1 Simple Turing Machine operations ($\star$)

1. Give a formal description of a Turing Machine that shifts its entire input one position to the right, leaving the first cell blank. The head should end up on the new blank cell before the shifted input.
2. Give an "implementation-level" description (i.e. no need to be fully formal) of a Turing Machine that reverses its input string.

2 Pushdown Automata with k stacks ($\star\star$)

Let a k-PDA be a pushdown automaton with access to k stacks. A 1-PDA is a standard PDA, and you have seen that 2-PDAs can recognise any Turing-recognisable language. Show that, for any $k \geq 0$, any language recognised be a k-PDA is Turing-recognisable.

3 Closure properties

Show that the class of Turing-recognisable languages is closed under the operations

1. union ($\star$)
2. intersection ($\star$)
3. concatenation ($\star\star$)
4. Kleene star ($\star\star$)

4 Tape alphabet ($\star\star$)

Let $\mathcal{L} \subseteq \{0,1\}^*$. Show that, if $\mathcal{L}$ is Turing-recognisable, it is recognised by a Turing machine with tape alphabet $\{0, 1, \textvisiblespace\}$.

5 Doubly infinite tape ($\star\star$)

A Turing Machine with *doubly infinite tape* has a tape that is infinite to the left as well as the right (so it will never encounter a tape end). The head starts on the leftmost character of the input and the tape contains blanks on both sides of the input. Computation is otherwise defined as for normal Turing Machines.

Show that Turing Machines with doubly infinite tape recognise the class of Turing-recognisable languages, just like standard Turing Machines.