic charge of Maxwell field. When $k=1$, the metric Eq. (1) is just the Reissner–Nordström–de Sitter solution. For general M and Q, the equation $f(r)=0$ may have four real roots. Three of them are real, the largest is the cosmological horizon r_c, the smallest is the inner (Cauchy) horizon of black hole, the one in between is the outer horizon r_+ of the black hole. And the fourth is negative and has no physical meaning. The case $M=Q=0$ reduces to the de Sitter space with a cosmological horizon $r_c=l$.

When $k=0$ or $k<0$, there is only one positive real root of $f(r)$, and this locates the position of cosmological horizon r_c.

In the case of $k = 0$, $\gamma_{ij}\, dx^i\, dx^j$ is an n-dimensional Ricci flat hypersurface, when $M = Q = 0$, the solution Eq. (1) goes to pure de Sitter space

$$ds^2 = \frac{r^2}{l^2} dt^2 - \frac{l^2}{r^2} dr^2 + r^2\, dx_n^2 \tag{3}$$

in which r becomes a timelike coordinate.

When $Q = 0$ and $M \to -M$ the metric Eq. (1) is the TdS (topological de Sitter) solution,[32,33] which have a cosmological horizon and a naked singularity, for this type of solution, the Cardy–Verlinde formula also work well.

In the BBM prescription,[22] the gravitational mass, subtracted the anomalous Casimir energy, of the TRNdS solution is

$$E = -M = -\frac{r_c^{n-1}}{\omega_n}\left(k - \frac{r_c^2}{l^2} + \frac{n\omega_n^2 Q^2}{8(n-1)r_c^{2n-2}}\right). \tag{4}$$

Some thermodynamic quantities associated with the cosmological horizon are

$$\begin{aligned} T &= \frac{1}{4\pi r_c}\left(-(n-1)k + (n+1)\frac{r_c^2}{l^2} + \frac{n\omega_n^2 Q^2}{8r_c^{2n-2}}\right), \\ S &= \frac{r_c^n\, \mathrm{Vol}(\sigma)}{4G}, \\ \phi &= -\frac{n}{4(n-1)}\frac{\omega_n Q}{r_c^{n-1}}, \end{aligned} \tag{5}$$

where ϕ is the chemical potential conjugate to the charge Q.

The Casimir energy E_c, defined as $E_c = (n+1)E - nTS - n\phi Q$ in this case, is found to be

$$E_c = -\frac{2nkr_c^{n-1}\, \mathrm{Vol}(\sigma)}{16\pi G}, \tag{6}$$

when $k = 0$, the Casimir energy vanishes, as the case of asymptotically AdS space. When $k = \pm 1$, we see from Eq. (6) that the sign of energy is just contrast to the case of TRNAdS space.[30]

Thus we can see that the entropy Eq. (5) of the cosmological horizon can be rewritten as

$$S = \frac{2\pi l}{n}\sqrt{\left|\frac{E_c}{k}\right|(2(E - E_q) - E_c)}, \tag{7}$$

where

$$E_q = \frac{1}{2}\phi Q = -\frac{n}{8(n-1)}\frac{\omega_n Q^2}{r_c^{n-1}}. \tag{8}$$

We note that the entropy expression (7) has a similar form as the case of TRNAdS black holes.[30]

For the black hole horizon, which is only for the case $k = 1$, the associated thermodynamic quantities are

$$\tilde{T} = \frac{1}{4\pi r_+}\left((n-1) - (n+1)\frac{r_+^2}{l^2} - \frac{n\omega_n^2 Q^2}{8r_+^{2n-2}}\right),$$
$$\tilde{S} = \frac{r_+^n \operatorname{Vol}(\sigma)}{4G}, \tag{9}$$
$$\tilde{\phi} = \frac{n}{4(n-1)}\frac{\omega_n Q}{r_+^{n-1}}.$$

The AD mass of TRNdS solution can be expressed in terms of black hole horizon radius r_+ and charge Q,

$$\tilde{E} = M = \frac{r_+^{n-1}}{\omega_n}\left(1 - \frac{r_+^2}{l^2} + \frac{n\omega_n^2 Q^2}{8(n-1)r_+^{2n-2}}\right). \tag{10}$$

In this case, the Casimir energy, defined as $\tilde{E}_c = (n+1)\tilde{E} - n\tilde{T}\tilde{S} - n\tilde{\phi}Q$, is

$$\tilde{E}_c = \frac{2nr_+^{n-1}\operatorname{Vol}(\sigma)}{16\pi G} \tag{11}$$

and the black hole entropy $\tilde{S}$ can be rewritten as

$$\tilde{S} = \frac{2\pi l}{n}\sqrt{\tilde{E}_c|2(\tilde{E} - \tilde{E}_q) - \tilde{E}_c|}, \tag{12}$$

where

$$\tilde{E}_q = \frac{1}{2}\tilde{\phi}Q = \frac{n\omega_n Q^2}{8(n-1)r_+^{n-1}}, \tag{13}$$

which is the energy of electromagnetic field outside the black hole horizon. Thus we demonstrate that the black hole horizon entropy of TRNdS solution can be expressed in a form as the Cardy–Verlinde formula. However, if one uses the BBM mass Eq. (4), the black hole horizon entropy $\tilde{S}$ cannot be expressed by a form like the Cardy–Verlinde formula. Our result is in favour of the dS/CFT correspondence.

3. Conclusion

The Cardy–Verlinde formula recently proposed by Verlinde,[31] relates the entropy of a certain CFT to its total energy and Casimir energy in arbitrary dimensions. In the spirit of dS/CFT correspondence, this formula has been shown to hold exactly for the cases of dS Schwarzschild, ds topological, ds Reissner–Nordström and dS Kerr black holes. In this paper we have further checked the Cardy–Verlinde formula with topological Reissner–Nordström de Sitter black hole.

For space–times of black holes in dS spaces, the total entropy is the sum of black hole horizon entropy and cosmological horizon entropy. If one uses the BBM mass of the asymptotically dS spaces, the black hole horizon entropy cannot be expressed by a form like the Cardy–Verlinde formula.[32] In this paper, we have found that if one uses the AD prescription to calculate the conserved charges of asymptotically dS spaces, the TRNdS black hole horizon entropy can also be rewritten in a form of Cardy–Verlinde formula, which indicates that the thermodynamics of black hole horizon in dS spaces can also be described by a certain CFT. Our result is also reminiscent of the Carlip's claim[34] that for black holes in any dimension the Bekenstein–Hawking entropy can be reproduced using the Cardy formula.[35]

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