

1 Context-free pumping (★★)

Prove, using the pumping lemma, that the following languages are not context-free.

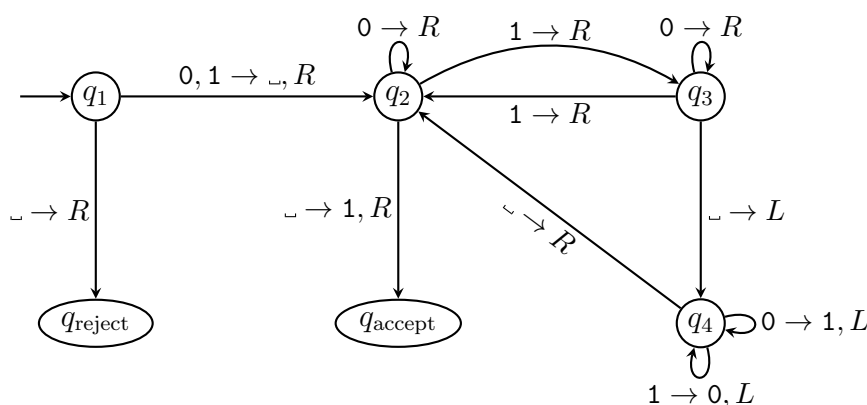
1. $\{a^n b^n a^n b^n \mid n \geq 0\}$
2. $\{a^n \# a^{2n} \# a^{3n} \mid n \geq 0\}$
3. $\{w \# t \mid w \text{ is a substring of } t \text{ and } w, t \in \{a, b\}^*\}$
4. $\{t_1 \# t_2 \# \dots \# t_k \mid k \geq 2, t_i \in \{a, b\}^* \text{ for all } i \text{ and for some } i \neq j, t_i = t_j\}$
5. The language of palindromes over $\{a, b\}$ that contain the same number of 'a's and 'b's.
6. (★★★) Over $\Sigma = \{1, 2, 3, 4\}$, the language of all words w where $N_1(w) = N_2(w)$ and $N_3(w) = N_4(w)$. Here $N_\sigma(w)$ denotes the number of times character σ appears in w .

2 Turing machine definition (★)

1. Can a Turing machine ever write the blank symbol \sqcup on its tape?
2. Can the tape alphabet Γ be equal to the input alphabet Σ of a Turing machine?
3. Can the tape head of a Turing machine ever be in the same location on two successive steps?
4. Can a Turing machine contain just a single state?

3 A mysterious Turing machine (★★)

Let M be the Turing machine described by the following transition diagram, with input alphabet $\{0, 1\}$, tape alphabet $\{0, 1, \sqcup\}$, start state q_1 , accepting state q_{accept} and rejecting state q_{reject} .



1. What is $L(M)$?
2. Does M halt on every possible input?

4 Tape alphabet (★★)

Let $\mathcal{L} \subseteq \{0, 1\}^*$. Show that, if \mathcal{L} is Turing-recognisable, it is recognised by a Turing machine with tape alphabet $\{0, 1, \sqcup\}$.

5 Doubly infinite tape (★★)

A Turing machine with *doubly infinite tape* has a tape that is infinite to the left as well as the right (so it will never encounter a tape end). The head starts on the leftmost character of the input and the tape contains blanks on both sides of the input. Computation is otherwise defined as for normal Turing machines.

Show that every language recognised by a Turing machine with doubly infinite tape is Turing-recognisable (i.e. is recognised by a standard Turing machine).