COMS10003

Mathematical Methods for Computer Scientists

Assignment - Portfolio 2

This assignment is worth 15% of the unit mark

2014-15

Instructions

1. You are required to provide solutions to the following five questions.
2. Each question is worth 10 marks.
3. You are expected to work independently.
4. Clearly state your name and your user name on your submission. The submission can be hand written or typed.
5. All answers should be clearly structured and you should fully explain and justify each and every step.
6. Marks will be deducted if justifications and explanations are not given. Answers without any justification or explanation will be given ZERO MARKS.
7. You should hand-in a hard copy of your answers to the MVSE School Office AND upload an electronic copy to SAFE (PF2). If you produce hand written answers, then these should be scanned and uploaded as a single PDF document. No other format will be accepted.
8. The DEADLINE for submission of both the hard copy and the electronic copy is

4pm on Friday 15th May 2015.

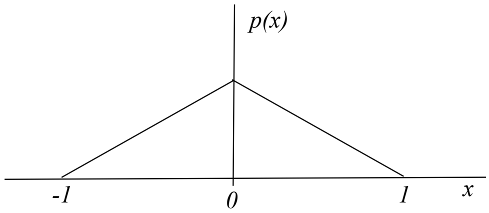
Question 1

a) A computer hosts only two web servers, A and B. Records show that the mean number of requests per hour to server A is 60 with standard deviation 10, whilst to server B it is 80 with standard deviation 5. Derive the mean total number of requests per hour to the computer and the associated standard deviation. State clearly any assumptions that you make.

[2 marks]

b) The probability density function of a continuous random variable x is shown below. Derive the following: (i) the expected value of x; (ii) the variance of x; and (iii) the probability that x has a value between 0.2 and 0.8.

[6 marks]



c) If the number of requests per hour to web server A in Question 1a is normally distributed, derive the percentage of time that you would expect there to be more than 70 requests per hour to the server.

[2 marks]

Question 2

Derive $\partial f/\partial x$, $\partial f/\partial y$ and $\partial f/\partial z$ for the following, clearly showing and explaining your working in each case (2 marks for each part):

a) $f(x, y, z) = x^3 + y^3 + z^3$

b) $f(x, y) = e^{x+y}$

c) $f(x, y) = xy \sin xy$

d) $f(x, y, z) = xy \ln z$

e) $f(x, y, z) = x \sin xyz$

[10 marks]

Question 3

Show by checking whether the following functions $f(t)$ are odd or even, explaining your reasoning clearly in each case (2 marks for each part):

a) $f(t) = \cos t$

b) $f(t) = t^3 + t$

c) $f(t) = 3t^2 \sin t$

d) $f(t) = t^3 + 2t^2 + t$

e) $f(t) = |t|$

[10 marks]

Question 4

a) The vectors $\mathbf{v}_1 = (-1, 0, 1)$ and $\mathbf{v}_2 = (1, -1, 2)$ span a subspace Ω. Determine a vector $\mathbf{v}_3$ which is orthogonal to $\mathbf{v}_1$ and which is also within the subspace Ω.

[3 marks]

b) Hence determine the projections of the vectors $\mathbf{w} = (1, -2, 5)$ and $\mathbf{z} = (-2, -2, 3)$ onto the subspace Ω.

[4 marks]

c) Which of the two vectors $\mathbf{w}$ and $\mathbf{z}$ is within the subspace Ω? Explain how you arrived at your answer. Use the other vector and its projection to determine a vector which is orthogonal to the subspace. Show that it is orthogonal to the subspace.

[3 marks]

Question 5

a) Derive the eigenvalues and eigenvectors of the following matrix

[5 marks]

$$A = \begin{bmatrix} 7 & -2 & 3 \\ 8 & -1 & 6 \\ 4 & -2 & 6 \end{bmatrix}$$

b) Hence use diagonalisation to derive the matrix $B = A^5$.

[5 marks]