Numerical Differentiation

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Fall 2020

Introduction

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At the beginning of the integration unit, we used calculus to integrate this equation to determine the vertical distance y the jumper has fallen after a time t.

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Now suppose that you were given the reverse problem. That is, you were asked to determine velocity based on the jumper's position as a function of time. Because it is the inverse of integration, differentiation could be used to make the determination:

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Beyond velocity, you might also be asked to compute the jumper's acceleration. To do this, we could either take the first derivative of velocity, or the second derivative of displacement:

$$a(t) = \frac{dv(t)}{dt} = \frac{d^2y(t)}{dt^2} = gsech^2\left(\sqrt{\frac{gc_d}{m}} t\right)$$

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- Mathematically, the derivative, which serves as the fundamental vehicle for differentiation, represents the rate of change of a dependent variable with respect to an independent variable.

Differentiation

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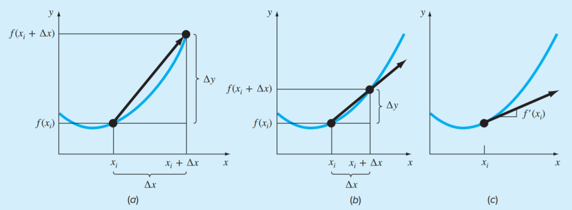

- As depicted in figure (a) above, the mathematical definition of the derivative begins with a difference approximation:

$$\frac{\Delta y}{\Delta x} = \frac{f(x_i + \Delta x) - f(x_i)}{\Delta x}$$

where y and $f(x)$ are alternative representatives for the dependent variable and x is the independent variable.

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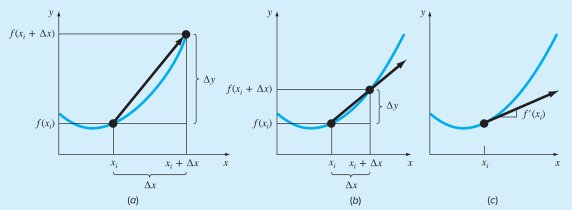

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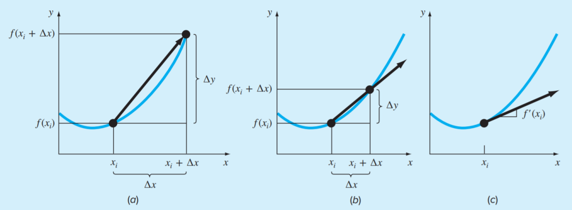

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- The derivative is the slope of the tangent line to to curve at x_i

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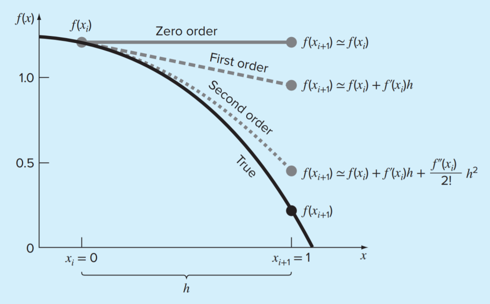
- The Taylor series then provides a means to express this idea mathematically in a form that can be used to generate practical results.



Taylor Series & Plan for the rest of the semester

On Wednesday, we will begin to use Taylor series expansions to derive the finite difference schemes used to evaluate derivatives.

We will wrap up differentiation on the 9th, supplementary material may be posted after for you to read if interested.

Class periods on the 11th, 13th, and 16th will be used for you to work on your projects.