University of BRISTOL | Department of Computer Science

PROGRAMMING and ALGORITHMS II

Week 16 - Worksheets

CYK, Prim’s and Dijkstra’s Algorithms

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Unit Code COMS10001

CYK Example 1

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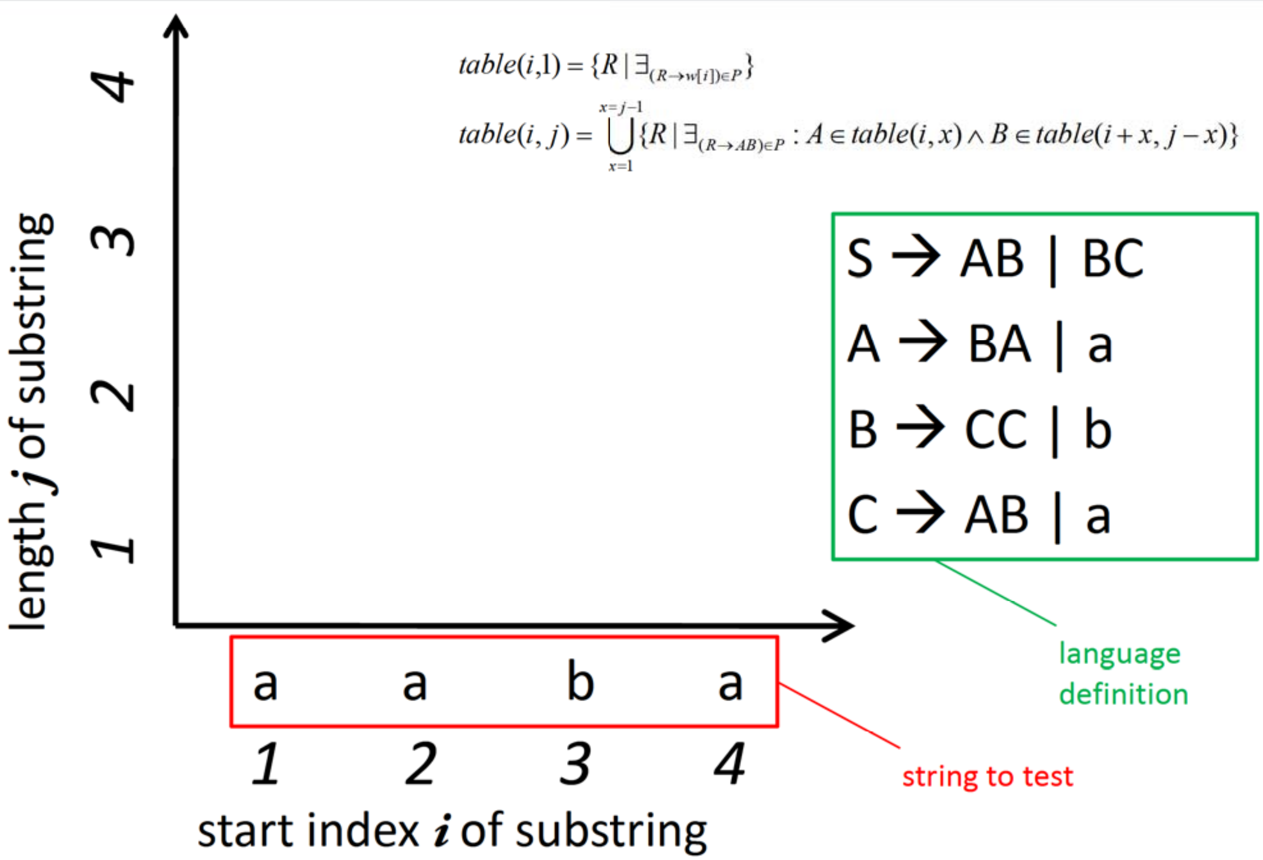
$$table(i,1) = \{R \mid \exists_{(R \to w[i]) \in P}\}$$

$$table(i,j) = \bigcup_{x=1}^{x=j-1} \{R \mid \exists_{(R \to AB) \in P} : A \in table(i,x) \wedge B \in table(i+x, j-x)\}$$



CYK Example 2

length j of substring: 1 2 3 4 5 6

S → DE
D → AB | AF
E → CE | c
F → DB
A → a
B → b
C → c

language definition

a	a	b	b	c	c
1	2	3	4	5	6

string to test

start index i of substring

CYK Example 3

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length j of substring: 1 2 3 4 5 6

S → SS |OA |OC
A → SC
O → [
C →]

Definition of the 'Dyck Language'

[	[	]	[	]	]
1	2	3	4	5	6

string to test

start index i of substring

Prim Example 1

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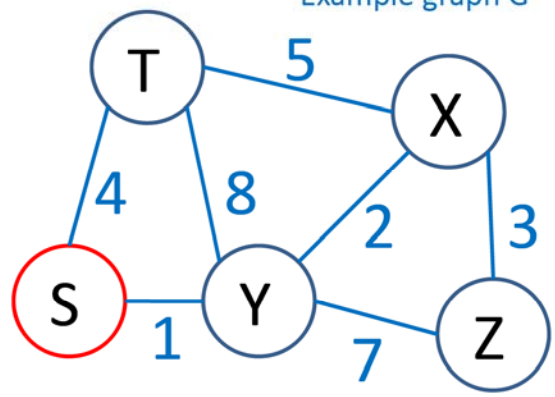
$G=(V,E)$	... graph with weights $d(v,w)\geq 0$ for all $v,w \in V$ (where $d(v,w)=\infty$ if $(v,w)\notin E$)
$W\subseteq V$	... set of visited nodes
$F\subseteq E$	... set of utilised edges for current tree
$s\in V$	... source vertex to start tree
$D(v)$	... immediate distance from current tree to v

1) Initialise: $W=\{s\};F=\{\};\forall_{v\in V}:D(v)=d(s,v)$
2) Select a new **current vertex** w in $V\setminus W$ with minimal $D(w)$
3) Add current vertex w to W, add related edge to F
4) Update distances: $\forall_{v\in V\setminus W}:D(v)=\min(D(v),d(w,v))$
5) If $V=W$ then exit, otherwise Goto 2)

Example graph G



	S	T	X	Y	Z
D:	0	4	∞	1	∞

F = { }

W = { S }

current next hop to S

visited nodes

current best distance

Prim Example 2

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$G=(V,E)$	... graph with weights $d(v,w)\geq 0$ for all $v,w \in V$ (where $d(v,w)=\infty$ if $(v,w)\notin E$)
$W\subseteq V$	... set of visited nodes
$F\subseteq E$	... set of utilised edges for current tree
$s\in V$	... source vertex to start tree
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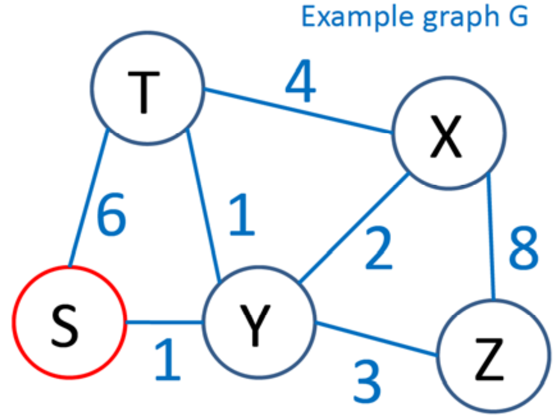
Example graph G



	S	T	X	Y	Z
D:	0	6	∞	1	∞

F = { }

W = { S }

current next hop to S

visited nodes

current best distance

Dijkstra Example 1

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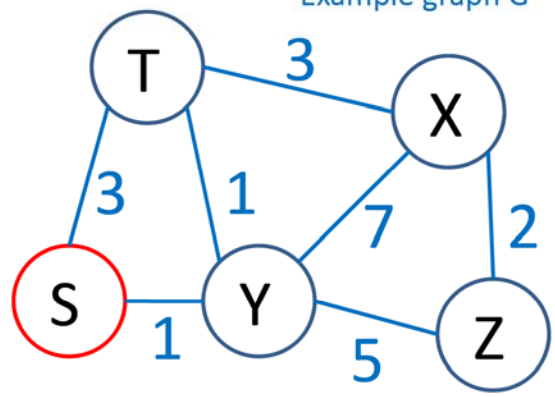
$G=(V,E)$	... graph with weights $d(v,w)\geq 0$ for all $v,w\in V$ (where $d(v,w)=\infty$ if $(v,w)\notin E$)
$W\subseteq V$	... set of visited nodes
$s\in V$	... source vertex to calculate distances to
$D(v)$	... current shortest distance estimate from s to v

1) Initialise: $W=\{s\};\forall_{v\in V}: D(v)=d(s,v)$
2) Select a new **current vertex** w in $V\backslash W$ with minimal $D(w)$
3) Add current vertex w to W
4) Update distances: $\forall_{v\in V\backslash W}: D(v)=\min(D(v),D(w)+d(w,v))$
5) If $V=W$ then exit, otherwise Goto 2)

Example graph G



	S	T	X	Y	Z
D:	0	3	∞	1	∞
R:	0	S	S	S	S

W = { S }

current next hop to S

visited nodes

current best distance

Dijkstra Example 2

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$G=(V,E)$	... graph with weights $d(v,w)\geq 0$ for all $v,w\in V$ (where $d(v,w)=\infty$ if $(v,w)\notin E$)
$W\subseteq V$	... set of visited nodes
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$D(v)$	... current shortest distance estimate from s to v

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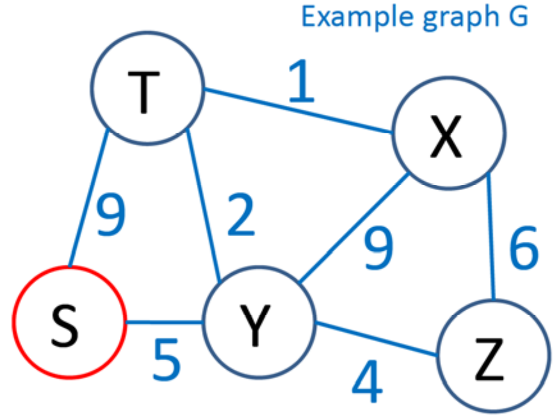
5) If $V=W$ then exit, otherwise Goto 2)

Example graph G



	S	T	X	Y	Z
D:	0	9	∞	5	∞
R:	0	S	S	S	S

W = { S }

current next hop to S

visited nodes

current best distance