## COMS21103: Data Structures and Algorithms
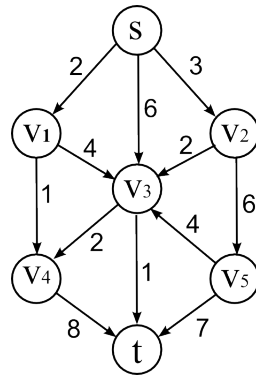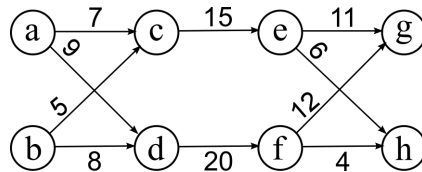
## Problem Sheet - Week 10

1. **Maximum Flow**

   (a) Calculate the *maximum flow* for the flow network below:

   

   (b) *optional:* Use the code stubs platform to test your solution for the maximum flow problem in (a).

   (c) Calculate the *maximum flow* for the flow network below:

   

   (d) For the flow network in (c), enumerate all possible cuts in the network, and calculate the capacity of each cut.

   (e) Suppose that, in addition to edge capacities, a flow network has vertex capacities. That is each vertex has a limit $\iota(v)$ on how much flow can pass through $v$. How can you convert such a network into a flow network without vertex capacities.

   (f) After finding the maximum flow for a network, suppose that we increased the capacity of a single edge $(u, v) \in E$ by 1. Give an O(V+E) algorithm to update the maximum flow.

   (g) For the image below ($3 \times 3$ pixels) with the number in the pixel indicating the greyscale pixel value (0-255), convert the image into a flow network. Consider the foreground model to be the Euclidean distance between the pixel value and 100, and the background model to be the Euclidean distance between the pixel value and 220. Can you find the minimum cut?

   