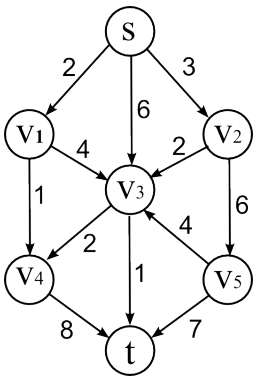
COMS21103: Data Structures and Algorithms

Problem Sheet - Week 10

1. **Maximum Flow**

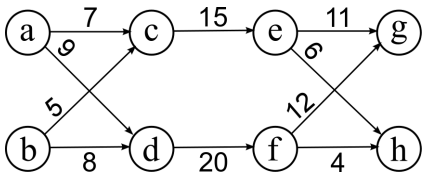
   (a) Calculate the *maximum flow* for the flow network below:



   Hint: The maximum flow is 7. Remember that you are searching for augmentation paths in the residual network G_f.

   (b) *optional:* Use the code stubs platform to test your solution for the maximum flow problem in (a).

   (c) Calculate the *maximum flow* for the flow network below:



   Hint: First you need to introduce a source and a sink. The maximum flow is 28.

   (d) For the flow network in (c), enumerate all possible cuts in the network, and calculate the capacity of each cut.

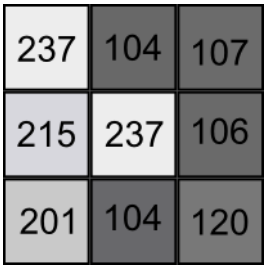
   (e) Suppose that, in addition to edge capacities, a flow network has vertex capacities. That is each vertex has a limit $\iota(v)$ on how much flow can pass through v. How can you convert such a network into a flow network without vertex capacities.

   Replace each vertex by two vertices with an edge between them carrying the edge weight $\iota(v)$

   (f) After finding the maximum flow for a network, suppose that we increased the capacity of a single edge $(u, v) \in E$ by 1. Give an O(V+E) algorithm to update the maximum flow.

   In the new residual network G_f, search for a path from s to t. The path must pass through the edge (u, v) as otherwise no paths were present. If such a path exists, augment the flow by 1, otherwise, the current flow is still the maximum. The time to find a path is O(V+E) using Breadth First Search for shortest path.

   (g) For the image below (3×3 pixels) with the number in the pixel indicating the greyscale pixel value (0-255), convert the image into a flow network. Consider the foreground model to be the Euclidean distance between the pixel value and 100, and the background model to be the Euclidean distance between the pixel value and 220. Can you find the minimum cut?



for each edge between two pixels use (1/Euclidean difference) as lower difference should indicate higher edge weight. For foreground and background models use (1/Euclidean distance).