COMS21202 – Symbols, Patterns and Signals

Problem Sheet: Representations and Transformations

1 – Using *sin(2πnx)*, demonstrate the concept of superposition as follows:

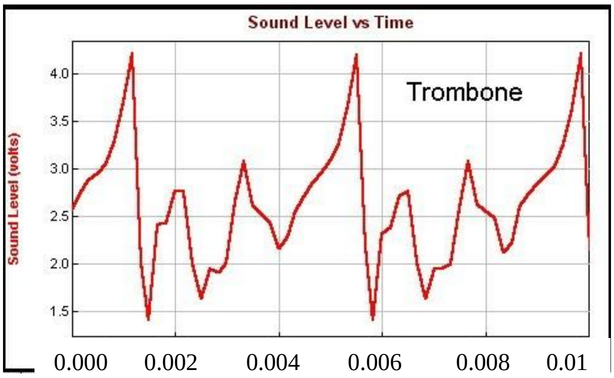
(a) first plot three sine functions over the range ±3 in steps of 0.1 using *n={1/4,1,2}*. Note, plots should appear in the same graph to give a better sense of what is happening.

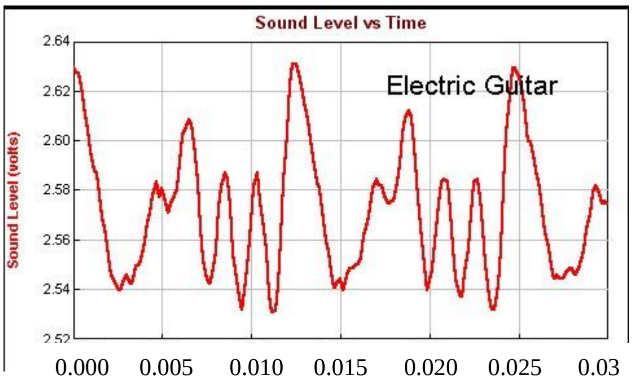
(b) Now plot in a different colour the sum of all the sines above.

(c) Add more sine functions over the same range and repeat step (b).

2 – What is White Light? Illustrate your answer with an approximate graph.

3 – The graphs below display the amplitude of the sound wave for a Trombone and an Electric Guitar as a function of time. The y-axis is the amplitude axis and the x-axis is the time axis. Notice that each one is plotted over a different length of time.





(a) Mark the period of the signal for each instrument.

(b) Approximately, how many periods are shown in these graphs for each instrument?

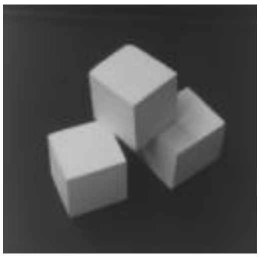
(c) Approximately, what is the peak amplitude in each case?

(d) Approximately, what is the frequency given the signal period in each case?

(e) Which signal contains higher frequency information? Why?

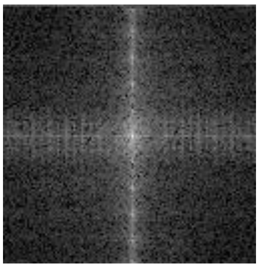
4 – Consider the two images (Sugar and Bricks) on the left. Identify which of the Fourier spaces (FS1 and FS2) on the right belongs to which image and explain clearly why.

Sugar

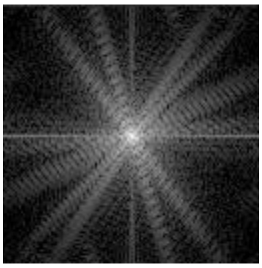


Bricks

FS1



FS2



5 – How would low pass filtering be achieved using the Fourier domain? In your answer describe what is meant by Cut-off Frequency.

6 – The following gene sequence contains significant frequencies. Design two different symbolic encodings and in each case apply your encoding to extract some of these frequencies.

ACAGAGATACAGAGATACAG......

7 – Calculate the result of the convolution *A**B* in each of the examples below by hand.

(i) $A = \begin{pmatrix} 1 & 2 & 1 \end{pmatrix}$ $\quad B = \begin{pmatrix} 2 & 2 & 3 & 3 & 2 \end{pmatrix}$

(ii) $A = \begin{pmatrix} 1 & 1 & 3 & 1 & 1 \end{pmatrix}$ $\quad B = \begin{pmatrix} 3 & 3 & -2 & -1 & 0 & 1 & 2 & 3 & 3 \end{pmatrix}$

(iii) $A = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix}$ $\quad B = \begin{pmatrix} 0 & 5 & 5 & 5 & 0 \\ 0 & 5 & 10 & 5 & 0 \\ 0 & 10 & 10 & 10 & 0 \\ 0 & 5 & 10 & 5 & 0 \\ 0 & 5 & 5 & 5 & 0 \end{pmatrix}$

Now verify your result using the *conv* family of functions in Matlab. Use *help conv* to determine what convention Matlab uses when convolving at the border points.

8 – Convolution in the spatial domain is equivalent to multiplication in the frequency domain, i.e. $f*g = FG$ (see Convolution lecture).

(a) Write a Matlab program that demonstrates $f*g = FG$ using any *NxN* image *f* and a *5x5* averaging filter *g* consisting of all *1*s. (Hint: you will need to pad *g* with zeros to make it the same size as the image before using fft2 in Matlab).
(b) Use Matlab's clock command to time how convolution in the spatial domain compares with multiplication in the Fourier domain.

9 – Determine the coordinates of point $S=(x_s,y_s)$ on the view plane as the perspective projection of point P=(44,75,50) in the world coordinate system. Assume the view plane is parallel to the XY plane and positioned along the Z-axis at the origin. The COP point is at C=(0,0,-25). Give the answer correct to 2 decimal places as well as rounded to integer screen coordinates.

10- a) In Figure 1, the top row shows an original image of a Clown and its Fourier space after an FFT operation. The rest of the figure shows four pairs of images, labelled (A, B, C, D). In each case one image is a filter mask and the other is the result of applying the mask to the Clown's Fourier space. What effect does each filter have on the frequencies in the Fourier space?

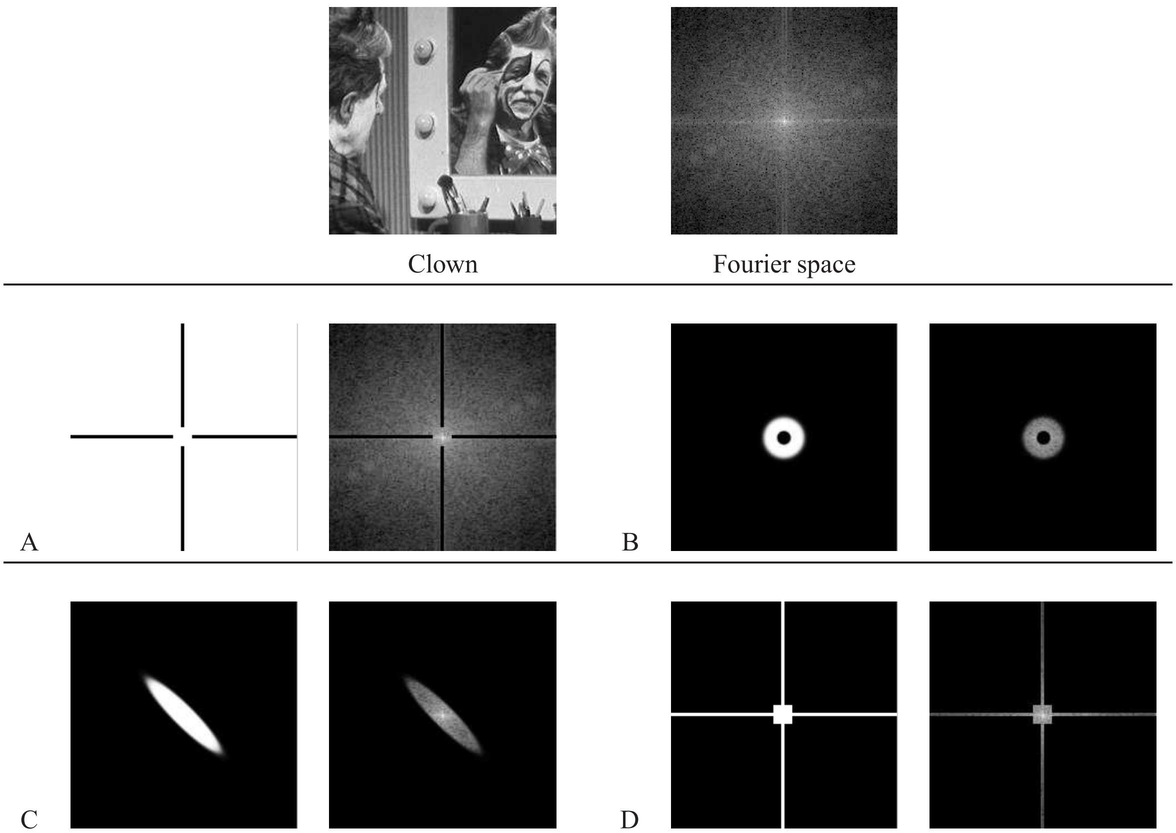


Figure 1: (top row) Clown image and its Fourier space, (next two rows) filter masks and the result after applying each mask to the Fourier space.

b) The four images in Figure 2, labelled (W, X, Y, Z) represent, *in an arbitrary order*, the inverse FFT of the Fourier spaces in (A, B, C, D) in Figure 1. Correctly state which inverse FFT image corresponds to which filtered Fourier image, and why.

W X Y Z

Figure 2: (top row) Clown image and its Fourier space, (next two rows) filter masks and the result after applying each mask to the Fourier space.