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Invariance image analysis using modified Zernike moments

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Abstract

Zernike moments play a vital role in feature extraction of digital images. Though computationally very complex compared to geometric and legendre moments, Zernike moments have proved to be better in terms of their feature representation capability, rotation invariance, fast computation, multi-level representation for describing the shapes of patterns and low noise sensitivity. However, there is a drawback in Zernike moments: they need to normalize an image to achieve scale invariance. The normalization process introduces some errors since it involves re-sampling and re-quantization of digital images and leads to inaccuracy of the classifier where the moments are being used. In this paper, a modification of Zernike moments is introduced to eliminate the above errors (including errors in rotation invariance). Invariance of the modified Zernike moments reflects significant improvement over the previous method.

Keywords: Oriya characters; Zernike moments; Geometric moments; Legendre moments; Scale invariance; Rotation invariance and translation invariance

1. Introduction

An essential issue in the field of pattern analysis is the recognition of objects and characters regardless of their position, size and orientation. This arises in a variety of situations such as inspection and packaging of manufactured parts, classification of chromosomes, target identification and scene analysis (Prokop and Reeves, 1992). Moments and the function of moments have been extensively employed as the invariant global features of an image in pattern recognition. Generally, these features are invariant under image translation, scale normalization and rotation only when they are computed from the original non-distorted analog two-dimensional image. In practice, one observes the digitized, quantized and often

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noisy version of the image and the invariance properties are satisfied only approximately. Some studies concerning the discretization error in the case of geometric moments were performed by Teh and Chin (1986). Regular moments have by far been the most popular type of moments. They are defined for digital image as

$$m_{pq} = \sum_{x} \sum_{y} x^p y^q f(x, y) \tag{1}$$

where m_{pq} is the $(p + q)$th order moment of the digital image function $f(x, y)$. Hu (1962) introduced seven nonlinear functions defined on regular moments which are translation, scale and rotation invariant. These seven so called moment invariants were used in a number of pattern recognition problems (Dudani et al., 1983; Maitra, 1979).

The definition of regular moments has the form of projection of $f(x, y)$ function onto the monomial $x^p y^q$. Unfortunately the basis set $x^p y^q$ is not orthogonal. Consequently, the recovery of image from these moments is quite difficult and computationally expensive. Moreover, it implies that the information content of m_{pq}s have a certain degree of redundancy. Teague (1980) has suggested orthogonal moments based on the theory of orthogonal polynomials to overcome the problems associated with the regular moments. However, Bailey and Srinath (1996) have used the polynomials, including Legender, Zernike and pseudo-Zernike to generate moment-based features which are invariant to location, size and (optionally) rotation (inputs are hand written digits). But they have presented a new approach to location invariance using minimum bounding circle and provided detailed analysis of the rotational properties of the moments. Zernike moments used in this study are a class of such orthogonal moments. The reason for selecting them from among the other orthogonal moments is that they possess a useful rotation invariance property. Rotating the image does not change the magnitudes of its Zernike moments. Hence, they could be used as rotation invariant features for image representation. These features could easily be constructed to an arbitrary high order. Another main property of Zernike moments is the ease of image reconstruction from them (Mukundan and Ramakrishnan, 1998; Patra, 2003). Hence, Zernike moments play a vital role in feature extraction of digital images. These Zernike moments have proved to be better in terms of their feature representation capability, rotation invariance, fast computation, multi-level representation for describing the shapes of patterns and low noise sensitivity (Teh and Chin, 1998; Belkasim, 1991; Khotanzad, 1990). However, Zernike moments have not been effectively used in image processing applications because of the drawback that they do not have scale invariance. At present, scale invariance can be achieved by scaling the original image such that its zeroth-order geometric moment is set equal to a predetermined value (Teh and Chin, 1998; Belkasim, 1991; Khotanzad, 1990; Kim and Kim, 1998, 2000). Zernike moments using this approach cannot well reflect the original shape in a digital image, because this approach is more suitable for an image in continuous space. For a digital image, scaling a shape introduces some errors since it involves the re-sampling and re-quantifying of digital image and leads to inaccuracy of the classifier. For example, when a circle shape in a digital image is shrunk to some degree, it may become a square shape.

This paper presents a technique in which the original Zernike moments are modified so as to eliminate the errors in both rotation and scale invariance arising out of re-sampling and re-quantifying of digital images. Invariance of the modified Zernike moments reflects noticeable improvement over the existing method.

The organization of the paper is as follows: Section 2 defines the Zernike moments and modified Zernike moments. Section 3 contains the experimental studies involving characters from the Oriya character data set. Conclusions are drawn in Section 4.

2. Zernike moments

2.1. Definition of Zernike moments

The kernel of Zernike moments is the set of orthogonal Zernike polynomials defined over the polar co-ordinates inside a unit circle. The form of these polynomials of order n repetition l is

$$V_{nl}(x,y) = V_{nl}(\rho,\theta) = R_{nl}(\rho)\exp(jl\theta) \tag{2}$$

where n is a positive integer or zero, l is an integer depicting the angular dependency or rotation subject to the conditions $n - |l| = \text{even}$, $|l| \leqslant n$, ρ is length of the vector from origin to pixel (x,y), θ is the angle between vector ρ and x-axis in counter clockwise direction, i.e., (ρ,θ) is defined over the unit disc, $j = \sqrt{-1}$ and R_{nl} is the radial polynomial defined by

$$\boldsymbol{R}_{nl}(\rho) = \sum_{s=0}^{(n-|l|)/2} (-1)^s \boldsymbol{F}(n,l,s,\rho) \tag{3}$$

where

$$\boldsymbol{F}(n,l,s,\rho) = \frac{(n-s)!}{s!\left(\frac{n+|l|}{2}-s\right)!\left(\frac{n-|l|}{2}-s\right)!}\rho^{n-2s}$$

Eq. (3) implies that $\boldsymbol{R}_{nl}(\rho) = \boldsymbol{R}_{n,-l}(\rho)$ and if the conditions $n - |l| = \text{even}$, $|l| \leqslant n$ are not met, then $\boldsymbol{R}_{nl}(\rho) = 0$. The above polynomials are orthogonal and satisfy

$$\int\int_{x^2+y^2\leqslant 1} [\boldsymbol{V}^*_{nl}(x,y)]\boldsymbol{V}_{pq}(x,y)\,\mathrm{d}x\,\mathrm{d}y = \frac{\pi}{(n+1)}\delta_{np}\delta_{lp} \tag{4}$$

with

$$\delta_{ab} = \begin{cases} & a = b, \\ & \text{otherwise} \end{cases}$$

Zernike moments are the projection of the image function onto these orthogonal basis functions. The Zernike moment of order n with repetition l for a digital image function $F(x,y)$($F(x,y)$ is the digital form of the continuous image $f(x,y)$) that vanishes outside the unit circle is as given in Eq. (5).

$$A_{nl} = \frac{n+1}{\pi}\sum_x\sum_y F(x,y)\boldsymbol{V}^*_{nl}(x,y), \quad x^2+y^2 \leqslant 1 \tag{5}$$

where * indicates the complex conjugate.

It is observed here that $\boldsymbol{A}^*_{nl} = \boldsymbol{A}_{n,-l}$.

To compute the Zernike moments of a given image, the image (or region of interest) is first mapped to the unit disc using polar co-ordinates, where the center of the image is the origin of the unit disc. Pixels falling outside the unit disc are not used in the computation. The features defined on the Zernike moments are derived by using rotational properties of these moments. If the image is rotated through angle ϕ, the relationship between the Zernike moments of the rotated image $\boldsymbol{A}'_{nl}$ and un-rotated one $\boldsymbol{A}_{nl}$ is

$$\boldsymbol{A}'_{nl} = \boldsymbol{A}_{nl}\exp(-jl\phi) \tag{6}$$

This relation shows that Zernike moments have simple rotational transformation properties; each Zernike moment merely acquires a phase shift on rotation. This simple property leads to the conclusion that the magnitudes of Zernike moments of a rotated image function remain identical to those before rotation. Thus, $|\boldsymbol{A}_{nl}|$, the magnitude of the Zernike moment can be taken as a rotation invariant feature of the underlying image function. Translation invariance is achieved by moving the origin to the center of the image by using the centralized moments. Scale invariance can be achieved by normalizing the image.

Fig. 1 shows the present approach of extracting Zernike moments from an image (Kim and Kim,

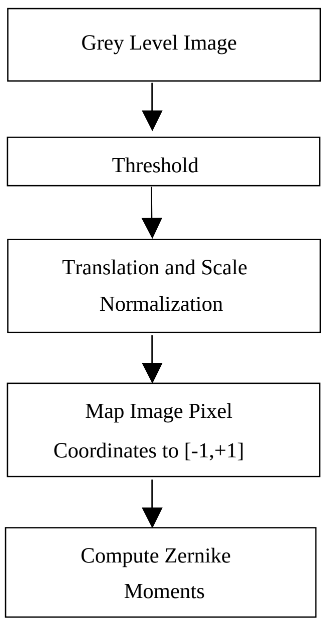

Fig. 1. Block diagram of extracting Zernike moments.

2000). First, the input image is binarized. Since the Zernike moments are defined over a unit disc, the radius of a circle is determined so as to enclose the shape completely from the centroid of the binarized shape in the image to the outermost pixel of the shape. The shape is then re-sampled to normalize it to the predetermined size. The Zernike moments are then extracted from the normalized image and the magnitudes are used as the features for pattern recognition.

The problem with the above approach is that the normalization of digital images generates errors of re-sampling and re-quantifying the image. If, instead the Zernike moments are first extracted, the effective error will be reduced, which will improve the ability for pattern recognition. This fact was the basis on which the modified Zernike moments have been developed and are experimented with. However, since $A_{n,-l} = A^*_{nl}$, it is observed that $|A_{nl}| = |A_{n,-l}|$. Hence, one can concentrate on $|A_{nl}|$ with $l \geqslant 0$ as far as the defined Zernike features are concerned.

2.2. Modified Zernike moments

Algorithm:

(i) Segment the input image to get binary image f.
(ii) Move the origin of the unit disc to the center of the shape.
(iii) Calculate the zeroth-order geometric moment $m_{00}^{(f)}$ of the image f.

$$m_{00}^{(f)} = \int\int_{R^2} f(x,y)\,\mathrm{d}x\,\mathrm{d}y \tag{7}$$

(iv) Calculate the various-order Zernike moments

$$A_{nl}^{(f)} = \frac{n+1}{\pi}\int\int_{x^2+y^2\leqslant 1} f(x,y)\,V^*_{nl}(x,y)\,\mathrm{d}x\,\mathrm{d}y$$

$$n = 0,1,2,\ldots,\infty \tag{8}$$

(v) Normalize the Zernike moments using $m_{00}^{(f)}$

$$A'^{(f)}_{nl} = \frac{A^{(f)}_{nl}}{m^{(f)}_{00}} \tag{9}$$

where $A'^{(f)}_{nl}$ are the modified Zernike moments.

(vi) Evaluate the magnitudes of modified Zernike moments $|A'^{(f)}_{nl}|$ as the features of shape in pattern recognition.

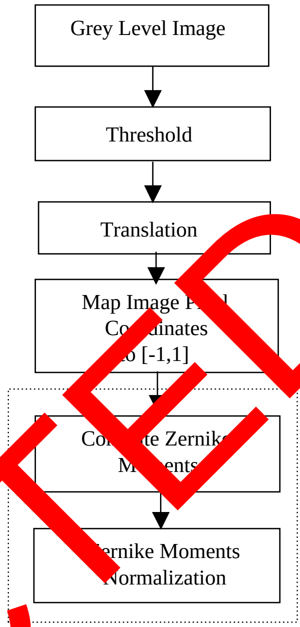

Fig. 2. Block diagram of extracting modified Zernike moments.

Fig. 2 shows the approach for extracting modified Zernike moments from an image. Lying within the dashed rectangle is the modified Zernike moment extraction.

3. Experimental studies

In this section, the results of applying both Zernike moments and the proposed modified Zernike moments technique on the given Oriya Characters are reported. Furthermore, the mean Error values of the modified Zernike moments are compared to those of the Zernike moments.

The test image database consists of 64×64 gray level noiseless images of Oriya Characters ('Ka' to 'Nya') in portable gray map format shown in Fig. 3. The experiments were conducted as follows:

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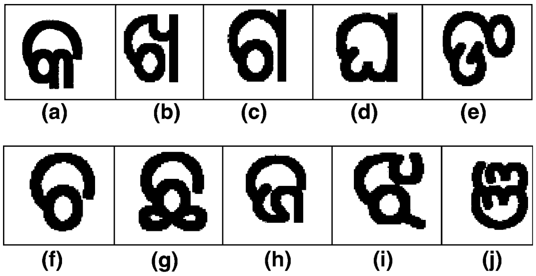

Fig. 3. 10 Oriya characters: (a) Ka; (b) Kha; (c) Ga; (d) Gha; (e) Una; (f) Cha; (g) Chha; (h) Ja; (i) Jha and (j) Nya.

Step 1 Transformation of images:
- (a) Rotation—the image is rotated by 5°, 30°, 50°, 60°, 120° and 130°.
- (b) Translation—the origin is moved to the centroid.
- (c) Scaling—the image is scaled by 500%, 230%, 120% and 90% factor.
- (d) Rotation/scaling—the image is first rotated by 30°, then scaled by 120%.

Step 2 Computation of modified and unmodified Zernike moments of transformed images. 20 moments (order $n = 0,1,\ldots,7$) are computed.

Step 3 Analysis of modified and unmodified Zernike moments in order to show the invariance properties.

In addition to the above noiseless character image set, three other sets of noisy images with SNR's of 50, 25 and 12 dB are also constructed from the normalized images of the noiseless set. This is done by randomly selecting some of the 4096 (64×64) pixels of a noiseless binary image and reversing their values from 0 to 1 or vice versa. The random pixel selection is done according to a uniform probability distribution between 1 and 4096. The SNR is equal to $20\,\mathrm{Log}_{10}[(4096 - L)/L]$, where L is the number of pixels, which are different between a noisy image and a noiseless version. Fig. 4 shows one image of character [$f(x)$] ("Ka") with different SNR's.

Figs. 5 and 6 depict the values of the modified and unmodified Zernike moments of characters 'Ka' and 'Kha' shown in Fig. 3(a) and (b) respectively. Here, we only show the values of moments of the original image, the image rotated by 30° and

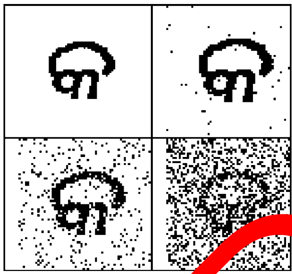

Fig. 4. One sample of image of character 'Ka' with different levels of noise. From top left to right SNR is noiseless, 50, 25 and 12 dB.

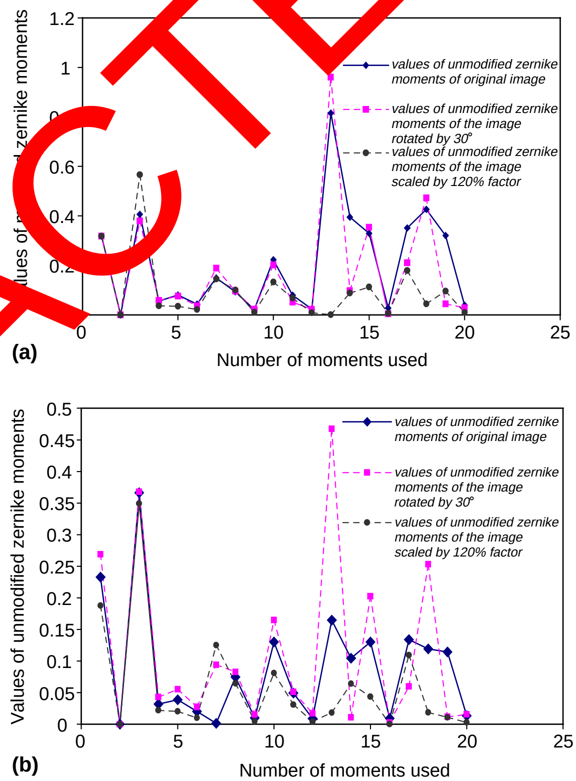

Fig. 5. Invariance of Zernike moments of the character 'Ka': (a) modified Zernike moments and (b) unmodified Zernike moments.

the image scaled by 120%. It is seen from Figs. 5(a) and 6(a) that the differences of the Zernike moments of the transformed images are very small and hence it is difficult to show all the discrepancy. However, in particular, the values for the image

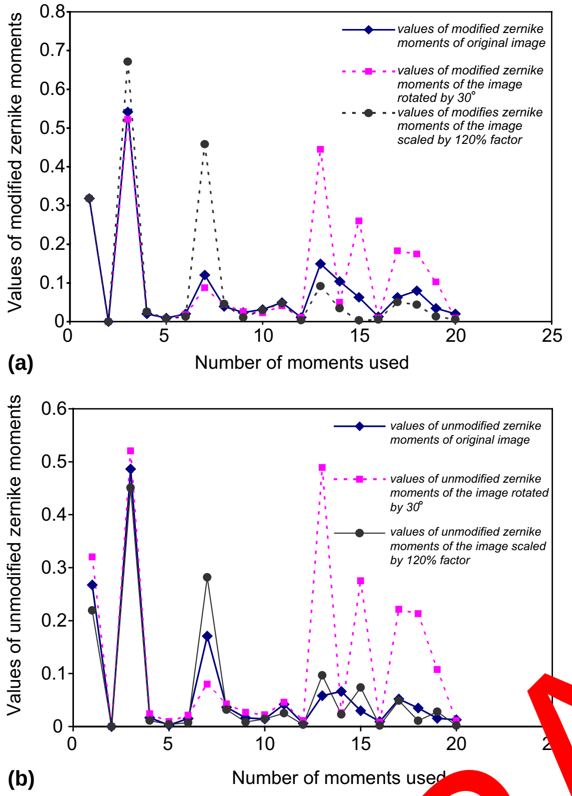

Fig. 6. Invariance of Zernike moments of the character 'Kha': (a) modified Zernike moments and (b) unmodified Zernike moments.

scaled by 120% are almost the same as those of the original image indicating that modified Zernike moments possess very good scale invariance. Figs. 5(b) and 6(b) show the values of unmodified Zernike moments of Fig. 3(a) and (b) respectively. Comparing these with Figs. 5(a) and 6(a), it is observed that the modified Zernike moments possess better rotation invariance and scale invariance properties than the unmodified Zernike moments. It is further interesting to note that except for $A'^{(f)}_{00} = 0.318$ and $A'^{(f)}_{11} = 0$, modified Zernike moments can be selected as good feature descriptors for pattern recognition.

Fig. 7 exhibits the mean error values of modified and unmodified Zernike moments of all the Characters in Fig. 3 rotated by 5°, 30°, 50°, 60°, 120°, 130° and scaled by 300%, 230%, 120%, 90% respectively. The significance of the mean error values of modified and unmodified Zernike

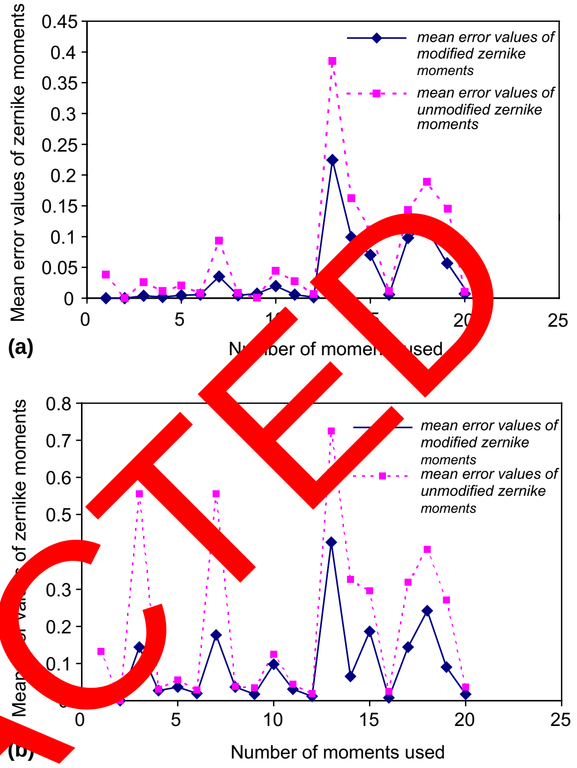

Fig. 7. Mean error values of: (a) modified and (b) unmodified Zernike moments.

moments is observed from the figure which shows that mean error values in modified case are smaller than those of the unmodified moments. This illustrates the fact that the proposed modified Zernike moments perform better than the unmodified ones

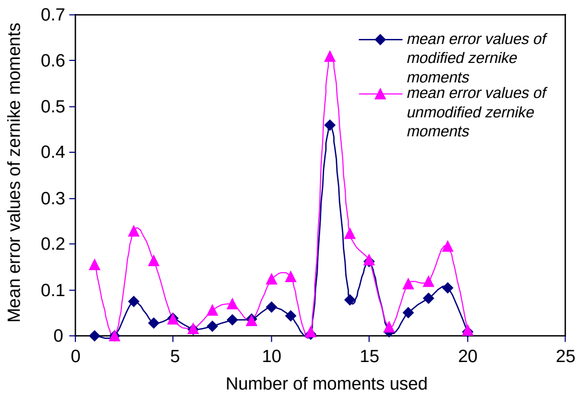

Fig. 8. Mean error values of modified and unmodified Zernike moments.

when used as a feature extraction tool for pattern recognition.

Fig. 8 shows the mean error values of modified and unmodified Zernike moments over the noisy image data set mentioned earlier (page 7). The mean error values in modified Zernike moments are observed to be less than mean error values of unmodified moments. It is concluded that modified Zernike moments are more immune to noise.

4. Conclusion

In this paper, a new set of features defined on modified Zernike moments, which are a mapping of an image function onto a set of orthogonal basis functions over the unit circle, has been developed. These features are proved to be rotation invariant. The orthogonal property makes the image reconstruction from its moments computationally simple. Moreover, it enables one to evaluate the image representation ability of each order moment as well as its contribution to the reconstruction process.

The drawbacks of the existing approach of extracting Zernike moments have been discussed here and an approach of modified Zernike moments has been proposed for better feature extraction. The performance of the modified Zernike moments is experimentally examined and based upon the obtained results, it is concluded that the invariance of the modified Zernike moments has been greatly improved over the existing technique for noiseless as well as noisy images.

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