University of Bristol
COMS21103: Data Structures and Algorithms
Problem Set 2

Remark: All the problems are from the textbook, and Problems with $\star$ are more challenging. However, we will mainly focus on Problem 1 to 4 during the problem class.

Problem 1: Problems from class. Give asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. Assume that $T(n)$ is constant for $n \leq 2$. Make your bounds as tight as possible, and justify your answers. *(These recurrences are from our Monday's class.)*

1. $T(n) = T(\lceil n/2 \rceil) + 1$
2. $T(n) = 2T(\lfloor n/2 \rfloor + 17) + n$
3. $T(n) = 2T(\sqrt{n}) + 1$

Problem 2: Recurrence examples. Give asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. Assume that $T(n)$ is constant for $n \leq 2$. Make your bounds as tight as possible, and justify your answers.

1. $T(n) = 2T(n/2) + n^3$
2. $T(n) = T(9n/10) + n$
3. $T(n) = 16T(n/4) + n^2$
4. $T(n) = 7T(n/3) + n^2$
5. $T(n) = 7T(n/2) + n^2$
6. $T(n) = 2T(n/4) + \sqrt{n}$
7. $T(n) = T(n-1) + n$
8. $T(n) = T(\sqrt{n}) + 1$

Problem 3: More recurrence examples. Give asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. Assume that $T(n)$ is constant for sufficient small n. Make your bunds as tight as possible, and justify your answers.

1. $T(n) = 3T(n/2) + n \lg n$.
2. $T(n) = 5T(n/5) + n/\lg n$.
3. $T(n) = 4T(n/2) + n^2\sqrt{n}$
4. $T(n) = 3T(n/3 + 5) + n/2$
5. $T(n) = 2T(n/2) + n/\lg n$
6. $T(n) = T(n/2) + T(n/4) + T(n/8) + n$
7. $T(n) = T(n-1) + 1/n$
8. $T(n) = T(n-1) + \lg n$

9. $T(n) = T(n-2) + 2\lg n$
10. $T(n) = \sqrt{n}T(\sqrt{n}) + n$

Problem 4: Use a recursion tree to give an asymptotically tight solution to the recurrence $T(n) = T(\alpha n) + T((1-\alpha)n) + cn$, where α is a constant in the range $0 < \alpha < 1$ and $c > 0$ is also a constant.

$\star$ **Problem 5:** Consider the regularity condition $af(n/b) \leq cf(n)$ for some constant $c < 1$, which is part of case 3 of the master theorem. Give an example of constants $a \geq 1$ and $b > 1$ and a function $f(n)$ that satisfies all the conditions in case 3 of the master theorem except the regularity condition.

$\star$ **Problem 6:** Show that case 3 of the master theorem is overstated, in the sense that the regularity condition $af(n/b) \leq cf(n)$ for some constant $c < 1$ implies that there exists a constant $\varepsilon > 0$ such that $f(n) = \Omega(n^{\log_b a + \varepsilon})$.