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Cardy–Verlinde formula and thermodynamics of black holes in de Sitter spaces

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Abstract

We continue the study of thermodynamics of black holes in de Sitter spaces. In a previous paper (hep-th/0111093), we have shown that the entropy of cosmological horizon in the Schwarzschild–de Sitter solutions and topological de Sitter solutions can be expressed in a form of the Cardy–Verlinde formula, if one adopts the prescription to compute the gravitational mass from data at early or late time infinity of de Sitter space. However, this definition of gravitational mass cannot give a similar expression like the Cardy–Verlinde formula for the entropy associated with the horizon of black holes in de Sitter spaces. In this paper, we first generalize the previous discussion to the cases of Reissner–Nordström–de Sitter solutions and Kerr–de Sitter solutions. Furthermore, we find that the entropy of black hole horizon can also be rewritten in terms of the Cardy–Verlinde formula for these black holes in de Sitter spaces, if we use the definition due to Abbott and Deser for conserved charges in asymptotically de Sitter spaces. We discuss the implication of our result. In addition, we give the first law of de Sitter black hole mechanics.

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1. Introduction

Recently much attention has been focused on studying the de Sitter (dS) space and asymptotically dS space. This is motivated at least by the following two aspects. First, recent analysis of astronomical data for supernova indicates that there is a positive cosmological constant in our universe [1–3]. Thus our universe might approach to a dS phase in the far future [4,5]. Second, defined in a manner analogous to the AdS/CFT correspondence, an interesting proposal, the so-called dS/CFT correspondence, has been

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suggested recently that there is a dual between quantum gravity on a dS space and a Euclidean conformal field (CFT) on a boundary of the dS space [6] (for earlier works on this proposal see [7–10]).

Unlike the case of asymptotically flat and asymptotically AdS spacetimes, however, it is not an easy matter to compute conserved charges associated with an asymptotically dS space because of the absence of spatial infinity and a globally timelike Killing vector in such a spacetime. Up to the best of the present author's knowledge, there are two prescriptions to calculate conserved charges of asymptotically dS spaces. One of them is the prescription proposed recently by Balasubramanian, de Boer and Minic (BBM) [11]. Using this prescription together with the surface counterterm method,[1] one can compute the boundary stress-energy tensor and conserved charges of asymptotically dS spaces from data at early or late time infinity. In this way the authors of [11] calculated the masses of three-, four- and five-dimensional Schwarzschild–dS (SdS) solutions. It is found that pure dS spaces are always more massive than SdS solutions in the corresponding dimensions: subtracting the anomalous Casimir energy of pure dS spaces in odd dimensions (in even-dimensional dS spaces there is no associated anomalous Casimir energy), the masses of SdS black hole solutions are always negative. This is also confirmed in higher-dimensional SdS solutions [14] and Kerr–dS solutions [15]. On the basis of this result, the authors of [11] proposed an intriguing conjecture: *any asymptotically dS space whose mass exceeds that of pure dS space contains a cosmological singularity*. This conjecture is partially verified within the topological dS solution and its dilatonic deformation in [16].

Adopting the definition of mass in the BBM prescription, in a previous paper [17], we have shown that the entropy of cosmological horizons in the SdS solutions and topological dS solutions can be expressed in terms of the Cardy–Verlinde formula [18], which is supposed to be an entropy formula of CFT in any dimension.[2] Thus our result provides support of the dS/CFT correspondence. However, there are two points to be well understood. The first is that the mass (energy of the dual CFT) is negative, and it is measured at the far past ($\mathcal{I}^-$) or far future ($\mathcal{I}^+$) boundary of dS space, which is outside the cosmological horizon. Therefore, the boundary is not accessible for observers inside the cosmological horizon. The other is that for the SdS black hole spacetime, except for the cosmological horizon, there is a black hole horizon, which has also associated Hawking radiation and entropy; the spacetime is believed to have total entropy which is the sum of black hole horizon entropy and cosmological horizon entropy.[3] In Ref. [17], it is found that if one adopts the mass definition in the BBM prescription, the entropy of black hole horizon cannot be rewritten in a form like the Cardy–Verlinde formula.

[1] The surface counterterm method in the asymptotically dS space has been also discussed in [12,13].

[2] In the AdS/CFT correspondence, the Cardy–Verlinde formula has been shown to hold in many cases, for instance, Schwarzschild–AdS black holes [18], Kerr–AdS black holes [19], hyperbolic AdS black holes [20], charged AdS black holes [20] and the Taub–Bolt AdS instanton solutions [21]. In the cosmological context, there are a lot of works on the discussion of the Cardy–Verlinde formula, for a complete list, for example, see the more recent reference [22].

[3] Even in the thermodynamic sense, there is no well-defined derivation for the total entropy being the sum of the black hole horizon entropy and cosmological horizon entropy. But when the temperature of black hole horizon is equal to that of cosmological horizon, it can be shown, for example, see [23] and references therein.

The other prescription to compute conserved charges of asymptotically dS spaces is developed by Abbott and Deser (AD) [24], by considering the deviation of metric from the pure dS space being defined as the vacuum (lowest energy state). In the AD prescription, the gravitational mass of asymptotically dS spaces is always positive, and coincides with the ADM mass in asymptotically flat spacetimes, when the cosmological constant goes to zero.

In this paper, we will generalize the discussion in [17] to the cases of Reissner–Nordström–dS (RNdS) and Kerr–dS (KdS) spacetimes. That is, we will show that the entropy of cosmological horizon in the RNdS and KdS solutions can also be rewritten in the form of Cardy–Verlinde formula. We then show that if one uses the AD prescription, the entropy of black hole horizons in dS spaces can also be expressed by the Cardy–Verlinde formula, but the extensive part of energy is found to be negative. We will also discuss the first law of dS black hole solutions. The organization of the paper is as follows. In Sections 2, 3, 4 we will consider the SdS, RNdS, and KdS black hole spacetimes, respectively. The case of pure dS space in a rotating coordinate system will also be discussed in Section 4. In Section 5 we summary and discuss our results.

2. Schwarzschild–de Sitter black holes

We start with an $(n+2)$-dimensional SdS black hole solution, whose metric is

$$ds^2 = -f(r)\,dt^2 + f(r)^{-1}\,dr^2 + r^2\,d\Omega_n^2, \tag{2.1}$$

where

$$f(r) = 1 - \frac{\omega_n M}{r^{n-1}} - \frac{r^2}{l^2}, \qquad \omega_n = \frac{16\pi G}{n\,\mathrm{Vol}(S^n)}. \tag{2.2}$$

Here G is the gravitational constant in $(n+2)$ dimensions, l is the curvature radius of dS space, $\mathrm{Vol}(S^n)$ denotes the volume of a unit n-sphere $d\Omega_n^2$, and M is an integration constant. When $M=0$, the solution (2.1) reduces to the pure dS space with a cosmological horizon at $r_{\mathrm{c}} = l$. When M increases with $M > 0$, a black hole horizon occurs and increases in size with M, while the cosmological horizon shrinks. Finally the black hole horizon r_+ touches the cosmological horizon r_{c} when

$$M = M_{\mathrm{N}} \equiv \frac{2}{\omega_n(n+1)}\left(\frac{n-1}{n+1}l^2\right)^{(n-1)/2}. \tag{2.3}$$

This is the Nariai black hole, the maximal black hole in dS space. When $M > M_{\mathrm{N}}$, both the two horizons disappear and the solution (2.1) describes a naked singularity. The cosmological horizon r_{c} and black hole horizon r_+ are two positive real roots of the equation, $f(r)=0$. The cosmological horizon is the larger one and the black hole horizon is smaller one. The thermodynamics associated with the cosmological horizon has been discussed in [17]. For completeness and convenience for discussing the RNdS and KdS cases below, we briefly give main properties here. The cosmological horizon of the

SdS solution has the Hawking temperature T and entropy S,

$$T = \frac{1}{4\pi r_c}\left((n+1)\frac{r_c^2}{l^2} - (n-1)\right), \qquad S = \frac{r_c^n \operatorname{Vol}(S^n)}{4G}. \tag{2.4}$$

Subtracting the anomalous Casimir energy of pure dS spaces in odd dimensions, in the BBM prescription the gravitational mass of the SdS black holes is [11,14]

$$E = -M = \frac{r_c^{n-1}}{\omega_n}\left(\frac{r_c^2}{l^2} - 1\right), \tag{2.5}$$

where the mass is expressed in terms of the cosmological horizon radius r_c. Following Ref. [18], the Casimir energy E_c (non-extensive part of total energy), defined as $E_c = (n+1)E - nTS$, is found to be [17]

$$E_c = -\frac{2nr_c^{n-1}\operatorname{Vol}(S^n)}{16\pi G}. \tag{2.6}$$

The negative definite Casimir energy is consistent with the argument that in the dS/CFT correspondence, the dual CFT is not unitary [6]. Further, one has

$$2E - E_c = \frac{2nr_c^{n+1}\operatorname{Vol}(S^n)}{16\pi G l^2}, \tag{2.7}$$

and the entropy S (2.4) of the cosmological horizon can be rewritten as [17][4]

$$S = \frac{2\pi l}{n}\sqrt{|E_c|(2E - E_c)}, \tag{2.8}$$

a form of the Cardy–Verlinde formula [18]. This result (2.8) provides evidence that the thermodynamics of cosmological horizon in the SdS solution can be described by a CFT. In other words, our result is in favor of the dS/CFT correspondence. For pure dS space, one has $E = 0$ and $r_c = l$, the formula (2.8) precisely reproduces the entropy of pure dS space as well.

In addition, it is easy to check that the BBM mass (2.5), Hawking temperature T and entropy S in (2.4) of the cosmological horizon satisfy the first law of thermodynamics

$$dE = T\,dS. \tag{2.9}$$

On the other hand, the black hole horizon r_+ in the SdS solution has also associated Hawking temperature $\widetilde{T}$ and entropy $\widetilde{S}$[5]

$$\widetilde{T} = \frac{1}{4\pi r_+}\left((n-1) - (n+1)\frac{r_+^2}{l^2}\right), \qquad \widetilde{S} = \frac{r_+^n \operatorname{Vol}(S^n)}{4G}. \tag{2.10}$$

The total entropy of the SdS solution (2.1) is the sum of the cosmological horizon entropy S and black hole horizon entropy $\widetilde{S}$. However, as noticed in Ref. [17], if one uses the

[4] For related discussions, see also [25,26], from another angle.

[5] Throughout this paper, notations with tildes will denote quantities associated with the black hole horizon.

BBM mass (2.5), the black hole horizon entropy $\widetilde{S}$ cannot be expressed by a form like the Cardy–Verlinde formula. Here we report that if we adopt the mass definition due to Abbott and Deser [24], the entropy $\widetilde{S}$ can also be rewritten in a Cardy–Verlinde form.[6] The AD mass $\widetilde{E}$ of the SdS solution is [24]

$$\widetilde{E} = M = \frac{r_+^{n-1}}{\omega_n}\left(1 - \frac{r_+^2}{l^2}\right), \tag{2.11}$$

which is expressed in terms of the black hole horizon radius r_+. In this case, the associated Casimir energy $\widetilde{E}_{\rm c}$, defined as $\widetilde{E}_{\rm c} = (n+1)\widetilde{E} - n\widetilde{T}\widetilde{S}$, is

$$\widetilde{E}_{\rm c} = \frac{2nr_+^{n-1}\,\mathrm{Vol}(S^n)}{16\pi G}, \tag{2.12}$$

and the extensive part of energy is

$$2\widetilde{E} - \widetilde{E}_{\rm c} = -\frac{2nr_+^{n+1}\,\mathrm{Vol}(S^n)}{16\pi G l^2}. \tag{2.13}$$

It is then easy to see that the black hole horizon entropy in (2.10) can be rewritten as

$$\widetilde{S} = \frac{2\pi l}{n}\sqrt{\widetilde{E}_{\rm c}\left|2\widetilde{E} - \widetilde{E}_{\rm c}\right|}, \tag{2.14}$$

once again, a form of the Cardy–Verlinde formula. However, here it is worthwhile to notice that the extensive part (2.13) of energy is negative.

The AD mass obeys the first law of thermodynamics associated with the black hole horizon

$$d\widetilde{E} = \widetilde{T}\,d\widetilde{S}. \tag{2.15}$$

Considering the fact that $\widetilde{E} = -E = M$ and combining (2.15) and (2.9), one has

$$\widetilde{T}\,d\widetilde{S} + T\,dS = 0. \tag{2.16}$$

However, it is not suitable to see it as the first law of thermodynamics for the SdS black hole solution, because in general both the two temperatures associated with the cosmological and black hole horizons are not equal to each another, and then the SdS black hole solution is unstable quantum-mechanically. Instead an appropriate way is to rewrite (2.16) as

$$\tilde{\kappa}\,d\widetilde{A} + \kappa\,dA = 0, \tag{2.17}$$

where $\tilde{\kappa}$ (κ) and $\widetilde{A}$ (A) are the surface gravity and area of black hole (cosmological) horizon, respectively. Eq. (2.17) is just the first law of SdS black hole mechanics. For another derivation of the first law of four-dimensional SdS black holes see [28].

[6] We will make some remarks on the possible implications of this result in Section 5.

3. Reissner–Nordström–de Sitter black holes

In this section we extend the previous discussions to the case of Reissner–Nordström–dS black hole solutions, whose metric is

$$ds^2 = -f(r)\,dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_n^2,$$
$$f(r) = 1 - \frac{\omega_n M}{r^{n-1}} + \frac{n\omega_n^2 Q^2}{8(n-1)r^{2n-2}} - \frac{r^2}{l^2}, \tag{3.1}$$

where Q is the electric/magnetic charge of Maxwell field. For general M and Q, the equation $f(r) = 0$ may have four real roots. Three of them are real: the largest one is the cosmological horizon r_c, the smallest is the inner (Cauchy) horizon of black hole, the middle one is the outer horizon r_+ of black hole. And the fourth is negative and has no physical meaning. The classification of the RNdS solution has been made in Ref. [29] (see also [23]). Some thermodynamic quantities associated with the cosmological horizon are

$$T = \frac{1}{4\pi r_c}\left(-(n-1) + (n+1)\frac{r_c^2}{l^2} + \frac{n\omega_n^2 Q^2}{8 r_c^{2n-2}}\right),$$
$$S = \frac{r_c^n \,\mathrm{Vol}(S^n)}{4G}, \qquad \phi = -\frac{n}{4(n-1)}\frac{\omega_n Q}{r_c^{n-1}}, \tag{3.2}$$

where ϕ is the chemical potential conjugate to the charge Q. In the BBM prescription, the gravitational mass, subtracted the anomalous Casimir energy, of the RNdS solution is

$$E = -M = -\frac{r_c^{n-1}}{\omega_n}\left(1 - \frac{r_c^2}{l^2} + \frac{n\omega_n^2 Q^2}{8(n-1)r_c^{2n-2}}\right). \tag{3.3}$$

The Casimir energy E_c, defined as $E_c = (n+1)E - nTS - n\phi Q$ in this case, is found to be

$$E_c = -\frac{2nr_c^{n-1}\,\mathrm{Vol}(S^n)}{16\pi G}, \tag{3.4}$$

which has a same form as the case of SdS solution. Thus we can see that the entropy (3.2) of the cosmological horizon can be rewritten as[7]

$$S = \frac{2\pi l}{n}\sqrt{|E_c|\big(2(E - E_q) - E_c\big)}, \tag{3.5}$$

where

$$E_q = \frac{1}{2}\phi Q = -\frac{n}{8(n-1)}\frac{\omega_n Q^2}{r_c^{n-1}}. \tag{3.6}$$

We note that the entropy expression (3.5) has a similar form as the case of charged AdS black holes [17], there it is also found that the energy of electromagnetic field has to be subtracted from the total energy. Furthermore, the first law of thermodynamics of the

[7] For related discussion, see also [27], from another angle.

cosmological horizon is

$$dE = T\,dS + \phi\,dQ. \tag{3.7}$$

For the black hole horizon, associated thermodynamic quantities are

$$\widetilde{T} = \frac{1}{4\pi r_+}\left((n-1) - (n+1)\frac{r_+^2}{l^2} - \frac{n\omega_n^2 Q^2}{8r_+^{2n-2}}\right),$$
$$\widetilde{S} = \frac{r_+^n \mathrm{Vol}(S^n)}{4G}, \qquad \tilde{\phi} = \frac{n}{4(n-1)}\frac{\omega_n Q}{r_+^{n-1}}. \tag{3.8}$$

The AD mass of RNdS solution can be expressed in terms of black hole horizon radius r_+ and charge Q,

$$\widetilde{E} = M = \frac{r_+^{n-1}}{\omega_n}\left(1 - \frac{r_+^2}{l^2} + \frac{n\omega_n^2 Q^2}{8(n-1)r_+^{2n-2}}\right). \tag{3.9}$$

In this case, the Casimir energy, defined as $\widetilde{E}_c = (n+1)\widetilde{E} - n\widetilde{T}\widetilde{S} - n\tilde{\phi}Q$, is

$$\widetilde{E}_c = \frac{2nr_+^{n-1}\mathrm{Vol}(S^n)}{16\pi G}, \tag{3.10}$$

and the black hole entropy $\widetilde{S}$ can be rewritten as

$$\widetilde{S} = \frac{2\pi l}{n}\sqrt{\widetilde{E}_c\left|2(\widetilde{E} - \widetilde{E}_q) - \widetilde{E}_c\right|}, \tag{3.11}$$

where

$$\widetilde{E}_q = \frac{1}{2}\tilde{\phi}Q = \frac{n\omega_n Q^2}{8(n-1)r_+^{n-1}}, \tag{3.12}$$

which is the energy of electromagnetic field outside the black hole horizon. Thus we demonstrate that the black hole horizon entropy of RNdS solution can be expressed in a form as the Cardy–Verlinde formula. The AD mass and thermodynamic quantities of black hole horizon satisfy the first law,

$$d\widetilde{E} = \widetilde{T}\,d\widetilde{S} + \tilde{\phi}\,dQ. \tag{3.13}$$

Considering $\widetilde{E} = -E = M$ and combining (3.7) with (3.13), one has

$$\widetilde{T}\,d\widetilde{S} + T\,dS + \tilde{\phi}\,dQ + \phi\,dQ = 0. \tag{3.14}$$

As the case of SdS solution, we can rewrite the above as

$$\tilde{\kappa}\,d\widetilde{A} + \kappa\,dA + 8\pi G\triangle\phi\,dQ = 0, \tag{3.15}$$

where $\triangle\phi \equiv \tilde{\phi} + \phi = \frac{n\omega_n Q}{4(n-1)}(1/(r_+^{n-1}) - 1/(r_c^{n-1}))$ is the chemical potential difference between the black hole horizon and the cosmological horizon. Eq. (3.15) is the first law of RNdS black hole mechanics.

4. Kerr–de Sitter black holes

The higher-dimensional Kerr–AdS black hole solution has been given in Ref. [30], from which one can obtain an $(n+2)$-dimensional Kerr–dS black hole solution with a single angular momentum parameter by replacing l^2 by $-l^2$. The metric of KdS solution is

$$\begin{aligned} ds^2 = &-\frac{\triangle_r}{\rho^2}\left(dt - \frac{a}{\Xi}\sin^2\theta\, d\phi\right)^2 + \frac{\rho^2}{\triangle_r}dr^2 + \frac{\rho^2}{\triangle_\theta}d\theta^2 \\ &- \frac{\triangle_\theta \sin^2\theta}{\rho^2}\left(a\,dt - \frac{r^2+a^2}{\Xi}d\phi\right)^2 + r^2\cos^2\theta\, d\Omega_{n-2}^2, \end{aligned} \tag{4.1}$$

where

$$\begin{aligned} \triangle_r &= \left(r^2+a^2\right)\left(1-\frac{r^2}{l^2}\right) - 2Mr^{3-n}, \qquad && \triangle_\theta = 1 + \frac{a^2}{l^2}\cos^2\theta, \\ \Xi &= 1 + \frac{a^2}{l^2}, && \rho^2 = r^2 + a^2\cos^2\theta. \end{aligned} \tag{4.2}$$

The KdS solution has the same horizon structure as the RNdS solution. The cosmological horizon has associated thermodynamic quantities

$$\begin{aligned} T &= \frac{r_{\rm c}}{4\pi(r_{\rm c}^2+a^2)}\left(-(n-1)\left(1-\frac{a^2}{l^2}\right) - (n-3)\frac{a^2}{r_{\rm c}^2} + (n+1)\frac{r_{\rm c}^2}{l^2}\right), \\ S &= \frac{\mathrm{Vol}(S^n)}{4G\Xi} r_{\rm c}^{n-2}\left(r_{\rm c}^2+a^2\right), \\ J &= \frac{a\,\mathrm{Vol}(S^n)}{8\pi G\Xi^2} r_{\rm c}^{n-3}\left(r_{\rm c}^2+a^2\right)\left(1-\frac{r_{\rm c}^2}{l^2}\right), \\ \Omega &= -\frac{a\Xi}{r_{\rm c}^2+a^2}, \end{aligned} \tag{4.3}$$

where J is the angular momentum of the KdS solution in the BBM prescription [15] and Ω is the chemical potential conjugate to J, which can be explained as the angular velocity of the cosmological horizon. After subtracting the anomalous Casimir energy of pure dS space in the rotating coordinates, the BBM mass of KdS is [15][8]

$$E = -\frac{n\,\mathrm{Vol}(S^n)}{16\pi G\Xi} r_{\rm c}^{n-3}\left(r_{\rm c}^2+a^2\right)\left(1-\frac{r_{\rm c}^2}{l^2}\right), \tag{4.4}$$

which is expressed in terms of the cosmological horizon radius $r_{\rm c}$ and angular momentum parameter a. For the thermodynamics of cosmological horizon, we find that the Casimir energy, defined as $E_{\rm c} = (n+1)E - nTS - n\Omega J$, is

$$E_{\rm c} = -\frac{2n\,\mathrm{Vol}(S^n)}{16\pi G\Xi} r_{\rm c}^{n-3}\left(r_{\rm c}^2+a^2\right), \tag{4.5}$$

[8] In Ref. [15] the gravitational mass and angular momentum are calculated, in the BBM prescription, for four-, five- and seven-dimensional KdS solutions, respectively. We expect that the expression (4.4) and the angular momentum in (4.3) always hold in any dimension.

and the extensive part of energy

$$2E - E_{\rm c} = \frac{2nr_{\rm c}^{n-1}\,{\rm Vol}(S^n)}{16\pi G\Xi l^2}\big(r_{\rm c}^2 + a^2\big). \tag{4.6}$$

Thus it is easy to see that the entropy (4.3) of cosmological horizon can be re-expressed as

$$S = \frac{2\pi l}{n}\sqrt{|E_{\rm c}|(2E - E_{\rm c})}, \tag{4.7}$$

a completely same form as the case of SdS solution. For the cosmological horizon, its first law of thermodynamics is

$$dE = T\,dS + \Omega\,dJ. \tag{4.8}$$

The AD mass $\widetilde{E}$ and angular momentum $\tilde{J}$ and other thermodynamic quantities associated with the black hole horizon are

$$\begin{aligned}
\widetilde{T} &= \frac{r_+}{4\pi(r_+^2 + a^2)}\left((n-1)\left(1 - \frac{a^2}{l^2}\right) + (n-3)\frac{a^2}{r_+^2} - (n+1)\frac{r_+^2}{l^2}\right),\\
\widetilde{S} &= \frac{{\rm Vol}(S^n)}{4G\Xi} r_+^{n-2}\big(r_+^2 + a^2\big),\\
\tilde{J} &= \frac{a\,{\rm Vol}(S^n)}{8\pi G\Xi^2} r_+^{n-3}\big(r_+^2 + a^2\big)\left(1 - \frac{r_+^2}{l^2}\right),\\
\widetilde{E} &= \frac{{\rm Vol}(S^n)}{16\pi G\Xi} r_+^{n-3}\big(r_+^2 + a^2\big)\left(1 - \frac{r_+^2}{l^2}\right),\\
\widetilde{\Omega} &= \frac{a\Xi}{r_+^2 + a^2}.
\end{aligned} \tag{4.9}$$

Note that here we have $\widetilde{E} = -E$ and $\tilde{J} = J$. The AD mass and angular momentum satisfy

$$d\widetilde{E} = \widetilde{T}\,d\widetilde{S} + \widetilde{\Omega}\,d\tilde{J}. \tag{4.10}$$

Combining (4.10) with (4.8), we obtain the first law of KdS black hole mechanics,

$$\tilde{\kappa}\,d\widetilde{A} + \kappa\,dA + 8\pi G \triangle\Omega\,dJ = 0, \tag{4.11}$$

where $\triangle\Omega \equiv \widetilde{\Omega} + \Omega = a\Xi(1/(r_+^2 + a^2) - 1/(r_{\rm c}^2 + a^2))$ is the angular velocity of black hole horizon with respect to the cosmological horizon.

For the thermodynamics of black hole horizon, the Casimir energy, defined as $\widetilde{E}_{\rm c} = (n+1)\widetilde{E} - n\widetilde{T}\widetilde{S} - n\widetilde{\Omega}\tilde{J}$, is found to be

$$\widetilde{E}_{\rm c} = \frac{2n\,{\rm Vol}(S^n)}{16\pi G\Xi} r_+^{n-3}\big(r_+^2 + a^2\big), \tag{4.12}$$

and the extensive part of energy is

$$2\widetilde{E} - \widetilde{E}_{\rm c} = -\frac{2nr_+^{n-1}\,{\rm Vol}(S^n)}{16\pi G\Xi l^2}\big(r_+^2 + a^2\big). \tag{4.13}$$

Once again, the extensive energy is negative. The entropy $\widetilde{S}$ of black hole horizon can be rewritten as

$$\widetilde{S} = \frac{2\pi l}{n}\sqrt{\widetilde{E}_c|2\widetilde{E} - \widetilde{E}_c|}, \tag{4.14}$$

a form as the case of SdS black hole solutions.

In the KdS solution (4.1), a special case is the one where $M = 0$. In this case, it can be shown that the metric (4.1) with $M = 0$ describes a pure dS space in rotating coordinates [36]. That is, the solution (4.1) with $M = 0$ can be changed into the pure dS space in the static coordinates (namely the solution (2.1) with $M = 0$) by a coordinate transformation. For the pure dS space in rotating coordinates, the cosmological horizon is still at $r_c = l$, the BBM mass (4.4) and angular momentum in (4.3) vanish. The Hawking temperature and entropy are

$$T = \frac{1}{2\pi l}, \qquad S = \frac{l^n \operatorname{Vol}(S^n)}{4G}. \tag{4.15}$$

The Casimir energy from (4.5) is

$$E_c = -\frac{2nl^{n-1}\operatorname{Vol}(S^n)}{16\pi G}. \tag{4.16}$$

These quantities are the same as those in the case of the static coordinates. Therefore the entropy of cosmological horizon of the pure dS space in the rotating coordinates can also be given using the Cardy–Verlinde formula. Further, it seems to indicate that although the vacuum states might be different in the static coordinates and rotating coordinates, the dual CFT should give rise to the same thermo-excitation for the pure dS space.

5. Conclusion and discussion

Holographic principle says that a theory with gravity in D dimensions can be equivalent to a theory in $(D-1)$ dimensions without gravity [31]. The AdS/CFT correspondence [32] is a beautiful realization of the principle. Recently it has been proposed that quantum gravity on a dS space is dual to a Euclidean CFT on a boundary of the dS space (dS/CFT correspondence) [6]. Unlike the AdS/CFT correspondence, however, the understanding to the dS/CFT correspondence so far acquired is quite incomplete. In order to establish the dS/CFT correspondence, one has to first collect more theoretic data. To well understand the gravity of asymptotically dS spaces is one of important topics.

In a previous paper [17], adopting the BBM prescription [11] to compute boundary stress-energy tensor and conserved charges of asymptotically dS spaces, we have found that the entropy associated with the cosmological horizon in the SdS solutions and topological dS solutions can be rewritten in terms of the Cardy–Verlinde formula, which is supposed to be an entropy formula of CFTs in any dimension. In this paper we have shown that the conclusion also holds for the RNdS solutions and KdS solutions (see (3.5) and (4.7)). This result therefore provides support of the dS/CFT correspondence.

For spacetimes of black holes in dS spaces, however, the total entropy is the sum of black hole horizon entropy and cosmological horizon entropy. If one uses the BBM mass of the asymptotically dS spaces, the black hole horizon entropy cannot be expressed by a form like the Cardy–Verlinde formula. In this paper, we have found that if one uses the AD prescription to calculate conserved charges of asymptotically dS spaces, the (SdS, RNdS, KdS) black hole horizon entropy can also be rewritten (see (2.14), (3.11) and (4.14)) in a form of Cardy–Verlinde formula, which indicates that the thermodynamics of black hole horizon in dS spaces can be also described by a certain CFT.

Our result seems to imply that we need two different CFTs associated, respectively, with the black hole horizon and cosmological horizon to describe the dS black hole spacetimes. It looks reasonable because generically the black hole horizon and cosmological horizon have different temperatures, one do not expect that a same field theory can describe simultaneously the thermodynamics of black hole horizon and of cosmological horizon. Our result is also reminiscent of the Carlip's claim [33] that for black holes in any dimension the Bekenstein–Hawking entropy can be reproduced using the Cardy formula [34]. Carlip obtained his result by considering general relativity on a manifold with boundary. He found that the constraint algebra of general relativity may acquire a central extension, which can be calculated using covariant phase techniques. When the boundary is a (local) Killing horizon, a natural set of boundary conditions leads to a Virasoro subalgebra with a calculable central charge. He then used conformal field theory methods to determine the density of states at the boundary, which yields the expected entropy of black holes. The Carlip's method is also applicable to the cosmological horizon. For the case of pure dS spaces, an analysis has been made in [35]. Applying the Carlip's method to black holes in de Sitter spaces, obviously one cannot give the total entropy using a same conformal field theory because in this method the central charges associated with the black hole horizon and cosmological horizon are different, which also implies that one needs two different CFTs for the black holes in dS spaces. As a result, the holography dual to the black holes in dS spaces might be quite complicated. In order to establish the hologram for black holes in dS spaces, many issues remain to be investigated.

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