Theory of Computation – COMS11700
Regular languages - Study Guide

March 3, 2014

Study guide

This section is intended as a study guide. The material on regular languages is split in a number of topics. For each topic, a list of specific items and problems are provided. If you solve these problems and understand the issues you will do very well in the course. The references to exercises are for Sipser, 2nd international edition.

1 Deterministic Finite Automata (DFA)

Topics The notion and definition of DFA, presentation of DFA by transition diagrams and formal notation, the class of regular languages, techniques for designing DFAs, and closure operations.

Designing DFAs

1. Exercises 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, (pages 83-84)
2. For each of the following regular expressions, draw a DFA recognizing the corresponding language
   (a) $(0 \cup 1)^*110^*$
   (b) $(11 \cup 10)^*$
   (c) $(1 \cup 110)^*0$
3. Exercise 1.22
4. Draw a DFA that recognizes the language of all strings of 0's and 1's of length greater than 1 that, if they were interpreted as binary representations of integers, would represent integers divisible by 3. Leading 0's are permissible.
5. Problems 1.32, 1.33

2 Nondeterministic Finite Automata (NFA)

Topics The notion of nondeterminism, definition of acceptnace of NFAs, economy of states using NFA, equivalence of DFAs and NFAs, conversion of an NFA to an equivalent DFA.

The notion of non-determinism Exercise 1.11 (page 85), 1.14

Designing NFAs Exercise 1.7

Conversion of NFAs to DFAs Exercise 1.16

3 Regular expressions

Topics: The definition of regular expressions, writing regular expressions, equivalence with finite automata: every regular expression has an equivalent finite automaton and every finite automaton has an equivalent regular expression.

Basic stuff

1. What is the shortest string of a's and b's not in the language corresponding to the regular expression $b^*(abb^*)^*a^*$
2. Consider the following two regular expressions: $R_1 = a^* \cup b^*$ and $R_2 = ab^* \cup ba^* \cup b^*a \cup (a^*b)^*$
   - Find a string corresponding to R_1 but not to R_2
   - Find a string corresponding to R_2 but not to R_1
   - Find a string corresponding to both R_1 and R_2
   - Find a string that does not correspond to either R_1 or R_2
3. What is true of the language corresponding to a regular expression that does not involve the operators $*$ or $+$. [1] Why?

4 Non-regular languages

Topics Pumping lemma, examples of nonregular languages and applications of the pumping lemma.

Applications of the Pumping Lemma

1. Using the Pumping Lemma show that each of the following languages is not regular.
   (a) $L = \{a^n ba^{2n} \mid n \geq 0\}$
   (b) $L = \{a^i b^j c^k \mid k > i + j\}$
   (c) $L = \{x \in \{a,b\}^* \mid N_a(x) \leq N_b(x)\}$, where $N_a(x)$ is the number of occurences of the letter a in x. The definition of $N_b(x)$ is analogous.
   (d) $L = \{x \in \{a,b\}^* \mid \text{no initial substring of } x \text{ has more } b's \text{ than } a's\}$
2. Problems 1.46(page 90), 1.53 (page 91).

[1] Recall that R^+ is shorthand for RR^*. This means a word is in the language described by R^+ if it is the concantenation of one or more words from the language of R.

5 Closure Properties of Regular Languages

For each statement bellow, decide whether it is true or false. If it is true, prove it; if not, give a counterexample. All parts refer to languages over $\{a, b\}$.

1. If $L_1 \subseteq L_2$ and L_1 is not regular, then L_2 is not regular.
2. If $L_1 \subseteq L_2$ is not regular, then L_1 is regular.
3. If L_1 and L_2 are nonregular, then $L_1 \cup L_2$ is nonregular.
4. If L_1 and L_2 are nonregular, then $L_1 \cap L_2$ is nonregular.
5. If L is not regular, then $\overline{L}$, the complement of L is not regular.
6. If L_1 is regular, L_2 is nonregular, and $L_1 \cap L_2$ is nonregular, then $L_1 \cup L_2$ is nonregular.
7. If $L_1, L_2, \ldots$ are regular, then $L = \cup_{n=1}^{\infty} L_n$ is regular.