Lecture 2

Computer memory and the binary number system

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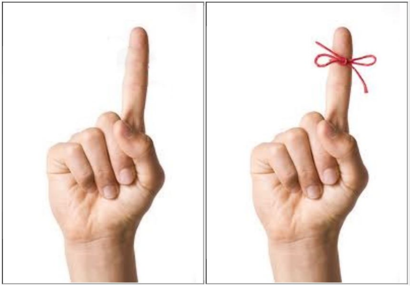
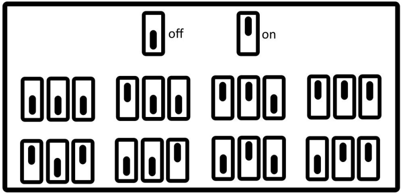
Memory device

A memory device is a nifty gadget that allows us to store and recall information. This device has more than one state.



Example

Think of a light switch. It can either be on or off - one switch has 2 states.
A row of n switches can be in 2^n states. For example, lets consider a row of 3 switches.



$2^3 = 8$ and we can see in this figure, there are 8 possible states. We can number these possible states 0-7 and represent them using the binary number system.

Binary

The binary number system, or base-two number system, uses 2 digits to encode a number - 0 or 1.

Let's compare base-two with base-ten. In base ten, decimal, you have columns for $10^0 = 1$ (ones), $10^1 = 10$ (tens), $10^2 = 100$ (hundreds), $10^3 = 1000$ (thousands), etc. In base two, you have columns for $2^0 = 1$ (ones), $2^1 = 2$ (twos), $2^2 = 4$ (fours), $2^3 = 8$ (eights) and so on.

decimal (base 10)	binary (base 2)	expansion
0	0	0 ones
1	1	1 one
2	10	1 two and zero ones
3	11	1 two and 1 one
4	100	1 four, 0 twos, and 0 ones
5	101	1 four, 0 twos, and 1 one
6	110	1 four, 1 two, and 0 ones
7	111	1 four, 1 two, and 1 one
8	1000	1 eight, 0 fours, 0 twos, and 0 ones
9	1001	1 eight, 0 fours, 0 twos, and 1 ones
10	1010	1 eight, 0 fours, 1 two, and 0 ones
11	1011	1 eight, 0 fours, 1 two, and 1 one
12	1100	1 eight, 1 four, 0 twos, and 0 ones
13	1101	1 eight, 1 four, 0 twos, and 1 one
14	1110	1 eight, 1 four, 1 two, and 0 ones
15	1111	1 eight, 1 four, 1 two, and 1 one
16	10000	1 sixteen, 0 eights, 0 fours, 0 twos, and 0 ones

Light switch example

Let's go back to our light switch example. I'll give you a few minutes to fill in the rest. NOTE: The only coefficients you may use are 0 or 1.

0: $0 * 2^0 \rightarrow 0$

1: $1 * 2^0 \rightarrow 1$

2: $1 * 2^1 + 0 * 2^0 \rightarrow 10$

3:

4:

5:

6:

7:

Converting larger base ten numbers to base two numbers

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\end{aligned}$$

Now start with the last whole number and move up through the remainders.

$375_{10} = 101100101_2$

Converting larger base ten numbers to base two numbers

Now let's check that this is correct: $375_{10} = 101100101_2$. The first thing we want to do is number the digits of 101100101from right to left - keep in mind the right-most digit is the remainder from the first divide.

digits:	1	0	1	1	0	0	1	0	1
numbering:	8	7	6	5	4	3	2	1	0

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Now we can write out the series

$$101100101_2 = 1 * 2^8 + 0 * 2^7 + 1 * 2^6 + 1 * 2^5 + 0 * 2^4 + 0 * 2^3 + 1 * 2^2 + 0 * 2^1 + 1 * 2^0$$

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$$101100101_2 = 256 + 0 + 64 + 32 + 0 + 0 + 4 + 0 + 1 = 357_{10}$$

Class Activity

Please determine how these base ten numbers would be written as base two numbers:

- 714_{10}

- 179_{10}

- 1428_{10}

Class Activity

Show the following are true:

- $1011001010_2 \rightarrow 714_{10}$

- $10110011_2 \rightarrow 179_{10}$

- $10110010100_2 \rightarrow 1428_{10}$