Spontaneous excitation of an accelerated atom

in a spacetime with a reflecting plane boundary

Hongwei Yu

CCAST(World Lab.), P. O. Box 8730, Beijing, 100080,

P. R. China and Department of Physics and Institute of Physics,

Hunan Normal University, Changsha, Hunan 410081, China[*]

Shizhuan Lu

Department of Physics and Institute of Physics,

Hunan Normal University, Changsha, Hunan 410081, China

Abstract

We study a two-level atom in interaction with a real massless scalar quantum field in a spacetime with a reflecting boundary. The presence of the boundary modifies the quantum fluctuations of the scalar field, which in turn modifies the radiative properties of atoms. We calculate the rate of change of the mean atomic energy of the atom for both inertial motion and uniform acceleration. It is found that the modifications induced by the presence of a boundary make the spontaneous radiation rate of an excited inertial atom to oscillate near the boundary and this oscillatory behavior may offer a possible opportunity for experimental tests for geometrical (boundary) effects in flat spacetime. While for accelerated atoms, the transitions from ground states to excited states are found to be possible even in vacuum due to changes in the vacuum fluctuations induced by both the presence of the boundary and the acceleration of atoms, and this can be regarded as an actual physical process underlying the Unruh effect.

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[*] Mailing address

I. INTRODUCTION

Spontaneous emission is one of the most important features of atoms and so far mechanisms such as vacuum fluctuations [1, 2], radiation reaction [3], or a combination of them [4] have been put forward to explain why spontaneous emission occurs. The ambiguity in physical interpretation arises from different choices of ordering of commuting operators of atom and field in a Heisenberg picture approach to the problem. Significant progress has been made by Dalibard, Dupont-Roc and CohenTannoudji(DDC), who argued in Ref.[5] and Ref.[6] that there exists a symmetric operator ordering that the distinct contributions of vacuum fluctuations and radiation reaction to the rate of change of an atomic observable are separately Hermitian. If one demands such a ordering, each contribution can possess an independent physical meaning. The DDC prescription resolves the problem of stability for ground-state atoms when only radiation reaction is considered and the problem of "spontaneous absorption" of atoms when only vacuum fluctuations are taken into account. Using this prescription one can show that for ground-state atoms, the contributions of vacuum fluctuations and radiation reaction to the rate of change of the mean excitation energy cancel exactly and this cancellation forbids any transitions from the ground state and thus ensures atom's stability. While for any initial excited state, the rate of change of atomic energy acquires equal contributions from vacuum fluctuations and from radiation reaction.

Recently, Audretsch, Müeller and Holzmann [7, 8, 9] have generalized the formalism of DDC [6] to evaluate vacuum fluctuations and radiation reaction contributions to the spontaneous excitation rate and radiative energy shifts of an accelerated two-level atom interacting with a scalar field in a unbounded Minkowski space. In particular, their results show that when an atom is accelerated, then the delicate balance between vacuum fluctuations and radiation reaction is altered since the contribution of vacuum fluctuations to the rate of change of the mean excitation energy is modified while that of the radiation reaction remains the same. Thus transitions to excited states for ground-state atoms become possible even in vacuum. This result not only is consistent with the Unruh effect [10], but also provides a physically appealing interpretation of it. The Unruh effect, which is closely related to the Hawking radiation of black holes, predicts that a linearly accelerated two-level particle detector becomes excited when moving through the Minkowski vacuum and it behaves as if it were immersed (inertial) in a bath of thermal radiation at the Unruh temperature proportional to its acceleration, when coupling with massless scalar fields is considered. However, it is worth noting that the equality between the behavior of uniformly accelerated two-level particle detectors coupled with massless scalar fields and the inertial detectors lying at rest in a thermal bath may not be valid in general when the massless scalar field is replaced by other fields [11]. Let us illustrate this as follows. Couple an Unruh-DeWitt detector (two-level monopole) with energy gap ΔE to a massive scalar field with mass $m > \Delta E$. The excitation per proper time of this detector when it is uniformly accelerated with proper

acceleration a =constant) goes for $\Delta E << m$ as

$$P/T \sim a \int_0^\infty dx x K_0^2[\sqrt{x^2 + (m/a)^2}] \,, \tag{1}$$

where $K_\nu(z)$ is a Bessel function of imaginary argument. This is clearly non-vanishing because the external agent is making work on the detector. On the other hand, the same detector lying inertial(!) at rest in a thermal bath with temperature $T = a/2\pi$ is unable to excite because $\Delta E < m \leq \omega$, where ω is the energy of the massive scalar particles. Of course, this fact does not challenge by any means the Unruh effect because what the Unruh effect does state is that Eq. (1) can be recovered by using Fulling's quantization in conjunction with the fact that the Minkowski vacuum is a thermal state of Rindler particles [11].

In this sense, the Unruh effect is intrinsically related to the effects of modified vacuum fluctuations induced by the acceleration of the atom (or detector) in question. On the other hand, however, It is well-known that the presence of boundaries in a flat spacetime also modifies the vacuum fluctuations of quantum fields, and it has been demonstrated that this modification (or changes) in vacuum fluctuations can lead to a lot of novel effects, such as the Casimir effect [12], the light-cone fluctuations when gravity is quantized [13, 14, 15, 16], and the Brownian (random) motion of test particles in electromagnetic vacuum [17, 18], just to name a few. Therefore, it remains interesting to see what happens to the radiation properties of accelerated atoms found in Ref.[7] when the vacuum fluctuations are further modified by the presence of boundaries. In this paper, following the formalism developed by Audretsch and Müller[7], we will calculate the effects of modified vacuum fluctuations and radiation reaction due to the presence of a reflecting plane boundary upon the spontaneous excitation of both an inertial and a uniformly accelerated atom interacting with a quantized real massless scalar field. Let us note here that the response rate of a uniformly accelerated source interacting with a massless real scalar field in the presence of boundaries has recently been discussed [19].

The paper is organized as follows, we will review the formalism developed in Refs. [7] in Sec. II, then apply it to the case of an inertial atom in Sec. III and to the case of an accelerated atom in Sec. IV. Finally we will conclude with some discussions in Sec. V

II. THE GENERAL FORMALISM FOR VACUUM FLUCTUATIONS AND RADIATION REACTION

To study how the spontaneous emission of atoms is modified by the presence of a reflecting plane boundary, we examine a simple case: a two-level atom in interaction with a real massless scalar quantum field which obeys the Dirichlet boundary condition $\phi(x)|_{z=0} = 0$. Here we have assumed that the reflecting plane boundary is located at $z = 0$ in space. Let us consider a pointlike two-level atom on a stationary space-time trajectory $x(\tau)$, where τ

denotes the proper time on the trajectory. The stationary trajectory guarantees the existence of stationary atomic states, $|+\rangle$ and $|-\rangle$, with energies $\pm\frac{1}{2}\omega_0$ and a level spacing ω_0. The atom's Hamiltonian which controls the time evolution with respect to τ is given, in Dicke's notation [20], by

$$H_A(\tau) = \omega_0 R_3(\tau), \tag{2}$$

where $R_3 = \frac{1}{2}|+\rangle\langle+| - \frac{1}{2}|-\rangle\langle-|$ is the pseudospin operator commonly used in the description of two-level atoms[20]. The free Hamiltonian of the scalar quantum field that governs its time evolution with respect to τ is

$$H_F(\tau) = \int d^3k\, \omega_{\vec{k}}\, a_{\vec{k}}^\dagger a_{\vec{k}} \frac{dt}{d\tau}. \tag{3}$$

Here $a_{\vec{k}}^\dagger$, $a_{\vec{k}}$ are the creation and annihilation operators with momentum $\vec{k}$. Following Ref. [7], we assume that the interaction between the atom and the quantum field is described by a Hamiltonian

$$H_I(\tau) = \mu R_2(\tau)\phi(x(\tau)), \tag{4}$$

where μ is a coupling constant which we assume to be small, $R_2 = \frac{1}{2}i(R_- - R_+)$, and $R_+ = |+\rangle\langle-|$, $R_- = |-\rangle\langle+|$. The coupling is effective only on the trajectory $x(\tau)$ of the atom.

We can now write down the Heisenberg equations of motion for the atom and field observables. The field is always considered to be in its vacuum state $|0\rangle$. We will separately discuss the two physical mechanisms that contribute to the rate of change of atomic observables: the contribution of vacuum fluctuations and that of radiation reaction. For this purpose, we can split the solution of field ϕ of the Heisenberg equations into two parts: a free or vacuum part ϕ^f, which is present even in the absence of coupling, and a source part ϕ^s, which represents the field generated by the interaction between the atom and the field. Following DDC[5, 6], we choose a symmetric ordering between atom and field variables and consider the effects of ϕ^f and ϕ^s separately in the Heisenberg equations of an arbitrary atomic observable G. Then, we obtain the individual contributions of vacuum fluctuations and radiation reaction to the rate of change of G. Since we are interested in the spontaneous emission of the atom, we will concentrate on the mean atomic excitation energy $\langle H_A(\tau)\rangle$. The contributions of vacuum fluctuations(vf) and radiation reaction(rr) to the rate of change of $\langle H_A\rangle$ can be written as (cf. Ref.[5, 6, 7])

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{vf} = 2i\mu^2 \int_{\tau_0}^\tau d\tau'\, C^F(x(\tau), x(\tau')) \frac{d}{d\tau}\chi^A(\tau, \tau'), \tag{5}$$

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{rr} = 2i\mu^2 \int_{\tau_0}^\tau d\tau'\, \chi^F(x(\tau), x(\tau')) \frac{d}{d\tau}C^A(\tau, \tau'), \tag{6}$$

with $|\rangle = |a, 0\rangle$ representing the atom in the state $|a\rangle$ and the field in the vacuum state $|0\rangle$. Here the statistical functions of the atom, $C^A(\tau, \tau')$ and $\chi^A(\tau, \tau')$, are defined as

$$C^A(\tau, \tau') = \frac{1}{2}\langle a|\{R_2^f(\tau), R_2^f(\tau')\}|a\rangle, \tag{7}$$

4

$$\chi^A(\tau, \tau') = \frac{1}{2}\langle a|[R_2^f(\tau), R_2^f(\tau')]|a\rangle \tag{8}$$

and those of the field are

$$C^F(x(\tau), x(\tau')) = \frac{1}{2}\langle 0|\{\phi^f(x(\tau)), \phi^f(x(\tau'))\}|0\rangle, \tag{9}$$

$$\chi^F(x(\tau), x(\tau')) = \frac{1}{2}\langle 0|[\phi^f(x(\tau)), \phi^f(x(\tau'))]|0\rangle. \tag{10}$$

C^A is called the symmetric correlation function of the atom in the state $|a\rangle$, χ^A its linear susceptibility. C^F and χ^F are the Hadamard function and Pauli-Jordan or Schwinger function of the field respectively.

The explicit forms of the statistical functions of the atom are given by

$$C^A(\tau, \tau') = \frac{1}{2}\sum_b |\langle a|R_2^f(0)|b\rangle|^2 \left(e^{i\omega_{ab}(\tau-\tau')} + e^{-i\omega_{ab}(\tau-\tau')}\right), \tag{11}$$

$$\chi^A(\tau, \tau') = \frac{1}{2}\sum_b |\langle a|R_2^f(0)|b\rangle|^2 \left(e^{i\omega_{ab}(\tau-\tau')} - e^{-i\omega_{ab}(\tau-\tau')}\right), \tag{12}$$

where $\omega_{ab} = \omega_a - \omega_b$ and the sum runs over a complete set of atomic states. Using the method of images, at a distance z from the boundary, the statistical functions of the field can be written as

$$
\begin{aligned}
C^F(x(\tau), x(\tau')) = \frac{1}{8\pi^2}\Bigg\{ &-\frac{1}{(\Delta t + i\epsilon)^2 - |\Delta\vec{x}|^2} - \frac{1}{(\Delta t - i\epsilon)^2 - |\Delta\vec{x}|^2} \\
&+\frac{1}{(\Delta t + i\epsilon)^2 - [(x-x')^2 + (y-y')^2 + (z+z')^2]} \\
&+\frac{1}{(\Delta t - i\epsilon)^2 - [(x-x')^2 + (y-y')^2 + (z+z')^2]}\Bigg\},
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
\chi^F(x(\tau), x(\tau')) = \frac{i}{4\pi}\epsilon(\Delta t)\{&\delta(\Delta t^2 - ((x-x')^2 + (y-y')^2 + (z+z')^2)) \\
&-\delta(\Delta t^2 - |\Delta\vec{x}|^2)\},
\end{aligned}
\tag{14}
$$

where $\Delta t = t(\tau) - t(\tau')$, $\Delta\vec{x} = \vec{x}(\tau) - \vec{x}(\tau')$, and

$$\epsilon(\Delta t) = \begin{cases} +1 & for \quad \Delta t > 0 \\ -1 & for \quad \Delta t < 0 \end{cases} \tag{15}$$

is the sign function.

III. SPONTANEOUS EMISSION FROM A UNIFORMLY MOVING ATOM

In the present Section, we apply the formalism given in the proceeding Section to study the spontaneous emission of an inertial atom in the presence of a reflecting plane boundary.

For an inertial atom moving in the x-direction with a constant velocity v at a distance z from the plane, one has

$$t(\tau) = \gamma\tau, \qquad x(\tau) = x_0 + v\gamma\tau, \qquad y(\tau) = y_0, \qquad z(\tau) = z \qquad (16)$$

where $\gamma = (1 - v^2)^{-\frac{1}{2}}$. From the general forms Eq.(13) and Eq.(14), we can easily obtain the statistical functions of the field

$$
\begin{aligned}
C^F(x(\tau), x(\tau')) &= -\frac{1}{8\pi^2}\left(\frac{1}{(\tau - \tau' + i\epsilon)^2} - \frac{1}{(\tau - \tau' + i\epsilon)^2 - 4z^2}\right. \\
&\left.\quad + \frac{1}{(\tau - \tau' - i\epsilon)^2} - \frac{1}{(\tau - \tau' - i\epsilon)^2 - 4z^2}\right), \qquad (17)
\end{aligned}
$$

$$\chi^F(x(\tau), x(\tau')) = -\frac{i}{4\pi}\epsilon(\gamma(\tau - \tau'))\{\delta((\tau - \tau')^2) - \delta((\tau - \tau')^2 - 4z^2)\}. \qquad (18)$$

We can now evaluate Eq.(5) and Eq.(6) using the statistical functions given above, With a substitution $u = \tau - \tau'$, we get, for the contribution of the vacuum fluctuations to the rate of change of atomic excitation energy,

$$
\begin{aligned}
\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{vf} &= \frac{\mu^2}{8\pi^2}\sum_b \omega_{ab}|\langle a|R_2^f(0)|b\rangle|^2 \int_{-\infty}^{+\infty} du \left(\frac{1}{(u + i\epsilon)^2} + \frac{1}{(u - i\epsilon)^2}\right. \\
&\left.\quad - \frac{1}{(u - 2z + i\epsilon)(u + 2z + i\epsilon)} - \frac{1}{(u - 2z - i\epsilon)(u - 2z - i\epsilon)}\right)e^{i\omega_{ab}u}, \quad (19)
\end{aligned}
$$

and for that of radiation reaction

$$
\begin{aligned}
\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{rr} &= i\frac{\mu^2}{4\pi}\sum_b \omega_{ab}|\langle a|R_2^f(0)|b\rangle|^2\left\{ \int_{-\infty}^{+\infty} du \frac{\delta(u)}{u}e^{i\omega_{ab}u}\right. \\
&\quad - \int_0^\infty du \frac{\epsilon(\gamma u)}{4z}[\delta(u + 2z) + \delta(u - 2z)]e^{i\omega_{ab}u} \\
&\left.\quad + \int_{-\infty}^0 du \frac{\epsilon(-\gamma u)}{4z}[\delta(-u + 2z) - \delta(-u - 2z)]e^{i\omega_{ab}u}\right\}, \qquad (20)
\end{aligned}
$$

where we have extended the range of integration to infinity for sufficiently long times $\tau - \tau_0$. The integrals in Eq (19) and Eq (20) can be evaluated via the residue theorem to get

$$
\begin{aligned}
\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{vf} &= -\frac{\mu^2}{2\pi}\sum_{\omega_a > \omega_b}|\langle a|R_2^f(0)|b\rangle|^2\left[\frac{1}{2}\omega_{ab}^2 - \frac{\omega_{ab}}{4z}\sin(2z\omega_{ab})\right] \\
&\quad + \frac{\mu^2}{2\pi}\sum_{\omega_a < \omega_b}|\langle a|R_2^f(0)|b\rangle|^2\left[\frac{1}{2}\omega_{ab}^2 - \frac{\omega_{ab}}{4z}\sin(2z\omega_{ab})\right], \qquad (21)
\end{aligned}
$$

for the contribution of vacuum fluctuation to the rate of change of the atomic excitation energy and

$$
\begin{aligned}
\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{rr} &= -\frac{\mu^2}{2\pi}\sum_{\omega_a > \omega_b}|\langle a|R_2^f(0)|b\rangle|^2\left[\frac{1}{2}\omega_{ab}^2 - \frac{\omega_{ab}}{4z}\sin(2z\omega_{ab})\right] \\
&\quad - \frac{\mu^2}{2\pi}\sum_{\omega_a < \omega_b}|\langle a|R_2^f(0)|b\rangle|^2\left[\frac{1}{2}\omega_{ab}^2 - \frac{\omega_{ab}}{4z}\sin(2z\omega_{ab})\right]. \qquad (22)
\end{aligned}
$$

6

for that of radiation reaction

A few comments are now in order here. One can see from Eq. (21) that for an atom initially in the excited state ($|a\rangle = |+\rangle$), only the first term ($\omega_a > \omega_b$) contributes and one has $\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{vf} < 0$. While for an atom initially in the ground state ($|a\rangle = |-\rangle$), only the second term survives ($\omega_a > \omega_b$) and so $\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{vf} > 0$. Therefore, if only contributions of vacuum fluctuations are considered, both spontaneous excitation and de-excitation would equally occur. This leads to the well-known problem of spontaneous absorption for a ground state atom in vacuum. On the other hand, Eq. (22) shows that radiation reaction always makes the atom to lose energy since $\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{rr} < 0$, no matter if it is in the ground or excited state. This leads to a problem similar to the instability of atoms in classical electrodynamics. However, by adding the contributions of vacuum fluctuations and radiation reaction, we obtain the total rate of change of the atomic excitation energy

$$\left\langle \frac{dH_A}{d\tau} \right\rangle_{tot} = \left\langle \frac{dH_A}{d\tau} \right\rangle_{vf} + \left\langle \frac{dH_A}{d\tau} \right\rangle_{rr}$$

$$= -\frac{\mu^2}{2\pi} \left(\sum_{\omega_a > \omega_b} |\langle a|R_2^f(0)|b\rangle|^2 (\omega_{ab}^2 - \frac{\omega_{ab}}{2z} \sin(2z\omega_{ab})) \right). \tag{23}$$

It follows that for an atom in the ground state ($\omega_a < \omega_b$), the effects of both contributions exactly cancel, since each term in $\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{vf}$ is canceled exactly by the corresponding term in $\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{rr}$. Therefore, the presence of a plane boundary conspires to modify the vacuum fluctuations and radiation reaction in such a way that the delicate balance between the vacuum fluctuations and radiation reaction found in Ref. [7] in absence of boundaries remains and this ensures the stability of ground-state inertial atoms in vacuum with a reflecting boundary.

Eq. (23) gives the radiation rate of an excited atom. The corrections induced by the presence of the boundary are represented by z dependent terms in all the above results and they are oscillating functions of z and ω_{ab}. Let us note first that corrections only change the rate of change of atomic energy quantitatively but not qualitatively since we always have $\omega_{ab}^2 - \frac{\omega_{ab}}{2z} \sin(2z\omega_{ab}) \geq 0$. Secondly, as z, distance of the atom from the boundary, approaches infinity, our results reduce to those of the unbounded Minkowski space [7] as expected. Thirdly, for a given atom, the radiation rate is a function of z and it could either be enhanced or be weakened as compared with the case without any boundary, depending on the atom's distance to the plane boundary. Finally, the radiation rate becomes zero, as the atom is placed closer and closer to the boundary. This can be understood as a result of the fact that the scalar field vanishes on the boundary and so does the interaction Hamiltonian Eq.(4).

Let us now calculate the Einstein A coefficient for the spontaneous emission of inertially moving atoms in the presence of the boundary. For this purpose, following Ref. [7], we can

obtain a differential equation for the atomic excitation energy in order μ^2,

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{tot} = -\frac{\mu^2}{8\pi}\omega_0 \left(\frac{1}{2}\omega_0 + \langle H_A(\tau)\rangle\right)\left(1 - \frac{\sin(2z\omega_0)}{2z\omega_0}\right). \tag{24}$$

The solution of (24) is

$$\langle H_A(\tau)\rangle = -\frac{1}{2}\omega_0 + \left(\langle H_A(0)\rangle + \frac{1}{2}\omega_0\right)e^{-A\tau}, \tag{25}$$

the familiar exponential decay to the atomic ground state $\langle H_A\rangle = -\frac{1}{2}\omega_0$. The spontaneous emission rate is given by the Einstein A coefficient of the scalar theory:

$$A = \frac{\mu^2}{8\pi}\omega_0 \left(1 - \frac{\sin(2z\omega_0)}{2z\omega_0}\right). \tag{26}$$

We see once again that the rate of spontaneous emission is modified by the presence of the boundary and the Einstein coefficient is function of z for a given atom.

IV.   UNIFORMLY ACCELERATED ATOM

Let us now turn to the case in which the atom is uniformly accelerated in a direction parallel to the reflecting plane boundary. We assume that the atom is at a distance z from the boundary and is being accelerated in the x direction with a proper acceleration a. Specifically, the atom's trajectory is described by

$$t(\tau) = \frac{1}{a}\sinh a\tau, \qquad x(\tau) = \frac{1}{a}\cosh a\tau, \qquad z(\tau) = z, \qquad y(\tau) = 0. \tag{27}$$

The statistical functions of the field for the trajectory Eq. (27) can be evaluated from their general forms Eq. (13) and Eq. (14). After some calculations, we obtain

$$C^F(x(\tau), x(\tau')) = -\frac{a^2}{32\pi^2}\left(\frac{1}{\sinh^2[\frac{a}{2}(\tau - \tau') + i\epsilon]} + \frac{1}{\sinh^2[\frac{a}{2}(\tau - \tau') - i\epsilon]}\right.$$
$$\left. -\frac{1}{\sinh^2[\frac{a}{2}(\tau - \tau') + i\epsilon] - (az)^2} - \frac{1}{\sinh^2[\frac{a}{2}(\tau - \tau') - i\epsilon] - (az)^2}\right) \tag{28}$$

$$\chi^F(x(\tau), x(\tau')) = -\frac{i}{8\pi}\frac{a}{\sinh\frac{a}{2}(\tau - \tau')}\left(\delta(\tau - \tau')\right.$$
$$-\frac{1}{2\sqrt{1 + (az)^2}}\delta(\tau - \tau' - \frac{2}{a}\sinh^{-1}(az))$$
$$\left. +\frac{1}{2\sqrt{1 + (az)^2}}\delta(\tau - \tau' + \frac{2}{a}\sinh^{-1}(az))\right). \tag{29}$$

8

With the help of the following integral, which can be readily evaluated by residues,

$$\int_{-\infty}^{\infty} \left(\frac{1}{\sinh^2(\frac{a}{2}u + i\epsilon) - (az)^2} + \frac{1}{\sinh^2(\frac{a}{2}u - i\epsilon) - (az)^2} \right) e^{i\omega_{ab}u} du$$

$$= \left(1 + \frac{2}{e^{2\pi|\omega_{ab}|/a} - 1} \right) \frac{4\pi \sin\left(\frac{2\omega_{ab} \sinh^{-1}(az)}{a} \right)}{a^2 z\sqrt{1 + (az)^2}} , \tag{30}$$

we can calculate the contribution of vacuum fluctuation to the rate of change of the atomic excitation energy to get

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{vf} = -\frac{\mu^2}{2\pi} \Bigg[\sum_{\omega_a > \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 \, f(\omega_{ab}, a, z) \left(\frac{1}{2} + \frac{1}{e^{\frac{2\pi}{a}\omega_{ab}} - 1} \right)$$

$$- \sum_{\omega_a < \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 \, f(\omega_{ab}, a, z) \left(\frac{1}{2} + \frac{1}{e^{\frac{2\pi}{a}|\omega_{ab}|} - 1} \right) \Bigg] , \tag{31}$$

where

$$f(\omega_{ab}, a, z) = 1 - \frac{1}{2\omega_{ab}z\sqrt{1 + (az)^2}} \sin\left(\frac{2\omega_{ab}z \sinh^{-1}(az)}{az} \right) . \tag{32}$$

Comparing the above result with Eq. (56) of Ref. [7], one can see that the function $f(\omega_{ab}, a, z)$ gives the modification induced by the presence of the boundary. When $z \to \infty$, the function $f(\omega_{ab}, a, z)$ approaches 1 and we recover the result obtained in Ref. [7] for a uniformly accelerated atom in a unbounded Minkowski space as expected. On the other hand, if $(az) \to 0$, one has

$$f(\omega_{ab}, a, z) \approx 1 - \frac{\sin(2\omega_{ab}z)}{2\omega_{ab}z} . \tag{33}$$

Let us note that $(az) \to 0$ can be fulfilled either by keeping a at fixed finite value and letting z approach zero or keeping z fixed and letting a go zero. For the former case, $f(\omega_{ab}, a, z) \approx 0$. This reveals that as the atom gets closer and closer to the boundary the contribution of the vacuum fluctuations to the rate of change of the atomic excitation energy dies off in an oscillatory manner, no matter if the atom is in inertial motion (refer to Eq. (21)) or is accelerated as long as the proper accelerated is finite. While for the latter case, plugging Eq. (33) into Eq. (31), one recovers the result for an inertially moving atom, i.e., Eq. (21).

Similarly, one has for the contribution of radiation reaction,

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{rr} = -\frac{\mu^2}{2\pi} \sum_{\omega_a > \omega_b} |\langle a|R_2^f(0)|b\rangle|^2 \Bigg[\frac{1}{2}\omega_{ab}^2 - \frac{\omega_{ab}}{4z\sqrt{1 + (az)^2}}$$

$$\times \sin\left(\frac{2\omega_{ab}z \sinh^{-1}(az)}{az} \right) \Bigg] - \frac{\mu^2}{2\pi} \sum_{\omega_a < \omega_b} |\langle a|R_2^f(0)|b\rangle|^2$$

$$\times \Bigg[\frac{1}{2}\omega_{ab}^2 - \frac{\omega_{ab}}{4z\sqrt{1 + (az)^2}} \sin\left(\frac{2\omega_{ab}z \sinh^{-1}(az)}{az} \right) \Bigg]. \tag{34}$$

9

The above result reduces to that of an inertially moving atom, i.e., Eq. (22), when a goes to zero and it vanishes if the boundary is approached. However, the most distinct feature with the presence of a boundary is that the contribution of radiation reaction now depends on the acceleration of the atom, in sharp contrast to the unbounded Minkowski space where it has been shown that for accelerated atoms on arbitrary stationary trajectory, the contribution of radiation reaction is generally not altered from its inertial value [9].

Adding up the two contributions, one finds the total rate of change of the atomic excitation energy

$$\left\langle \frac{dH_A}{d\tau} \right\rangle_{tot} = \frac{\mu^2}{2\pi} \left[-\sum_{\omega_a > \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 \ f(\omega_{ab}, a, z) \ \left(1 + \frac{1}{e^{\frac{2\pi}{a}\omega_{ab}} - 1} \right) \right. $$
$$\left. + \sum_{\omega_a < \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 \ f(\omega_{ab}, a, z) \ \frac{1}{e^{\frac{2\pi}{a}|\omega_{ab}|} - 1} \right]. \tag{35}$$

For an excited atom, only $\omega_a > \omega_b$ contributes. One can see that the spontaneous emission is modified by the appearance of the thermal term as compared to an inertial atom near a reflecting boundary on one hand, and modified by the appearance of $f(\omega_{ab}, a, z)$ when compared to a uniformly accelerated atom in an unbounded Minkowski space on the other. However, for a ground-state atom, the delicate balance between the vacuum fluctuations and radiation reaction no longer exists, although both contributions of the vacuum fluctuations and radiation are altered for accelerated atoms in the presence of the boundary, as opposed to no change in the contribution of radiation reaction in absence of boundaries. There is a positive contribution from the $\omega_a < \omega_b$ term, therefore transitions of ground-state atoms to excited states are allowed to occur even in vacuum. The presence of the boundary modulates the transition rate with the function $f(\omega_{ab}, a, z)$ and makes the rate a function of z, the atom distance from the boundary. It is interesting to note that the spontaneous excitation rate of accelerated atoms (or the Unruh effect) becomes smaller and smaller as the atom is placed closer and closer to the boundary, since $f(\omega_{ab}, a, z)$ approaches 0 as $z \to 0$ for any finite value of a.

Now we wan to evaluate the Einstein coefficient. In the present case, we have two competing spontaneous processes, i.e., the spontaneous excitation and de-excitation. Thus there are two Einstein coefficients $A_\downarrow$ and $A_\uparrow$ which describe the corresponding transition rates. Consider an ensemble of N atoms. Let N_1 denote the number of atoms in the ground state, N_2 the number in the excited state. The rate equations are given by

$$\frac{dN_2}{d\tau} = -\frac{dN_1}{d\tau} = A_\uparrow N_1 - A_\downarrow N_2 \tag{36}$$

with

$$\langle H_A \rangle = \frac{1}{N} \left(-\frac{1}{2}\omega_0 N_1 + \frac{1}{2}\omega_0 N_2 \right). \tag{37}$$

The solution of the above equations is

$$\langle H_A(\tau) \rangle = -\frac{1}{2}\omega_0 + \omega_0 \frac{A_\uparrow}{A_\uparrow + A_\downarrow} + \left(\langle H_A(0) \rangle + \frac{1}{2}\omega_0 - \frac{A_\uparrow}{A_\uparrow + A_\downarrow}\omega_0 \right) e^{-(A_\uparrow + A_\downarrow)\tau} . \qquad (38)$$

On the other hand, we can simplify Eq. (35) to obtain a differential equation for $\langle H_A \rangle$

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle = -\frac{\mu^2}{4\pi}\omega_0 \left(\frac{1}{4}\omega_0 + f(\omega_0, a, z) \left(\frac{1}{2} + \frac{1}{e^{\frac{2\pi}{a}\omega_0} - 1} \right) \langle H_A(\tau) \rangle \right) , \qquad (39)$$

the solution of which gives the time evolution of the mean atomic excitation energy

$$\begin{aligned}
\langle H_A(\tau) \rangle &= -\frac{1}{2}\omega_0 + \frac{\omega_0}{e^{\frac{2\pi}{a}\omega_0} + 1} + \left(\langle H_A(0) \rangle + \frac{1}{2}\omega_0 - \frac{\omega_0}{e^{\frac{2\pi}{a}\omega_0} + 1} \right) . \\
&\quad \exp\left[-\frac{\mu^2}{4\pi}\omega_0 \left(\frac{1}{2} + \frac{1}{e^{\frac{2\pi}{a}\omega_0} - 1} \right) f(\omega_0, a, z)\tau \right] .
\end{aligned} \qquad (40)$$

This indicates that the atom evolves with a modified decay parameter towards the equilibrium value

$$\langle H_A \rangle = -\frac{1}{2}\omega_0 + \frac{\omega_0}{e^{\frac{2\pi}{a}\omega_0} + 1}, \qquad (41)$$

revealing a thermal excitation with temperature $T = a/2\pi$ above the ground state. The $+1$ in the denominator of the second term suggests that the atom obeys Fermi-Dirac statistics in thermal equilibrium. This remarkable feature can be understood as a result of the fermionic nature of a two-level system (for example, the atomic raising and lowering operators obey the anticommutation relation $\{R_+, R_-\} = 1$).

The Einstein coefficients $A_\downarrow$ and $A_\uparrow$ for an accelerated atom near a reflecting plane boundary readily follows from (40) and (38),

$$A_\downarrow = \frac{\mu^2}{8\pi}\omega_0 \left(1 + \frac{1}{e^{\frac{2\pi}{a}\omega_0} - 1} \right) f(\omega_0, a, z), \qquad A_\uparrow = \frac{\mu^2}{8\pi}\omega_0 \frac{f(\omega_0, a, z)}{e^{\frac{2\pi}{a}\omega_0} - 1}. \qquad (42)$$

A comparison of the coefficient $A_\downarrow$ for spontaneous emission from an accelerated atom near the plane boundary with the corresponding Einstein coefficients for an inertial atom near a plane boundary obtained in the last section and for an accelerated atom in an unbounded Minkowski space found in Ref. [7] shows that rate of spontaneous emission is enhanced by the thermal contribution as compared to the inertial case with the presence of the boundary and is modulated by the function $f(\omega_0, a, z)$ as compared to the accelerating case in an unbounded flat space. Since $f(\omega_0, a, z)$ is an oscillating function, the spontaneous emission rate can either be enhanced or weakened as compared to the case of an accelerated atom without the presence of the boundary, depending on the value of $f(\omega_0, a, z)$. At the meantime, it is easy to see that the transition rate $A_\uparrow$ for the spontaneous excitation is nonzero as long as $a \neq 0$ or $z \neq 0$ and it vanishes as $a \to 0$ as expected.

11

V. CONCLUSIONS

In conclusion, assuming a dipole like interaction between the atom and a scalar quantum field, we have studied the spontaneous emission of a two-level atom in a space with a reflecting plane boundary and examined both the contributions of vacuum fluctuations and radiation reaction for both inertial motion and uniform acceleration following the method developed in Refs. [5, 6, 7].

In the case of an inertial atom, our results show that for ground-state atoms, the contributions of vacuum fluctuations and radiation reaction to the rate of change of the mean excitation energy $\langle H_A(\tau) \rangle$ cancel exactly and the cancellation ensures that there are no upward radiative transitions in vacuum. For any initial excited state, the rate of change of atomic energy acquires equal contributions from vacuum fluctuations and from radiation reaction regardless of the distance from the plane boundary (refer to Eq.(21) and Eq.(22)). Therefore, the presence of a plane boundary does not change the delicate balance between the effects of vacuum fluctuations and radiation reaction and these two different effects seem to be equally important both in the unbounded Minkowski space and the space a plane boundary. Although the corrections induced by the presence of a boundary does not change the physical picture qualitatively, it does quantitatively. At distances far from the plane $(z\omega_{ab} \gg 1)$, the corrections become negligible as one would expect. It is interesting to note that close to the plane$(z\omega_{ab} \ll 1)$, the corrections becomes so large that the total radiation rate of the atom diminishes to zero in a oscillatory manner as the boundary is approached. Finally, let us note that the oscillatory behavior of the spontaneous radiation rate of an excited atom near a reflecting boundary may offer a possible opportunity for experimental tests for geometrical (boundary) effects in flat spacetime.

In the case of a uniformly accelerated atom, both contributions of the vacuum fluctuations and radiation reaction are altered by the presence of a reflecting plane boundary and the delicate balance between these two contributions existing in the case of inertial ground-state atoms is disturbed, making possible the spontaneous excitations from ground states. There are some interesting features to be noted as compared to the case without any boundary. First, the rate of change of the mean atomic excitation energy is now a function of the distance to the boundary and it dies off in an oscillatory way the boundary is approached, and second, the contribution of radiation reaction is now dependent on the acceleration of the atom, in sharp contrast to the unbounded Minkowski space where it has been shown that for accelerated atoms on arbitrary stationary trajectory, the contribution of radiation reaction is generally not altered from its inertial value [9].

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