$$
\begin{aligned}
\mathcal{A}[\![n]\!]s &= \mathcal{N}[\![n]\!] \\
\mathcal{A}[\![x]\!]s &= s\ x \\
\mathcal{A}[\![a_1 + a_2]\!]s &= \mathcal{A}[\![a_1]\!]s + \mathcal{A}[\![a_2]\!]s \\
\mathcal{A}[\![a_1 \star a_2]\!]s &= \mathcal{A}[\![a_1]\!]s \star \mathcal{A}[\![a_2]\!]s \\
\mathcal{A}[\![a_1 - a_2]\!]s &= \mathcal{A}[\![a_1]\!]s - \mathcal{A}[\![a_2]\!]s
\end{aligned}
$$

Table 1.1: The semantics of arithmetic expressions

$$
\begin{aligned}
\mathcal{B}[\![\texttt{true}]\!]s &= \mathbf{tt} \\
\mathcal{B}[\![\texttt{false}]\!]s &= \mathbf{ff} \\
\mathcal{B}[\![a_1 = a_2]\!]s &= \begin{cases} \mathbf{tt} & \text{if } \mathcal{A}[\![a_1]\!]s = \mathcal{A}[\![a_2]\!]s \\ \mathbf{ff} & \text{if } \mathcal{A}[\![a_1]\!]s \neq \mathcal{A}[\![a_2]\!]s \end{cases} \\
\mathcal{B}[\![a_1 \leq a_2]\!]s &= \begin{cases} \mathbf{tt} & \text{if } \mathcal{A}[\![a_1]\!]s \leq \mathcal{A}[\![a_2]\!]s \\ \mathbf{ff} & \text{if } \mathcal{A}[\![a_1]\!]s > \mathcal{A}[\![a_2]\!]s \end{cases} \\
\mathcal{B}[\![\neg\ b]\!]s &= \begin{cases} \mathbf{tt} & \text{if } \mathcal{B}[\![b]\!]s = \mathbf{ff} \\ \mathbf{ff} & \text{if } \mathcal{B}[\![b]\!]s = \mathbf{tt} \end{cases} \\
\mathcal{B}[\![b_1 \wedge b_2]\!]s &= \begin{cases} \mathbf{tt} & \text{if } \mathcal{B}[\![b_1]\!]s = \mathbf{tt} \text{ and } \mathcal{B}[\![b_2]\!]s = \mathbf{tt} \\ \mathbf{ff} & \text{if } \mathcal{B}[\![b_1]\!]s = \mathbf{ff} \text{ or } \mathcal{B}[\![b_2]\!]s = \mathbf{ff} \end{cases}
\end{aligned}
$$

Table 1.2: The semantics of boolean expressions

$$
\begin{aligned}
&\mathcal{S}_{\text{ds}}[\![x := a]\!]s = s[x \mapsto \mathcal{A}[\![a]\!]s] \\
&\mathcal{S}_{\text{ds}}[\![\texttt{skip}]\!] = \text{id} \\
&\mathcal{S}_{\text{ds}}[\![S_1\ ;\ S_2]\!] = \mathcal{S}_{\text{ds}}[\![S_2]\!] \circ \mathcal{S}_{\text{ds}}[\![S_1]\!] \\
&\mathcal{S}_{\text{ds}}[\![\texttt{if}\ b\ \texttt{then}\ S_1\ \texttt{else}\ S_2]\!] = \text{cond}(\mathcal{B}[\![b]\!], \mathcal{S}_{\text{ds}}[\![S_1]\!], \mathcal{S}_{\text{ds}}[\![S_2]\!]) \\
&\mathcal{S}_{\text{ds}}[\![\texttt{while}\ b\ \texttt{do}\ S]\!] = \text{FIX}\ F \\
&\qquad\qquad \text{where } F\ g = \text{cond}(\mathcal{B}[\![b]\!], g \circ \mathcal{S}_{\text{ds}}[\![S]\!], \text{id})
\end{aligned}
$$

Table 4.1: Denotational semantics for **While**

$[\mathrm{ass}_{\mathrm{ns}}]$	$\langle x := a,\, s\rangle \to s[x \mapsto \mathcal{A}[\![a]\!]s]$
$[\mathrm{skip}_{\mathrm{ns}}]$	$\langle \texttt{skip},\, s\rangle \to s$
$[\mathrm{comp}_{\mathrm{ns}}]$	$\dfrac{\langle S_1,\, s\rangle \to s',\ \langle S_2,\, s'\rangle \to s''}{\langle S_1;S_2,\, s\rangle \to s''}$
$[\mathrm{if}^{\mathrm{tt}}_{\mathrm{ns}}]$	$\dfrac{\langle S_1,\, s\rangle \to s'}{\langle \texttt{if}\ b\ \texttt{then}\ S_1\ \texttt{else}\ S_2,\, s\rangle \to s'}$ if $\mathcal{B}[\![b]\!]s = \mathbf{tt}$
$[\mathrm{if}^{\mathrm{ff}}_{\mathrm{ns}}]$	$\dfrac{\langle S_2,\, s\rangle \to s'}{\langle \texttt{if}\ b\ \texttt{then}\ S_1\ \texttt{else}\ S_2,\, s\rangle \to s'}$ if $\mathcal{B}[\![b]\!]s = \mathbf{ff}$
$[\mathrm{while}^{\mathrm{tt}}_{\mathrm{ns}}]$	$\dfrac{\langle S,\, s\rangle \to s',\ \langle \texttt{while}\ b\ \texttt{do}\ S,\, s'\rangle \to s''}{\langle \texttt{while}\ b\ \texttt{do}\ S,\, s\rangle \to s''}$ if $\mathcal{B}[\![b]\!]s = \mathbf{tt}$
$[\mathrm{while}^{\mathrm{ff}}_{\mathrm{ns}}]$	$\langle \texttt{while}\ b\ \texttt{do}\ S,\, s\rangle \to s$ if $\mathcal{B}[\![b]\!]s = \mathbf{ff}$

Table 2.1: Natural semantics for **While**

$[\mathrm{ass}_{\mathrm{sos}}]$	$\langle x := a,\, s\rangle \Rightarrow s[x \mapsto \mathcal{A}[\![a]\!]s]$
$[\mathrm{skip}_{\mathrm{sos}}]$	$\langle \texttt{skip},\, s\rangle \Rightarrow s$
$[\mathrm{comp}^{1}_{\mathrm{sos}}]$	$\dfrac{\langle S_1,\, s\rangle \Rightarrow \langle S'_1,\, s'\rangle}{\langle S_1;S_2,\, s\rangle \Rightarrow \langle S'_1;S_2,\, s'\rangle}$
$[\mathrm{comp}^{2}_{\mathrm{sos}}]$	$\dfrac{\langle S_1,\, s\rangle \Rightarrow s'}{\langle S_1;S_2,\, s\rangle \Rightarrow \langle S_2,\, s'\rangle}$
$[\mathrm{if}^{\mathrm{tt}}_{\mathrm{sos}}]$	$\langle \texttt{if}\ b\ \texttt{then}\ S_1\ \texttt{else}\ S_2,\, s\rangle \Rightarrow \langle S_1,\, s\rangle$ if $\mathcal{B}[\![b]\!]s = \mathbf{tt}$
$[\mathrm{if}^{\mathrm{ff}}_{\mathrm{sos}}]$	$\langle \texttt{if}\ b\ \texttt{then}\ S_1\ \texttt{else}\ S_2,\, s\rangle \Rightarrow \langle S_2,\, s\rangle$ if $\mathcal{B}[\![b]\!]s = \mathbf{ff}$
$[\mathrm{while}_{\mathrm{sos}}]$	$\langle \texttt{while}\ b\ \texttt{do}\ S,\, s\rangle \Rightarrow$ $\langle \texttt{if}\ b\ \texttt{then}\ (S;\ \texttt{while}\ b\ \texttt{do}\ S)\ \texttt{else}\ \texttt{skip},\, s\rangle$

Table 2.2: Structural operational semantics for **While**

$$
\begin{array}{ll}
[\mathrm{ass_p}] & \{\ P[x{\mapsto}\mathcal{A}[\![a]\!]]\ \}\ x := a\ \{\ P\ \} \\[1ex]
[\mathrm{skip_p}] & \{\ P\ \}\ \texttt{skip}\ \{\ P\ \} \\[1ex]
[\mathrm{comp_p}] & \dfrac{\{\ P\ \}\ S_1\ \{\ Q\ \},\quad \{\ Q\ \}\ S_2\ \{\ R\ \}}{\{\ P\ \}\ S_1;\ S_2\ \{\ R\ \}} \\[2ex]
[\mathrm{if_p}] & \dfrac{\{\ \mathcal{B}[\![b]\!] \land P\ \}\ S_1\ \{\ Q\ \},\quad \{\ \neg\mathcal{B}[\![b]\!] \land P\ \}\ S_2\ \{\ Q\ \}}{\{\ P\ \}\ \texttt{if}\ b\ \texttt{then}\ S_1\ \texttt{else}\ S_2\ \{\ Q\ \}} \\[2ex]
[\mathrm{while_p}] & \dfrac{\{\ \mathcal{B}[\![b]\!] \land P\ \}\ S\ \{\ P\ \}}{\{\ P\ \}\ \texttt{while}\ b\ \texttt{do}\ S\ \{\ \neg\mathcal{B}[\![b]\!] \land P\ \}} \\[2ex]
[\mathrm{cons_p}] & \dfrac{\{\ P'\ \}\ S\ \{\ Q'\ \}}{\{\ P\ \}\ S\ \{\ Q\ \}}\quad \text{if } P \Rightarrow P' \text{ and } Q' \Rightarrow Q
\end{array}
$$

Table 6.1: Axiomatic system for partial correctness

$$
\begin{array}{ll}
[\mathrm{ass_t}] & \{\ P[x{\mapsto}\mathcal{A}[\![a]\!]]\ \}\ x := a\ \{\ \Downarrow P\ \} \\[1ex]
[\mathrm{skip_t}] & \{\ P\ \}\ \texttt{skip}\ \{\ \Downarrow P\ \} \\[1ex]
[\mathrm{comp_t}] & \dfrac{\{\ P\ \}\ S_1\ \{\ \Downarrow Q\ \},\quad \{\ Q\ \}\ S_2\ \{\ \Downarrow R\ \}}{\{\ P\ \}\ S_1;\ S_2\ \{\ \Downarrow R\ \}} \\[2ex]
[\mathrm{if_t}] & \dfrac{\{\ \mathcal{B}[\![b]\!] \land P\ \}\ S_1\ \{\ \Downarrow Q\ \},\quad \{\ \neg\mathcal{B}[\![b]\!] \land P\ \}\ S_2\ \{\ \Downarrow Q\ \}}{\{\ P\ \}\ \texttt{if}\ b\ \texttt{then}\ S_1\ \texttt{else}\ S_2\ \{\ \Downarrow Q\ \}} \\[2ex]
[\mathrm{while_t}] & \dfrac{\{\ P(\mathbf{z}{+}\mathbf{1})\ \}\ S\ \{\ \Downarrow P(\mathbf{z})\ \}}{\{\ \exists \mathbf{z}.P(\mathbf{z})\ \}\ \texttt{while}\ b\ \texttt{do}\ S\ \{\ \Downarrow P(\mathbf{0})\ \}} \\
& \text{where } P(\mathbf{z}{+}\mathbf{1}) \Rightarrow \mathcal{B}[\![b]\!],\ P(\mathbf{0}) \Rightarrow \neg\mathcal{B}[\![b]\!] \\[1ex]
& \text{and } \mathbf{z} \text{ ranges over natural numbers (that is } \mathbf{z}{\geq}\mathbf{0}) \\[2ex]
[\mathrm{cons_t}] & \dfrac{\{\ P'\ \}\ S\ \{\ \Downarrow Q'\ \}}{\{\ P\ \}\ S\ \{\ \Downarrow Q\ \}}\quad \text{where } P \Rightarrow P' \text{ and } Q' \Rightarrow Q
\end{array}
$$

Table 6.2: Axiomatic system for total correctness