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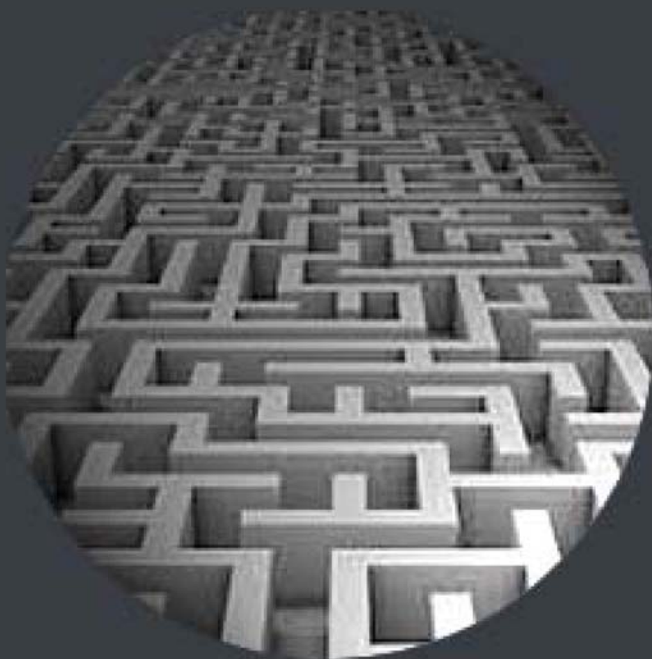
Department of Computer Science

PROGRAMMING and ALGORITHMS II



Brief Recap on

COMPLEXITY BASICS

Dr Tilo Burghardt

Unit Code COMS10001

Eulerian Cycles

(use each edge once to cycle the graph)

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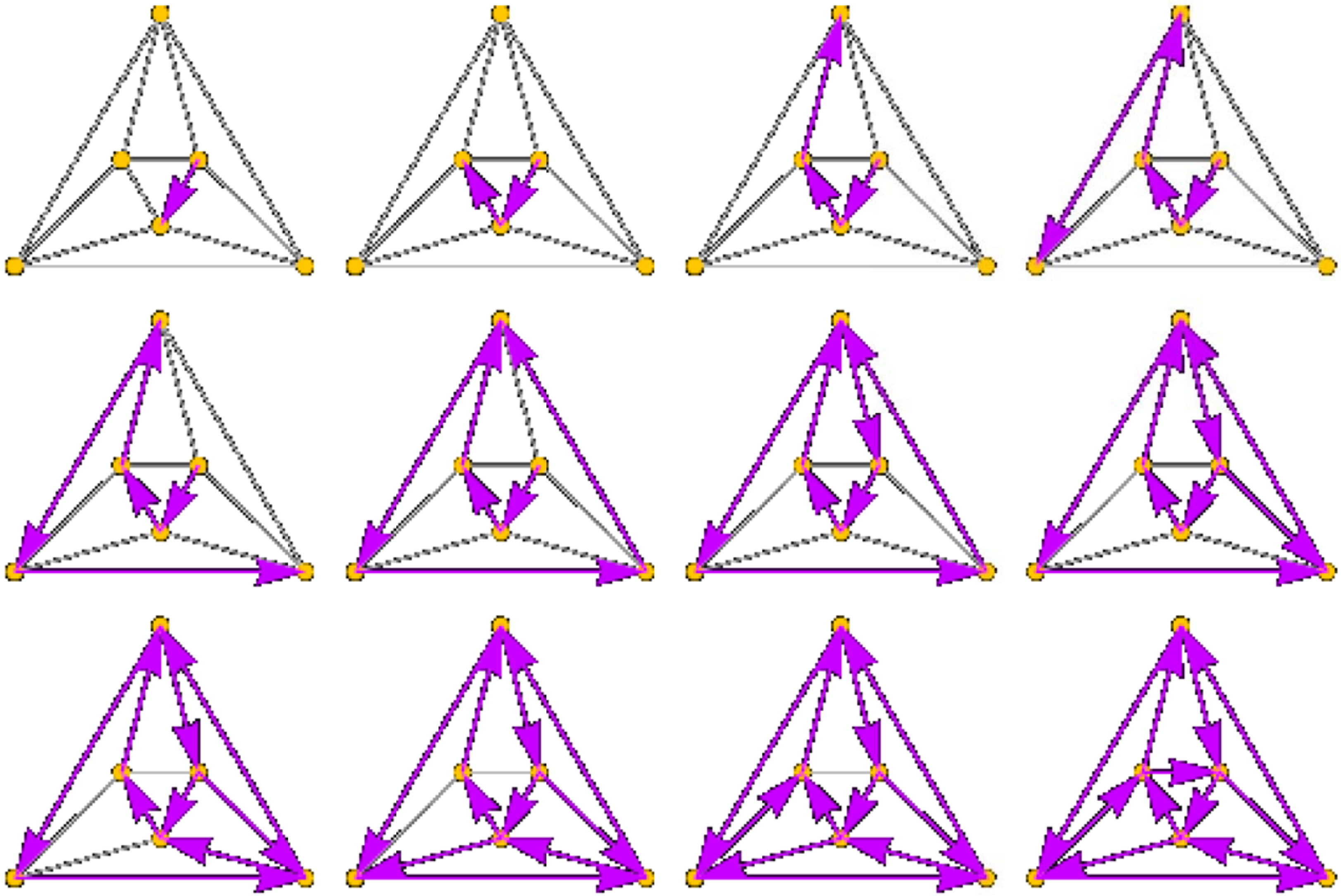
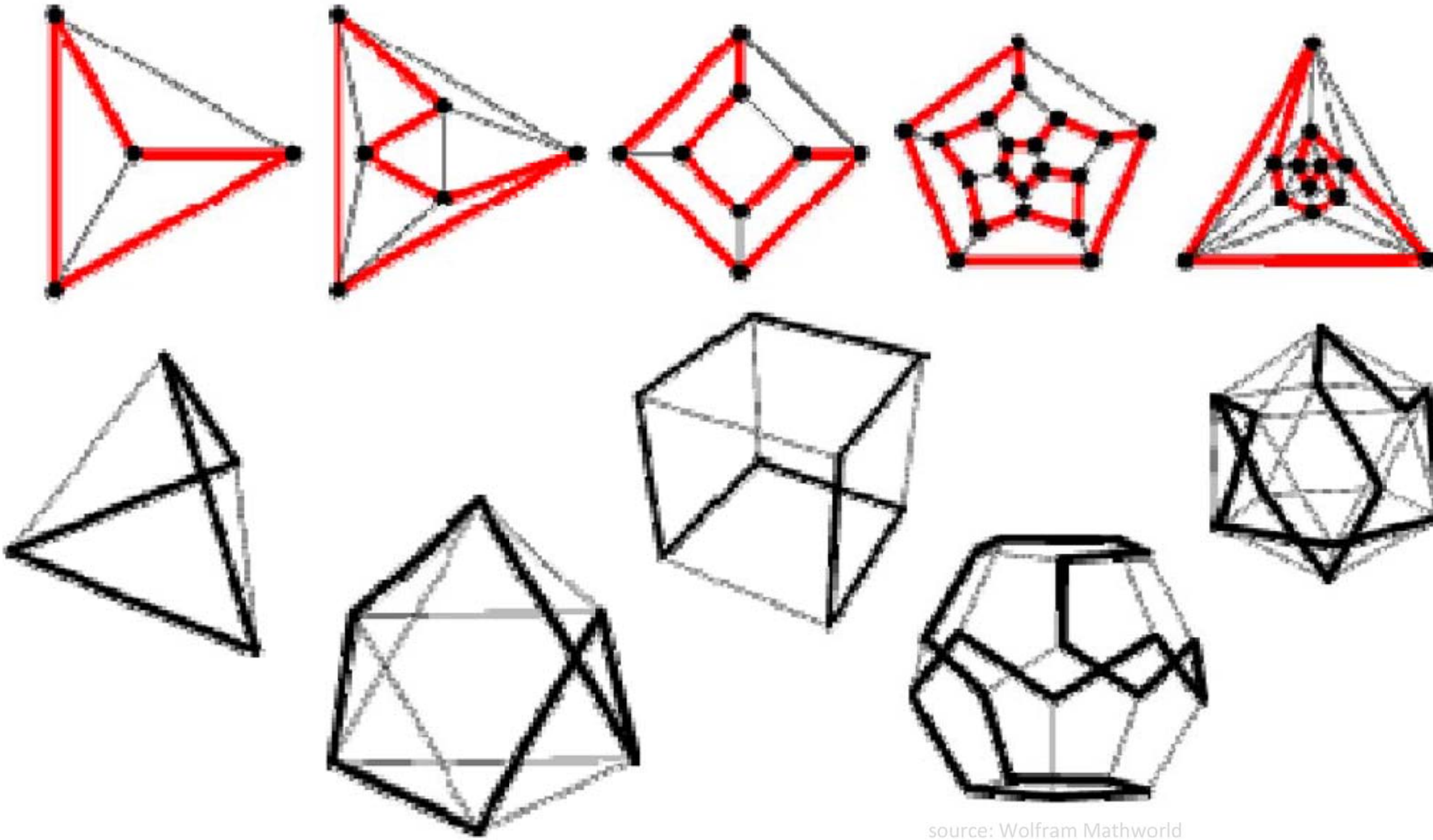


source: Wolfram Mathworld

Hamiltonian Cycles

(use each node once to cycle the graph)



source: Wolfram Mathworld

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Computability Classes...

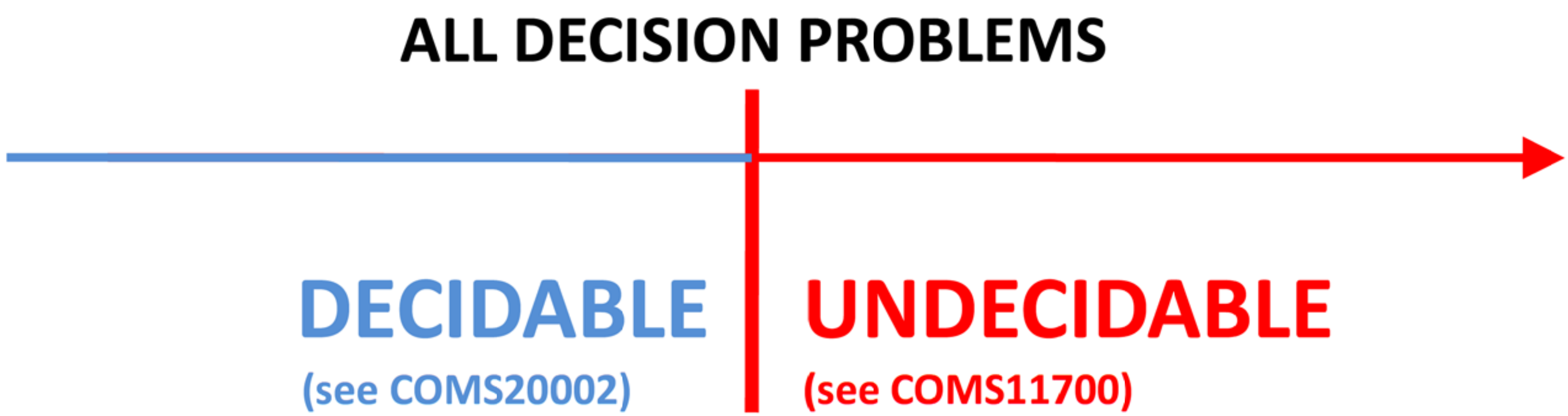
describe sets of problems (or languages) that can be solved (decided/recognised) **at all** by a given machine (e.g. a Turing Machine)...

Complexity Classes...

describe sets of problems (or languages) that can be solved by a given machine (e.g. a Turing Machine) in a **bounded amount** of time, space or other resource...

Computability Theory

- Computability Theory aims at working out what can be computed at all
- Not everything can be computed! – some problems/languages cannot be decided by a Turing Machine ... e.g. halting problem (board)



Recap: Chomsky Hierarchy

Undecidable

TM

- Type 0: recursively enumerable
  $X \rightarrow Y$

Decidable

- Type 1: context-sensitive
  $XAY \rightarrow XZY$

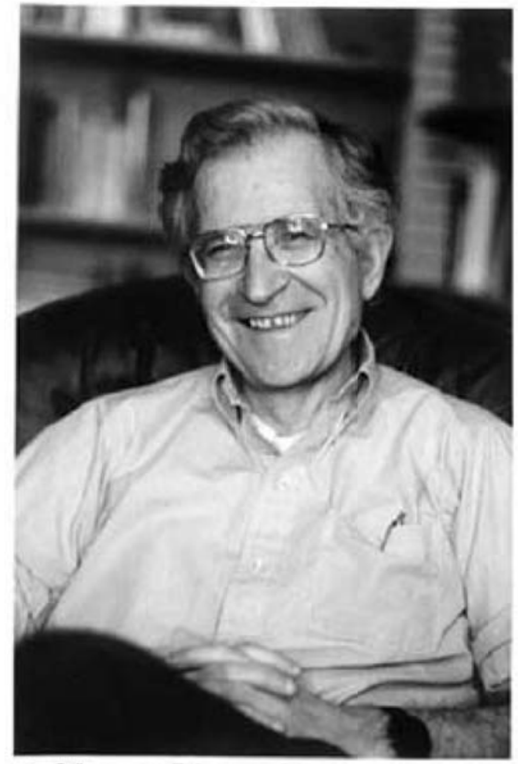
PDA

- Type 2: context-free
  $A \rightarrow X$

DFA

- Type 3: regular
  $A \rightarrow a\, ; A \rightarrow aB$

trivial



A Noam Chomsky

$X, Y, Z \ldots$ strings of terminals and non-terminals

a, b, c … terminals

$A, B, C \ldots$ non-terminals

Modelling Problem Difficulty

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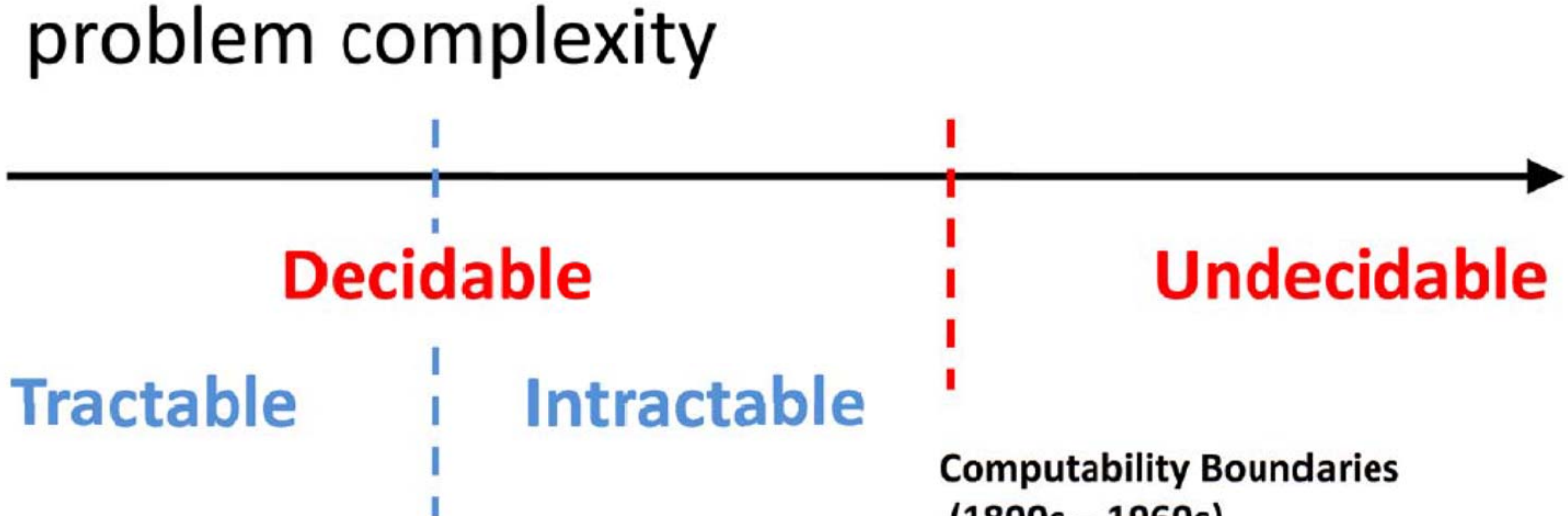


Complexity Boundaries (1960s – today)

1960s: Hartmanis and Stearns: Complexity classes
1971: Cook/Levin, Karp: $P=NP$?
1976: Knuth's O, Ω, Θ

Computability Boundaries (1800s – 1960s)

1900: Hilbert's Problems
1936: Turing's *Computable Numbers*
1957: Chomsky's *Syntactic Structures*

Recap: Big-O and Big-Ω

$f(n) = O(g(n))$ defines an upper bound and means:

There are *positive* constants c and n_0 such that

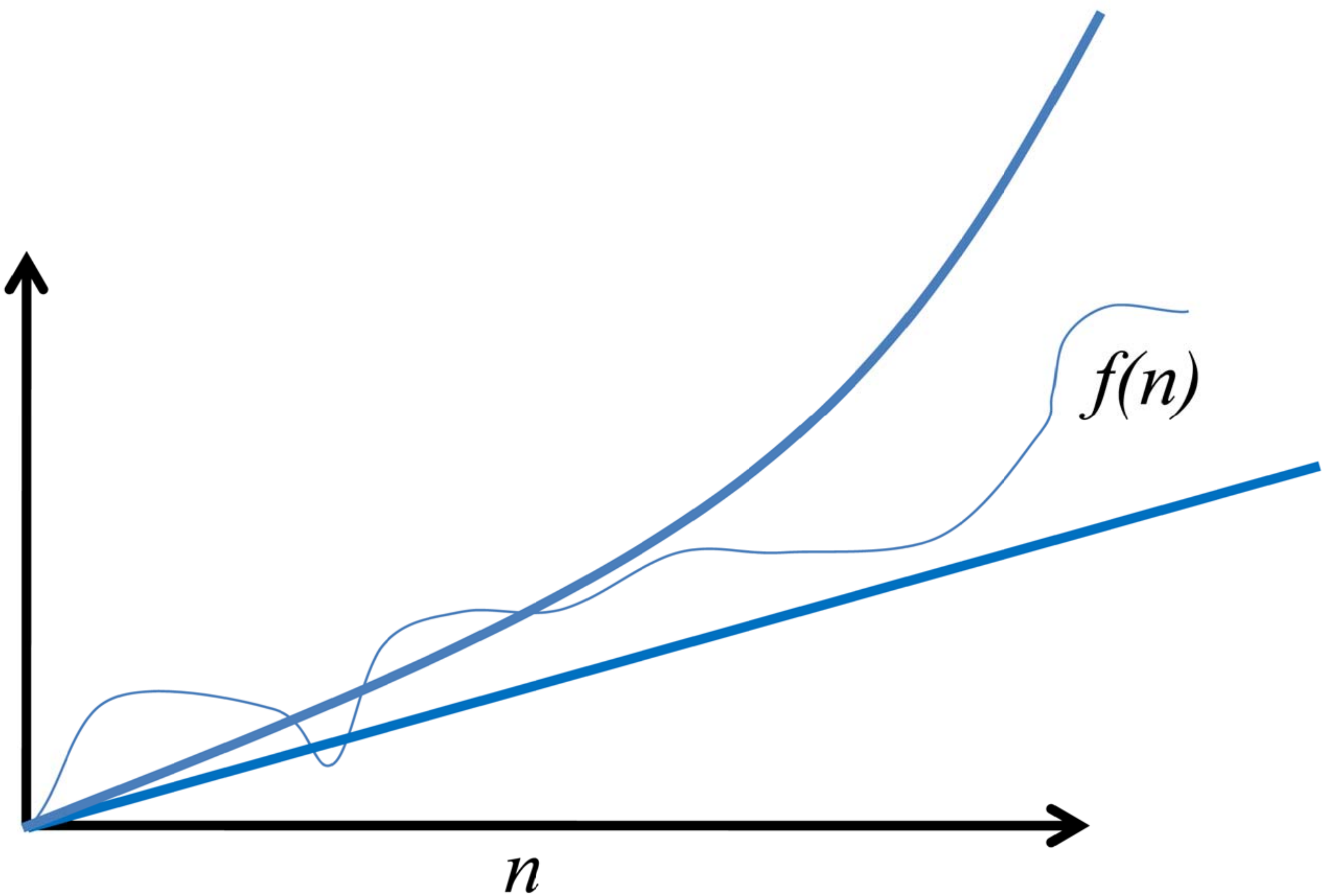
$$f(n) \leq cg(n) \quad \text{for all } n \geq n_0$$

$f(n) = \Omega(g(n))$ defines a lower bound and means:

There are *positive* constants c and n_0 such that

$$f(n) \geq cg(n) \quad \text{for all } n \geq n_0$$

Recap: Big-O and Big-Ω



Theta (`order of’) Notation

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Intuitively: $f(n) = \Theta(g(n))$ stipulates a set of functions that <u>grow as fast as</u> f that is:

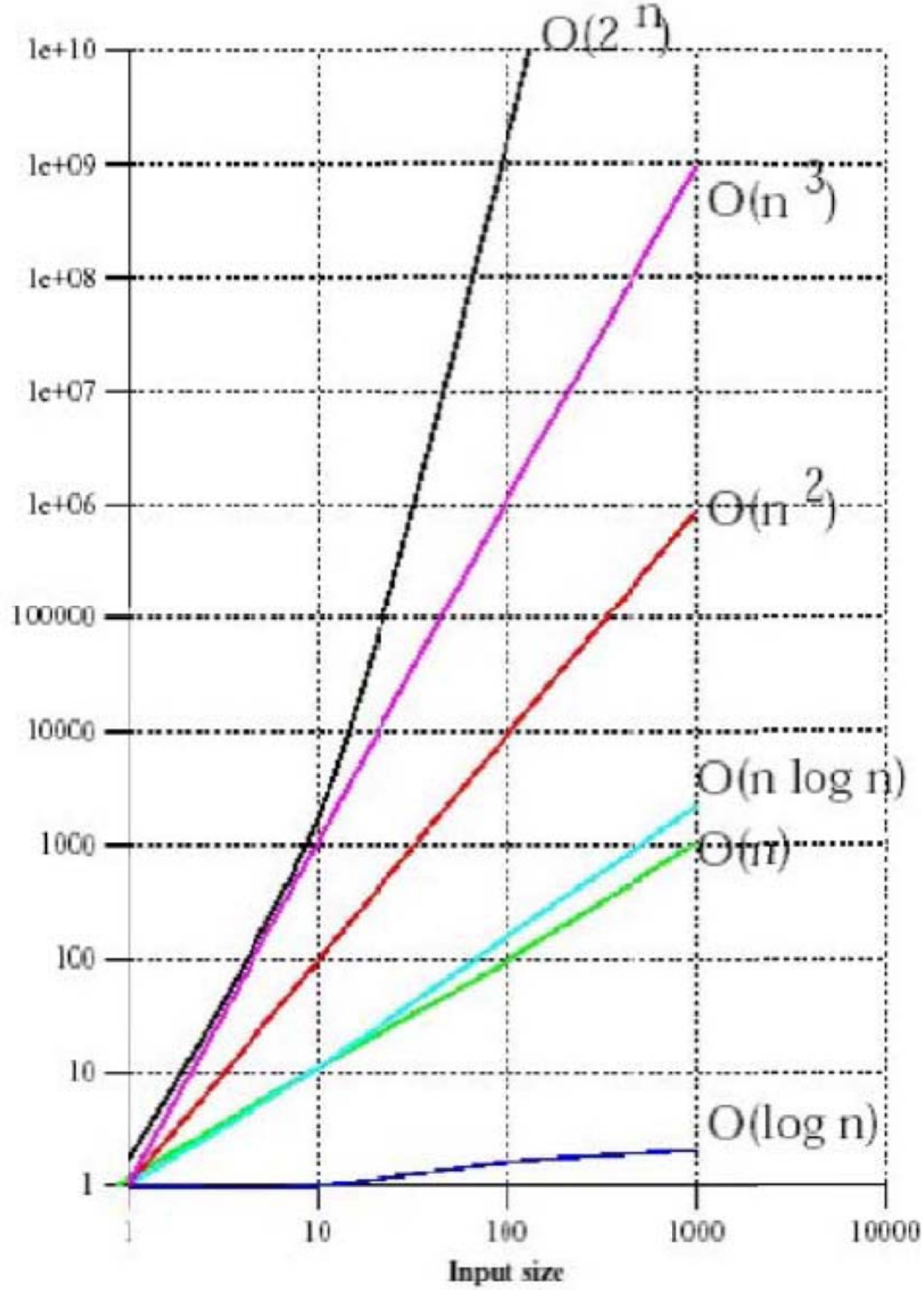
Formally: $f(n) = \Theta(g(n))$ if and only if:

$$f(n) = O(g(n))$$

and

$$f(n) = \Omega(g(n))$$

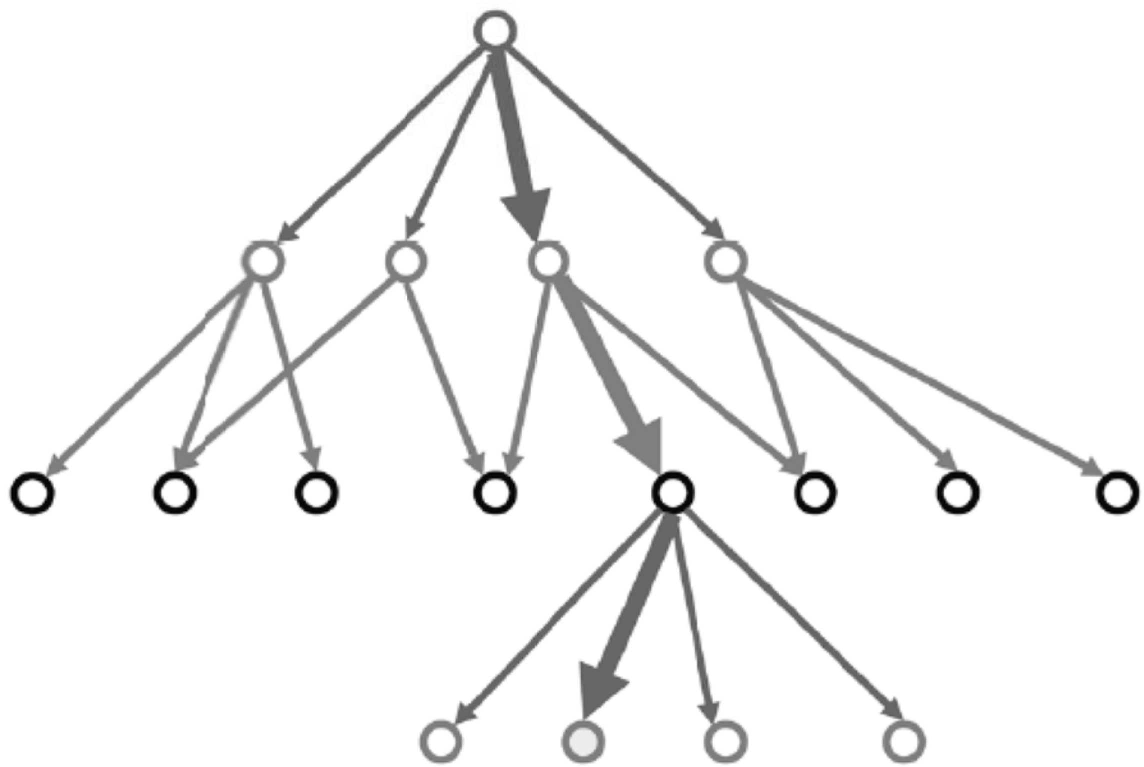
We then say `f is of order g’



O(2 n)
O(n 3)
O(n 2)
O(n log n)
O(n)
O(log n)
1e+10
1e+09
1e+08
1e+07
1e+06
100000
10000
1000
100
10
1
1
10
100
1000
10000
Input size

Recap: Game Verifiers

Given a path through a game, can you check if it is a valid winning path in polynomial time?



Oracles

Determinism vs Non-Determinism

Example Problem:

Is `x=96` part of the list `A=(9,4,55,67,...,96,8)` ?

Deterministic Algorithm

```
• for i=1 to n {
    if (A(i) = x) return true;
  }
  return false;
```

Non-deterministic Algorithm

```
• j ← oracle_choice (1:n)
  if {A(j) = x} return true
   else return false;
```

Deterministic Machines...

running on a particular input will always produce the same output ... and the underlying machine model (e.g. a Turing Machine) will always pass through the same pre-determined sequence of states based on the input data and program.

Non-deterministic Machines (with an oracle) ...

can `guess correctly’ at decision points and their output can be verified quickly by checking the produced certificate in polynomial time.

A decision problem is in P ...

if and only if it can be decided by a deterministic polynomial time Turing Machine.

A decision problem is in NP ...

if and only if it can be decided by a non-deterministic polynomial time Turing Machine

A language is in NP ...

if and only if it has a polynomial time verifier. That is, there is a certificate which can check that a string is part of the language in polynomial time.

Problem Complexity Overview

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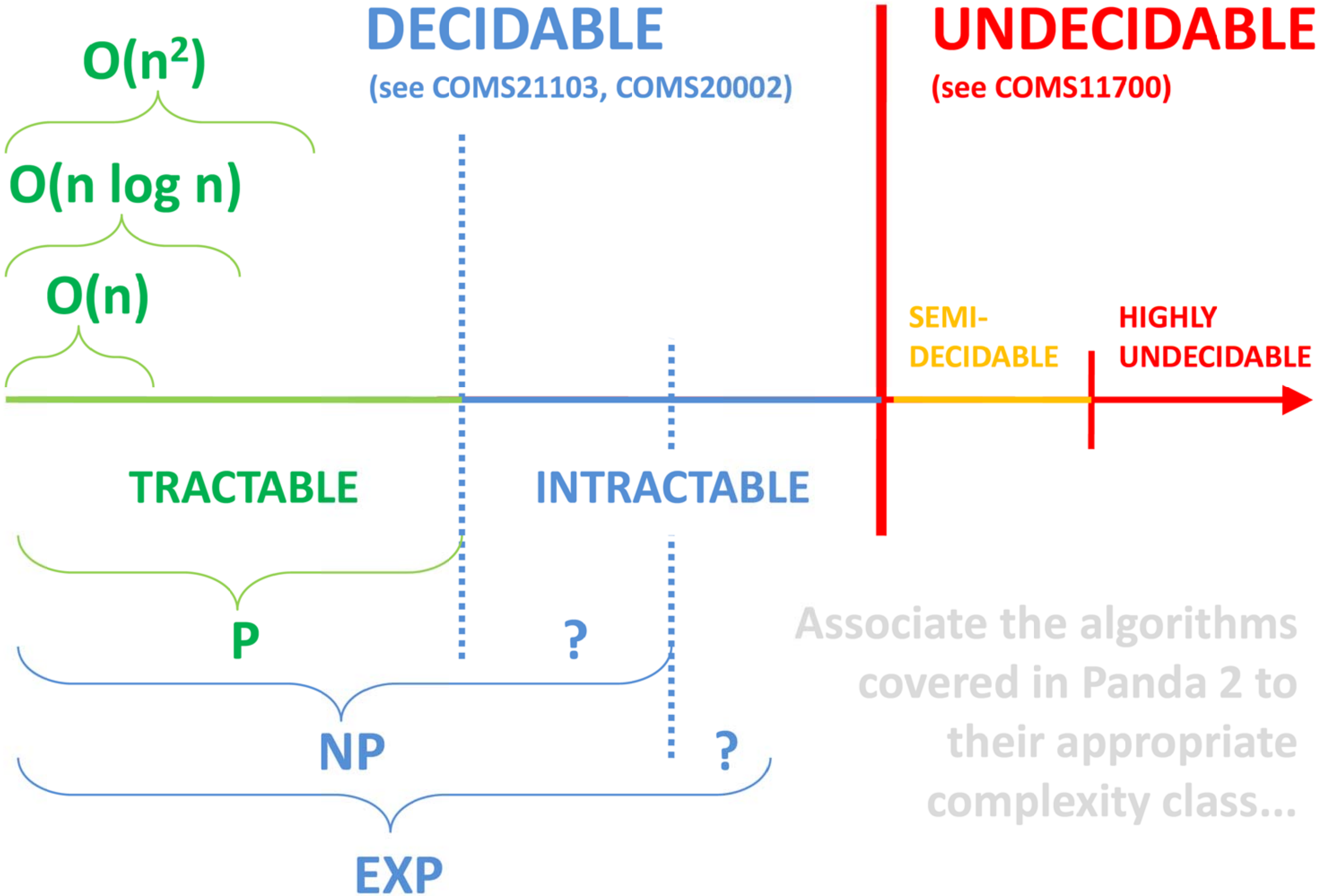

Associate the algorithms covered in Panda 2 to their appropriate complexity class...