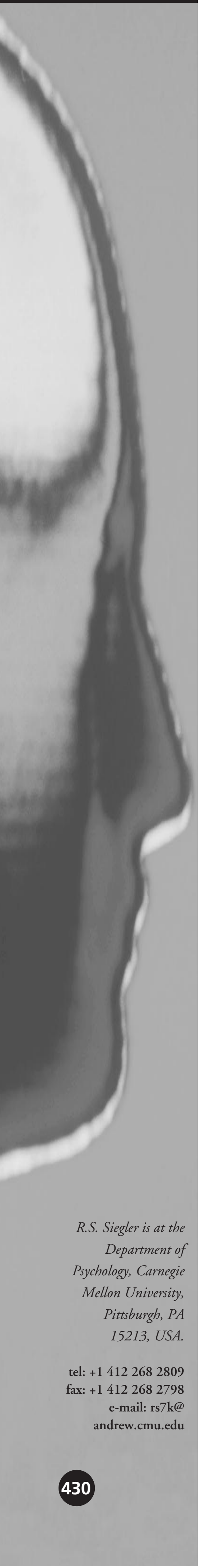

Strategic development

Robert S. Siegler

Strategic development is more diverse, multifaceted and eventful than previously realized. Individual children know and use multiple strategies for solving a given kind of problem; they choose adaptively among available alternatives; and they frequently discover new strategies that enhance their problem-solving abilities. A recently formulated computer simulation (SCADS) indicates how associative and metacognitive processes work together to produce adaptive choices among existing strategies and discovery of useful new approaches.

From infancy to old age, people solve problems by using strategies. Faced with the need to descend down a ramp to their beckoning mother, infants sometimes crawl down the ramp, sometimes slide down feet first, sometimes slide down head first, and sometimes descend in a sitting position[1]. Faced with the need to solve two-digit multiplication problems, 70-year-olds sometimes use a calculator, sometimes use pencil and paper, and sometimes multiply in their head[2]. This strategic variability is present within individuals as well as between them; the same infant will use varied strategies to descend down the ramp, and the same senior citizen will use varied strategies to multiply numbers. Thus, varied strategy use seems to be a basic characteristic of cognition and action throughout the life span.

Because children must constantly devise strategies for coping with unfamiliar problems, strategy choice and strategy discovery play particularly large roles in their lives. Recent research on these topics has led to fundamental shifts in views of strategic development, and indeed of cognitive development in general. Simply put, strategic development is more diverse, more multifaceted, and more eventful than had previously been imagined. Cross-sectional studies of strategy use at different ages, microgenetic studies of children in the process of discovering novel strategies, and computer simulations of strategic development all converge on the same conclusions.

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Two visual metaphors for development

Implicit metaphors shape our thinking about many topics. An implicit metaphor underlying ideas about cognitive development in general, and strategic development in particular, was made explicit by Robbie Case when he titled his 1992 book *The Mind's Staircase*[3]. As shown in Fig. 1A, within the 'staircase' metaphor, children are depicted as thinking in a given way for an extended period of time (a tread on the staircase); then their thinking undergoes a sudden upward shift (a riser on the staircase); then they think in a different, 'higher' way for another extended period (the next tread); and so on.

Although this depiction of development is most closely associated with the Piagetian and neo-Piagetian traditions, it also underlies competing approaches to development. For example, theory–theory approaches rest on a similar metaphor. Two-year-olds are said to have a desire theory of mind, whereas three-year-olds are said to have a belief–desire theory of mind; five-year-olds are said to have a psychological theory of biology until around age 10, when they generate a truly biological theory; and so on[4,5]. Information-processing descriptions also often reflect the staircase metaphor. Thus, five-year-olds are said to solve simple addition problems by counting from one, seven-year-olds by counting from the larger addend, and nine-year-olds by retrieving the answer from memory[6].

Most staircase depictions have been based on data aggregated across many children and many trials. More recently, investigators have begun to assess individual children's strategy use on each trial. They have done this by videotaping the child's overt behavior during the trial and, if the child is old enough, asking immediately after the trial 'How did you solve that problem?'

The results yielded by such trial-by-trial assessments have been both consistent and surprising. Regardless of whether the tasks have involved problem solving, reasoning, memory, language, or motor activity, and regardless of whether the children have been infants, toddlers, preschoolers, elementary schoolers, or adolescents, children use a variety of strategies[7–10].

The strategic variability is not due to one child using one strategy and another child a different one. Individual children have been found to use at least three strategies in such varied domains as arithmetic, spelling, scientific experimentation, and recall of previously presented information[11]. Even when children are presented the identical problem on two successive days, they frequently use a different strategy the second day than the first[12]. Indeed, a single presentation of a problem can elicit one strategy in speech and a different one in gesture (see Box 1).

The fact that children use a variety of strategies over prolonged periods of time does not mean that strategy choices are random or that strategic development is directionless. Even infants and toddlers usually employ strategies that fit the demands of the context[1,10]. With age and experience, their (and older children's) strategy choices become even more adaptive. Thus, children most often use strategies that yield fast and accurate performance on the particular problem, though they use other strategies as well.

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These findings with children have inspired investigators to examine whether adults also used varied strategies to solve a single type of problem. The answer is clear: they do. Such variability in adults' strategies has been documented on spatial reasoning, scientific experimentation, mental arithmetic, sentence verification, and other tasks[13–15].

Such findings indicate that the 'overlapping waves' depiction, shown in Fig. 1B, might be a more useful way of thinking about strategic development than the staircase metaphor. Within the overlapping waves depiction, children typically know and use a variety of strategies at any one time. With age and experience, the relative frequency of each strategy changes, with some strategies becoming less frequent (Strategy 1), some becoming more frequent (Strategy 5), some becoming more frequent and then less frequent (Strategy 2), and some never becoming very frequent (Strategy 3). In addition to changes in relative frequencies of existing strategies, new strategies are discovered (Strategies 3 and 5), and some older strategies cease to be used (Strategy 1).

The overlapping waves approach to strategic development has several advantages over staircase approaches. Most obviously, it fits the data better; studies that have examined strategy use on a trial-by-trial basis have consistently revealed substantial variability. It also better captures the dynamic, continually changing character of development. Moreover, its assumption that discovery of new strategies is a frequent occurrence raises a variety of important issues. What processes lead up to strategy discoveries? What is the experience of strategy discovery like? How are new strategies generalized beyond their initial context? Microgenetic methods have made it possible to investigate all of these issues.

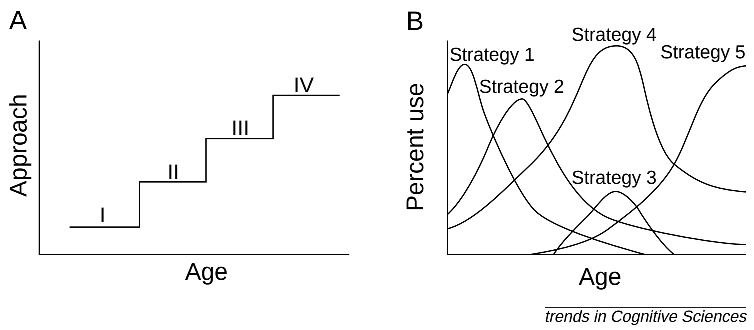

Fig. 1. Models of cognitive development. (A) The staircase model of development: prolonged periods of thinking at a certain level, followed by sudden transitions to new, higher levels of thinking. **(B)** The overlapping waves model of development: use of varied strategies at all times, with continuously changing frequencies of existing strategies and occasional discovery of new strategies.

Discovering new strategies

Microgenetic methods are techniques used to examine changes while they are occurring. They have three key characteristics: (1) observations span as large a portion as possible of the period from the beginning of the change to the time at which competence reaches a relatively stable level; (2) the density of observations is high relative to the rate of change of the phenomenon; and (3) observations are subjected to trial-by-trial analysis with the goal of inferring the processes that gave rise to the changes[16,17]. The approach of following children's thinking on a trial-by-trial basis as they have experiences that are likely to elicit discoveries produces a more detailed database on the discovery process than would otherwise be possible.

A microgenetic study performed by Siegler and Jenkins illustrates these advantages[18]. The study focused on four- and five-year-olds' discovery of the 'min strategy' (counting from

Box 1. Using variable strategies

Variability in strategy use sometimes is evident not only from one trial to the next but even within a single trial. Especially at points of cognitive transition, children often express more advanced thinking through hand motions than the thinking that they express through speech. Even more surprising, children who show such within-trial cognitive variability are more likely to learn from instruction than children who express the same strategy in gesture as in speech (see Goldin-Meadow, this issue, pp. 419–429).

Goldin-Meadow, Alibali, and their colleagues have demonstrated this phenomenon on several tasks. The most common context has involved problems of the following form: 3+4+5=__+5. Solving such problems requires understanding of mathematical equality, the concept that values on each side of the equal sign must be equivalent. Alibali and Goldin-Meadow presented fourth graders with a pretest on such problems, then an instructional procedure, and finally a post-test (Ref. a). The results suggested that children first express the same incorrect understanding in gesture that they do in speech, that they then express a more advanced understanding in gesture but not in speech, and that their understanding in speech then catches up with the understanding that they were expressing in gesture.

Why is expressing different ideas in gesture and in speech positively related to learning? Goldin-Meadow has recently suggested that gesture provides an ideal forum for experimenting with new ideas, leading children unconsciously to gravitate towards it (Ref. b). Teachers and other adults can interpret children's gestures quite accurately (Ref. c). This allows the adults to shape what they say to the ideas expressed in the child's gestures. If the child expresses a good idea in gesture, the adult can make the idea explicit and can encourage the child in that way of thinking. If the child expresses a flawed idea in gesture, the adult can address the likely source of the misunderstanding and provide a better way of thinking about the problem without directly contradicting the child. In contrast, if the child expressed a similar misunderstanding in speech, the adult would be more likely to say 'No, that's wrong', or provide negative feedback in some other relatively direct form. Thus, gestures provide a non-threatening way for children to try out new ways of thinking.

References

a Alibali, M. and Goldin-Meadow, S. (1993) Gesture-speech mismatch and mechanisms of learning: what the hands reveal about a child's state of mind *Cognit. Psychol.* 25, 468–573

b Goldin-Meadow, S. Giving the mind a hand: the role of gesture in cognitive change, in *Mechanisms of Cognitive Development: Behavioral and Neural Perspectives* (McClelland, J.L. and Siegler, R.S., eds), Erlbaum (in press)

c Goldin-Meadow, S. and Sandhofer, C. (1999) Gestures convey substantive information about a children's thoughts to ordinary listeners *Dev. Sci.* 2, 67–74

Table 1. Addition strategies used by preschoolers

Strategy	Use of strategy to solve 3+5
Count from 1	Put up 3 fingers, usually accompanied by saying'1,2,3'; Put up 5 fingers, usually accompanied by saying '1,2,3,4,5'; Count all fingers, saying '1,2,3,4,5,6,7,8'.
Shortcut sum	Say '1,2,3,4,5,6,7,8' perhaps simultaneously putting up fingers.
Min	Say '5,6,7,8' or '6,7,8' perhaps simultaneously putting up fingers on each count.
Count from first	Count from first addend, saying '3,4,5,6,7,8' or '4,5,6,7,8'.
Retrieval	Say an answer and explain by saying, e.g. 'I just knew it'.

the larger addend, for example, solving 3+8 by counting '8, 9, 10, 11'). The children did not yet know the min strategy but did know how to add by counting from one. They were presented simple addition problems for 11 weeks (three sessions per week, seven problems per session). Strategy use on each trial was assessed by videotaping children's audible and visible behavior during the trial and asking immediately after each trial, 'How did you solve that problem?' There were eight main findings:

(1) Most of the preschoolers used at least six strategies for adding numbers (the most common of which are described in Table 1).

(2) Children chose adaptively among the strategies; they used each approach most often on problems on which it produced fast and accurate performance relative to alternative strategies.

(3) Given practice, most of the four- and five-year-olds discovered the min strategy.

(4) A short-lived transition strategy, the 'shortcut sum strategy', began to be used by most children shortly before they discovered the min strategy and appeared to be transitional to it. The view that the shortcut sum strategy was transitional was based not only on the temporal proximity between its discovery and discovery of the min strategy but also on the fact that it incorporated aspects of both of the most common previous approach, counting from one, and the min strategy. As shown in Table 1, the shortcut sum strategy involves starting counting at one; in this respect it is like the counting from one strategy. Also, however, the shortcut sum involves proceeding through the numbers only once; in this way, therefore, it is like the min strategy (and unlike counting from one).

(5) In contrast to the usual view that 'necessity is the mother of invention', discoveries of the min strategy often occurred without prior failure. They were made following correct answers, as well as following errors, and occurred on easy as well as hard problems.

(6) Generalization proceeded slowly. After discovering the min strategy, children continued to count from one more often than they used the min approach.

(7) To promote greater generalization, challenge problems were presented in the eighth week of the study. These were problems such as 3+22, which were easy to solve via the min strategy and almost impossible for the preschoolers to solve by counting from one or retrieval. When presented such challenge problems, the preschoolers used the min strategy much more often than previously, and they continued to use it thereafter on the types of problems that had been presented in the first seven weeks (i.e. problems with addends of 5 or less).

(8) Discovery did not require trial and error. Children generated new strategies without ever trying conceptually flawed approaches, such as counting the first addend twice.

This last point might require some explanation. Both commonsense views of discovery and formal models[19] suggest that when people make discoveries, they at first try a mix of legitimate and flawed new strategies and that they then gradually stop using the flawed approaches. This is not what happened, however. Children actually never tried strategies that violated the basic principles of addition. Such illegal strategies could easily be imagined – for example, counting the first addend twice – but they were never used. Instead, conceptual understanding of the properties that are necessary for legitimate strategies in the domain appeared to constrain the strategies that children tried. The relevant conceptual understanding seemed to take the form of a goal sketch, an outline of the goals that strategies in the domain must meet. The goal sketch for solving addition problems was hypothesized to specify two main goals: that both addends be represented and that the answer subsume the representations of both addends.

To test whether preschoolers possessed such conceptual understanding, Kevin Crowley and I performed another experiment[20]. In it, a puppet demonstrated three addition strategies: counting from one (a strategy that all of the children already used), the min strategy (which the children did not yet use), and the illegal strategy of counting the first addend twice. The preschoolers' task was to judge the 'smartness' of each strategy.

The preschoolers judged the illegal strategy, which they did not use, as 'not smart'; in contrast, they judged the min strategy, which they also did not use, as just as smart as their familiar counting from one strategy. The ability to accurately judge the value of strategies that they did not yet use was not due to anything special about arithmetic or number. Other children in the Siegler and Crowley study showed comparable understanding of the potential value of tic-tac-toe strategies that they did not yet know[20]. Thus, it seems likely that as they gain experience in a domain, children develop conceptual understanding that helps them avoid flawed strategies and helps them discover useful ones.

Modeling strategy discovery

The detailed information provided by microgenetic studies is invaluable for evaluating how well alternative computer simulations fit empirical data on people's discoveries. The information also indicates the features of discoveries that simulations must generate. Both of these advantages can be illustrated in the context of discovery of the min strategy.

Most models of discovery can be classified into one of two categories: metacognitive or associative. Metacognitive models make discoveries through application of high level heuristics; associative models make discoveries by extracting correlations among tasks, actions and outcomes, and forming new procedures based on the correlations. Each of these approaches can produce discoveries of the same type that people do; the question is whether they do so in the same way as people.

The HPM (Heuristic Procedure Modification) model of Neches[21] discovers the min strategy primarily through applying 21 heuristics to data generated while solving arithmetic problems. Among these heuristics are ones for identifying redundant parts of strategies, for identifying parts of strategies that are irrelevant to the solution, and for comparing the cognitive effort needed to execute alternative strategies. HPM starts knowing only the sum strategy (i.e. counting from one). Then it recognizes the redundancy of this counting, and begins to count from the first addend. Then it applies an efficiency heuristic and recognizes that counting from the larger of the two addends (the min strategy) is faster, regardless of whether the larger addend is first or second. Following this, the min strategy is used on all problems.

HPM is an unusually well-specified metacognitive model, and it fits some of Siegler and Jenkins' microgenetic data. In particular, at the beginning of its run, it knows the sum strategy but not the min approach; it goes on to discover the min strategy; and it makes the discovery without trying any illegal approaches. However, HPM's performance also deviates from that of children in some important ways. It generates counting from the first addend as a transitional approach to the min strategy, which children do not do. It fails to generate the shortcut sum strategy, which is transitional for most children. It also consistently uses the min strategy once it is discovered, whereas children's generalization of it is slow and halting. Finally, it always makes discoveries in the same order, whereas children vary considerably in their order.

Jones and Van Lehn's GIPS (General Inductive Problem Solving) model[22] relies on associative mechanisms rather than metacognitive ones. GIPS uses rules to solve problems. Successful uses of a rule lead to increases in the rule's strength; unsuccessful uses have the opposite effect. Like HPM, GIPS at first always uses the sum strategy. Also like HPM, it soon notes the redundant counting within the sum strategy; unlike HPM, however, it generates the correct transition strategy, the shortcut sum approach, rather than counting from the first addend. GIPS goes on to learn that counting is more accurate when the larger addend is counted first, and the model thereafter consistently proceeds in that order until it detects that simply saying the first number always yields a value equal to counting from one up to it. At this point, GIPS generates the min strategy, and it consistently uses it thereafter.

The discoveries of GIPS fit several aspects of the children's data. The model initially relies on the sum strategy, goes on to discover the min strategy, and makes the transition through use of the shortcut sum strategy. However, it fails to fit other aspects of the data. Its initial use of the sum strategy is too consistent, its generalization of the min strategy is too rapid, and unless the human operator intervenes, it generates illegal strategies before discovering the min approach. Jones and Van Lehn indicate that GIPS could be modified to address the first two problems, but it is unclear how it could ever overcome the third.

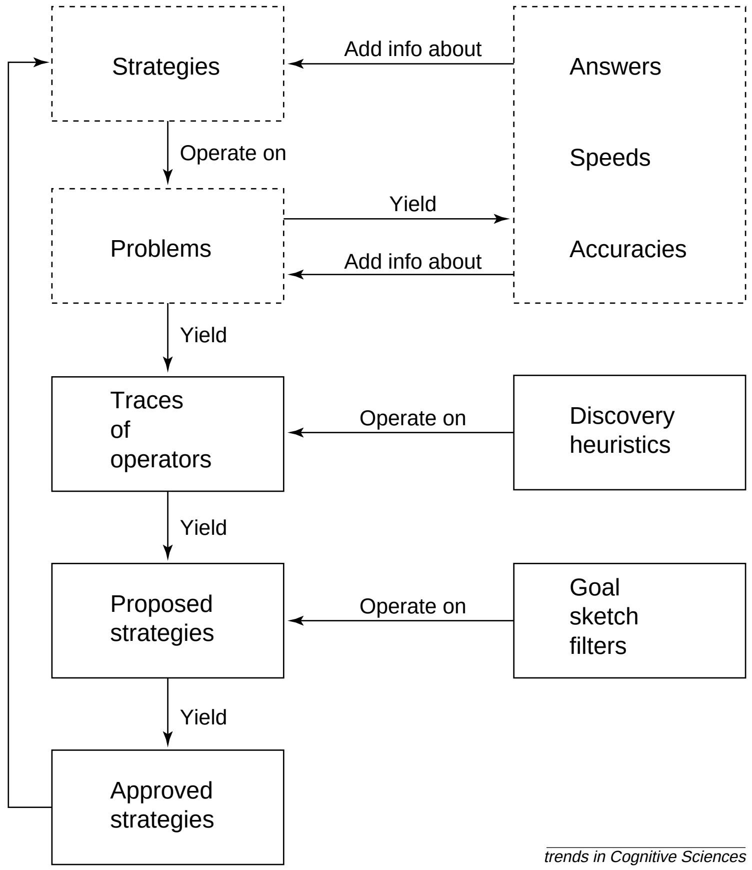

Fig. 2. Flow of control within the SCADS model of strategy choice and strategy discovery. The three boxes with dotted outlines at the top of the figure illustrate the associative processes that lead to adaptive strategy choices. The five boxes with solid outlines at the bottom correspond to the metacognitive processes that, together with the data from the associative part of the model, lead to discovery of new strategies.

The strengths and weaknesses of the two models suggest that a model combining metacognitive and associative mechanisms could be more useful than a model including only one or the other. Metacognitive mechanisms seem crucial for avoiding generation of strategies that violate principles of the domain. Associative mechanisms seem crucial for providing the varied strategy use and slow generalization that children exhibit.

To test this view, Shrager and Siegler generated a model that included both associative and metacognitive mechanisms. They labeled it SCADS (Strategy Choice And Discovery Simulation)[23]. The flow of control within SCADS is illustrated in Fig. 2. The metacognitive processes (shown by boxes with solid outlines) operate on the data generated by the associative parts of the model (boxes with dotted outlines) to discover new strategies.

As shown at the top of the Fig. 2, SCADS uses strategies to solve problems, the strategies yield data about the speed and accuracy with which the strategy solves the problem and about answers that are generated, and this information is fed back to the databases of the relevant strategy and problem. Choices among alternative strategies reflect the relative strengths of the strategies. These strengths reflect the past success of each strategy in solving problems in general, problems that share features with that problem and the particular problem (if it has been presented before). Thus, the faster and more accurate the strategy has been in the past, relative to alternative strategies, the more often it is chosen.

In addition to this primarily associative part of the model, SCADS includes a metacognitive system that analyses the sequence of operators in existing strategies, identifies potential improvements, and recombines operators from existing approaches to form new strategies. The metacognitive system within SCADS includes three components: the attentional spotlight, the strategy-discovery heuristics, and the goal sketch filters.

The 'attentional spotlight' (not shown in Fig. 2) focuses cognitive resources on the goal that is being pursued and thus increases the likelihood of successful attainment of that goal. Early in the running of SCADS, each strategy consists of a loosely associated set of operators. The attentional spotlight is used to supervise execution of these operators, thus improving the likelihood of the operators being executed in the correct order. As the links among operators become stronger, the cognitive resources of the attentional spotlight are shifted increasingly to the strategy-discovery heuristics.

Execution of a strategy creates a working memory trace. When sufficient attentional resources are available, the 'strategy-discovery heuristics' are applied to the working memory trace of the just-executed strategy. SCADS includes two strategy-discovery heuristics: (1) if a redundant set of operators is detected, delete one of the sets of operators that produced the redundancy; and (2) if statistics on a strategy indicate that the strategy is more effective when executed in a particular order, then create a specialized version of the strategy that will always be executed in that order.

Outstanding questions

- Does strategic variability change systematically with age and experience? Does it steadily increase as children learn additional strategies; does it steadily decrease as they use the optimal approach more often; or does it initially increase as they learn new approaches and then decrease as they focus on the optimal one?
- Are there systematic individual differences in the amount of variability? Do some children consistently prefer to use varied approaches, and others prefer to stick with one or a few approaches?
- What processes lead children to discover new strategies when their existing approaches are generating successful performance?
- When are new strategies generalized relatively quickly, and when is generalization slower? The difference in effectiveness between existing and new approaches is one important influence on the rate of generalization, but are there others?
- Conceptual constraints guide strategy discovery in domains in which children know strategies that are useful, even if not optimal. Are discoveries also constrained in domains that are unfamiliar, and if so, how do children identify relevant domains from which to draw analogies?
- What are the educational implications of research in strategy choice and strategy discovery? For example, when is presentation of challenging problems most advantageous: at the outset of learning in the domain or when learning is already fairly far along?
- What neural mechanisms underlie strategic variability? Do different neural systems underlie different strategies, and if so, how does competition among them generate strategy choices?

These two heuristics allow the system to propose a variety of strategies: some useful, some inefficient but legal, some illegal. The third metacognitive mechanism, the 'goal sketch', helps the system to avoid trying the illegal approaches. The goal sketch consists of two filters, both of which must be passed before a strategy is tried. One filter requires that both addends be represented; the other filter requires that the answer include representations corresponding to both addends.

All of these features of SCADS are consistent with current understanding of children's thinking. Children at first represent strategies as loosely linked sequences of operations, and only gradually bind them together into a single unit[24]. By the age of four years, perhaps sooner, children form working memory representations of strategies that they just executed[25]. Focused attention enhances performance from infancy onward[26]. With experience in executing strategies, cognitive resources become available for a variety of metacognitive purposes[27]. Thus, SCADS is a developmentally plausible model.

SCADS was run 30 times, with each run including 500 trials. (The repeated runs were necessary because the system's behavior varied somewhat from run to run, just as different children vary.) On each run, SCADS began with two strategies: retrieval and counting from one (the first strategy that parents usually teach)[28]. It applied these two strategies, and eventually others that it discovered, to a series of 500 items (20 times through the 25 problems with addends of 1–5). Counting from one dominated early choices, within each run, because problem-answer associations were too weak for retrieval to be chosen often. These early executions of counting from one also required substantial attentional resources, because the operators within the strategy were not tightly linked.

After 70–80 trials, SCADS typically had freed enough attentional resources to begin proposing new strategies. Some of the proposed strategies violated one or both parts of the goal sketch and therefore were never tried. Other proposed strategies are legal but inefficient; they are tried, but their inefficiency led to their not being used much. On all 30 runs, SCADS also generated the shortcut sum strategy, the approach that was transitional to the min strategy among the children in Siegler and Jenkins. Because the shortcut sum strategy is more efficient than the sum strategy, the model used it reasonably often. Also as with children, discovery of the shortcut sum strategy is followed relatively soon by discovery of the min strategy.

SCADS generated seven of the eight major findings of children's strategy choices and strategy discoveries listed above: it used multiple strategies in all phases of learning; it chose adaptively among the strategies; it discovered the min strategy; it usually discovered the shortcut sum strategy shortly before the min strategy; it discovered new strategies on trials that followed correct answers as well as on trials following errors; it generalized its use of the min strategy quite slowly; it never attempted to execute illegal strategies.

Having shown that SCADS could generate seven of the eight basic phenomena, Shrager and Siegler tested whether it could generate the eighth: the positive effects of challenge problems on generalization of the min strategy. To test whether the model would show a similar effect, we replaced some trials with items similar or identical to the challenge problems presented by Siegler and Jenkins: items with one addend of 5 or less and the other addend of 20 or more (e.g. 3121).

Presentation of the challenge problems increased generalization in SCADS, as it had with children. SCADS chose the min strategy more often on the challenge problems than on standard problems that had been presented on corresponding trials in the previous runs. It also continued to choose the min strategy more often on subsequent standard problems with addends of 5 or less.

Thus, SCADS generated all eight of the key characteristics of strategy development in this domain. More generally, it demonstrated that combining associative and metacognitive mechanisms could account for a complex set of phenomena that probably could not be accounted for by either type of mechanism alone.

Conclusions

Strategy use is highly variable throughout the lifespan. This variability allows children as well as adults to choose strategies that fit the demands of particular problems and surrounding circumstances. It also facilitates discovery of new strategies, because existing strategies provide pieces from which new approaches can be assembled. From early childhood onward, discovery of new strategies is a frequent event. The new strategies that are tried are far from random; instead, they generally reflect conceptual understanding of the requisites of appropriate strategies in the particular domain. Discovery of new strategies does not depend on the failure of existing ones; instead, discoveries come in the context of success as well as following failures. Thus, strategic variability, adaptive choices among the varying approaches, and discovery of new approaches seem to be basic characteristics of human cognition.

Acknowledgements

Preparation of this paper and much of the research described were supported by grants from the National Institutes of Health (HD-19011) and from the Spencer Foundation.

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