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Language Engineering

Dr Oliver Ray
(csxor@Bristol.ac.uk)

Department of Computer Science
University of Bristol

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Natural Operational Semantics p19-32

- Natural operational semantics (aka big step semantics) is some-where in between the structural and denotational semantics
- It specifies axiom and rule schemata that relate configurations directly to their completed semantic values
- For convenience we use a different arrow ($\rightarrow$) to distinguish the natural semantics from the operational semantics ($\Rightarrow$) exactly as done in the textbook

Natural Semantics: Arithmetics

- Using our subscript notation, the semantics of numerals is easy

$$\frac{}{\langle n_i , \sigma \rangle \to i}$$

- And variables evaluate to integers (with no numerals required)

$$\frac{}{\langle v , \sigma \rangle \to \sigma(v)}$$

- The semantics of the arithmetic operators are easier than in the structural semantics since we only require the end result

$$\frac{\langle a , \sigma \rangle \to i \qquad \langle b , \sigma \rangle \to j}{\langle a + b , \sigma \rangle \to i{+}j}$$

- In these rules $\sigma \in$ State $n_i \in$ Num $v \in$ Var $a,b \in$ Aexp

- Analogous rules apply to the other arithmetic operators

Example: Arithmetics

- Suppose σ_{12} is a state that maps x to 1 and y to 2
- Then we have $\langle (x+5)+y, \sigma_{12}\rangle \to 8$ by the following proof tree

$$\dfrac{\dfrac{\dfrac{}{\langle x, \sigma_{12}\rangle \to 1} \quad \dfrac{}{\langle 5, \sigma_{12}\rangle \to 5}}{\langle x+5, \sigma_{12}\rangle \to 6} \qquad \dfrac{}{\langle y, \sigma_{12}\rangle \to 2}}{\langle (x+5)+y, \sigma_{12}\rangle \to 8}$$

Natural Semantics: Booleans

- The rules for Booleans are similarly defined with two base cases

$$\frac{}{\langle \text{true}, \sigma\rangle \to tt} \qquad \frac{}{\langle \text{false}, \sigma\rangle \to ff}$$

- There are two cases for inequality (and analogous rules for equality)

$$\frac{\langle a, \sigma\rangle \to i \quad \langle c, \sigma\rangle \to j}{\langle a \le c, \sigma\rangle \to tt} \text{ if } i \le j \qquad \frac{\langle a, \sigma\rangle \to i \quad \langle c, \sigma\rangle \to j}{\langle a \le c, \sigma\rangle \to ff} \text{ if } i > j$$

- Two cases for negation

$$\frac{\langle b, \sigma\rangle \to ff}{\langle \neg b, \sigma\rangle \to tt} \qquad \frac{\langle b, \sigma\rangle \to tt}{\langle \neg b, \sigma\rangle \to ff}$$

- It is convenient to define three cases for conjunction

$$\frac{\langle b, \sigma\rangle \to tt \quad \langle d, \sigma\rangle \to tt}{\langle b \wedge d, \sigma\rangle \to tt} \qquad \frac{\langle b, \sigma\rangle \to ff}{\langle b \wedge d, \sigma\rangle \to ff} \qquad \frac{\langle d, \sigma\rangle \to ff}{\langle b \wedge d, \sigma\rangle \to ff}$$

- In these rules $\sigma \in$ State b,d$\in$ Bexp a,c $\in$ Aexp

Example: Booleans

- Suppose σ_{12} is a state that maps x to 1 and y to 2
- Then we have $\langle \neg(x \leq 5), \sigma_{12} \rangle \to ff$ by the following proof tree

$$\dfrac{\dfrac{\dfrac{}{\langle x, \sigma_{12} \rangle \to 1} \qquad \dfrac{}{\langle 5, \sigma_{12} \rangle \to 5}}{\langle x \leq 5, \sigma_{12} \rangle \to tt}}{\langle \neg(x \leq 5), \sigma_{12} \rangle \to ff}$$

Natural Semantics: Commands p20

$$\frac{}{\langle \text{skip}, \sigma\rangle \to \sigma}$$

$$\frac{\langle a, \sigma\rangle \to i}{\langle x{:=}a, \sigma\rangle \to \sigma[x \mapsto i]}$$

$$\frac{\langle S_1, \sigma\rangle \to \sigma' \qquad \langle S_2, \sigma'\rangle \to \sigma''}{\langle S_1 ; S_2, \sigma\rangle \to \sigma''}$$

$$\frac{\langle b, \sigma\rangle \to \mathrm{tt} \qquad \langle S_1, \sigma\rangle \to \sigma'}{\langle \text{if } b \text{ then } S_1 \text{ else } S_2, \sigma\rangle \to \sigma'}$$

$$\frac{\langle b, \sigma\rangle \to \mathrm{ff} \qquad \langle S_2, \sigma\rangle \to \sigma'}{\langle \text{if } b \text{ then } S_1 \text{ else } S_2, \sigma\rangle \to \sigma'}$$

$$\frac{\langle b,\sigma\rangle \to \mathrm{tt} \qquad \langle S,\sigma\rangle \to \sigma' \qquad \langle \text{while } b \text{ do } S, \sigma'\rangle \to \sigma''}{\langle \text{while } b \text{ do } S, \sigma\rangle \to \sigma''}$$

$$\frac{\langle b,\sigma\rangle \to \mathrm{ff}}{\langle \text{while } b \text{ do } S, \sigma\rangle \to \sigma}$$

Example: Commands cf. ex 2.1 p22-3

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\langle x , \sigma_{570} \rangle \to 5}}{\langle z:=x , \sigma_{570} \rangle \to \sigma_{575}} \quad \dfrac{\dfrac{}{\langle y , \sigma_{575} \rangle \to 7}}{\langle x:=y , \sigma_{575} \rangle \to \sigma_{775}}}{\langle z:=x ; x:=y , \sigma_{570} \rangle \to \sigma_{775}} \quad \dfrac{\dfrac{}{\langle z , \sigma_{775} \rangle \to 5}}{\langle y:=z , \sigma_{775} \rangle \to \sigma_{755}}}{\langle (z:=x ; x:=y) ; y:=z , \sigma_{570} \rangle \to \sigma_{755}}$$

Termination and Looping p25

- The execution of statement S in state σ *terminates* iff there exists a state σ' such that $\langle S , \sigma\rangle \rightarrow \sigma'$
- The execution of statement S in state σ *loops* iff there exists no state σ' such that $\langle S , \sigma\rangle \rightarrow \sigma'$
- A statement S always *terminates* iff its execution terminates in all states σ
- A statement S always *loops* iff its execution loops in all states σ

Determinism and Equivalence p26-8

- A natural semantics is deterministic iff $\langle S, \sigma \rangle \to \sigma'$ and $\langle S, \sigma \rangle \to \sigma''$ imply that $\sigma' = \sigma''$ for all $\sigma, \sigma', \sigma'' \in$ State and for all $S \in$ Stm

- Two statements S_1 and S_2 are *semantically equivalent* (under the natural operational semantics) whenever it holds that
  - $\langle S_1, \sigma \rangle \to \sigma'$ iff $\langle S_2, \sigma \rangle \to \sigma'$ for all $\sigma, \sigma' \in$ State