University of Bristol

COMS21103: Data Structures and Algorithms

Problem Set 1

Problem 1: Prove that $o(g(n)) \cap \omega(g(n))$ is the empty set.

Problem 2: Asymptotic notation properties

Let $f(n)$ and $g(n)$ be asymptotically positive functions. Prove or disprove each of the following conjectures.

1. $f(n) = O(g(n))$ implies $g(n) = O(f(n))$.
2. $f(n) + g(n) = \Theta(\min(f(n), g(n)))$.
3. $f(n) = O(g(n))$ implies $\lg(f(n)) = O(\lg(g(n)))$, where $\lg(g(n)) \geq 1$ and $f(n) \geq 1$ for all sufficiently large n.
4. $f(n) = O(g(n))$ implies $2^{f(n)} = O(2^{g(n)})$.
5. $f(n) = O((f(n))^2)$.
6. $f(n) = O(g(n))$ implies $g(n) = \Omega(f(n))$.
7. $f(n) = \Theta(f(n/2))$.
8. $f(n) + o(f(n)) = \Theta(f(n))$.

Problem 3: Relative asymptotic growths

Indicate, for each pair of expressions (A, B) in the table below, whether A is O, o, Ω, ω, or Θ of B. Assume that $k \geq 1$, $\epsilon > 0$, and $c > 1$ are constants. Your answer should be in the form of the table with "yes" or "no" written in each box.

	A	B	O	o	Ω	ω	Θ
a.	$\lg^k n$	n^ϵ					
b.	n^k	c^n					
c.	$\sqrt{n}$	$n^{\sin n}$					
d.	2^n	$2^{n/2}$					
e.	$n^{\lg c}$	$c^{\lg n}$					
f.	$\lg(n!)$	$\lg(n^n)$					