

COMS10003 Work Sheet 22

Random Variables I

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1. Continuous random variable X and discrete random variable Y have probability density function (pdf) and probability mass function (pmf) given below. In each case determine and sketch the corresponding cumulative distribution function (cdf).

$$p(x) = \begin{cases} 1/a & 0 \leq x \leq a \\ 0 & \text{else} \end{cases} \quad P(y) = \frac{1}{4} \sum_{i=1}^4 \delta(y - i)$$

where $\delta(y) = 1$ if $y = 0$ and zero otherwise.

2. A discrete random variable X can be one of three values with probabilities given by its pmf $P(-1) = 0.2$, $P(2) = 0.5$, $P(6) = 0.3$. Compute the expected value, the variance, $Var(X)$ and the standard deviation of X .
3. Determine the sample mean and the sample variance of the following set $\{2, 5, 3, 7, 8, 12, 3, 2\}$
4. The joint pmf for two discrete random variables X and Y taking values 0, 1, 3 and $-1, 1, 2$, respectively, is shown below.

$X : Y$	-1	1	2
0	0.12	0.08	0.1
1	0.2	0.04	0.25
3	0.08	0.1	0.03

- (a) Compute the marginal mass functions
 - (b) Are X and Y independent? Justify your answer.
 - (c) Compute $E(X)$ and $E(Y)$.
 - (d) Compute the pmf for the RV $Z = X + Y$
 - (e) Compute the conditional pmf $P(X|Y = 2)$.
5. Prove the following for scalar a and random variables X and Y :

$$E(aX) = aE(X) \quad E(X + Y) = E(X) + E(Y) \quad Var(aX) = a^2 Var(X)$$

6. If X and Y are independent random variables, prove the following:

$$E(XY) = E(X)E(Y) \quad Var(X + Y) = Var(X) + Var(Y)$$

$$Var(XY) = E(X^2)E(Y^2) - E(X)^2E(Y)^2$$

7. A fair die is thrown 10 times. Compute the probability that the die will show a number less than 5 on exactly 3 of the 10 throws.
8. Over one week, Alice received 50 spam emails and 150 non-spam emails. She then receives 10 emails during one day in the following week. Estimate the probability that exactly 6 of those will be spam and the probability that she will receive less than 3 spam emails.
9. Prove that the expected value and variance of a random variable which follows the binomial distribution is $E(X) = np$ and $Var(X) = np(1 - p)$.
10. (Buffon's Needle - version taken from the Wikipedia entry - try not to peep!) A needle of length L is dropped onto a plane which is ruled with parallel lines T units apart. Show that the probability that the needle will lie across a line is given by $2L/T\pi$. *Hint: consider the problem in terms of two random variables - the distance of the needle centre from one of the parallel lines and the angle the needle makes with the parallel lines. Assume that they are independent (are they really?) and then determine the joint pdf. That should get you started.*