COMS10003: Class Test on Logic

Kerstin Eder

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Instructions

For this class test you are required to answer four questions on logic.

You are expected to work independently.

You may use your logic reference card.

Before you start, familiarise yourself with all the questions.

Clearly state your name and your user name on the answer sheet.

All answers should be clearly structured and fully justified.

Question 1: For each of the statements listed below, determine whether or not it is **logically equivalent to the negation** of the statement:

"If you go home on the weekend, then you should bring your lecture notes and study."

Start by expressing the above statement in the language of propositional logic, clearly stating what meaning you assign to propositional variables.

Proceed by formalising each of the listed statements.

Answer: *The above can be formalised as follows:*

p	$=$	*you go home on the weekend*
q	$=$	*you should bring your lecture notes*
r	$=$	*you should study*

Based on this formalisation, the above statement can be written as the compound proposition $p \Rightarrow (q \wedge r)$.

The negation of this compound proposition is $p \wedge (\neg q \vee \neg r) \equiv (p \wedge \neg q) \vee (p \wedge \neg r)$.

We need to establish whether or not the formalisation of each statement below is logically equivalent to this statement.

Once formalised, it helps to transform each statement into normal form (DNF or CNF), or to provide a truth table.

The following shows solutions that aim to establish logical equivalence using symbolic manipulation. An alternative, based on truth tables is given afterwards.

1. If you don't go home on the weekend, then you shouldn't bring your lecture notes and you shouldn't study.

   Answer: *This can be formalised to:* $\neg p \Rightarrow \neg q \wedge \neg r \equiv p \vee (\neg q \wedge \neg r)$, *which is not logically equivalent to the negated statement above.*

2. If you don't go home on the weekend, then you shouldn't bring your lecture notes or you shouldn't study.

   Answer: *This can be formalised to:* $\neg p \Rightarrow \neg q \vee \neg r \equiv p \vee (\neg q \vee \neg r)$. *Again, this is not logically equivalent to the negated statement above.*

3. If you don't go home on the weekend, then you should bring your lecture notes and study.

   Answer: *This can be formalised to:* $\neg p \Rightarrow q \wedge r \equiv p \vee (q \wedge r)$. *Again, this is not logically equivalent to the negated statement above.*

4. You don't go home on the weekend, and you shouldn't bring your lecture notes and you shouldn't study.

Answer: *This can be formalised to:* $\neg p \wedge \neg q \wedge \neg r$*. Again, this is not logically equivalent to the negated statement above.*

5. You don't go home on the weekend, and you shouldn't bring your lecture notes or you shouldn't study.

   Answer: *This can be formalised to:* $\neg p \wedge (\neg q \vee \neg r)$*. Again, this is not logically equivalent to the negated statement above.*

6. You don't go home on the weekend, or you should bring your lecture notes and you should study.

   Answer: *This can be formalised to:* $\neg p \vee (q \wedge r)$*. This also is not logically equivalent to the negated statement above.*

7. You go home on the weekend and you shouldn't bring your lecture notes, or you go home on the weekend and you shouldn't study.

   Answer: *This can be formalised to:* $(p \wedge \neg q) \vee (p \wedge \neg r)$*, which clearly is logically equivalent to the negated statement above.*

8. You go home on the weekend, and you shouldn't bring your lecture notes and you shouldn't study.

   Answer: *This can be formalised to:* $p \wedge \neg q \wedge \neg r$*. This, again, is not logically equivalent to the negated statement above.*

9. You go home on the weekend, and you shouldn't bring your lecture notes or you shouldn't study.

   Answer: *This can be formalised to:* $p \wedge (\neg q \vee \neg r)$*, which is logically equivalent to the negated statement above.*

Clearly explain and justify your answer with reference to your formalisation. You may use either symbolic manipulation or truth tables.

Answer: *Let* S *denote the original statement, and let* $\neg S$ *denote its negation in:*

p	q	r	S	$\neg S$	*1.*	*2.*	*3.*	*4.*	*5.*	*6.*	*7.*	*8.*	*9.*
T	T	T	T	**F**	T	T	T	F	F	T	**F**	F	**F**
T	T	F	F	**T**	T	T	T	F	F	F	**T**	F	**T**
T	F	T	F	**T**	T	T	T	F	F	F	**T**	F	**T**
T	F	F	F	**T**	T	T	T	F	F	F	**T**	T	**T**
F	T	T	T	**F**	F	F	T	F	F	T	**F**	F	**F**
F	T	F	T	**F**	F	T	F	F	T	T	**F**	F	**F**
F	F	T	T	**F**	F	T	F	F	T	T	**F**	F	**F**
F	F	F	T	**F**	T	T	F	T	T	T	**F**	F	**F**

Based on the truth table above, it can be seen that only 7. and 9. are logically equivalent to $\neg S$*.*

Note, that it would be sufficient to find one value assignment to the propositions that results in a truth value of the overall statement which differs from the one in $\neg S$. *However, it should be clearly stated that finding one such value assignment, i.e. a counter example, is enough to demonstrate that the two statements are not logically equivalent.*

[9 marks, one for each statement]

Question 2: Demonstrate that $\{\wedge, \neg\}$ is a functionally complete set of connectives. Explain clearly what you need to do and show each step of your work.

Hint: You may make reference to normal forms.

Answer: *We already know that* $\{\wedge, \vee, \neg\}$ *is functionally complete (reference to CNF/DNF), and that every statement can be transformed into these normal forms. [1] So now we have to express* $\vee$ *using only* $\{\wedge, \neg\}$.

1. $p \vee q = \neg\neg(p \vee q) = \neg(\neg p \wedge \neg q)$ *[1]*

[2 marks]

Question 3: Transform the following proposition into an equivalent proposition in Disjunctive Normal Form:

$$(((\neg p \Rightarrow q) \vee (p \wedge \neg r)) \Leftrightarrow \neg q)$$

Minimise your answer. Is the minimised proposition still in Disjunctive Normal Form? Explain.

[3 marks]

Answer:

This can be done either using a truth table, or by symbolic manipulation.

p	q	r	$f = (\neg p \Rightarrow q)$	$g = (p \wedge \neg r)$	$h = (f \vee g)$	$h \Leftrightarrow \neg q$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	1	0	1	0
0	1	1	1	0	1	0
1	0	0	1	1	1	1
1	0	1	1	0	1	1
1	1	0	1	1	1	0
1	1	1	1	0	1	0

Based on the truth table, an equivalent formula in DNF is $(p \wedge \neg q \wedge \neg r) \vee (p \wedge \neg q \wedge r)$. *[1]*

This can be minimized to $(p \wedge \neg q)$. *[1] Yes, this is still (trivially) in DNF, as it is a single conjunction. [1]*

Question 4: Demonstrate that $((p \lor (q \lor r)) \land (r \lor \neg p)) \equiv ((q \land \neg p) \lor r)$ using:

a) a truth table, and

Answer:

p	q	r	$f = (p \lor q \lor r)$	$g = (r \lor \neg p)$	$f \land g$	$h = (q \land \neg p)$	$h \lor r$
0	0	0	0	1	0	0	0
0	0	1	1	1	1	0	1
0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	1	0	0	0	0
1	0	1	1	1	1	0	1
1	1	0	1	0	0	0	0
1	1	1	1	1	1	0	1

Because the columns for $f \land g$ and $h \lor r$ have the same truth values [1], both propositions are logically equivalent. [1]

b) logical equivalences and symbolic manipulation. Justify each step.

Answer:

	$(p \lor (q \lor r)) \land (r \lor \neg p)$	| *Associativity*
$\equiv$	$((p \lor q) \lor r) \land (r \lor \neg p)$	| *Commutativity*
$\equiv$	$(r \lor (p \lor q)) \land (r \lor \neg p)$	| *Distributive Law, r [1]*
$\equiv$	$r \lor ((p \lor q) \land \neg p)$	| *Distributive Law, $\neg p$ [1]*
$\equiv$	$r \lor ((p \land \neg p) \lor (q \land \neg p))$	| $p \land \neg p \equiv \mathbb{F}$, *i.e.* $p \land \neg p \equiv \mathbb{F}$
		| *is a Contradiction [1]*
$\equiv$	$r \lor (\mathbb{F} \lor (q \land \neg p))$	| *Identity Law [1]*
$\equiv$	$r \lor (q \land \neg p)$	| *Commutativity*
$\equiv$	$(q \land \neg p) \lor r$	

Therefore, the two propositions are logically equivalent.

[1 mark lost for missing answer sentence.]

[6 marks]