1 CFG recap ($\star$)

Let G be the following CFG.

$$\begin{aligned} R &\to XRX \mid S \\ S &\to \mathtt{a}T\mathtt{b} \mid \mathtt{b}T\mathtt{a} \\ T &\to XTX \mid X \mid \varepsilon \\ X &\to \mathtt{a} \mid \mathtt{b} \end{aligned}$$

1. What are the variables, terminals and the start variable of G?
2. Give three strings in $L(G)$ and three strings not in $L(G)$.
3. Which of the following nine statements are true?

(1.) $T \Rightarrow \mathtt{aba}$	(4.) $T \overset{*}{\Rightarrow} T$	(7.) $T \overset{*}{\Rightarrow} XX$
(2.) $T \overset{*}{\Rightarrow} \mathtt{aba}$	(5.) $XXX \overset{*}{\Rightarrow} \mathtt{aba}$	(8.) $T \overset{*}{\Rightarrow} XXX$
(3.) $T \Rightarrow T$	(6.) $X \overset{*}{\Rightarrow} \mathtt{aba}$	(9.) $S \overset{*}{\Rightarrow} \varepsilon$

4. ($\star\star$) Describe $L(G)$ in English, and give a direct set-theoretic characterisation of $L(G)$.

2 Arithmetic ($\star$)

Consider the following CFG.

$$\begin{aligned} E &\to E + T \mid T \\ T &\to T \times F \mid F \\ F &\to (E) \mid \mathtt{a} \end{aligned}$$

Give parse trees and derivations for

(1.) `a` (2.) `a+a` (3.) `a+a+a` (4.) `((a))`

3 Constructing CFGs ($\star$)

Give CFGs for the following languages over the alphabet $\Sigma = \{0, 1\}$.

1. $\{w \mid w$ contains at least three '`1`'s $\}$
2. $\{w \mid w$ starts and ends with the same symbol $\}$
3. $\{w \mid$ the length of w is odd $\}$
4. $\{w \mid$ the length of w is odd and the middle symbol is a `0` $\}$
5. $\{w \mid w$ is a palindrome $\}$
6. The empty set.

4 Another Context-Free Language ($\star\star$)

Give a CFG for the following language.

$$L = \{x\texttt{\#}y \mid x, y \in \{\texttt{0}, \texttt{1}\}^* \text{ and } |x| \neq |y|\}$$

5 Chomsky Normal Form

1. ($\star$) Convert the following CFG into Chomsky normal form using the standard procedure.

$$\begin{aligned} A &\to BAB \mid B \mid \varepsilon \\ B &\to \texttt{00} \mid \varepsilon \end{aligned}$$

2. ($\star\star$) Show that for a grammar G in Chomsky normal form, for any string $w \in L(G)$ of length $n \geq 1$ any derivation of w requires exactly $2n - 1$ steps.

6 Seeing Stars ($\star\star$)

The following construction does *not* prove that the class of context-free languages is closed under the star operation. Give a counterexample to show why.
"Let A be a context-free language. Let $G = (V, \Sigma, R, S)$ be a CFG for A. Add the new rule $S \to SS$. This is a CFG for A^*."

7 The C Programmer's Nightmare

Consider the following CFG for a fragment of a programming language.

$$\begin{aligned} \langle \textsc{Stmt} \rangle &\to \langle \textsc{Assign} \rangle \mid \langle \textsc{IfThen} \rangle \mid \langle \textsc{IfThenElse} \rangle \\ \langle \textsc{IfThen} \rangle &\to \texttt{if condition then } \langle \textsc{Stmt} \rangle \\ \langle \textsc{IfThenElse} \rangle &\to \texttt{if condition then } \langle \textsc{Stmt} \rangle \texttt{ else } \langle \textsc{Stmt} \rangle \\ \langle \textsc{Assign} \rangle &\to \texttt{a:=1} \end{aligned}$$

$$\begin{aligned} \Sigma &= \{\texttt{if}, \texttt{condition}, \texttt{then}, \texttt{else}, \texttt{a:=1}\} \\ V &= \{\langle \textsc{Stmt} \rangle, \langle \textsc{IfThen} \rangle, \langle \textsc{IfThenElse} \rangle, \langle \textsc{Assign} \rangle\} \end{aligned}$$

1. ($\star$) Show that this grammar is ambiguous.

2. ($\star\star$) Give an unambiguous grammar for the same language.