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Observer-based passive control of linear time-delay systems with parametric uncertainty ☆

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Abstract

This paper deals with the problem of observer-based passive control of a class of uncertain linear systems with delayed state and parameter uncertainties. This problem aims at designing the linear state observers such that, for all admissible parameter uncertainties, the observation process remains robustly stable and passive, independently of the time delay. The time delay is assumed to be unknown, and the parameter uncertainties are allowed to be norm-bounded and appear in all the matrices of the state-space model. An effective matrix inequality methodology is developed to solve the proposed problem. We derive the conditions for the existence of the desired robust passive observers, and then characterize the analytical expression of these observers in terms of some free parameters. A numerical example demonstrates the validity and applicability of the present approach.

1. Introduction

Since the introduction of the notion of passivity, many researchers have considered the passive control problem for linear time-delay systems [4,17,18,25,26]. Using classical definitions of passivity and positive realness [1,7,10], the problem of passive control of a class of uncertain linear systems with delayed state and parameter uncertainties is considered [2,3]. The robust passivity synthesis problem for a class of uncertain systems with multiple state delays is investigated in [5]. Passivity of delay-dependent neutral-type systems is considered by using a descriptor model transformation [11]. In particular, when the time delay factor is known, it is emphasized that delay-dependent passivity yields less conservative performance results. These results show that passivity-based methods are highly effective is producing robust controllers to classes of time-delay with parametric uncertainties.

On the other hand, various methods such as algebraic, geometric, inversion approaches, generalized inverse, singular-value decomposition, and the Kronecker canonical form techniques have been used in the observer design [12,15,21]. Furthermore, since the system uncertainties and exogenous disturbance input are unavoidable in modeling, the robust state observer design problem have been studied for many years [13,14,22]. For the controller design case,

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which are independent of the time delay and thus suitable for the systems with unknown time delay, have been developed in [6,23] by using a Riccati equation approach, and the delay-dependent design methods, which are suitable for systems with the time delay being of known size, have been proposed in [8] based on a Riccati equation approach and in [16] based on a linear matrix inequality (LMI) approach. However, little attention has been paid to the state observer design problems in the simultaneous presence of time delay, exogenous disturbance input and parametric uncertainty. This motivates the present research on designing robust passive observers for linear systems against unknown time delay and admissible norm-bounded uncertainties.

This paper deals with the problem of robust passive observer design for a class of uncertain linear systems with delayed state and parameter uncertainties. This problem aims at designing the linear state observers such that, for all admissible parameter uncertainties, the observation process remains robustly stable and passive independently of the time delay. The time delay is assumed to be unknown, and the parameter uncertainties are allowed to be norm-bounded and appear in all the matrices of the state-space model, and may be time varying. A new, simple, algebraic parameterized approach is exploited, which enables us to derive the existence conditions and characterize the set of expected robust passive observers in terms of some free parameters, for a class of uncertain linear state delayed systems. A special feature of the results obtained in the present paper is that the set of the design observer gains, when it is not empty, must be very large because of the free design parameters in the expression of observer gains, and much explicit freedom is subsequently offered which gives the possibility for directly achieving further performance requirements on the observation process. It is shown that a desired solution is related to two Riccati matrix equations, or two quadratic matrix inequalities which are not difficult to solve.

2. Problem formulation and assumptions

We consider a linear uncertain continuous-time state delayed system described by

$$\dot{x}(t) = (A + \Delta A)x(t) + (A_d + \Delta A_d)x(t-d) + D_1 w(t), \tag{1}$$

$$y(t) = (C_1 + \Delta C_1)x(t) + D_2 w(t), \tag{2}$$

$$z(t) = (C_2 + \Delta C_2)x(t) + D_3 w(t), \tag{3}$$

$$x(t) = \varphi(t), \quad \forall t \in [-d, 0], \tag{4}$$

where $x(t) \in R^n$ is the state, $w(t) \in R^r$ is the square integrable exogenous disturbance, $y(t) \in R^p$ is the controlled output. For brevity, we have omitted known input terms in (1)–(3) since it is well known this does not affect the generality of the discussion on the observer design. A, A_d, D_1, C_1, D_2, C_2, D_3 are known constant matrices with appropriate dimensions, d denotes the unknown state delay, $\varphi(t)$ is a continuous vector-valued initial function. ΔA, ΔA_d, ΔC_1, ΔC_2 are real-valued matrix functions representing norm-bounded parameter uncertainties and satisfy:

$$\begin{bmatrix} \Delta A \\ \Delta C_1 \\ \Delta C_2 \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} F N_1, \quad \Delta A_d = M_1 F N_2, \tag{5}$$

where $F \in R^{i \times j}$, which may be time-varying, is a real uncertain matrix with Lebesgue measurable elements and meets

$$FF^{\mathrm{T}} \leqslant I \tag{6}$$

and M_1, M_2, M_3, N_1, N_2 are known real constant matrices of appropriate dimensions parameters in F enter the nominal matrices A, A_d, C_1, C_2. The uncertainties ΔA, ΔA_d, ΔC_1, ΔC_2 are said to be admissible if both (5) and (6) are satisfied. The reason for assuming the structure given in (5) and (6) can be found in many papers dealing with robust control and estimation problems, see [19], and references therein.

Throughout this paper, we will make the following assumptions.

Assumption 1. The system matrix A is asymptotically stable.

Assumption 2. The matrix M_2, M_3 is of full row rank.

It is noted that Assumption 2 does not lose any generality. In this paper, the full-order linear state observer under consideration is of the form:

$$\dot{\hat{x}}(t) = G\hat{x}(t) + Ky(t), \tag{7}$$

where the constant matrices G and K are observer parameters to be designed.

Define the error state $e(t) = x(t) - \hat{x}(t)$, then it follows from (1)–(3) and (7) that

$$\dot{e}(t) = Ge(t) + [(A + \Delta A) - K(C_1 + \Delta C_1) - G]x(t) + (A_d + \Delta A_d)x(t-d) + (D_1 - KD_2)w(t). \tag{8}$$

Now, by defining

$$x_f = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}, \quad A_f = \begin{bmatrix} A & 0 \\ A - G - KC & G \end{bmatrix}, \quad A_{df} = \begin{bmatrix} A_d & 0 \\ A_d & 0 \end{bmatrix}, \tag{9}$$

$$D_f = \begin{bmatrix} D_1 \\ D_1 - KD_2 \end{bmatrix}, \quad C_f = [C_2 \quad 0], \tag{10}$$

$$\Delta A_f = M_f F N_f, \quad \Delta A_{df} = M_{df} F N_{df}, \quad \Delta C_f = M_3 F N_f, \tag{11}$$

$$M_f = \begin{bmatrix} M_1 \\ M_1 - KM_2 \end{bmatrix}, \quad N_f = [N_1 \quad 0], \quad M_{df} = \begin{bmatrix} M_1 \\ M_1 \end{bmatrix}, \quad N_{df} = [N_2 \quad 0] \tag{12}$$

and combining (1)–(3) and (5)–(8), we obtain the following augmented system:

$$\dot{x}_f(t) = (A_f + \Delta A_f)x(t) + (A_{df} + \Delta A_{df})x(t-d) + D_f w(t), \tag{13}$$

$$z(t) = (C_f + \Delta C_f)x(t) + D_3 w(t). \tag{14}$$

Definition 1 [2]. The dynamical systems (13) and (14) is called passive if

$$\int_0^{t_p} w^{\tau}(s)z(s)\,ds \geqslant \beta, \quad \forall w \in L_2[0,\infty),$$

where β is some constant which depends on the initial condition of the system. It is said to be strictly passive (SP) if it is passive and $D_3 + D_3^T > 0$.

The objective of this paper is to design the observer parameters G and K, such that for all admissible parameter uncertainties ΔA_f, ΔA_{df}, ΔC_f the augmented system (13) and (14) is asymptotically stable and passive.

The following lemmas will be useful in designing an expected robust passive observer for the uncertain linear time-delay system (1), (3) and (4).

Lemma 1 [20]. *Let A, L, E and F be real matrices of appropriate dimensions, with F satisfying $FF^T \leqslant I$. Then we have:*

(a) *For any scalar $\varepsilon > 0$,*

$LFE + E^T F^T L^T \leqslant \varepsilon^{-1} LL^T + \varepsilon E^T E.$

(b) *For any matrix $P > 0$ and scalar $\varepsilon > 0$ such that $\varepsilon I - EPE^T > 0$,*

$(A + LFE)P(A + LFE)^T \leqslant A(P - \varepsilon^{-1}EE^T)^{-1}A^T + \varepsilon LL^T.$

(c) *For any matrix $P > 0$ and scalar $\varepsilon > 0$ such that $P - \varepsilon LL^T > 0$,*

$(A + LFE)^T P(A + LFE) \leqslant A^T(P - \varepsilon LL^T)^{-1}A + +\varepsilon^{-1}EE^T.$

Lemma 2 [illegible]. *For a positive-definite matrix Q, if there exists a positive-definite matrix P satisfying the following algebraic Riccati inequality (ARI):*

$$(A_f + \Delta A_f)^T P + P(A_f + \Delta A_f) + Q + P(A_{df} + \Delta A_{df})Q^{-1}(A_{df} + \Delta A_{df})^T P$$
$$+ \left(D_f^T P - C_f - \Delta C_f\right)^T (D_3 + D_3^T)^{-1}\left(D_f^T P - C_f - \Delta C_f\right) < 0 \tag{15}$$

or equivalently satisfying the linear matrix inequality (LMI):

$$\begin{bmatrix} (A_f + \Delta A_f)^T P + P(A_f + \Delta A_f) + Q & P(A_{df} + \Delta A_{df}) & PD_f - C_f^T \\ (A_{df} + \Delta A_{df})^T P & -Q & 0 \\ D_f^T P - C_f & 0 & -(D_3 + D_3^T) \end{bmatrix} < 0 \tag{16}$$

for all admissible parameter uncertainties ΔA_f, ΔA_{df}, ΔC_f, then the system (13) and (14) is robustly asymptotically stable and passive.

3. Main results and proofs

The following theorem offers the theoretical basis for achieving the desired design goal.

Theorem 1. *Let δ_1, δ_2, σ be sufficiently small positive constants and $Q_1 > 0$ be a positive-definite matrix. If there exist positive scalars ε_1, $\varepsilon_2(N_2 Q^{-1} N_2^{\mathrm{T}} < \varepsilon_2 I)$, $\varepsilon_3(M_3 M_3^{\mathrm{T}} < \varepsilon_3(D_3 + D_3^{\mathrm{T}}))$ and a matrix $S \in R^{n\times p}$ such that the following two Riccati equations have positive-definite solutions $P_1 > 0$ and $P_2 > 0$; respectively;*

$$\left[A - C_2^{\mathrm{T}}\left(D_3 + D_3^{\mathrm{T}} - \varepsilon_3^{-1} M_3 M_3^{\mathrm{T}}\right)^{-1} D_1^{\mathrm{T}}\right]^{\mathrm{T}} P_1 + P_1\left[A - D_1\left(D_3 + D_3^{\mathrm{T}} - \varepsilon_3^{-1} M_3 M_3^{\mathrm{T}}\right)^{-1} C_2\right]$$
$$+ P_1\left[\varepsilon_1 M_1 M_1^{\mathrm{T}} + \phi + D_1\left(D_3 + D_3^{\mathrm{T}} - \varepsilon_3^{-1} M_3 M_3^{\mathrm{T}}\right)^{-1} D_1^{\mathrm{T}}\right] P_1$$
$$+ C_2^{\mathrm{T}}\left(D_3 + D_3^{\mathrm{T}} - \varepsilon_3^{-1} M_3 M_3^{\mathrm{T}}\right)^{-1} C_2 + \left(\varepsilon_1^{-1} + \varepsilon_3\right) N_1^{\mathrm{T}} N_1 + Q_1 + \delta_1 I = 0, \tag{17}$$

$$(\widehat{A} - \Omega^{\mathrm{T}} R^{-1} \widehat{C})^{\mathrm{T}} P_2 + P_2(\widehat{A} - \Omega^{\mathrm{T}} R^{-1} \widehat{C}) + P_2\left[D_1\left(D_3 + D_3^{\mathrm{T}} - \varepsilon_3^{-1} M_3 M_3^{\mathrm{T}}\right)^{-1} D_1^{\mathrm{T}}\right.$$
$$\left. + \varepsilon_1 M_1 M_1^{\mathrm{T}} + \phi - \Omega^{\mathrm{T}} R^{-1} \Omega\right] P_2 + S S^{\mathrm{T}} - \widehat{C}^{\mathrm{T}} R^{-1} \widehat{C} + (\sigma + \delta_2) I = 0, \tag{18}$$

where

$$\phi = A_d\left(Q_1 - \varepsilon_2^{-1} N_2^{\mathrm{T}} N_2\right)^{-1} A_d^{\mathrm{T}} + \varepsilon_2 M_1 M_1^{\mathrm{T}}, \tag{19}$$

$$\widehat{A} = A + \varepsilon_1 M_1 M_1^{\mathrm{T}} P_1 + \phi P_1 + D_1\left(D_3 + D_3^{\mathrm{T}} - \varepsilon_3^{-1} M_3 M_3^{\mathrm{T}}\right)^{-1}\left(D_1^{\mathrm{T}} P_1 - C_2\right), \tag{20}$$

$$\widehat{C} = C + \varepsilon_1 M_2 M_1^{\mathrm{T}} P_1 + D_2\left(D_3 + D_3^{\mathrm{T}} - \varepsilon_3^{-1} M_3 M_3^{\mathrm{T}}\right)^{-1}\left(D_1^{\mathrm{T}} P_1 - C_2\right), \tag{21}$$

$$R = \varepsilon_1 M_2 M_2^{\mathrm{T}} + D_2\left(D_3 + D_3^{\mathrm{T}} - \varepsilon_3^{-1} M_3 M_3^{\mathrm{T}}\right)^{-1} D_2^{\mathrm{T}}, \tag{22}$$

$$\Psi = \widehat{C} + \varepsilon_1 M_2 M_1^{\mathrm{T}} P_2 + D_2\left(D_3 + D_3^{\mathrm{T}} - \varepsilon_3^{-1} M_3 M_3^{\mathrm{T}}\right)^{-1} D_1^{\mathrm{T}} P_2, \tag{23}$$

$$\Omega = \varepsilon_1 M_2 M_1^{\mathrm{T}} + D_2\left(D_3 + D_3^{\mathrm{T}} - \varepsilon_3^{-1} M_3 M_3^{\mathrm{T}}\right)^{-1} D_1^{\mathrm{T}}, \tag{24}$$

then the observer (7) with parameters

$$K = P_2^{-1}[\Psi^{\mathrm{T}} R^{-1} + S U R^{-1/2}], \tag{25}$$

$$G = \widehat{A} - K\widehat{C}, \tag{26}$$

where $U \in R^{p\times p}$ is arbitrary orthogonal (i.e., $UU^{\mathrm{T}} = I$), will be such that, independently of the unknown time-delay d, then the augmented system (13) and (14) is asymptotically stable and passive.

Proof. Since the matrix M_2, M_3 is of full row rank, R^{-1} then exists. In view of Lemma 1, we have

$$(\Delta A_f)^{\mathrm{T}} P + P(\Delta A_f) \leqslant \varepsilon_1 P M_f M_f^{\mathrm{T}} P + \varepsilon^{-1} N_f^{\mathrm{T}} N_f, \tag{27}$$

$$(A_{df} + \Delta A_{df}) Q^{-1} (A_{df} + \Delta A_{df})^{\mathrm{T}} \leqslant A_{df}\left(Q - \varepsilon_2^{-1} N_{df}^{\mathrm{T}} N_{df}\right)^{-1} A_{df}^{\mathrm{T}} + \varepsilon_2 M_{df} M_{df}^{\mathrm{T}}, \tag{28}$$

$$\left(D_f^{\mathrm{T}} P - C_f - \Delta C_f\right)^{\mathrm{T}} \left(D_3 + D_3^{\mathrm{T}}\right)^{-1} \left(D_f^{\mathrm{T}} P - C_f - \Delta C_f\right)$$
$$\leqslant \left(D_f^{\mathrm{T}} P - C_f\right)^{\mathrm{T}} \left(D_3 + D_3^{\mathrm{T}} - \varepsilon_3^{-1} M_3 M_3^{\mathrm{T}}\right)^{-1} \left(D_f^{\mathrm{T}} P - C_f\right) + \varepsilon_3 N_f^{\mathrm{T}} N_f. \tag{29}$$

We set

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} > 0, \quad Q = \begin{bmatrix} Q_1 & 0 \\ 0 & \sigma I \end{bmatrix} > 0 \tag{30}$$

and consider definitions (9)–(11) and (13), (20)–(24), then we have

$$\begin{aligned}\Sigma &= (A_f+\Delta A_f)^{\mathrm T}P + P(A_f+\Delta A_f) + Q + P(A_{df}+\Delta A_{df})Q^{-1}(A_{df}+\Delta A_{df})^{\mathrm T}P \\ &\quad + \left(D_f^{\mathrm T}P - C_f - \Delta C_f\right)^{\mathrm T}(D_3+D_3^{\mathrm T})^{-1}\left(D_f^{\mathrm T}P - C_f - \Delta C_f\right) \\ &\leqslant A_f^{\mathrm T}P + PA_f + \varepsilon_1 PM_fM_f^{\mathrm T}P + \varepsilon^{-1}N_f^{\mathrm T}N_f + A_{df}\left(Q - \varepsilon_2^{-1}N_{df}^{\mathrm T}N_{df}\right)^{-1}A_{df}^{\mathrm T} \\ &\quad + \varepsilon_2 M_{df}M_{df}^{\mathrm T} + \left(D_f^{\mathrm T}P - C_f\right)^{\mathrm T}\left(D_3 + D_3^{\mathrm T} - \varepsilon_3^{-1}M_3M_3^{\mathrm T}\right)^{-1}\left(D_f^{\mathrm T}P - C_f\right) + \varepsilon_3 N_f^{\mathrm T}N_f = \begin{bmatrix}\Sigma_{11} & \Sigma_{12}\\ \Sigma_{12}^{\mathrm T} & \Sigma_{22}\end{bmatrix},\end{aligned} \tag{31}$$

where

$$\begin{aligned}\Sigma_{11} &= A^{\mathrm T}P_1 + P_1A + P_1\varepsilon_1M_1M_1^{\mathrm T}P + P_1\phi P_1 + \varepsilon_1^{-1}N_1^{\mathrm T}N_1 + \varepsilon_3N_1^{\mathrm T}N_1 + Q_1 \\ &\quad + \left(P_1D_1 - C_2^{\mathrm T}\right)\left(D_3 + D_3^{\mathrm T} - \varepsilon_3^{-1}M_3M_3^{\mathrm T}\right)^{-1}\left(D_1^{\mathrm T}P_1 - C_2\right),\end{aligned} \tag{32}$$

$$\begin{aligned}\Sigma_{12} &= (A - KC - G)^{\mathrm T}P_2 + \varepsilon_1P_1M_1(M_1 - KM_2)^{\mathrm T}P_2 + P_1\phi P_2 \\ &\quad + \left(P_1D_1 - C_2^{\mathrm T}\right)\left(D_3 + D_3^{\mathrm T} - \varepsilon_3^{-1}M_3M_3^{\mathrm T}\right)^{-1}(D_1 - KD_2)P_2,\end{aligned} \tag{33}$$

$$\begin{aligned}\Sigma_{22} &= G^{\mathrm T}P_2 + P_2G + \varepsilon_1P_2(M_1 - KM_2)(M_1 - KM_2)^{\mathrm T}P_2 + P_2\phi P_2\sigma I \\ &\quad + P_2(D_1 - KD_2)\left(D_3 + D_3^{\mathrm T} - \varepsilon_3^{-1}M_3M_3^{\mathrm T}\right)^{-1}(D_1 - KD_2)^{\mathrm T}P_2.\end{aligned} \tag{34}$$

It follows from (17) that $\Sigma_{11} = -\delta_1 I < 0$. By resorting to $G = \widehat{A} - K\widehat{C}$ and the definitions of R and Ψ, we have

$$\begin{aligned}\Sigma_{22} &= (\widehat{A} - K\widehat{C})^{\mathrm T}P_2 + P_2(\widehat{A} - K\widehat{C}) + P_2\phi P_2 + \sigma I + \varepsilon_1P_2\left(M_1M_1^{\mathrm T} - M_1M_2^{\mathrm T}K^{\mathrm T} - KM_2M_1^{\mathrm T} - KM_2M_2^{\mathrm T}K^{\mathrm T}\right)P_2 \\ &\quad + P_2\Big[D_1\left(D_3 + D_3^{\mathrm T} - \varepsilon_3^{-1}M_3M_3^{\mathrm T}\right)^{-1}D_1^{\mathrm T} - D_1\left(D_3 + D_3^{\mathrm T} - \varepsilon_3^{-1}M_3M_3^{\mathrm T}\right)^{-1}D_2^{\mathrm T}K^{\mathrm T} \\ &\quad - KD_2\left(D_3 + D_3^{\mathrm T} - \varepsilon_3^{-1}M_3M_3^{\mathrm T}\right)^{-1}D_1^{\mathrm T} + KD_2\left(D_3 + D_3^{\mathrm T} - \varepsilon_3^{-1}M_3M_3^{\mathrm T}\right)^{-1}D_2^{\mathrm T}K^{\mathrm T}\Big]P_2 = \widehat{A}^{\mathrm T}P_2 + P_2\widehat{A} + P_2\phi P_2 + \sigma I \\ &\quad + P_2\left[\varepsilon_1M_1M_1^{\mathrm T} + D_1\left(D_3 + D_3^{\mathrm T} - \varepsilon_3^{-1}M_3M_3^{\mathrm T}\right)^{-1}D_1^{\mathrm T}\right]P_2 - (P_2K)\Big[\widehat{C} + \varepsilon_1M_2M_1^{\mathrm T}P_2 \\ &\quad + D_2\left(D_3 + D_3^{\mathrm T} - \varepsilon_3^{-1}M_3M_3^{\mathrm T}\right)^{-1}D_1^{\mathrm T}P_2\Big] - \left[\widehat{C} + \varepsilon_1M_2M_1^{\mathrm T}P_2 + D_2\left(D_3 + D_3^{\mathrm T} - \varepsilon_3^{-1}M_3M_3^{\mathrm T}\right)^{-1}D_1^{\mathrm T}P_2\right]^{\mathrm T}(P_2K)^{\mathrm T} \\ &\quad + (P_2K)\left[\varepsilon_1M_2M_2^{\mathrm T} + D_2\left(D_3 + D_3^{\mathrm T} - \varepsilon_3^{-1}M_3M_3^{\mathrm T}\right)^{-1}D_2^{\mathrm T}\right](P_2K)^{\mathrm T} = \widehat{A}^{\mathrm T}P_2 + P_2\widehat{A} + P_2\phi P_2 + \sigma I \\ &\quad + P_2\left[\varepsilon_1M_1M_1^{\mathrm T} + D_1\left(D_3 + D_3^{\mathrm T} - \varepsilon_3^{-1}M_3M_3^{\mathrm T}\right)^{-1}D_1^{\mathrm T}\right]P_2 - (P_2K)\Psi - \Psi^{\mathrm T}(P_2K)^{\mathrm T} + (P_2K)R(P_2K)^{\mathrm T} \\ &= \widehat{A}^{\mathrm T}P_2 + P_2\widehat{A} + \sigma I + P_2\left[\phi + \varepsilon_1M_1M_1^{\mathrm T} + D_1\left(D_3 + D_3^{\mathrm T} - \varepsilon_3^{-1}M_3M_3^{\mathrm T}\right)^{-1}D_1^{\mathrm T}\right]P_2 - \Psi^{\mathrm T}R^{-1}\Psi \\ &\quad + [(P_2K)R^{1/2} - \Psi^{\mathrm T}R^{-1/2}][(P_2K)R^{1/2} - \Psi^{\mathrm T}R^{-1/2}]^{\mathrm T}.\end{aligned} \tag{35}$$

In the light of (25), it is easy to see that

$$[(P_2K)R^{1/2} - \Psi^{\mathrm T}R^{-1/2}][(P_2K)R^{1/2} - \Psi^{\mathrm T}R^{-1/2}]^{\mathrm T} = SS^{\mathrm T}. \tag{36}$$

Furthermore, we find that (18) can be rewritten as

$$\widehat{A}^{\mathrm T}P_2 + P_2\widehat{A} + P_2\left[\varepsilon_1M_1M_1^{\mathrm T} + \phi + D_1\left(D_3 + D_3^{\mathrm T} - \varepsilon_3^{-1}M_3M_3^{\mathrm T}\right)\right]P_2 + SS^{\mathrm T} - \Psi^{\mathrm T}R^{-1}\Psi + (\sigma + \delta_2)I = 0 \tag{37}$$

and thus (35)–(37) indicate that

$$\Sigma_{22} = -\delta_2 I < 0. \tag{38}$$

Moreover, substituting (26) into (33) immediately yields $\Sigma_{12} = 0$, and therefore $\Sigma < 0$. Finally, it follows from Lemma 2 that system (13) and (14) is robustly asymptotically stable and passive. This proves Theorem 1. □

Remark 1. The use of the sufficiently small positive scalars $\delta_1 > 0$, $\delta_2 > 0$ is just to ensure that $\Sigma_{11} < 0$ and $\Sigma_{22} < 0$ hold. As will be seen later, these two parameters can be removed when we use two quadratic matrix inequalities or two linear matrix inequalities to replace the Riccati-like matrix equations (17) and (18) and restate Theorem 1.

Remark 2. Theorem 1 shows that the robust passive stability constraint on the uncertain state delayed system (1)–(4) can be guaranteed when two positive-definite solutions P_1, P_2, respectively, to the algebraic Riccati equations (17) and (18) are known to exist for some positive scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$ and positive-definite matrix Q_1. It is seen that the existence of a positive-definite solution to (25) means that the system matrix A must be asymptotically stable, i.e., the Assumption 1 holds.

Remark 3. Note that Theorem 1 offers sufficient conditions for the existence of the expected robust passive observer design problem for time-delay systems. The result may be conservative mainly due to the introduction of the inequalities (27)–(29). However, the conservativeness in Theorem 1 can be reduced over the design parameters $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$. A related discussion can be found in [19] and references therein. Also, when the time-delay d is a known constant, a delay-dependent algorithm in [16] has to be developed in order to reduce the relevant conservativeness, which gives one of the further research topics.

Remark 4. It is worth mentioning that, although the corresponding synthesis problem [illegible] < 0) is more involved, it is firstly used to deal with the problem of observer-based passive control of linear systems. As is well known, for systems without uncertainty, we only need one observer parameter to be designed. However, since the observer structure (7) is uncertainty-independent, we have two observer parameters G and K here. It can be seen from (9)–(14) that, unlike [24,27], it is impossible to treat (combine) the parameters G and K as a unified "compact factor". Therefore, the parameterized techniques developed in [24,27] cannot be directly applied to the problem addressed in this paper. Alternatively, in Theorem 1, we develop a new parameterized approach which enables us to derive the existence conditions and characterize the set of expected robust observers in terms of some *free* parameters by solving two Riccati-like equations, or as will be shown in sequel, by solving two quadratic matrix inequalities or two linear matrix inequalities. The principle advantage of the approach presented in Theorem 1 is that a parameterization of the set of desired observer can be given which is compact over some free design parameters such as the orthogonal matrix $U(U \in R^{p\times p})$, and thus make it possible to directly consider other performance requirements besides the robust passive constraints. A further discussion on the use of design freedom is given in Remark 6.

The parameter-dependent Riccati equations (17) and (18), which play a key role in the design of expected observers, have the same type as those in [23], and thus in general they can be dealt with by using the approach proposed in [23]. Moreover, instead of the Riccati matrix Eqs. (17) and (18), we shall restate Theorem 1 in terms of two quadratic matrix inequalities (QMIs) or LMIs in a clearer sense, and subsequently reduce the complexity of computation. To achieve such a goal, we first give a proposition as follows which can be easily proved.

Proposition 1. *For a given negative definite matrix $\Gamma < 0$ $(\Gamma \in R^{n\times n})$; there always exists a matrix $S \in R^{n\times p} (p \leqslant n)$ such that*

$$\Gamma + SS^{\mathrm{T}} < 0.$$

Theorem 2. *Let σ be a sufficiently small positive constant and $Q_1 > 0$ be a positive-definite matrix. If there exist positive scalars ε_1, $\varepsilon_2(N_2 Q$[illegible]$N_2^{\mathrm{T}} <$ [illegible]$)$, $\varepsilon_3(M_3M_3^{\mathrm{T}} < \varepsilon_3(D_3 + D_3^{\mathrm{T}}))$ such that the following QMIs*:

$$\begin{aligned}
&\left[A - [\text{illegible}]C_2([\text{illegible}] + D_3^{\mathrm{T}} - \varepsilon_3^{-1}M_3M_3^{\mathrm{T}})^{-1}D_1^{\mathrm{T}}\right]^{\mathrm{T}}P_1 + P_1\left[A - D_1\left(D_3 + D_3^{\mathrm{T}} - \varepsilon_3^{-1}M_3M_3^{\mathrm{T}}\right)^{-1}C_2\right]\\
&\quad + P_1\left[[\text{illegible}] + \phi + D_1\left(D_3 + D_3^{\mathrm{T}} - \varepsilon_3^{-1}M_3M_3^{\mathrm{T}}\right)^{-1}D_1^{\mathrm{T}}\right]P_1\\
&\quad + [\text{illegible}]^{\mathrm{T}}\left(D_3 + D_3^{\mathrm{T}} - \varepsilon_3^{-1}M_3M_3^{\mathrm{T}}\right)^{-1}C_2 + \left(\varepsilon_1^{-1} + \varepsilon_3\right)N_1^{\mathrm{T}}N_1 + Q_1 < 0,
\end{aligned} \tag{39}$$

$$\begin{aligned}
\Gamma :=\ &(\widehat{A} - \Omega^{\mathrm{T}}R^{-1}\widehat{C})^{\mathrm{T}}P_2 + P_2(\widehat{A} - \Omega^{\mathrm{T}}R^{-1}\widehat{C}) + P_2\left[+D_1\left(D_3 + D_3^{\mathrm{T}} - \varepsilon_3^{-1}M_3M_3^{\mathrm{T}}\right)D_1^{\mathrm{T}}\right.\\
&\left.+\varepsilon_1M_1M_1^{\mathrm{T}} + \phi - \Omega^{\mathrm{T}}R^{-1}\Omega\right]P_2 - \widehat{C}^{\mathrm{T}}R^{-1}\widehat{C} + \sigma I < 0,
\end{aligned} \tag{40}$$

respectively; have positive-definite solutions $P_1 > 0$ and $P_2 > 0$, where the matrices ϕ, $\widehat{A}$, $\widehat{C}$, R, Ψ, Ω, respectively; are defined in (19)–(24), *then the observer* (7) *with parameters*

$$K = P_2^{-1}[\Psi^{\mathrm{T}}R^{-1} + SUR^{-1/2}], \tag{41}$$

$$G = \widehat{A} - K\widehat{C}, \tag{42}$$

where $U \in R^{p\times p}$ is arbitrary orthogonal (i.e., $UU^T = I$) and $S \in R^{n\times p}$ is an arbitrary matrix meeting $\Gamma + SS^T < 0$ and Γ is defined in (40), will be such that, independently of the unknown time-delay d, then the augmented system (13) and (14) is asymptotically stable and passive.

Proof. The proof is a direct combination of Theorem 1 and Proposition 1. □

Remark 5. Theorem 2 gives a QMI approach to the design of robust passive observers for linear uncertain time-delay systems. When we tackle with the QMIs (39) and (40), the local numerical searching algorithms suggested in [9] are effective for a relatively low-order model. A related discussion of the solving algorithm for quadratic matrix inequalities can also be found in [19]. Instead, in the case that $\sigma > 0$, $Q_1 > 0$, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$, are fixed, we can easily convert the two QMIs (39) and (40) into two LMIs by using the well-known results on Schur complement of a partitioned symmetric matrix, and it follows that the design problem can be efficiently solved [1].

Remark 6. It should be pointed out that, in the present design procedure of robust observers for time-delay systems, there still exists much explicit freedom, such as the choices of the positive-definite matrix $Q_1 > 0$, the free parameters $S(S \in R^{n\times p}$ satisfies $\Gamma + SS^T < 0)$ and orthogonal matrix $U(U \in R^{p\times p})$ in expressions (41) and (42). This remaining freedom provides the possibility for considering more performance constraints which requires further investigations.

4. Examples

In this section, we demonstrate the theory developed in this paper by means of a simple example. Consider the linear continuous uncertain time-delay system (1)–(3) with parameters given by

$$A = \begin{bmatrix} -2 & -0.5 \\ 0.5 & -3 \end{bmatrix},\quad A_d = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},\quad C_1 = [1 \quad 0],\quad C_2 = \begin{bmatrix} 1 & 0 \\ \text{[illegible]} & 1 \end{bmatrix},\quad D_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$D_2 = [0.5 \quad 0.8],\quad D_3 = \begin{bmatrix} 0 & 0.2 \\ 0 & 0.2 \end{bmatrix},\quad M_1 = \begin{bmatrix} 1 & \text{[illegible]} \\ \text{[illegible]} & 0.1 \end{bmatrix},\quad M_2 = [-0.2 \quad 0.8],$$

$$M_3 = \begin{bmatrix} 0.1 & 0.01 \\ 0.02 & 0.1 \end{bmatrix},\quad N_1 = \begin{bmatrix} 0.8 & 0 \\ \text{[illegible]} & 0.8 \end{bmatrix},\quad N_2 = \begin{bmatrix} 0.1 & 0.01 \\ \text{[illegible]} & 0.5 \end{bmatrix},\quad |d| \leqslant 0.05,$$

where $0 < d < 0.05$ is an unknown positive scalar.

The purpose is to design the robust passive observer being of the structure (7), which does not depend on both the uncertainties and time delay such that for all admissible parameter perturbations, the observation process is asymptotically stable and passive.

Subjected to the constraint N_2Q^{-1}[illegible] $< \varepsilon_2 I$, $M_3M_3^{\mathrm{T}} < \varepsilon_3(D_3 + D_3^{\mathrm{T}})$, we choose

$$\varepsilon_1 = 0.1,\quad \varepsilon_2 = 0.5,\quad \varepsilon_3 = 0.3,\quad \delta_1 = 0.01,\quad \sigma = 10,\quad Q_1 = I_2,\quad S = [0.3 \quad 0.5]^{\mathrm{T}}$$

and then obtain the positive-definite solution to Riccati equation (17) and thus $\widehat{A}$, $\widehat{C}$ and R respectively, as follows:

$$P_1 = \begin{bmatrix} \text{[illegible]} & \text{[illegible]} \\ \text{[illegible]} & 1.2387 \end{bmatrix},\quad \widehat{A} = \begin{bmatrix} -1.4507 & -0.3747 \\ 0.5671 & -1.6736 \end{bmatrix},\quad \widehat{C} = [-2.2484 \quad -1.9118],\quad R = 2.5639.$$

Then, solve Riccati equation (18) to give

$$P_2 = \begin{bmatrix} 2.1760 & -0.1713 \\ -0.1570 & 1.6854 \end{bmatrix},\quad \Psi = [-1.9702 \quad -1.5467].$$

Finally, since the dimension of measurement output is $p = 1$, the arbitrary matrix U can be only chosen to be 1 or -1. Therefore, in these two cases, the desired observer parameters $K1$, $G1$ (for $U = 1$) and $K2$, $G2$ (for $U = -1$) can be obtained from (25) and (26), respectively, as the following:

$$K_1 = \begin{bmatrix} -0.2827 \\ -0.1990 \end{bmatrix},\quad G_1 = \begin{bmatrix} -2.0864 & -0.9152 \\ 0.1197 & -2.0541 \end{bmatrix},\quad K_2 = \begin{bmatrix} -0.4856 \\ -0.5885 \end{bmatrix},\quad G_2 = \begin{bmatrix} -2.5425 & -1.3030 \\ -0.7560 & -2.7986 \end{bmatrix}.$$

It is not difficult to verify that the specified robust stability as well as passive disturbance rejection constraints are achieved.

5. Conclusions

This paper has studied the problem of robust passive state observer design for a class of continuous-time state delayed systems with parameter uncertainties in both state and measurement matrices. A linear observer structure has been adopted. A modified ARI equation approach has been developed to solve the above problem. Specifically, the conditions for the existence of the expected robust passive observers have been derived in terms of two ARI equation. Also, the analytical expression of the desired observers has been characterized. A numerical example has shown the effectiveness of the present design approach.

It has been demonstrated that the desired robust observers of time-delay systems, when they exist, are usually a large set, and the remaining freedom can be used to meet other expected performance requirements. The main results can also be extended to discrete-time systems and sampled-data systems, and systems with convex parameter uncertainties. These will be the subjects of further investigations.

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