COMS10003: Workshop on Proof

Propositional Logic and Inductive Proof

Kerstin Eder

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Introduction

In preparation for this workshop you've been asked to read up on *proof techniques in general and, in particular, on inductive proof.*

Note, this worksheet contains tasks on several topics related to Proof in Propositional Logic and Inductive Proof. Review the worksheet. Schedule your work so that you find an answer to those parts of the worksheet that enable you to solve the rest of the questions alone.

For each task and subtask, use the whiteboard to present and to discuss your solutions; take turns. Active participation is required from all group members.

Task 1: Proof in Propositional Logic

Task 1.1: Valid Arguments Determine whether the following arguments are valid.

Hint: First identify all propositions, then formalise the argument. Follow the approach demonstrated in the first lecture on proof.

a) To be great is to be misunderstood.
She is great.
She is misunderstood.

b) If Frank does not go running regularly he will not be able to complete the race.
Frank did not complete the race.
Frank did not go running regularly.

c) n is odd or n is even.
If n is odd, then $2n$ is even.
If n is even, then $2n$ is even.
$2n$ is even.

Task 1.2 Rules of Inference We have already used **Modus Ponens** as an inference rule. Find other inference rules and explain their intuitive meaning. In particular, make sure you include the following in your discussion:

- Modus Tollens

$$\begin{array}{c} \neg q \\ p \Rightarrow q \\ \hline \therefore \neg p \end{array}$$

- Conjunction introduction and conjunction elimination

$$\begin{array}{c} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

and

$$\begin{array}{c} p \wedge q \\ \hline \therefore p \end{array}$$

- Disjunction introduction

$$\begin{array}{c} p \\ \hline \therefore p \vee q \end{array}$$

- Hypothetical and disjunctive syllogism

$$\begin{array}{c} p \Rightarrow q \\ q \Rightarrow r \\ \hline \therefore p \Rightarrow r \end{array}$$

and

$$\begin{array}{r} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

Are the following arguments valid? If so, which inference rules have been used, if not, why not? Discuss. Use this opportunity to remember the names of the inference rules.

1. The taxi is either red or green. The taxi is not red, therefore the taxi is green.
2. If I can't make it tomorrow, then I will call you this evening. I have not called you this evening. Therefore I can make it tomorrow.
3. Speeding is illegal. Therefore it is prohibited by law.
4. If I can't make it tomorrow, then I will call you this evening. I can make it tomorrow. Therefore, I won't call you this evening.
5. Logic is fun and it is sunny today. Therefore, logic is fun.
6. If I go to London tomorrow, then I can't pick you up tomorrow. If I do not pick you up tomorrow, then grandma will pick you up. Therefore, if I go to London tomorrow, then grandma will pick you up tomorrow.
7. It is sunny today. Therefore, it is either sunny or raining today.
8. Whenever it rains, I go by car. I went by by car yesterday. Therefore, it rained yesterday.
9. If it rains, then I'll go by car. It rains today, therefore I go by car today.

Task 1.3 Logical Reasoning Which conclusions can you draw from the following statements? Clearly explain each step in each argument. Justify your reasoning. State the rules of inference used.

In particular, state whether or not you can get help with unloading the car?

"It is not snowing this morning. We will go skiing only if it is snowing. If we do not go skiing, then, if it is cold we will stay in. If we are in, then we can help you unload the car when you arrive. It is very cold today."

Task 2: Inductive Proof

Task 2.1: Use inductive proof to prove the following properties $P(n)$ for all $n \in \mathbb{N}^+$, where $\mathbb{N}^+ = \{1, 2, ...\}$:

1.
$$P(n) = \sum_{i=1}^{n} (2i - 1) = n^2 \tag{1}$$

   Hint: First try for a few examples to convince yourself that it works. Once you've gained confidence, proceed to drafting the proof.

2.
$$P(n) = \sum_{i=0}^{n} 2^i = 2^{n+1} - 1 \tag{2}$$

3.
$$P(n) = \sum_{i=0}^{n} r^i = \frac{1 - r^{n+1}}{1 - r} \qquad (r \neq 1) \tag{3}$$

Task 2.2:

1. Prove that $n! > 2^n$ for $n \in \mathbb{N}$ and $n \geq 4$.
2. Prove that $n^3 - n$ is divisible by 3 for all $n \in \mathbb{N}$.
3. Let $S_n = \{1, 2, ..., n\}$. Prove that $|\mathcal{P}(S_n)| = 2^n$ for $n \in \mathbb{N}^+$.
   ($\mathcal{P}(S)$ is the power set of S — every possible subset, including the empty set and S itself.)