Data Structures and Algorithms – COMS21103

2014/2015

Representing and Exploring Graphs

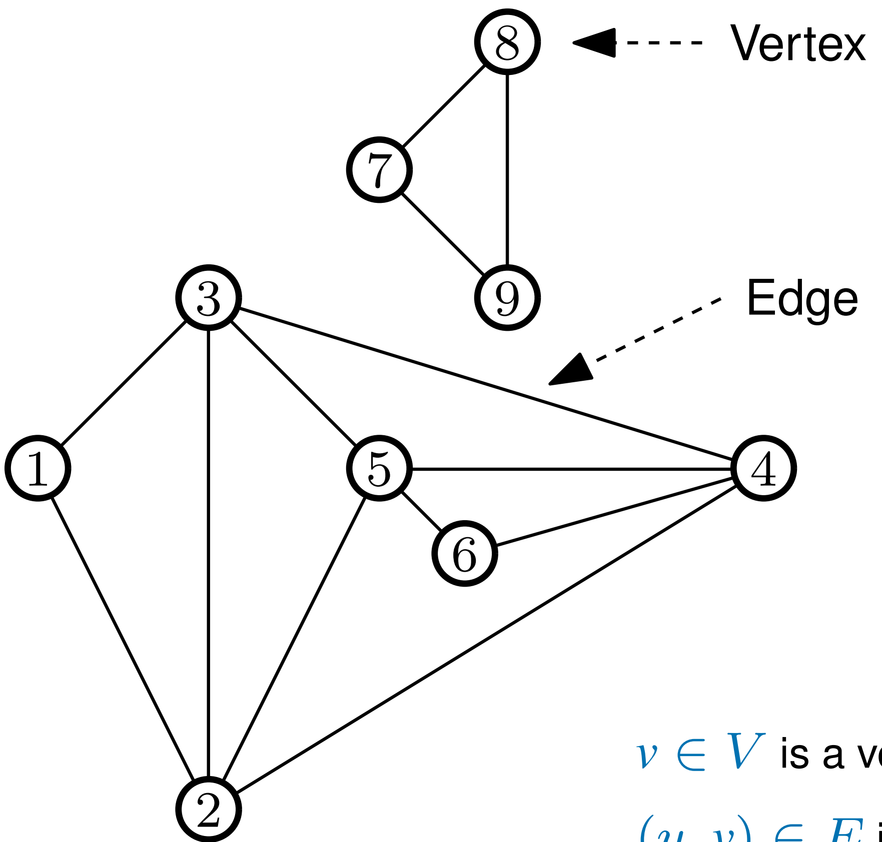
Depth First Search and Breadth First Search

Benjamin Sach

University of BRISTOL

Graph notation recap

G is a Graph,



V is the set of vertices

$|V|$ *is the number of vertices*

E is the set of edges

$|E|$ *is the number of edges*

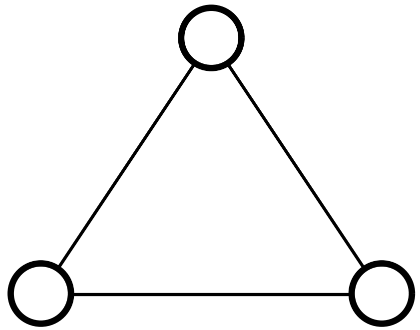
$v \in V$ is a vertex

$(u, v) \in E$ is an edge from u to v

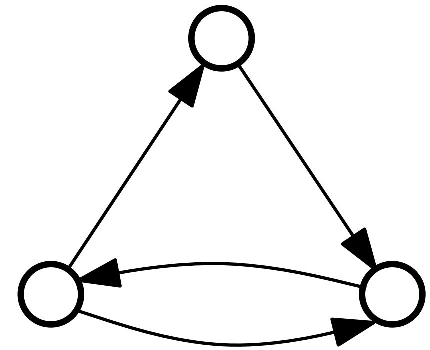
This lecture focuses on exploring **undirected** and **unweighted** graphs

though everything discussed today will work for directed, unweighted graphs too

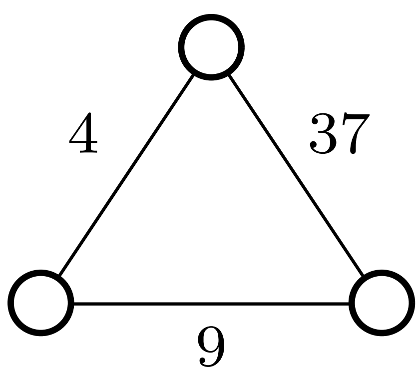
Graph notation recap



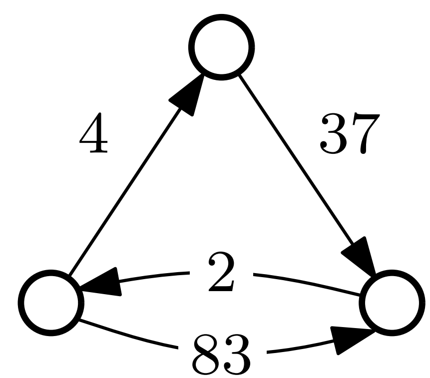
an **undirected** and **unweighted** graph



an **directed** and **unweighted** graph



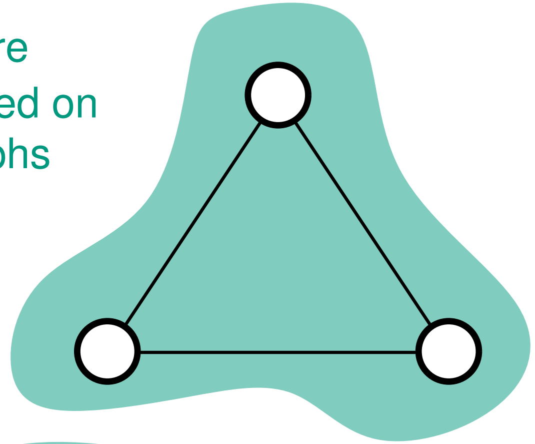
an **undirected** and **weighted** graph



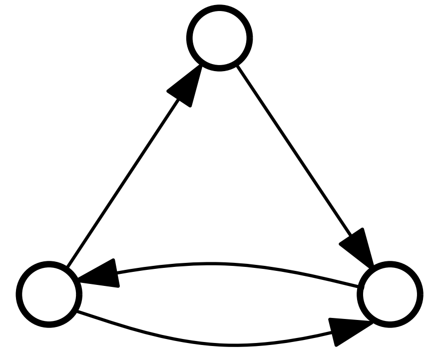
a **directed** and **weighted** graph

Graph notation recap

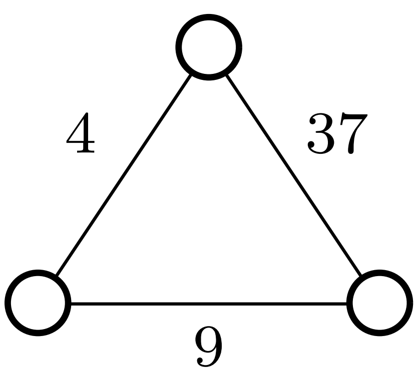
Today we're focused on these graphs



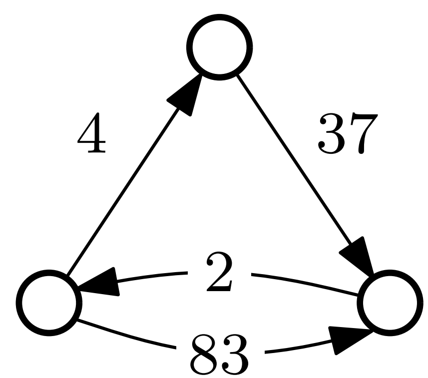
an **undirected** and **unweighted** graph



an **directed** and **unweighted** graph



an **undirected** and **weighted** graph



a **directed** and **weighted** graph

Graph notation recap

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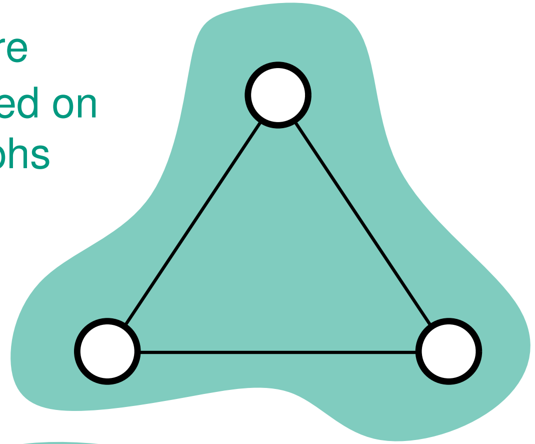
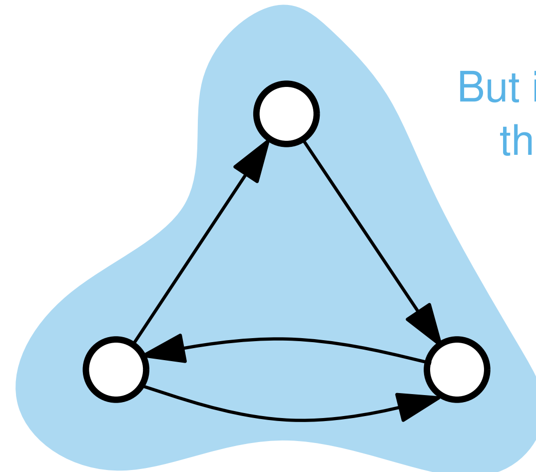
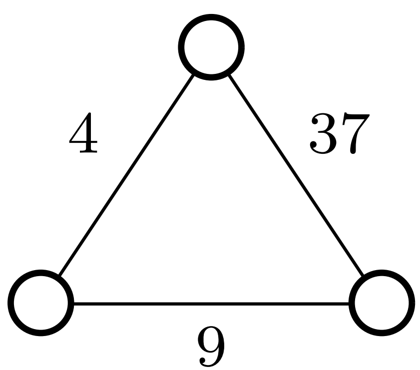


an **undirected** and **unweighted** graph

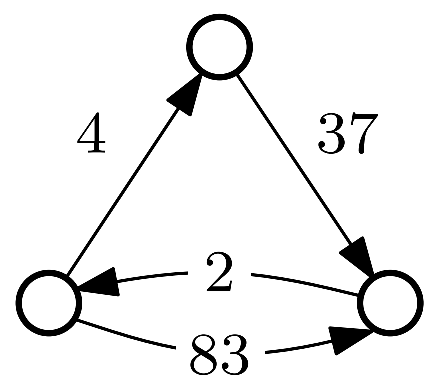
But it all works for these graphs too



an **directed** and **unweighted** graph



an **undirected** and **weighted** graph



a **directed** and **weighted** graph

V is the set of vertices
$|V|$ *is the number of vertices*

Graph representations

E is the set of edges
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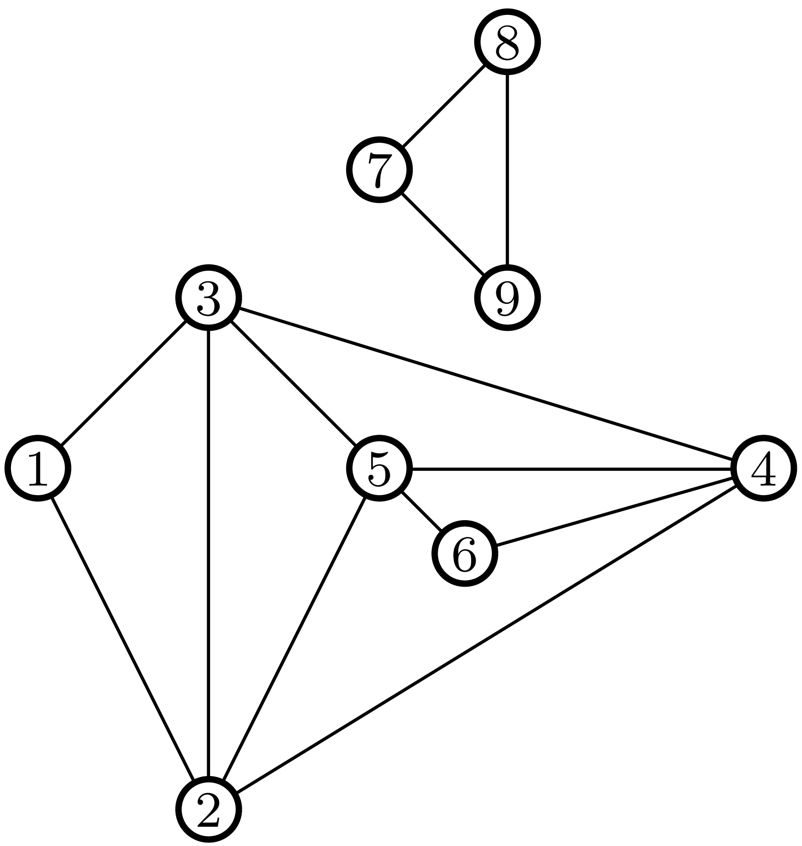

TO

FROM

$$\begin{array}{c|ccccccccc|}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\hline
1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
3 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
4 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
5 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\
6 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
8 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
9 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
\hline
\end{array}$$

$|V|$

$|V|$

Adjacency Matrix

We will in general assume that the vertices are numbered $1, 2, 3 \ldots |V|$

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Graph representations

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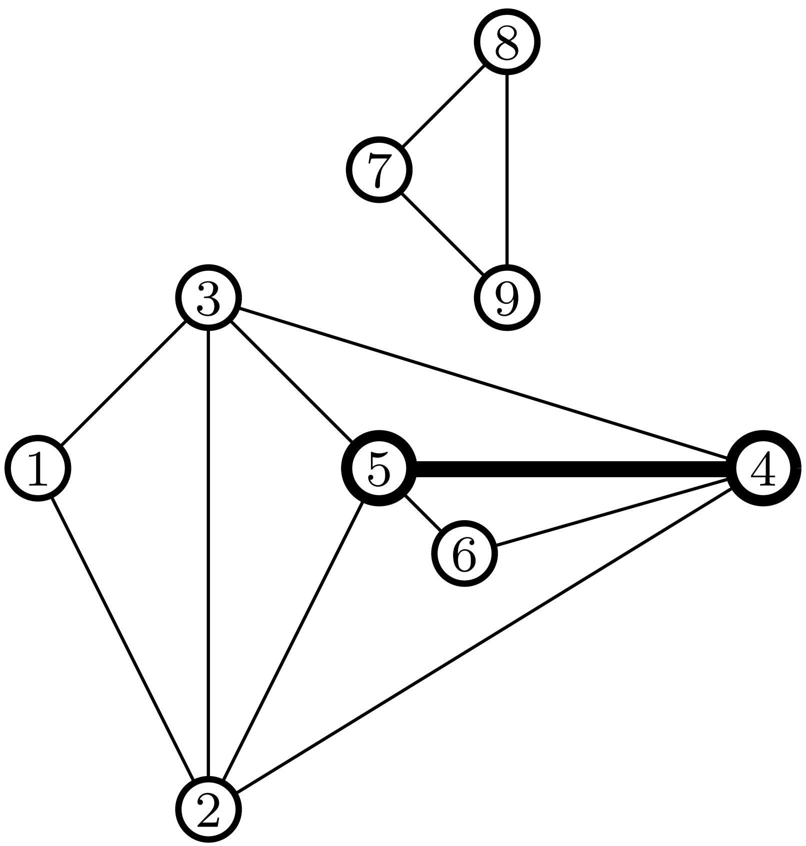

TO

FROM	1	2	3	4	5	6	7	8	9
1	0	1	1	0	0	0	0	0	0
2	1	0	1	1	1	0	0	0	0
3	1	1	0	1	1	0	0	0	0
4	0	1	1	0	1	1	0	0	0
5	0	1	1	1	0	1	0	0	0
6	0	0	0	1	1	0	0	0	0
7	0	0	0	0	0	0	0	1	1
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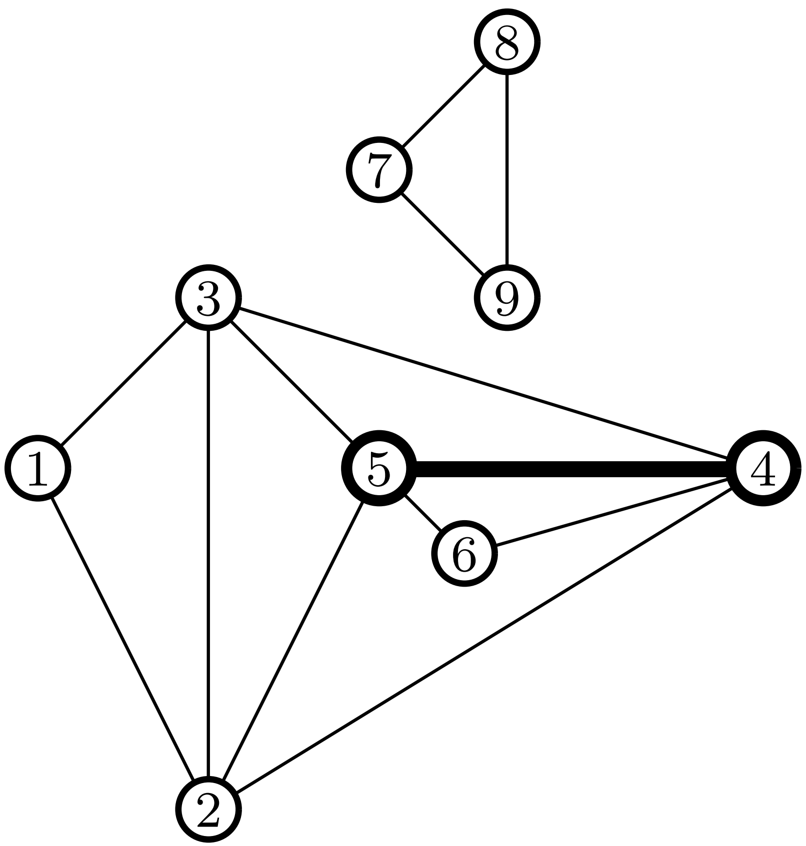

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1	0	1	1	0	0	0	0	0	0
2	1	0	1	1	1	0	0	0	0
3	1	1	0	1	1	0	0	0	0
4	0	1	1	0	1	1	0	0	0
5	0	1	1	1	0	1	0	0	0
6	0	0	0	1	1	0	0	0	0
7	0	0	0	0	0	0	0	1	1
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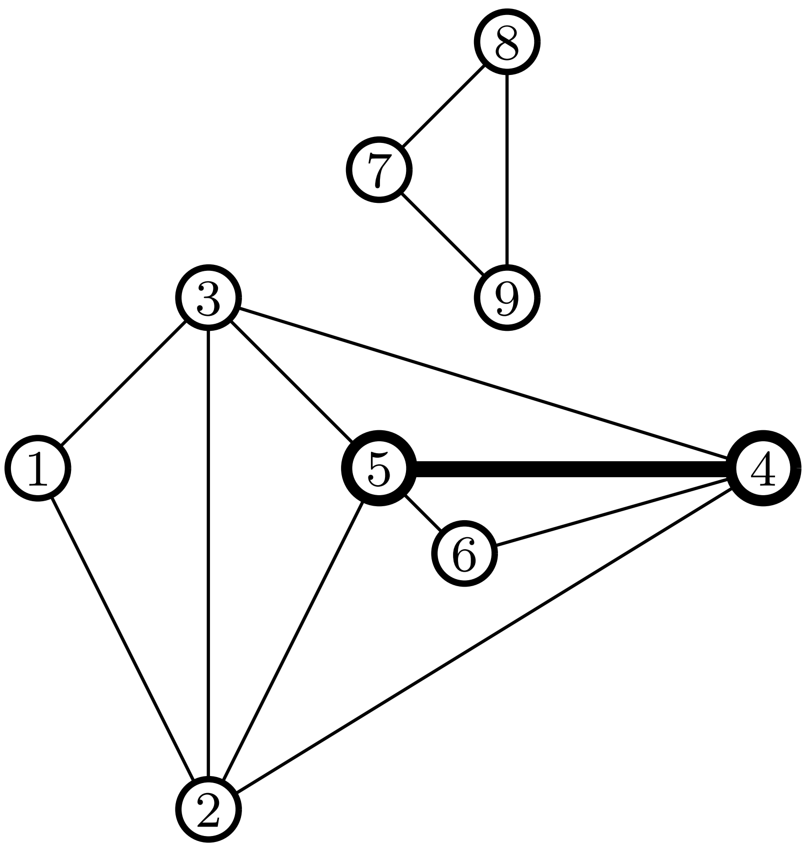

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4	0	1	1	0	1	1	0	0	0
5	0	1	1	1	0	1	0	0	0
6	0	0	0	1	1	0	0	0	0
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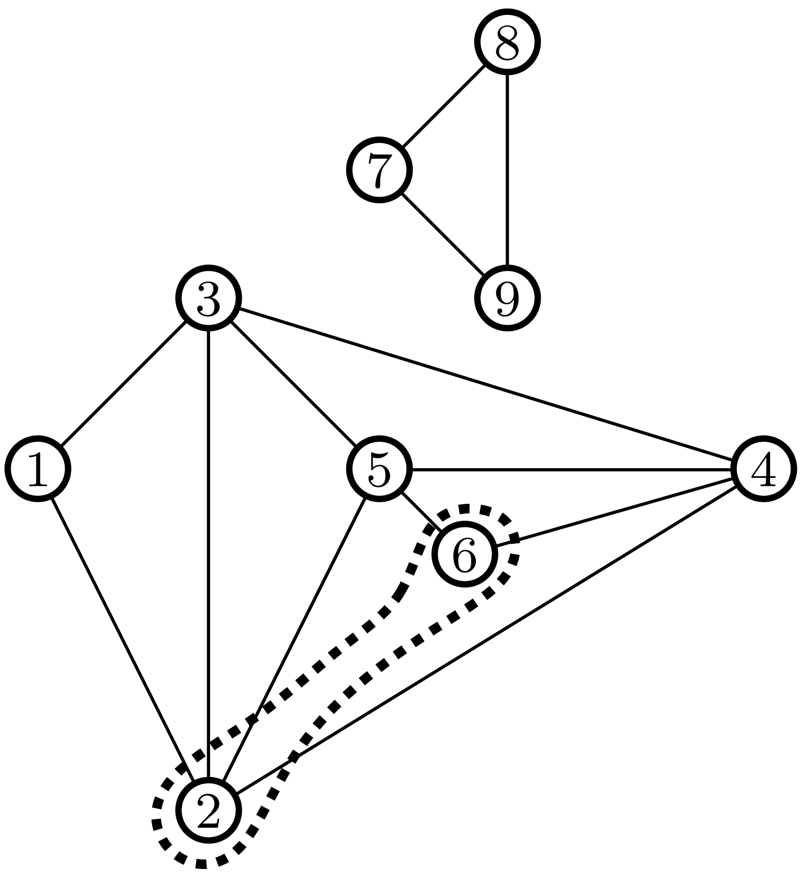

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4	0	1	1	0	1	1	0	0	0
5	0	1	1	1	0	1	0	0	0
6	0	0	0	1	1	0	0	0	0
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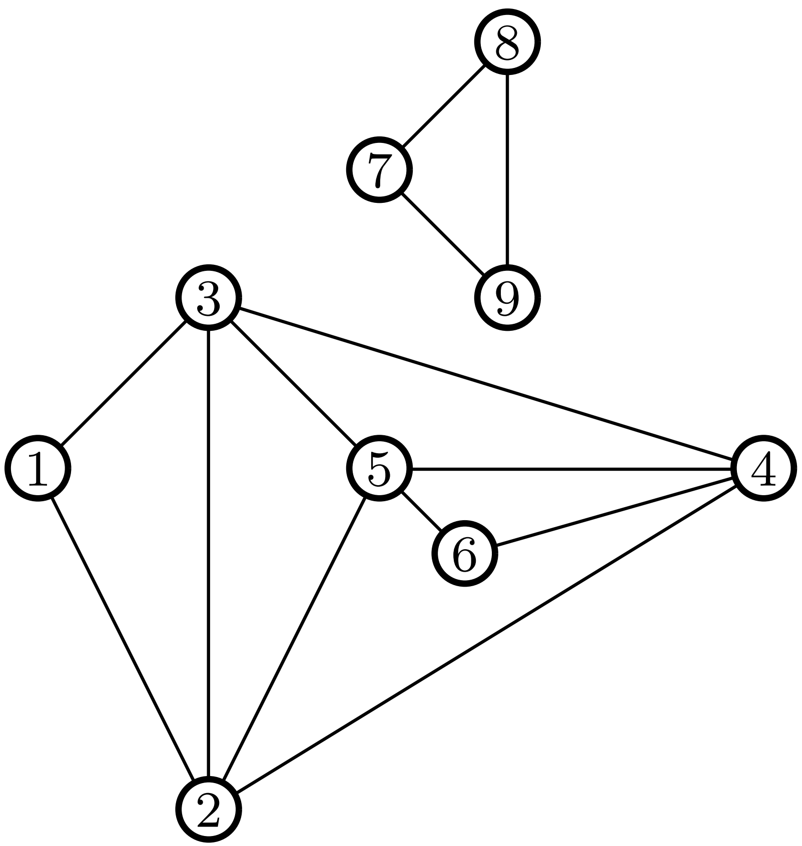

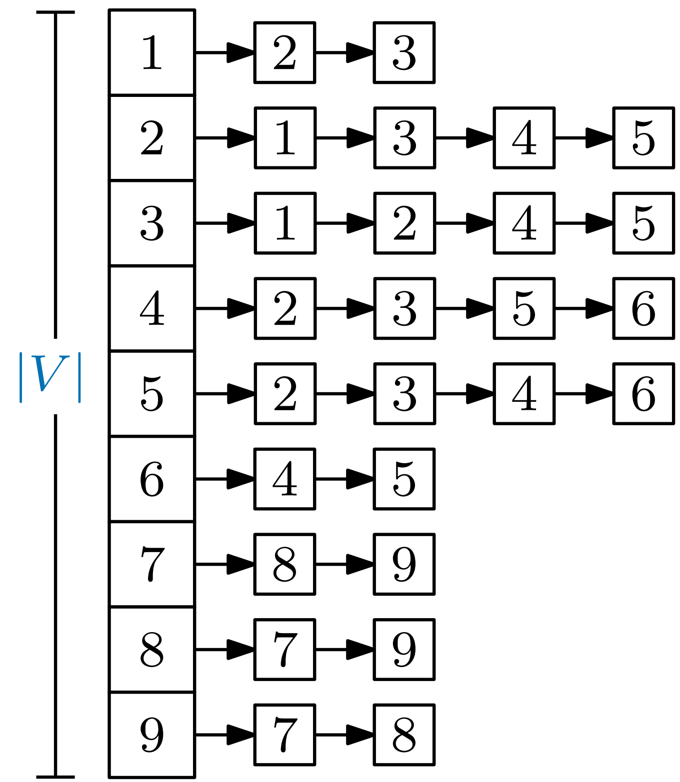

Adjacency List

(using linked lists)

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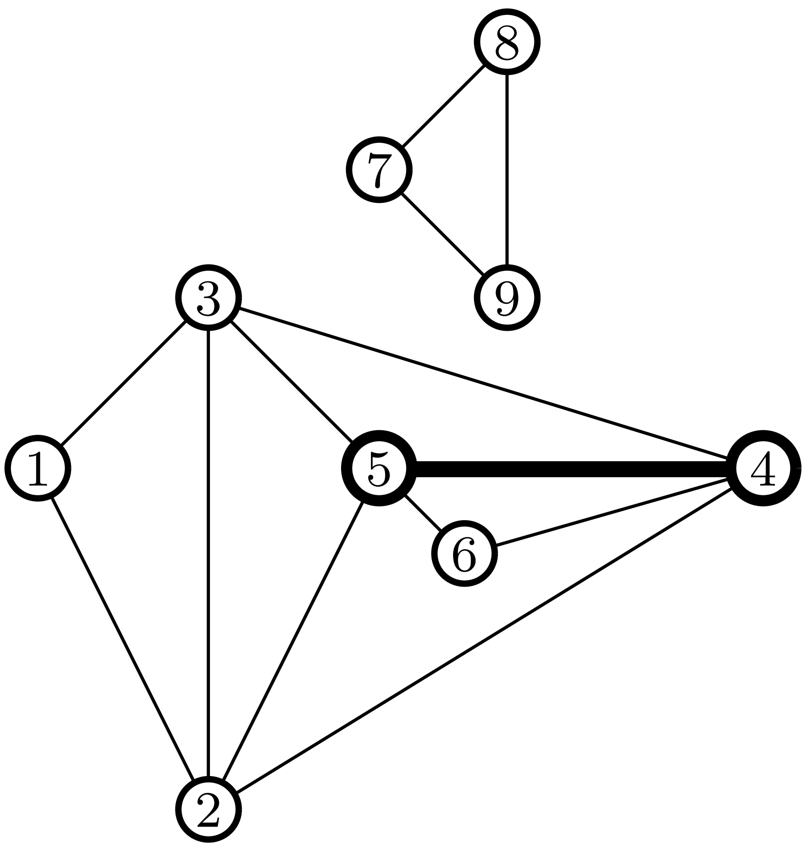

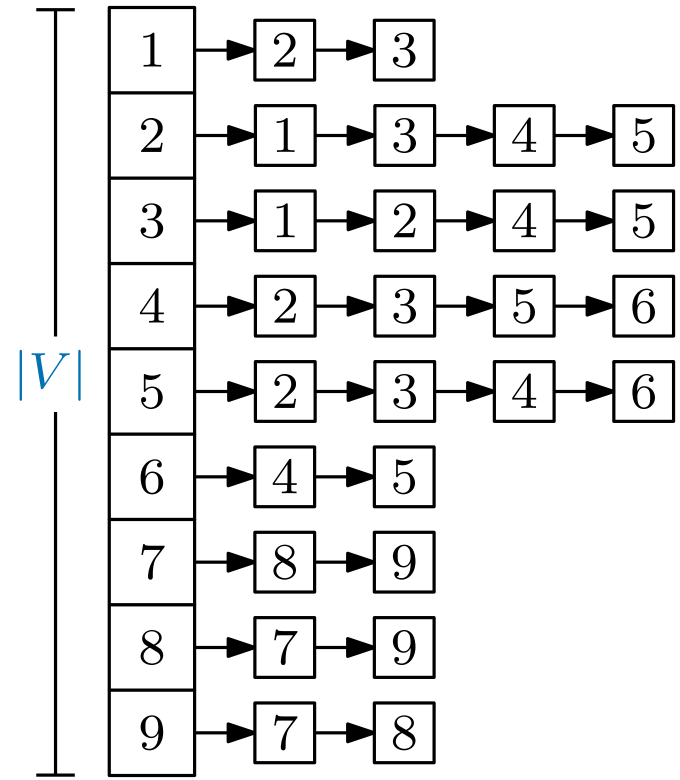

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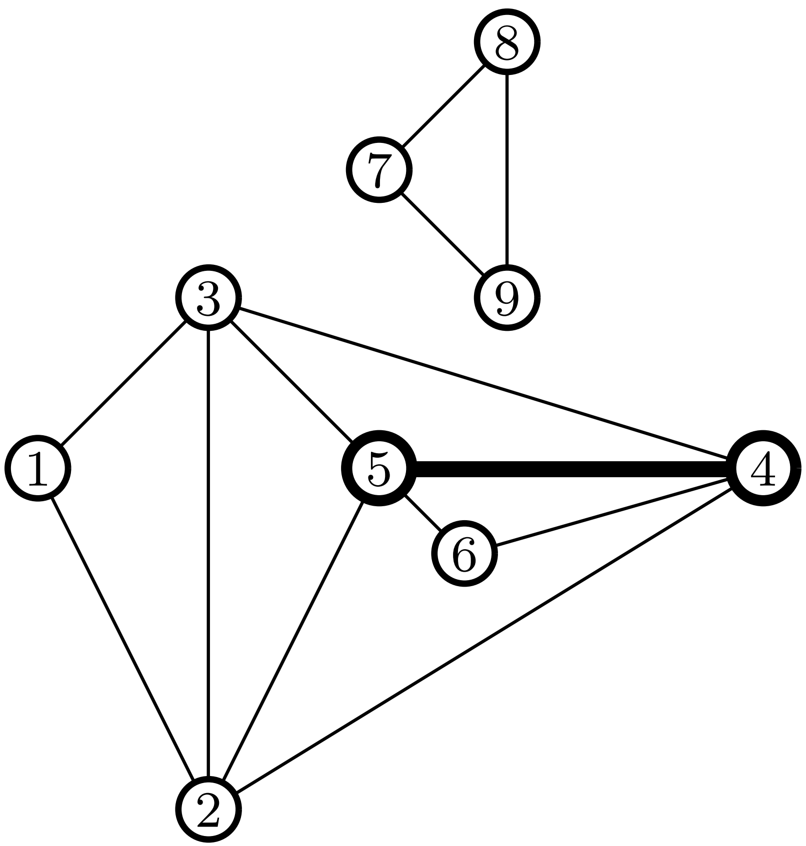

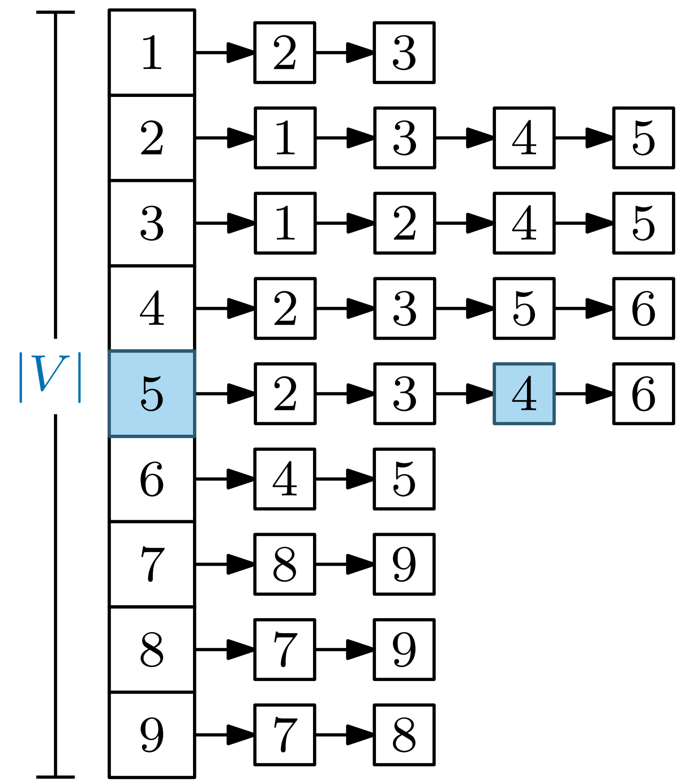

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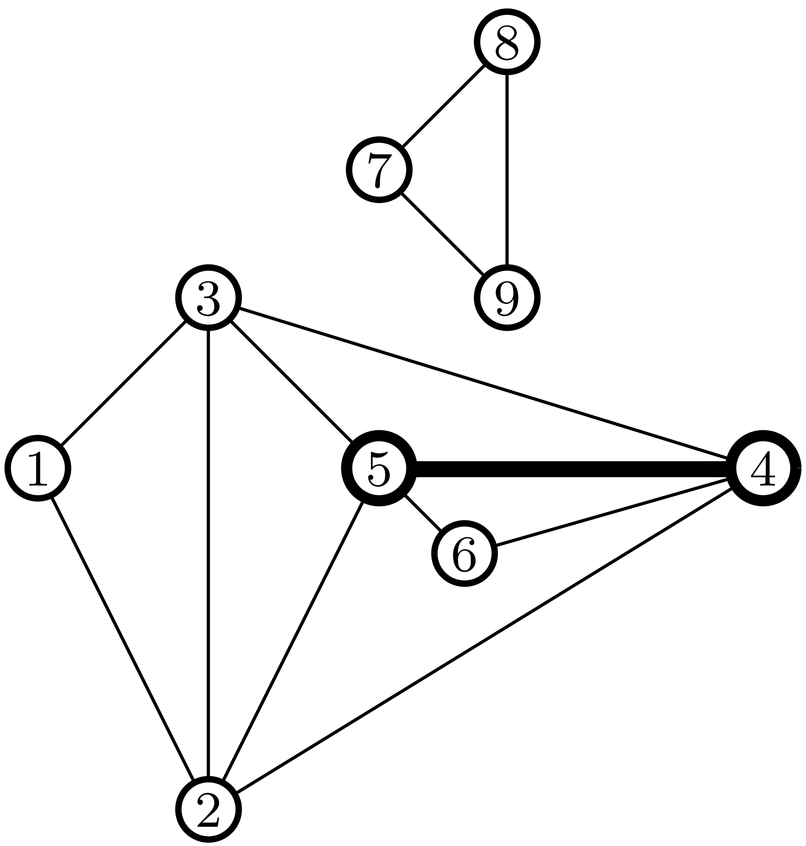

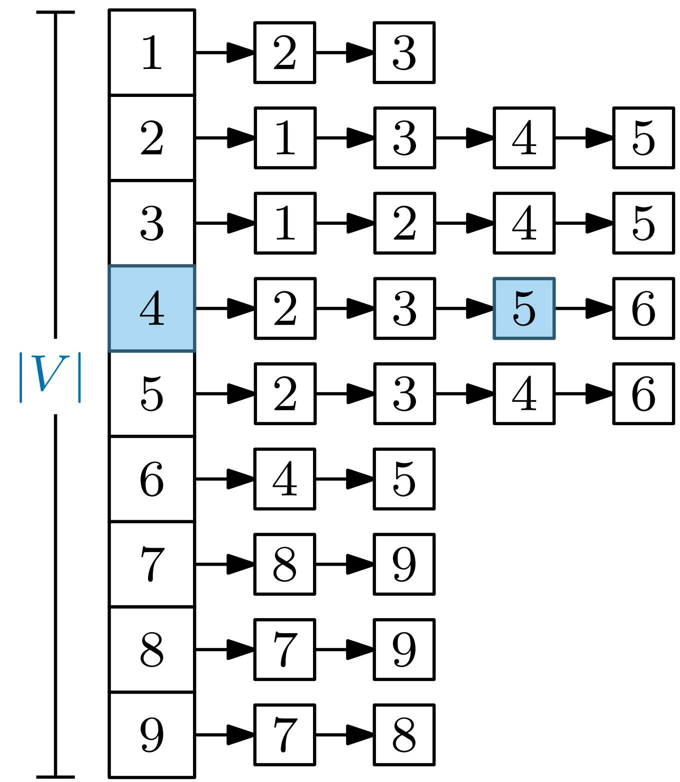

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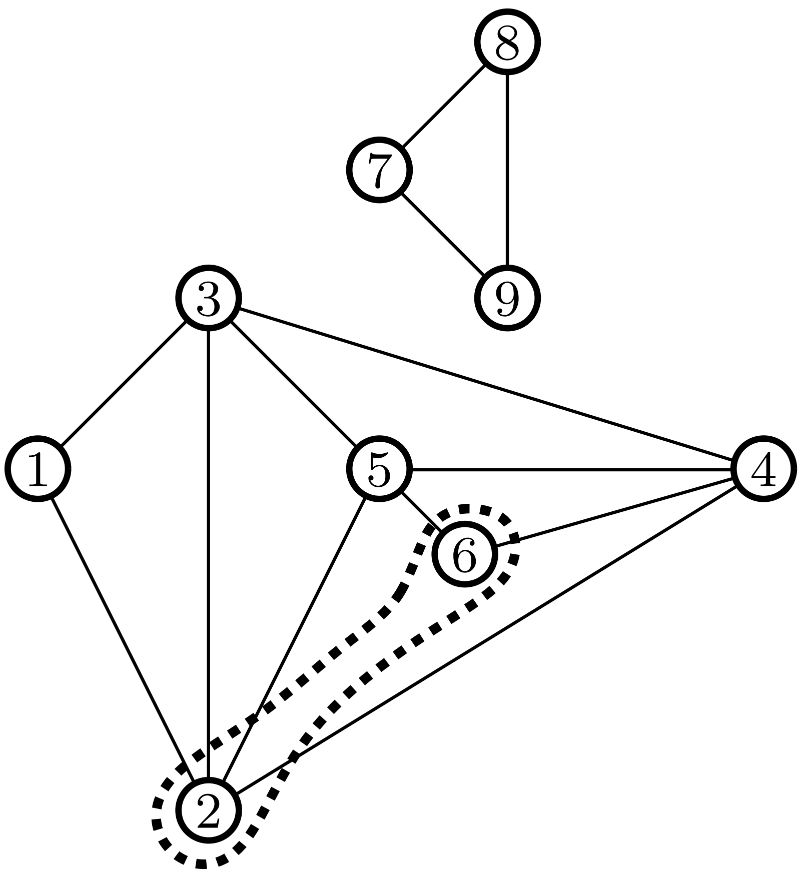

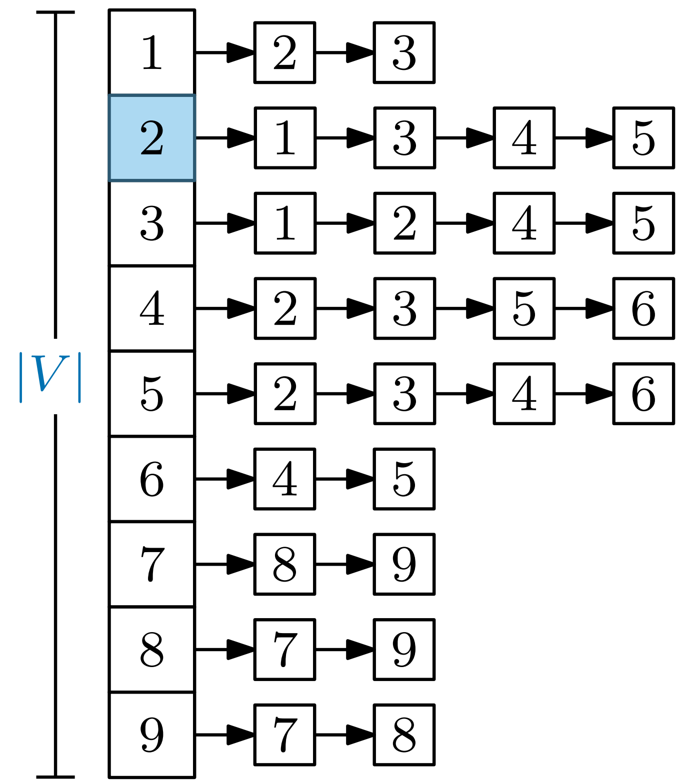

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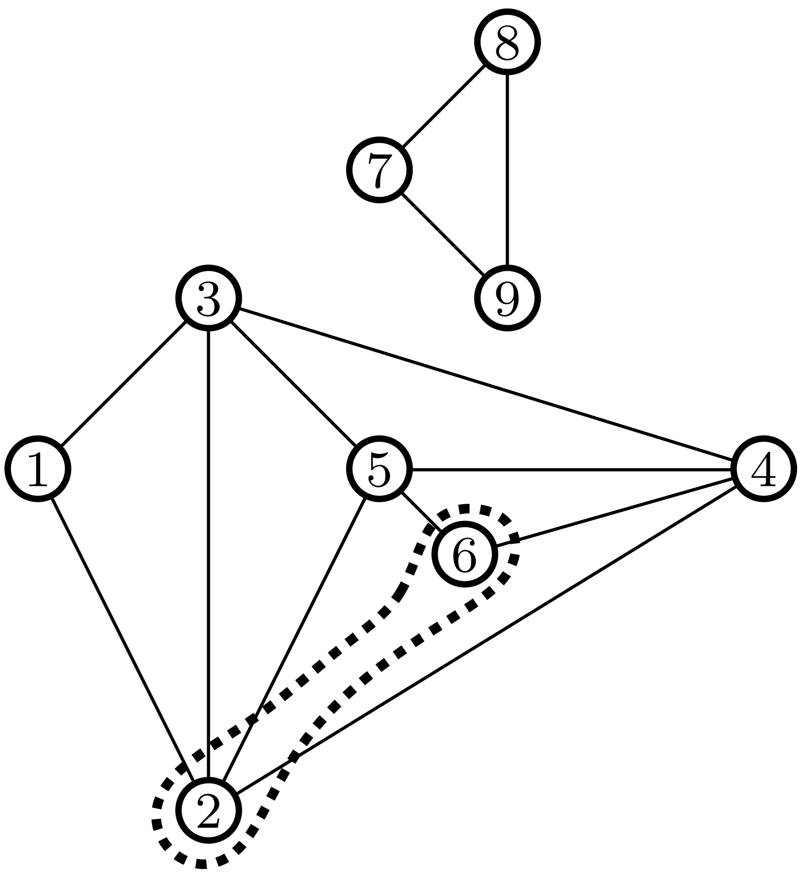

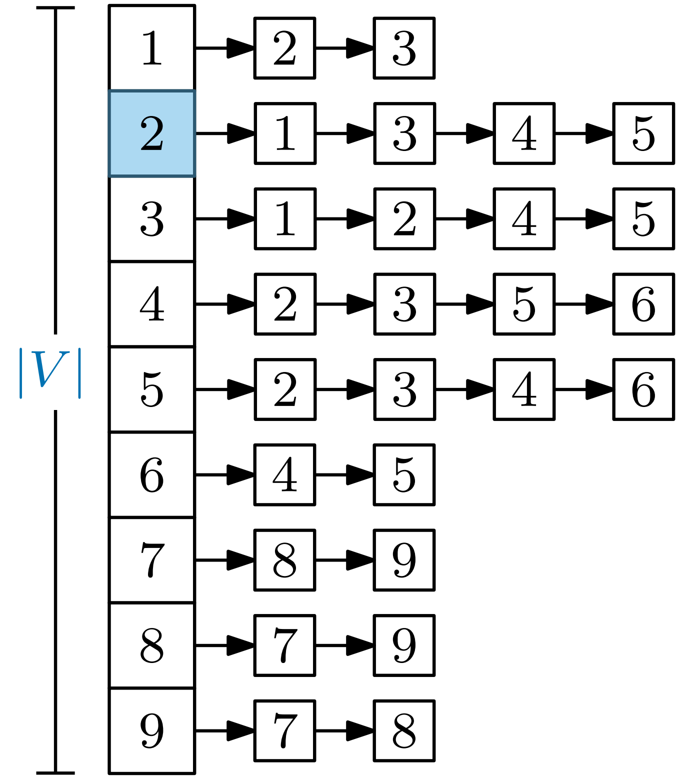

Adjacency List

(using linked lists)

We will in general assume that the vertices are numbered $1, 2, 3 \ldots |V|$

both representations are symmetric because the graphs are undirected

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$|V|$ *is the number of vertices*

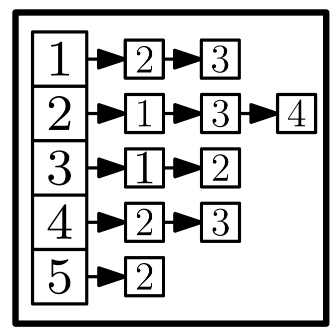
Basic operations

E is the set of edges
$|E|$ *is the number of edges*

	Adjacency Matrix	Adjacency List *(using linked lists)*
Space		
Is there an edge from vertex u to v?		
List all the edges leaving $u \in V$		

	1	2	3	4	5
1	0	1	1	0	0
2	1	0	1	1	1
3	1	1	0	1	1
4	0	1	1	0	1
5	0	1	1	1	0

Adjacency Matrix



Adjacency List
(using linked lists)

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Basic operations

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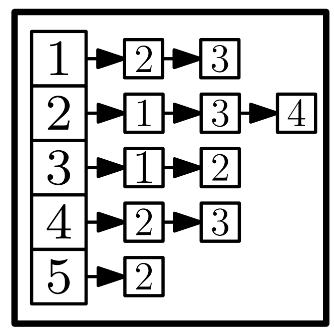
$|E|$ *is the number of edges*

	Adjacency Matrix	Adjacency List *(using linked lists)*
Space	$\Theta(\|V\|^2)$	$\Theta(\|V\| + \|E\|)$
Is there an edge from vertex u to v?		
List all the edges leaving $u \in V$		

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & 1 & 1 & 0 & 0 \\
2 & 1 & 0 & 1 & 1 & 1 \\
3 & 1 & 1 & 0 & 1 & 1 \\
4 & 0 & 1 & 1 & 0 & 1 \\
5 & 0 & 1 & 1 & 1 & 0
\end{array}
$$

Adjacency Matrix



Adjacency List
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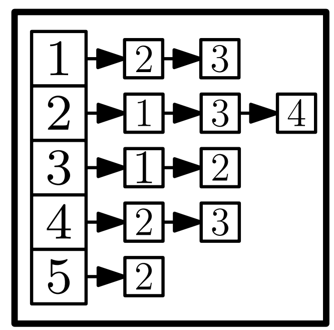
	Adjacency Matrix	Adjacency List *(using linked lists)*
Space	$\Theta(\lvert V\rvert^2)$	$\Theta(\lvert V\rvert + \lvert E\rvert)$
Is there an edge from vertex u to v?	$O(1)$ time	
List all the edges leaving $u \in V$		

$$
\begin{array}{c|ccccc|}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & 1 & 1 & 0 & 0 \\
2 & 1 & 0 & 1 & 1 & 1 \\
3 & 1 & 1 & 0 & 1 & 1 \\
4 & 0 & 1 & 1 & 0 & 1 \\
5 & 0 & 1 & 1 & 1 & 0 \\
\hline
\end{array}
$$

Adjacency Matrix



Adjacency List
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Space	$\Theta(\|V\|^2)$	$\Theta(\|V\| + \|E\|)$
Is there an edge from vertex u to v?	$O(1)$ time	$O(V)$ time
List all the edges leaving $u \in V$		

$$
\begin{array}{c|ccccc|}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
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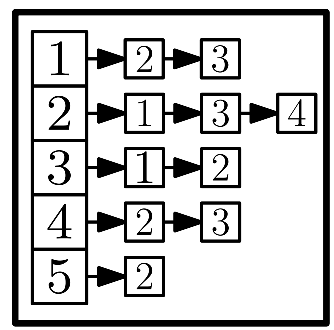
Adjacency Matrix



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Space	$\Theta(\lvert V\rvert^2)$	$\Theta(\lvert V\rvert + \lvert E\rvert)$
Is there an edge from vertex u to v?	$O(1)$ time	$O(\deg(u))$ time
List all the edges leaving $u \in V$		

	1	2	3	4	5
1	0	1	1	0	0
2	1	0	1	1	1
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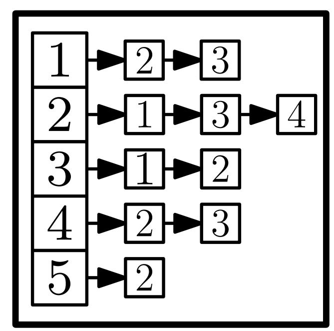
Adjacency Matrix



Adjacency List
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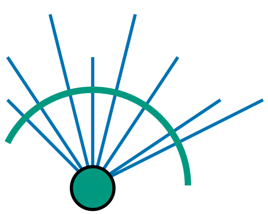
List all the edges leaving $u \in V$		

$\deg(u)$ *(the degree of u),*
is the number of edges leaving u



$$\begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 & 1 & 1 \\ 3 & 1 & 1 & 0 & 1 & 1 \\ 4 & 0 & 1 & 1 & 0 & 1 \\ 5 & 0 & 1 & 1 & 1 & 0 \\ \hline \end{array}$$

Adjacency Matrix

$1 \to 2 \to 3$
$2 \to 1 \to 3 \to 4$
$3 \to 1 \to 2$
$4 \to 2 \to 3$
$5 \to 2$

Adjacency List
(using linked lists)

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	Adjacency Matrix	Adjacency List *(using linked lists)*
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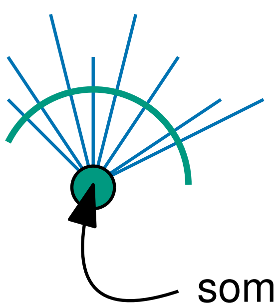
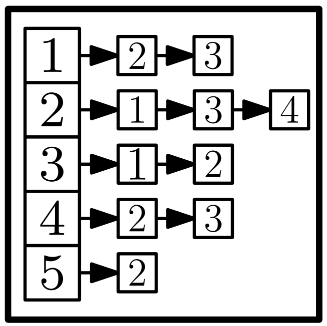


some vertex u

	1	2	3	4	5
1	0	1	1	0	0
2	1	0	1	1	1
3	1	1	0	1	1
4	0	1	1	0	1
5	0	1	1	1	0

Adjacency Matrix



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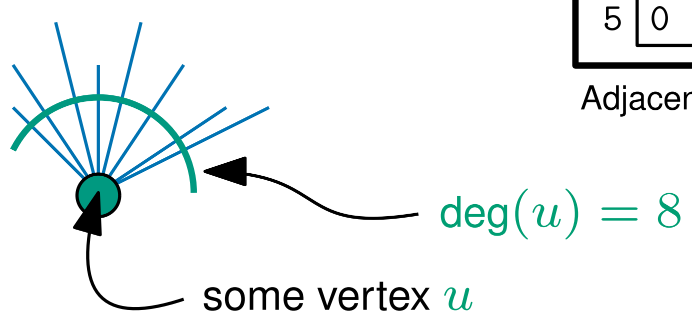
	Adjacency Matrix	Adjacency List *(using linked lists)*
Space	$\Theta(\|V\|^2)$	$\Theta(\|V\| + \|E\|)$
Is there an edge from vertex u to v?	$O(1)$ time	$O(\deg(u))$ time
List all the edges leaving $u \in V$		

$\deg(u)$ *(the degree of u),*
is the number of edges leaving u



$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & 1 & 1 & 0 & 0 \\
2 & 1 & 0 & 1 & 1 & 1 \\
3 & 1 & 1 & 0 & 1 & 1 \\
4 & 0 & 1 & 1 & 0 & 1 \\
5 & 0 & 1 & 1 & 1 & 0
\end{array}
$$

Adjacency Matrix

$1 \to 2 \to 3$
$2 \to 1 \to 3 \to 4$
$3 \to 1 \to 2$
$4 \to 2 \to 3$
$5 \to 2$

Adjacency List
(using linked lists)

V is the set of vertices
$|V|$ *is the number of vertices*

Basic operations

E is the set of edges
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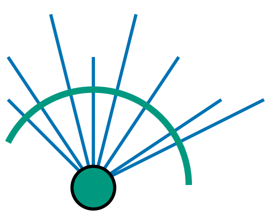
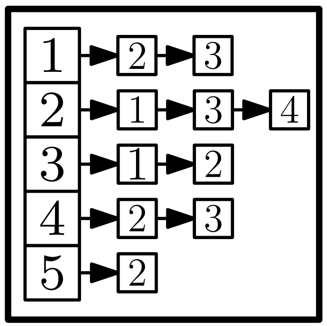


	1	2	3	4	5
1	0	1	1	0	0
2	1	0	1	1	1
3	1	1	0	1	1
4	0	1	1	0	1
5	0	1	1	1	0

Adjacency Matrix



Adjacency List
(using linked lists)

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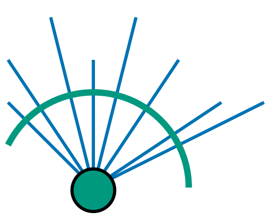
$|V|$ *is the number of vertices*

Basic operations

E is the set of edges
$|E|$ *is the number of edges*

	Adjacency Matrix	Adjacency List *(using linked lists)*
Space	$\Theta(\|V\|^2)$	$\Theta(\|V\| + \|E\|)$
Is there an edge from vertex u to v?	$O(1)$ time	$O(\deg(u))$ time
List all the edges leaving $u \in V$	$O(V)$ time	

$\deg(u)$ *(the degree of u),*
is the number of edges leaving u



	1	2	3	4	5
1	0	1	1	0	0
2	1	0	1	1	1
3	1	1	0	1	1
4	0	1	1	0	1
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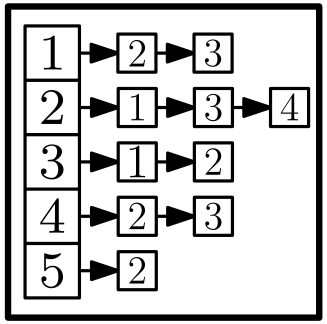
Adjacency Matrix



Adjacency List
(using linked lists)

University of BRISTOL

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List all the edges leaving $u \in V$	$O(V)$ time	$O(\deg(u))$ time

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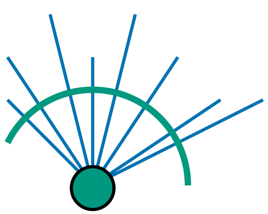
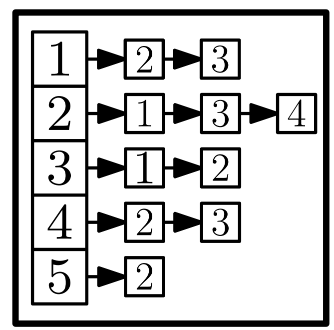


$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 & 1 & 1 \\ 3 & 1 & 1 & 0 & 1 & 1 \\ 4 & 0 & 1 & 1 & 0 & 1 \\ 5 & 0 & 1 & 1 & 1 & 0 \end{array}$$

Adjacency Matrix



Adjacency List
(using linked lists)

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	Adjacency Matrix	Adjacency List *(using linked lists)*	Adjacency List *(using hash tables)*
Space	$\Theta(\|V\|^2)$	$\Theta(\|V\| + \|E\|)$	$\Theta(\|V\| + \|E\|)$
Is there an edge from vertex u to v?	$O(1)$ time	$O(\deg(u))$ time	$O(1)$ time
List all the edges leaving $u \in V$	$O(V)$ time	$O(\deg(u))$ time	$O(\deg(u))$ time

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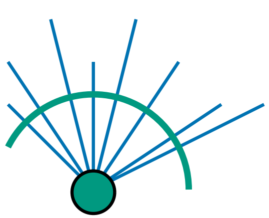

	1	2	3	4	5
1	0	1	1	0	0
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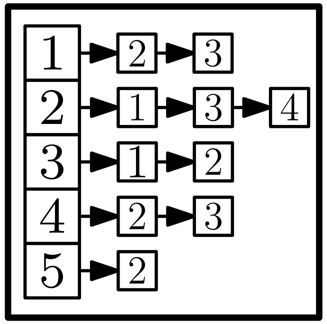
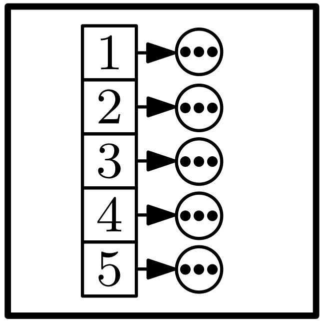
Adjacency Matrix



Adjacency List
(using linked lists)



Adjacency List
(using hash tables)

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Basic operations

E is the set of edges
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Space	$\Theta(\vert V\vert^2)$	$\Theta(\vert V\vert + \vert E\vert)$	$\Theta(\vert V\vert + \vert E\vert)$
Is there an edge from vertex u to v?	$O(1)$ time	$O(\deg(u))$ time	$O(1)$ time
List all the edges leaving $u \in V$	$O(V)$ time	$O(\deg(u))$ time	$O(\deg(u))$ time

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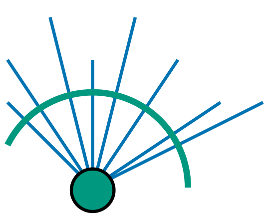
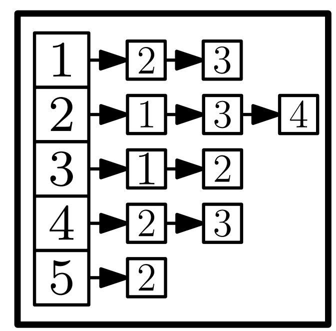
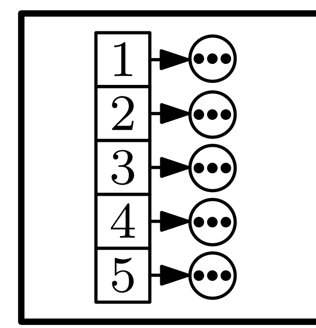


$$\begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 & 1 & 1 \\ 3 & 1 & 1 & 0 & 1 & 1 \\ 4 & 0 & 1 & 1 & 0 & 1 \\ 5 & 0 & 1 & 1 & 1 & 0 \\ \hline \end{array}$$

Adjacency Matrix



Adjacency List
(using linked lists)



Adjacency List
(using hash tables)

all three representations work for directed and/or weighted graphs too

(with the same complexities)

Basic operations

V is the set of vertices
$|V|$ *is the number of vertices*

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why don't we always use these?

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Space	$\Theta(\lvert V\rvert^2)$	$\Theta(\lvert V\rvert + \lvert E\rvert)$	$\Theta(\lvert V\rvert + \lvert E\rvert)$
Is there an edge from vertex u to v?	$O(1)$ time	$O(\deg(u))$ time	$O(1)$ time
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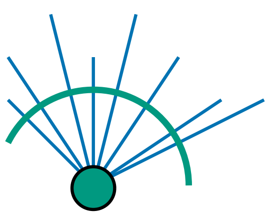

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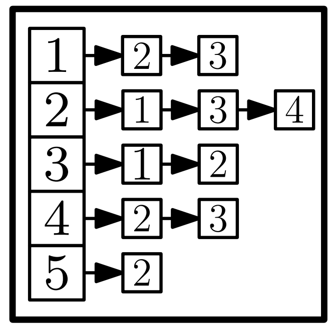
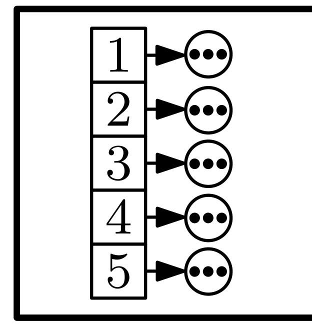
Adjacency Matrix



Adjacency List
(using linked lists)



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Basic hash tables give *expected* time complexities (constant time 'on average')

- no worst case guarantees against collisions

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Fortunately, few algorithms need to ask *"is there an edge?"*

Normally, *"tell me all the edges"* is fine

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For this course we'll stick to Adjacency Matrices and Lists (using linked lists)

(for today just Adjacency Lists)

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One practical reason to stick to standard representations is that

you may not have control over how your graph is stored.

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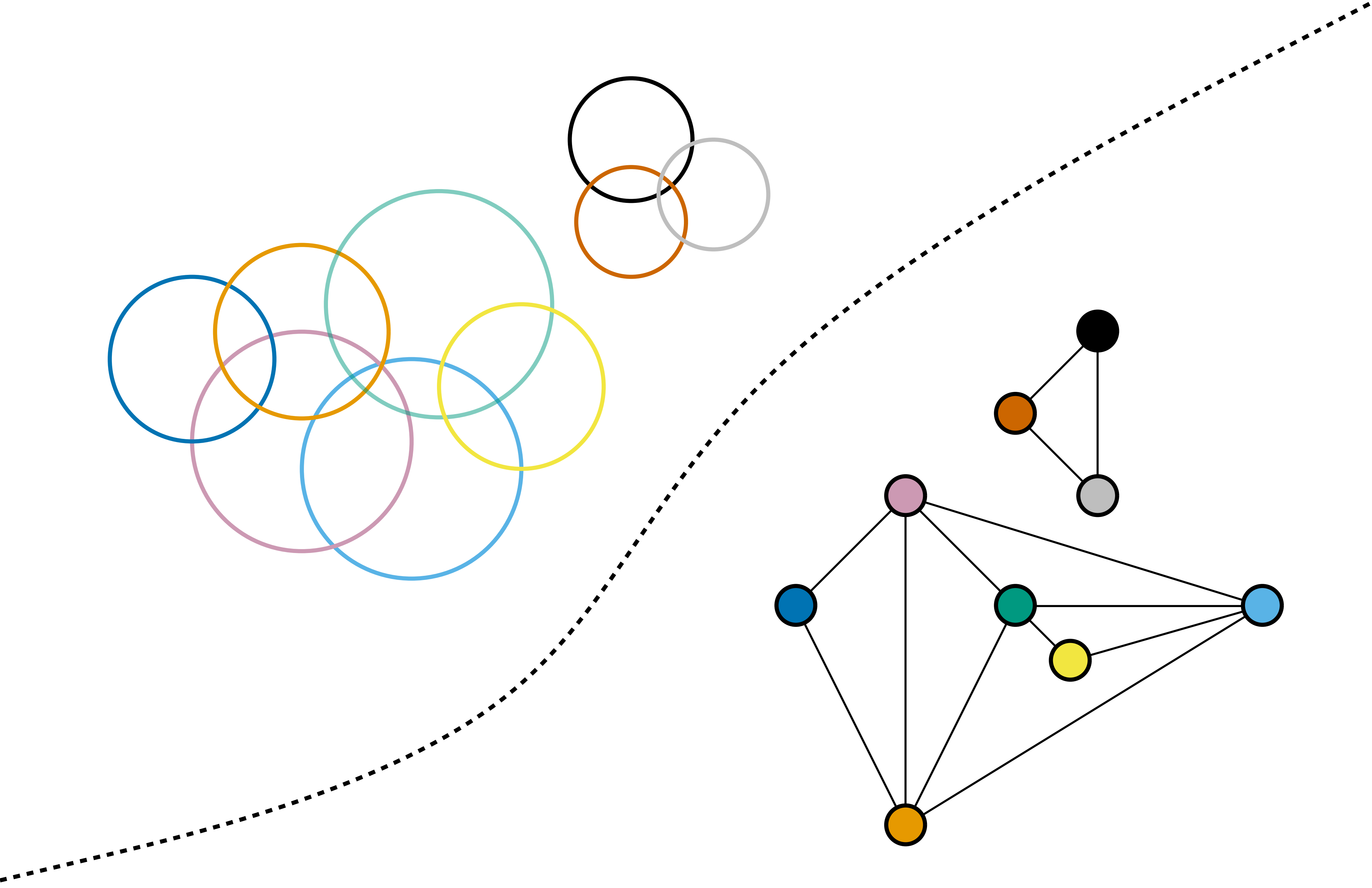
Fortunately in many cases, you can *pretend* your graph is stored as one of the above.

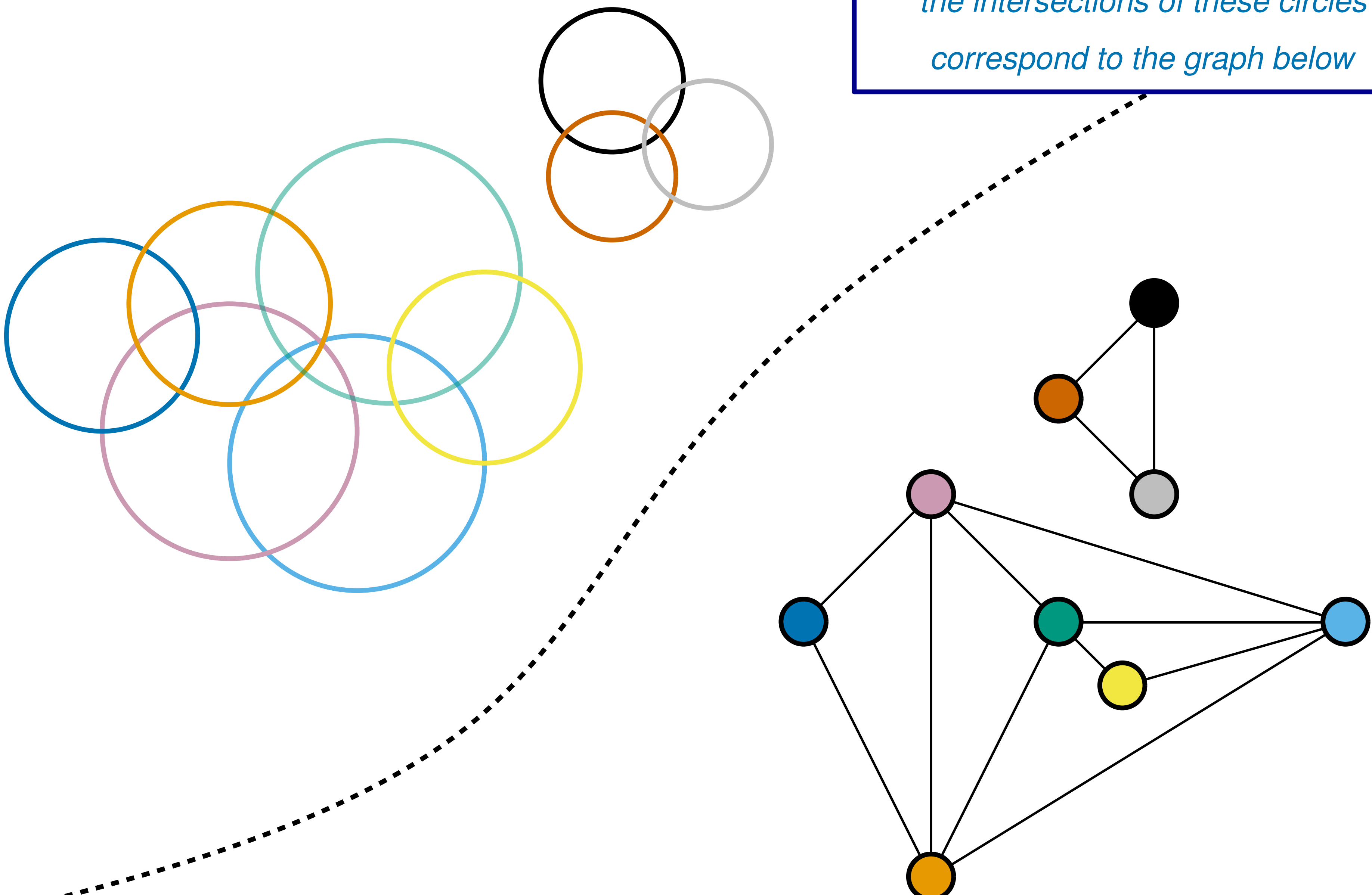
Intersection Graphs



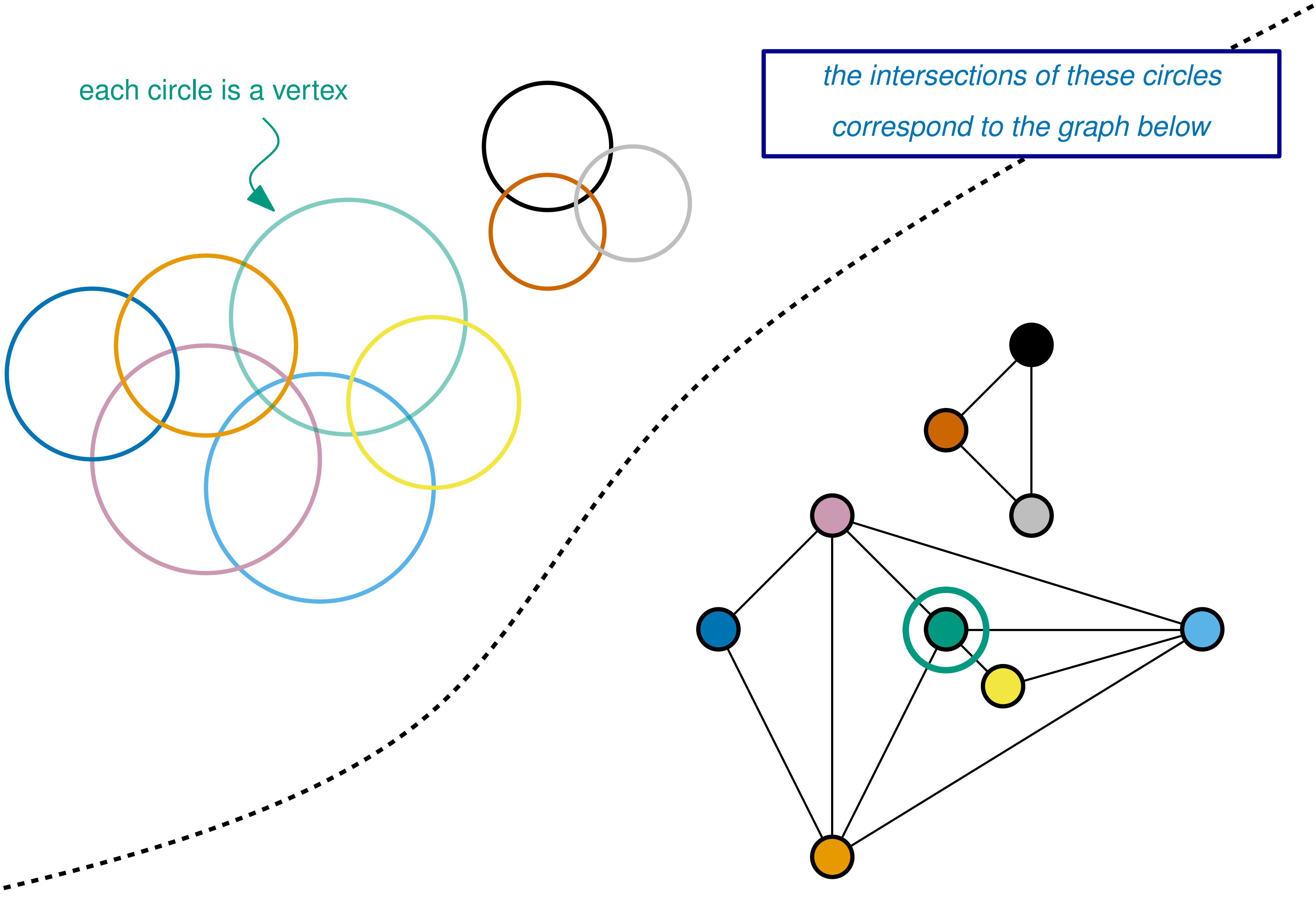
Intersection Graphs

the intersections of these circles

correspond to the graph below



Intersection Graphs



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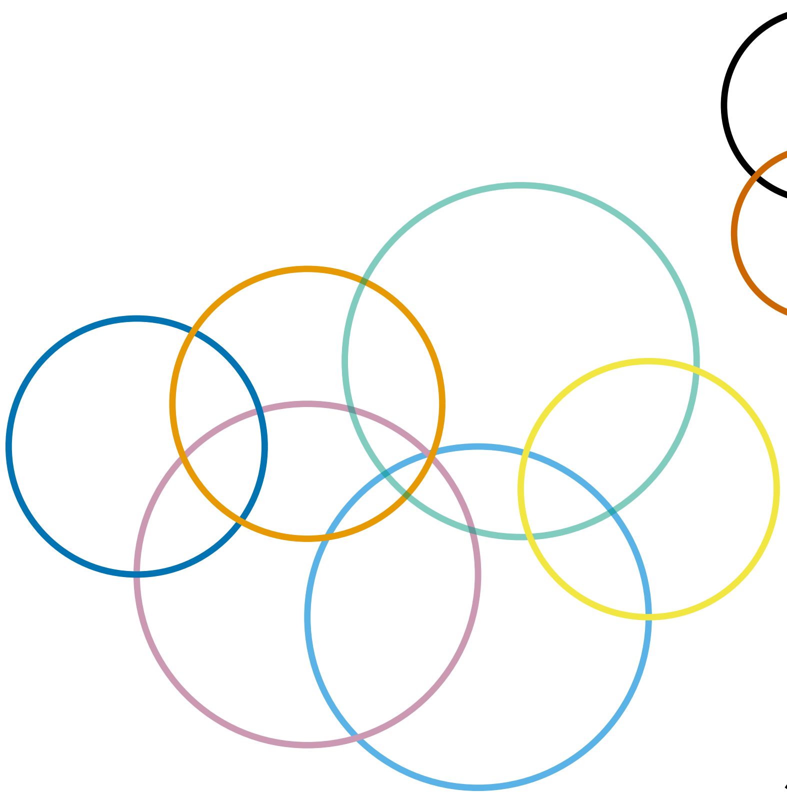

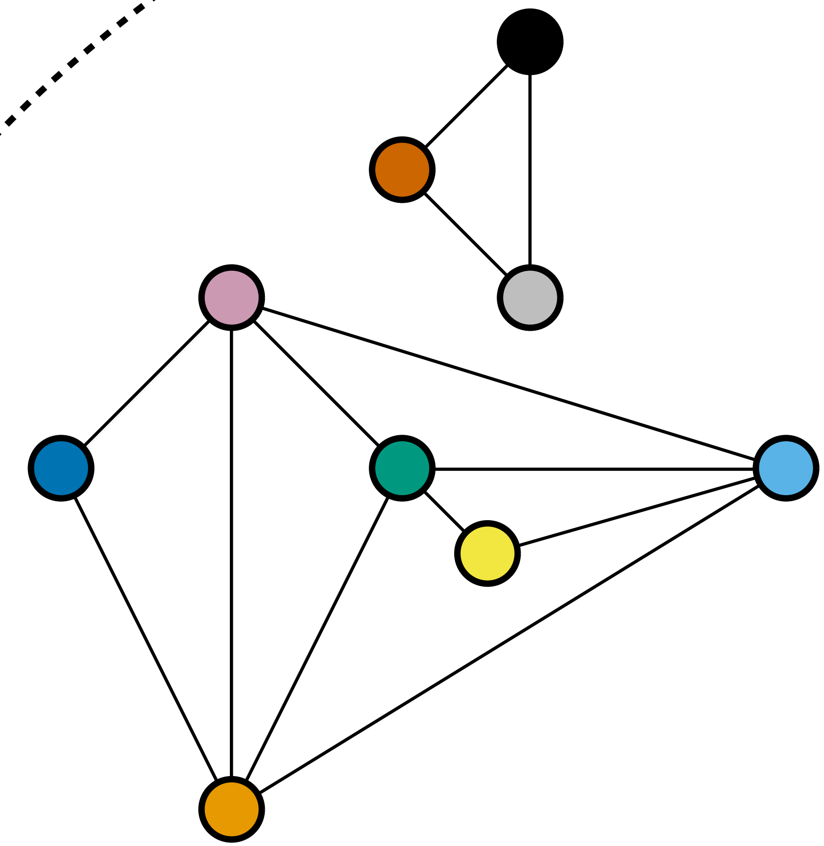

Intersection Graphs

each intersection

is an edge

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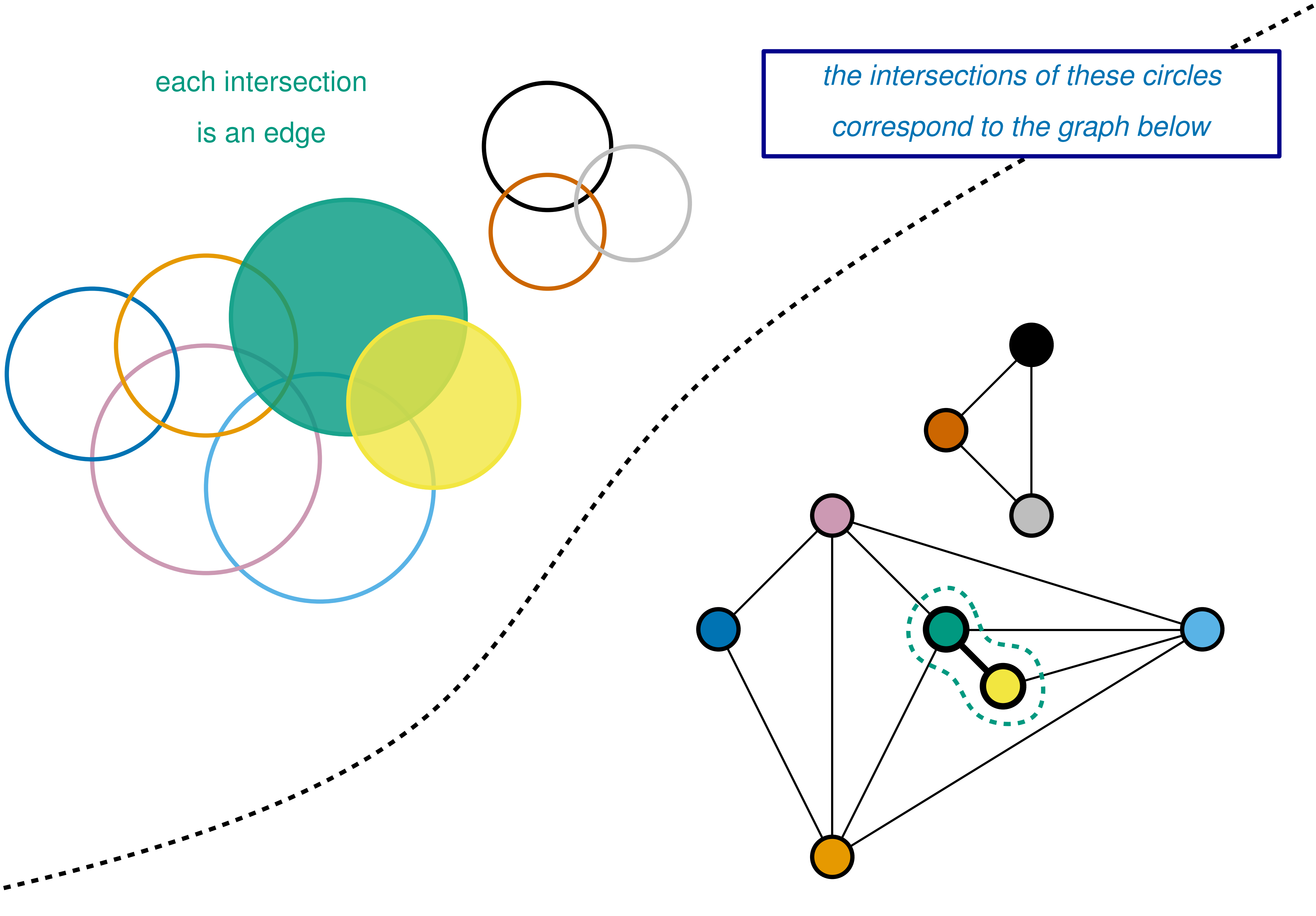

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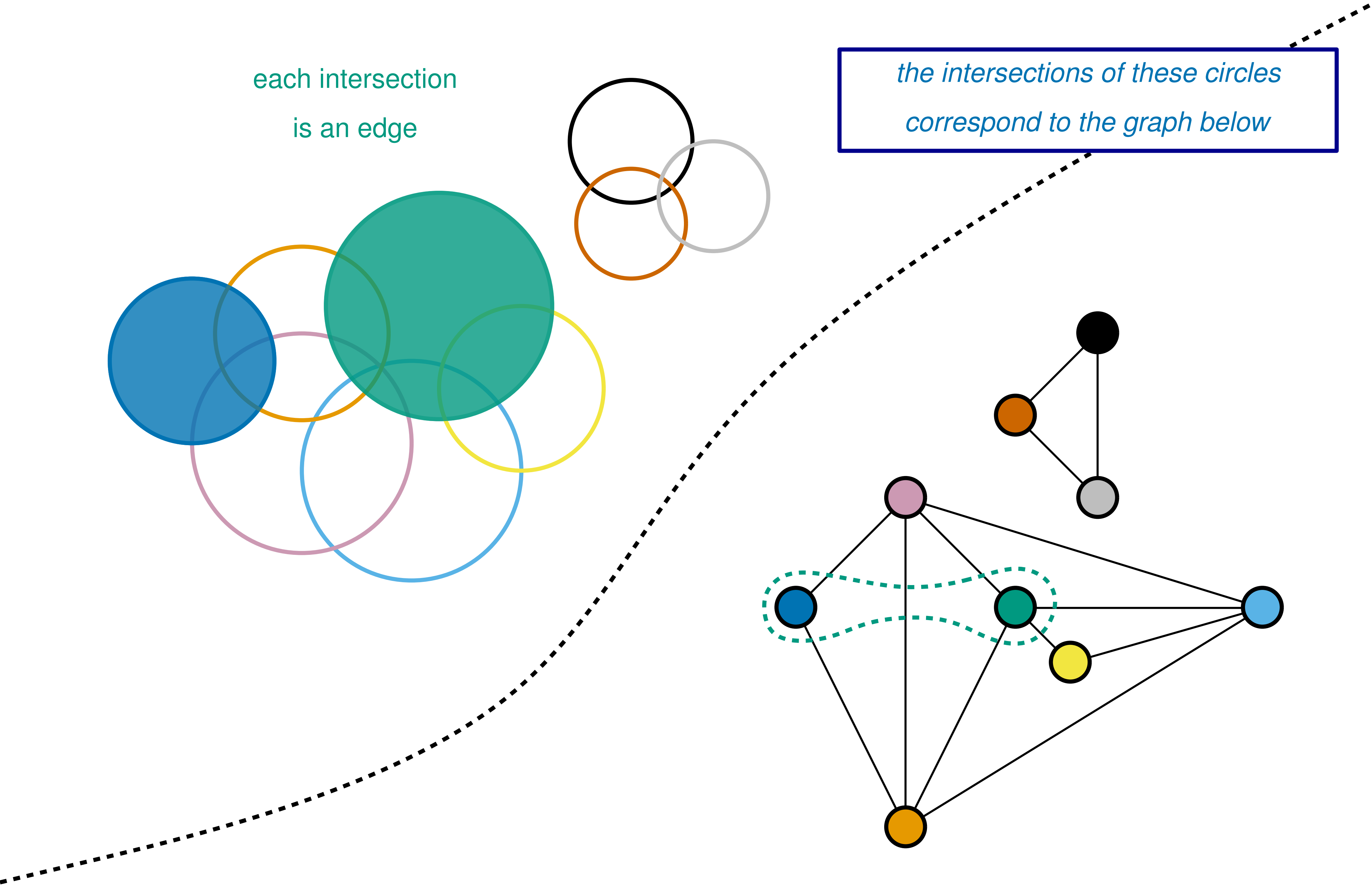

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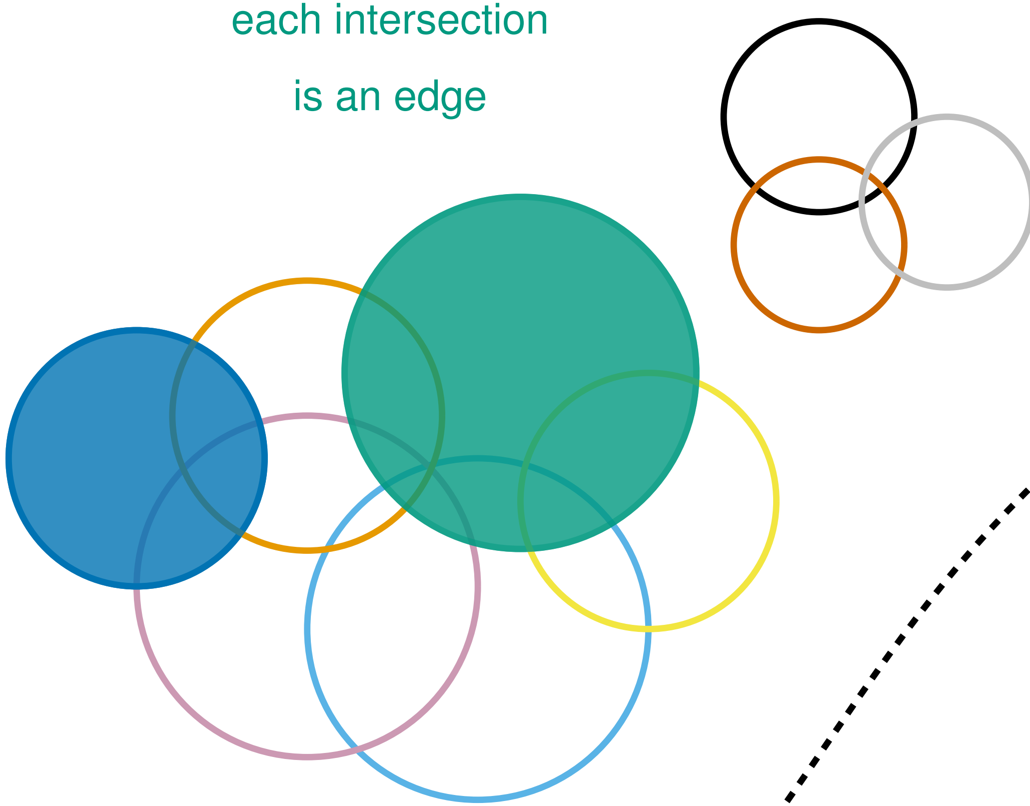

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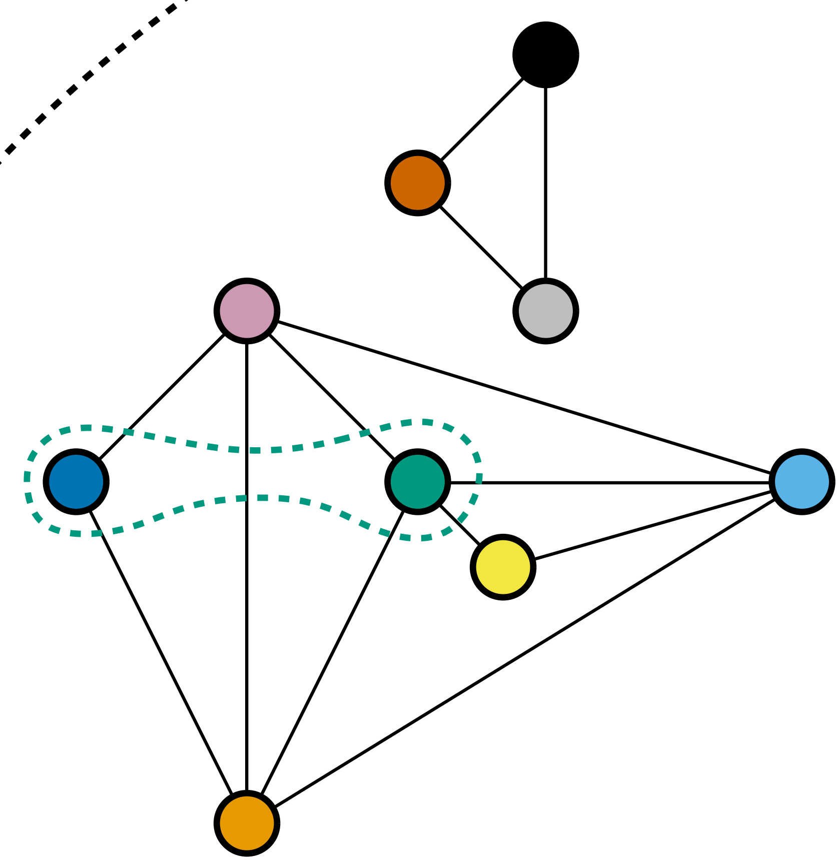


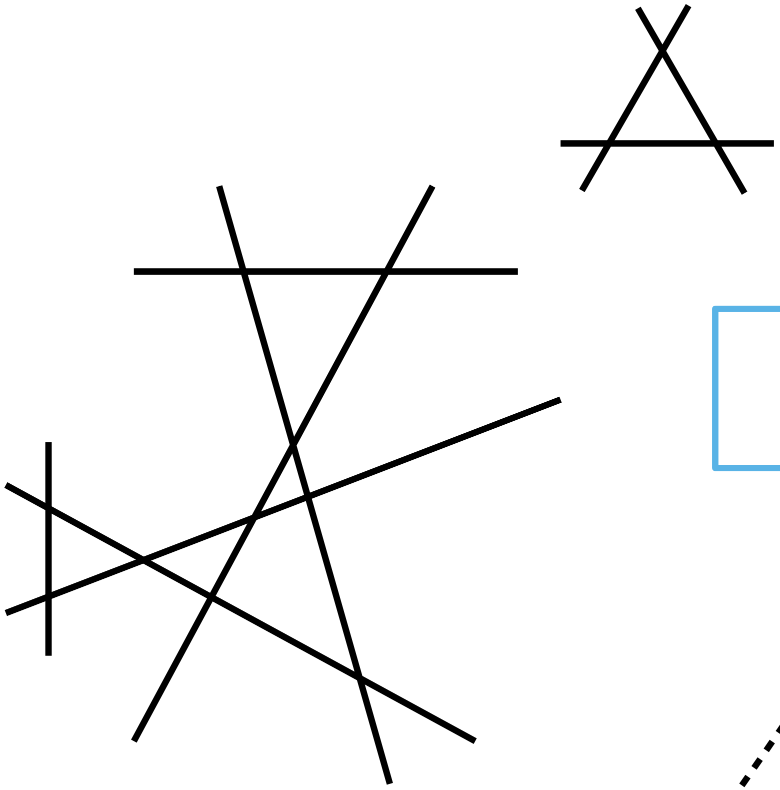
even if we only store the circles,

we can pretend we have

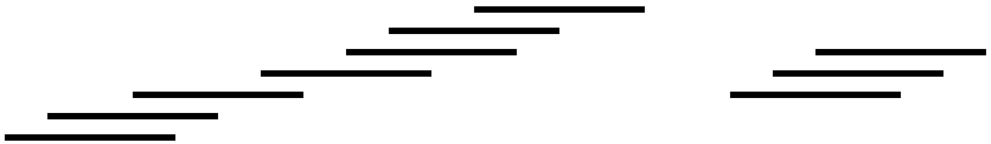
an **Adjacency Matrix**

without building one

Other intersection graphs...



both of these 'collections' also correspond to the example graph

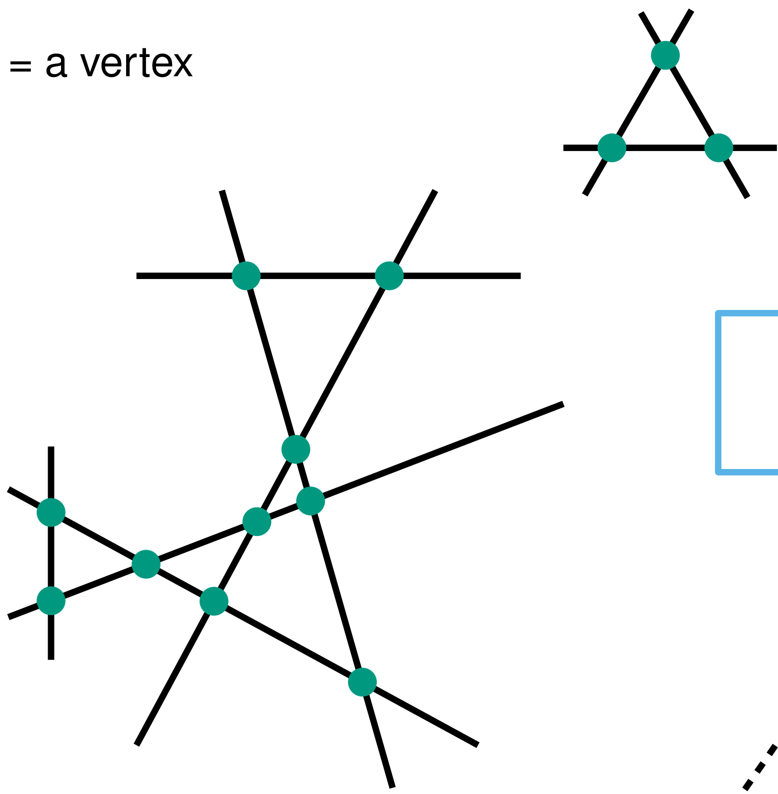


(these are 1D intervals on a line spread out for visual clarity)

Other intersection graphs. . .

● = an edge

╲ = a vertex



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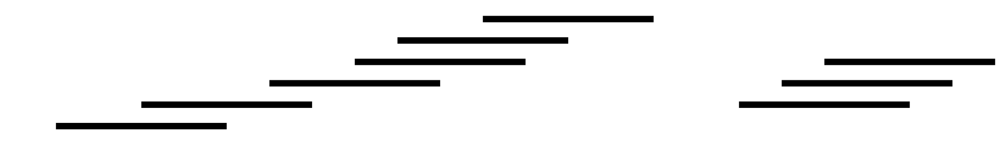

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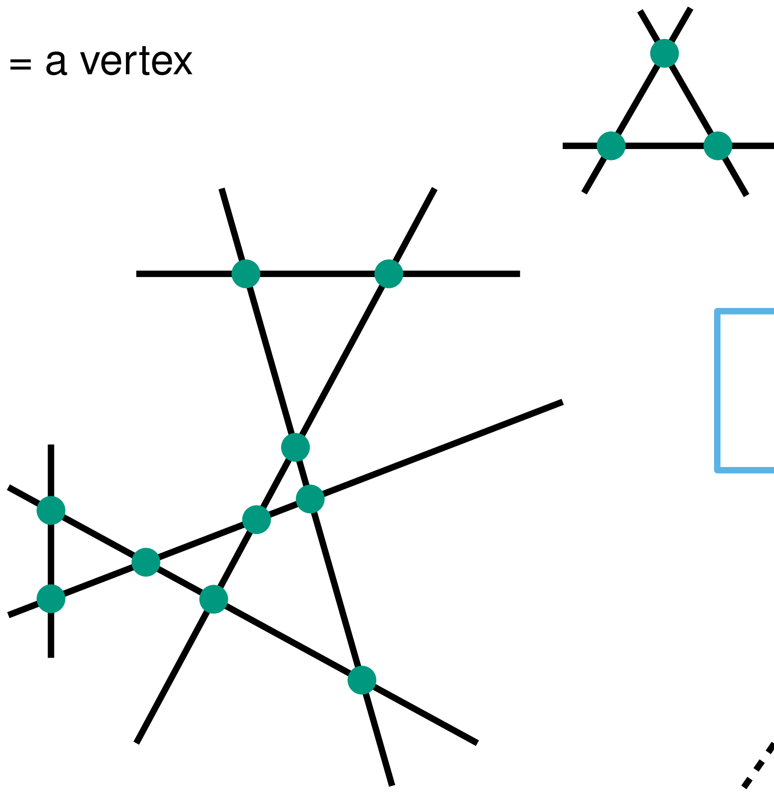

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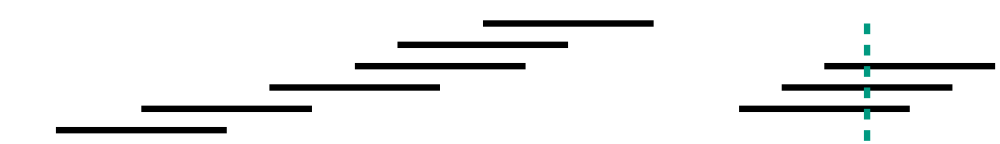
there is an edge if two intervals intersect



each interval is a vertex

(these are 1D intervals on a line spread out for visual clarity)

Other intersection graphs. . .

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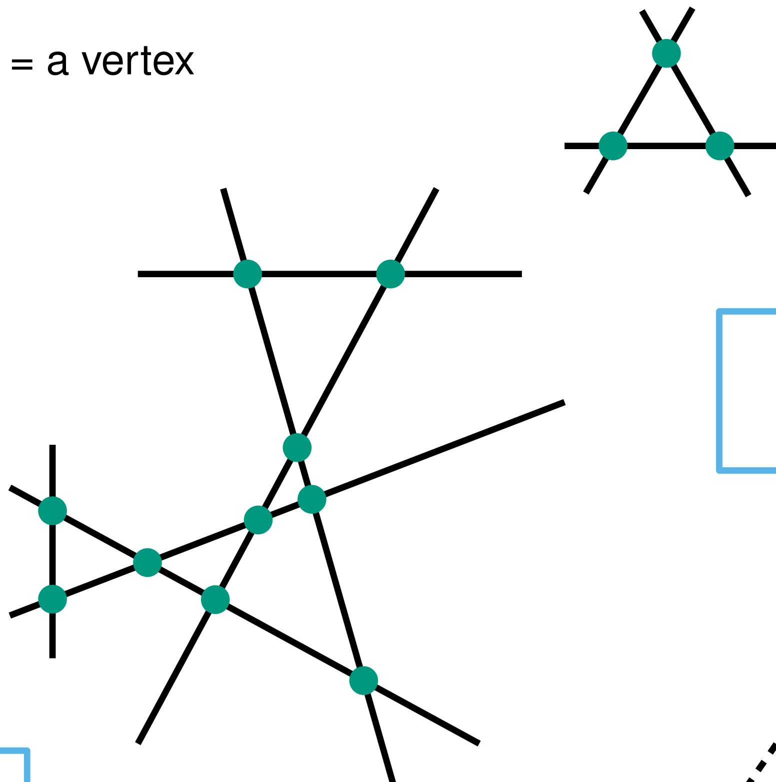
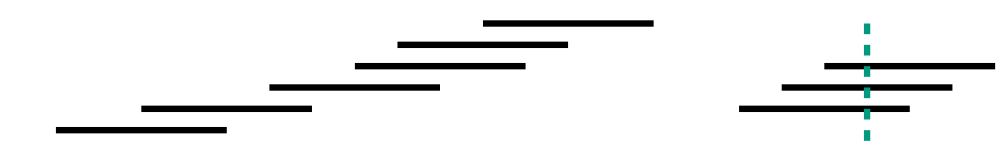


both of these ‘collections’ also correspond to the example graph

Again,

we can pretend we have an **Adjacency Matrix** without building one

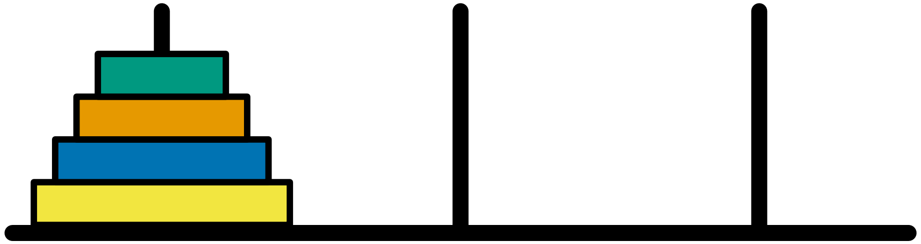
there is an edge if two intervals intersect



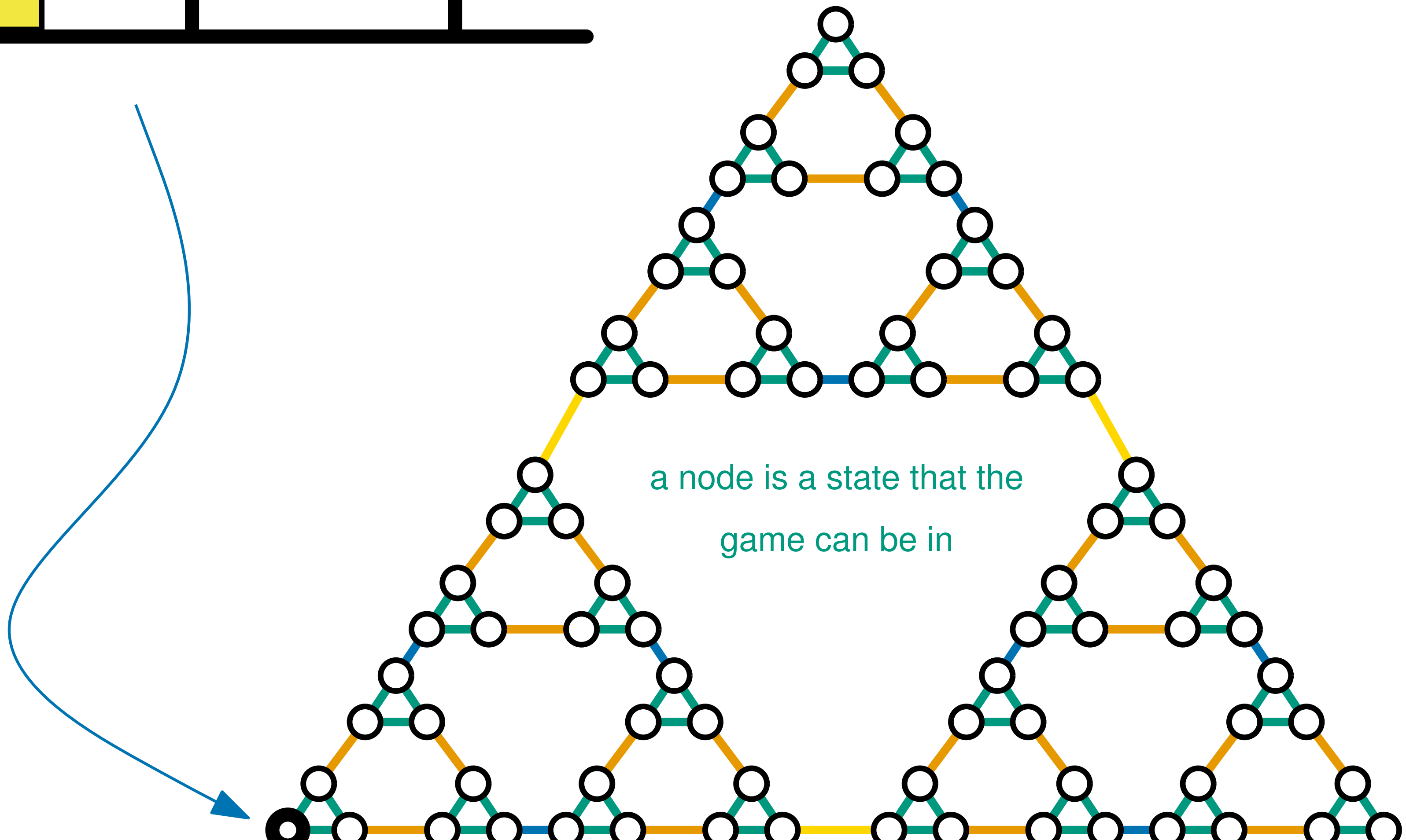
each interval is a vertex

(these are 1D intervals on a line spread out for visual clarity)

Configuration Graphs



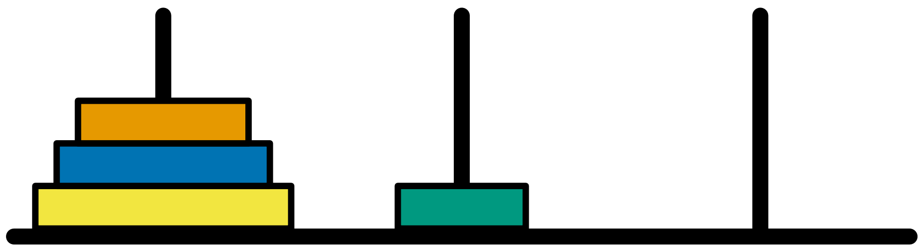
This is the configuration graph for the (4 disk) towers of Hanoi



a node is a state that the game can be in

there is an edge if you can get from one state to another in a single move

Configuration Graphs



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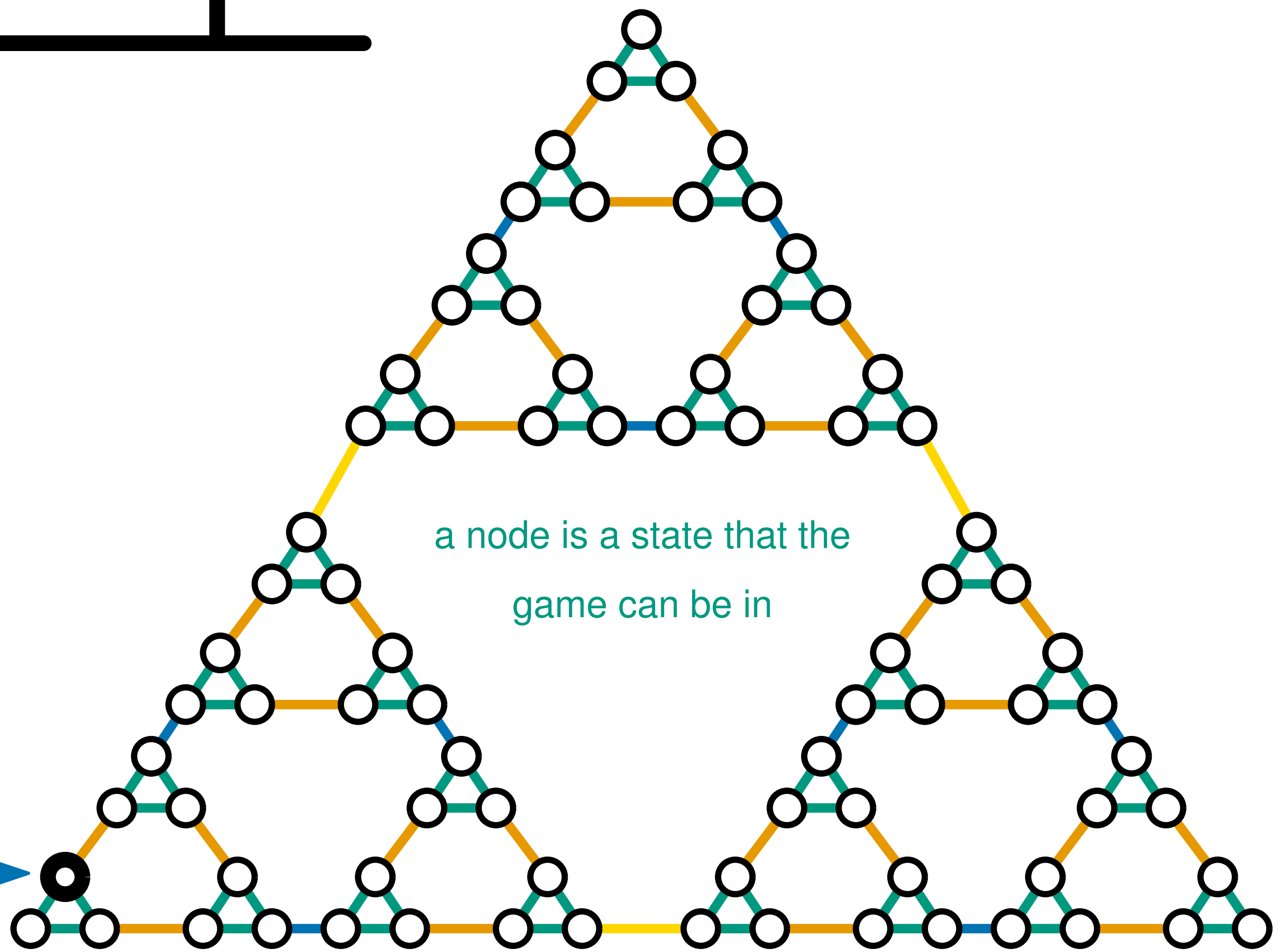

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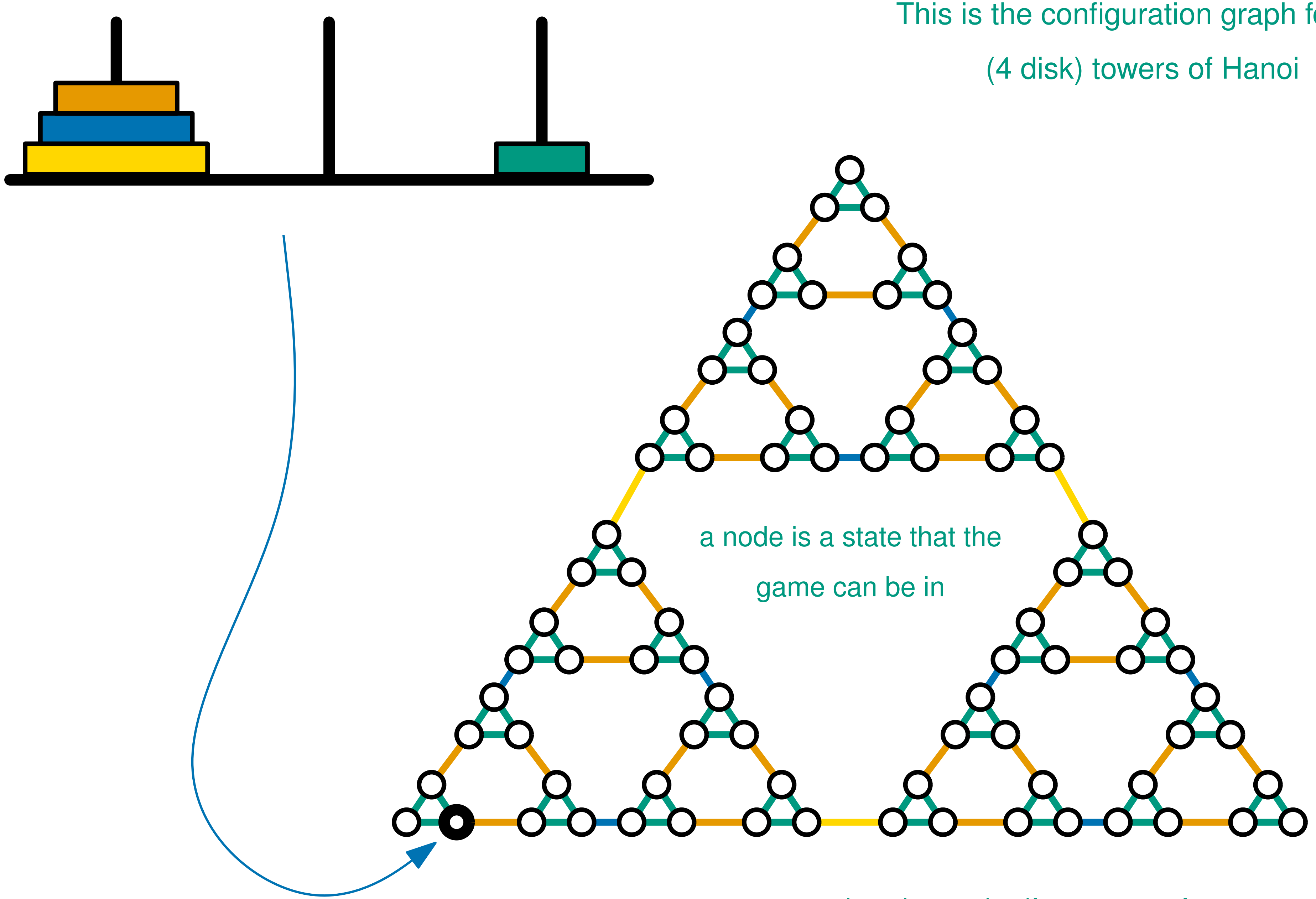

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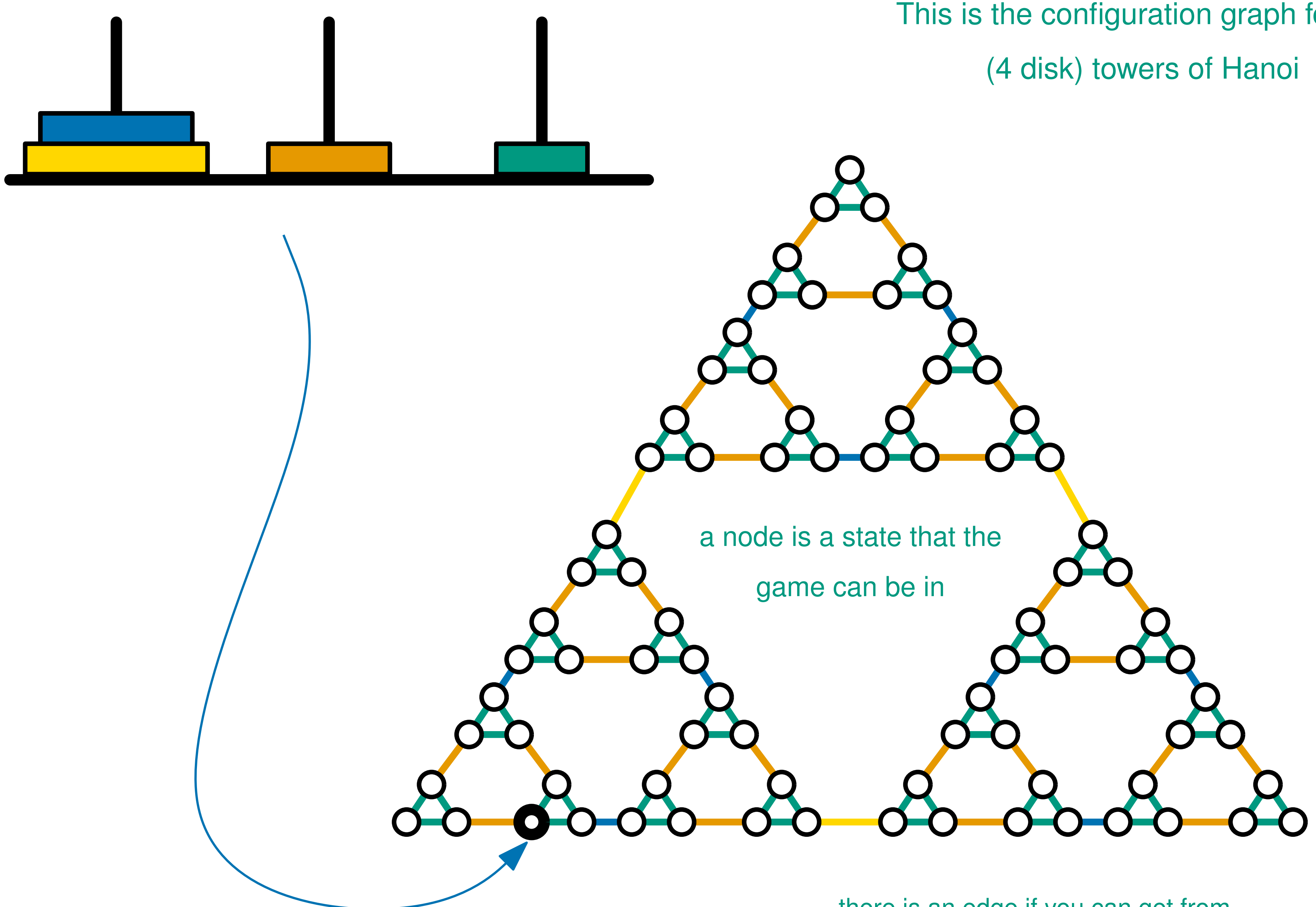

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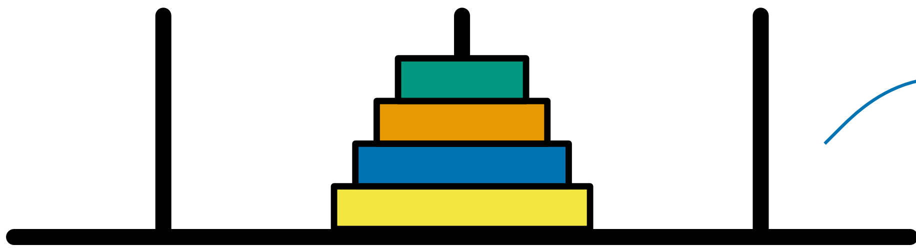

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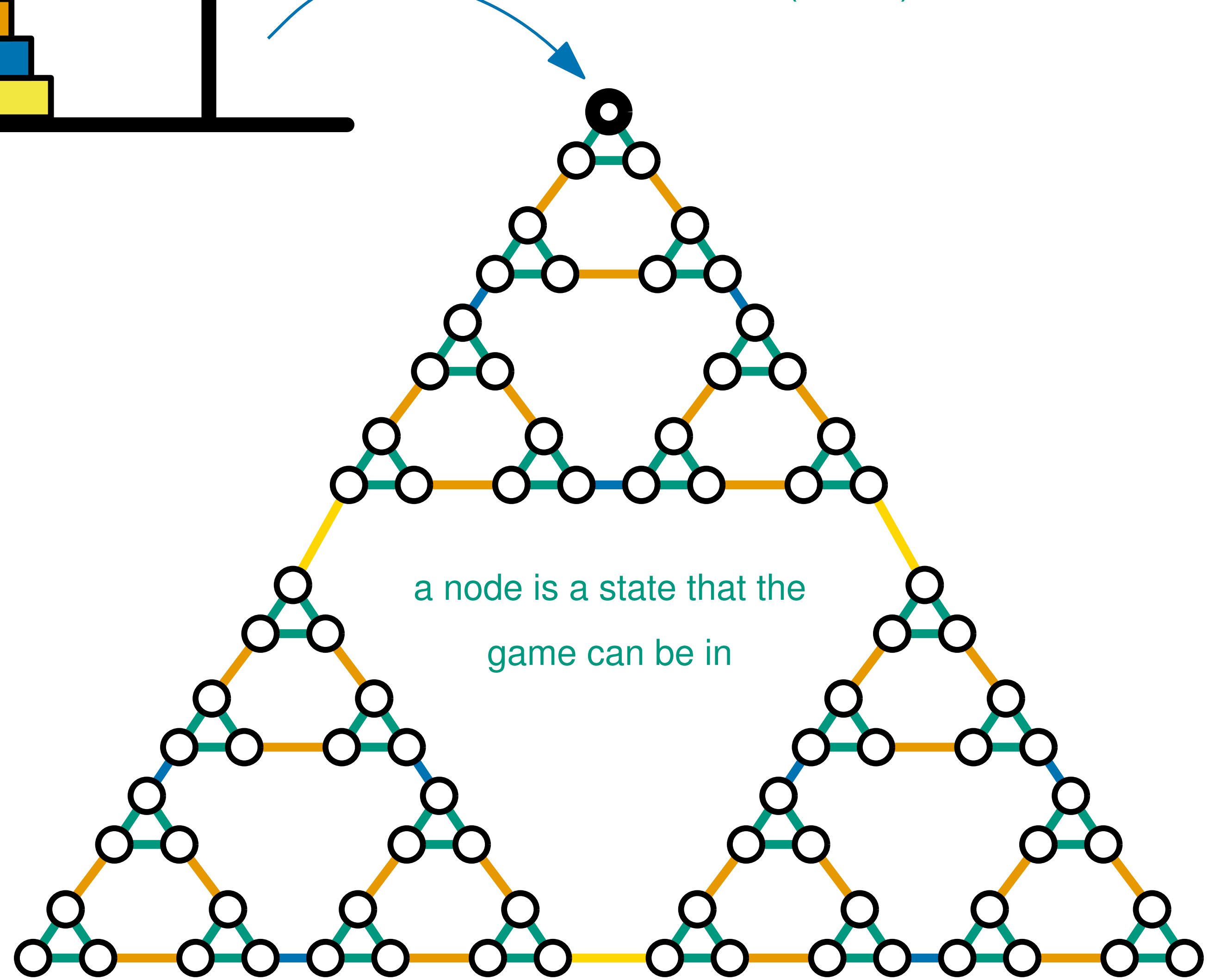

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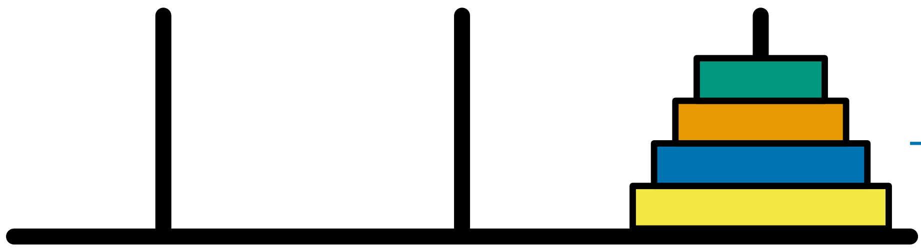

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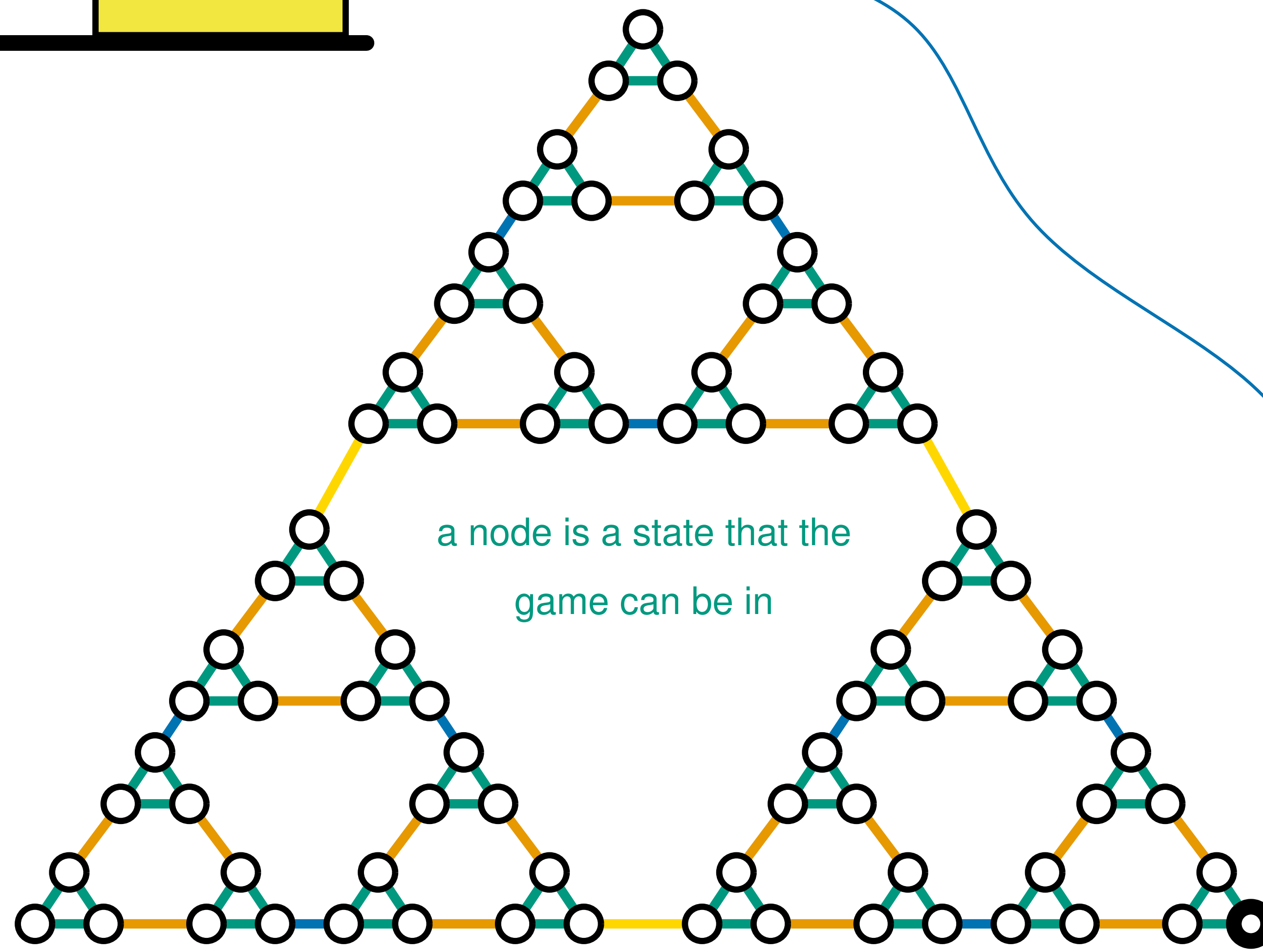

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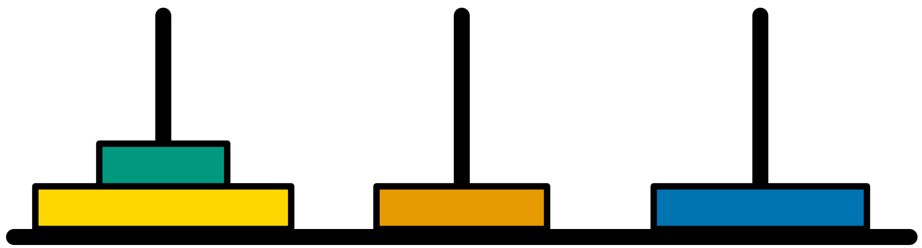

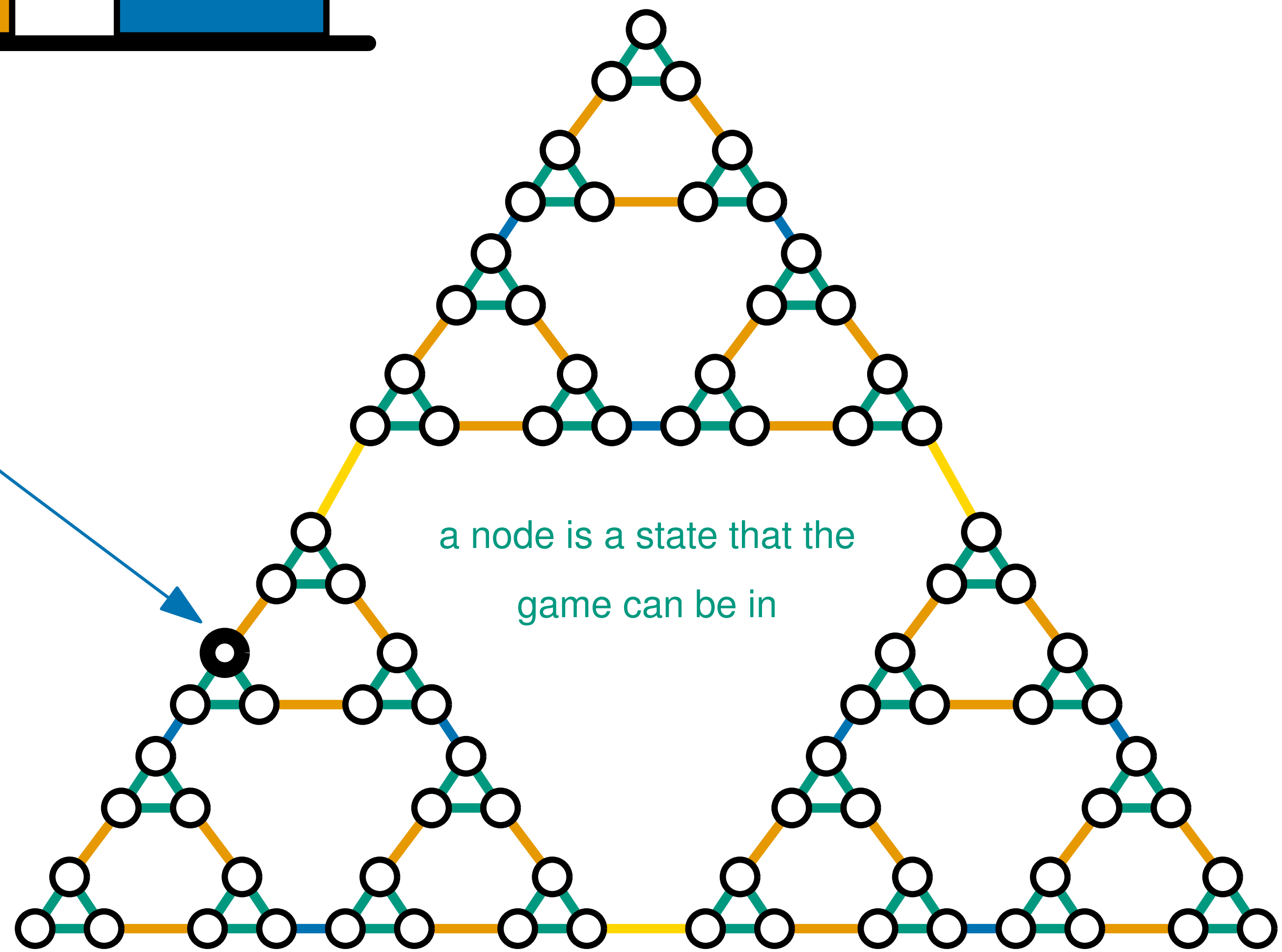

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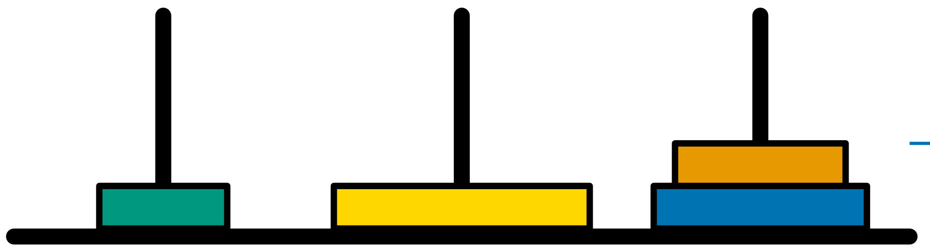

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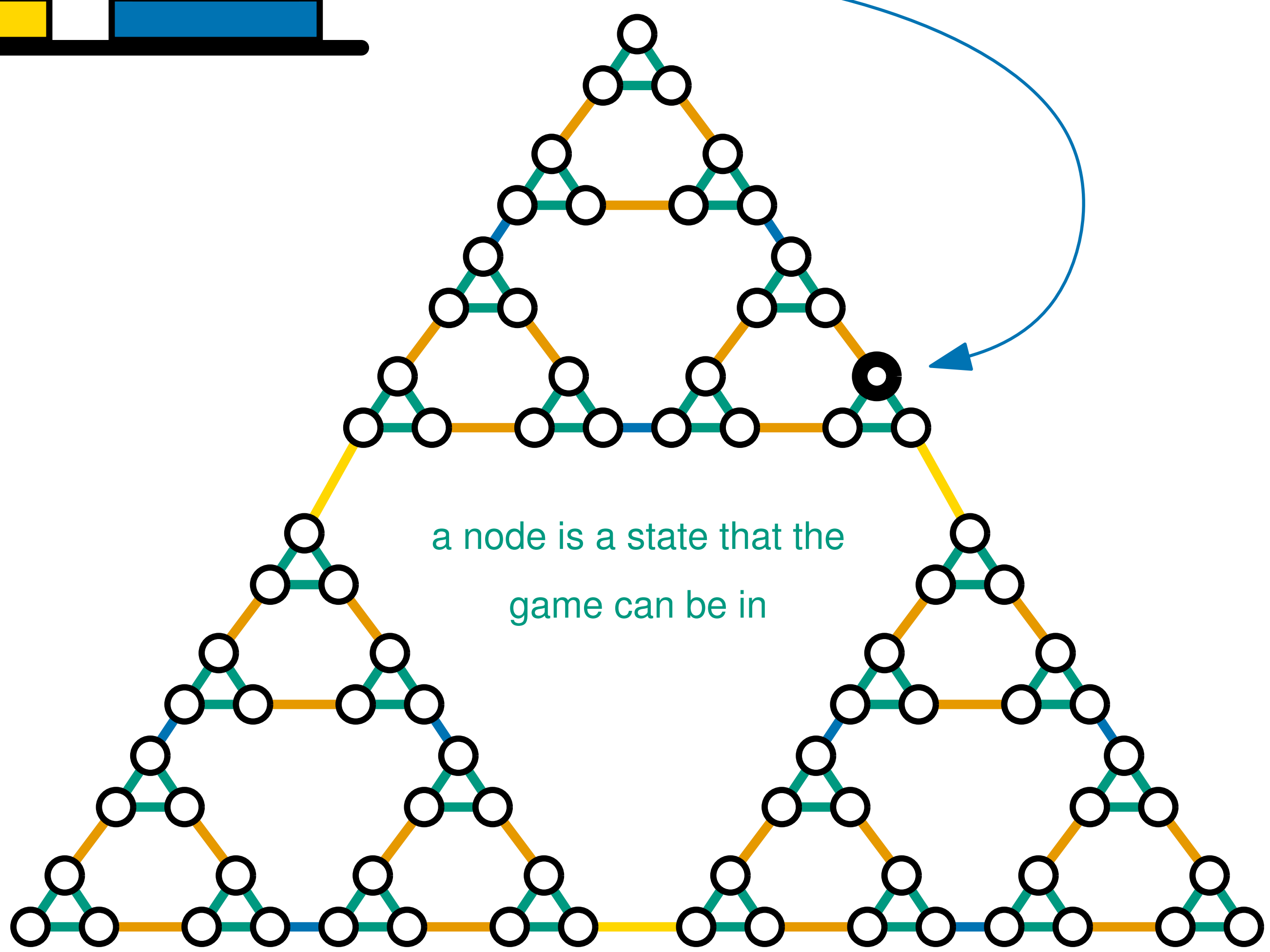

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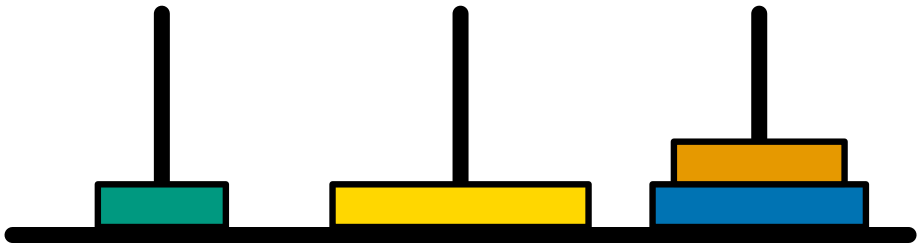

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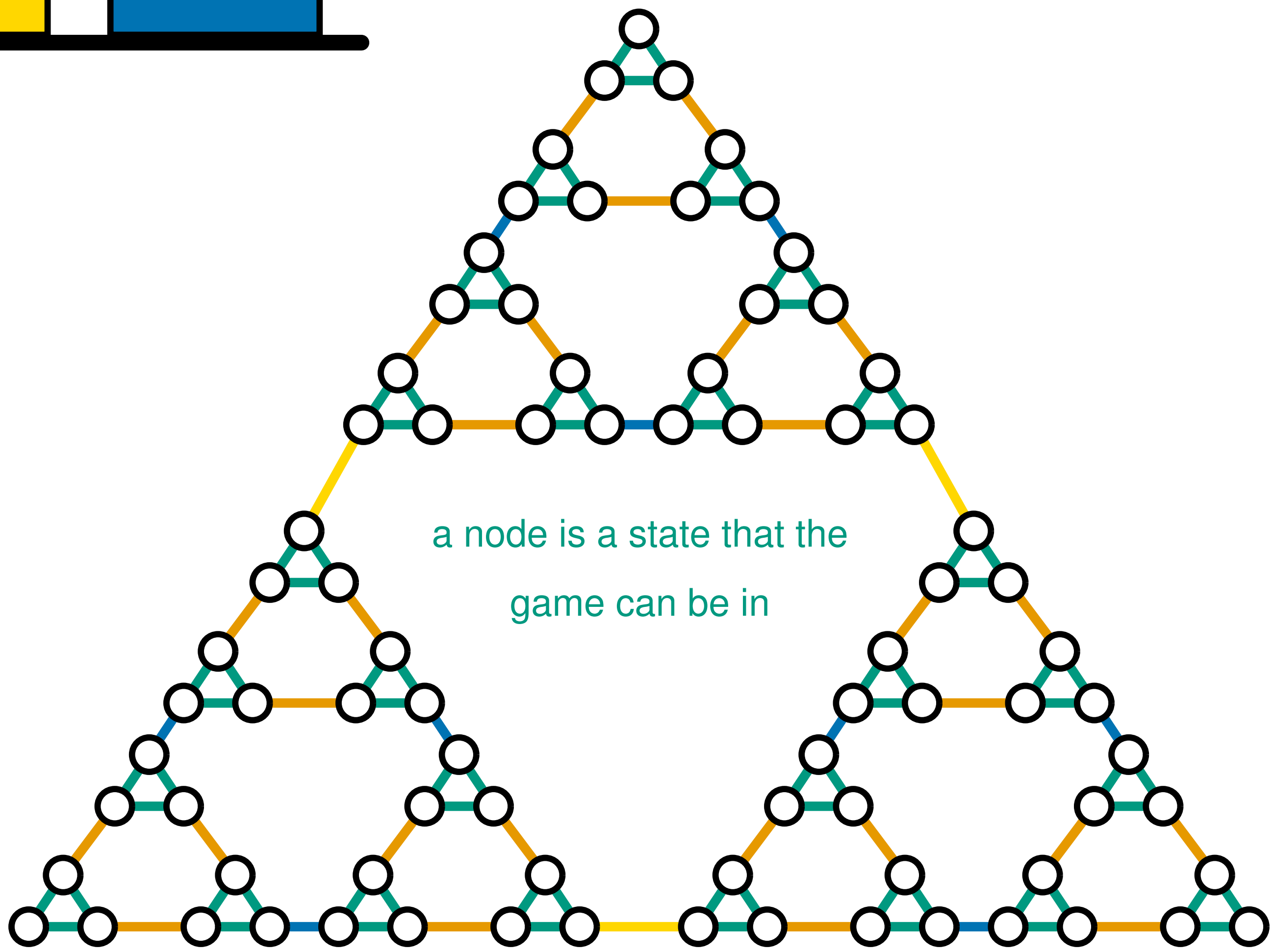

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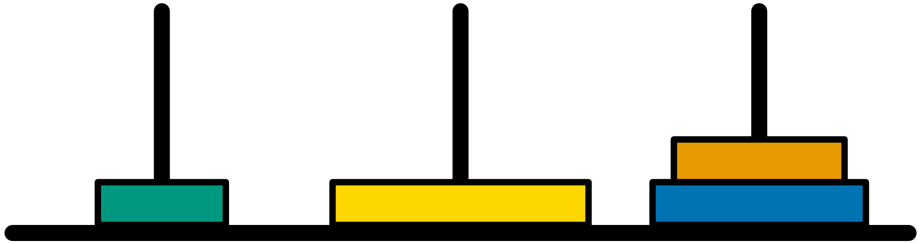
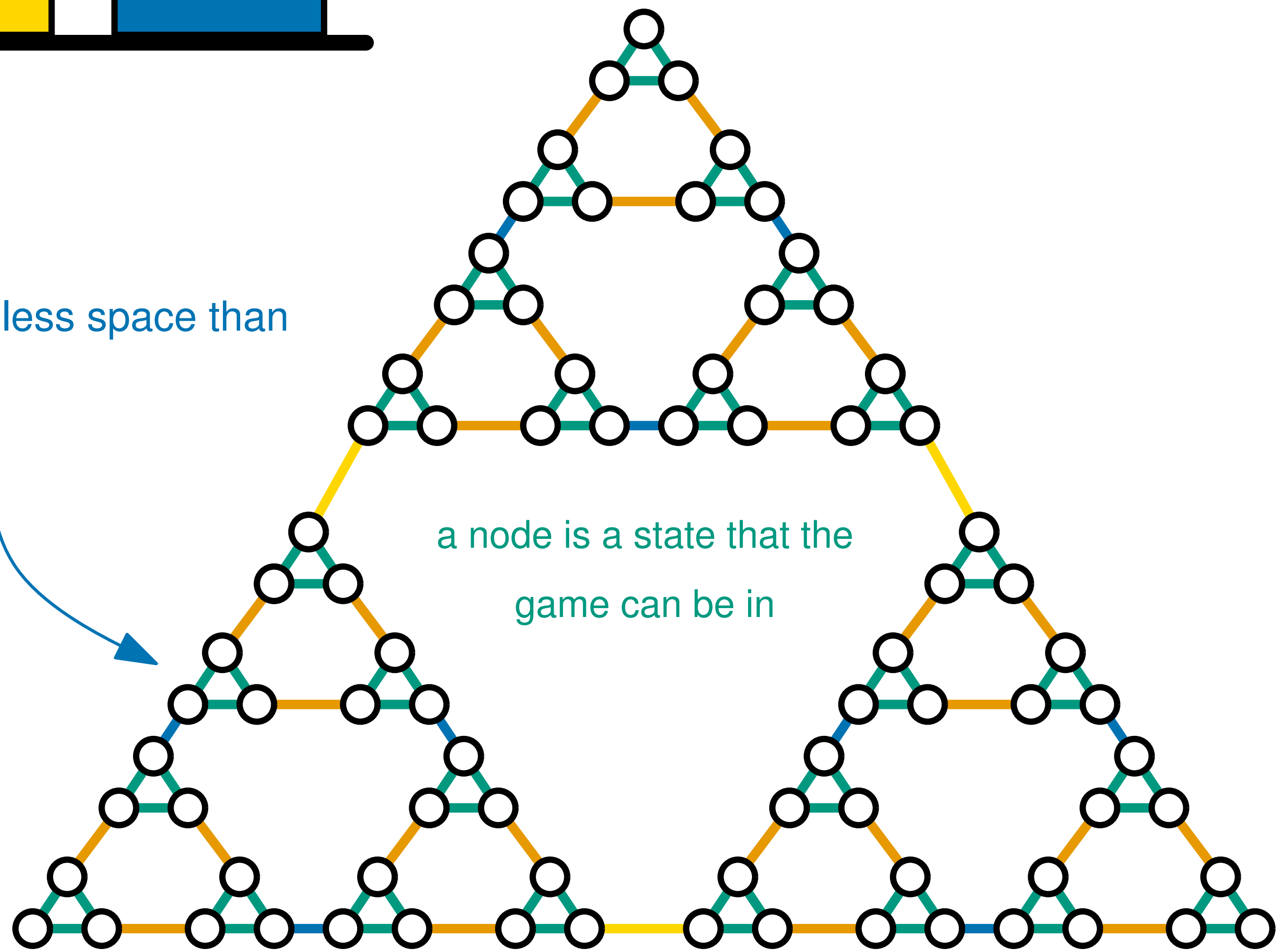


This is the configuration graph for the (4 disk) towers of Hanoi

storing **this** uses much less space than storing this **this** graph



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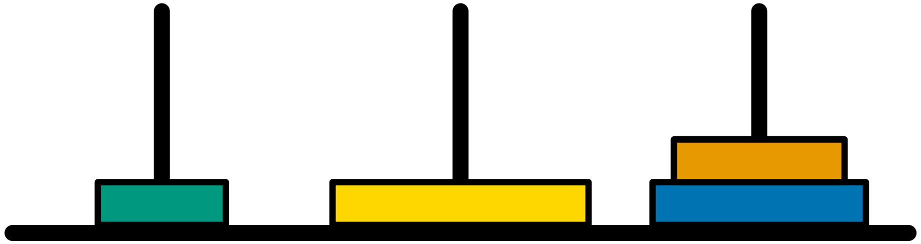

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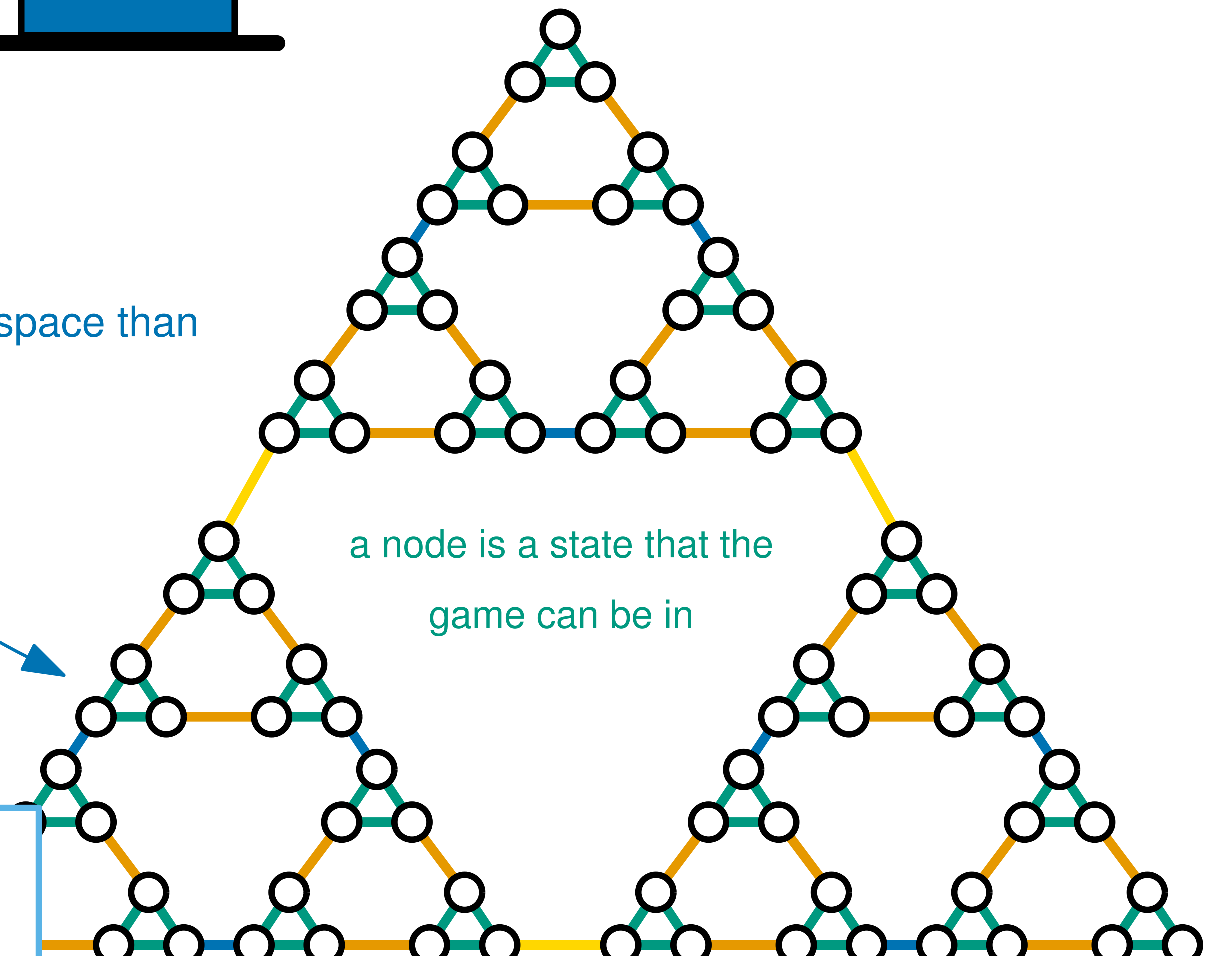

a node is a state that the

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we can pretend we have

an **Adjacency List**

without building one

there is an edge if you can get from

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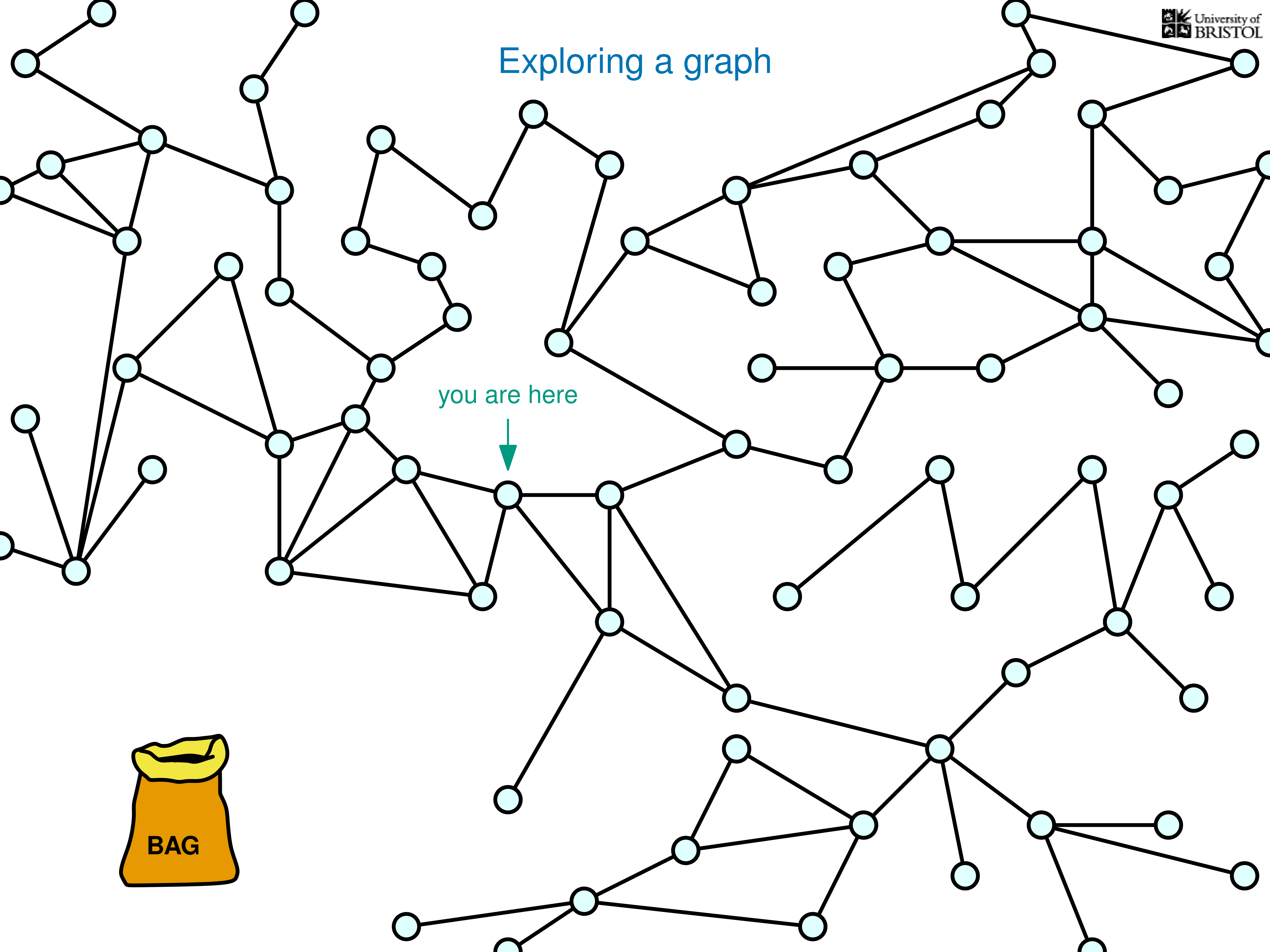
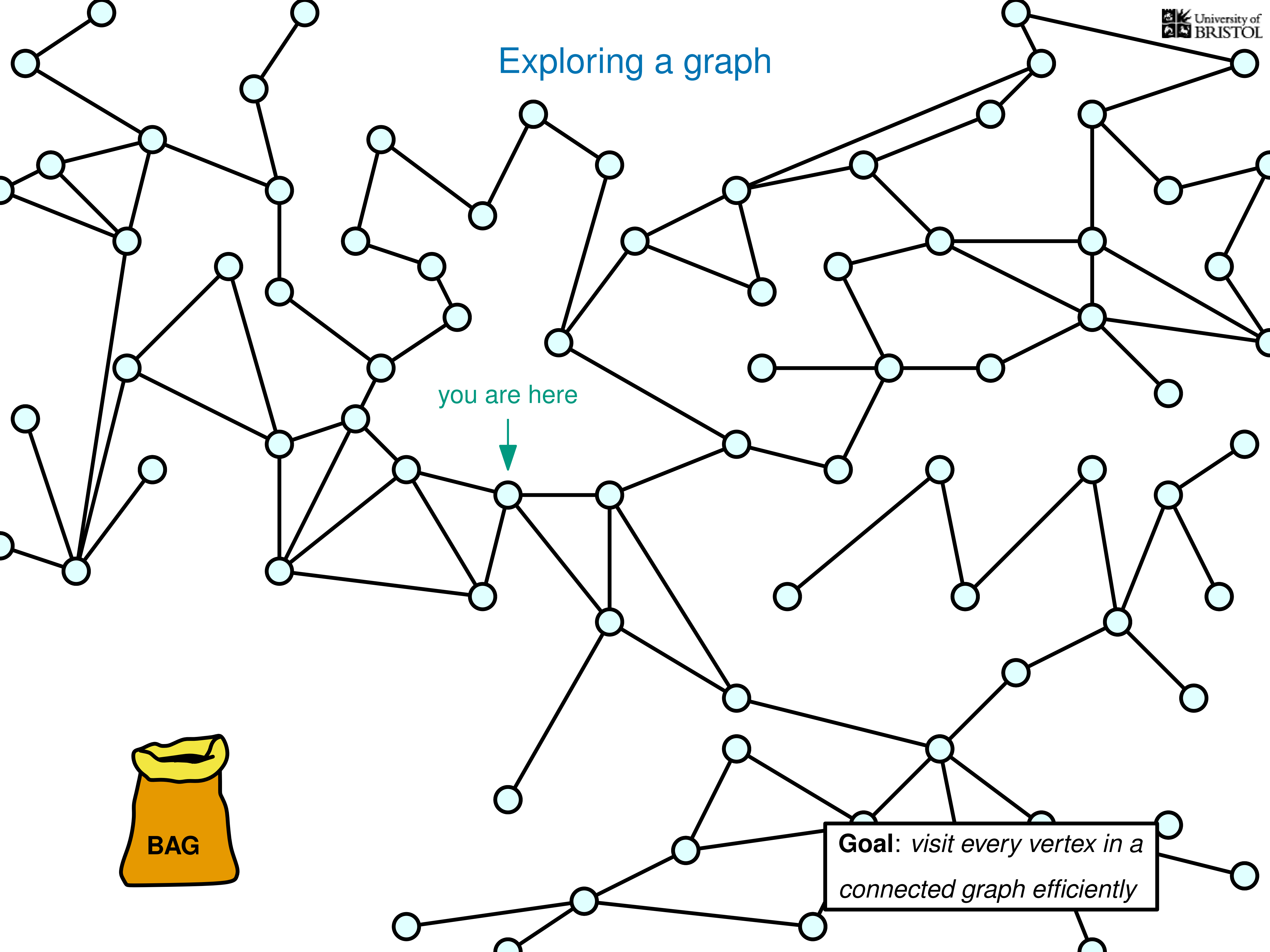
Exploring a graph

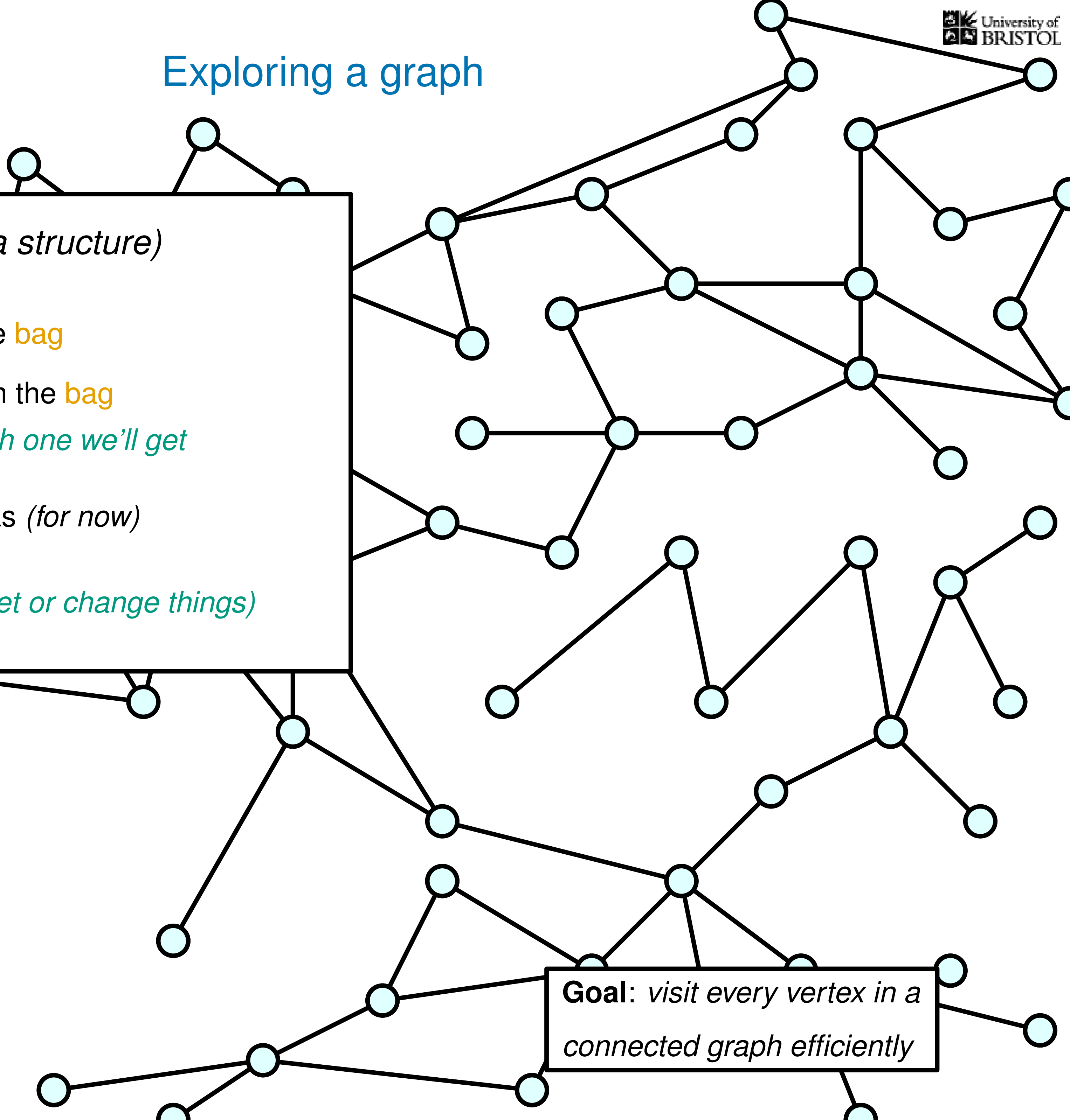


Exploring a graph



Goal: *visit every vertex in a connected graph efficiently*

Exploring a graph



The bag *(secretly a data structure)*

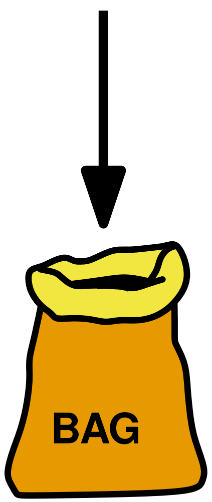
We can put a vertex in the bag

We can take a vertex from the bag

but we don't know which one we'll get

We don't care how it works *(for now)*

(the bag doesn't forget or change things)



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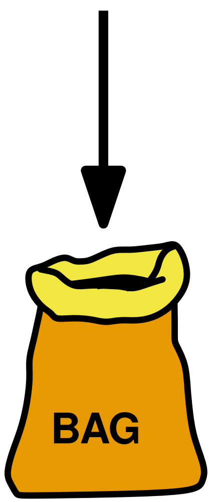
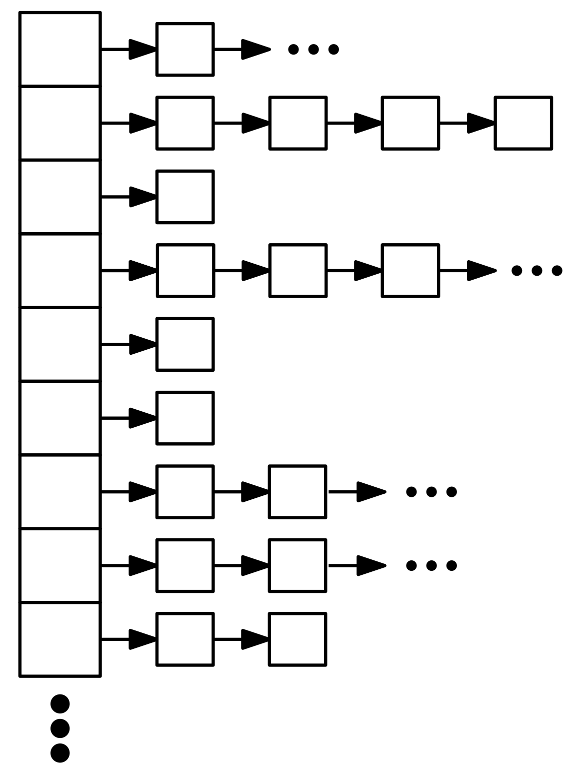


We going to use an adjacency list so the graph is really stored like this:



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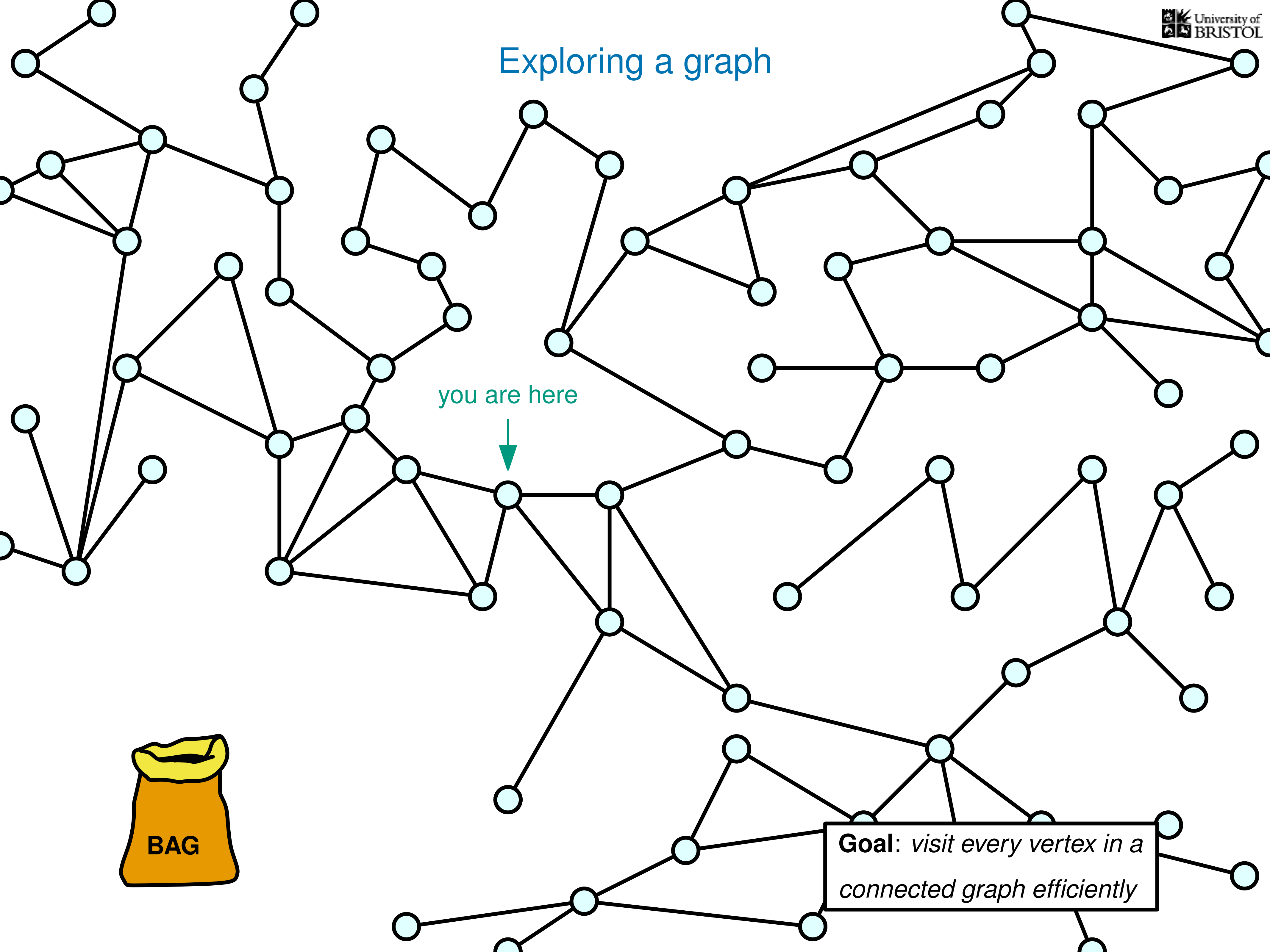

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Exploring a graph

TRAVERSE(s)

```
put s into the bag
while the bag is not empty
  take u from the bag
  if u unmarked
    mark u
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```

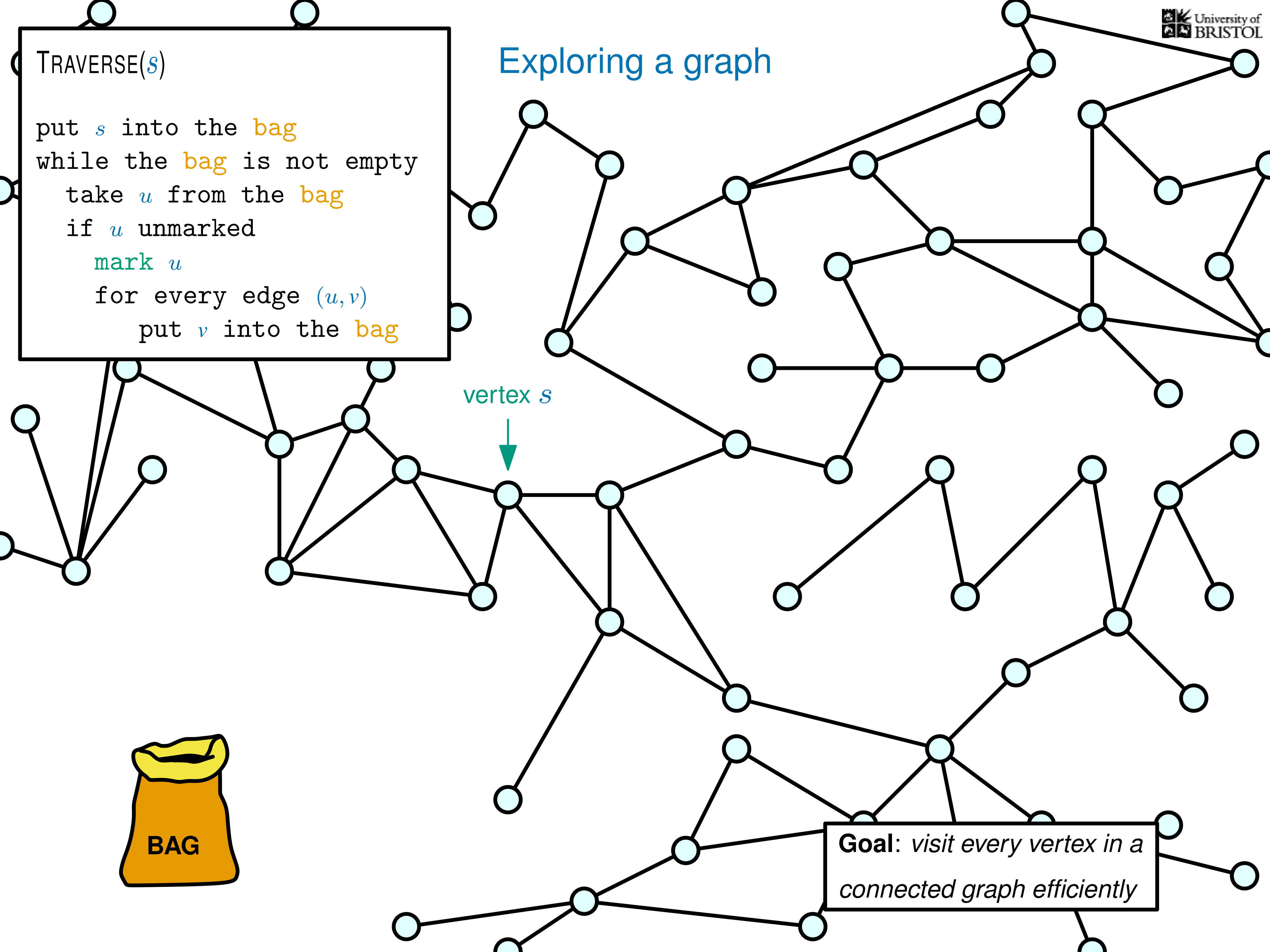

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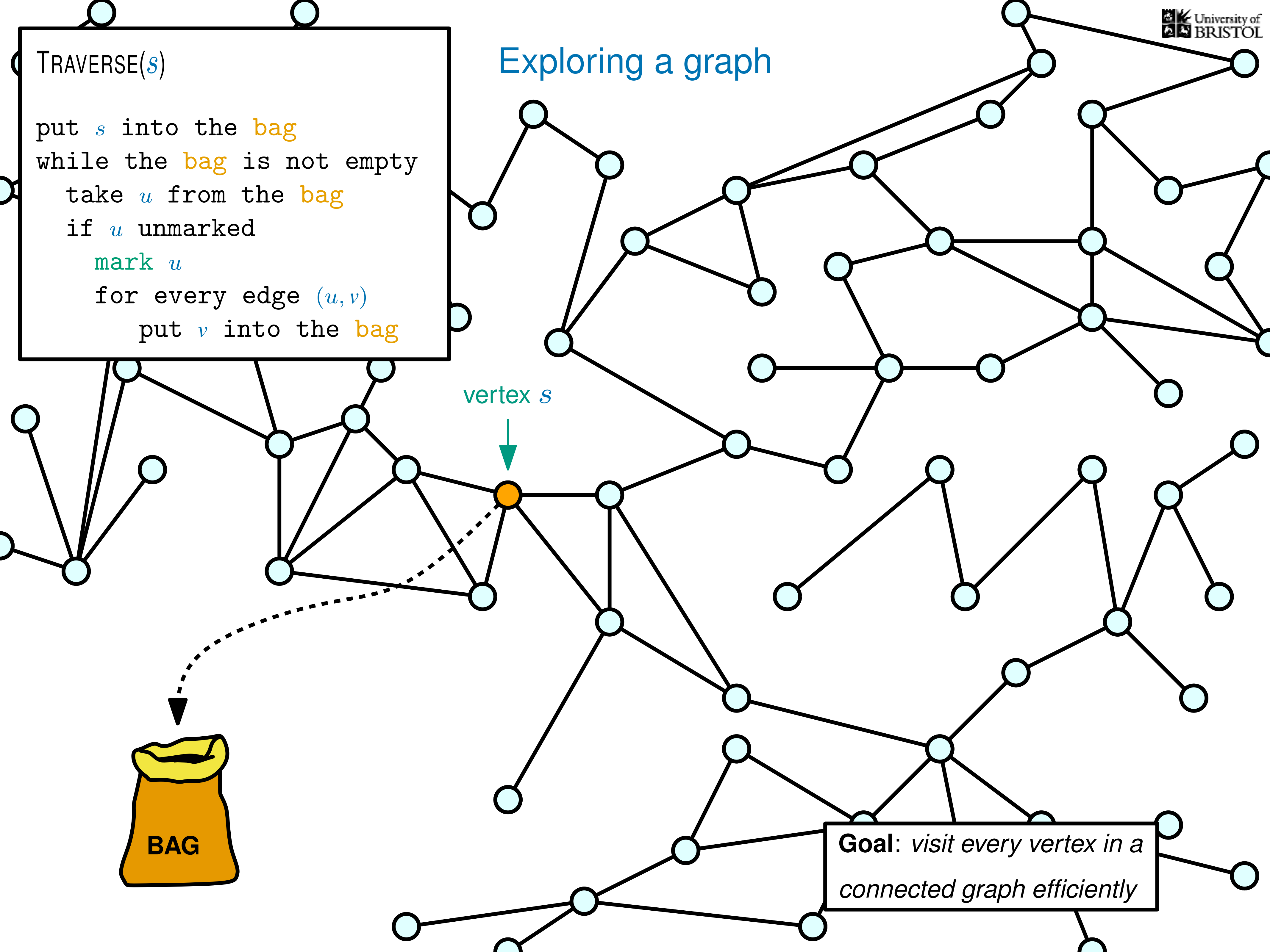

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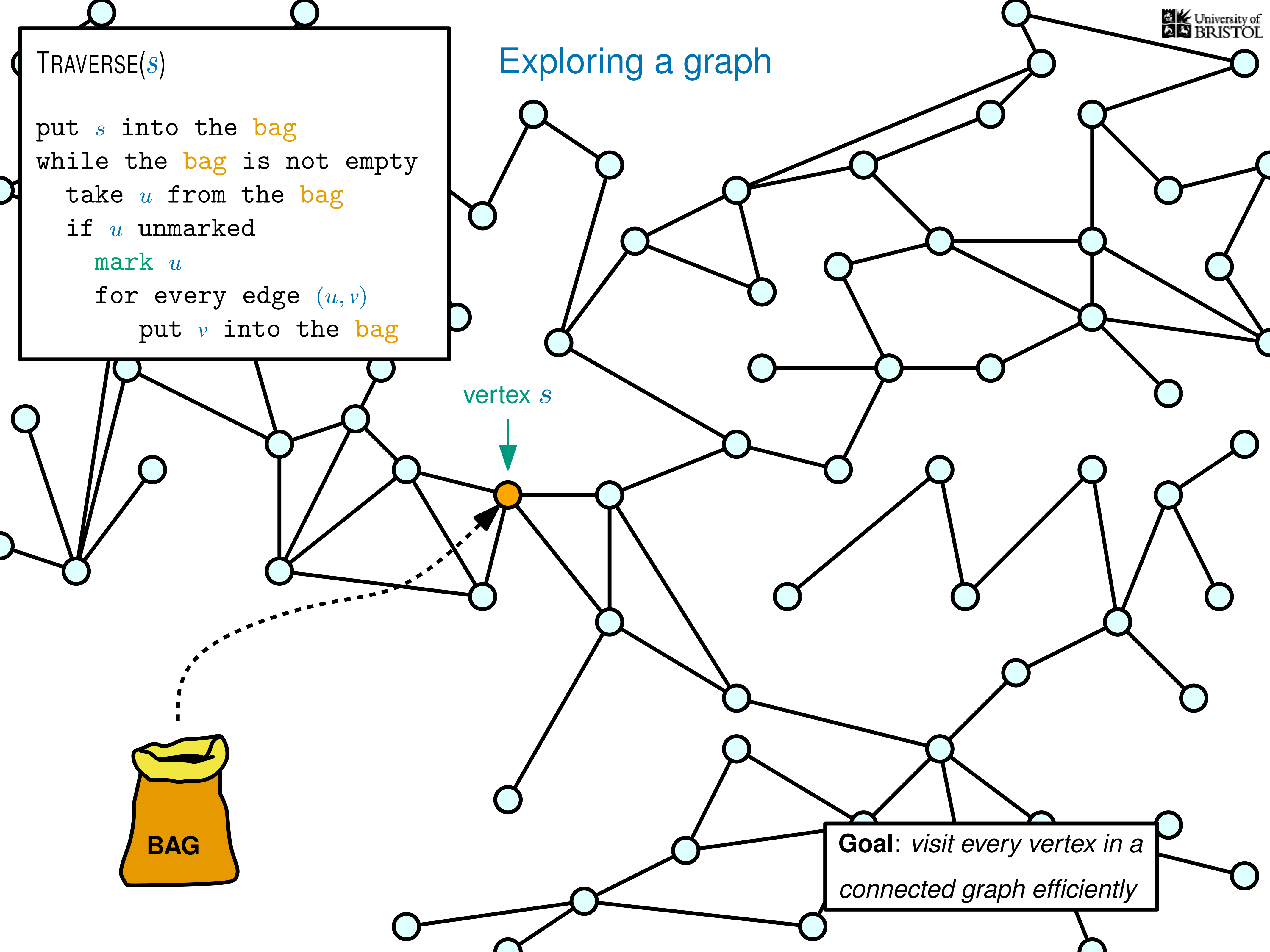

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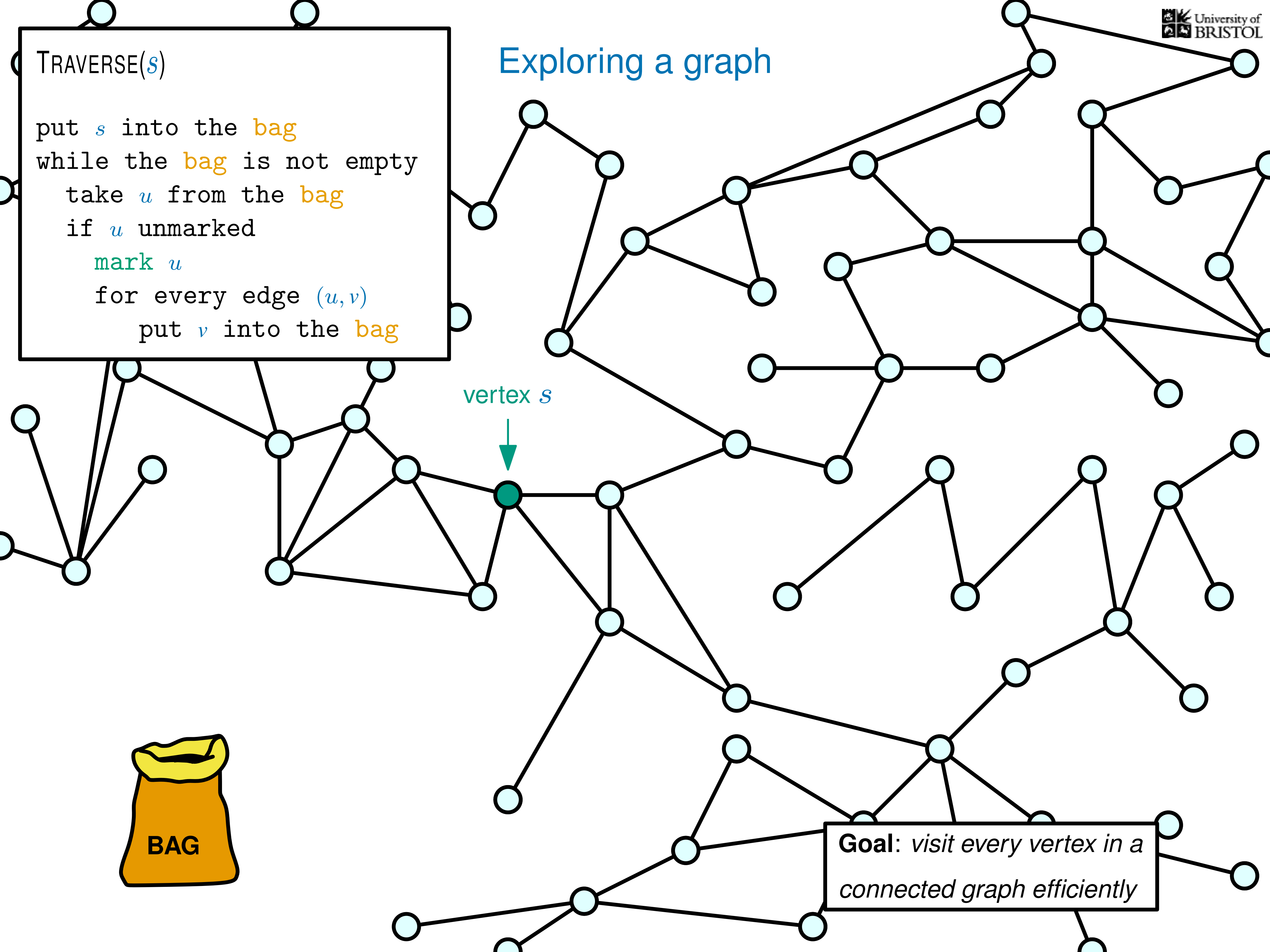

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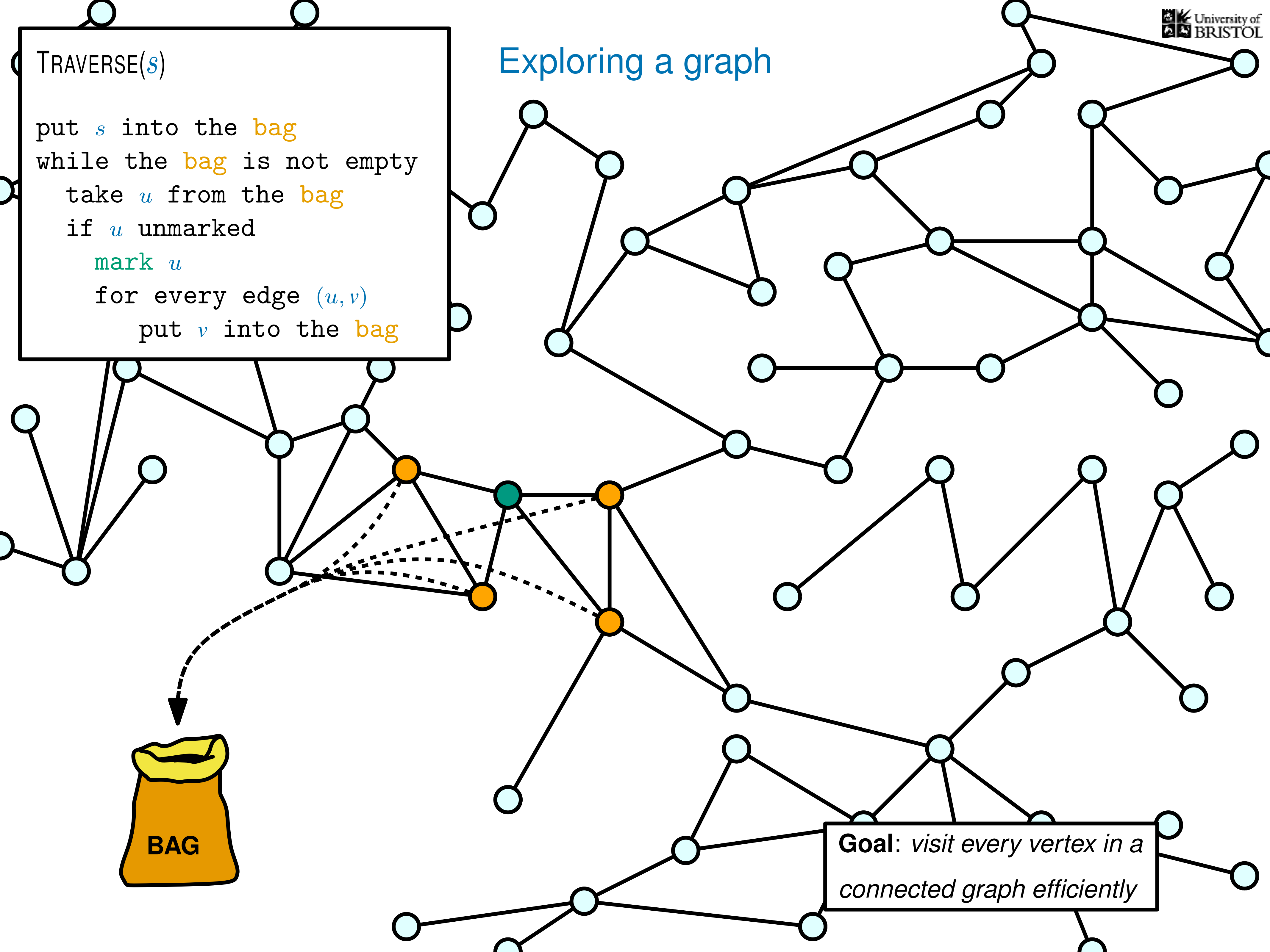

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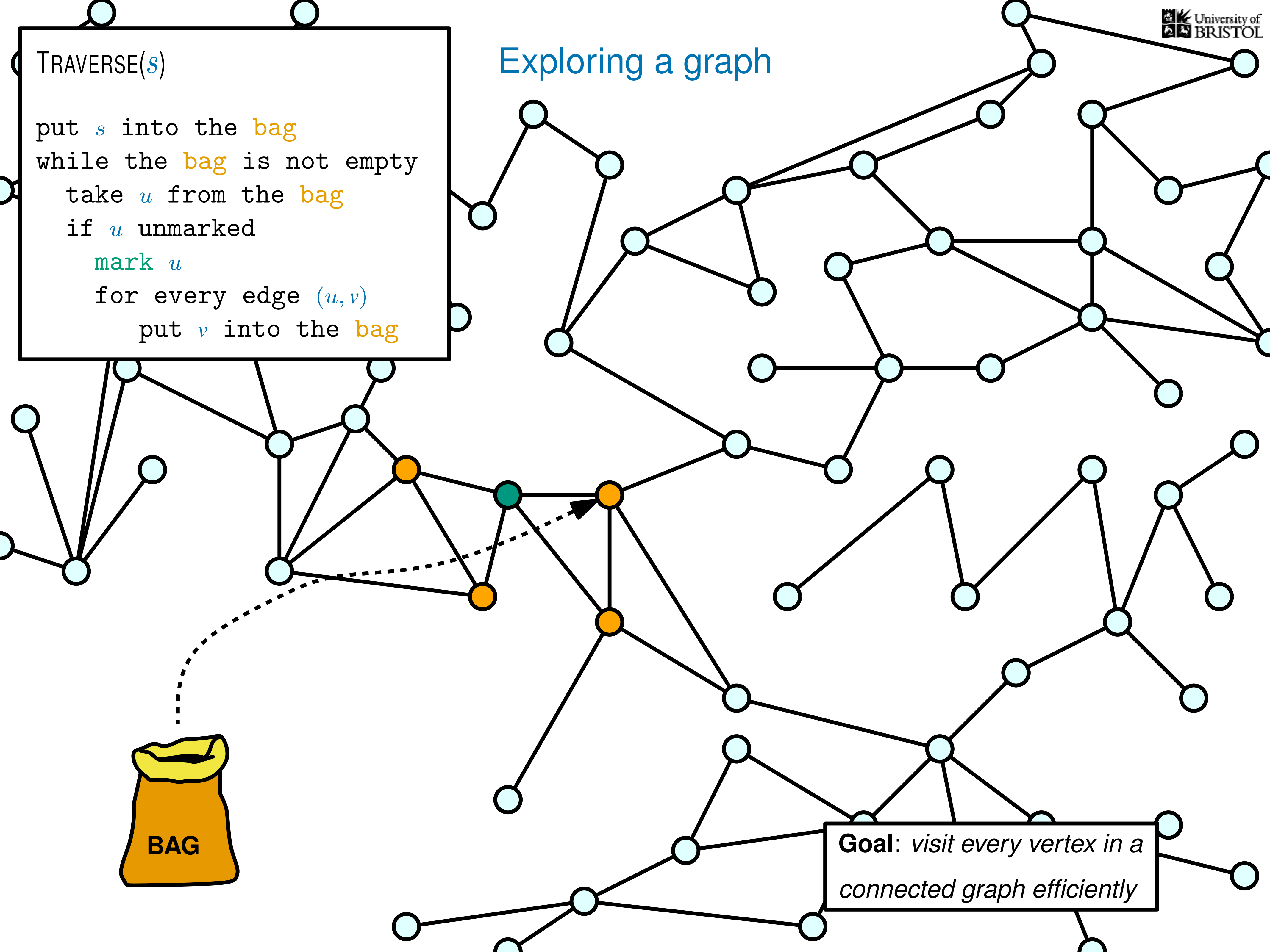

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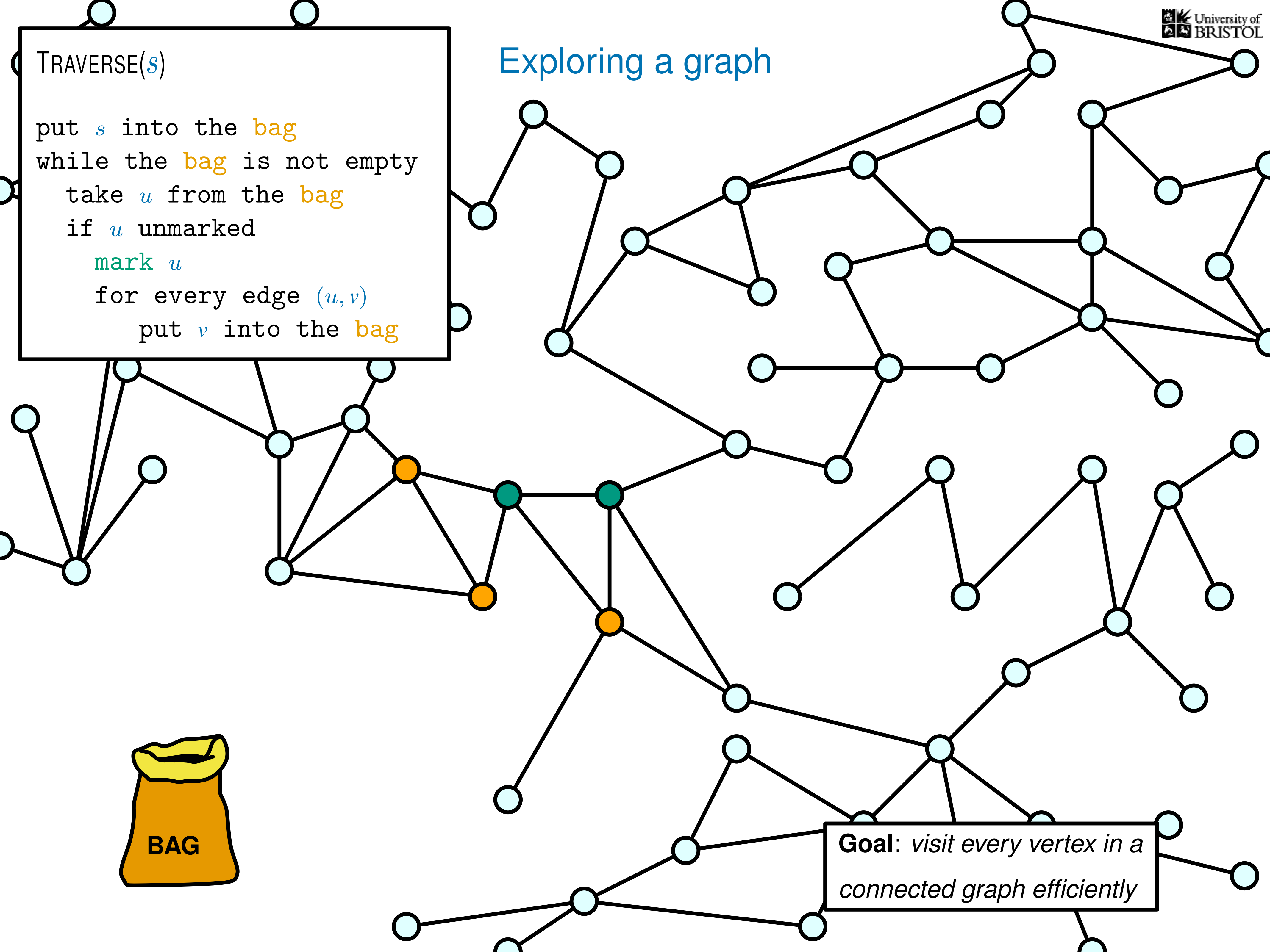

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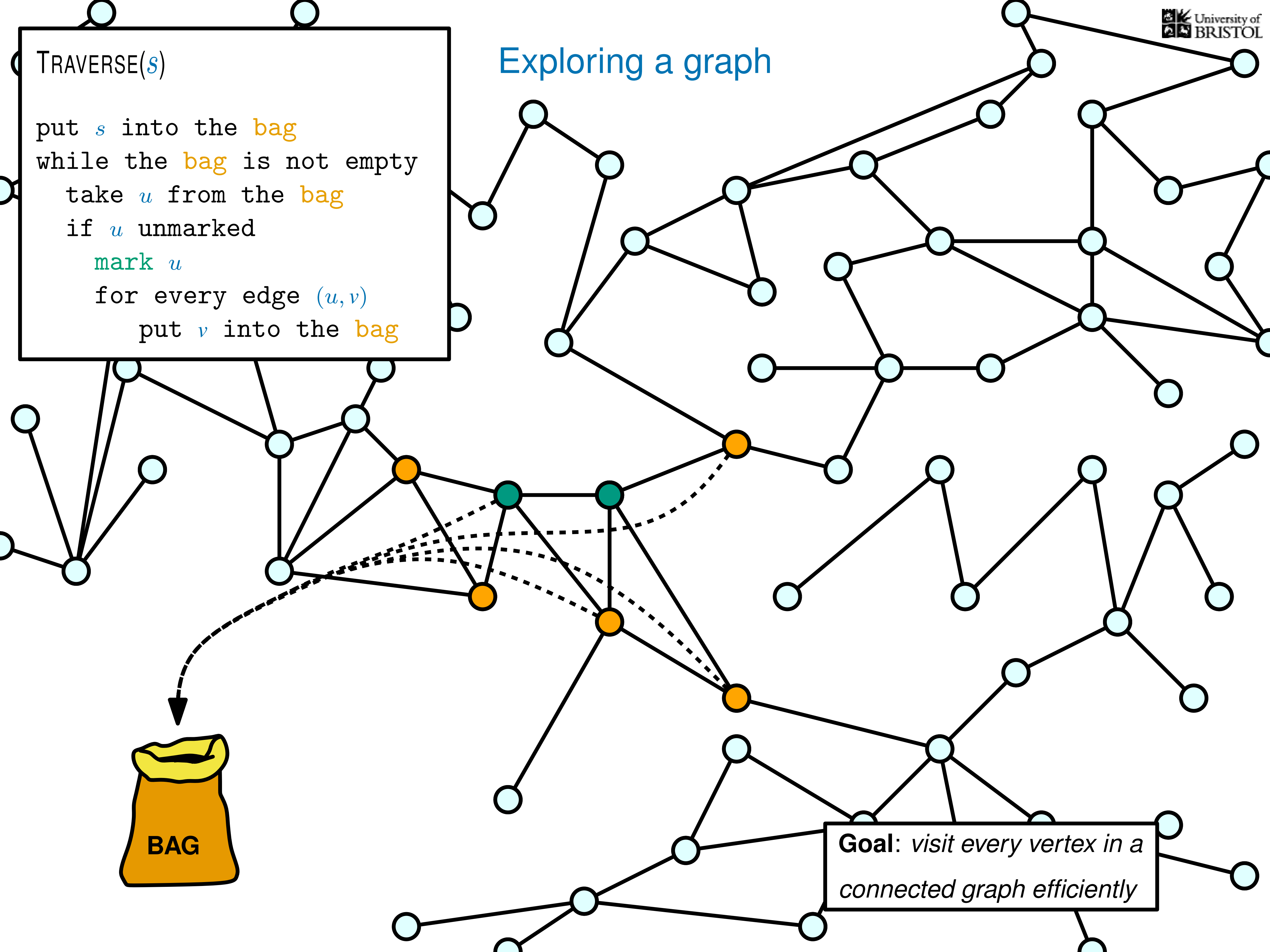

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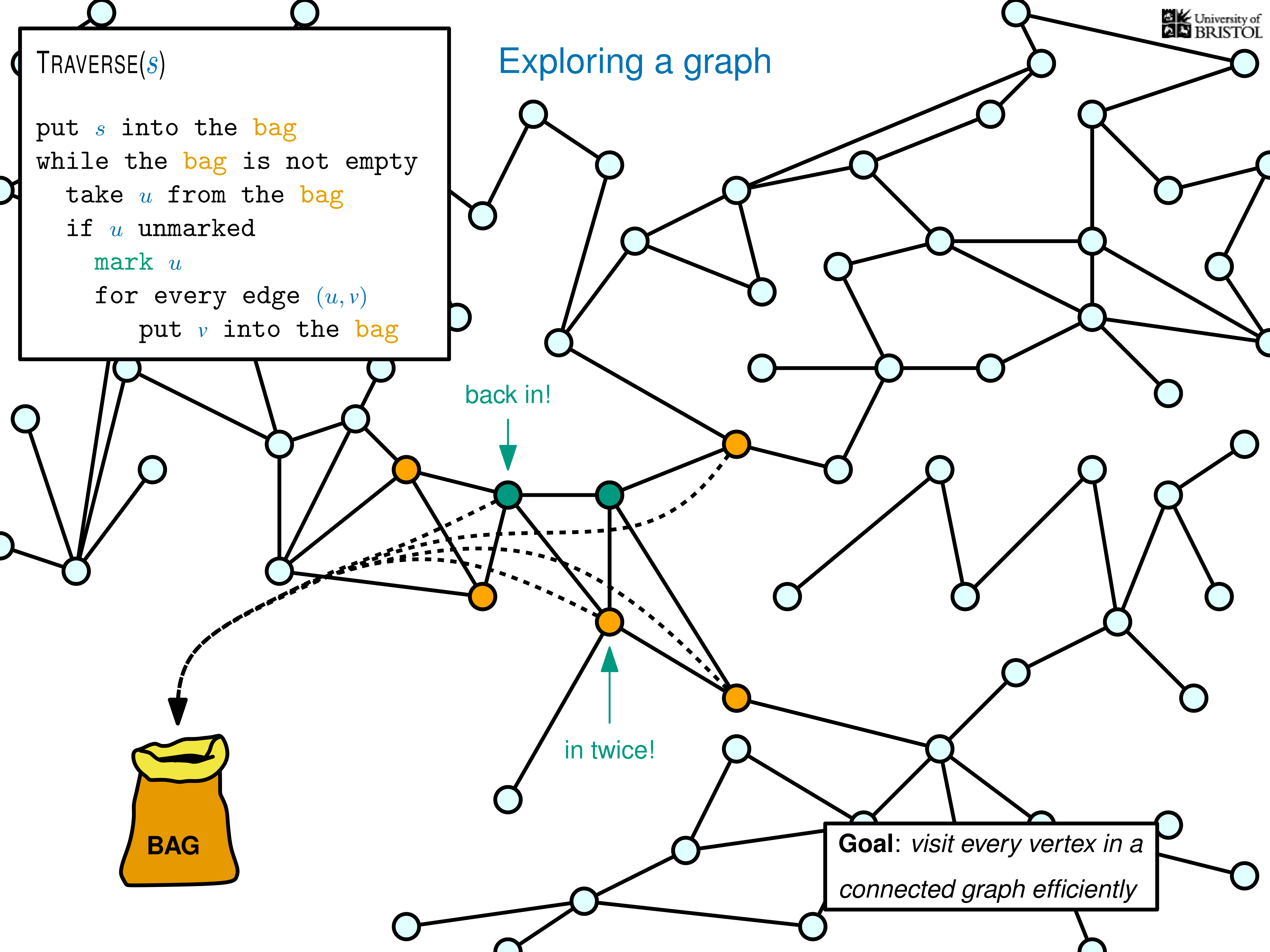

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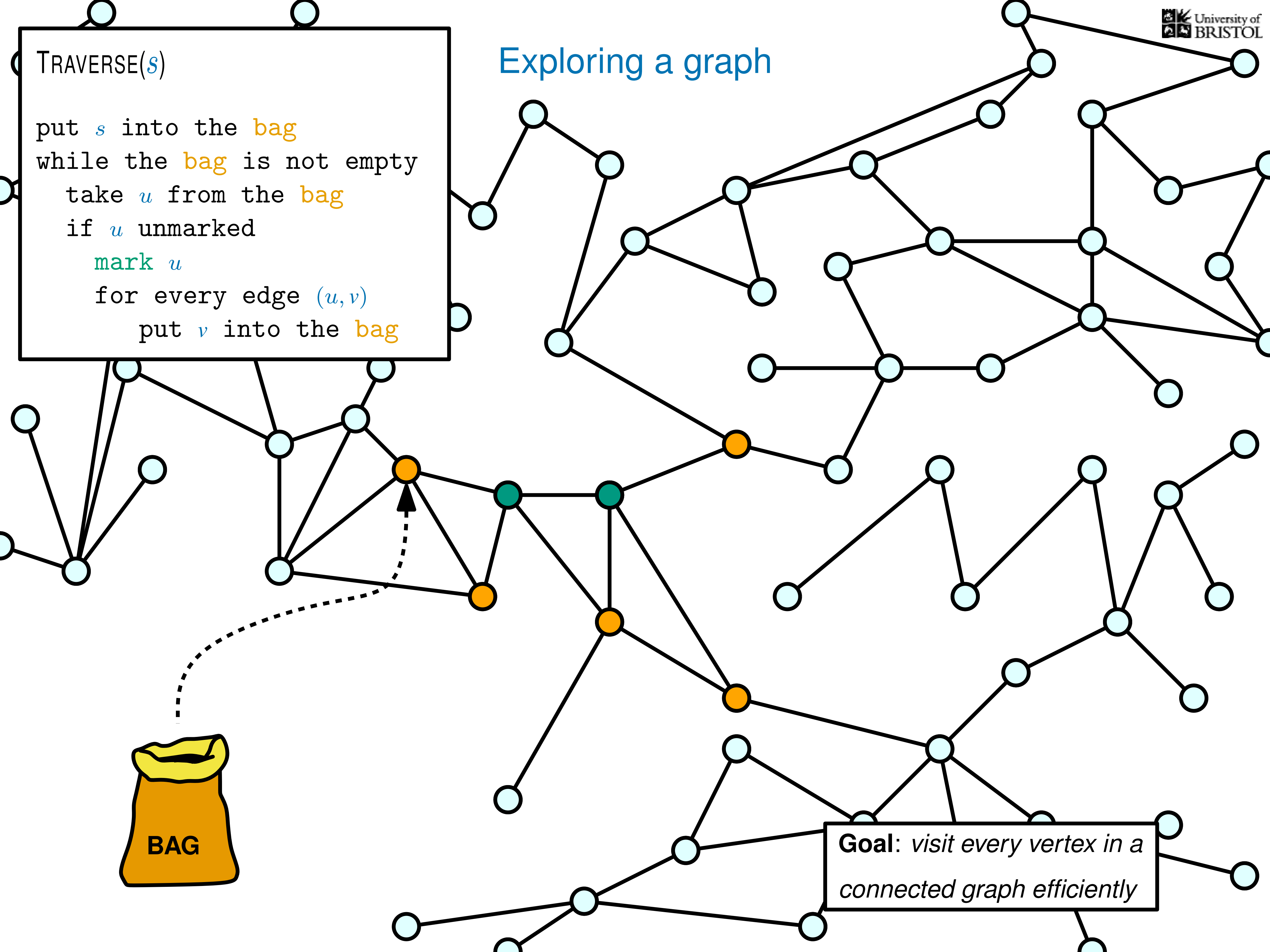

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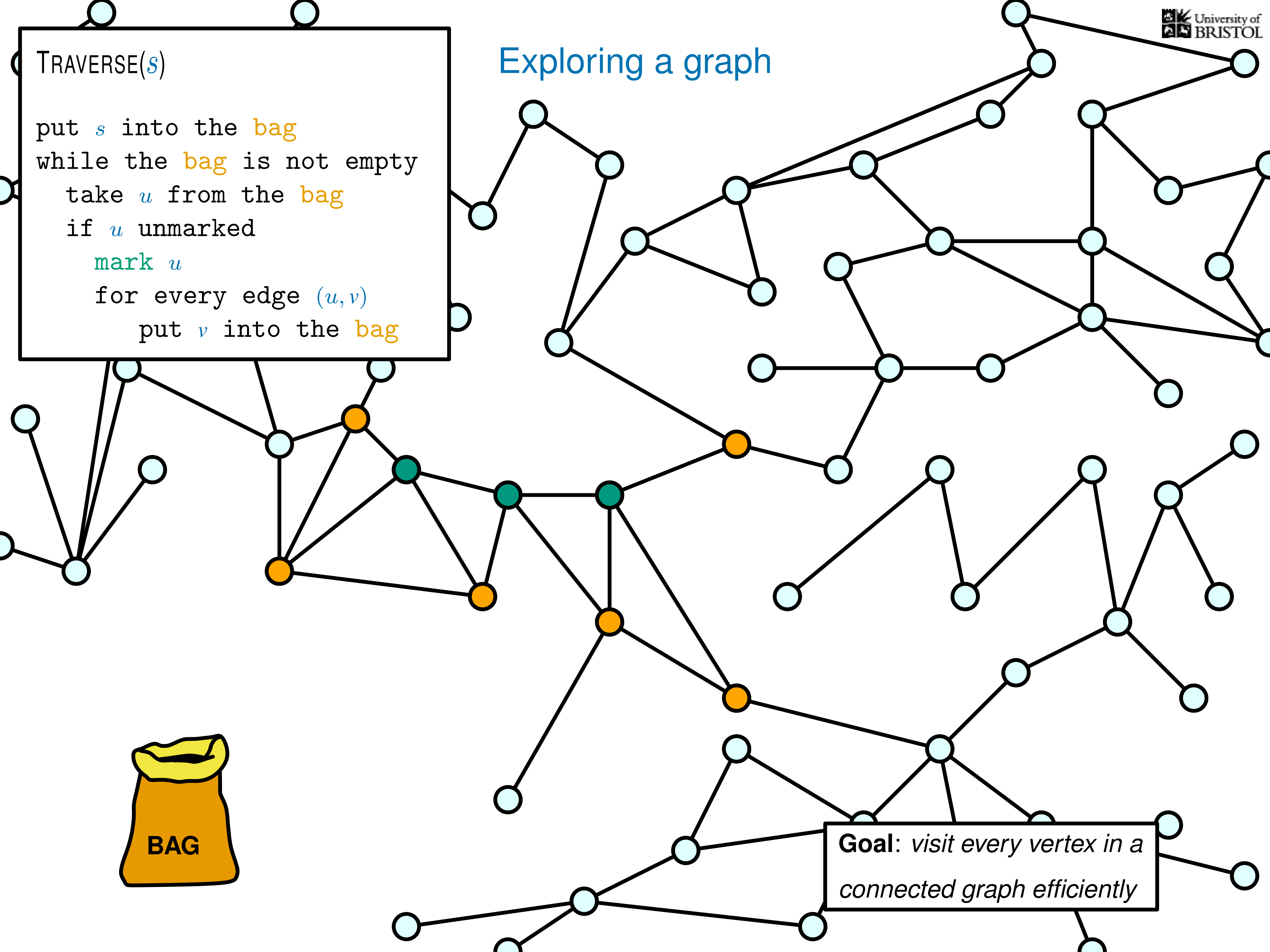

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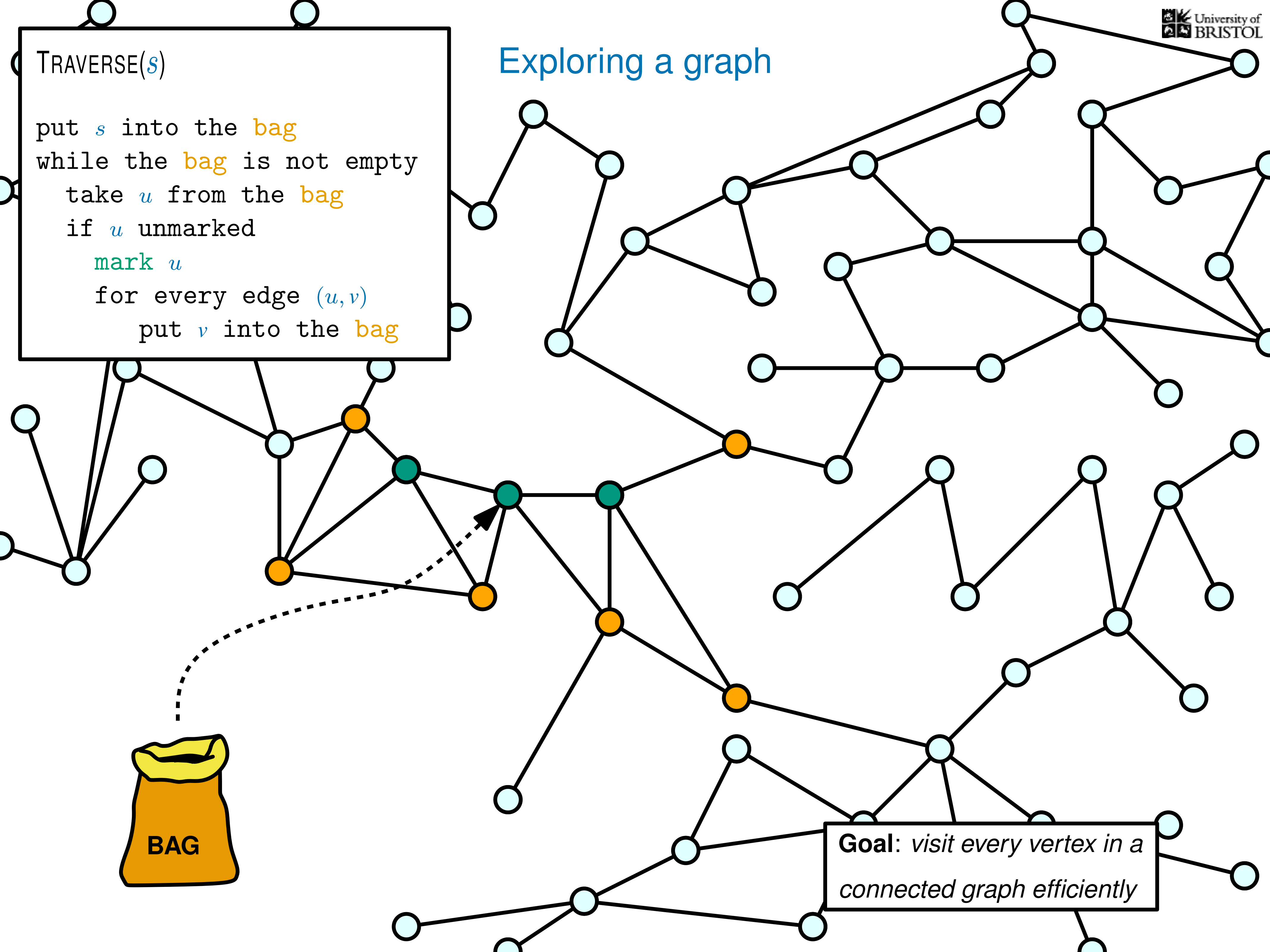

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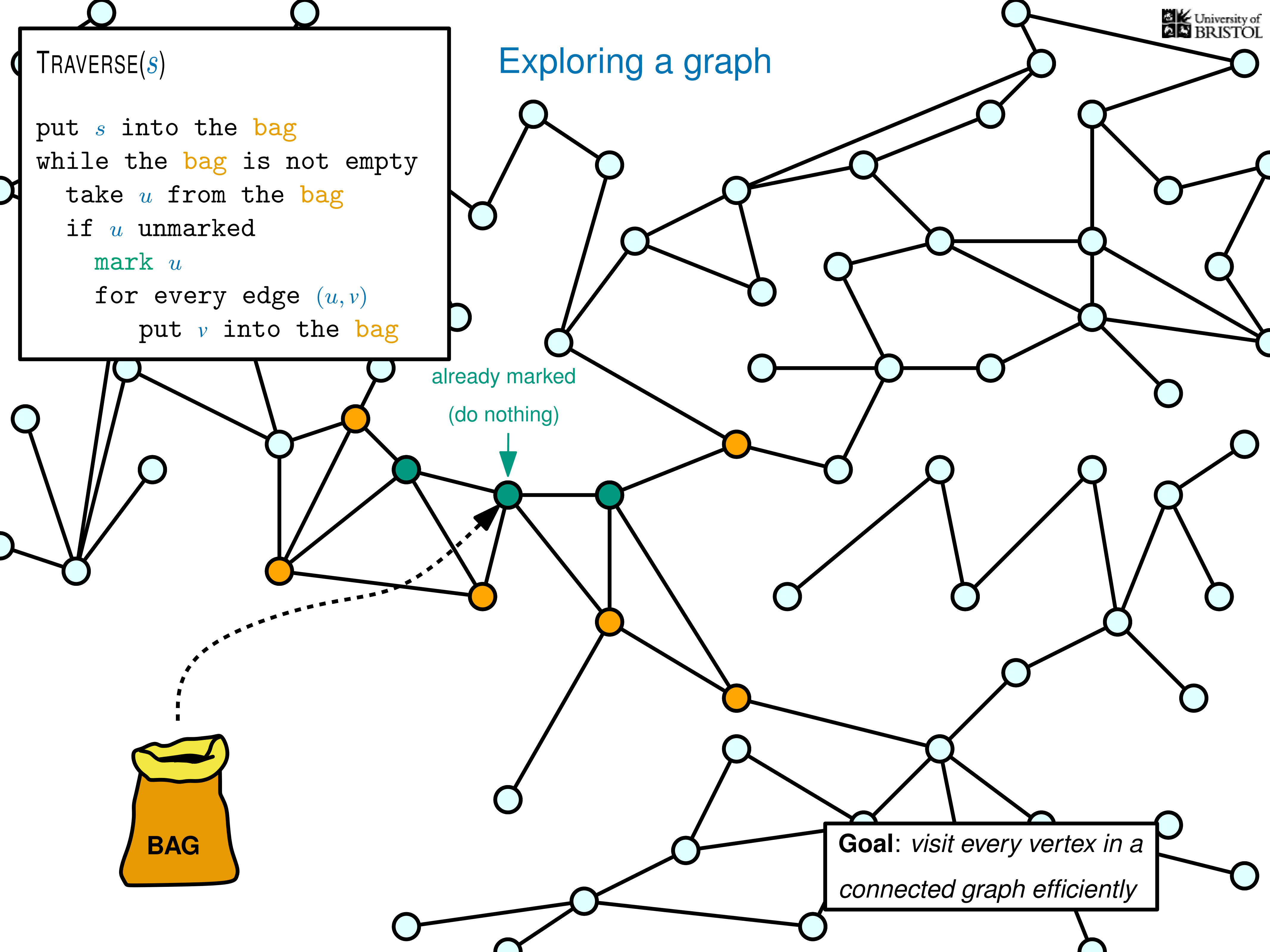

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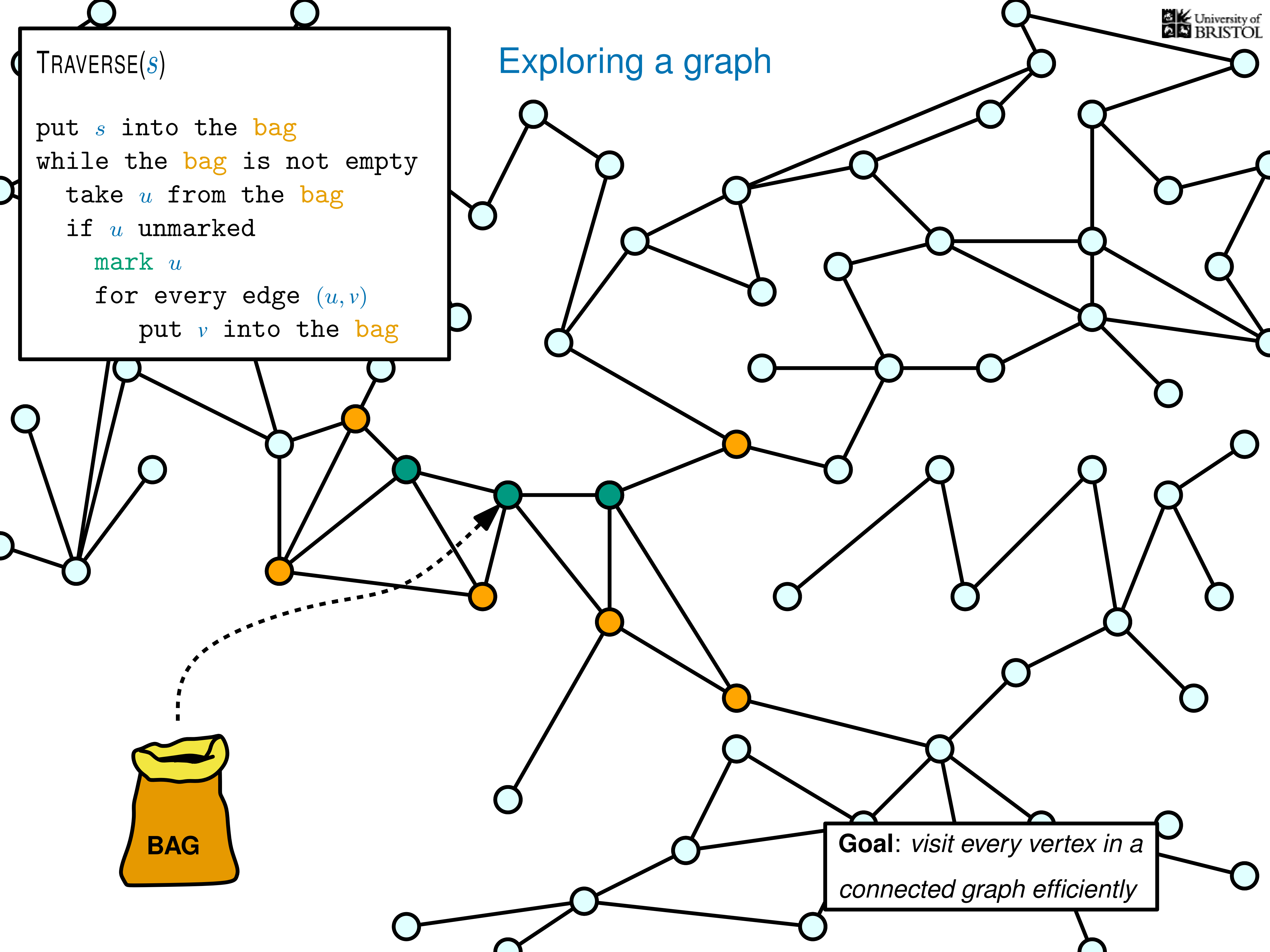

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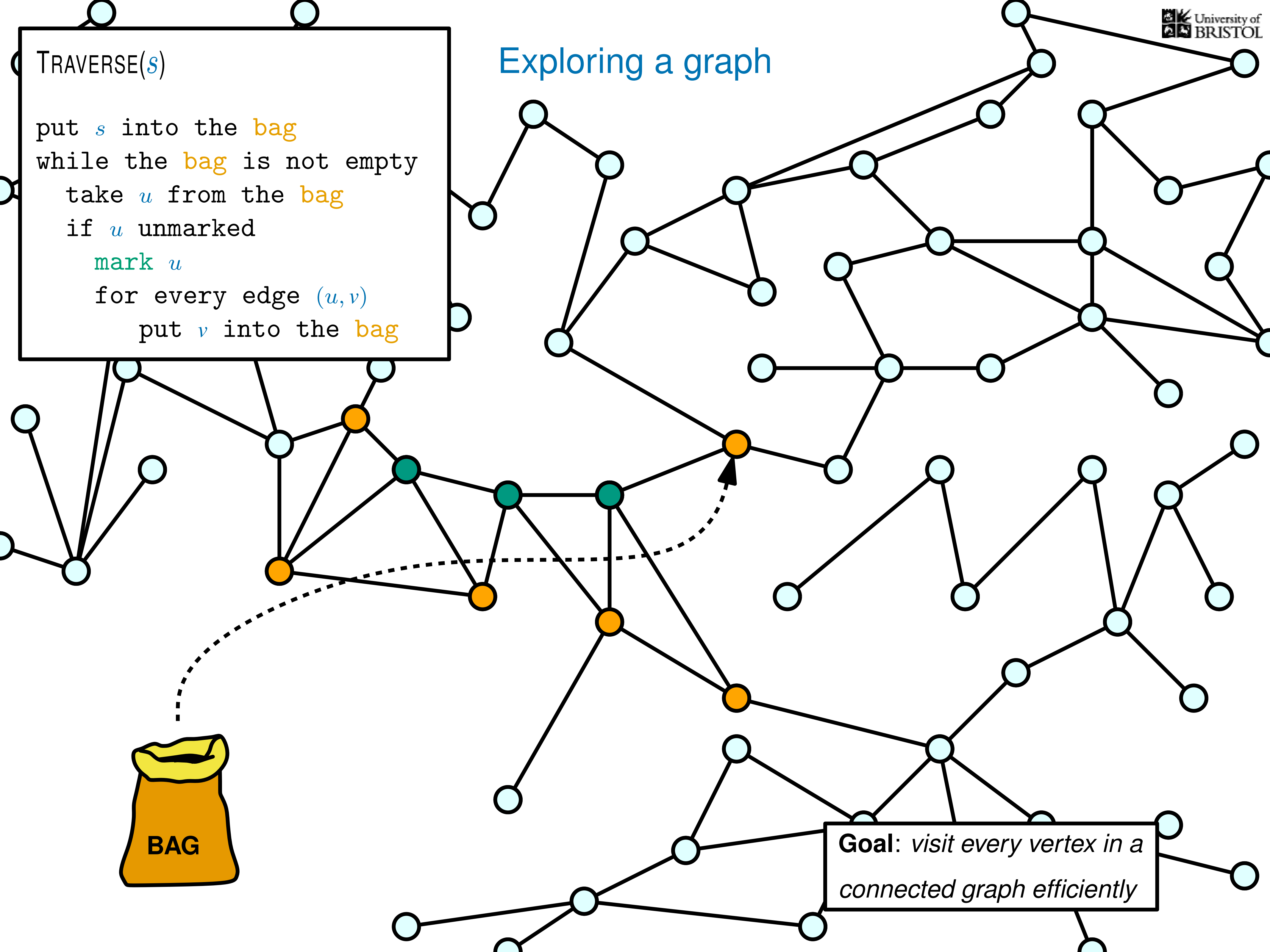

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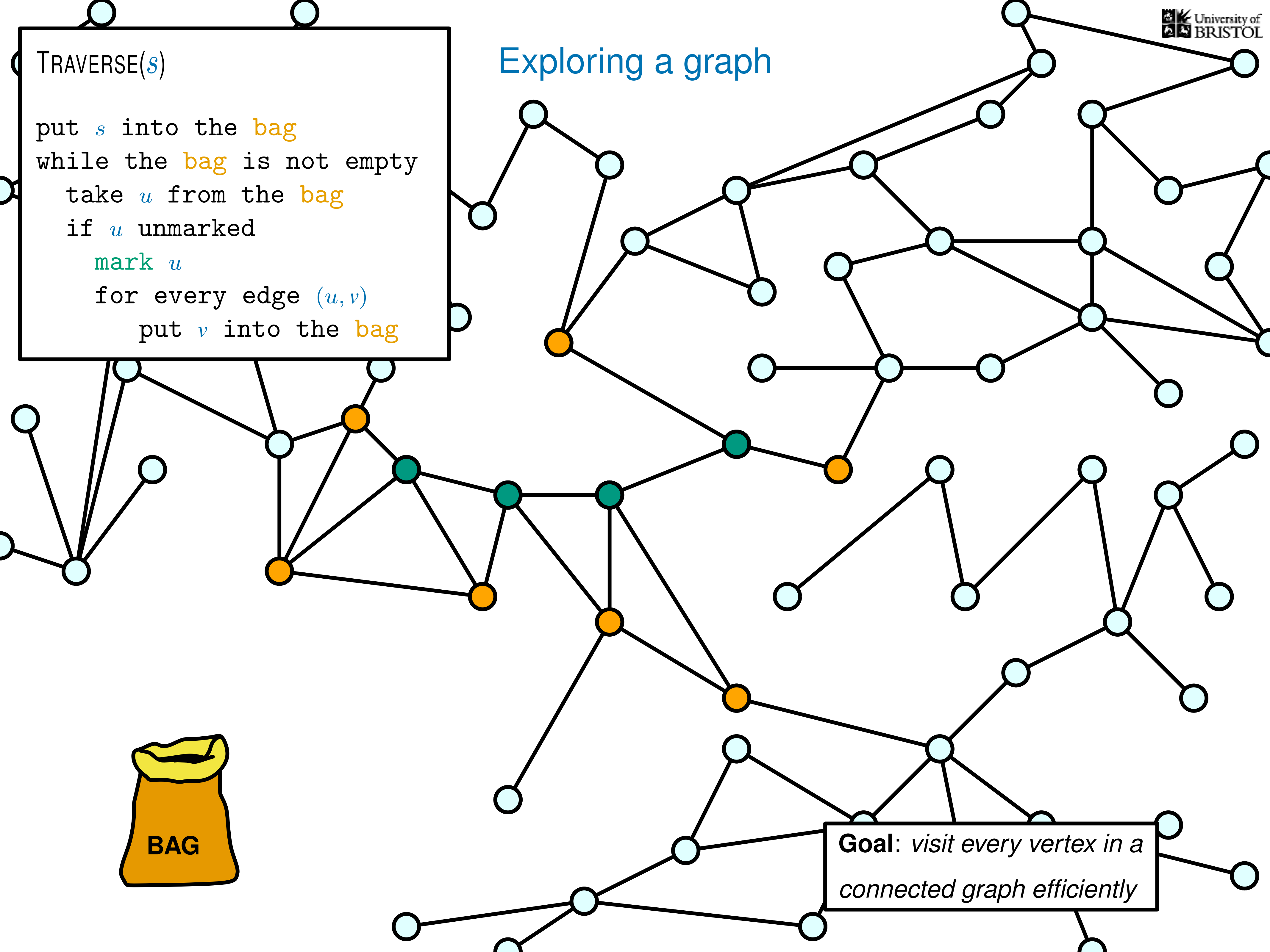

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University of BRISTOL

Exploring a graph

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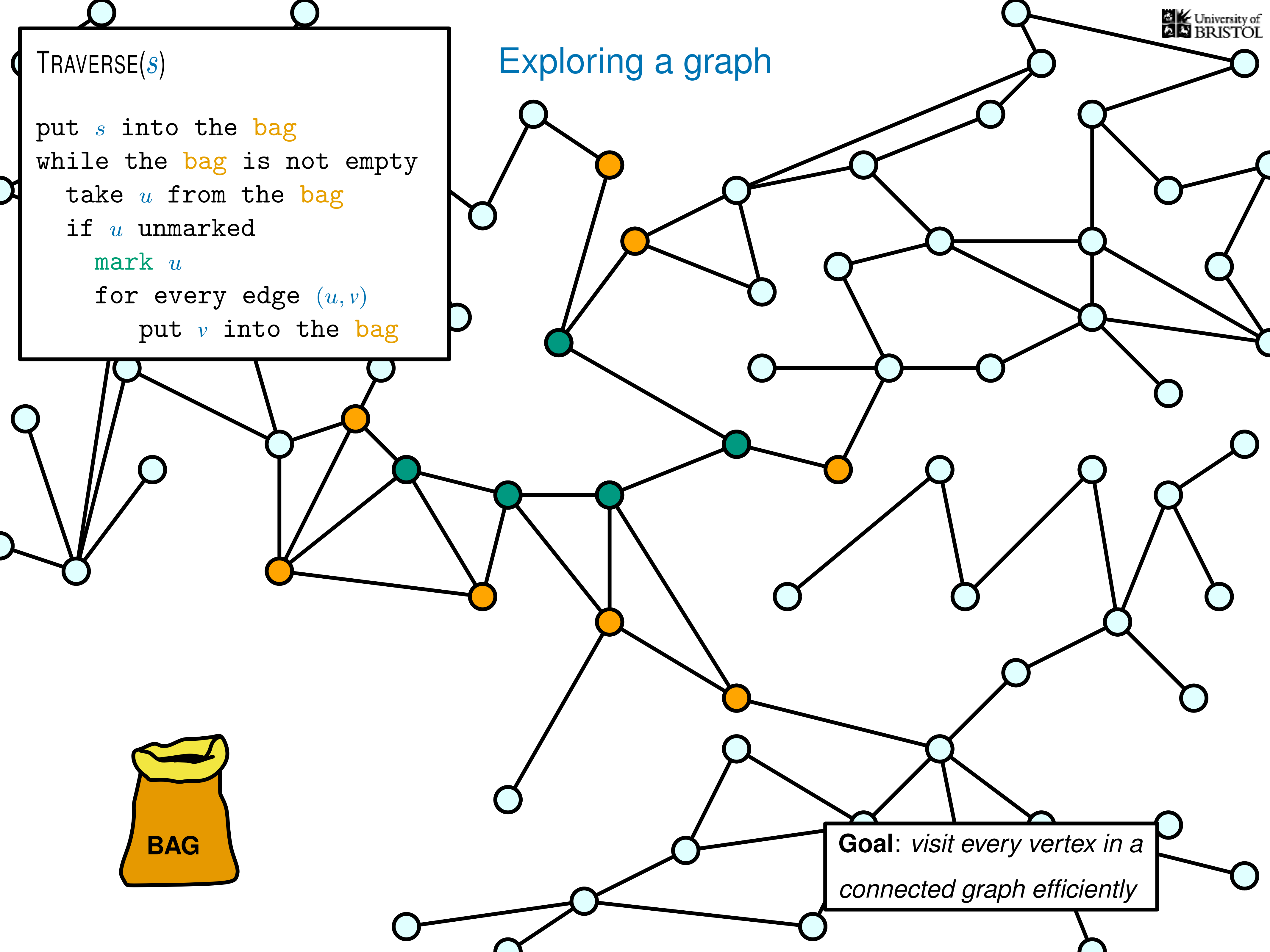

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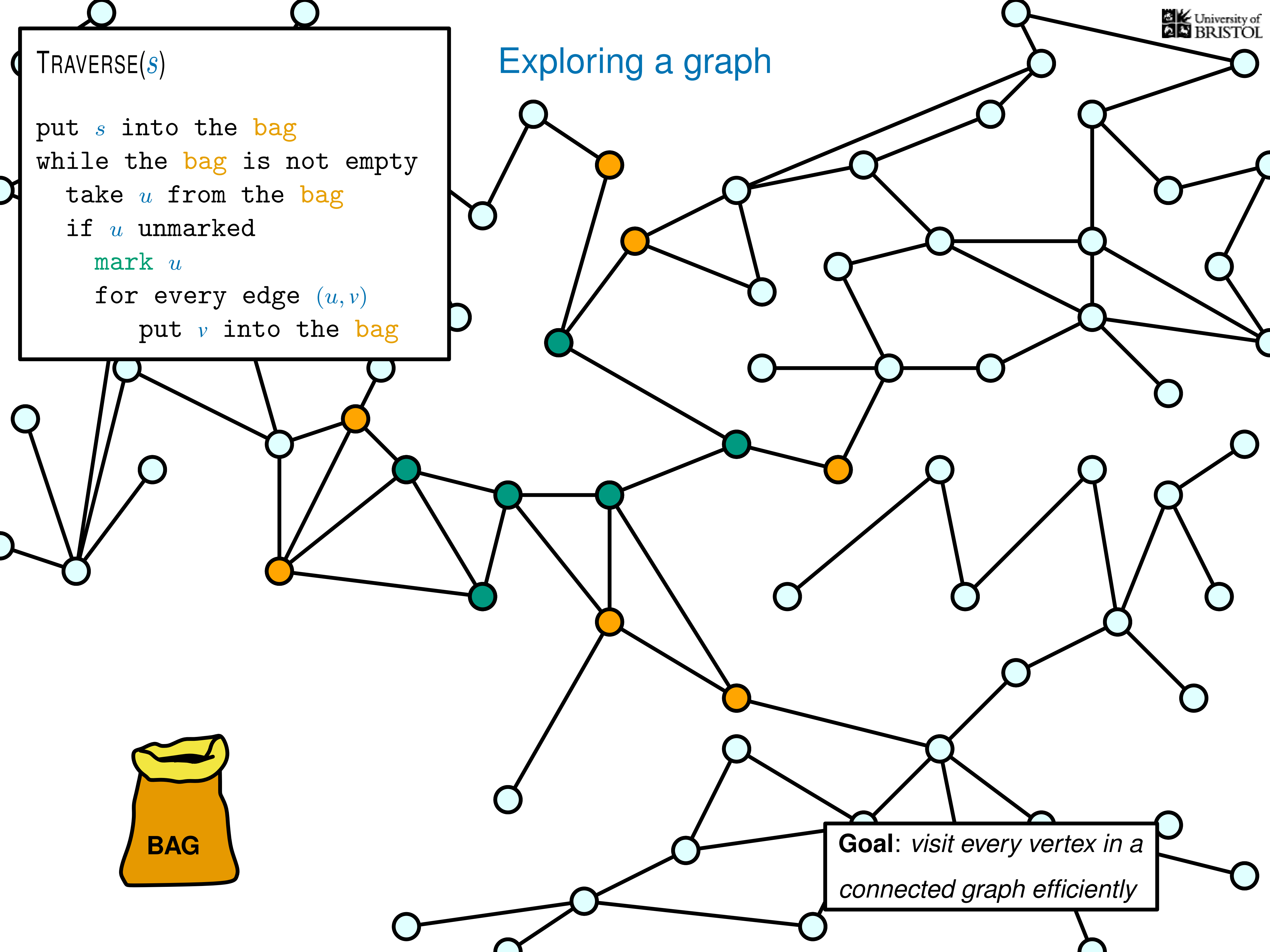

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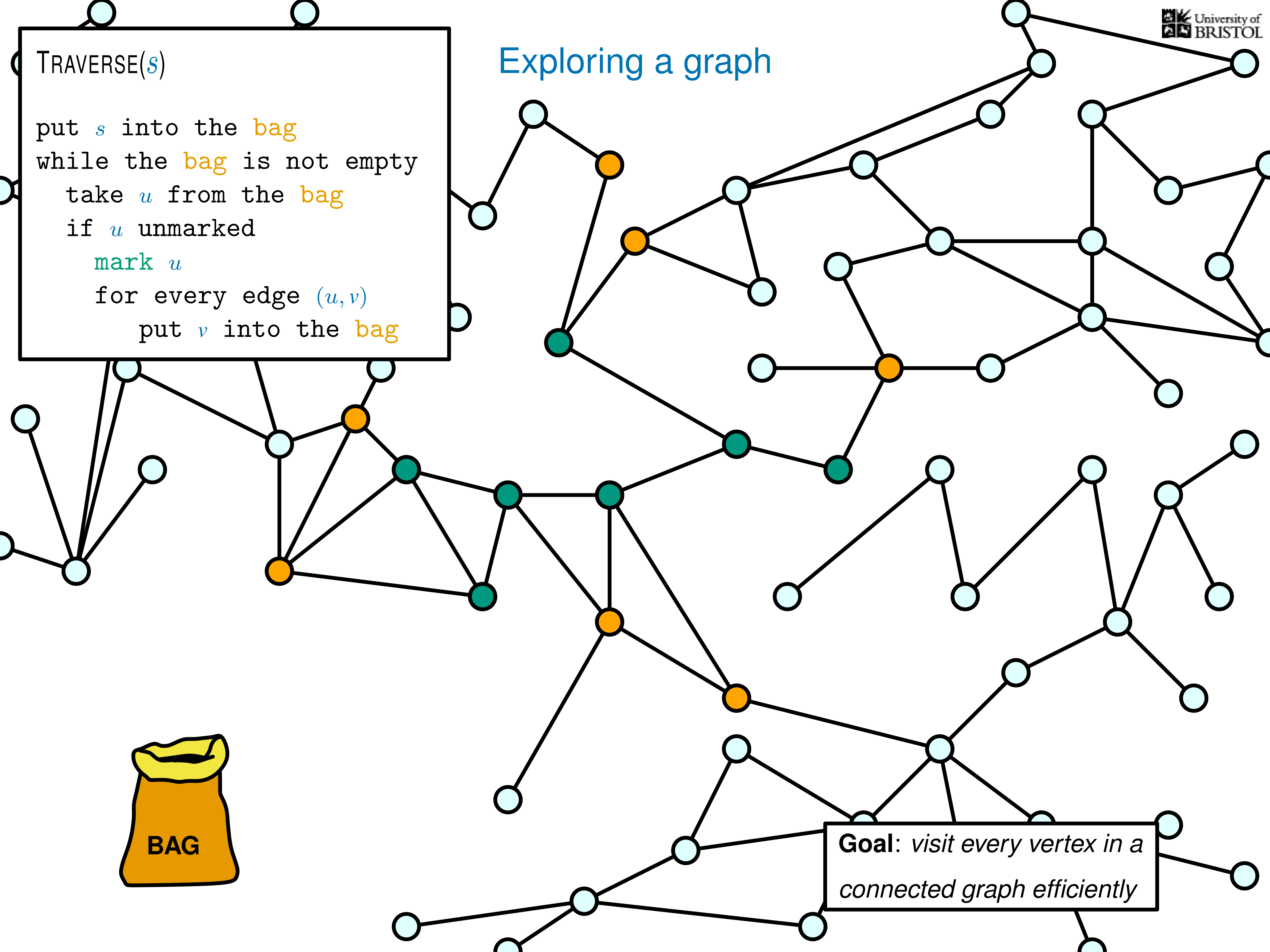

Goal: *visit every vertex in a connected graph efficiently*

Exploring a graph

Traverse(s)

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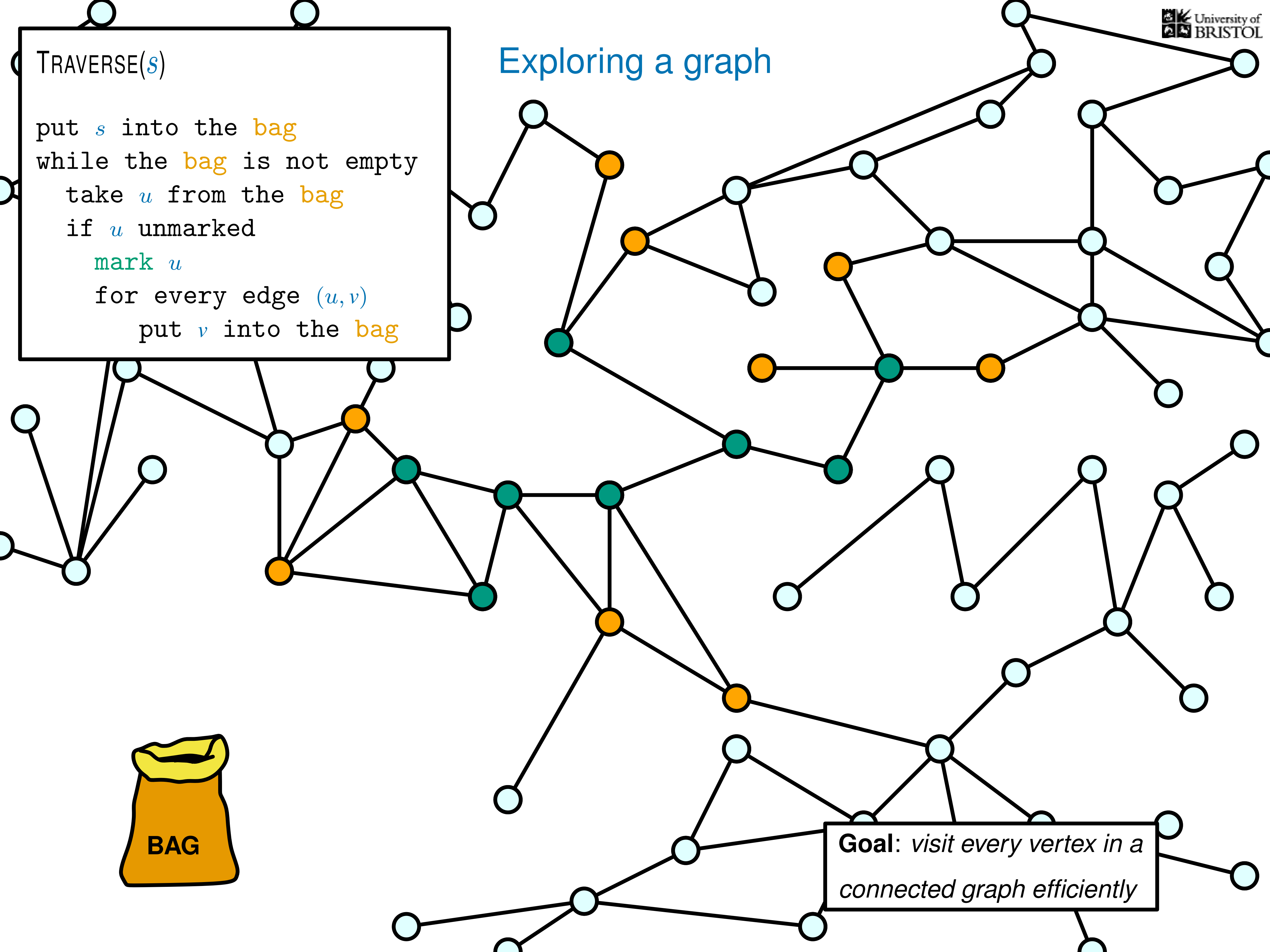

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Exploring a graph

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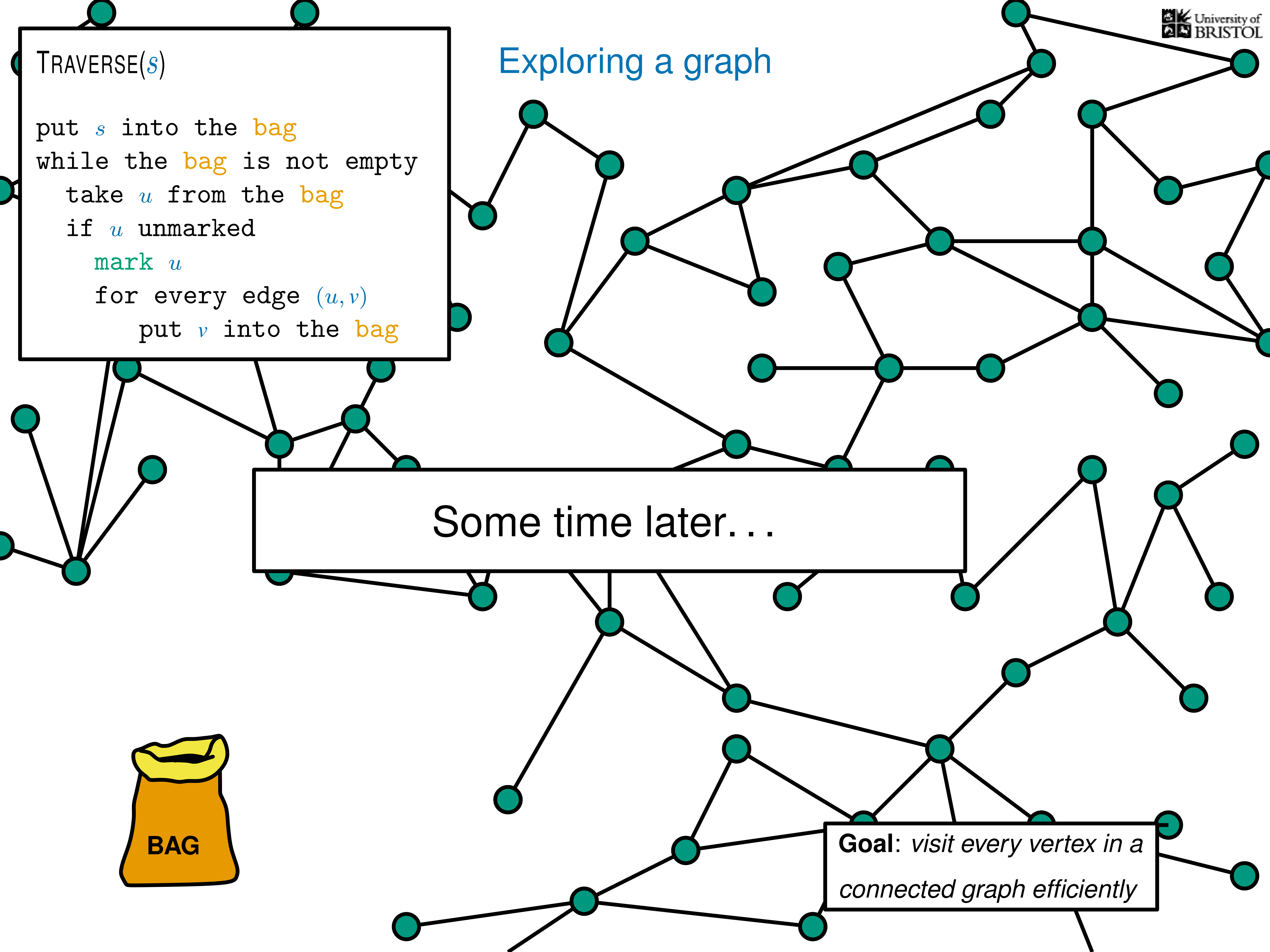

Some time later...

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Exploring a graph

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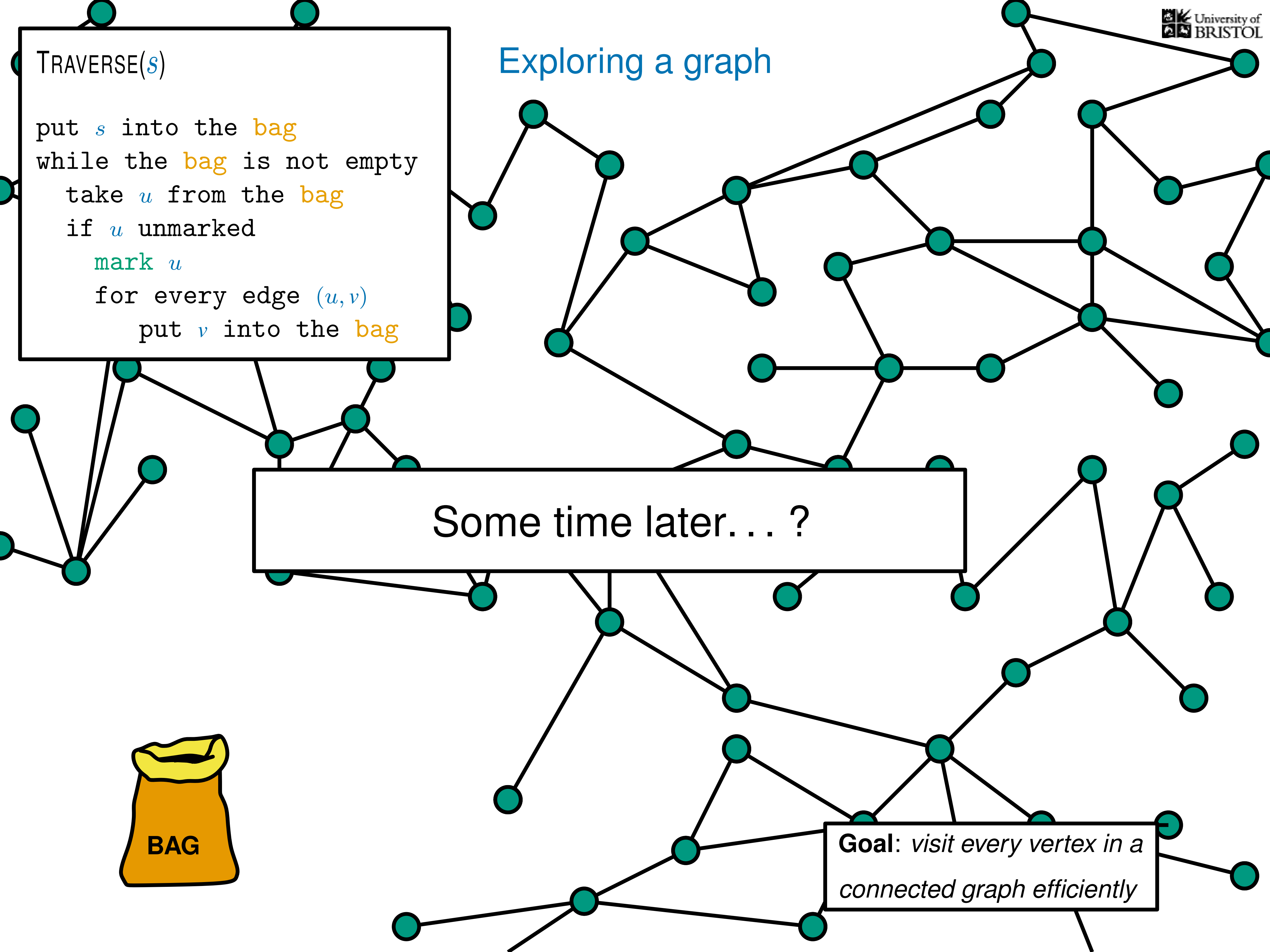

Goal: *visit every vertex in a connected graph efficiently*

Exploring a graph

TRAVERSE(s)

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Questions:

1. *Does* TRAVERSE *always stop?*
2. *Does it always visit every vertex?*
3. *How fast is it? (in the worst case)*

Some time later. . . ?

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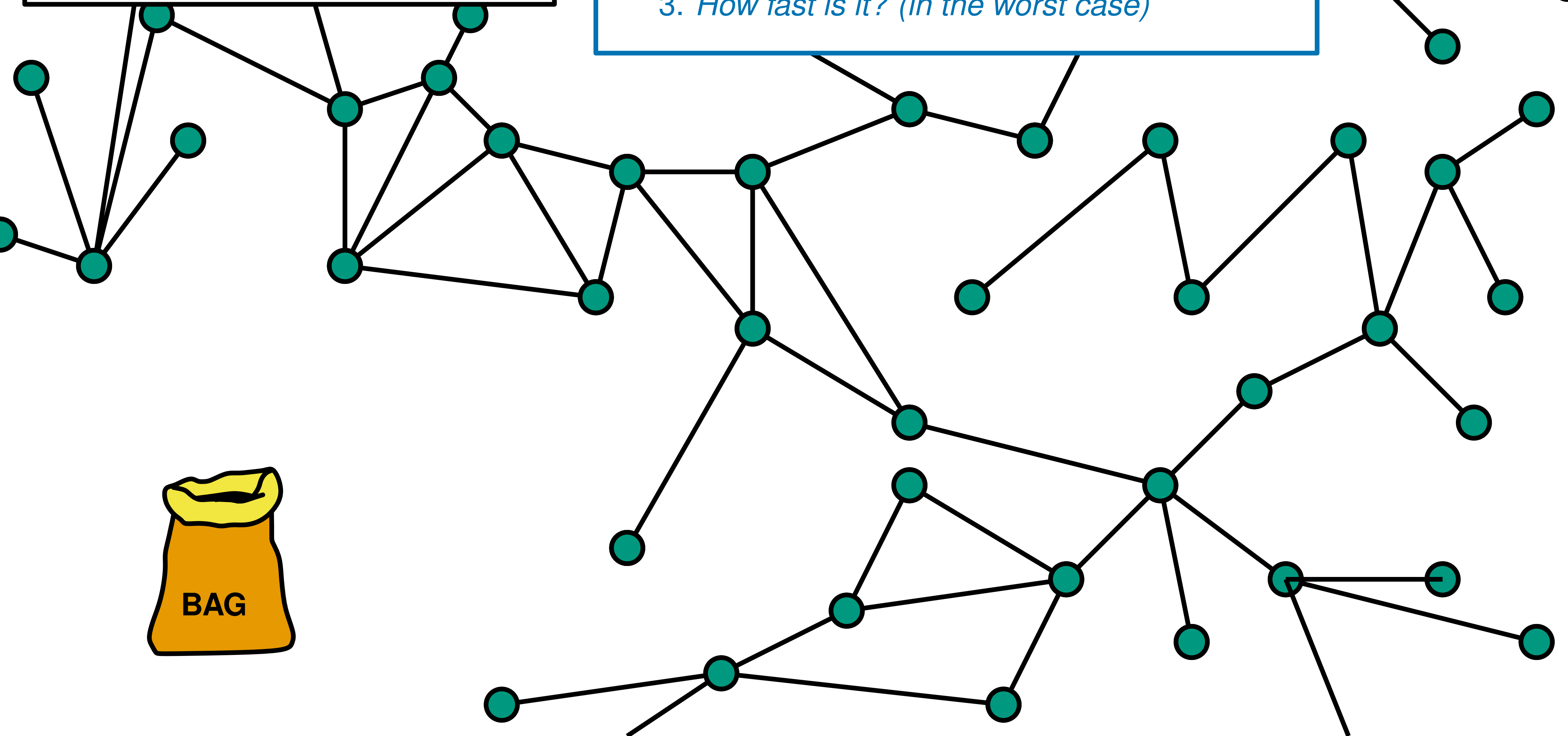

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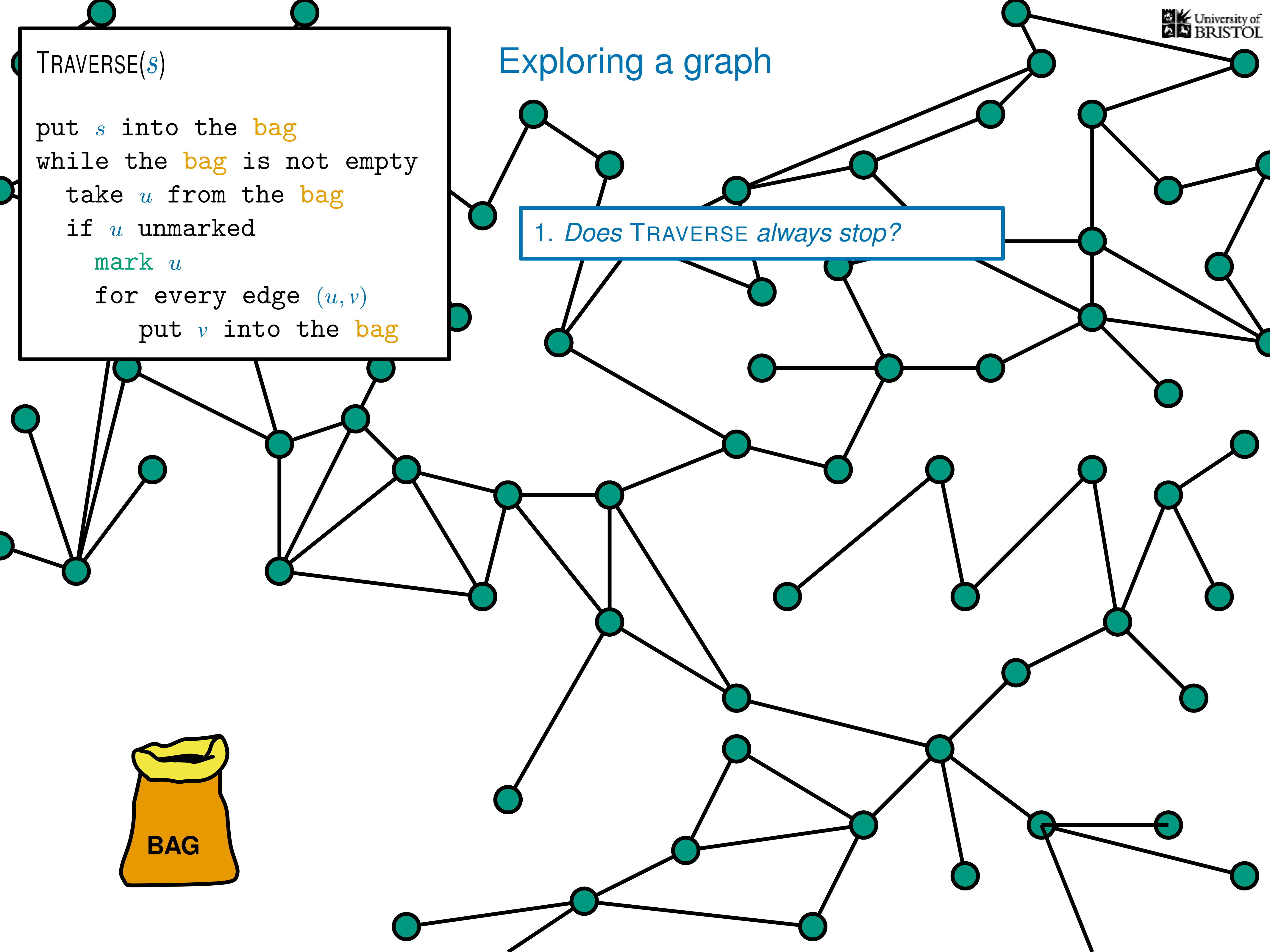

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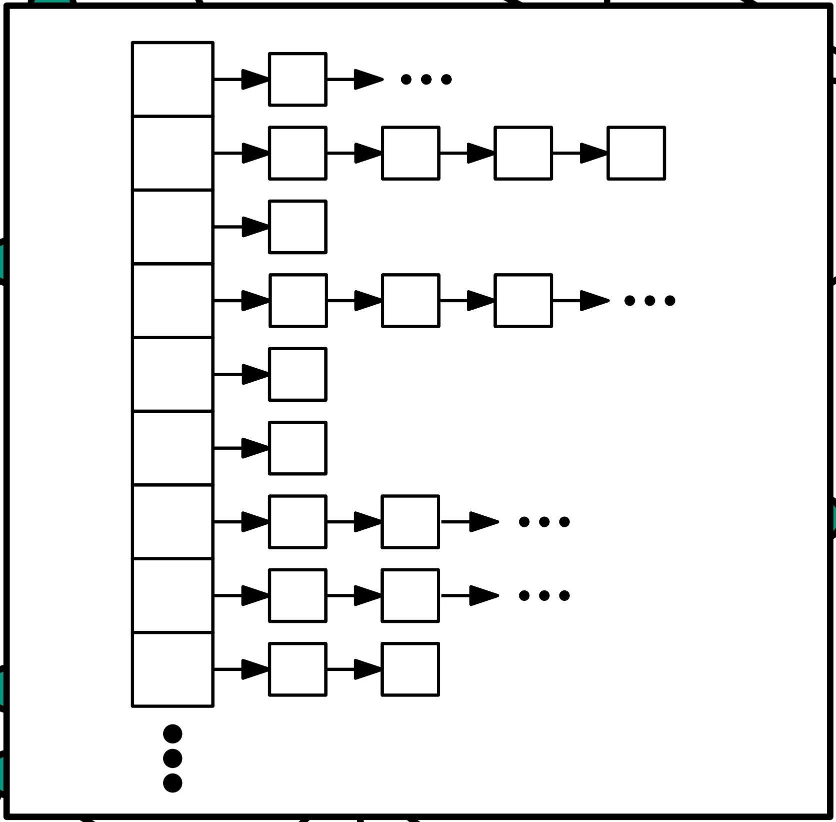

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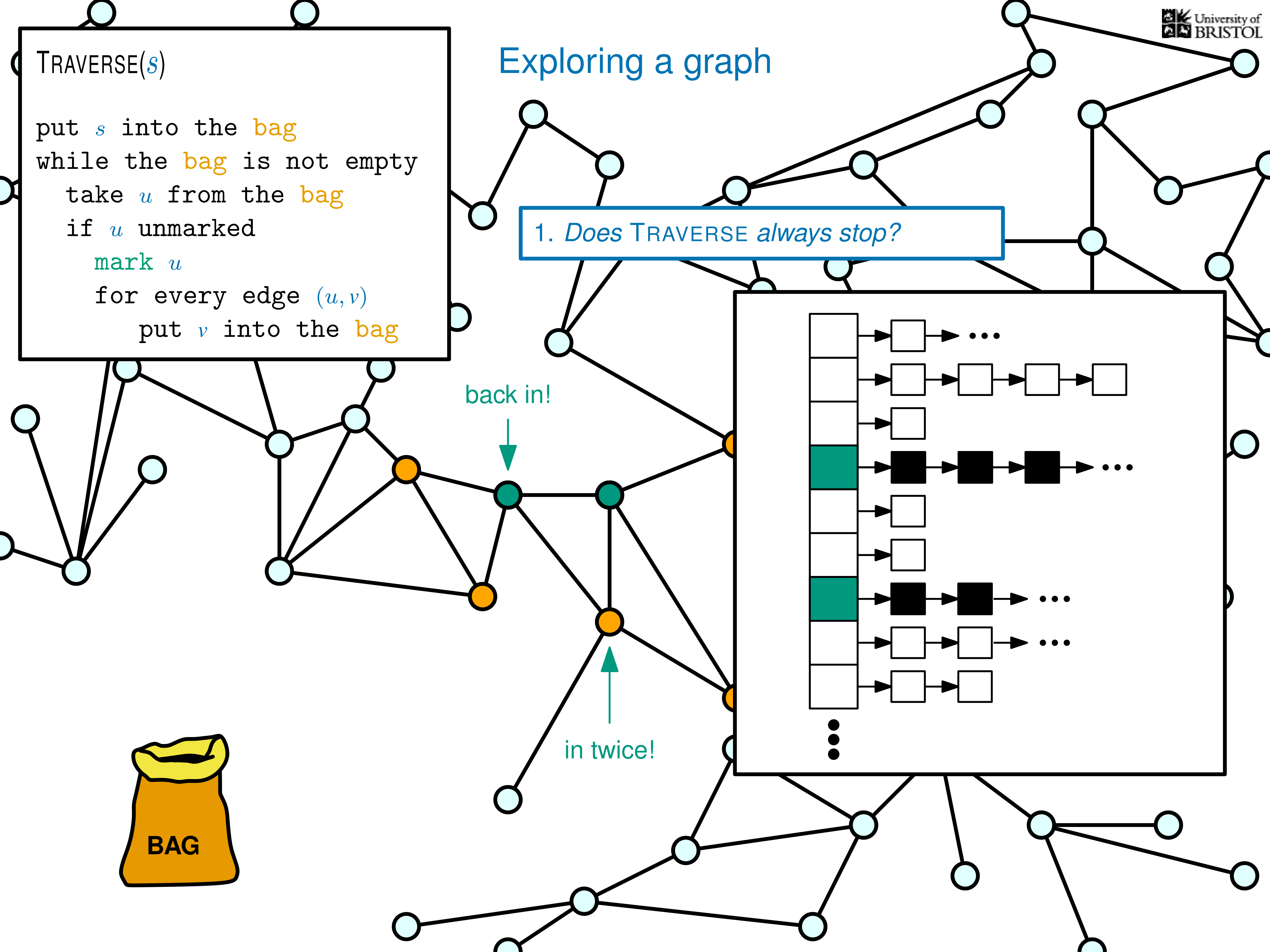

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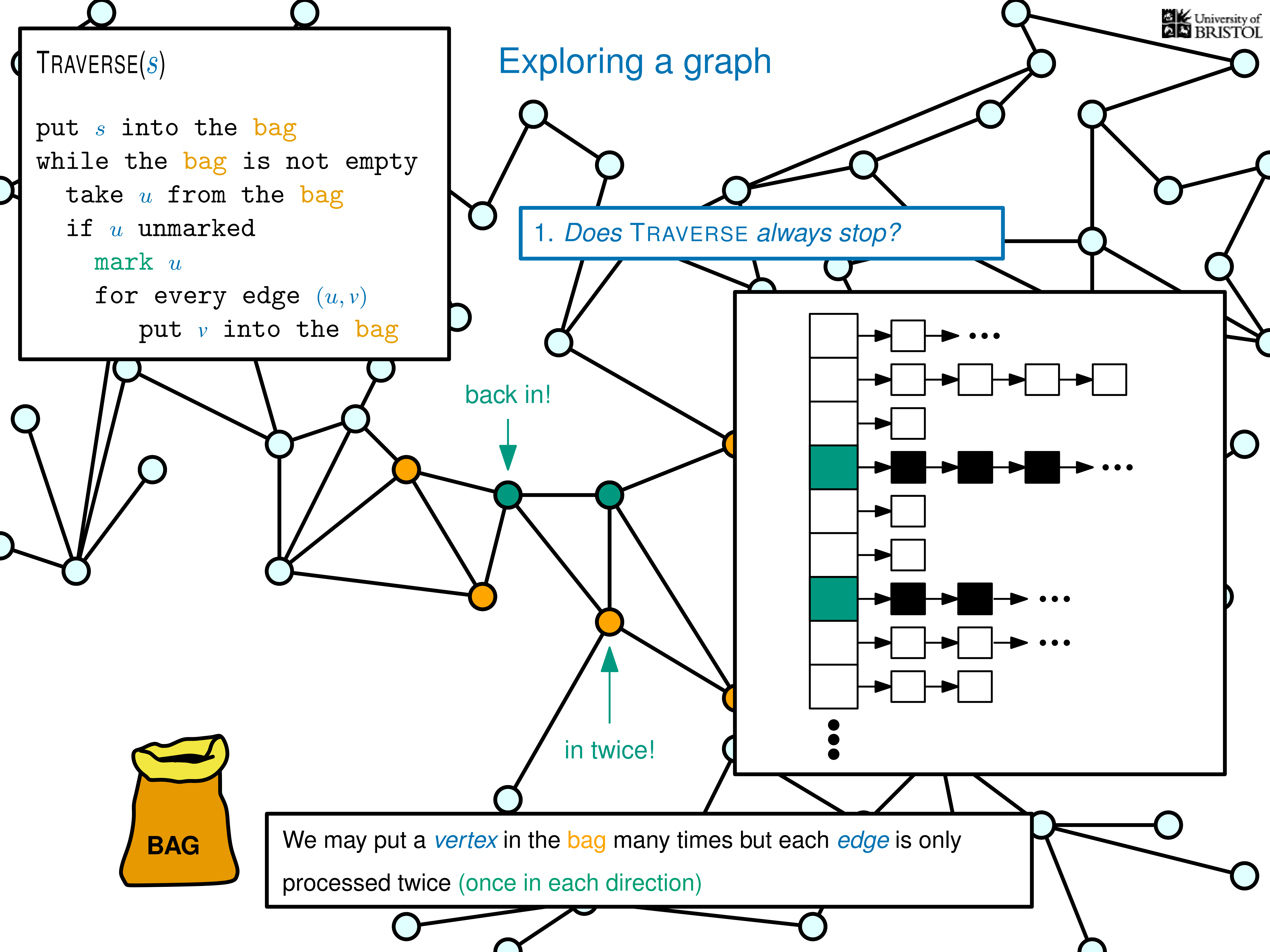

We may put a *vertex* in the bag many times but each *edge* is only processed twice (once in each direction)

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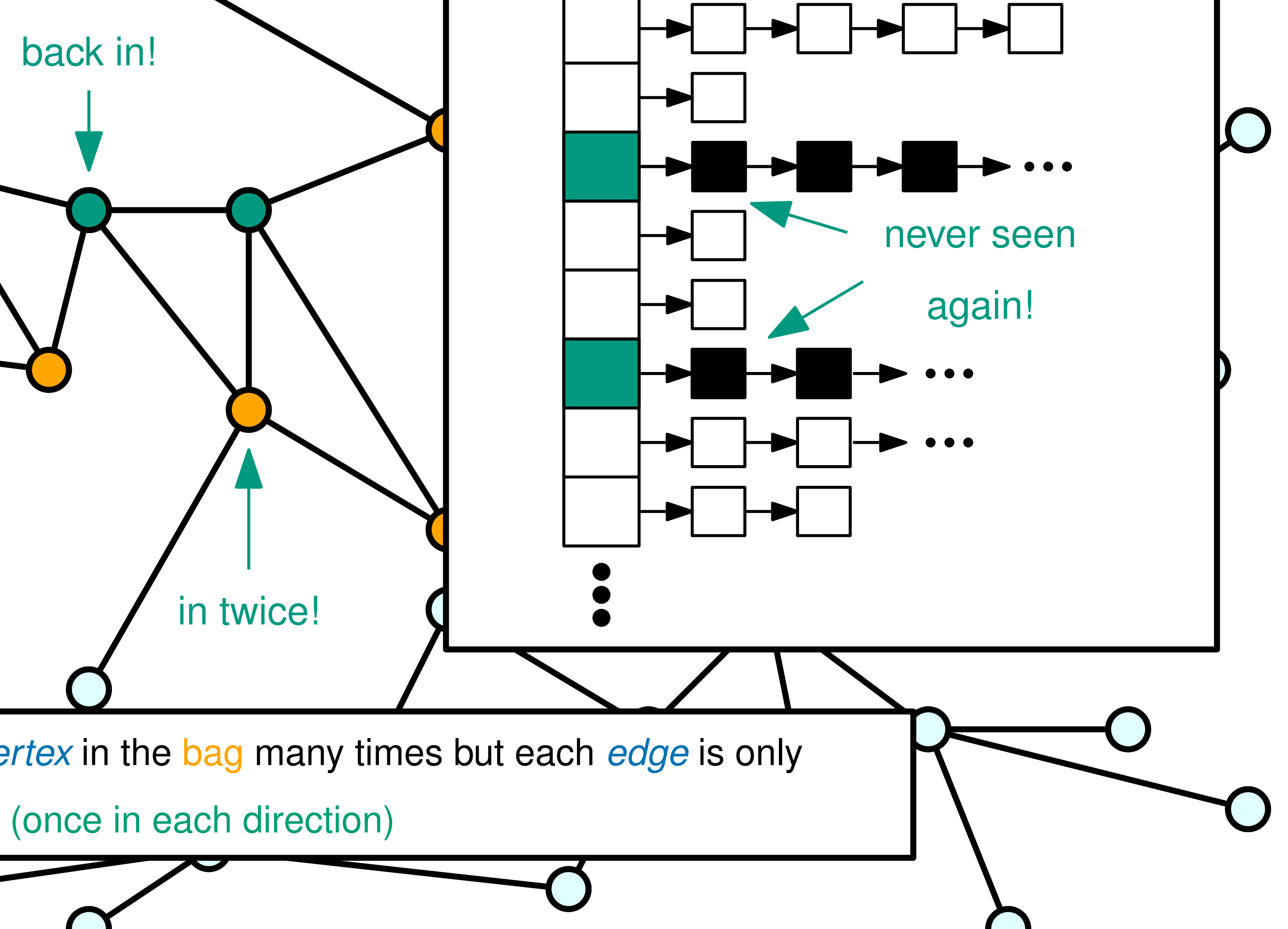

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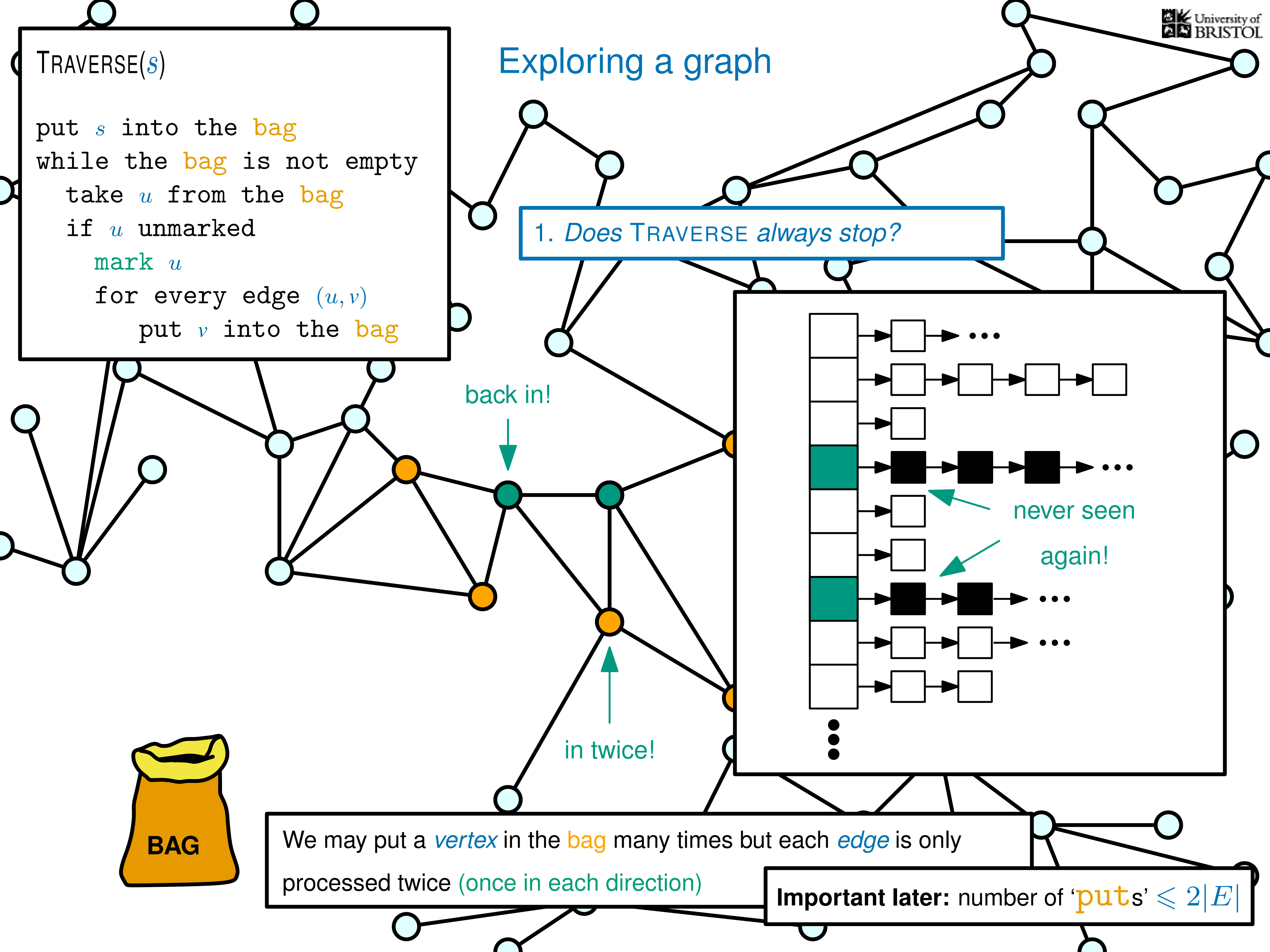

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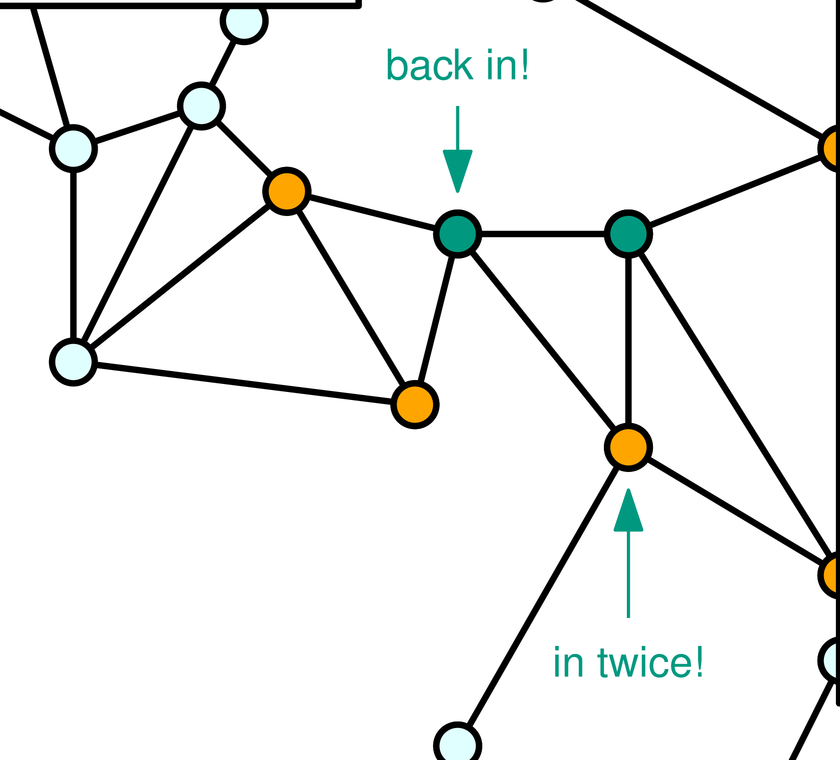

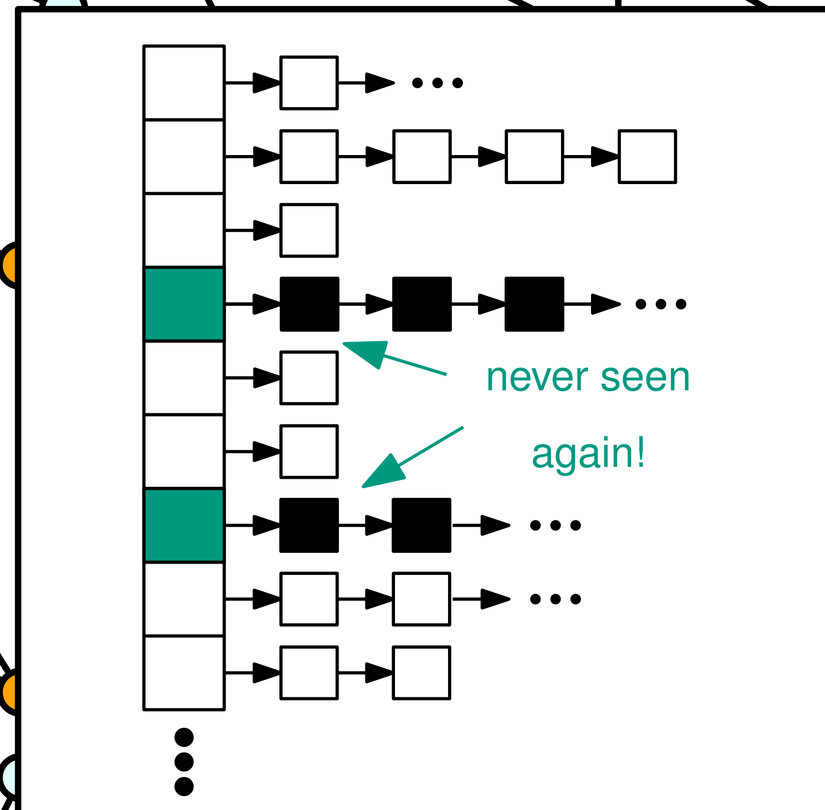

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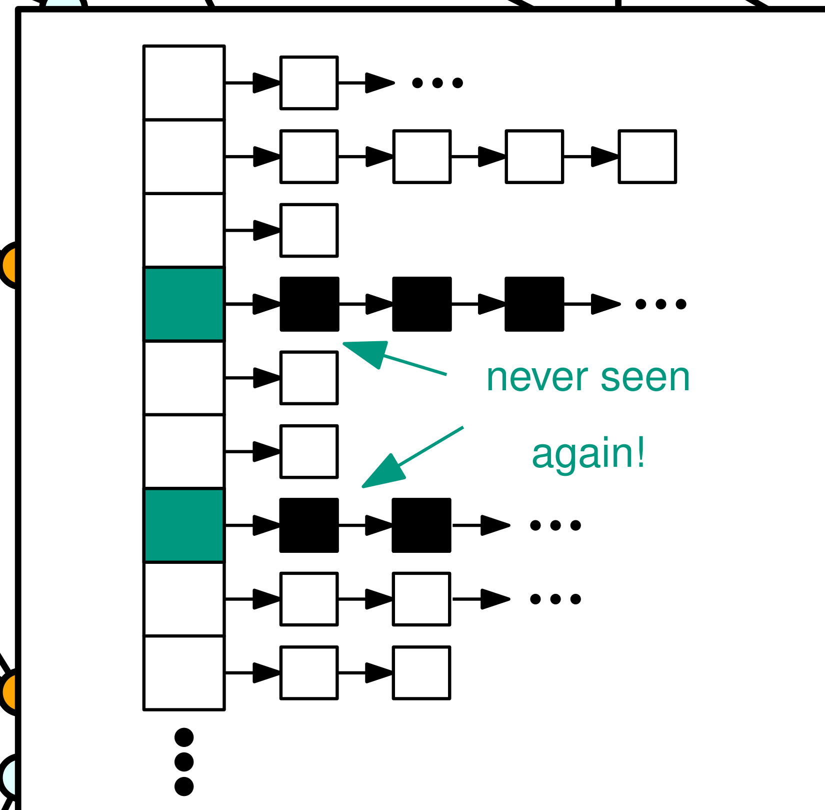
back in!

in twice!



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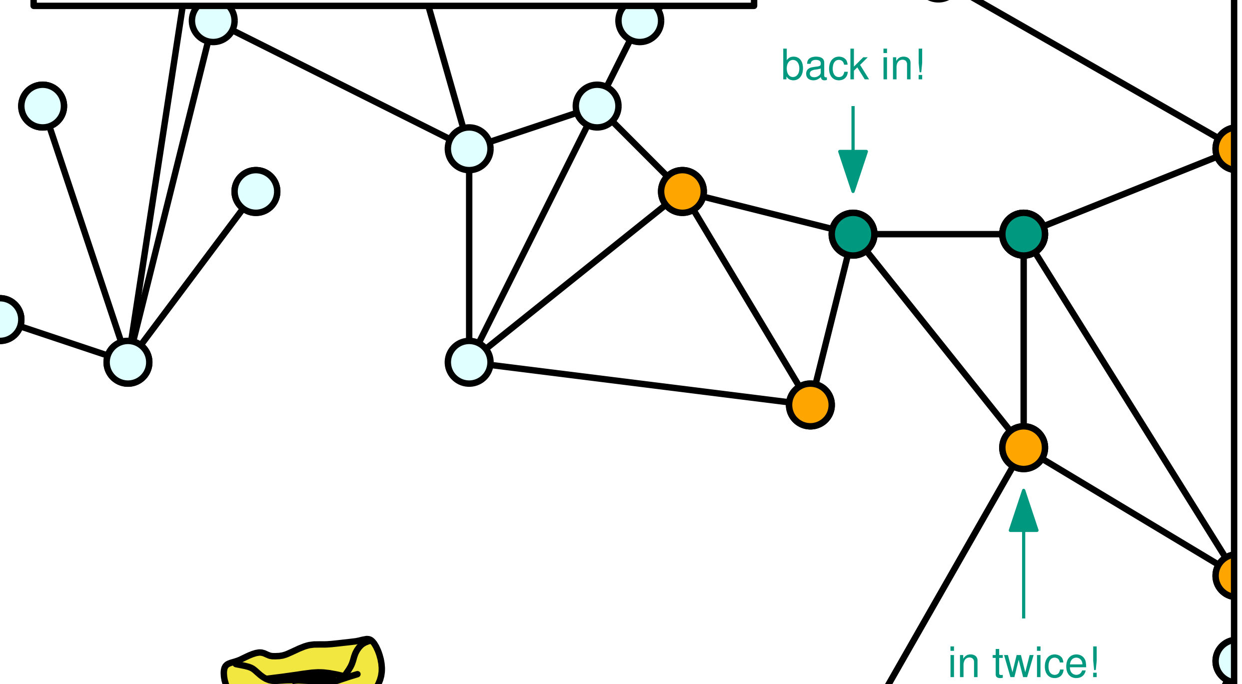

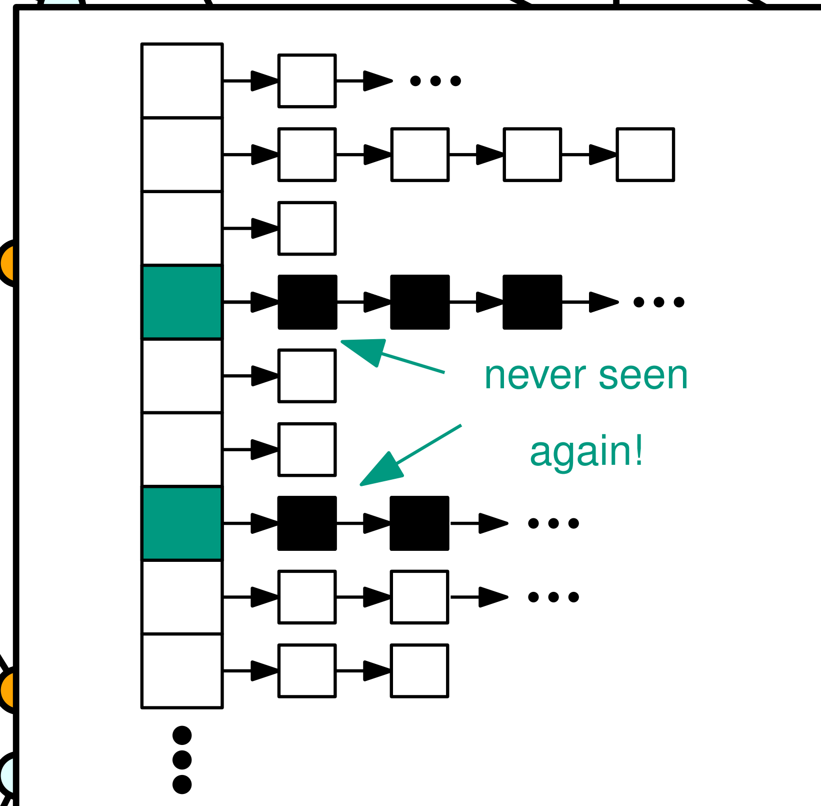

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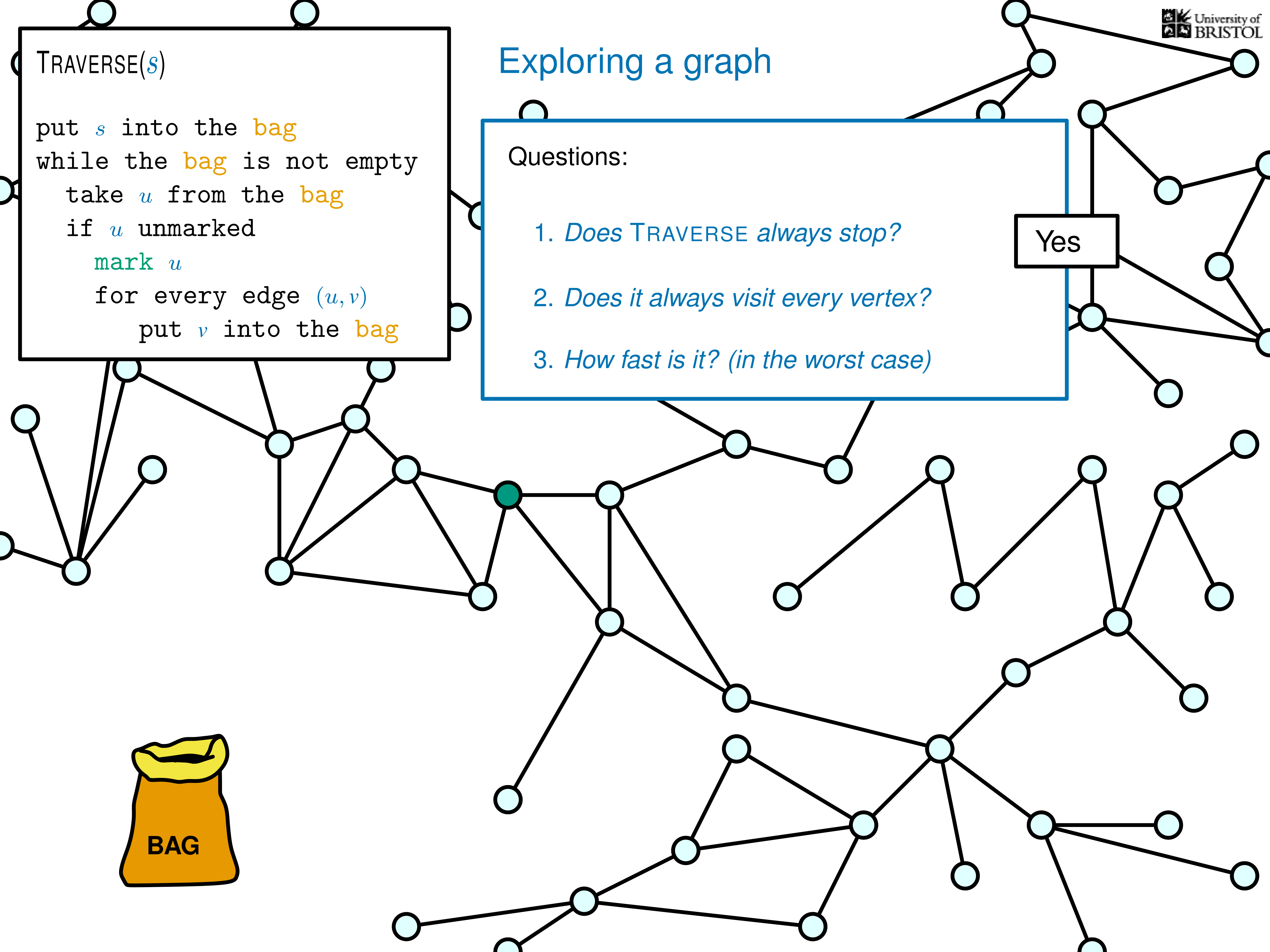

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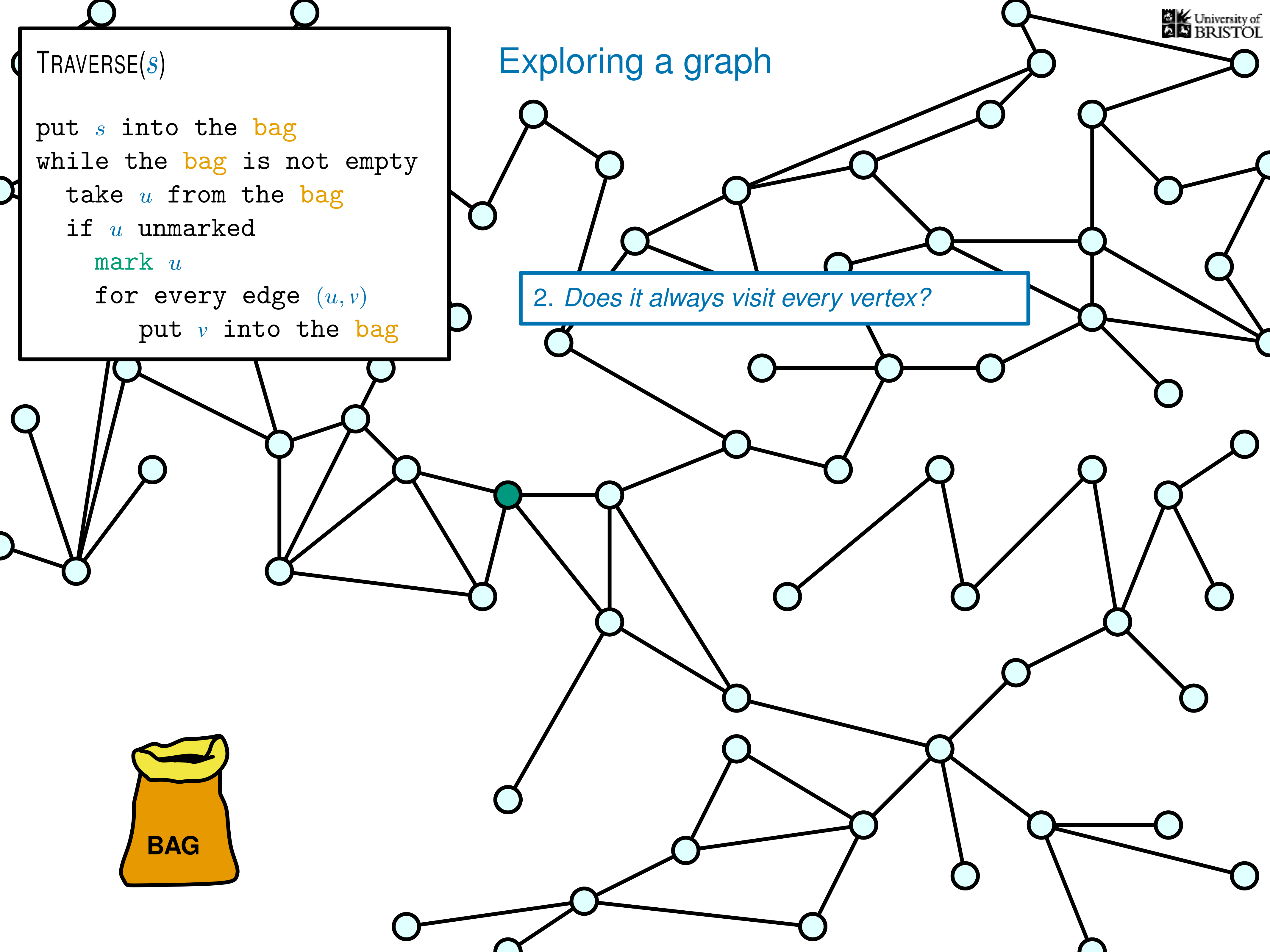
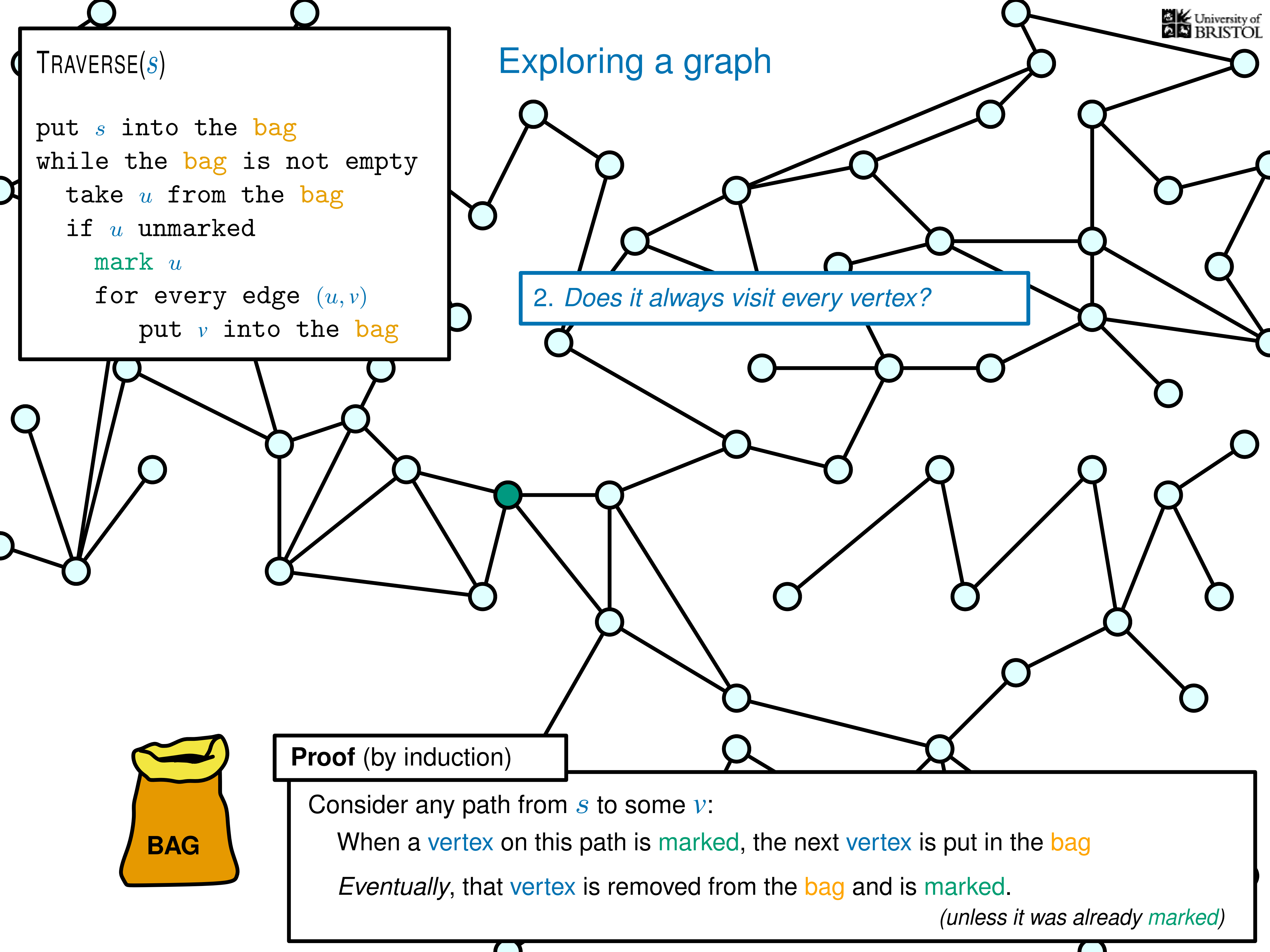


Exploring a graph



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BAG

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Proof (by induction)

Consider any path from s to some v:

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Exploring a graph

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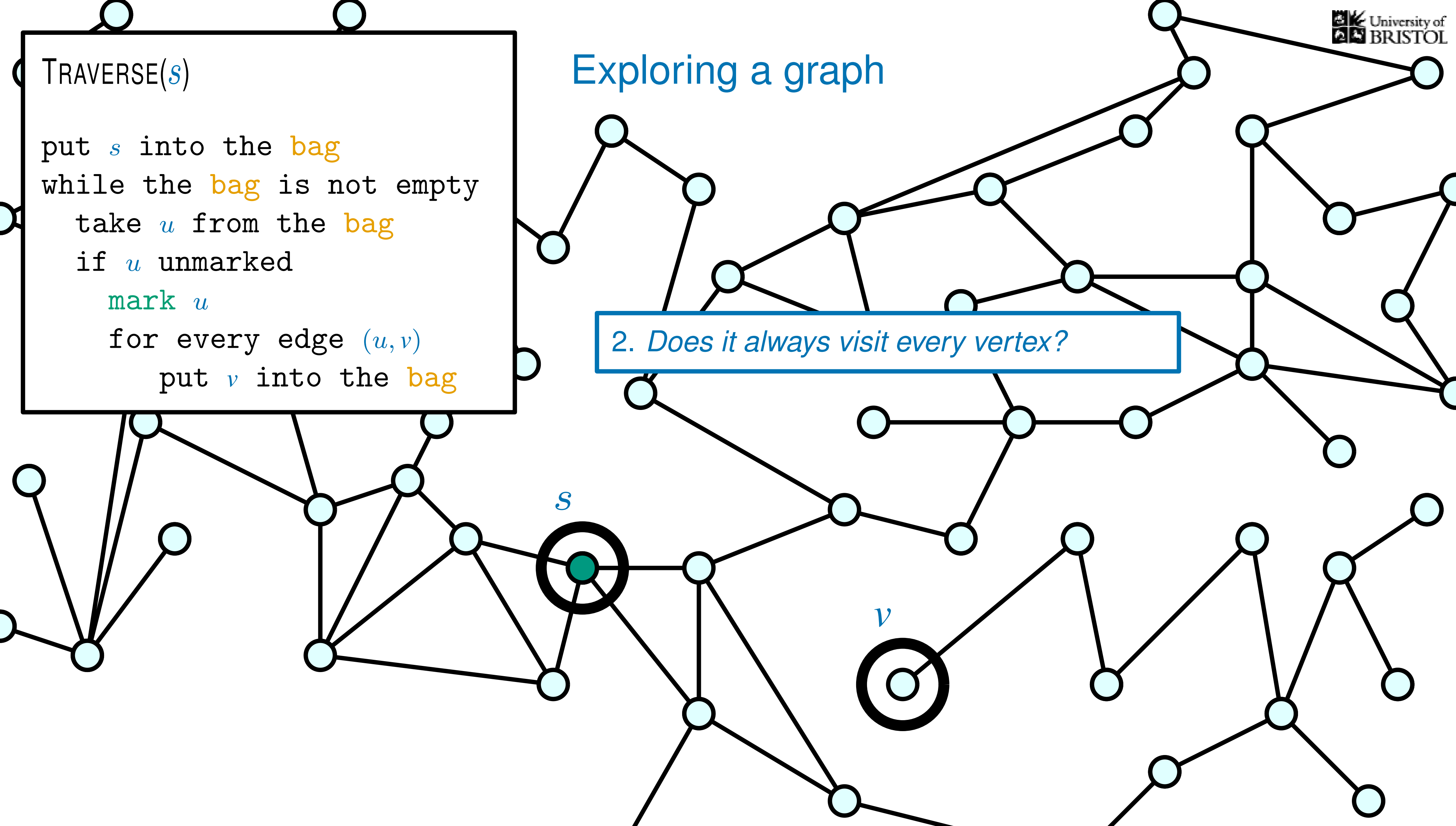

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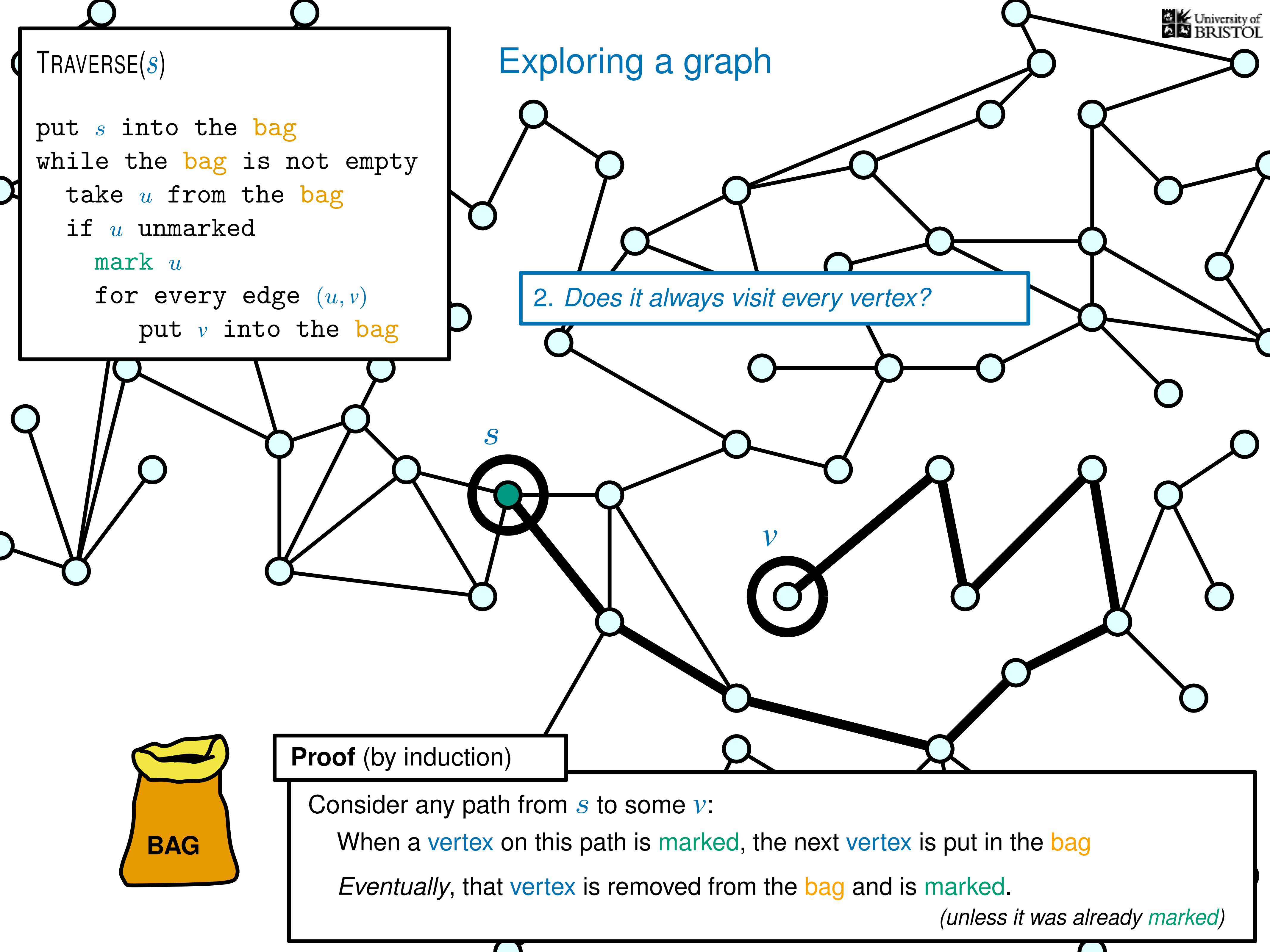

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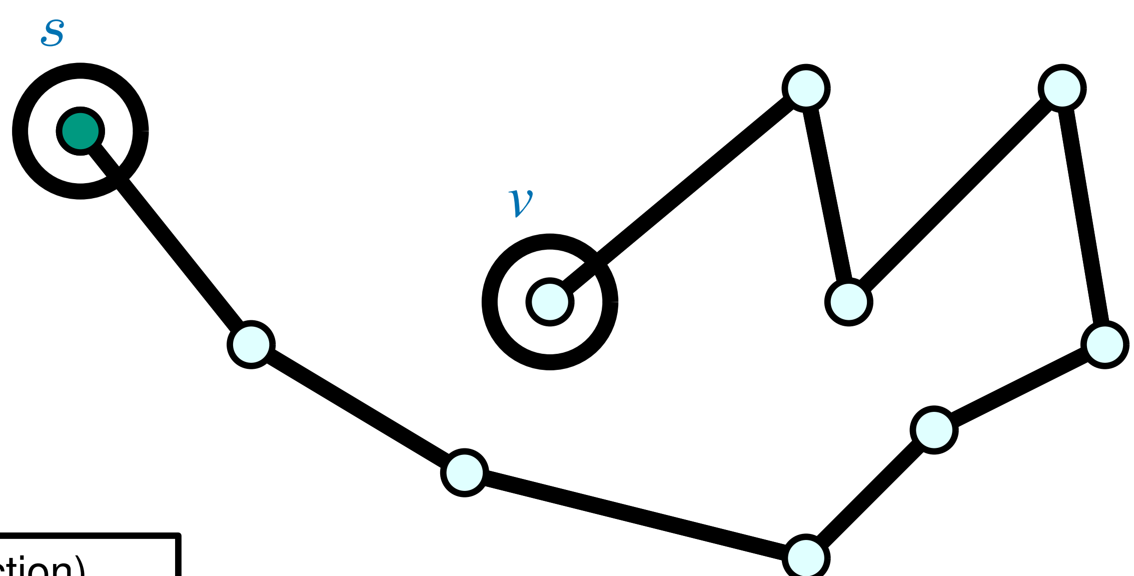

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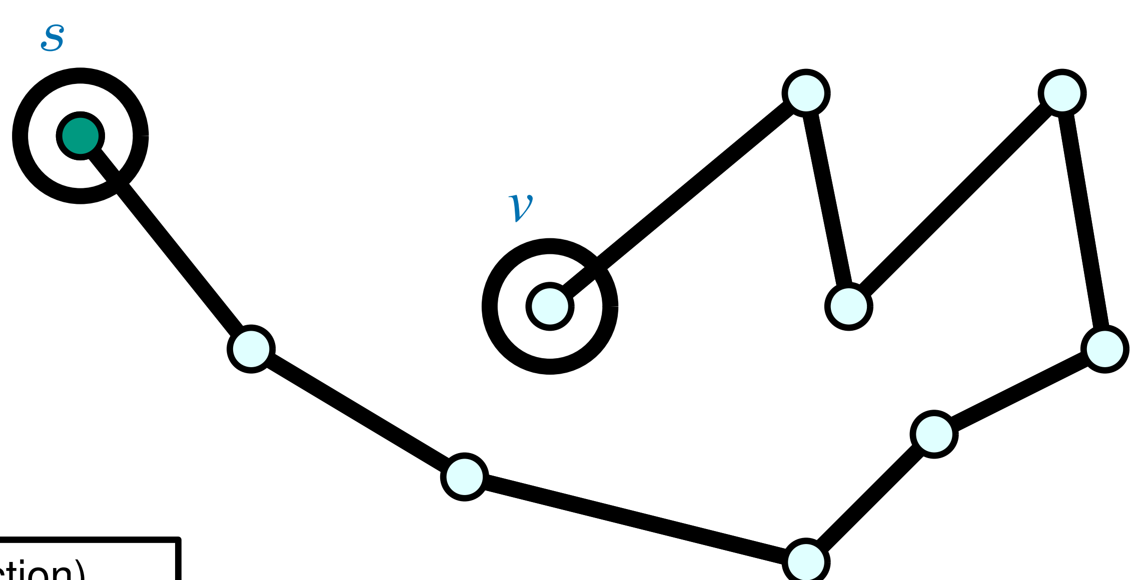

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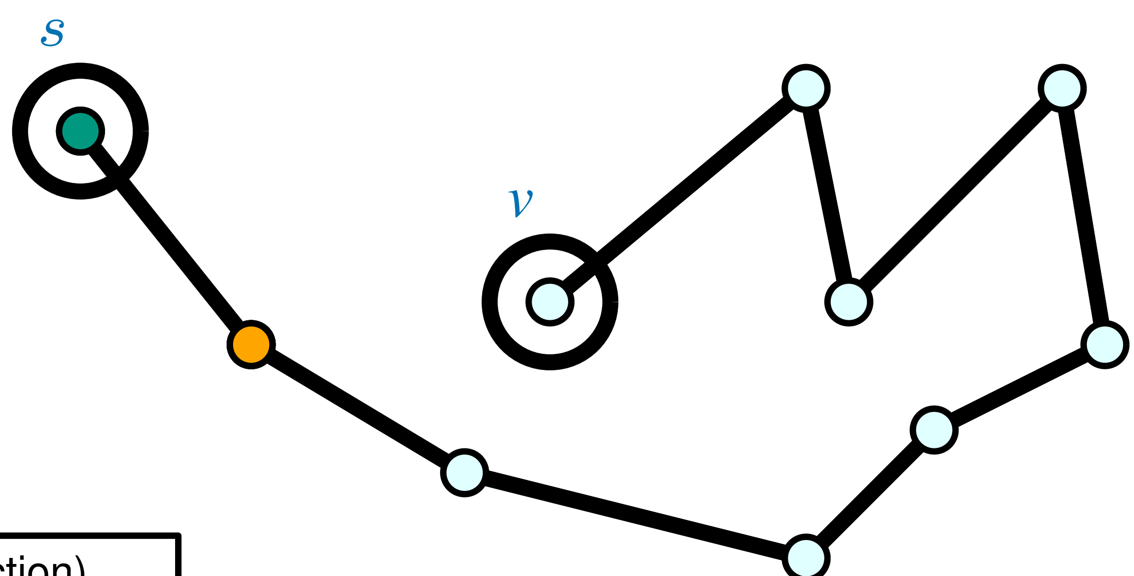

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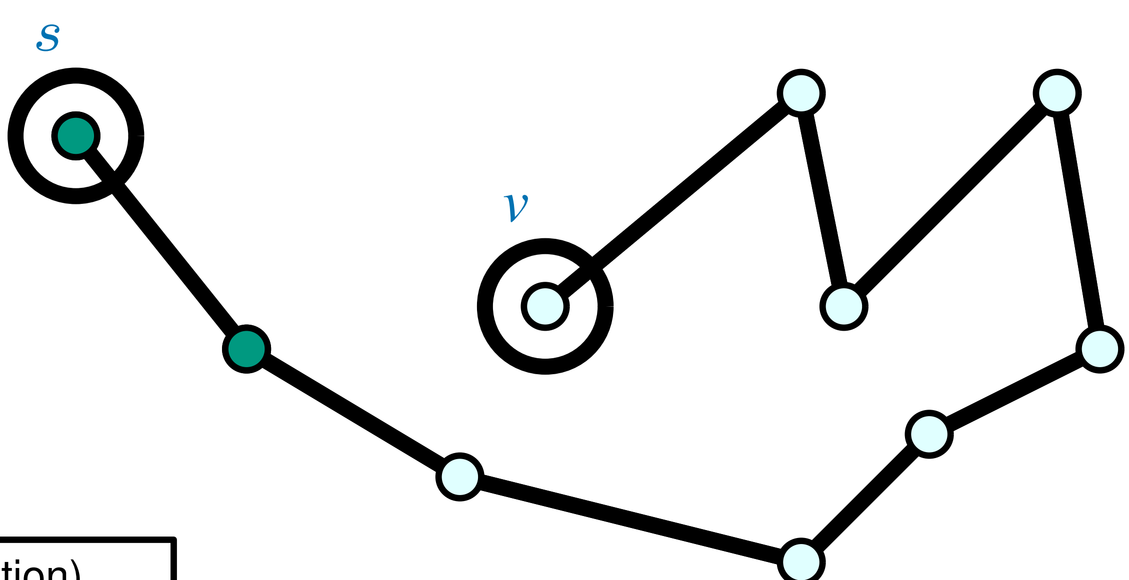

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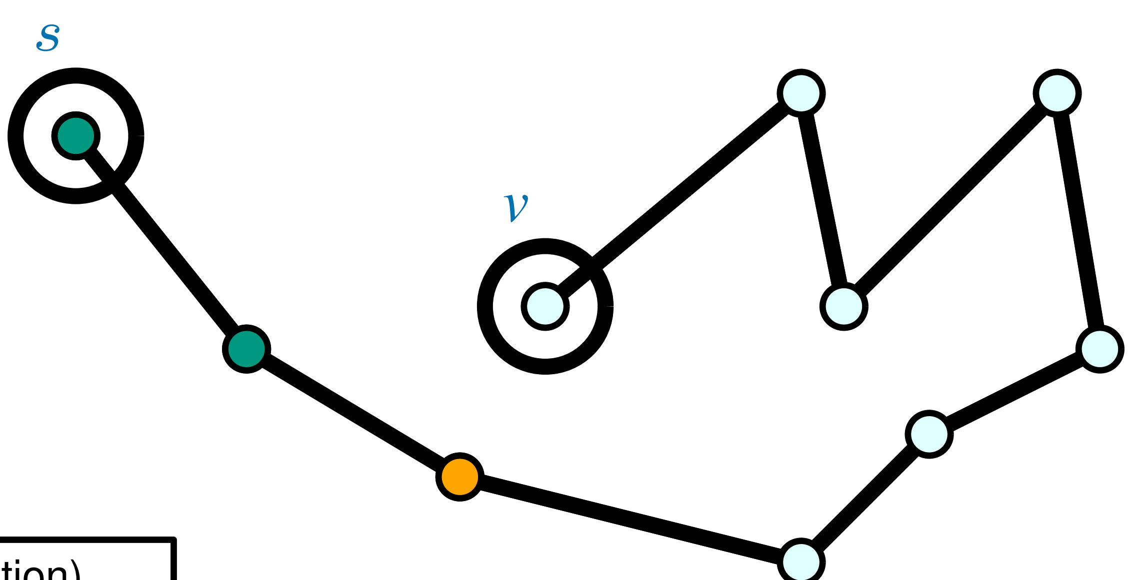

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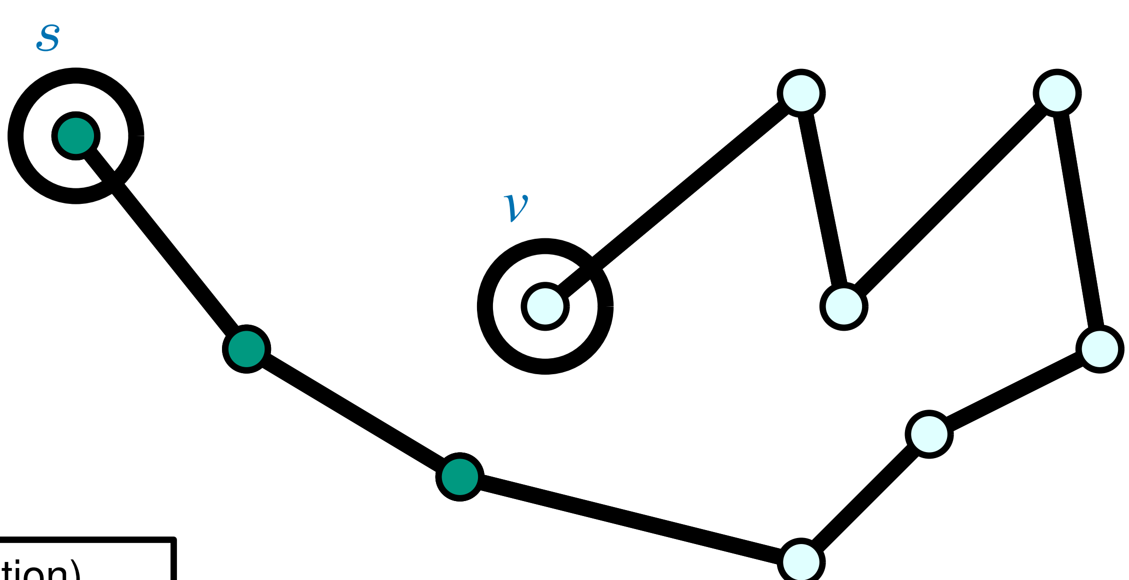

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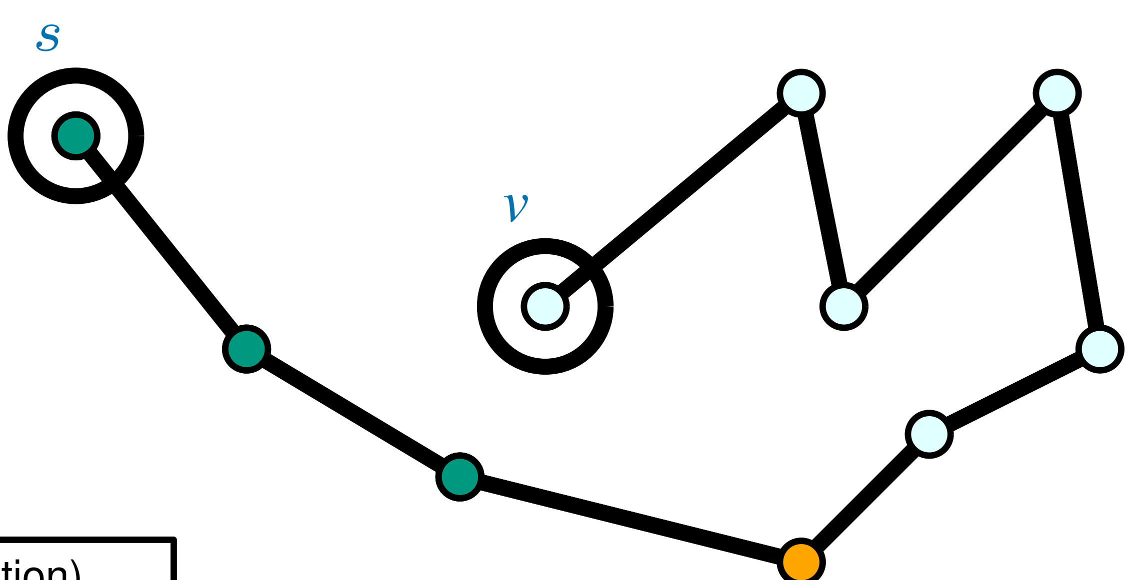

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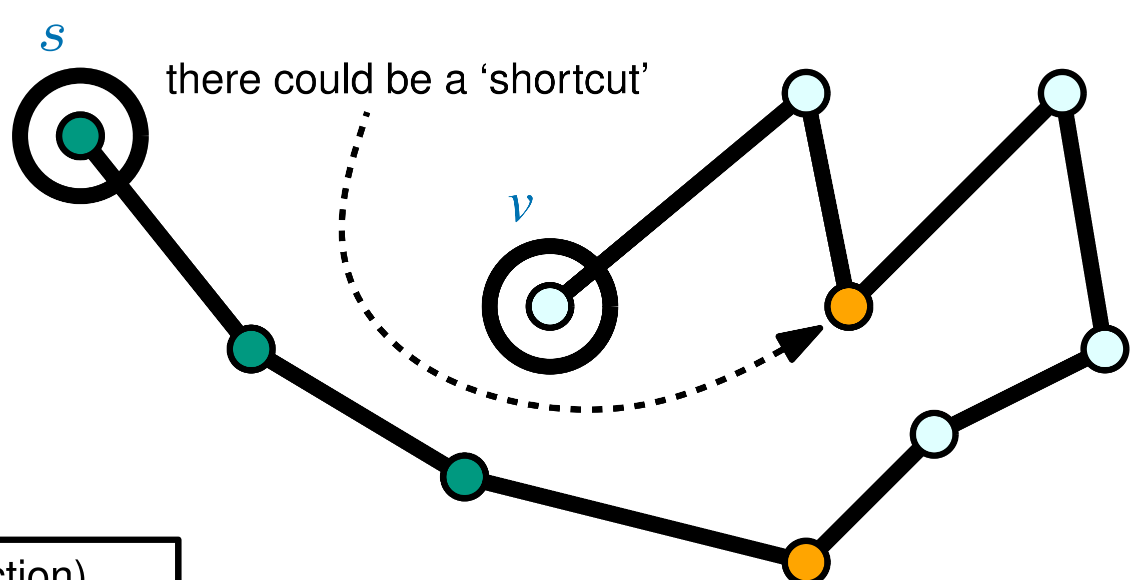

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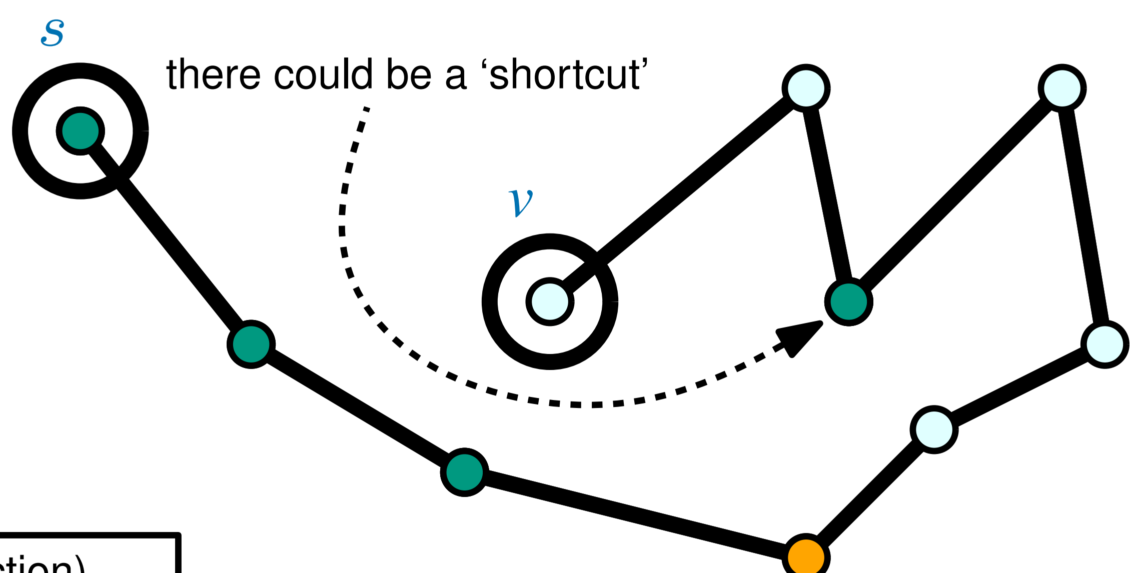

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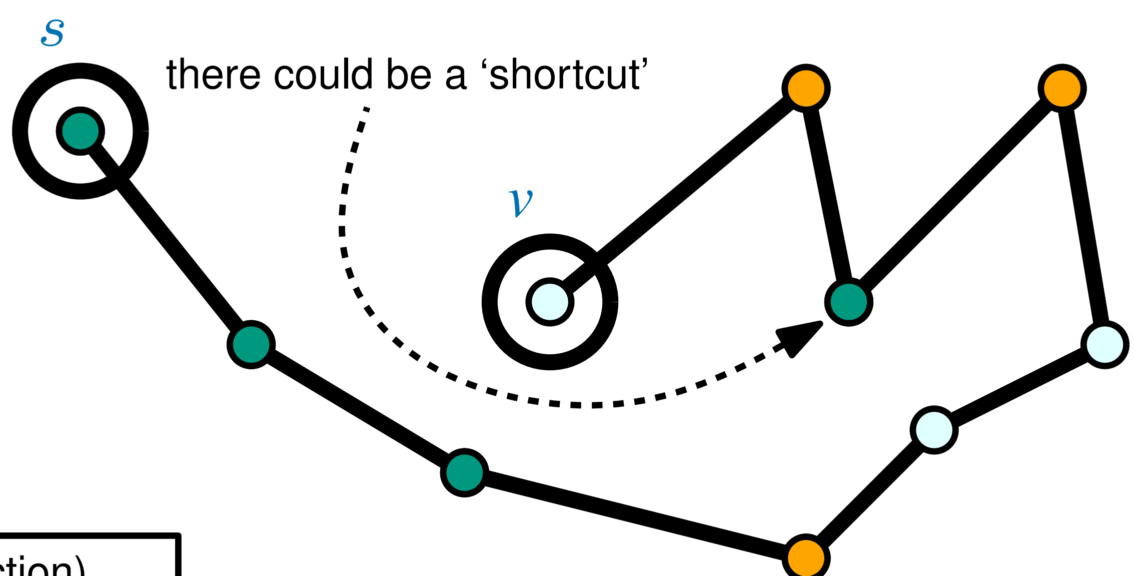

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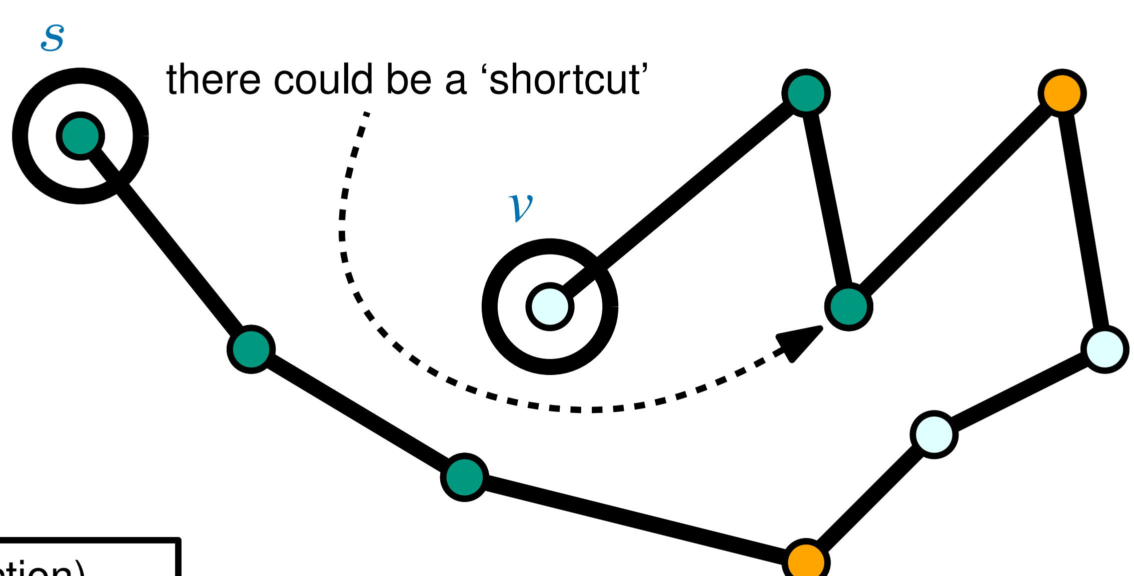

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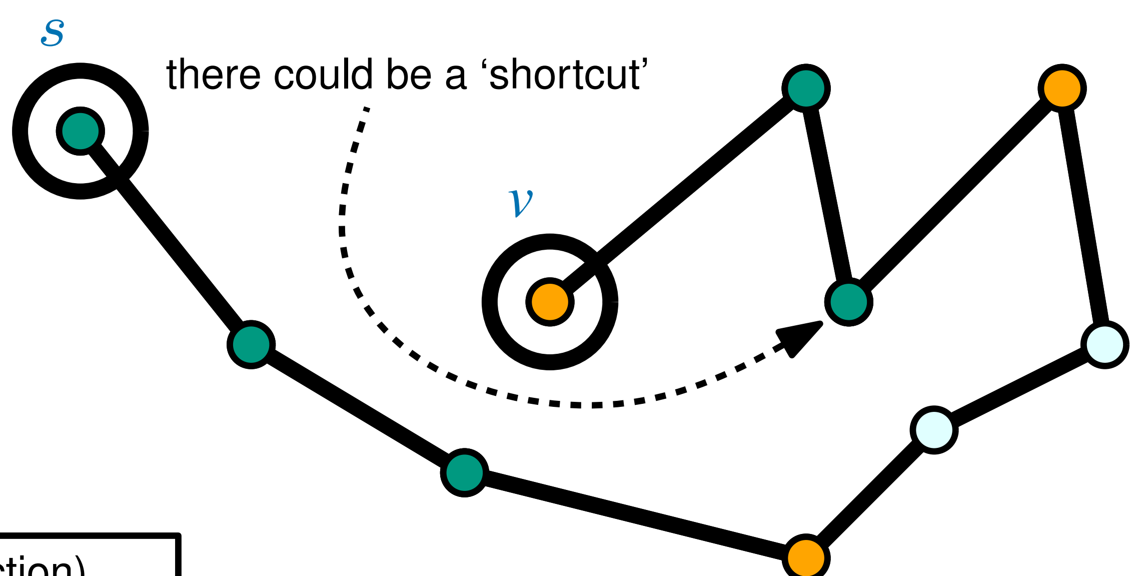

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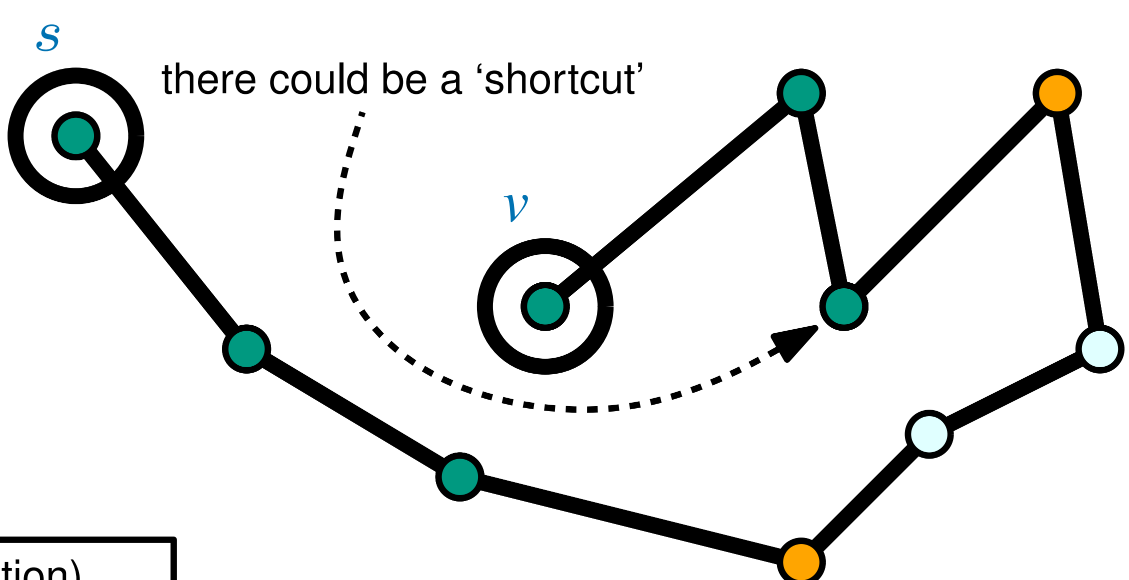

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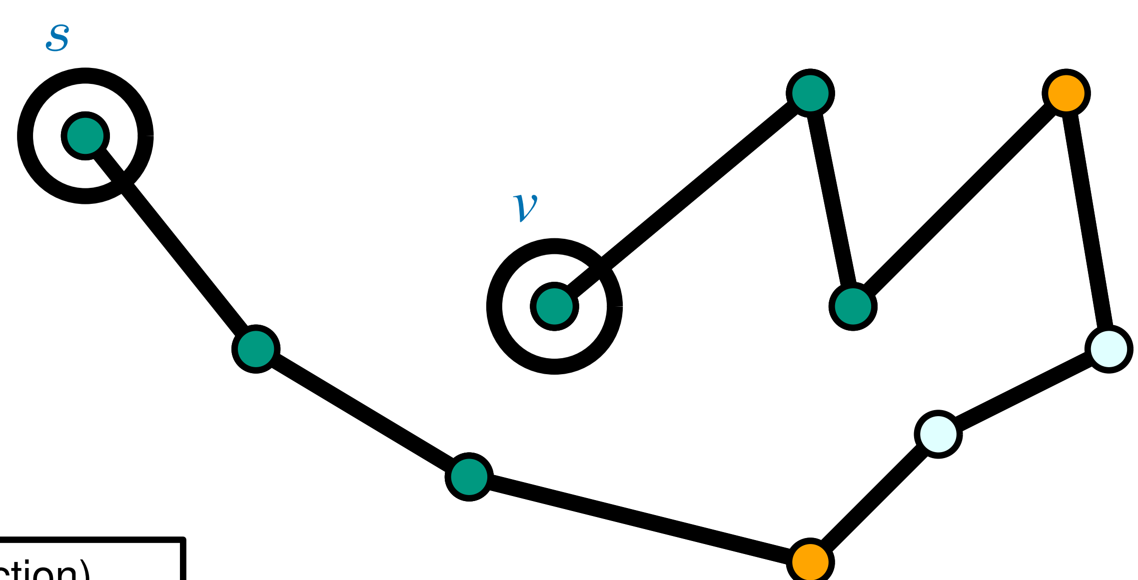

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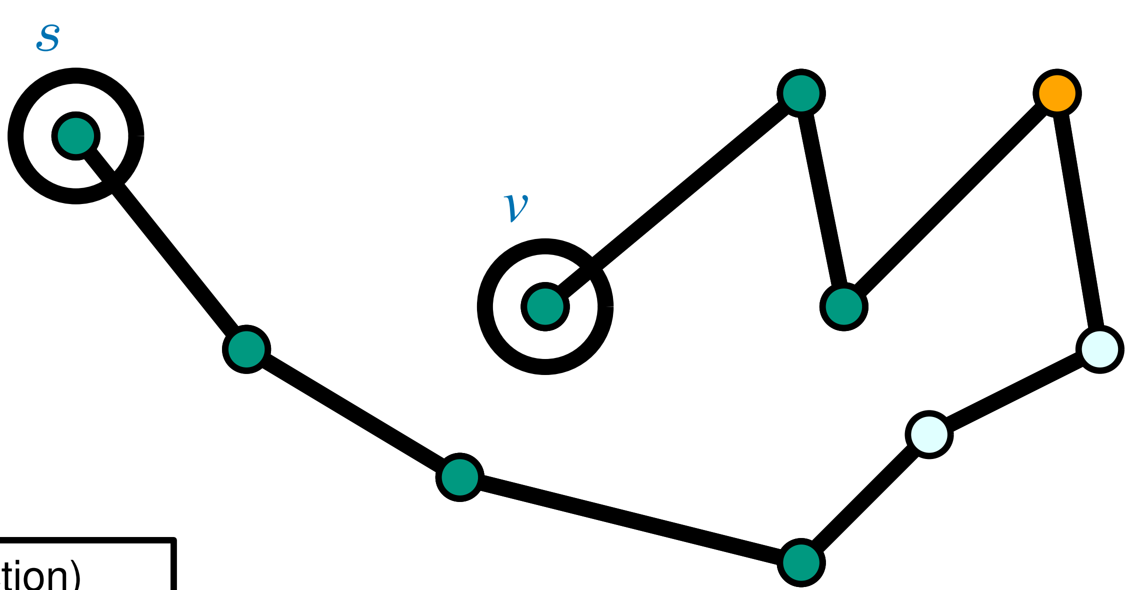

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Exploring a graph

$\textsc{Traverse}(s)$

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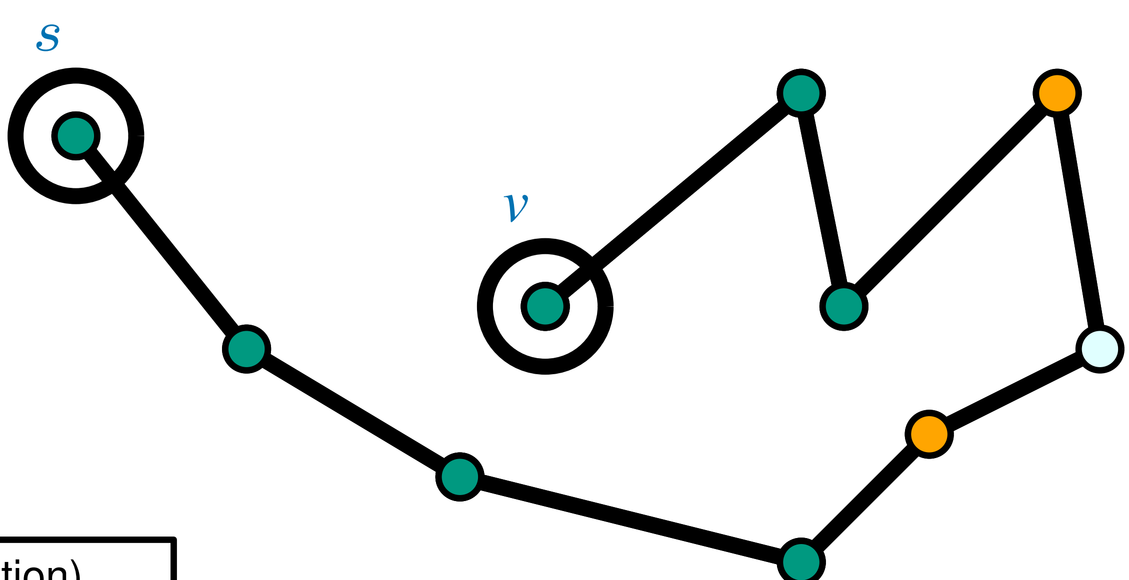

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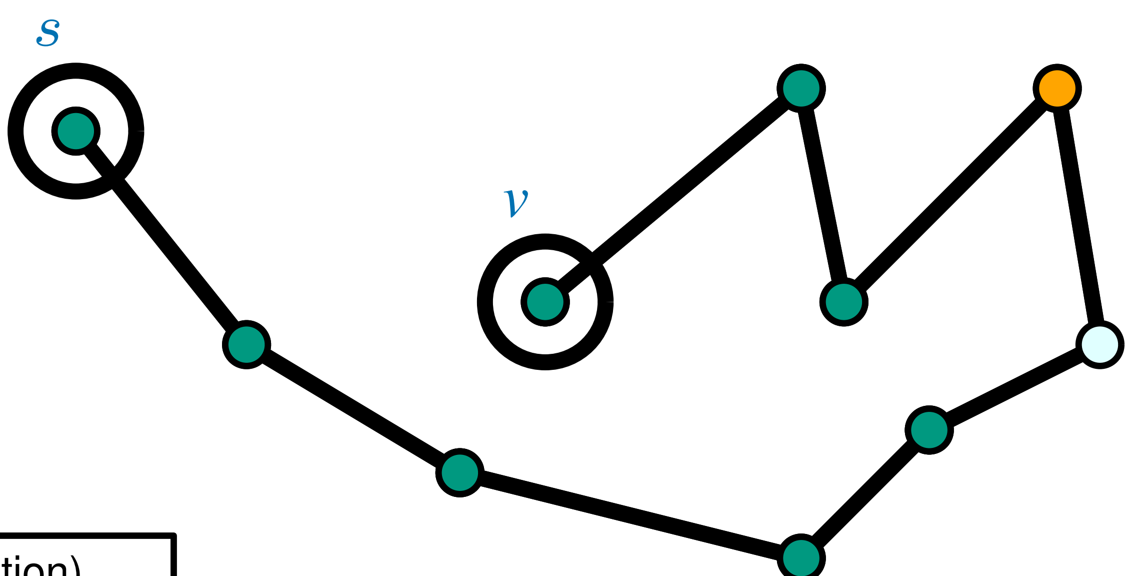

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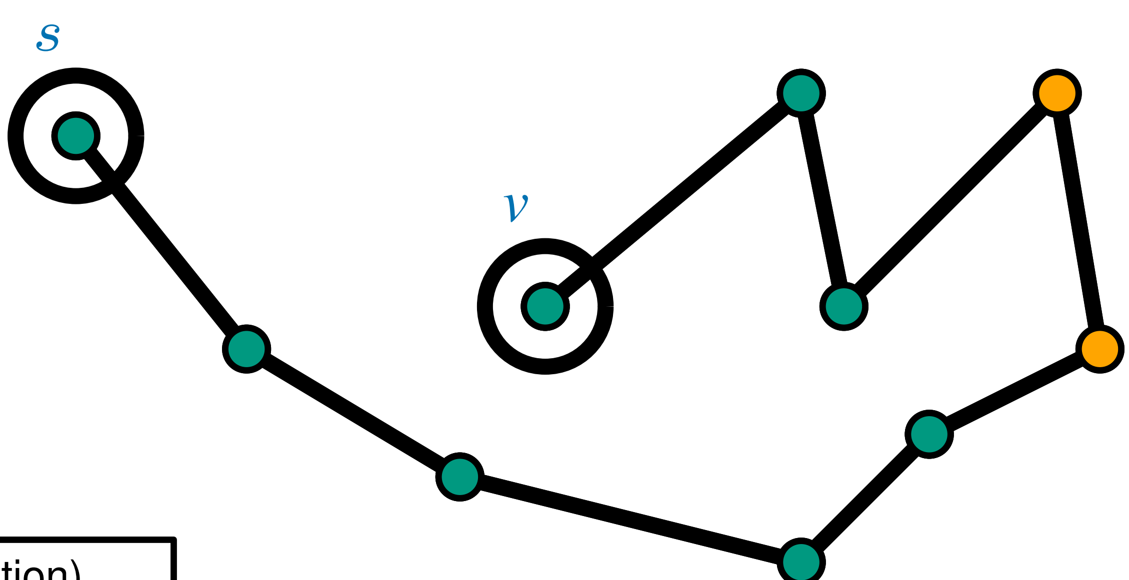
2. *Does it always visit every vertex?*



Proof (by induction)

Consider any path from s to some v:

When a vertex on this path is marked, the next vertex is put in the bag

Eventually, that vertex is removed from the bag and is marked.

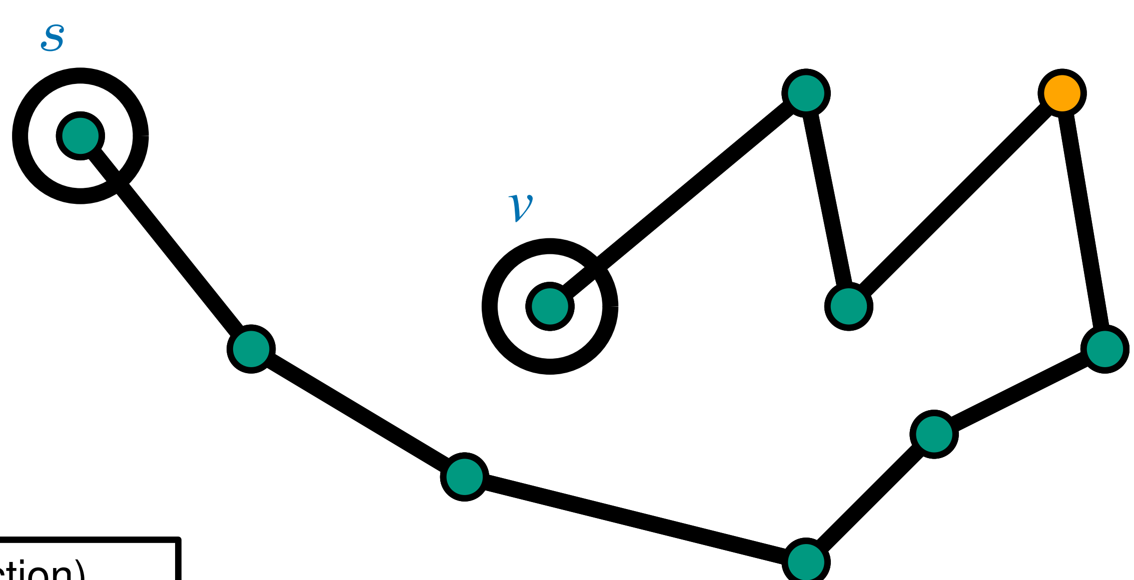
(unless it was already marked)

```
TRAVERSE(s)

put s into the bag
while the bag is not empty
  take u from the bag
  if u unmarked
    mark u
    for every edge (u, v)
       put v into the bag
```

Exploring a graph

2. *Does it always visit every vertex?*



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Exploring a graph

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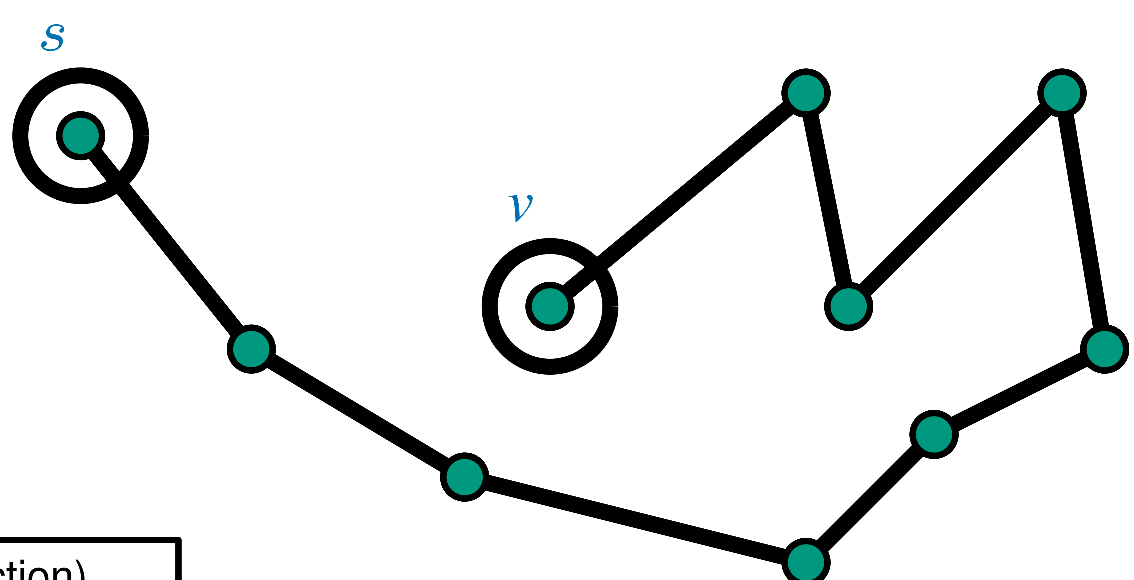

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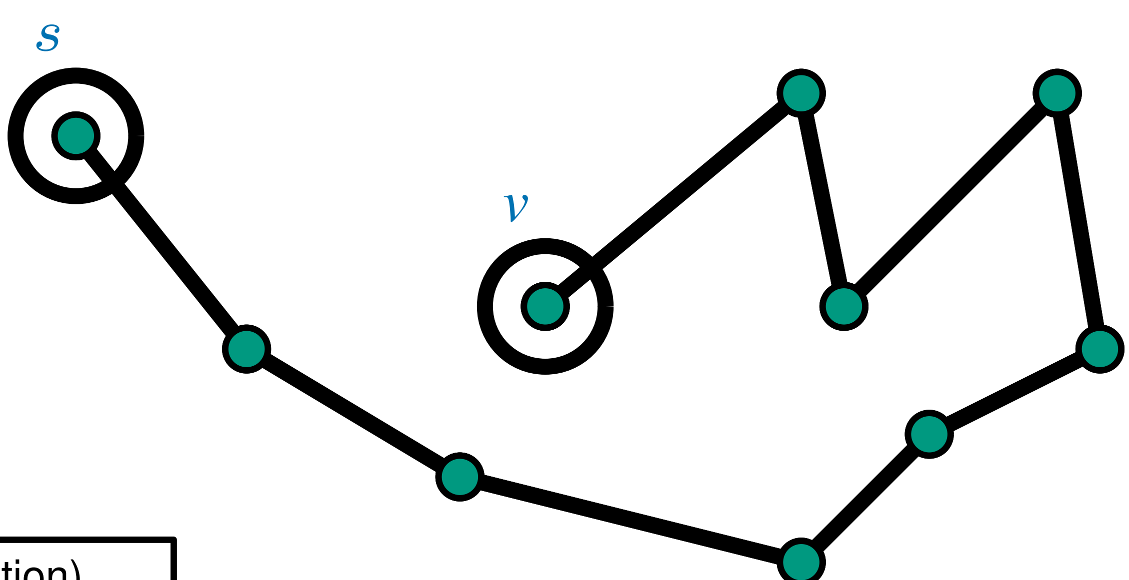
Exploring a graph

2. *Does it always visit every vertex?*

Yes



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Exploring a graph

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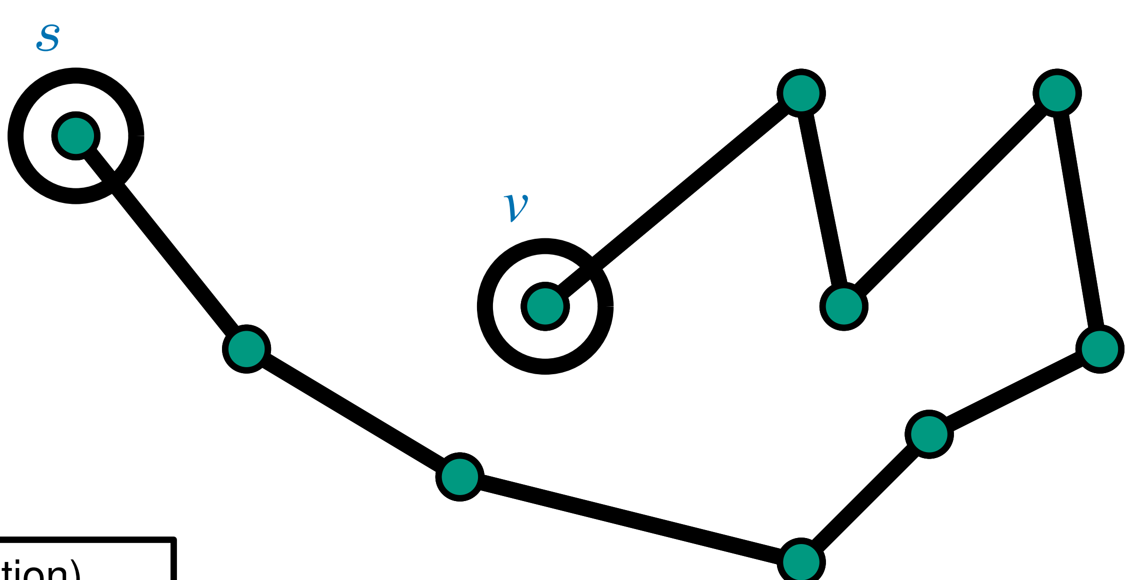
```

2. *Does it always visit every vertex?*

Yes

The correctness doesn't depend on how the bag works!



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Exploring a graph

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```
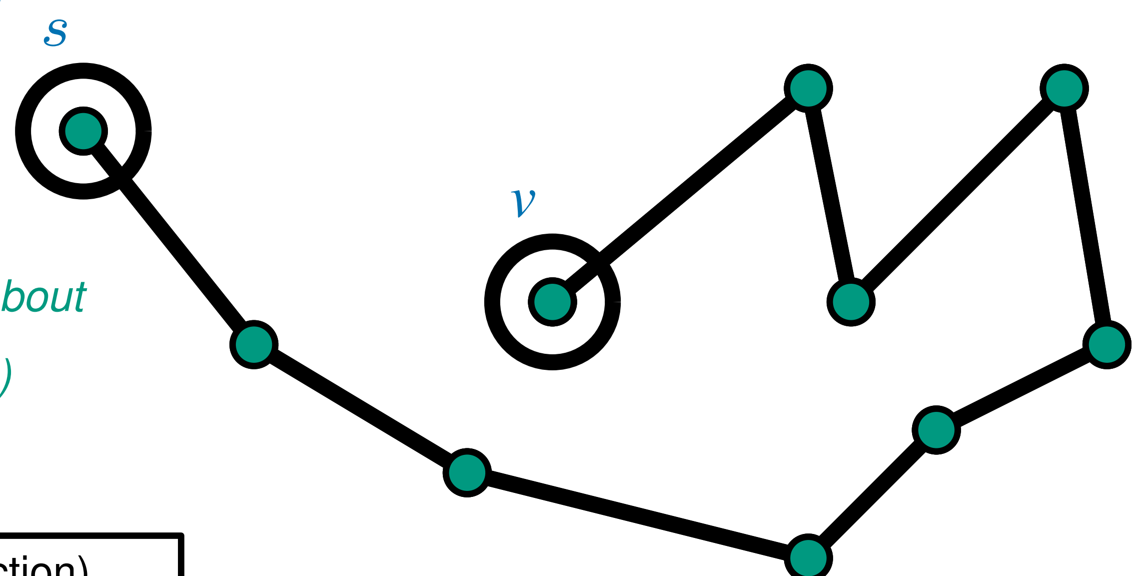

2\. *Does it always visit every vertex?*

Yes

The correctness doesn't depend on how the bag works!

(but this doesn't tell you anything about which order vertices are marked in)



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(unless it was already marked)

Exploring a graph

Questions:

1. *Does* TRAVERSE *always stop?*
2. *Does it always visit every vertex?*
3. *How fast is it? (in the worst case)*

Yes

Yes

Exploring a graph

Assumption

bag operations take $O(1)$ time

(we'll come back to this)

Questions:

1. *Does* TRAVERSE *always stop?* $\boxed{\text{Yes}}$
2. *Does it always visit every vertex?* $\boxed{\text{Yes}}$
3. *How fast is it? (in the worst case)*

V is the set of vertices
$|V|$ *is the number of vertices*

Exploring a graph

E is the set of edges
$|E|$ *is the number of edges*

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Time complexity:

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Exploring a graph

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Time complexity:

$O(1)$ time every time we `take` from the bag

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$|V|$ *is the number of vertices*

Exploring a graph

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         put v into the bag
```

Time complexity:

$O(1)$ time every time we `take` from the bag

(store an array where $\texttt{mark}[u] = 1$ *iff* u *is marked)*

Exploring a graph

V is the set of vertices
$|V|$ *is the number of vertices*

E is the set of edges
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Exploring a graph

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What is the total number of bag operations?

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Exploring a graph

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Questions:

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bag operations
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we do at most $2|E|$ `put` operations

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Exploring a graph

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Questions:

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bag operations
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(we'll come back to this)

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we do at most $2|E|$ `put` operations

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Exploring a graph

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bag operations take $O(1)$ time

(we'll come back to this)

Questions:

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The overall time complexity is $O(|E|)$

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Exploring a graph

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Questions:

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Assumption

bag operations
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(we'll come back to this)

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Exploring a graph

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```

What if we had used an adjacency matrix?

	1	2	3	4	5
1	0	1	1	0	0
2	1	0	1	1	1
3	1	1	0	1	1
4	0	1	1	0	1
5	0	1	1	1	0

V is the set of vertices
$|V|$ *is the number of vertices*

Exploring a graph

E is the set of edges
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Assumption

bag operations
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(we'll come back to this)

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finding all edges leaving u would have taken $O(|V|)$ time

Exploring a graph

V is the set of vertices
$|V|$ *is the number of vertices*

E is the set of edges
$|E|$ *is the number of edges*

University of Bristol

Assumption

bag operations
take $O(1)$ time

(we'll come back to this)

Questions:

1. *Does* TRAVERSE *always stop?* — Yes
2. *Does it always visit every vertex?* — Yes
3. *How fast is it? (in the worst case)* — $O(|E|)$

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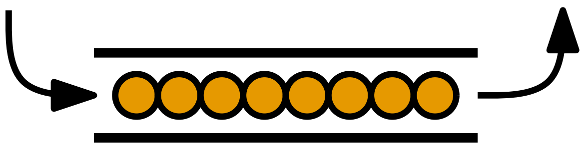
5	0	1	1	1	0

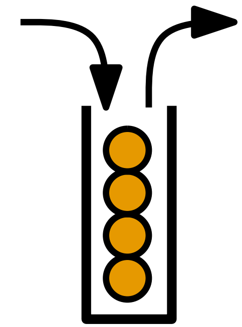
finding all edges leaving u would have taken $O(|V|)$ time

overall, the time complexity would have been $O(|V|^2)$ instead of $O(|E|)$.

How should we implement the bag?



Queue



Stack

Operations take $O(1)$ time

TRAVERSE visits every vertex in a connected graph in $O(|E|)$ time

but in different orders with different types of bag

with a Queue the algorithm is called

Breadth First Search

Applications

- **Shortest paths in unweighted graphs**
- Max-Flow
- Testing whether a graph is bipartite

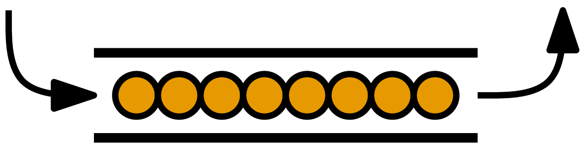
with a Stack the algorithm is called

Depth First Search

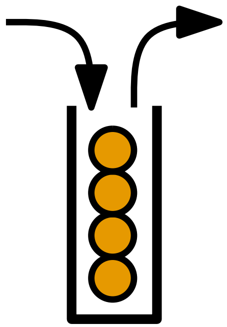
Applications

- Finding (strongly) connected components
- Topicologically sorting a Directed Acyclic Graph
- Testing for planarity

How should we implement the bag?



Queue



Stack

Operations take $O(1)$ time

TRAVERSE visits every vertex in a connected graph in $O(|E|)$ time

but in different orders with different types of bag

(and it works for directed graphs too)

with a Queue the algorithm is called

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Applications

- **Shortest paths in unweighted graphs**
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with a Stack the algorithm is called

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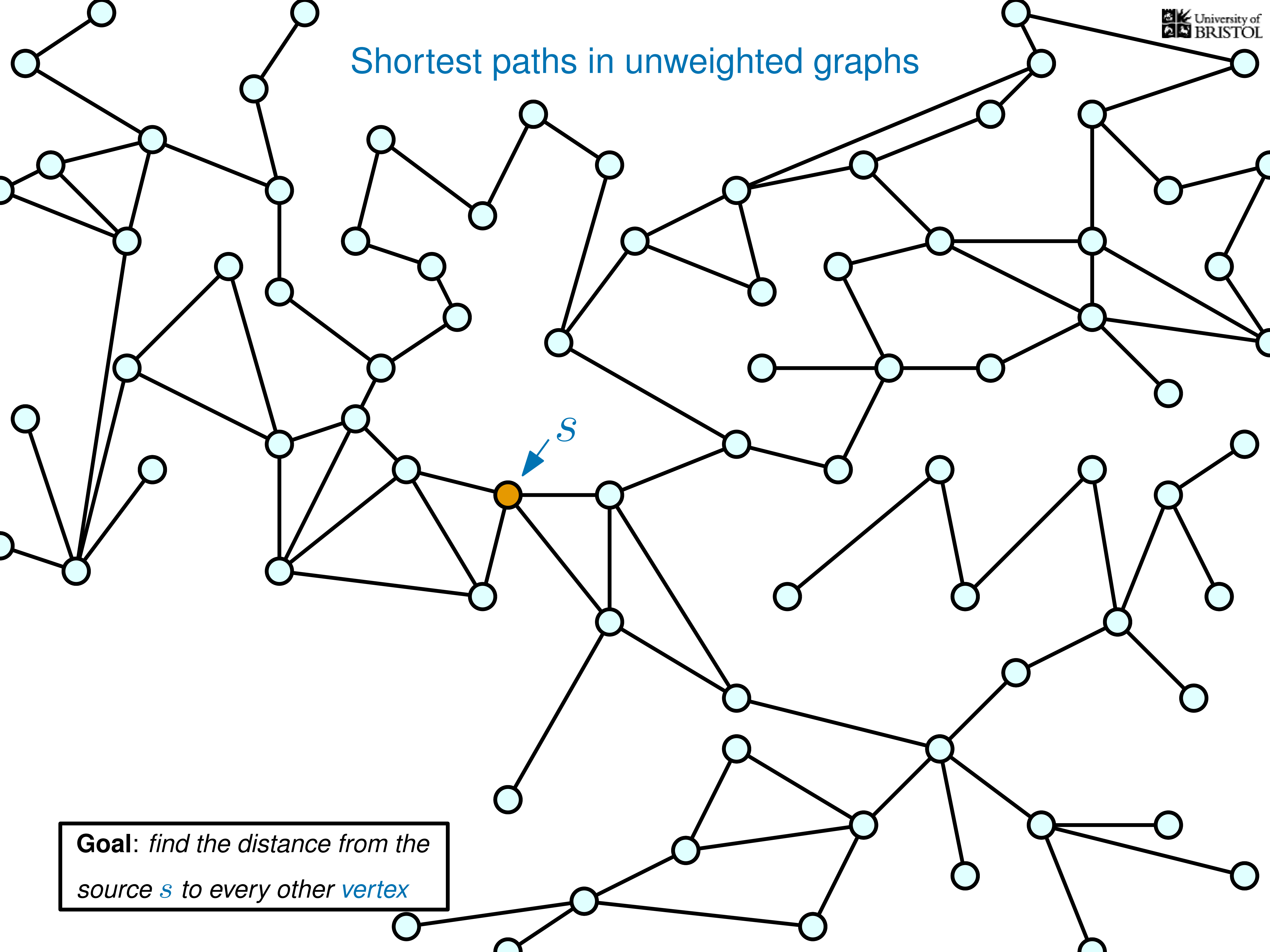
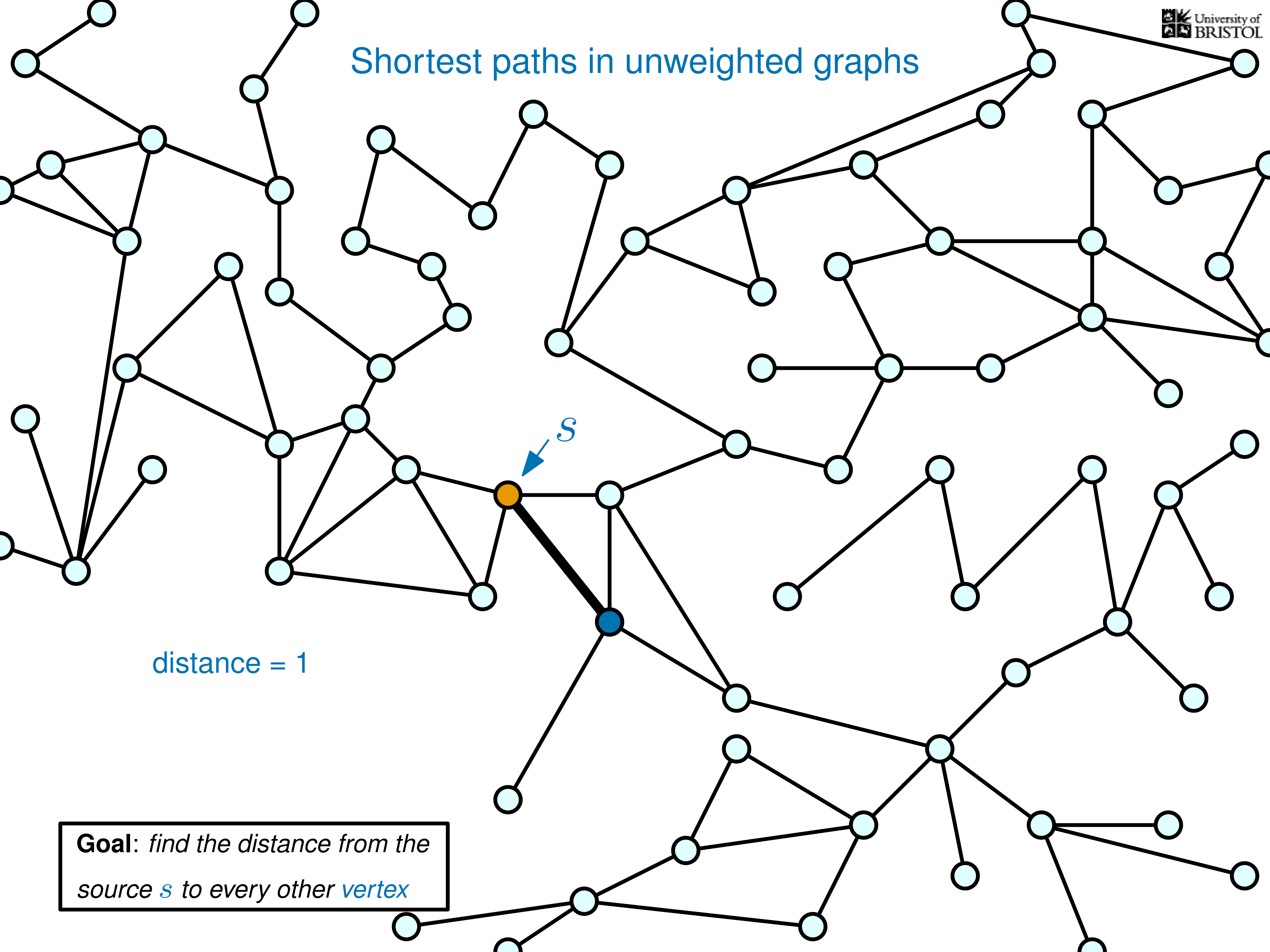
Shortest paths in unweighted graphs



Goal: *find the distance from the source s to every other vertex*

Shortest paths in unweighted graphs



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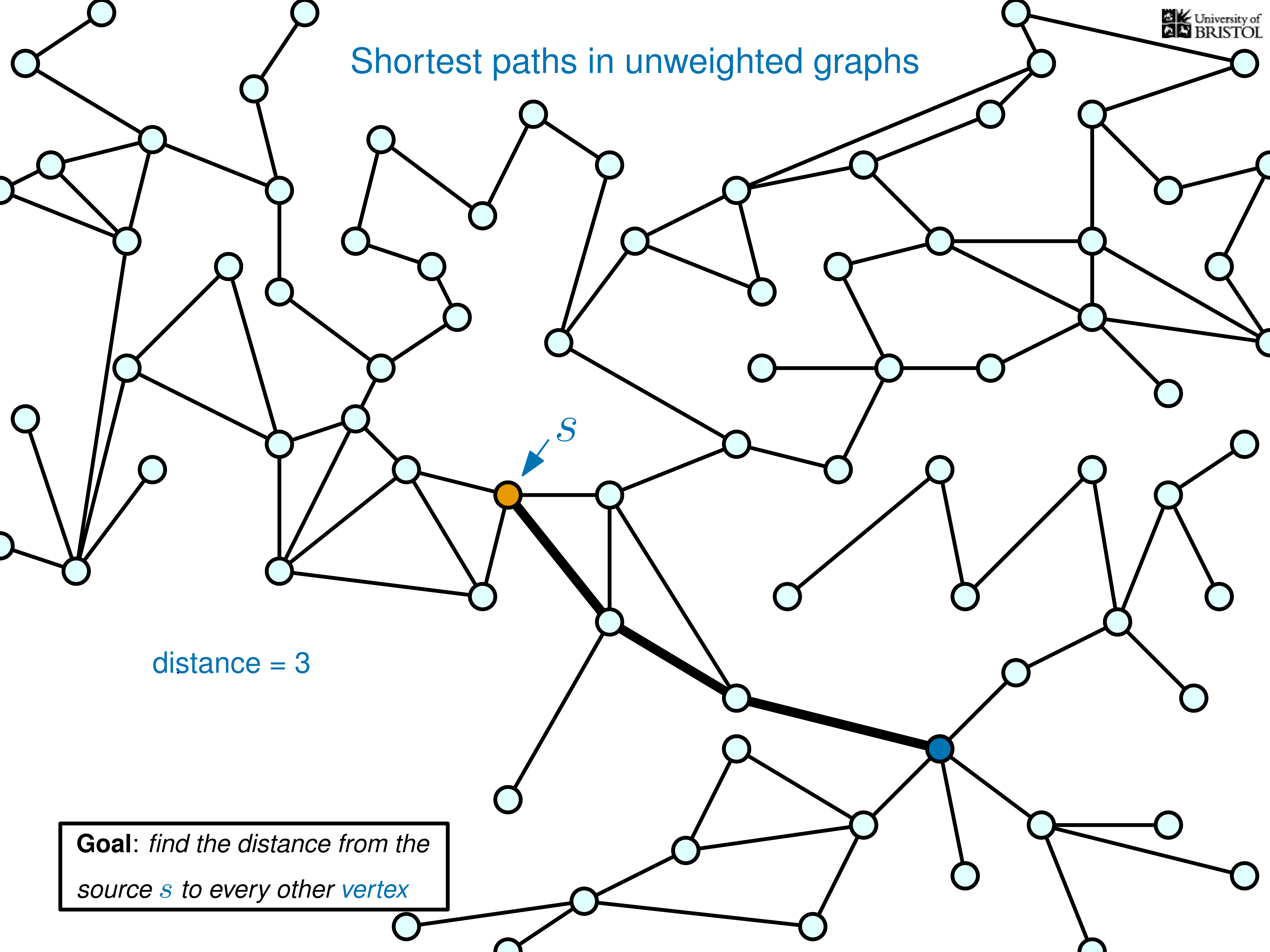
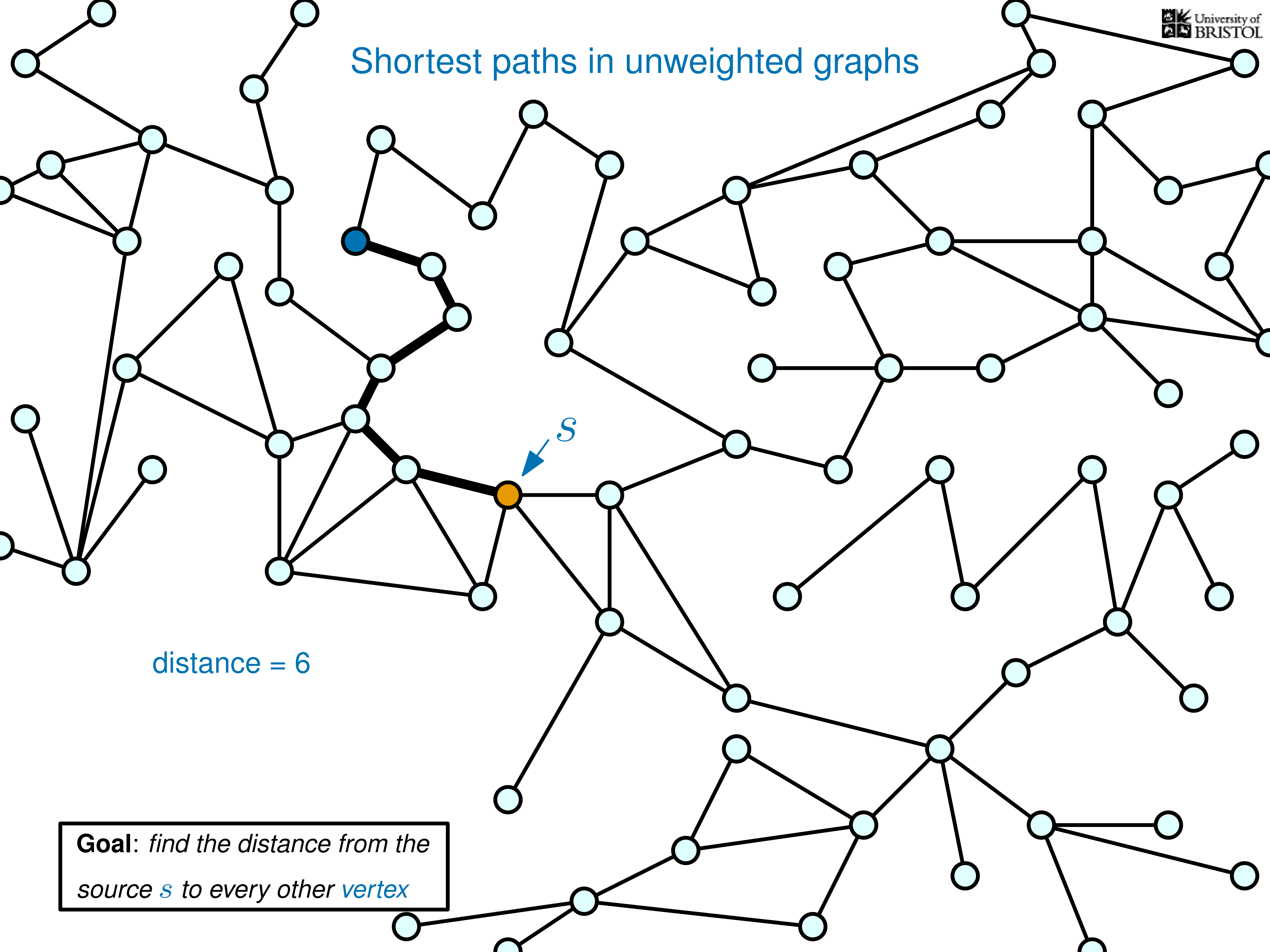
Shortest paths in unweighted graphs



distance = 3

Goal: *find the distance from the source s to every other vertex*

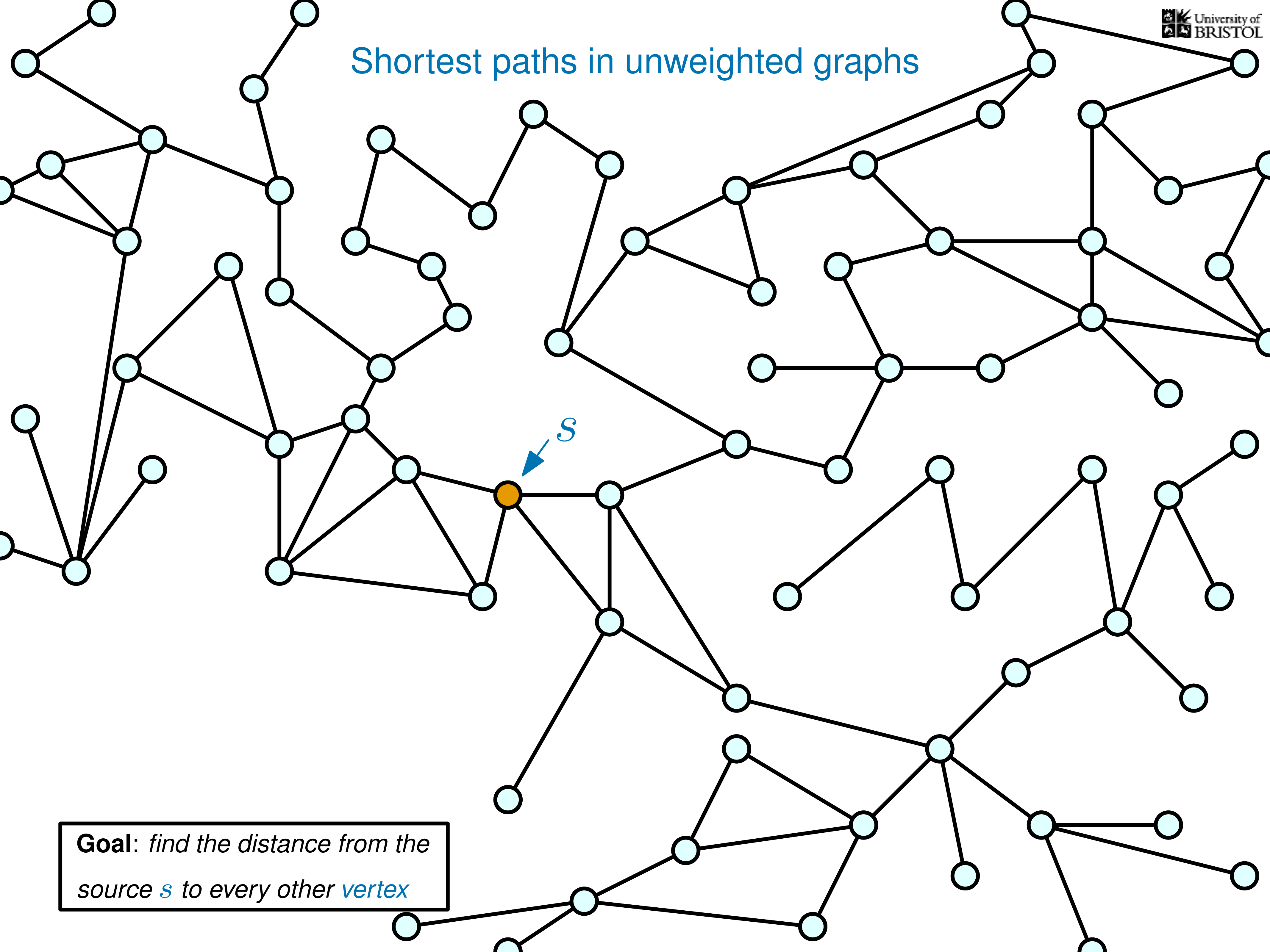
Shortest paths in unweighted graphs



distance = 6

Goal: *find the distance from the source s to every other vertex*

Shortest paths in unweighted graphs



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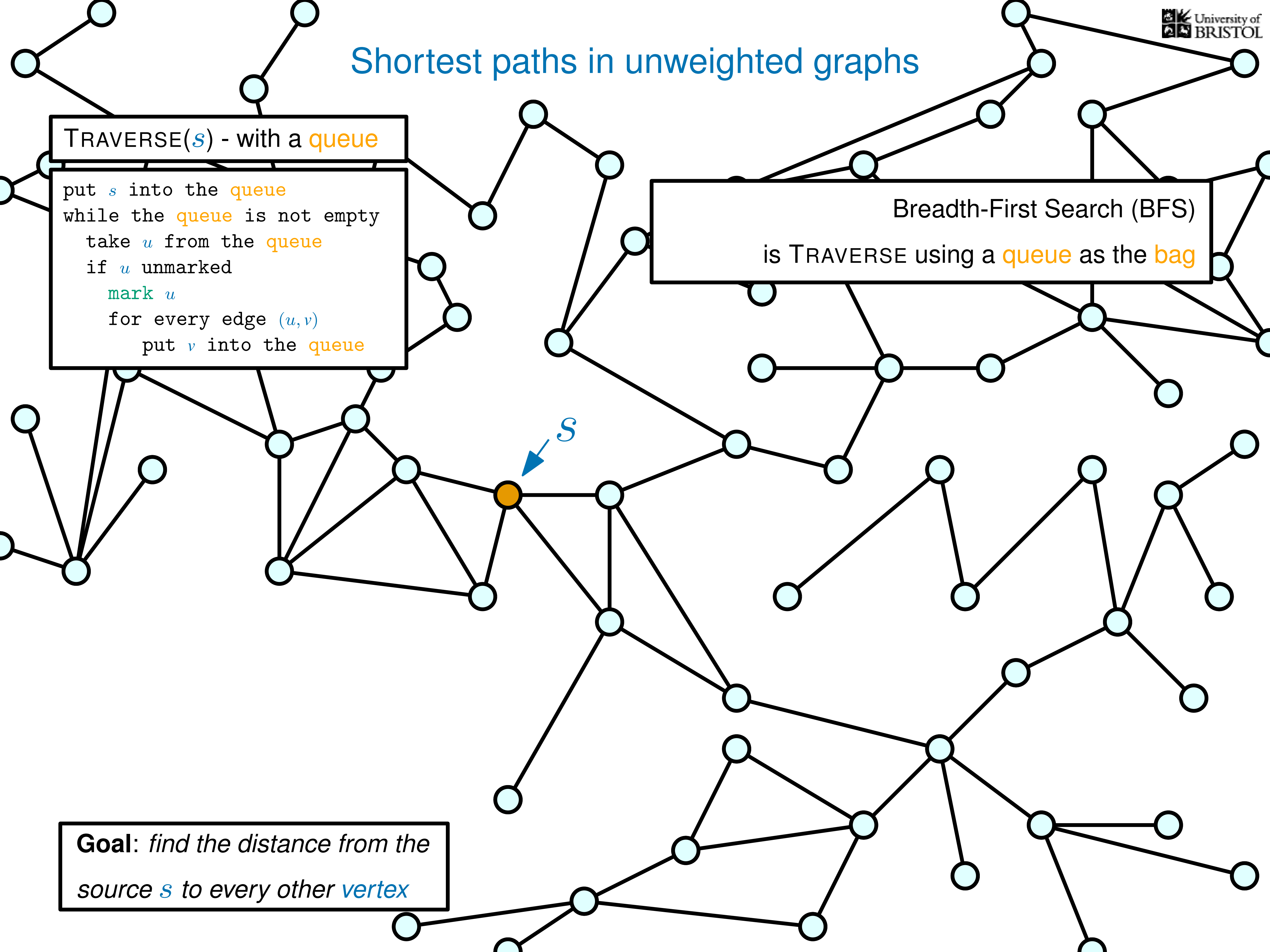
Shortest paths in unweighted graphs



TRAVERSE(s) - with a queue

```
put s into the queue
while the queue is not empty
  take u from the queue
  if u unmarked
    mark u
    for every edge (u, v)
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```
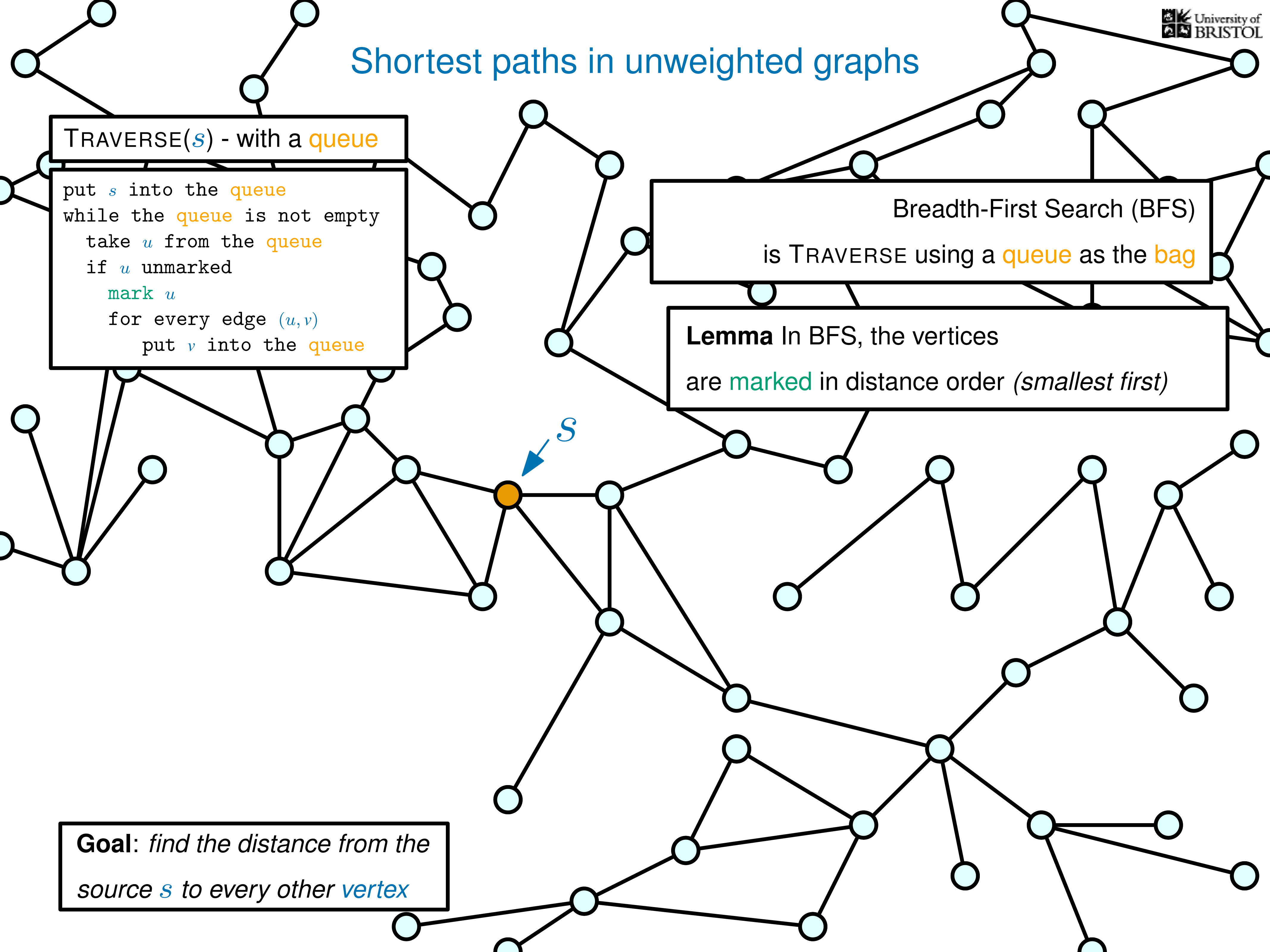

Breadth-First Search (BFS)

is TRAVERSE using a queue as the bag

Goal: *find the distance from the source s to every other vertex*

Shortest paths in unweighted graphs



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Breadth-First Search (BFS)

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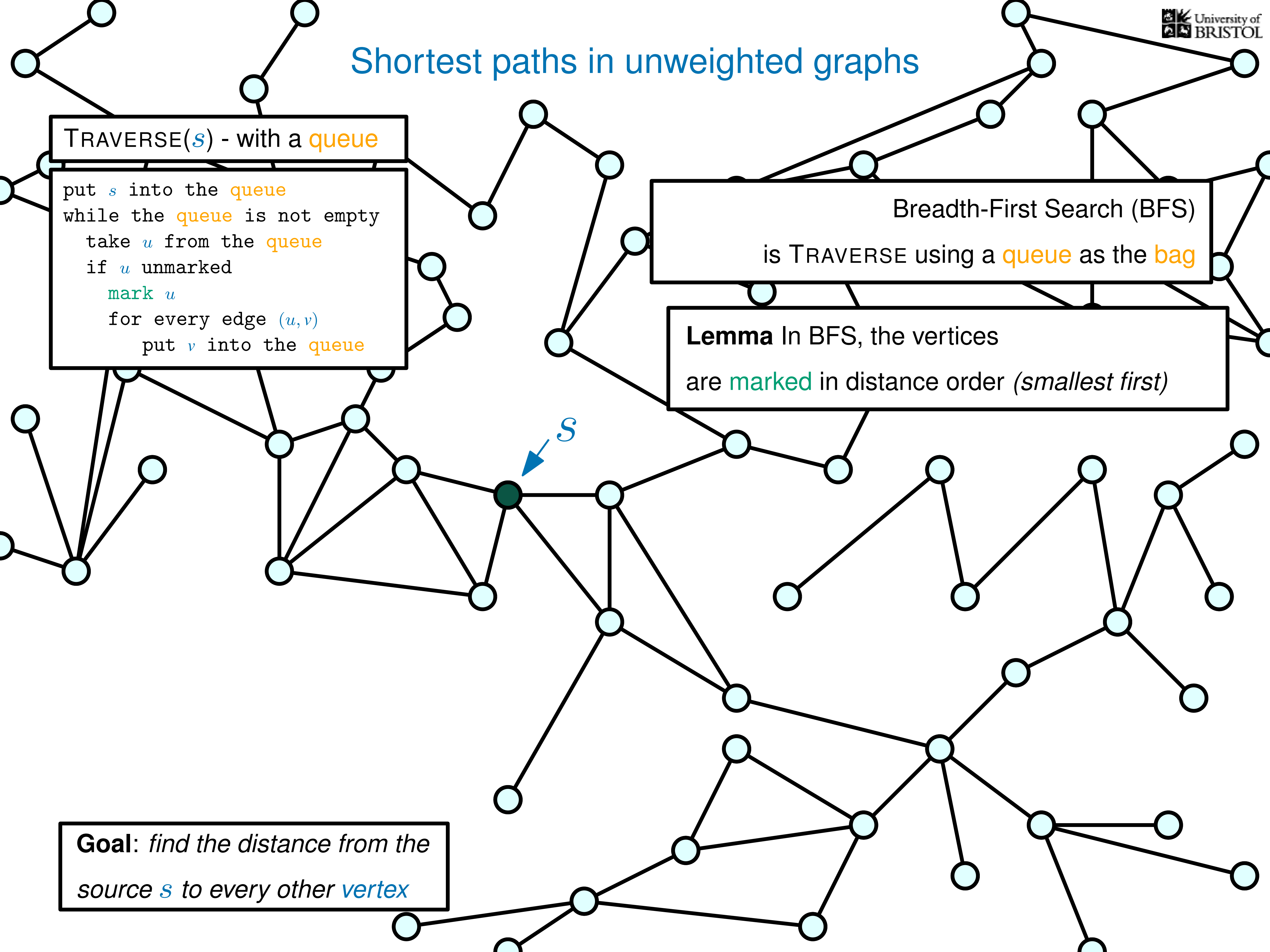
Lemma In BFS, the vertices are marked in distance order *(smallest first)*

Goal: *find the distance from the source s to every other vertex*

Shortest paths in unweighted graphs



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Breadth-First Search (BFS)

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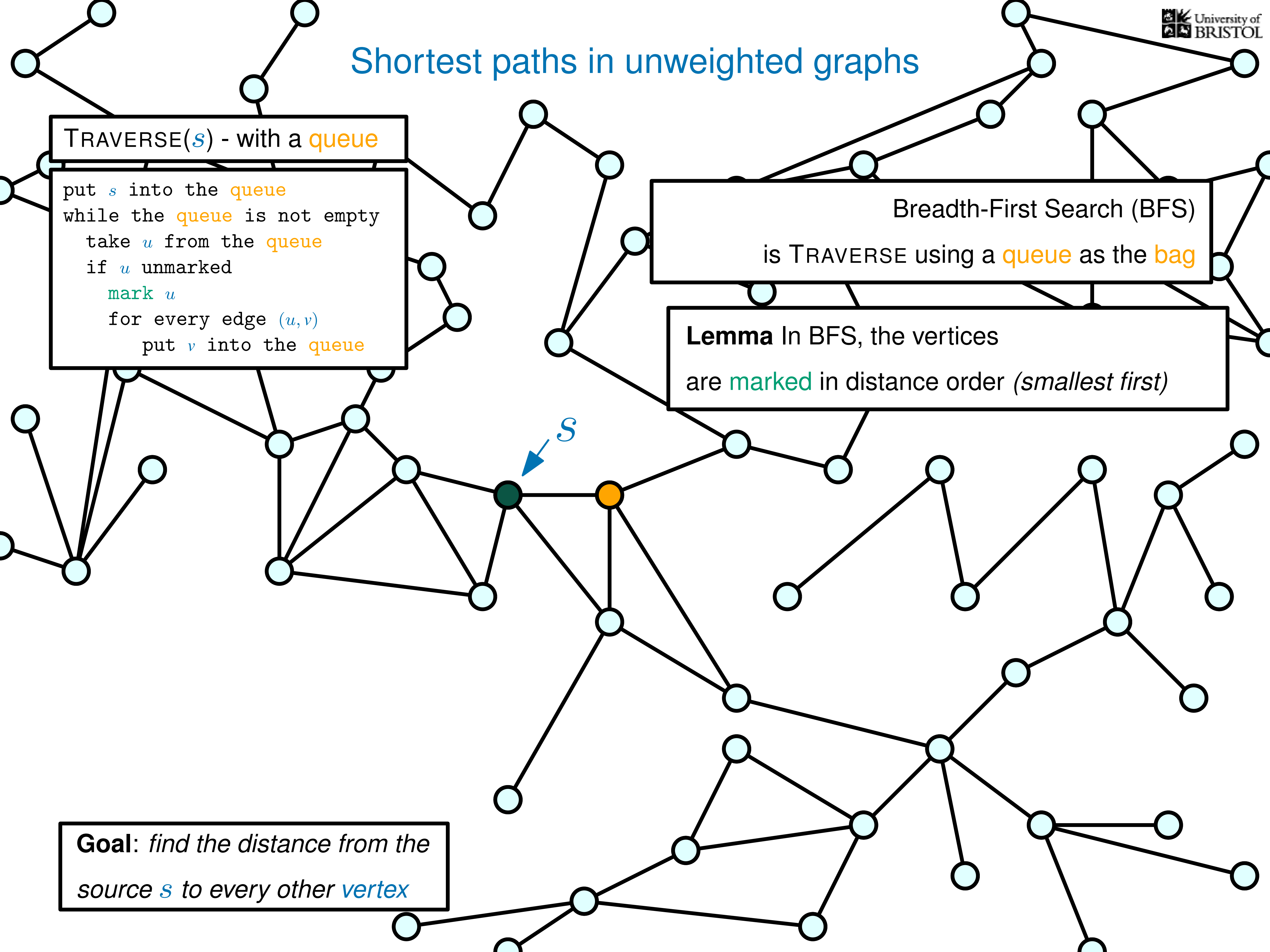
Lemma In BFS, the vertices are marked in distance order *(smallest first)*

Goal: *find the distance from the source* s *to every other vertex*

University of BRISTOL

Shortest paths in unweighted graphs



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Shortest paths in unweighted graphs

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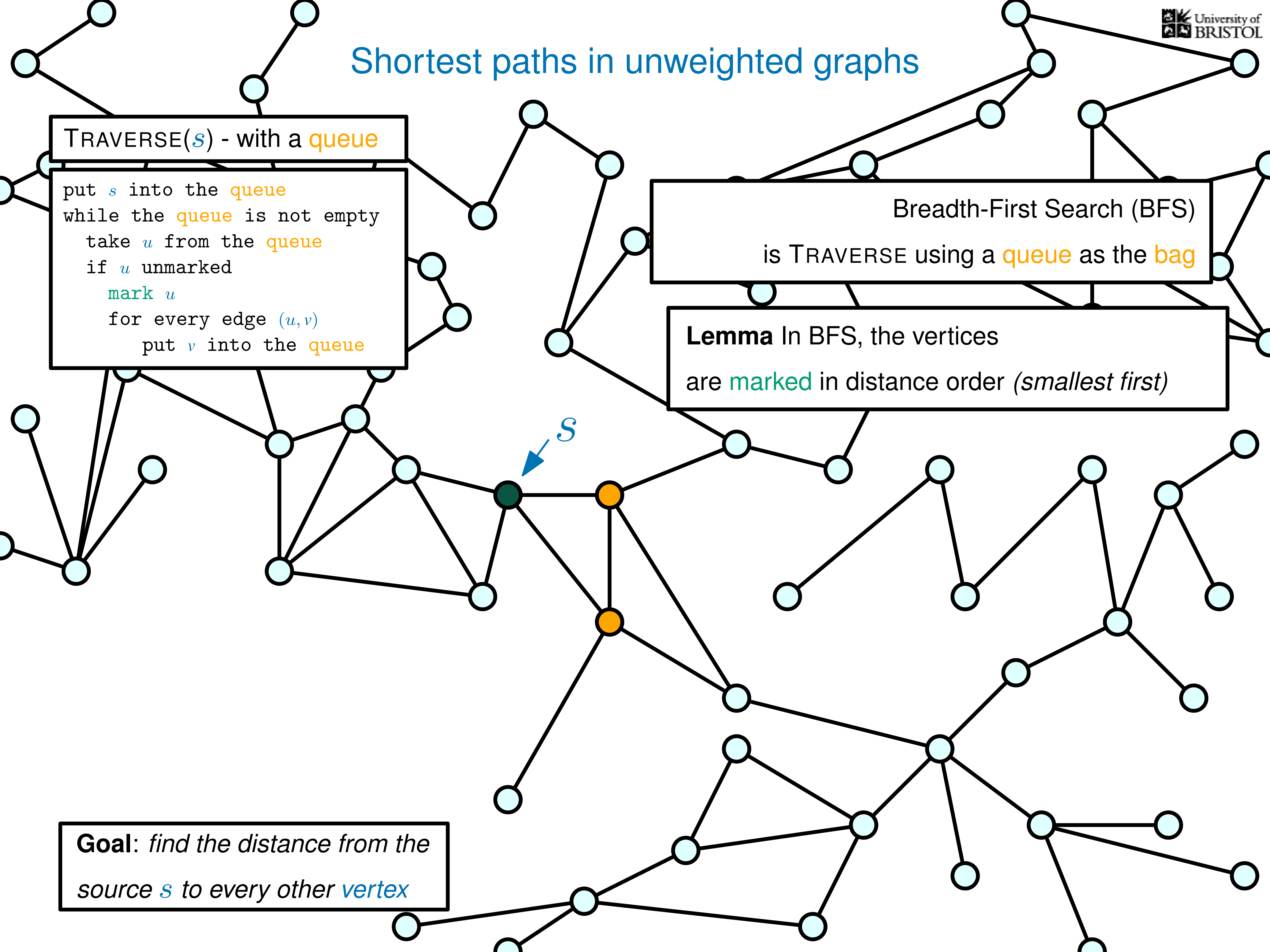

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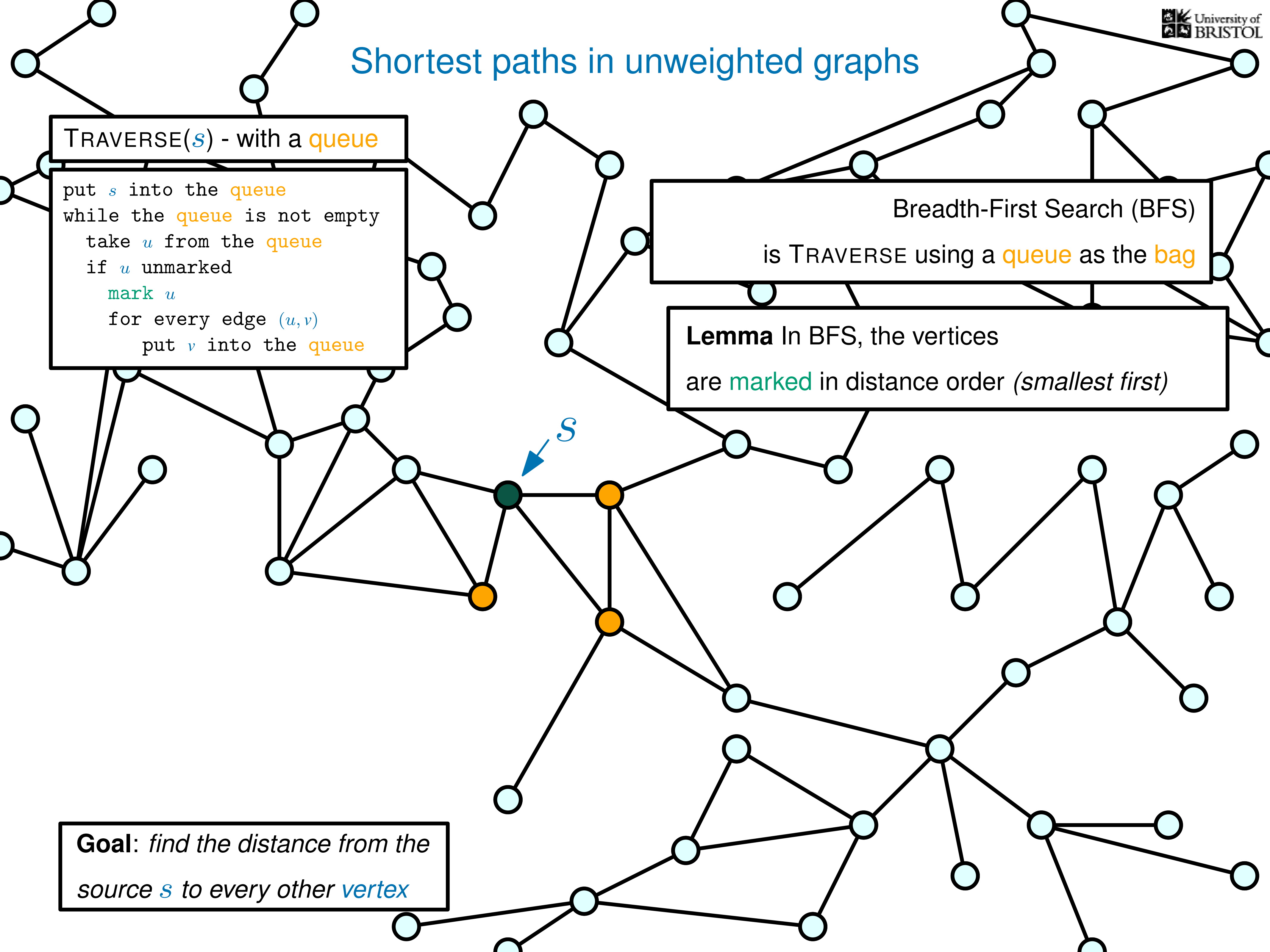
Shortest paths in unweighted graphs



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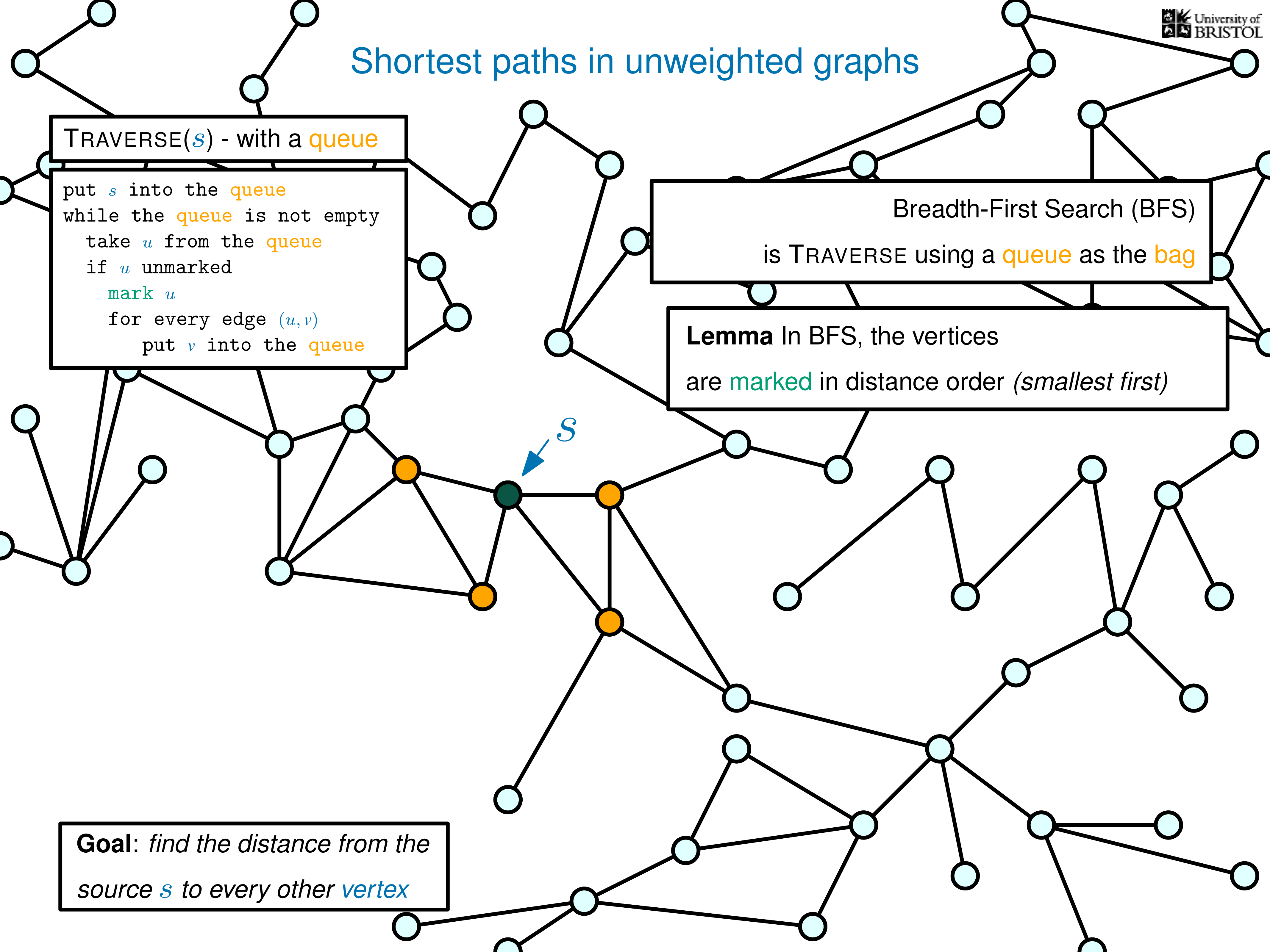

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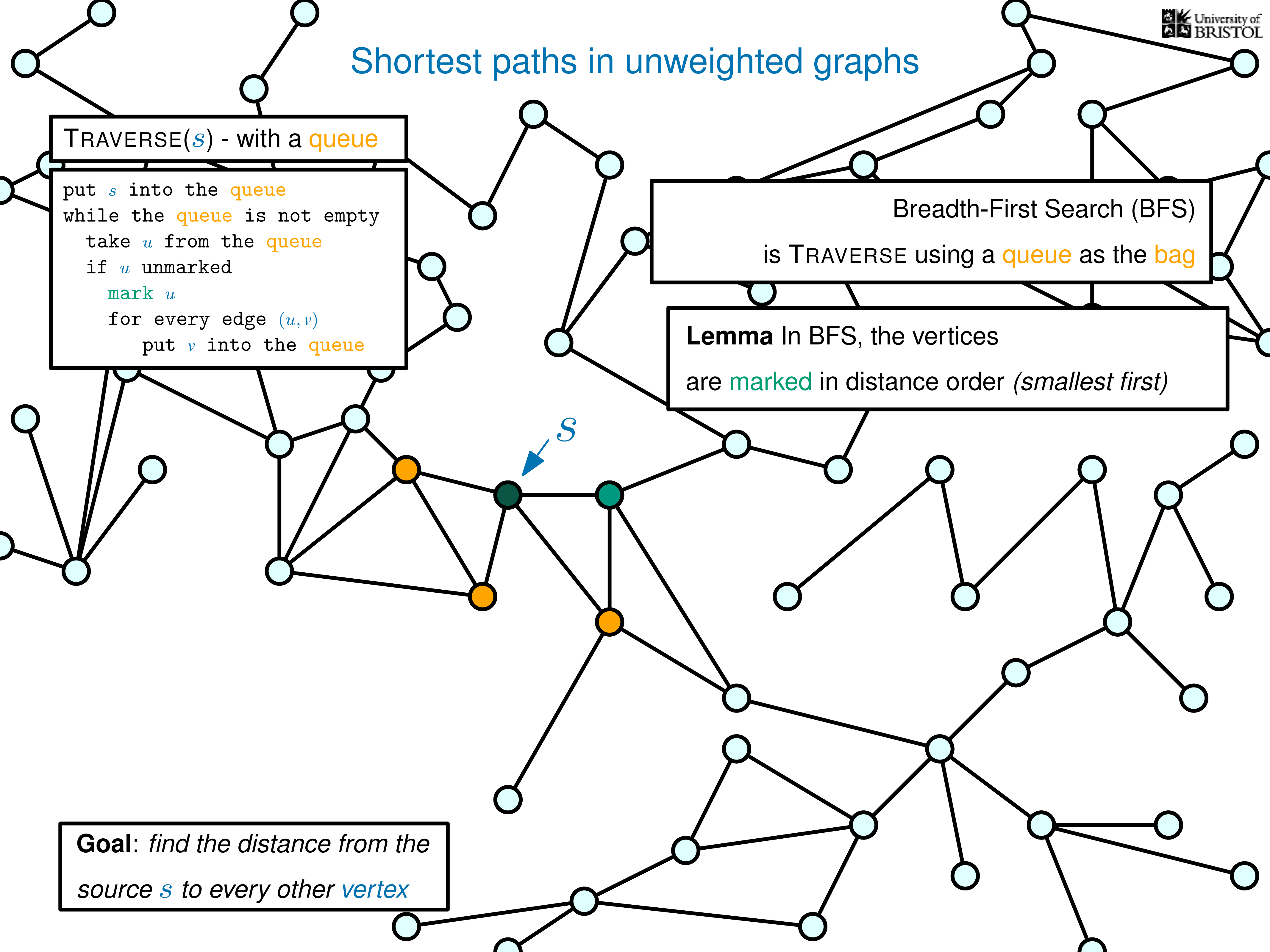

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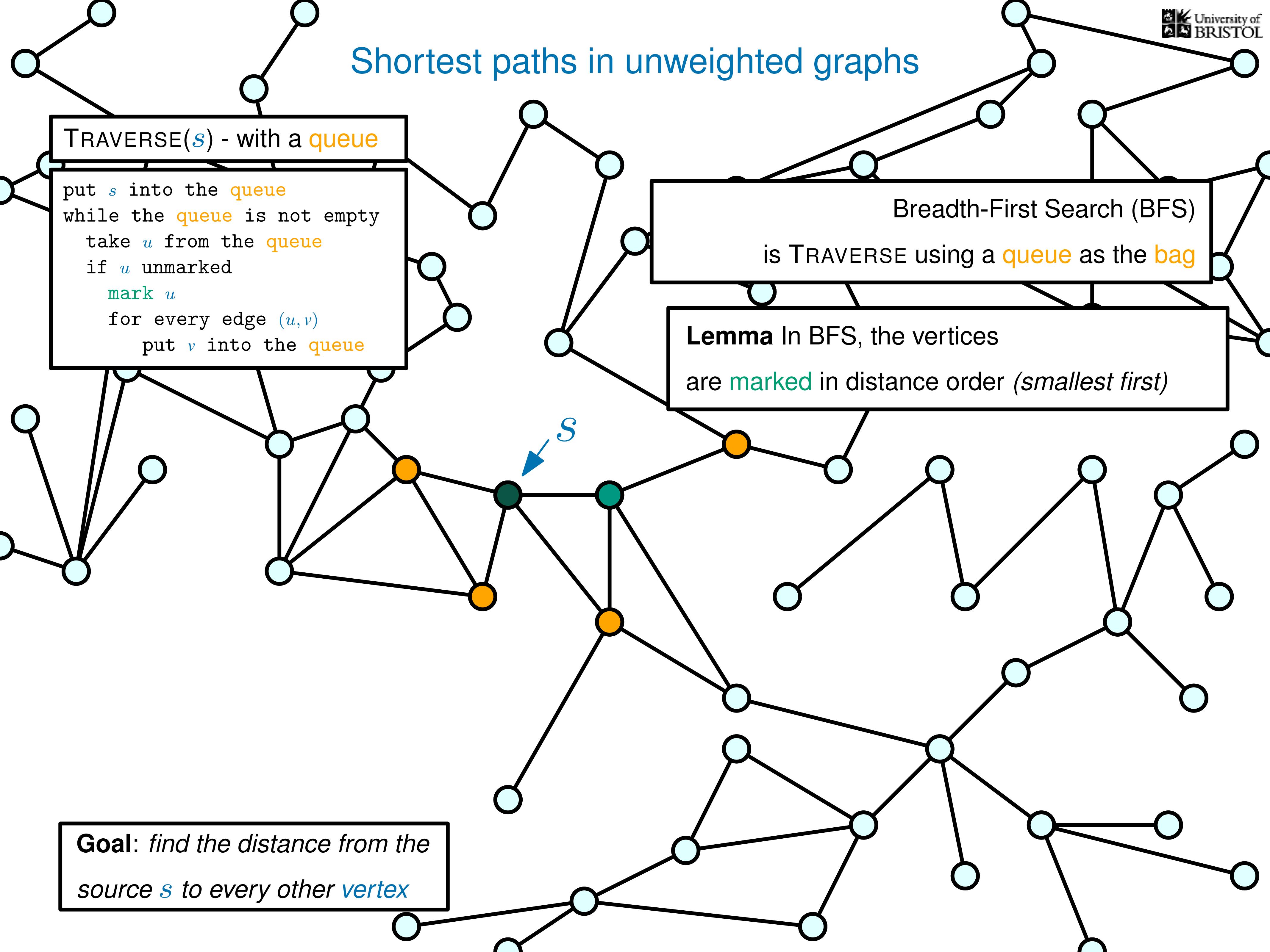

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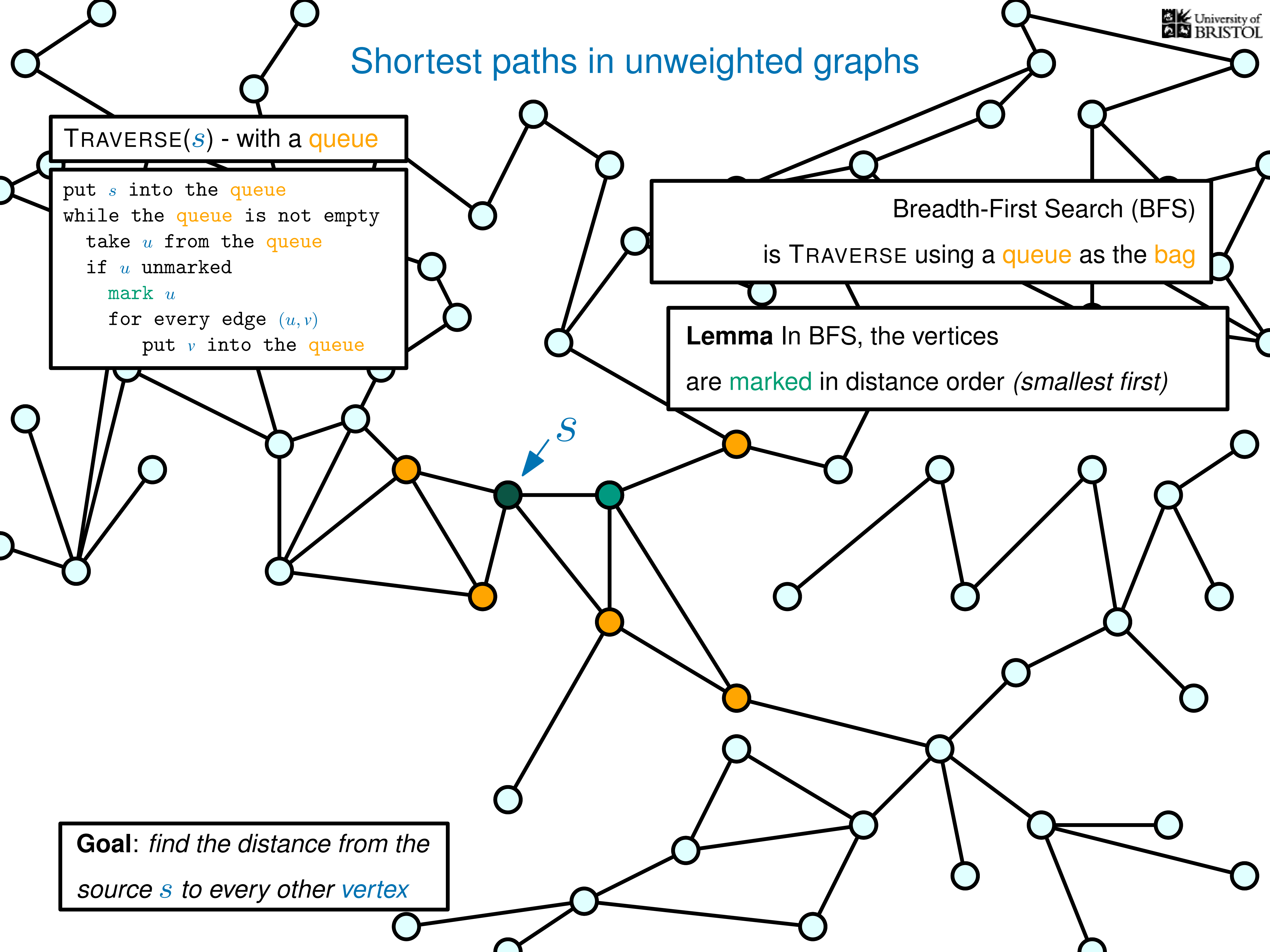

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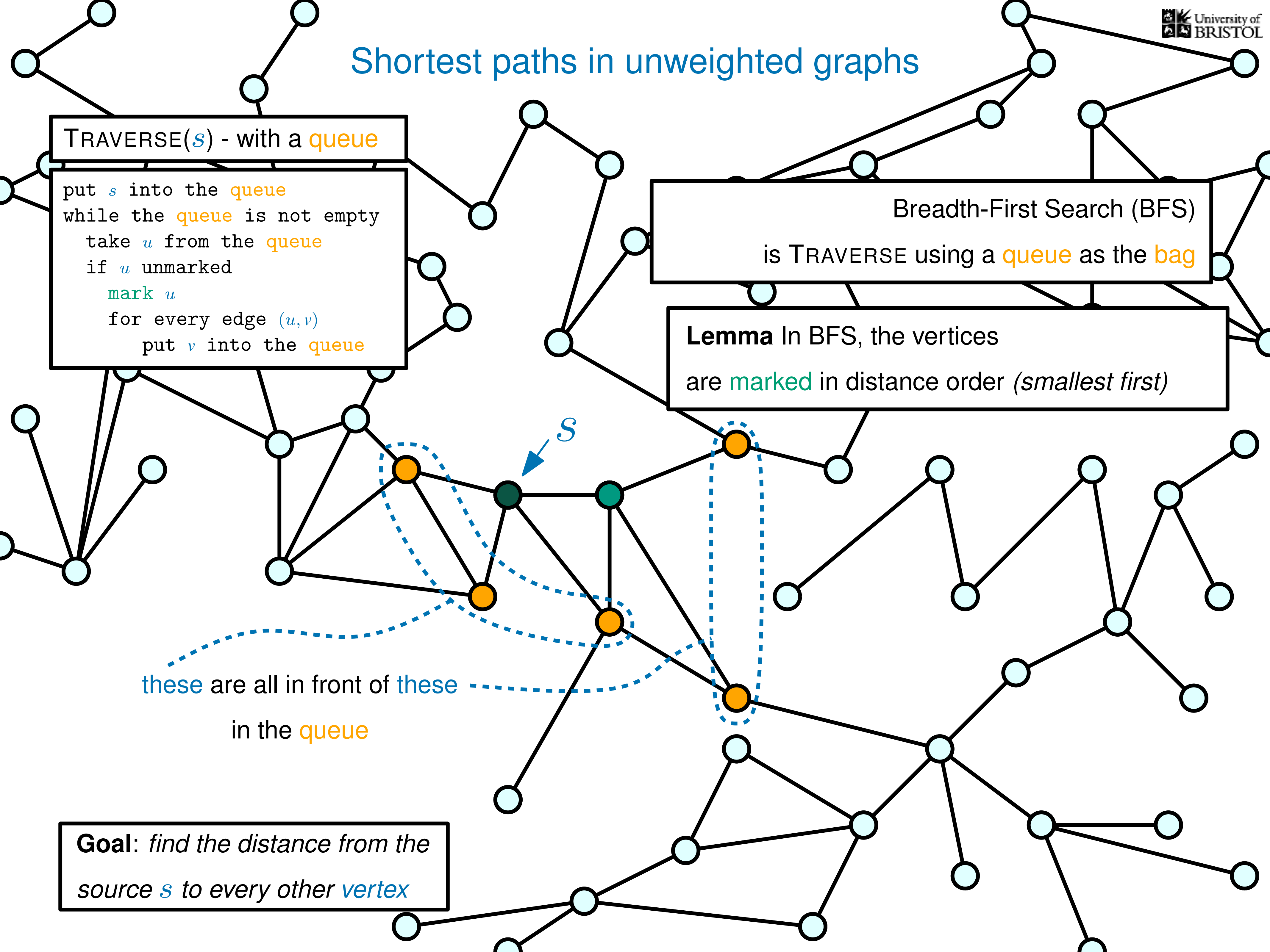

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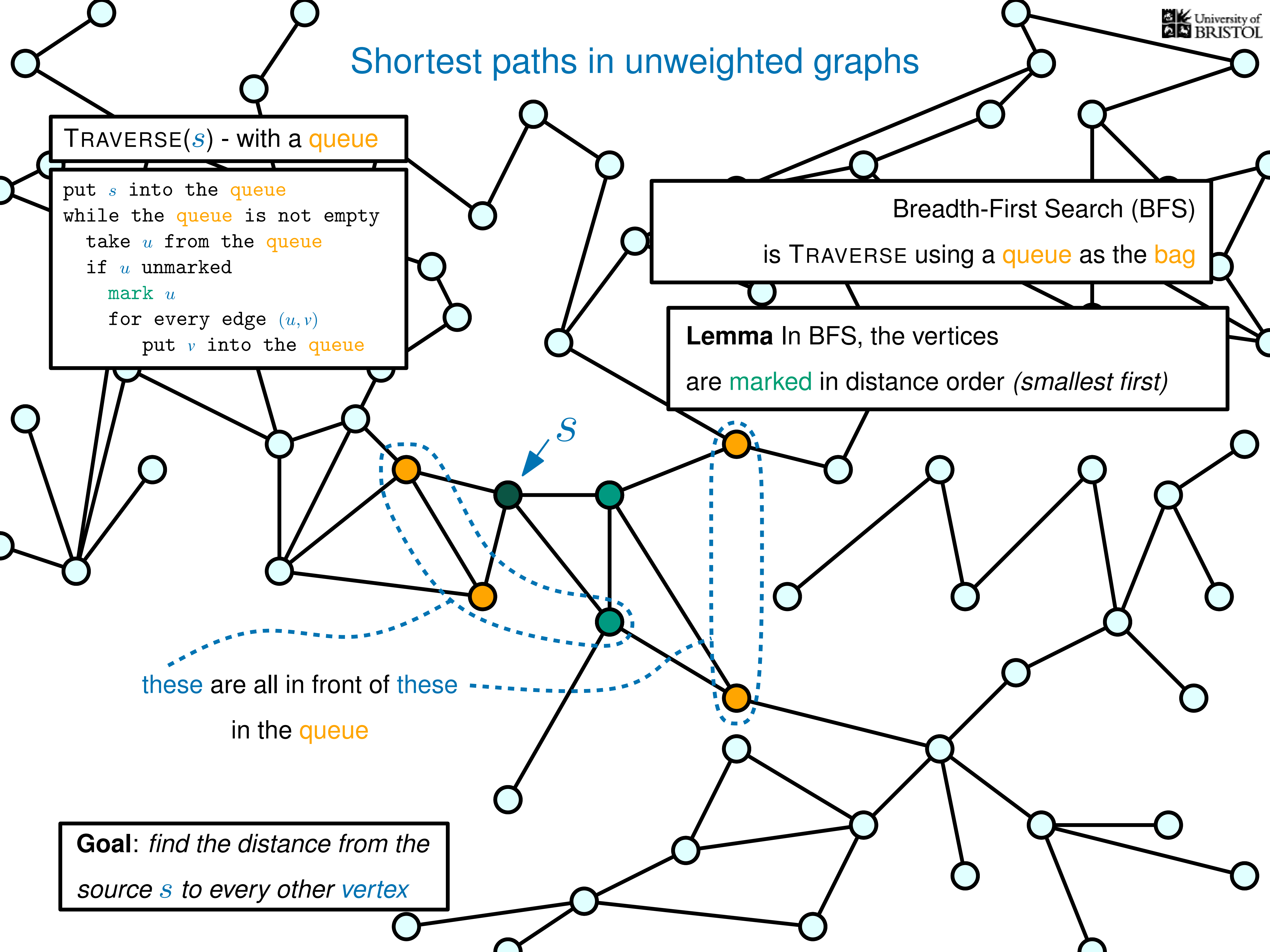

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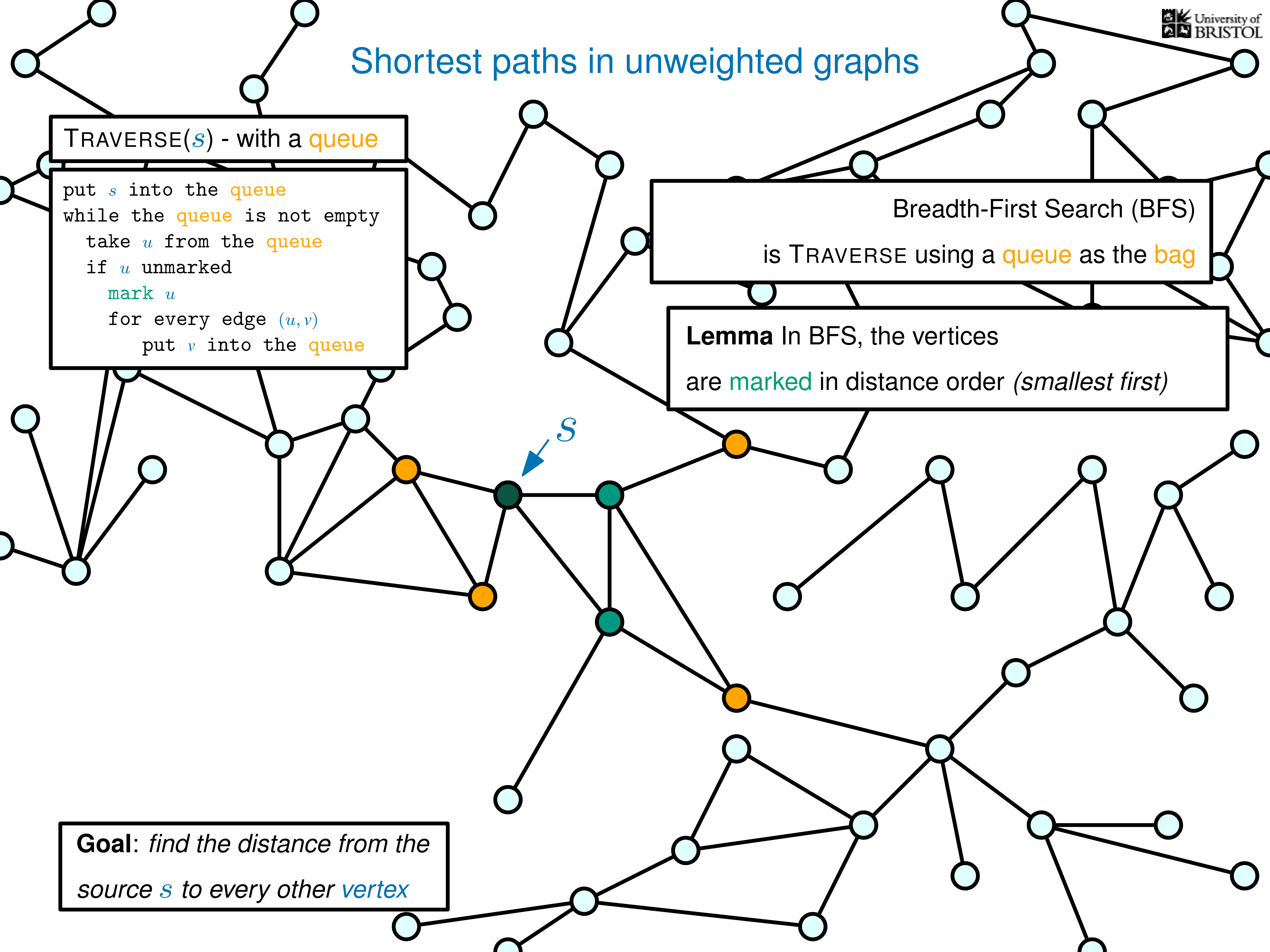

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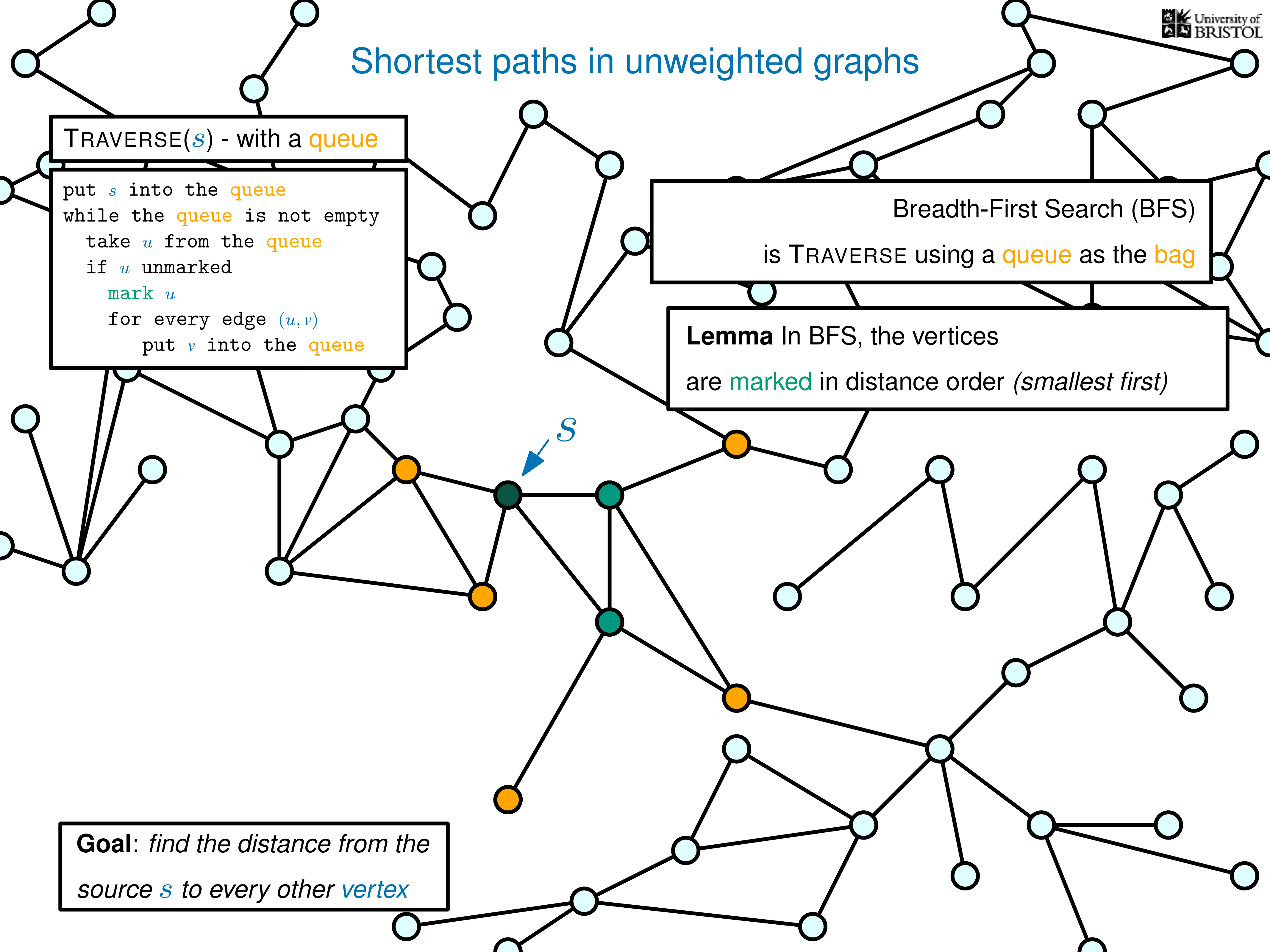

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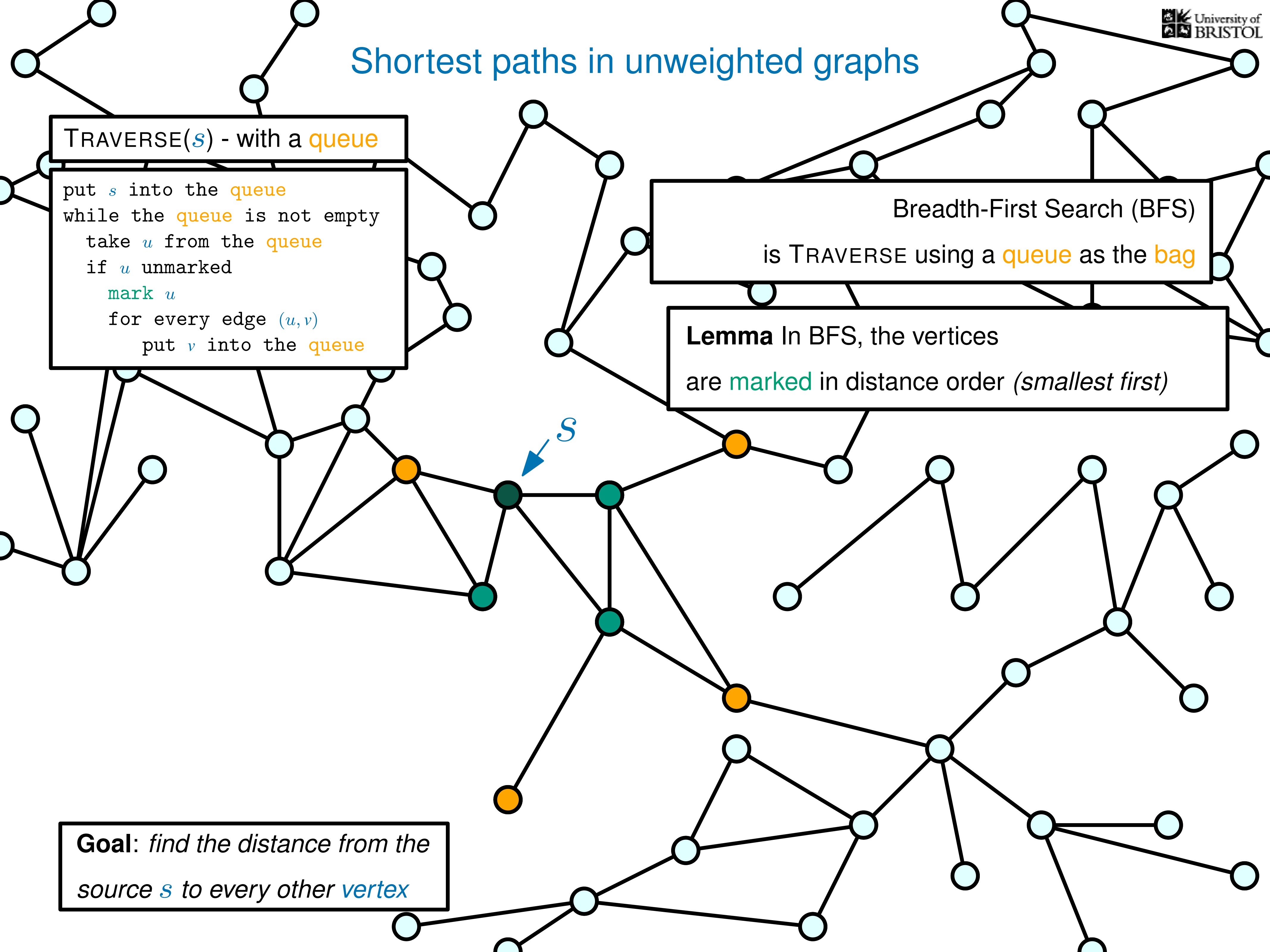

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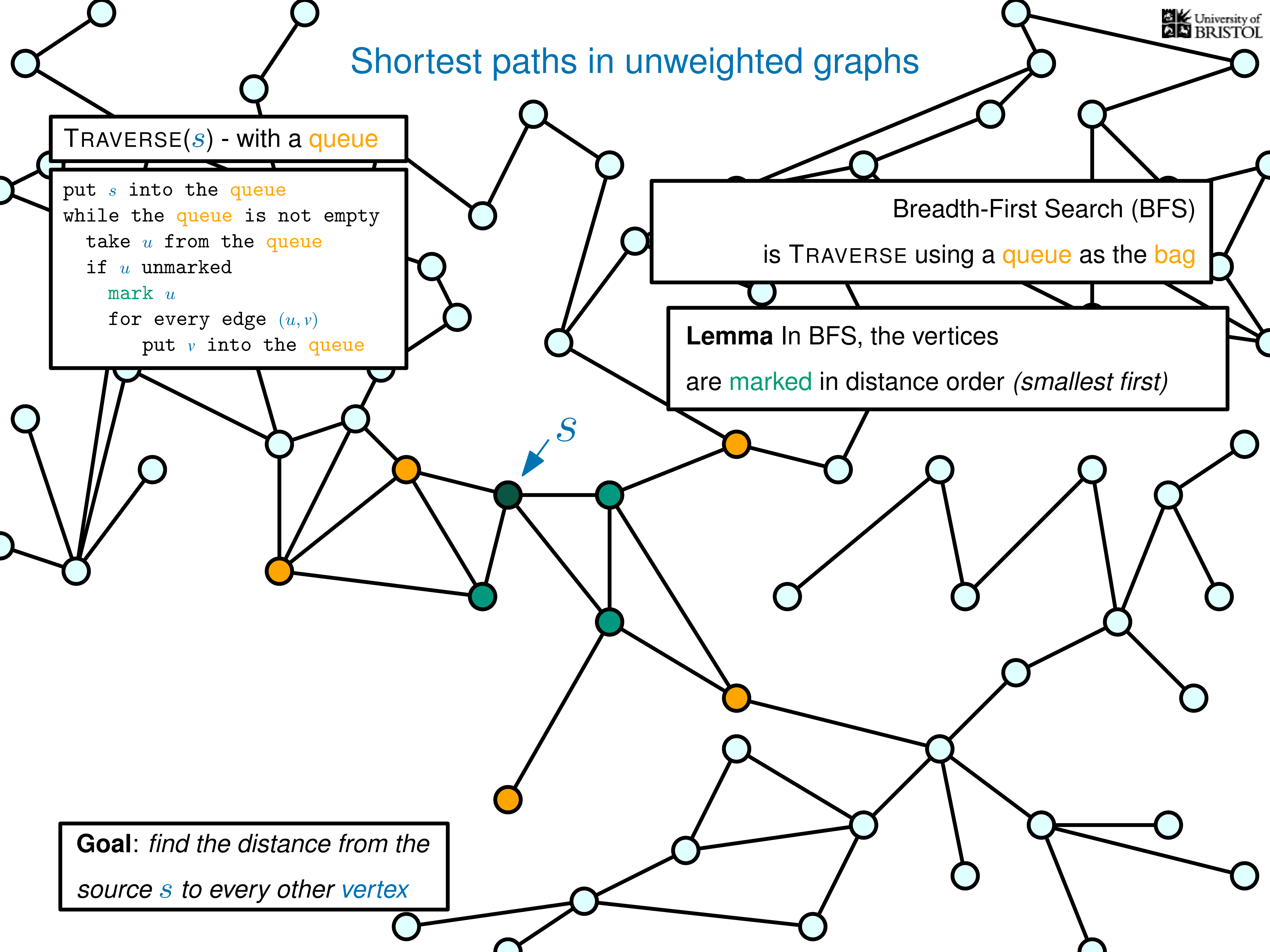

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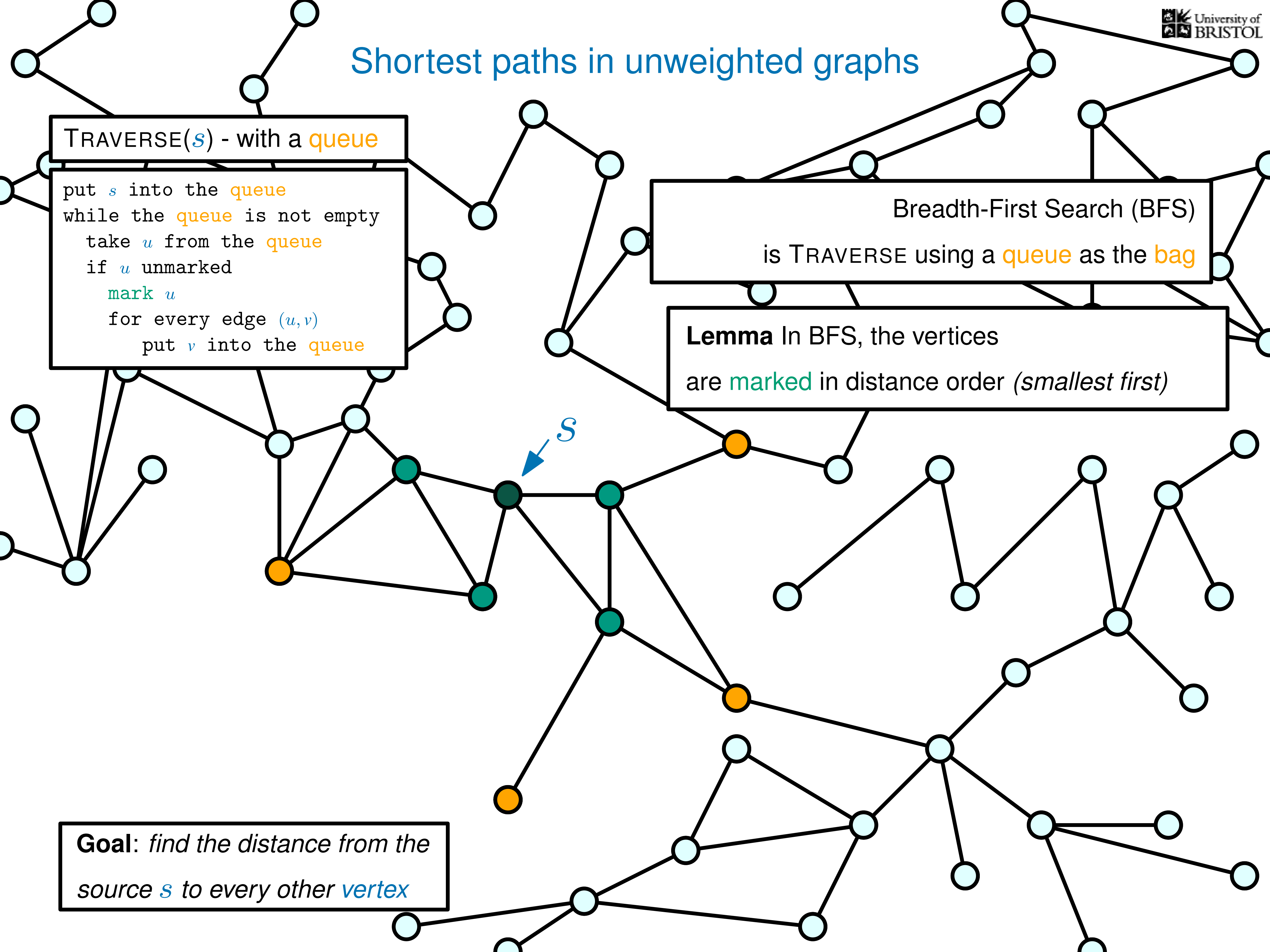

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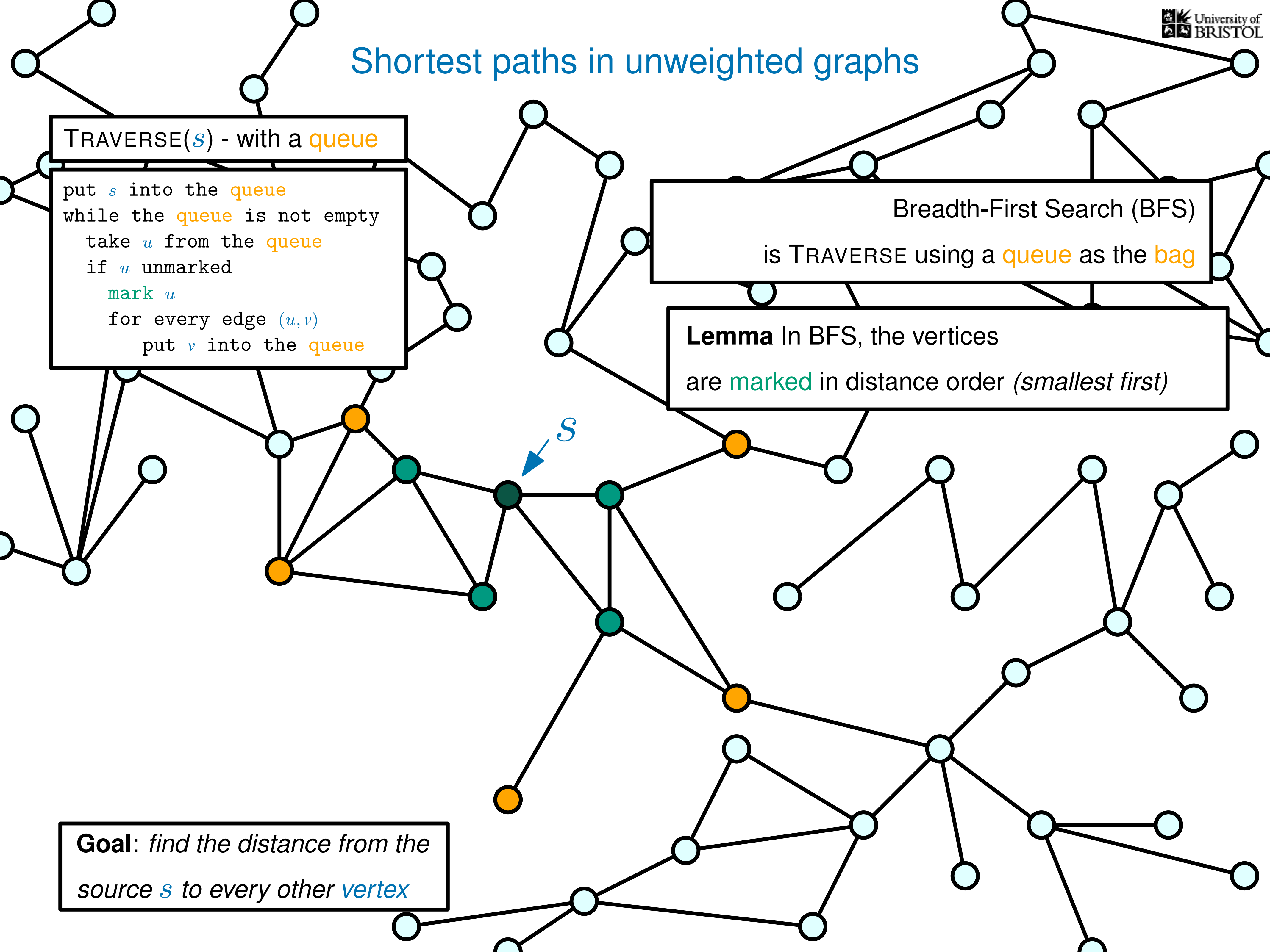

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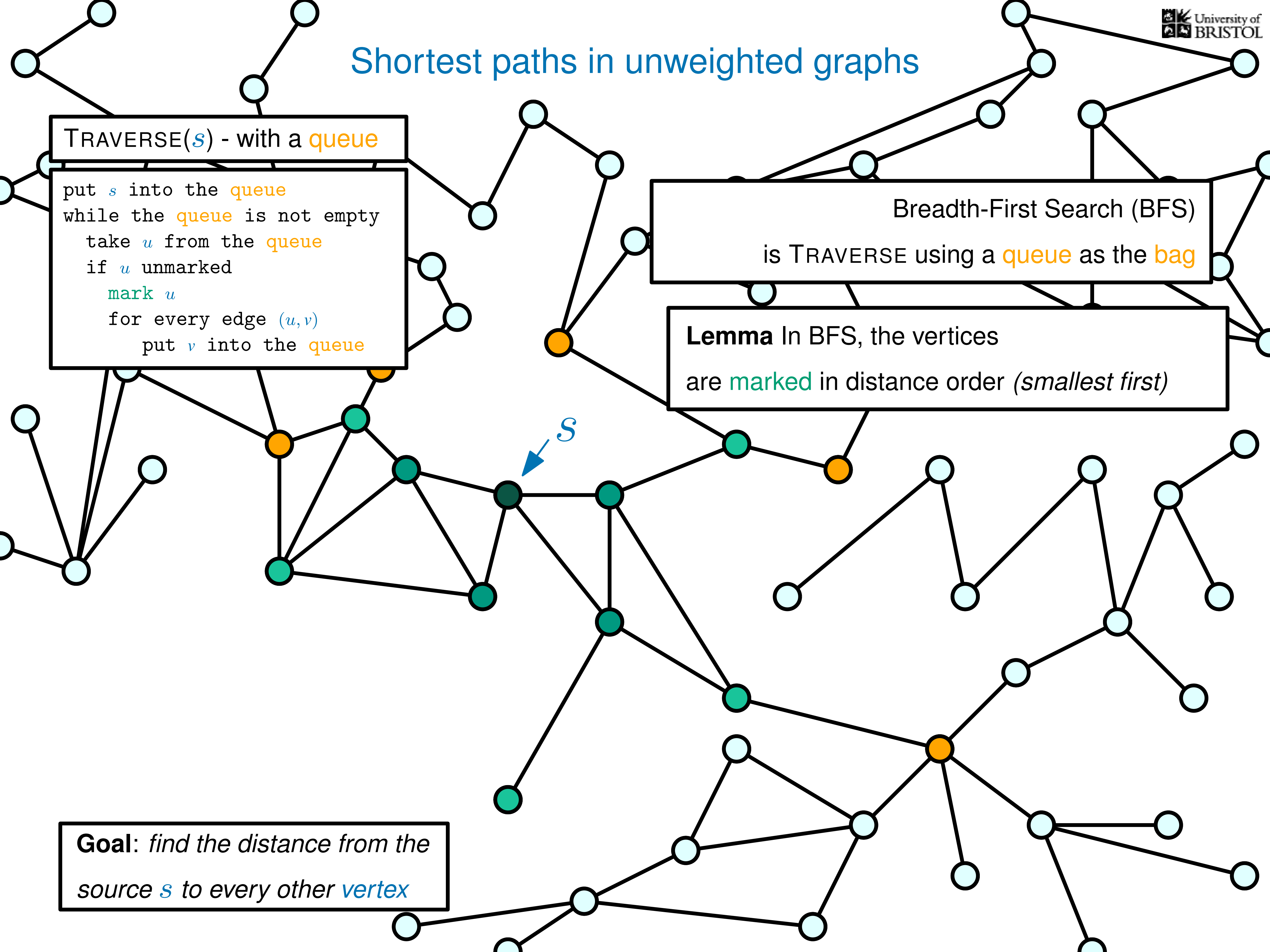

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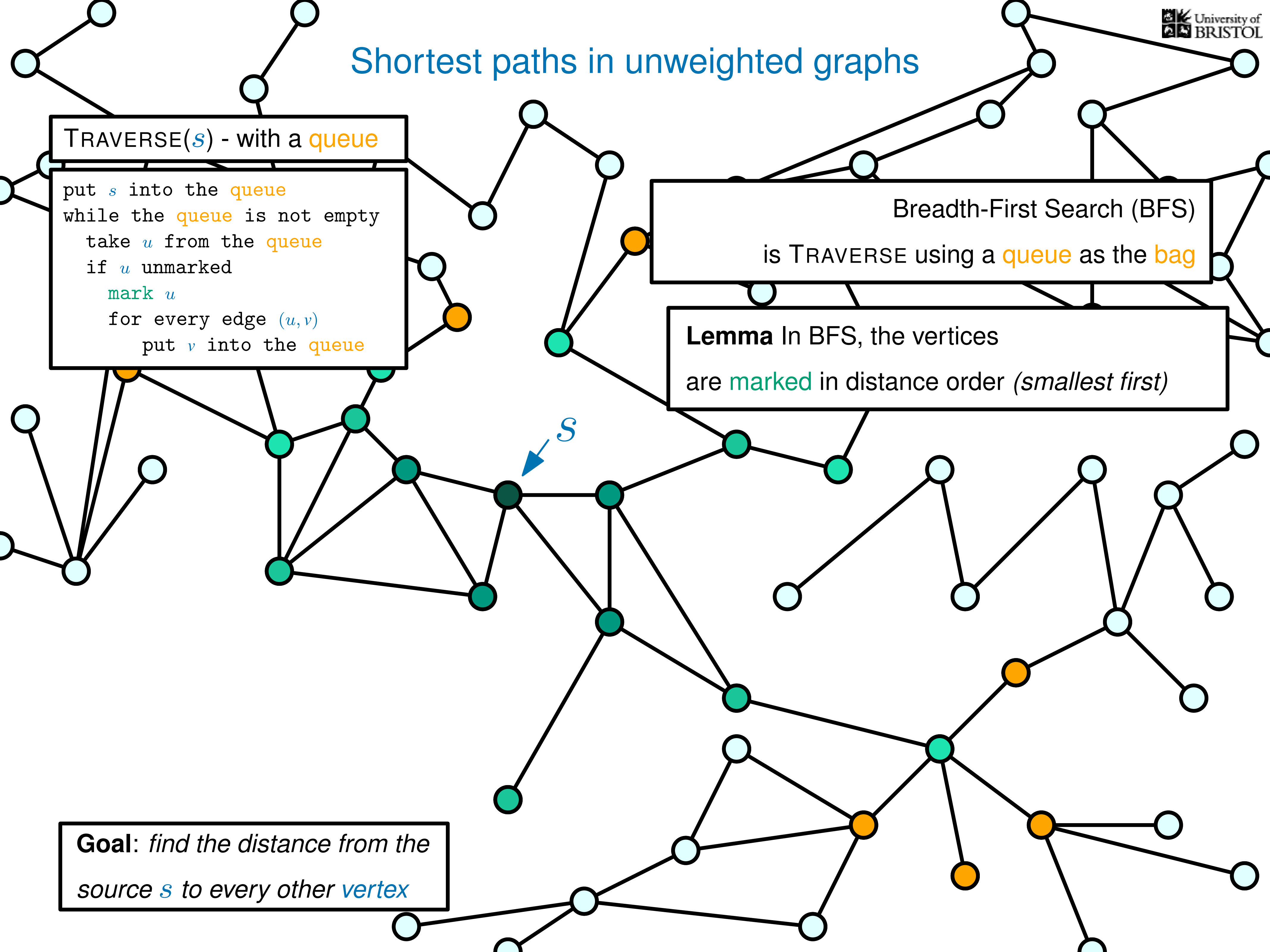

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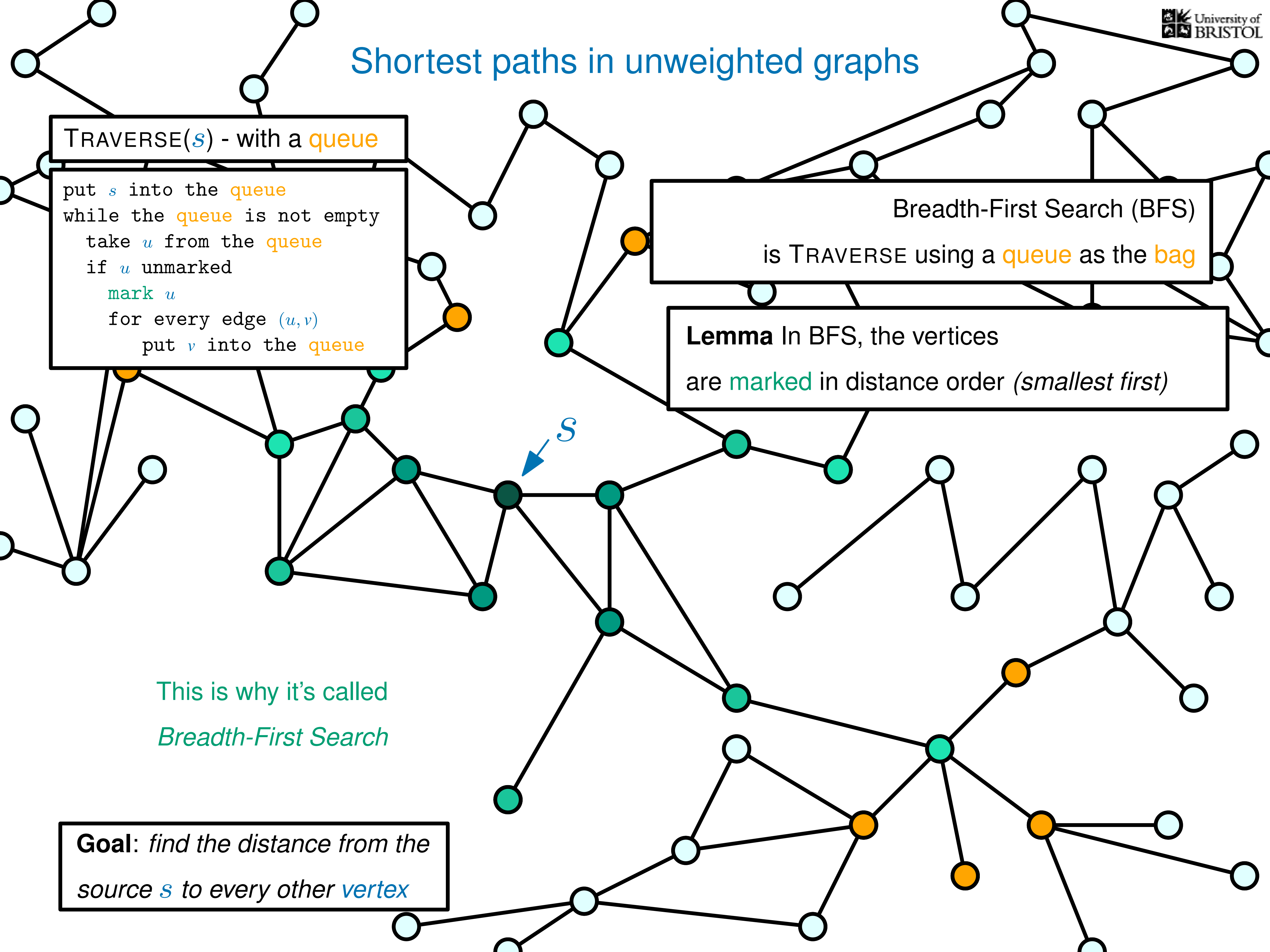

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This is why it's called *Breadth-First Search*

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Base Case: BFS marks all vertices at distance 0 from s before marking any vertex at distance > 0 .

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Therefore, *all* vertices at distance i will be marked before *any* vertex at distance $> i$

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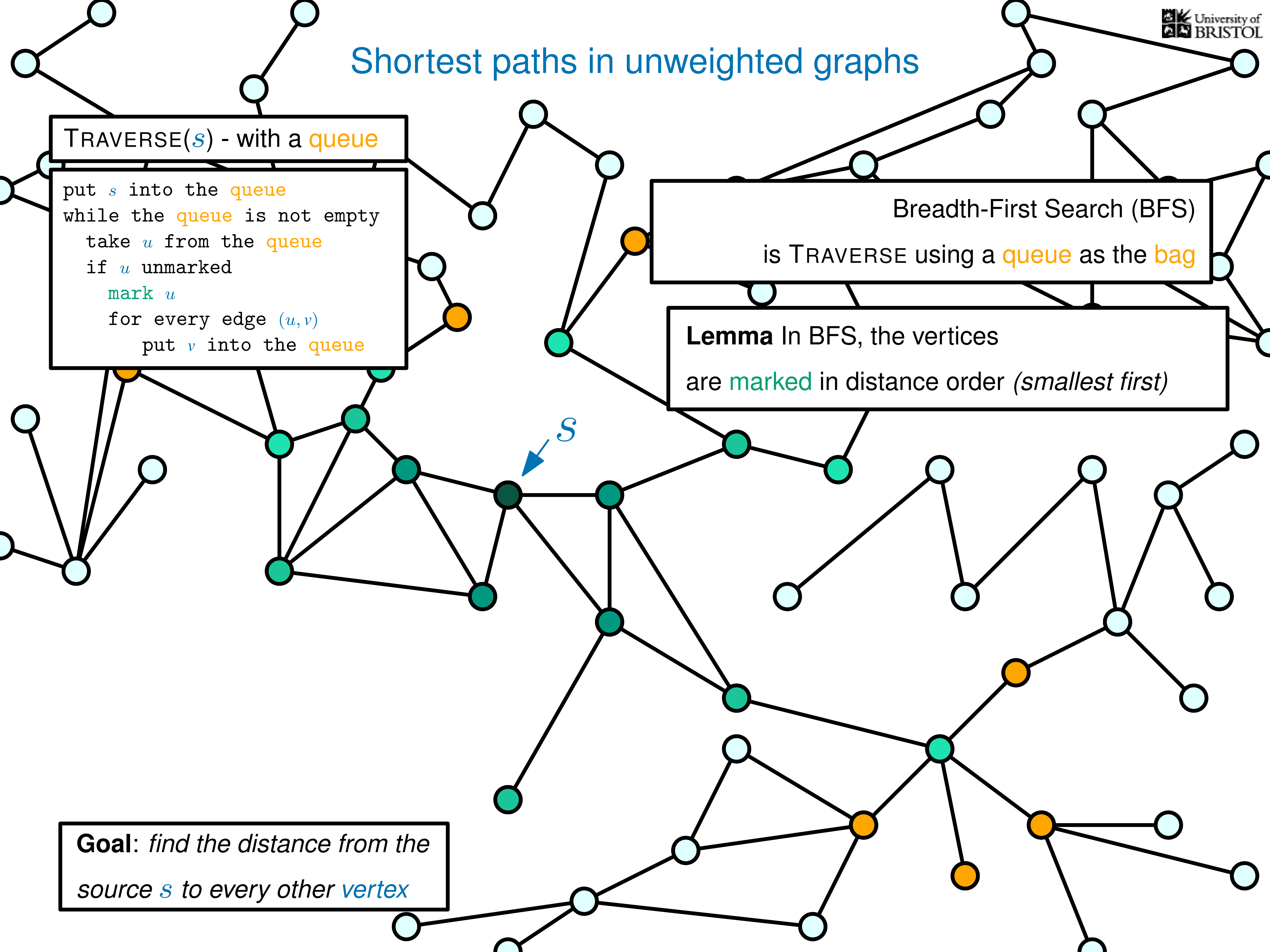
Shortest paths in unweighted graphs



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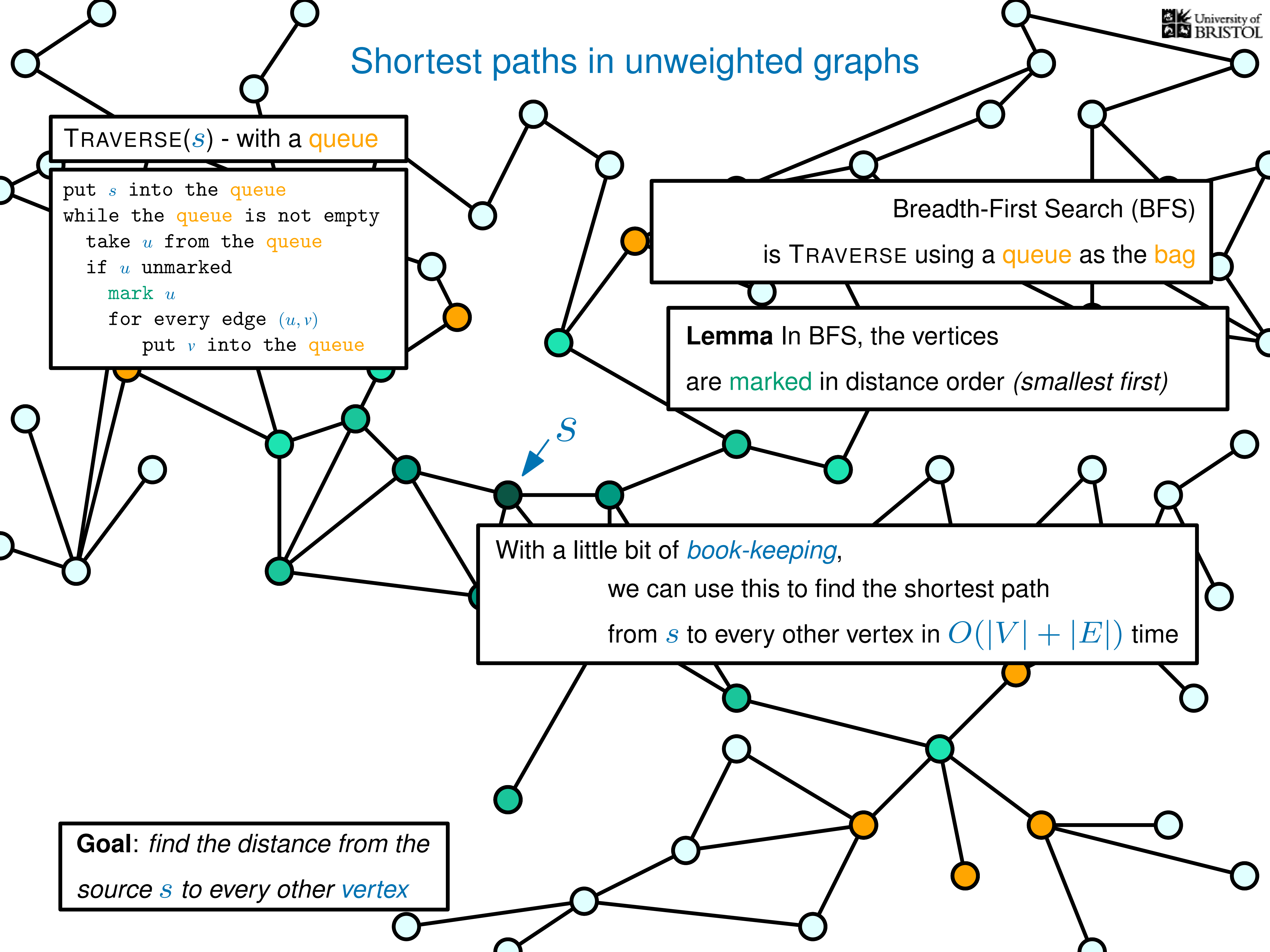

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With a little bit of *book-keeping*, we can use this to find the shortest path from s to every other vertex in $O(|V| + |E|)$ time

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It does not work for weighted graphs

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Shortest Paths using BFS

We've added four (NEW!) lines to TRAVERSE

to track the distances from s

BFS(s)

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for all v, set dist(v) = ∞ (NEW!)
set dist(s) = 0 (NEW!)
put s into the queue
while the queue is not empty
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       if dist(v) = ∞ (NEW!)
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$\mathtt{dist}(v)$ gives the distance

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How does this affect the time complexity?

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Time complexity:

$O(|V|)$ time to initialise an array dist

$O(1)$ time to set the distance to s

$\texttt{dist}(v)$ gives the distance between s and v

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$O(|V|)$ time to initialise an array dist

$O(1)$ time to set the distance to s

$O(1)$ time whenever we put into the queue

(so this doesn't change the complexity)

dist(v) gives the distance

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Shortest Paths using BFS

We've added four (NEW!) lines to TRAVERSE

to track the distances from s

How does this affect the time complexity?

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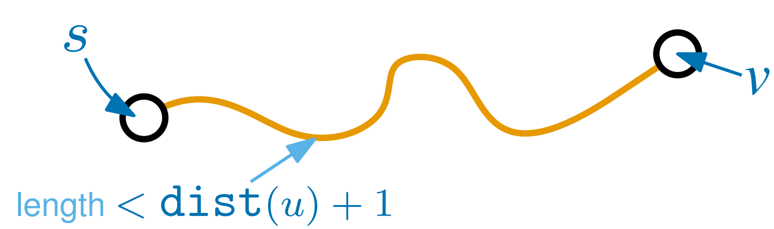

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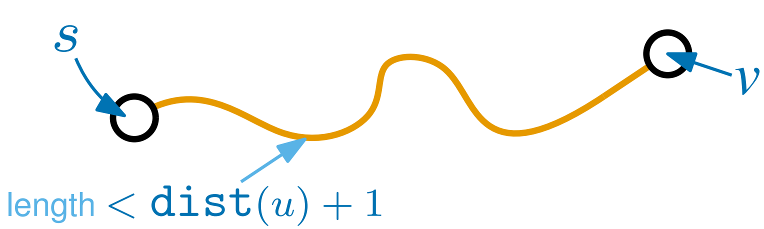

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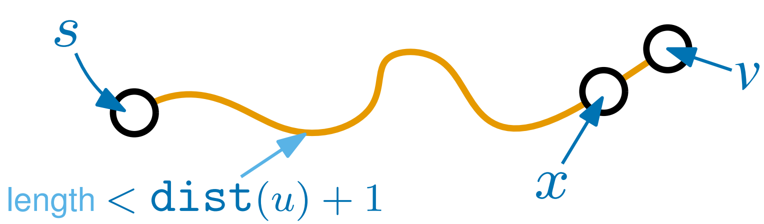

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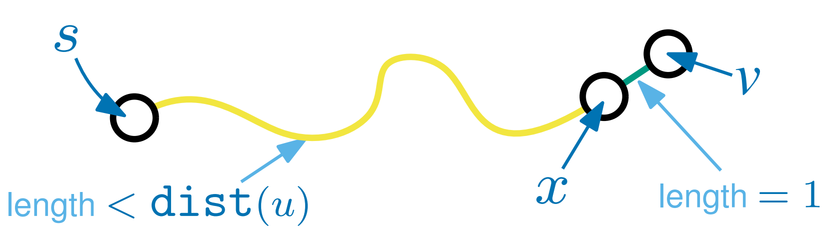

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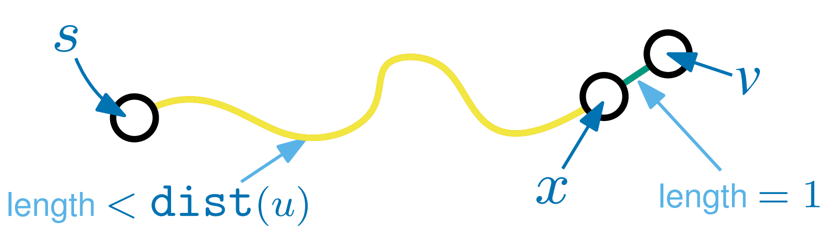

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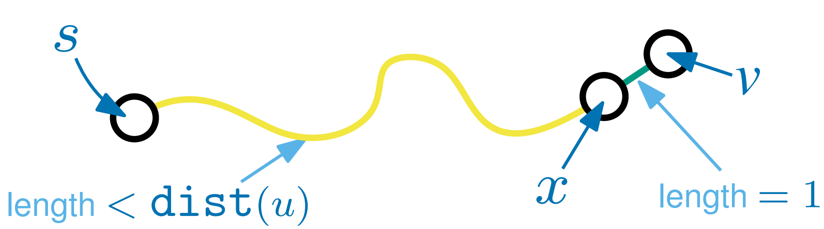

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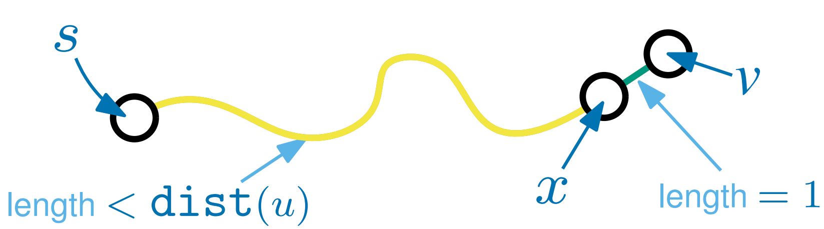

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Contradiction!

V is the set of vertices
$|V|$ *is the number of vertices*

Conclusion

E is the set of edges
$|E|$ *is the number of edges*

TRAVERSE visits every vertex in a connected graph in $O(|E|)$ time

(with an $O(1)$ time bag)

- the traversal order depends on the type of bag

(and it works for directed graphs too)

with a Queue the algorithm is called

Breadth First Search (BFS)

Applications

Max-Flow

Testing whether a graph is bipartite

Shortest paths in unweighted graphs

take $O(|V| + |E|)$ time using BFS

(works for directed graphs too)

with a Stack the algorithm is called

Depth First Search (DFS)

Applications

Finding (strongly) connected components

Topicologically sorting a Directed Acyclic Graph

Testing for planarity

Question

What does TRAVERSE do on an unconnected graph?