1: The Composite Trapezoid Rule (30 pts)

Add the Composite Trapezoid Rule to your library in a new class "Integration". Use it to determine the distance fallen by the free-falling bungee jumper in the first 3 seconds by evaluating the integral of

$$v(t) = \sqrt{\frac{gm}{c_d}} tanh\left(\sqrt{\frac{gc_d}{m}} t\right).$$

The second derivative of $v(t)$ is

$$v''(t) = -2g^{3/2}\sqrt{\frac{c}{m}} tanh\left(t\sqrt{\frac{cg}{m}}\right) sech^2\left(t\sqrt{\frac{cg}{m}}\right)$$

which is used to find the average of the second derivative

$$v''(t) = \frac{\int_0^3 -2g^{3/2}\sqrt{\frac{c}{m}} tanh\left(t\sqrt{\frac{cg}{m}}\right) sech^2\left(t\sqrt{\frac{cg}{m}}\right)}{3-0} = -0.8668$$

Apply the recursive composite trapezoid rule to this problem with a tolerance of 0.00001, and the following parameter values: $g = 9.81$ m/s2, $m = 68.1$ kg, and $c_d = 0.25$ kg/m. Report the distance fallen by the free-falling bungee jumper and the number of segments needed to reach the tolerance.

2: The Composite Simpson's 1/3 Rule (30 pts)

Add the Composite Simpson's 1/3 Rule to your library in the appropriate class. Use it to determine the distance fallen by the free-falling bungee jumper in the first 3 seconds by evaluating the integral of

$$v(t) = \sqrt{\frac{gm}{c_d}} tanh\left(\sqrt{\frac{gc_d}{m}} t\right).$$

The fourth derivative of $v(t)$ is

$$v^{(4)}(t) = \sqrt{\frac{gm}{c_d}}\left(\frac{16c_d^2 g^2 tanh\left(t\sqrt{\frac{c_d g}{m}}\right) sech^4\left(t\sqrt{\frac{c_d g}{m}}\right)}{m^2} - \frac{8c_d^2 g^2 tanh^3\left(t\sqrt{\frac{c_d g}{m}}\right) sech^2\left(t\sqrt{\frac{c_d g}{m}}\right)}{m^2}\right)$$

which is used to find the average of the fourth derivative

$$v^{(4)}(t) = \frac{\int_0^3 \sqrt{\frac{gm}{c_d}}\left(\frac{16c_d^2 g^2 tanh\left(t\sqrt{\frac{c_d g}{m}}\right) sech^4\left(t\sqrt{\frac{c_d g}{m}}\right)}{m^2} - \frac{8c_d^2 g^2 tanh^3\left(t\sqrt{\frac{c_d g}{m}}\right) sech^2\left(t\sqrt{\frac{c_d g}{m}}\right)}{m^2}\right)}{3-0} = 0.2001$$

NOTE: The average value for the 4th derivative is a positive value so you will have to change the if statement in your code to `if (abs(error)>tol)` to for your recursion code to work since $E_a < 0$. You should change this in both methods for your code to be robust. Apply the recursive composite Simpson's 1/3 rule to this problem with a tolerance of 0.00001, and the following parameter values: $g = 9.81$ m/s2, $m = 68.1$ kg, and $c_d = 0.25$ kg/m. Report the distance fallen by the free-falling bungee jumper and the number of segments needed to reach the tolerance.

3: Submit Your Library (40 pts)

By this point in time your library should contain the following routines:

- The Bisection method
- Newton-Raphson method
- Golden-Section Search Method
- Parabolic Interpolation
- The Trapezoid Rule
- Simpson's 1/3 Rule

Make sure that these routines work by testing them on the functions we covered in class, if they are not working you will not receive full credit for them. You are not required to submit any output for these tests, but we will check them to make sure they are working.