COMS22201: Language Engineering

Lab Exercises - Week 18 - Answers

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This worksheet will develop your understanding of the fixpoint characterisation of loops and the semantics of the statements of **While** as given by the semantic function from the last lecture with the following signature:

$$\mathcal{S}_{ds}[\![.]\!] : Stm \to (State \hookrightarrow State)$$

1. Explain whether $\mathcal{S}_{ds}[\![.]\!]$ is a *partial* or *total* function.

2. Give a direct characterisation of the semantics of the following program (by repeatedly applying the definition of $\mathcal{S}_{ds}[\![.]\!]$ and rearranging until all semantic brackets have been eliminated):

   ```
   if (x≤0) then x:=x*(0-1) else skip
   ```

3. Prove that there can be at most one *least fixpoint* of a any function f with respect to some *partial order* $\sqsubseteq$.

4. Find the least fixpoint of the function $f = (square \circ half \circ inc)$ obtained by composing the real-valued operators $square = \lambda x.(x * x)$, $half = \lambda x.(x/2)$ and $inc = \lambda x.(x + 1)$.

5. Give a direct characterisation of the functionals of the following loops:

   - `while ¬(x=0) do skip`
   - `while ¬(x=0) do x:=x-1`
   - `while ¬(x=0) do (y:=y*x; x:=x-1)`

   Find any fixpoints of the above and identify any least fixpoints.

6. Show that $\mathcal{S}_{ds}[\![\texttt{while true do skip}]\!] = \emptyset$.

7. Show that `S;skip` is semantically equivalent to `S`.

8. Show that `S1;(S2;S3)` is semantically equivalent to `(S1;S2);S3`.

9. Convince yourself that following Haskell function computes the factorial of its argument:

```
f = fix (\f n -> if n == 0 then 1 else n * f (n-1))
    where fix f = f (fix f)
```