COMS10003 Work Sheet 22

Linear Algebra: Solving Linear Equations and Inverting Matrices

Andrew Calway May 6, 2015

1. Use substitution to find solutions or otherwise to the following 2×2 linear systems. Sketch the geometric interpretation in each case.

$$\begin{aligned} x_1 - 2x_2 &= 1 \\ 2x_1 + x_2 &= 2 \end{aligned} \qquad \begin{aligned} x_1 + 2x_2 &= 3 \\ 3x_1 + 6x_2 &= 3 \end{aligned}$$

2. Use Gaussian elimination (GE) to find a solution to the following 3×3 system

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 2 \\ 2x_1 + x_2 - x_3 &= 1 \\ x_1 + 3x_2 + 2x_3 &= 3 \end{aligned}$$

3. Use GE to show that the following system does not have a solution. Describe in geometric terms why this is so.

$$\begin{aligned} 2x_1 + x_2 - 3x_3 &= 1 \\ 4x_1 - 2x_2 + x_3 &= 4 \\ 2x_1 - 3x_2 + 4x_3 &= 2 \end{aligned}$$

4. Use GE with matrix notation to solve the following linear system

$$\begin{aligned} x_1 + x_2 + x_3 &= 1 \\ 2x_1 + 2x_2 + x_3 &= 3 \\ 3x_1 + x_2 - x_3 &= 2 \end{aligned}$$

5. Use GE with matrix notation to find a general solution to the system

$$\begin{aligned} x_1 - 2x_2 + 3x_3 - x_4 &= 1 \\ 2x_1 - x_2 + x_3 + 3x_4 &= 1 \\ 4x_1 + x_2 - 2x_3 + 5x_4 &= 2 \\ 6x_1 - 3x_2 + 4x_3 + 3x_4 &= 4 \end{aligned}$$

6. Examine the GE algorithm and consider each division and each multiplication-subtraction as a single operation. Show that for an $n \times n$ system, GE is an $O(n^3)$ algorithm.

7. Use the Gauss-Jordan Method to find the inverses of the following matrices (if they exist). If you find one, confirm that it is correct using matrix multiplication.

$$\begin{bmatrix} 2 & 1 & -2 \\ 5 & -3 & 7 \\ 0 & -2 & -1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & -4 \\ -3 & 1 & -9 \\ 2 & -3 & 13 \end{bmatrix}$$