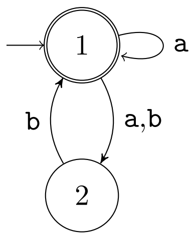
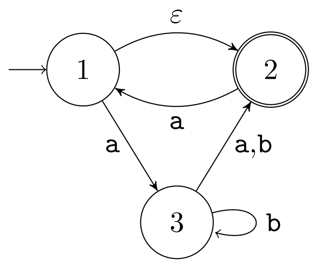
1 Converting NFAs to DFAs ($\star$)

Use the generic construction to convert the following two nondeterministic automata to equivalent deterministic finite automata.



(1)



(2)

2 Constructing & converting NFAs from regular expressions ($\star$)

1. Give an NFA recognizing the language $(\texttt{01} \cup \texttt{001} \cup \texttt{010})^*$.
2. Convert this NFA to an equivalent DFA. Give only the portion of the DFA that is reachable from the start state.

3 Binary addition and regular languages ($\star\star$)

Let

$$\Sigma_3 = \left\{ \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \ldots, \begin{bmatrix}1\\1\\1\end{bmatrix} \right\}.$$

Σ_3 contains all size 3 columns of 0s and 1s. A string of symbols in Σ_3 gives three rows of 0s and 1s. Consider each row to be a binary number and let

$$B = \{w \in \Sigma_3^* \mid \text{the bottom row of } w \text{ is the sum of the top two rows}\}.$$

For example,

$$\begin{bmatrix}0\\0\\1\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix} \in B, \quad \text{but} \quad \begin{bmatrix}0\\0\\1\end{bmatrix}\begin{bmatrix}1\\0\\1\end{bmatrix} \notin B.$$

Show that B is regular. (Hint: Working with $B^{\mathcal{R}}$ is easier. You may assume the result claimed in Problem 4 of Problem Sheet 2, i.e., if a language L is regular, then its 'reverse' language $L^{\mathcal{R}}$ is regular.)

4 Binary multiplication and regular languages ($\star\star\star$)

Let

$$\Sigma_2 = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}.$$

Here, Σ_2 contains all columns of 0s and 1s of height two. A string of symbols in Σ_2 gives two rows of 0s and 1s. Consider each row to be a binary number and let

$$C = \{w \in \Sigma_2^* \mid \text{the bottom row of } w \text{ is three times the top row}\}.$$

For example, $\begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \in C$, but $\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \notin C$. Show that C is regular. (You may assume the result claimed in Problem 4 of Problem Sheet 2.)

5 Regular expressions ($\star$)

For each of the following languages, give two strings that are members and two strings that are *not* members—a total of four strings for each part. Assume the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}\}$ in all parts.

1. $\mathtt{a}^*\mathtt{b}^*$
2. $\mathtt{a}(\mathtt{ba})^*\mathtt{b}$
3. $\mathtt{a}^* \cup \mathtt{b}^*$
4. $(\mathtt{aaa})^*$
5. $\Sigma^*\mathtt{a}\Sigma^*\mathtt{b}\Sigma^*\mathtt{a}\Sigma^*$
6. $\mathtt{aba} \cup \mathtt{bab}$
7. $(\varepsilon \cup \mathtt{a})\mathtt{b}$
8. $(\mathtt{a} \cup \mathtt{ba} \cup \mathtt{bb})\Sigma^*$