Data Structures and Algorithms – COMS21103

2015/2016

Single Source Shortest Paths

Priority Queues and Dijkstra's Algorithm

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University of BRISTOL

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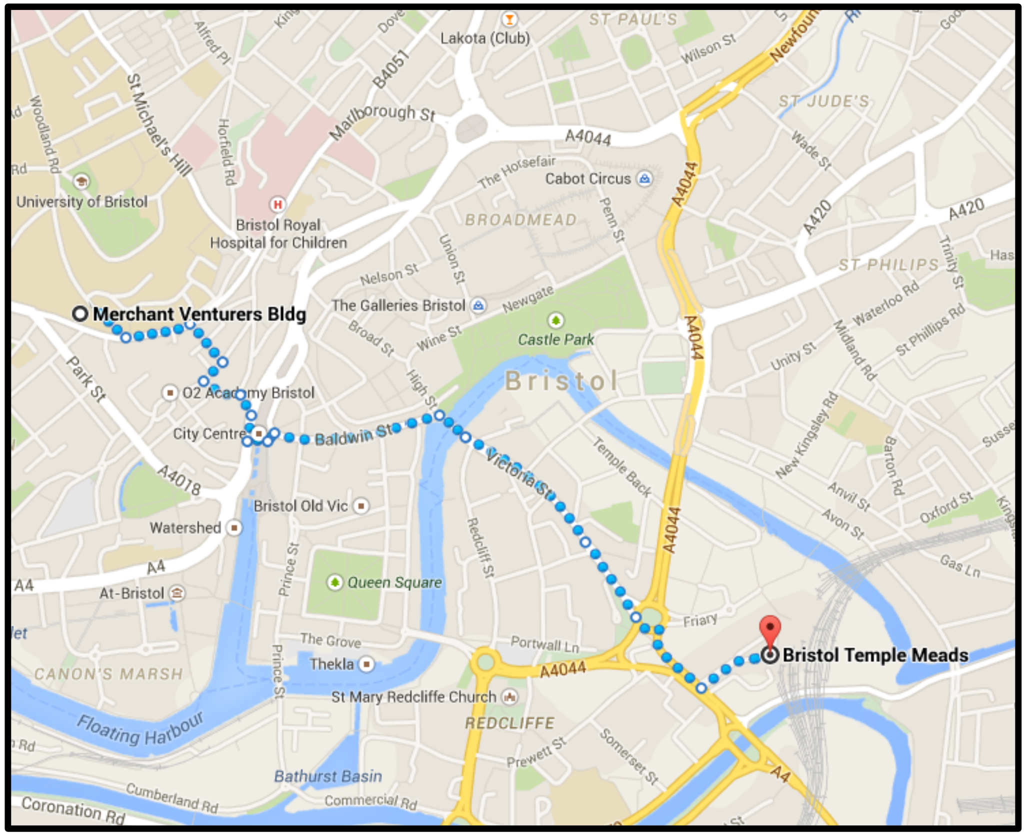
In today's lectures we'll be discussing the **single source shortest paths** problem
in a weighted, directed graph. . .



The shortest path from MVB to Temple Meads
(according to Google Maps)

In particular we'll be interested in **Dijkstra's Algorithm**

which is based on an abstract data structure called a priority queue

. . . which can be efficiently implemented as a binary heap

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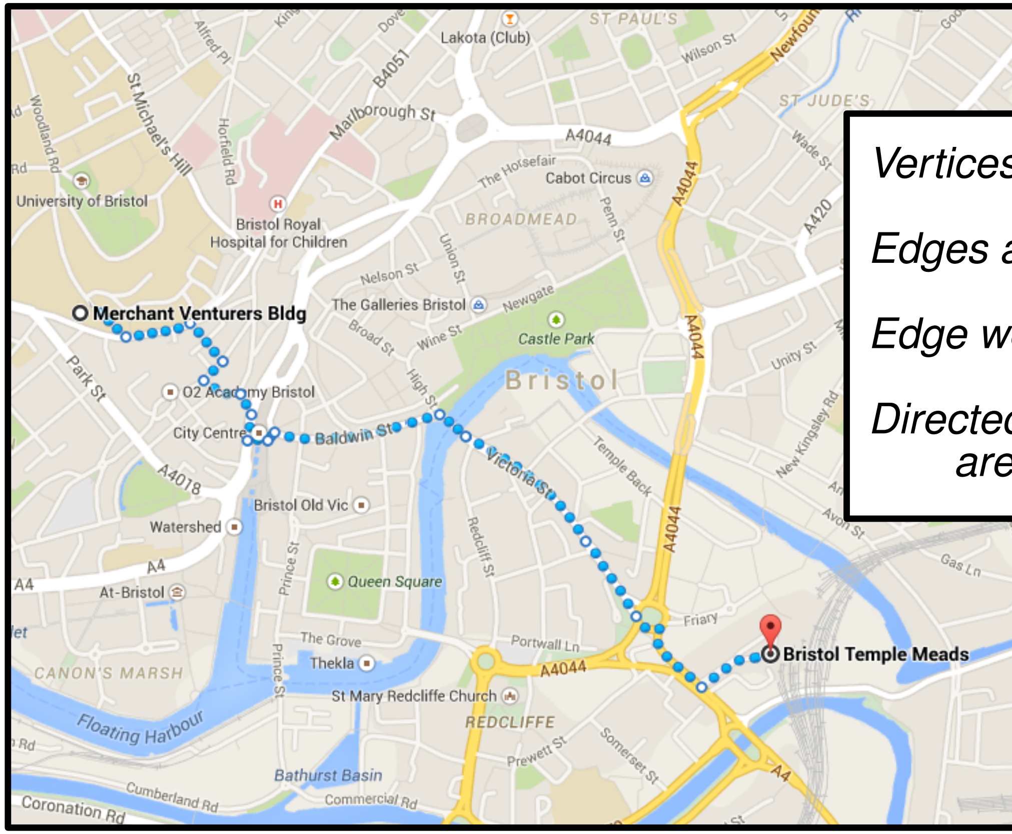

Vertices are junctions

Edges are roads

Edge weights are in miles

Directed edges are one-way streets

The shortest path from MVB to Temple Meads
(according to Google Maps)

In particular we'll be interested in **Dijkstra's Algorithm**

which is based on an abstract data structure called a priority queue

. . . which can be efficiently implemented as a binary heap

Part one

Priority Queues

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(you can forget all about graphs for the whole of part one)

Priority Queues

A **priority queue**, Q stores a set of distinct elements

Each element x has an associated value called its key - $x.\text{key}$

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(ties are broken arbitrarily)

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A **priority queue**: $\boxed{\textbf{Alice}\ 3}$ $\boxed{\textbf{Bob}\ 5}$ $\boxed{\textbf{Chris}\ 8}$

$\left(\boxed{x \quad x.\text{key}}\right)$

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A **priority queue**:

Alice	Bob	Chris
3	5	8

$\left(\begin{array}{|c|}\hline x \\ x.\text{key} \\ \hline\end{array}\right)$

$\textsc{Insert}(\text{Dawn}, 4)$

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Alice	Bob	Chris	Dawn
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5	8	4	6

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A **priority queue**:

Bob	Chris	Emma
5	8	6

$\left(\boxed{\begin{matrix} x \\ x.\text{key} \end{matrix}}\right)$

$\textsc{DecreaseKey}(\text{Bob}, 2)$

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A **priority queue**:

Bob	Chris	Emma
2	8	6

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DECREASEKEY(Bob, 2)

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A **priority queue**:

Bob 2	**Chris** 8	**Emma** 6	$\left(\boxed{x \; x.\text{key}} \right)$

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Bob 2 | **Chris** 8 | **Emma** 6

$\left(\boxed{x \quad x\text{.key}} \right)$

$\textsc{Insert}(\text{Alice}, 3)$

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3	2	8	6	x.key

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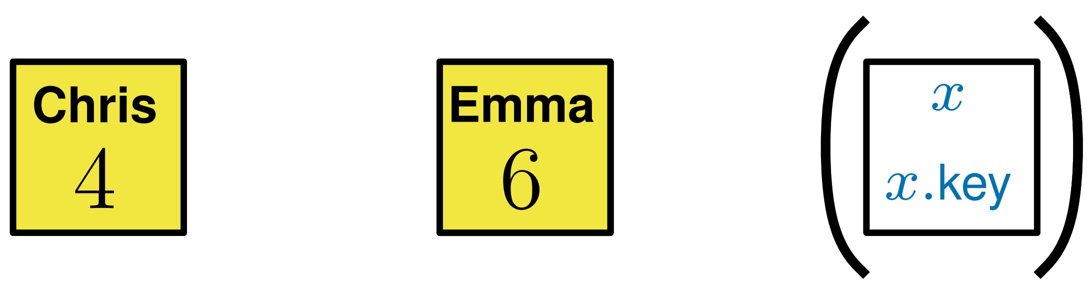

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Using a Linked List as a Priority Queue

There are many ways in which we could implement a priority queue...

but they aren't all efficient

Let n denote the number of elements in the queue

- our goal is to implement a queue with operations which scale well as n *grows*

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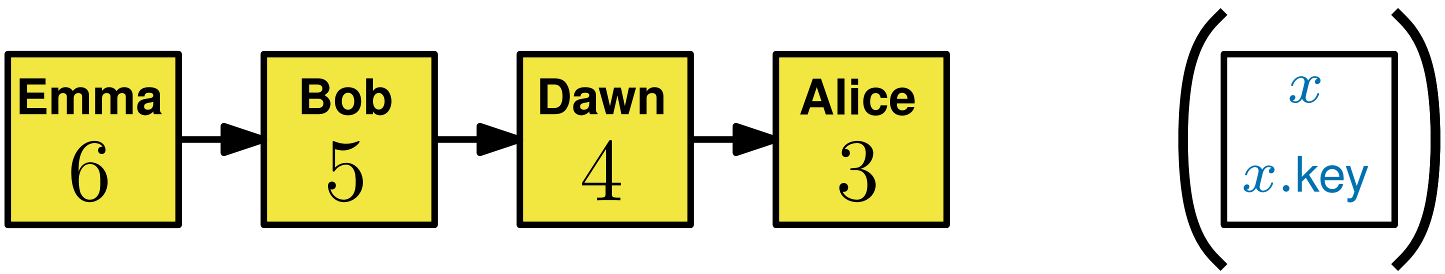

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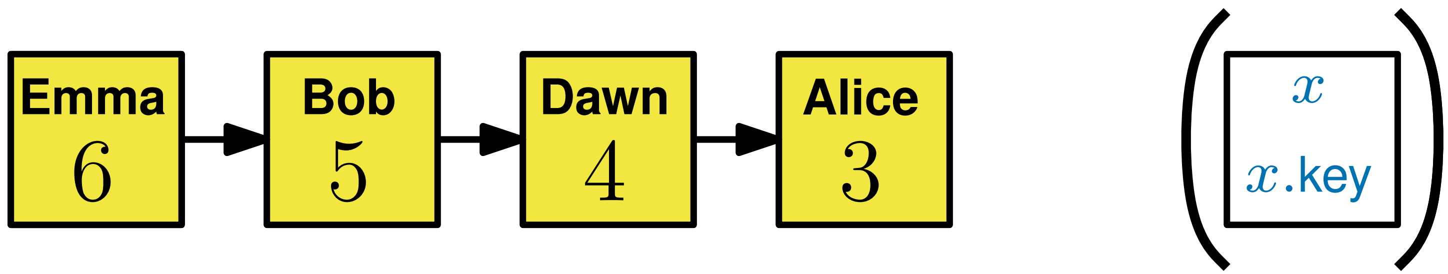

INSERT is very efficient,

- add the new item to the head of the list in $O(1)$ time

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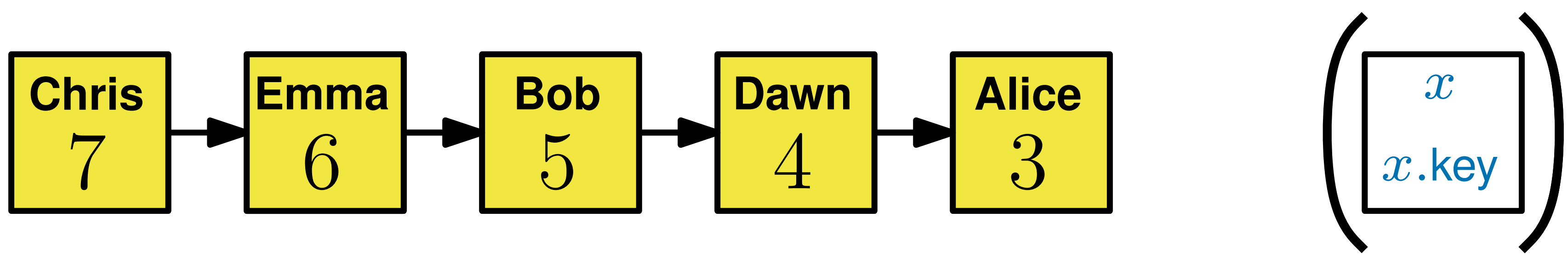

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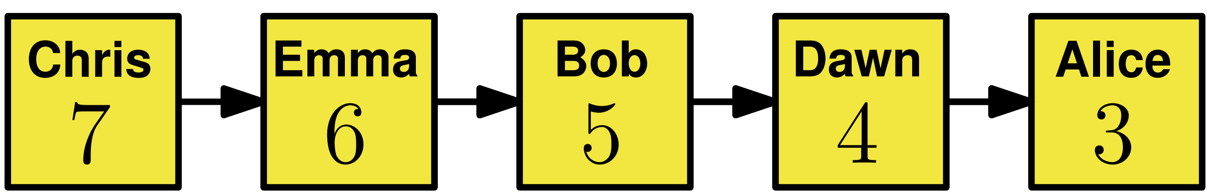

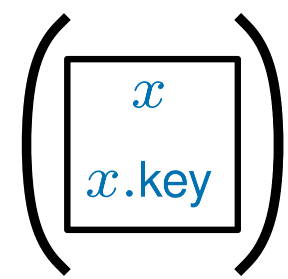

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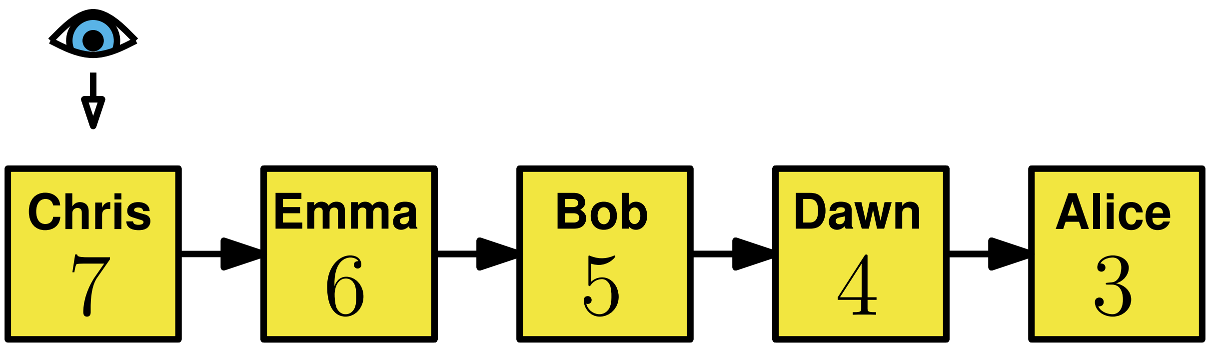

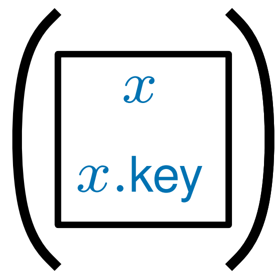

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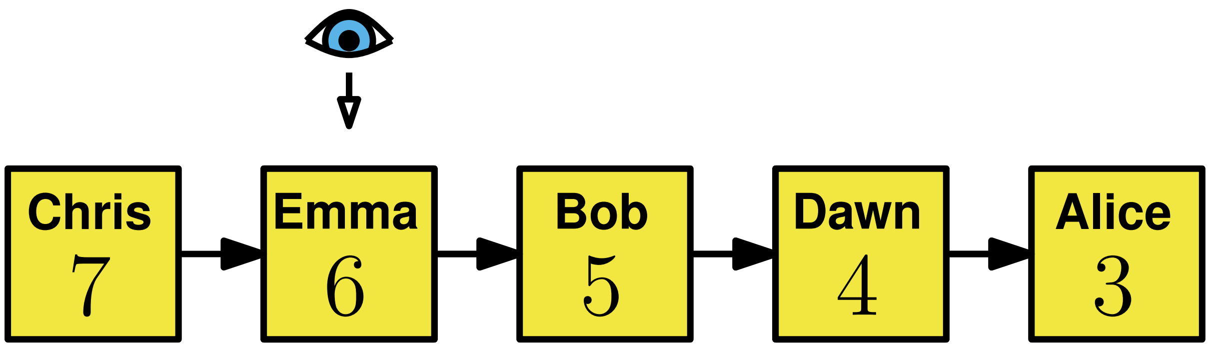

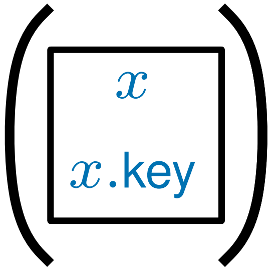

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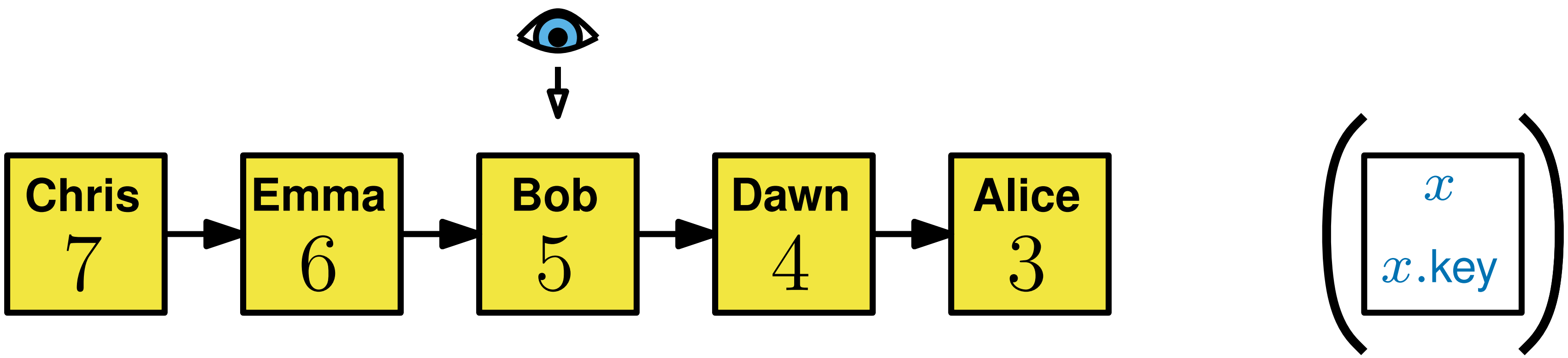

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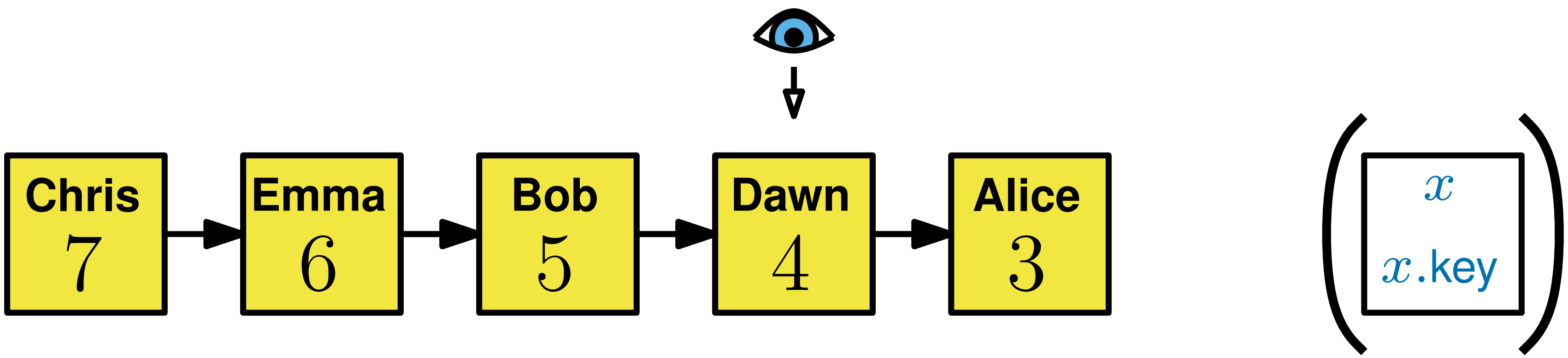

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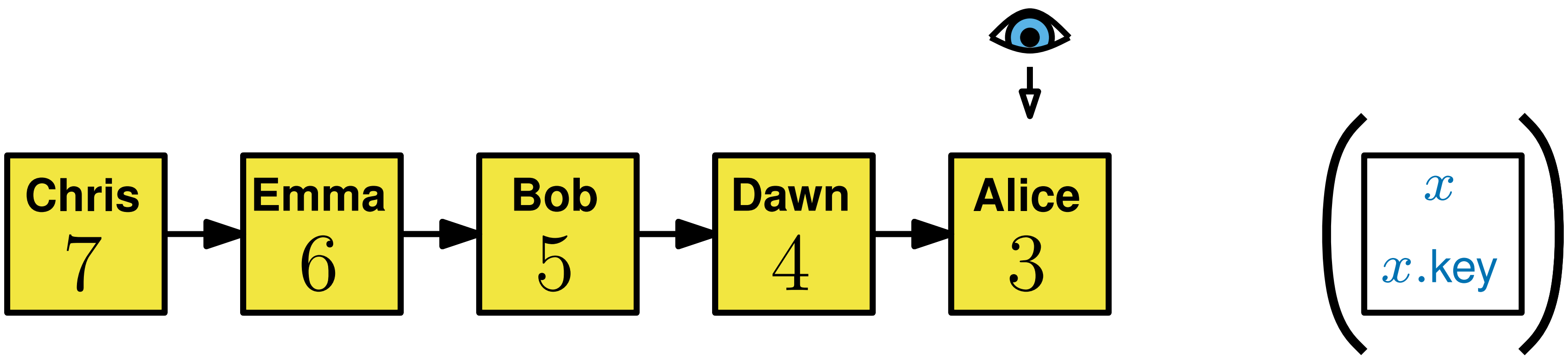

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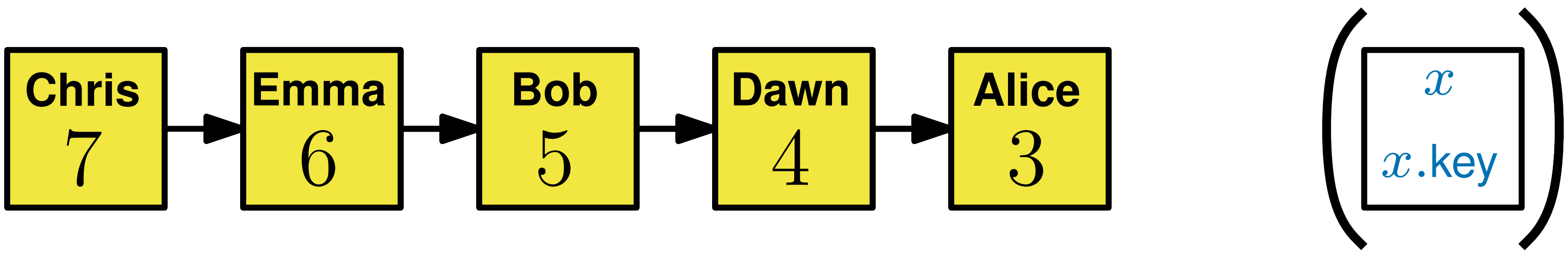

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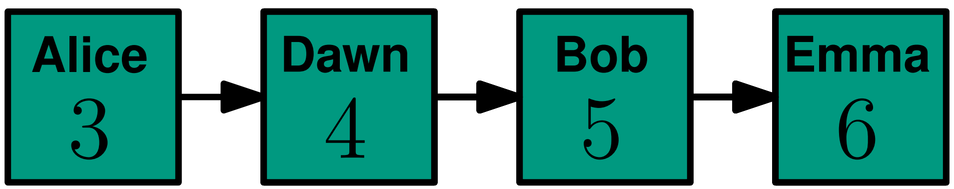

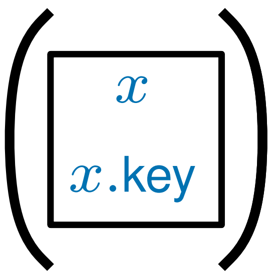

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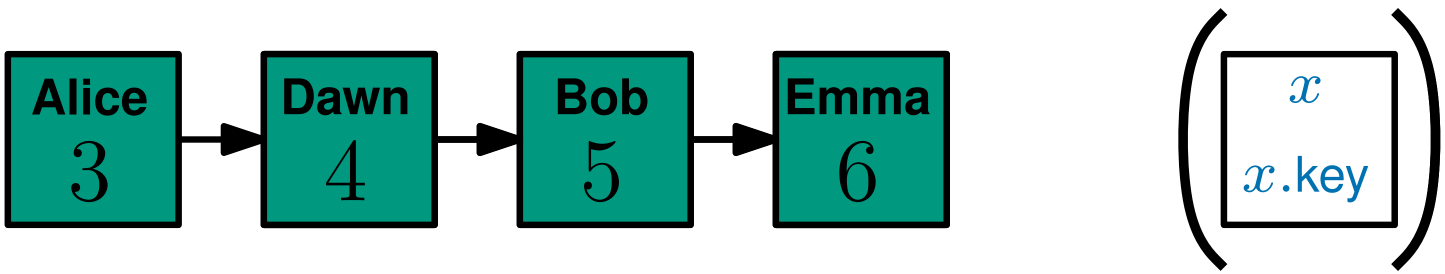

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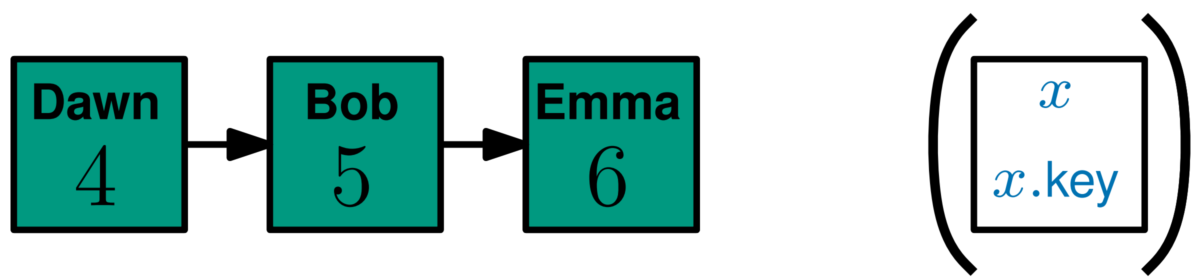

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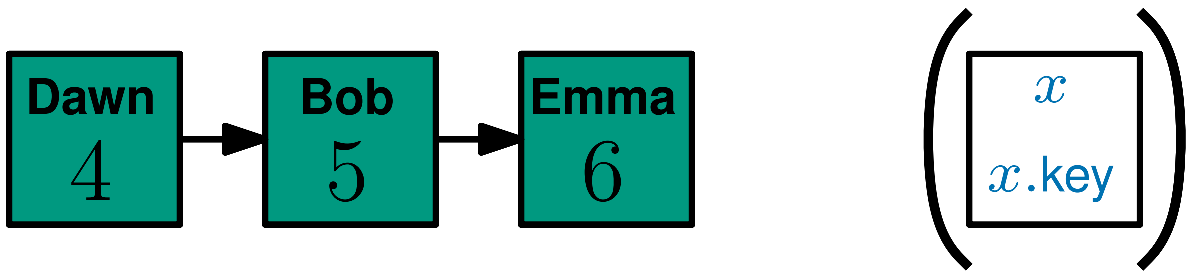

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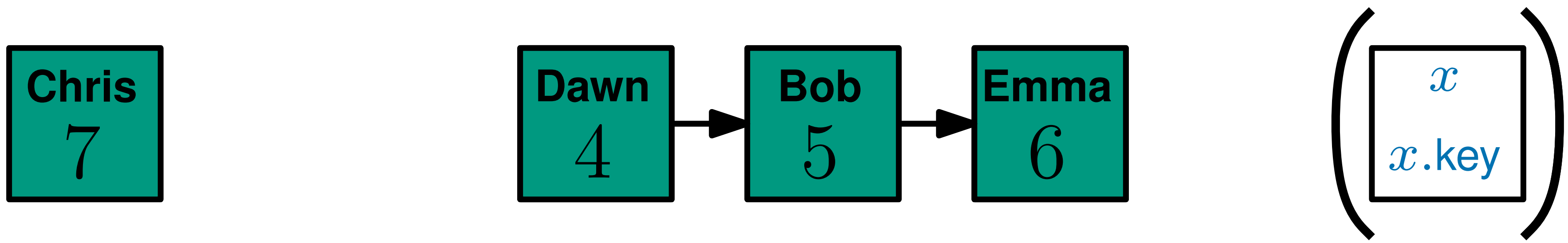

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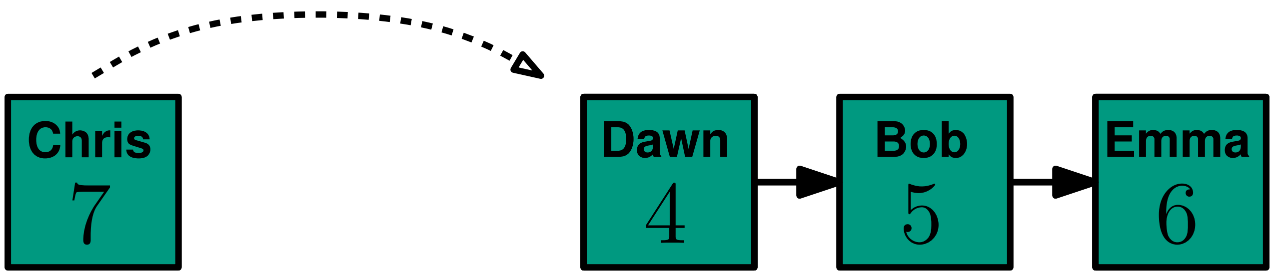

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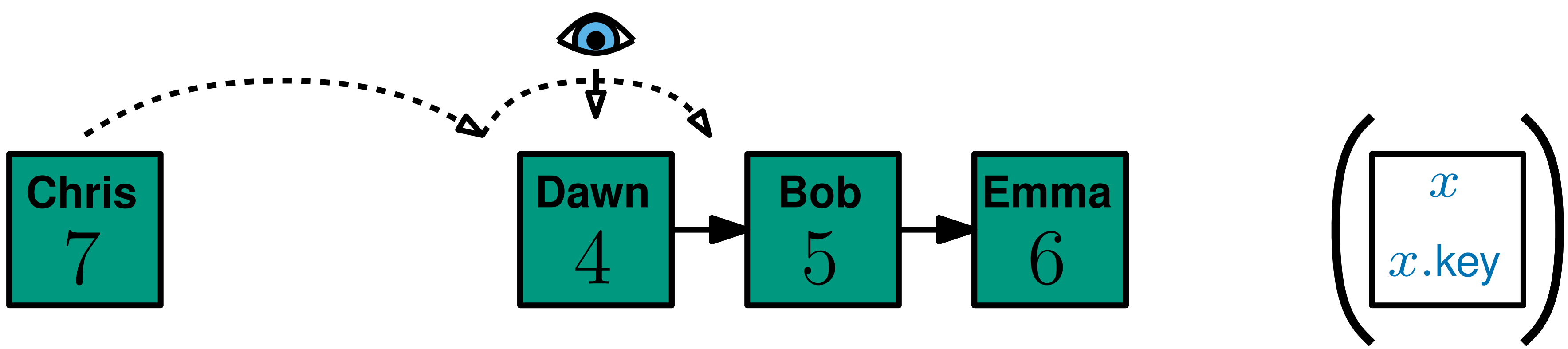

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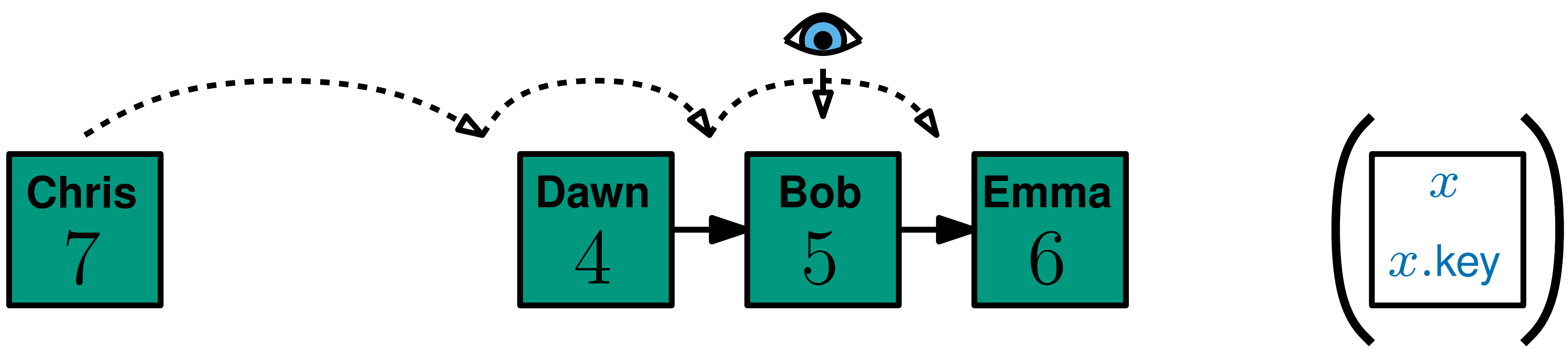

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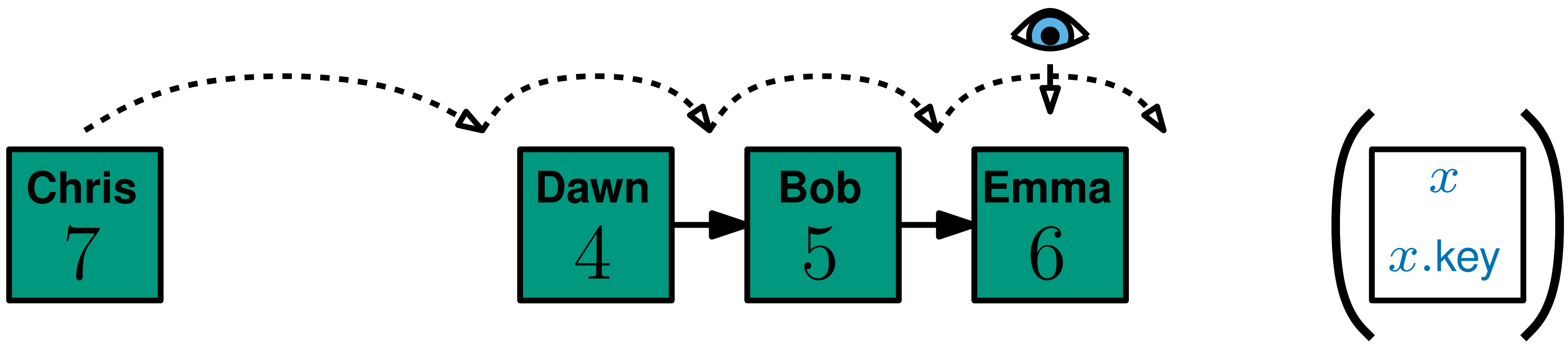

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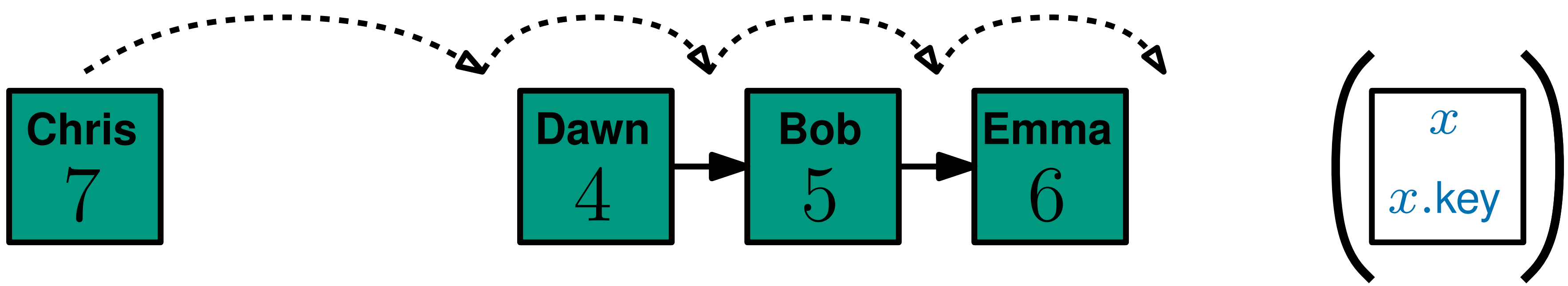

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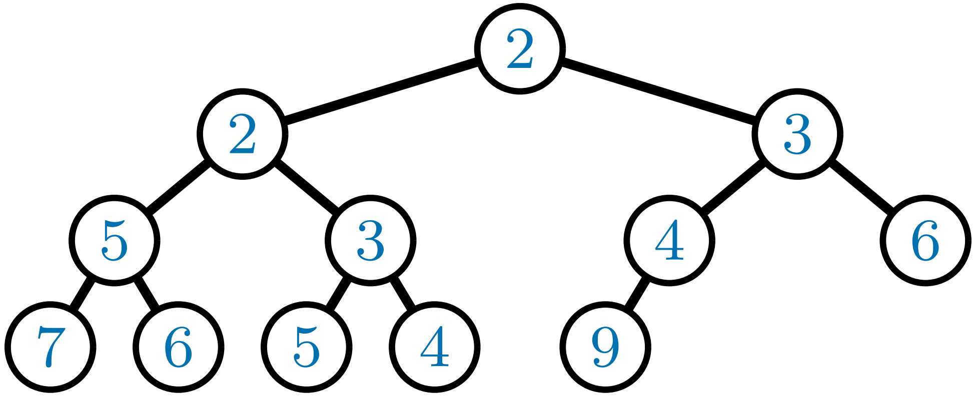

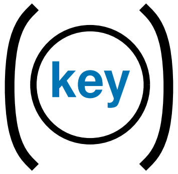

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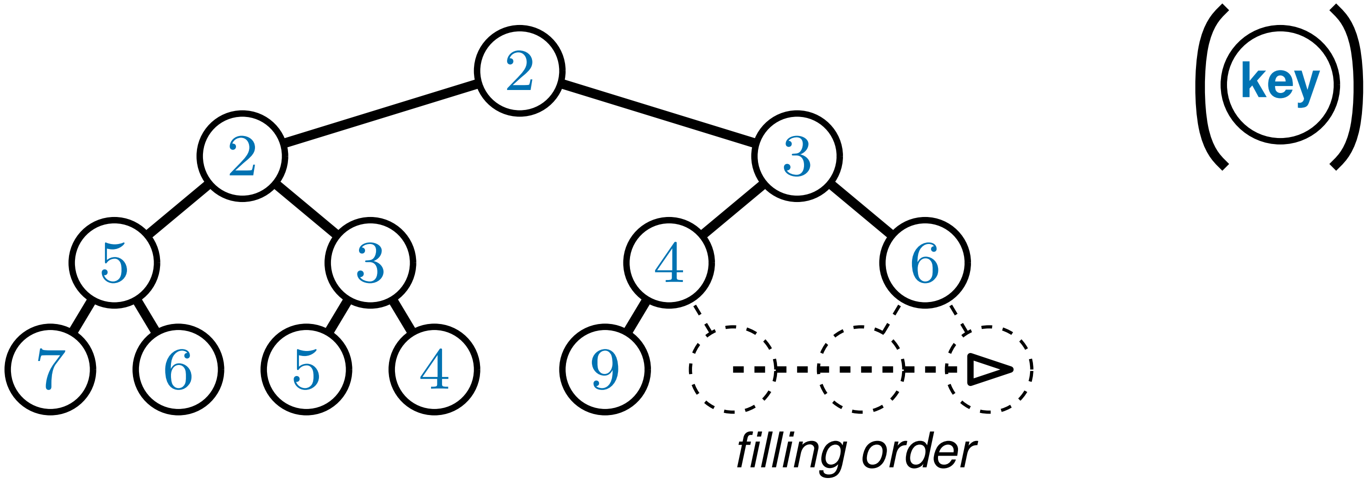

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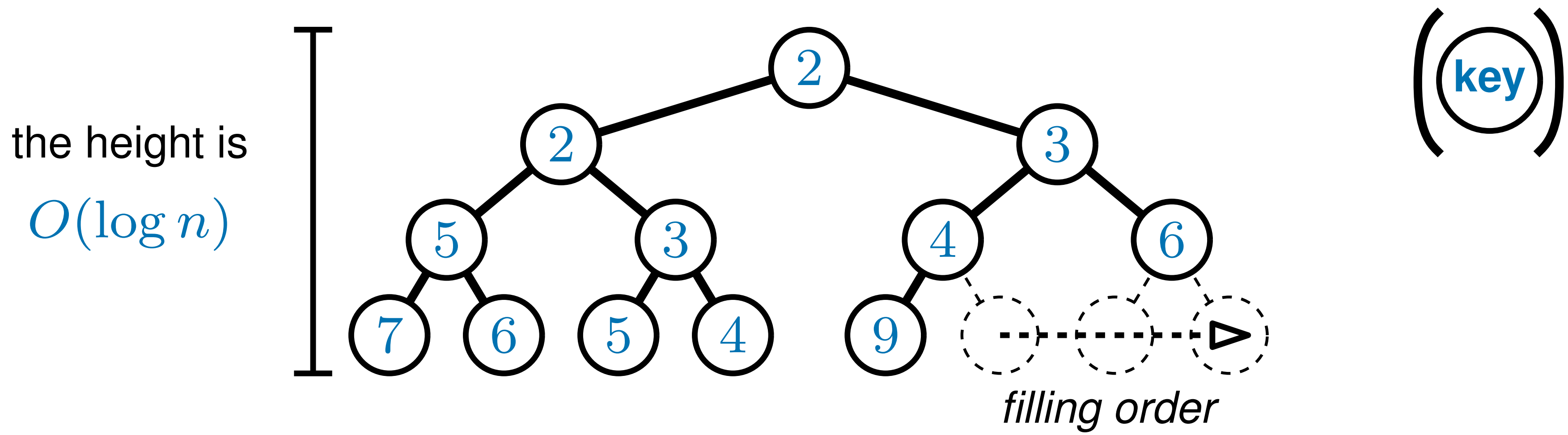

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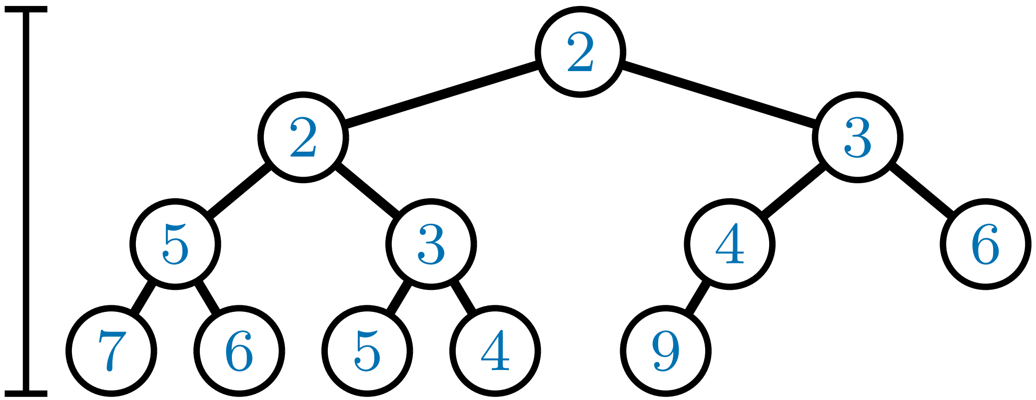

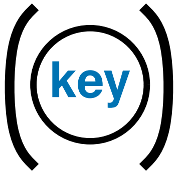

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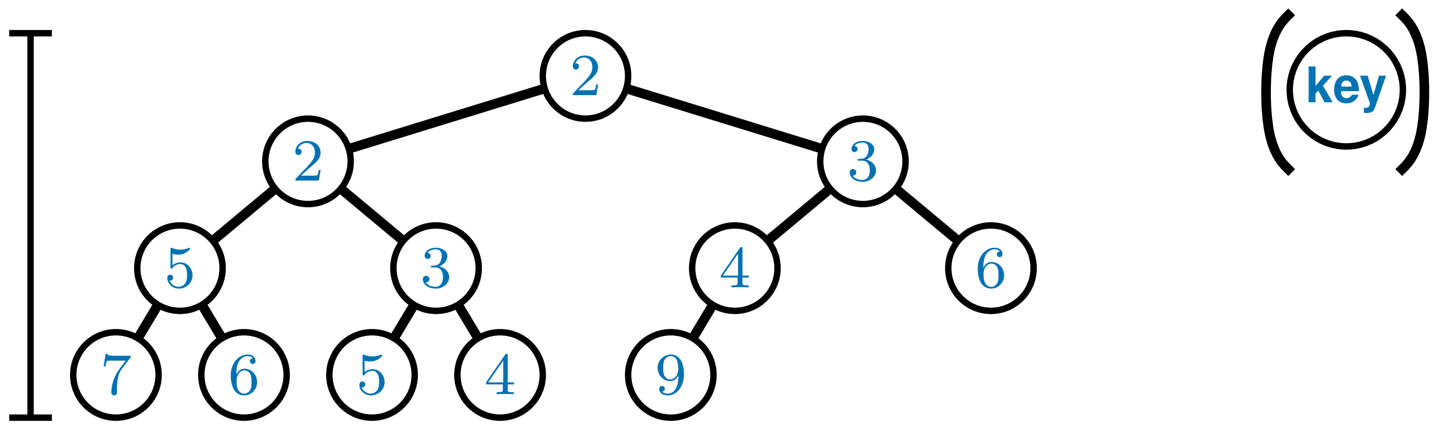

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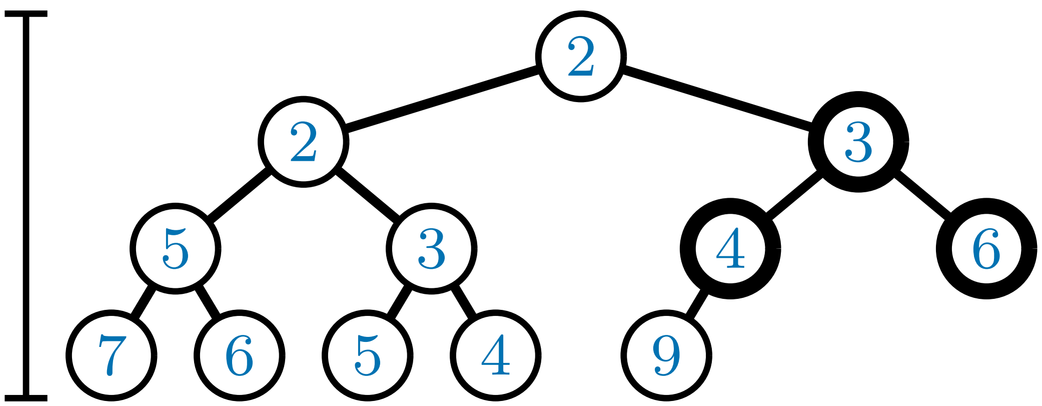

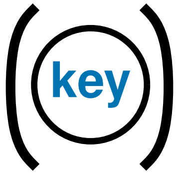

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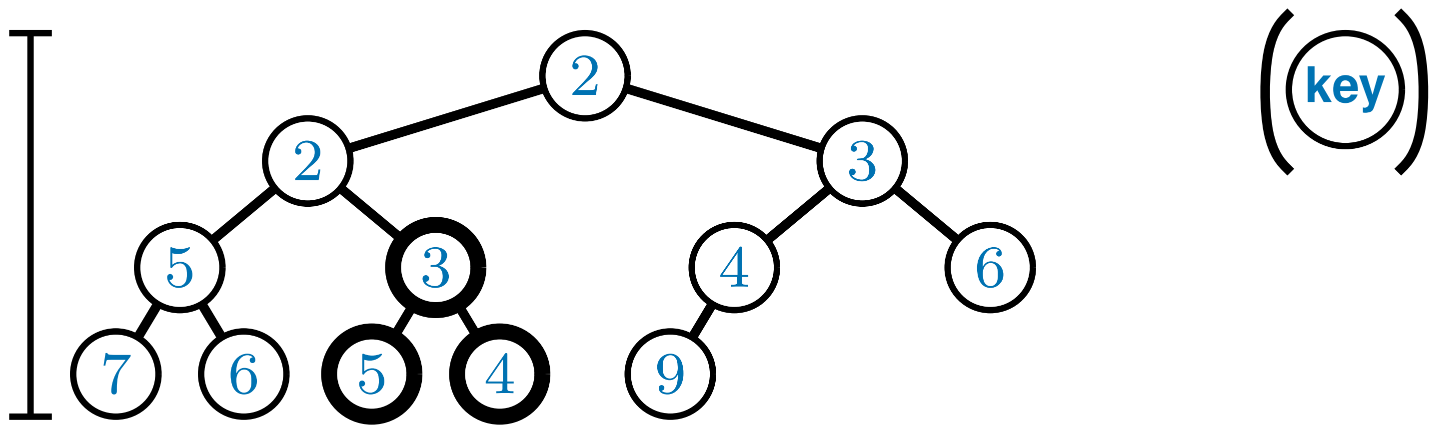

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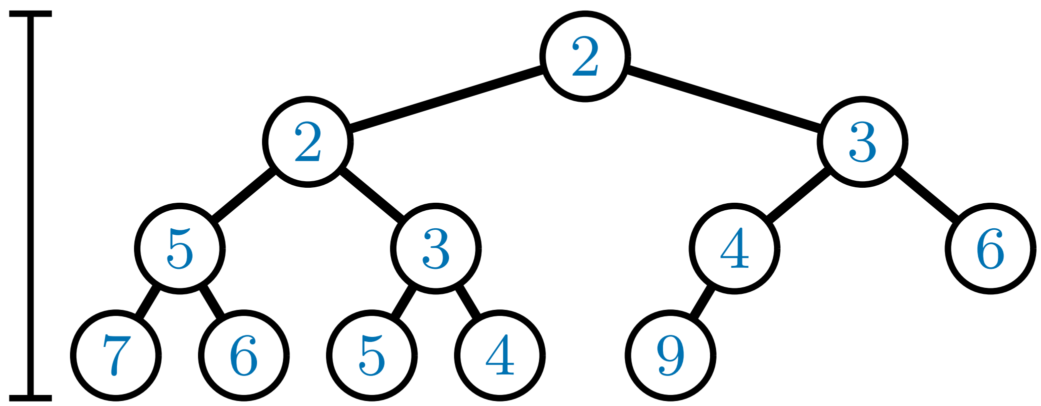

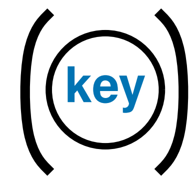

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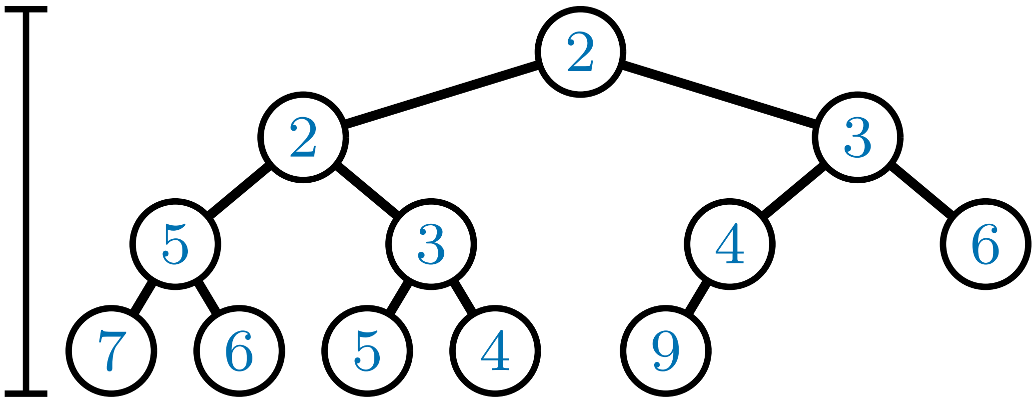

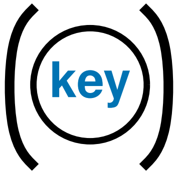

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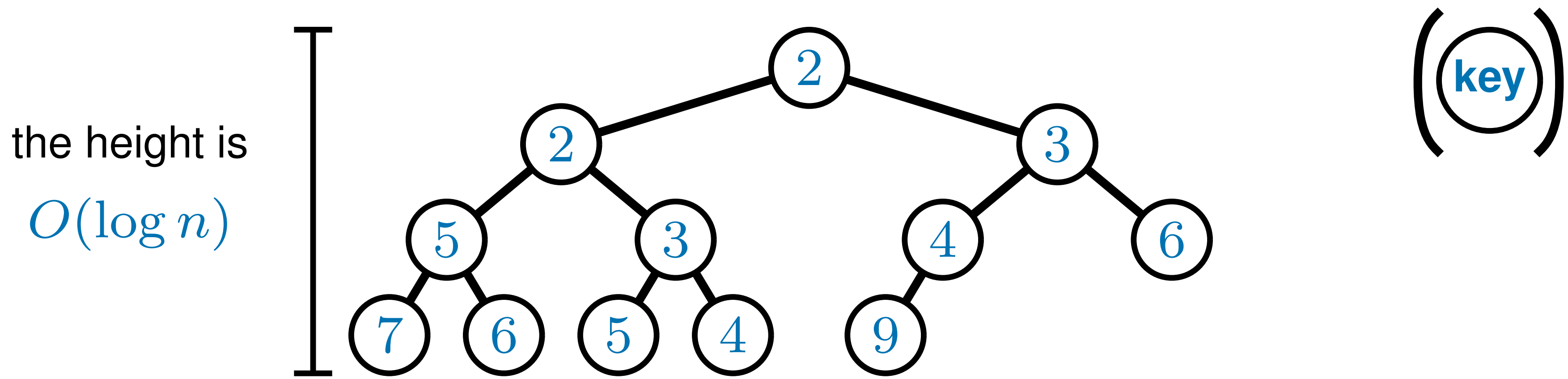

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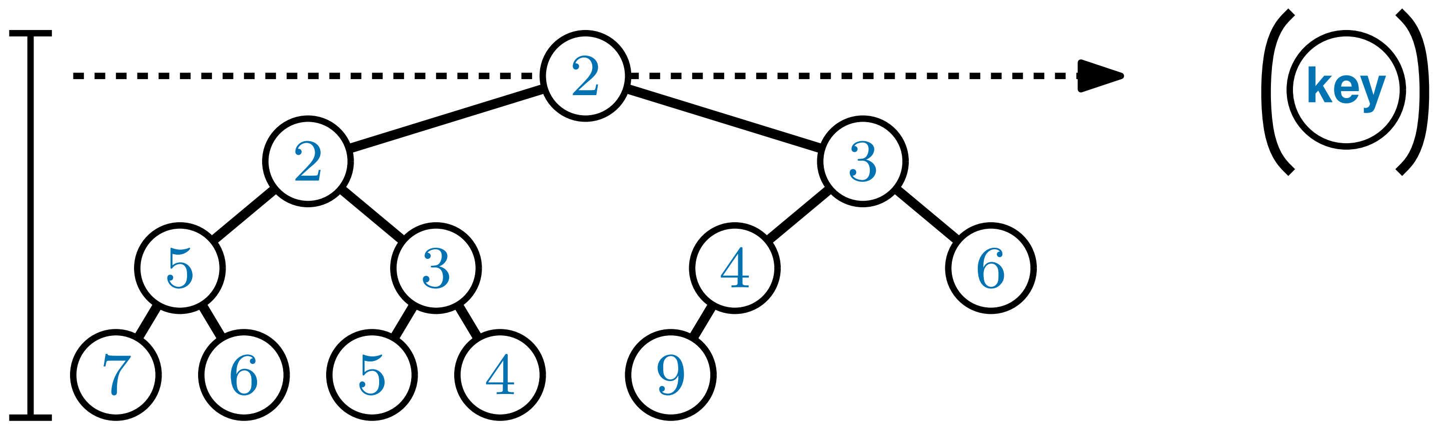

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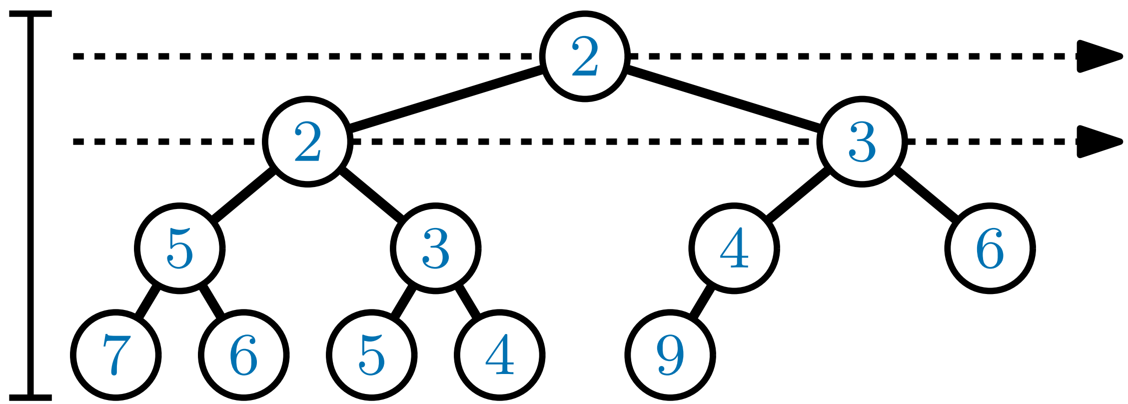

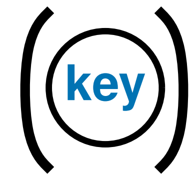

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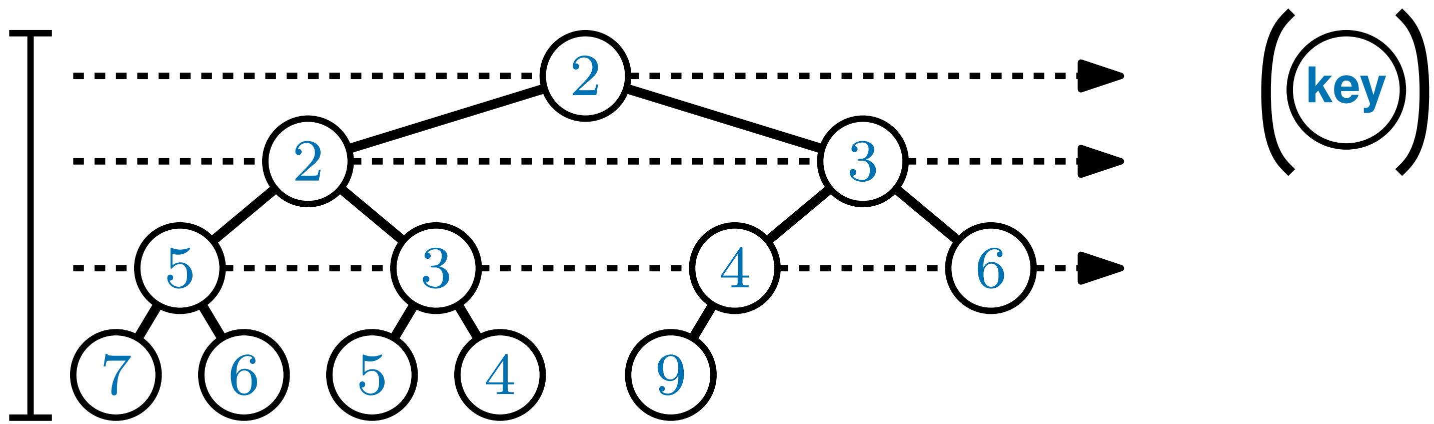

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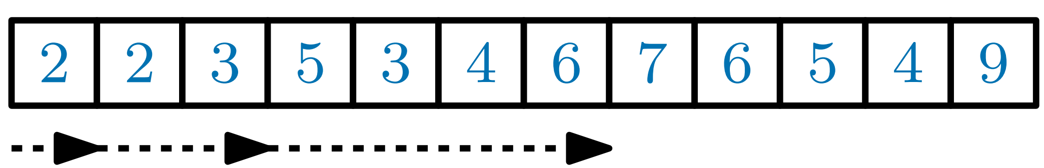

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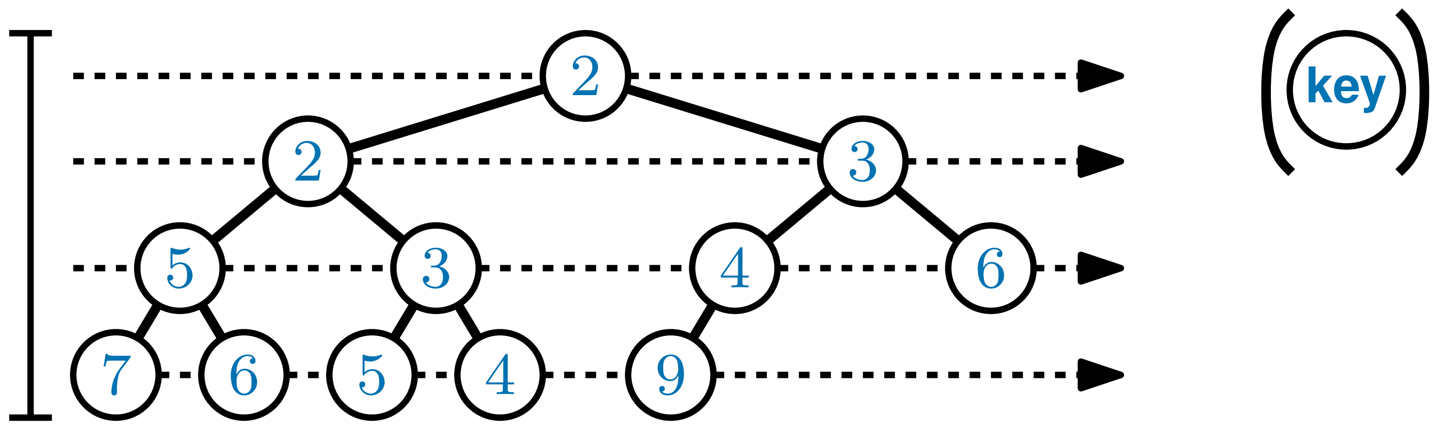

n is the number of elements in the Heap

Heap Property Any element has a key *less than or equal to* the keys of its children.

A binary heap can be efficiently stored implicitly as an array A:

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A binary heap is an ‘almost complete’ binary tree, where every level is full. . .
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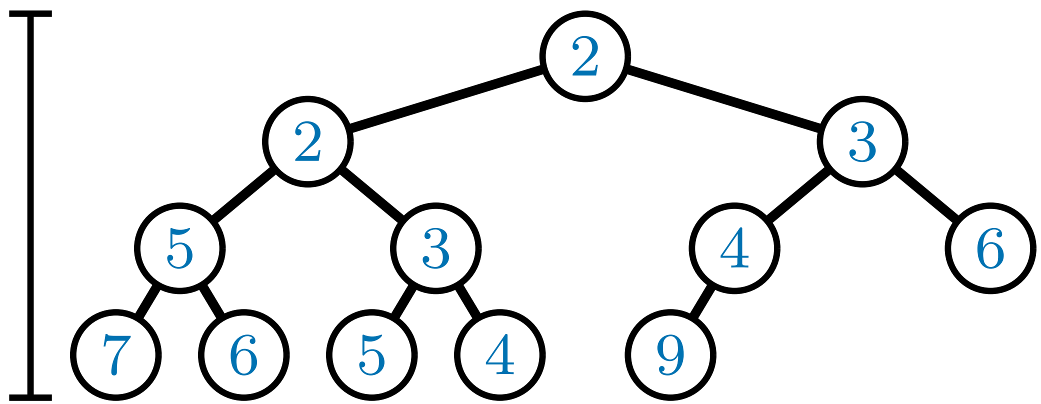

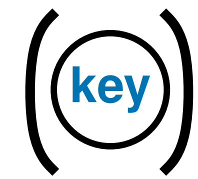

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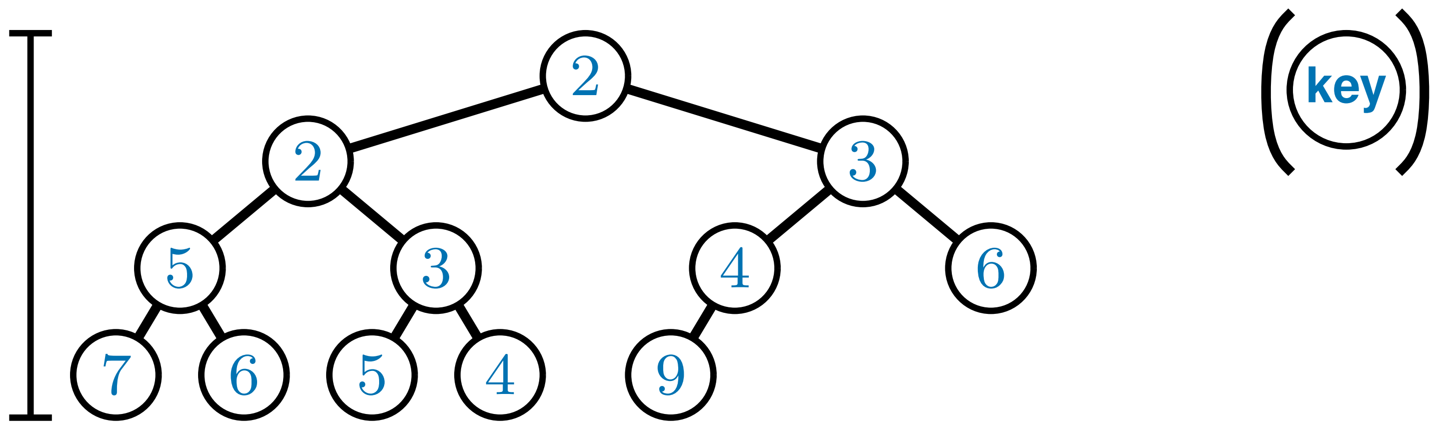

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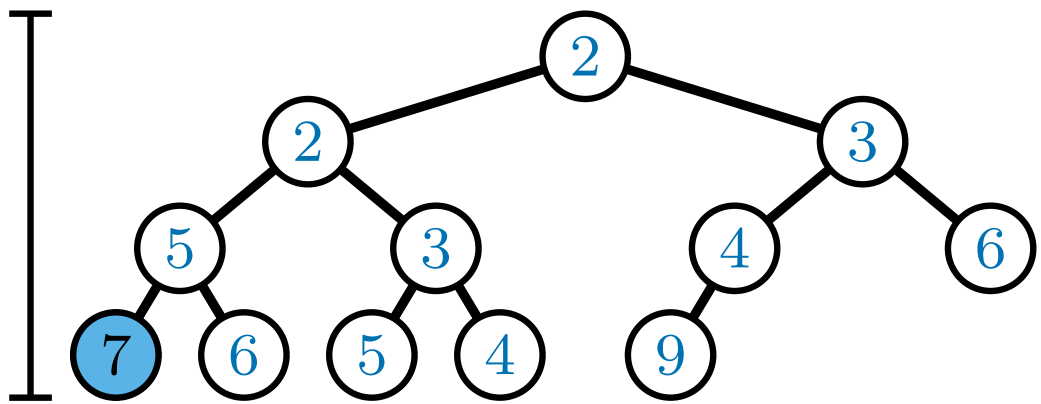

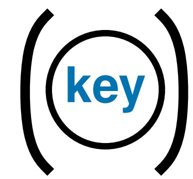

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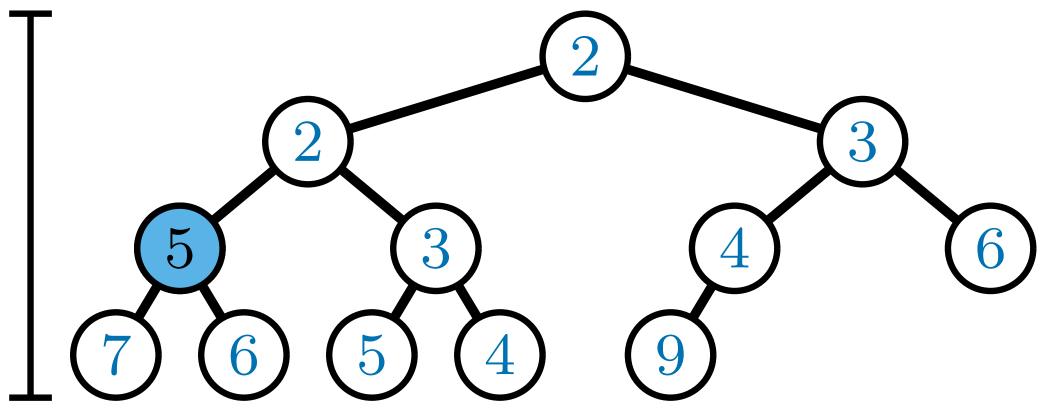

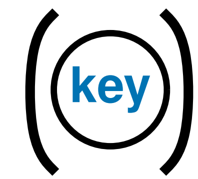

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Using a Binary Heap as a Priority Queue

We will now see how to use a Binary Heap to implement the required operations:

$\textsc{Insert}(x, k)$ - inserts x with x.key $= k$

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(ties are broken arbitrarily)

Each in $O(\log n)$ *time per operation*

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This is a little fiddly. . . we'll come back to it at the end of the lecture

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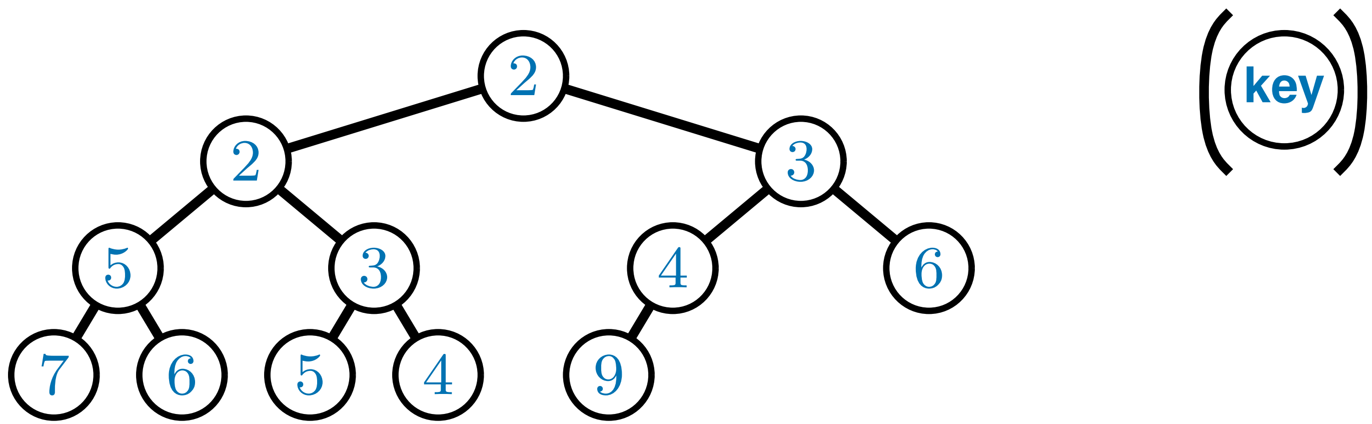

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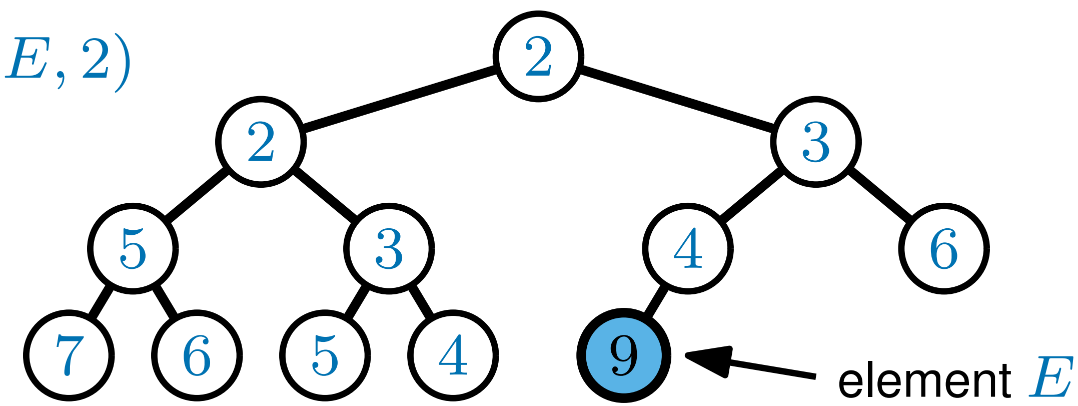

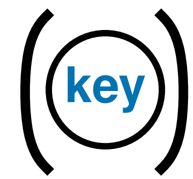

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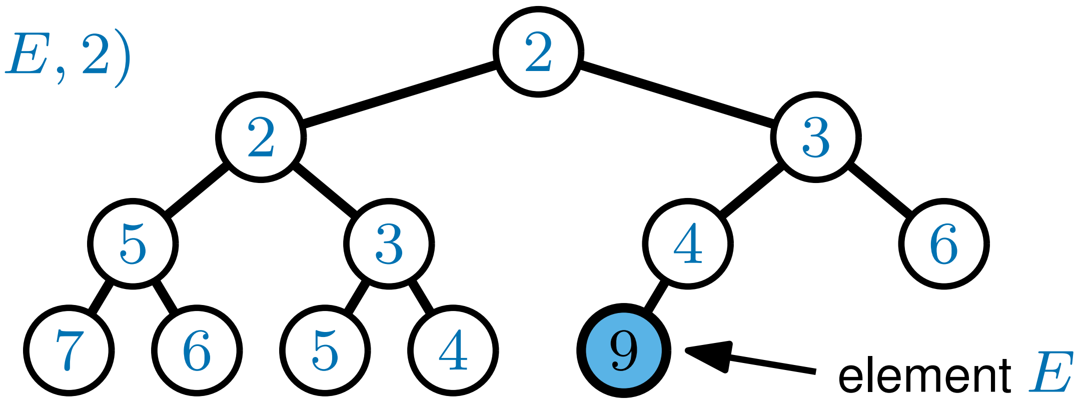

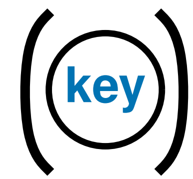

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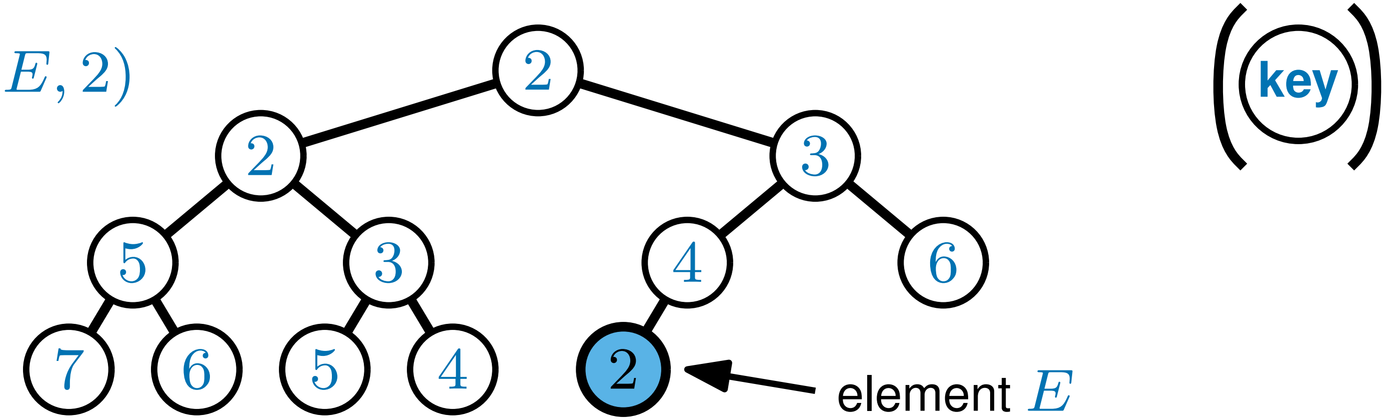

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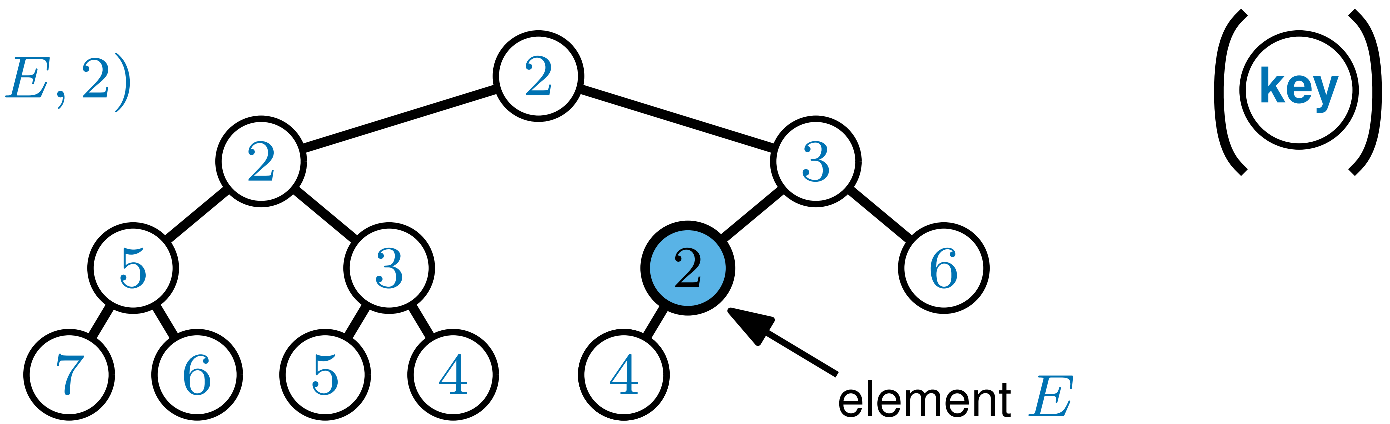

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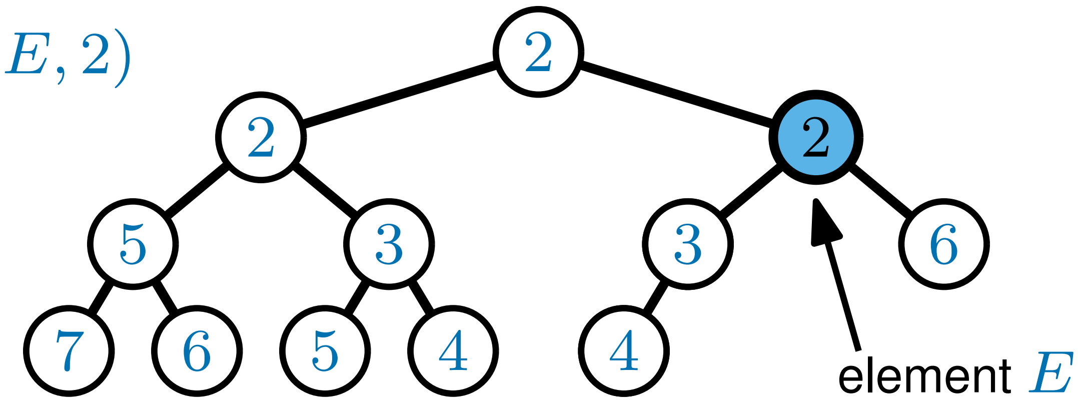

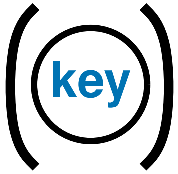

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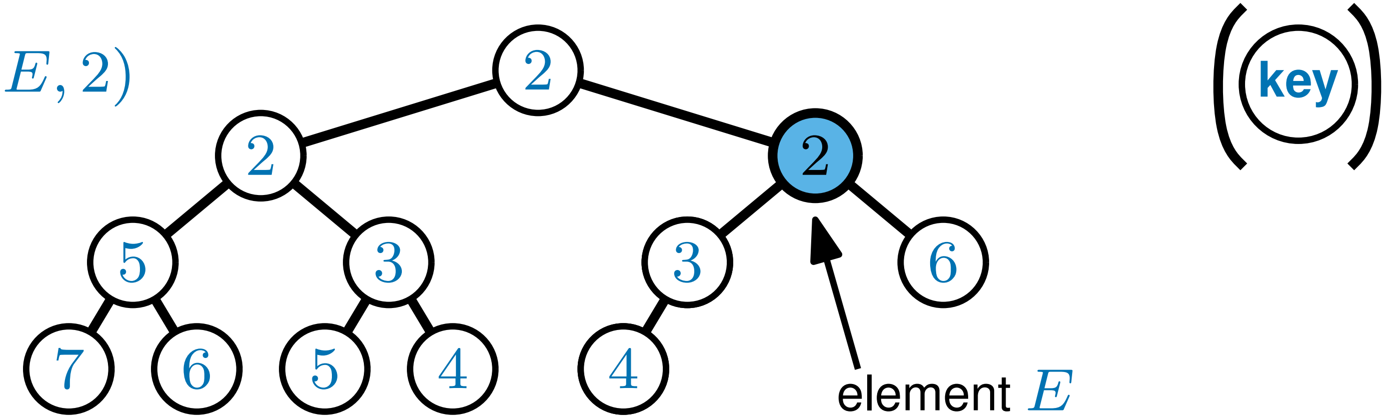
It's a heap again!



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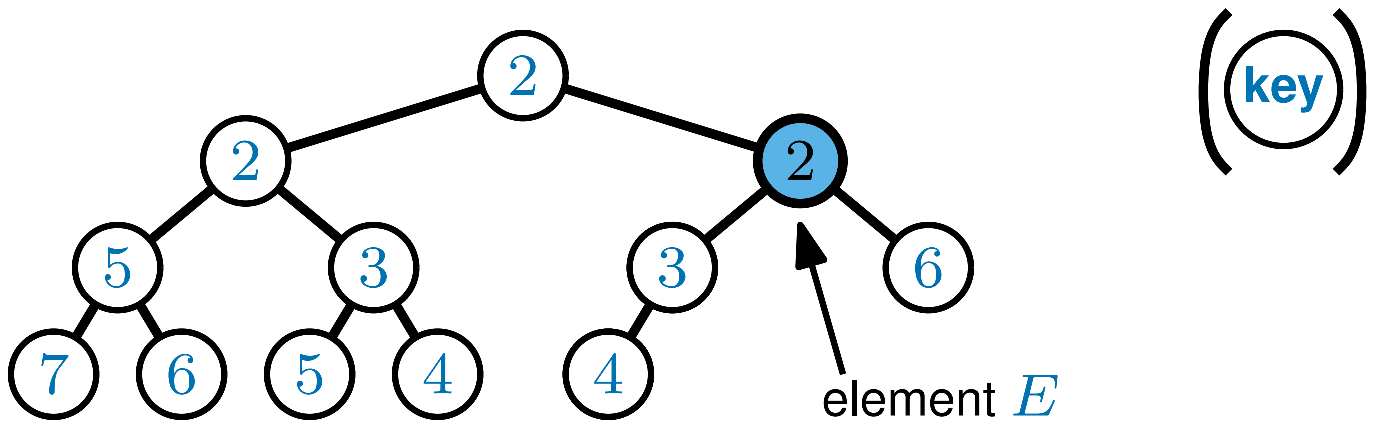

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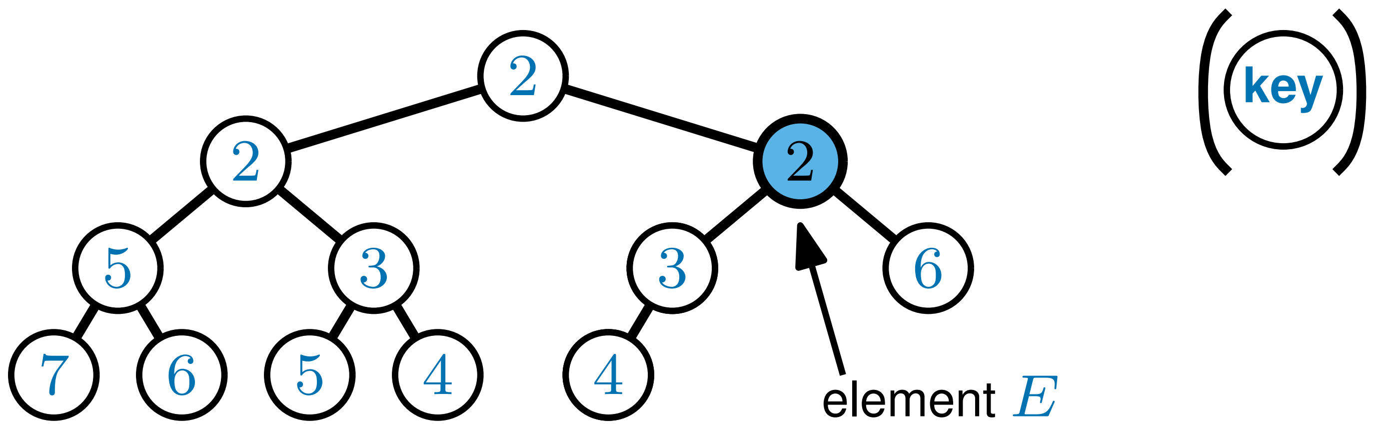

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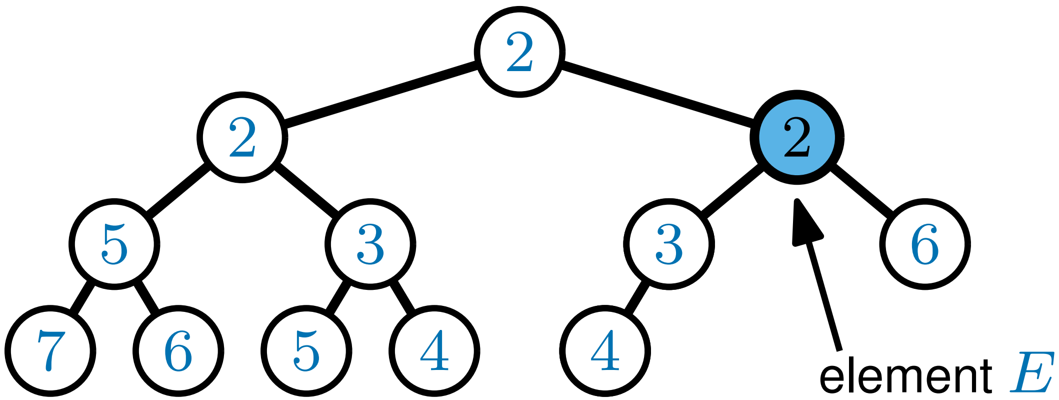

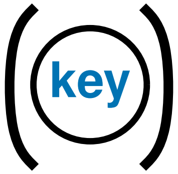

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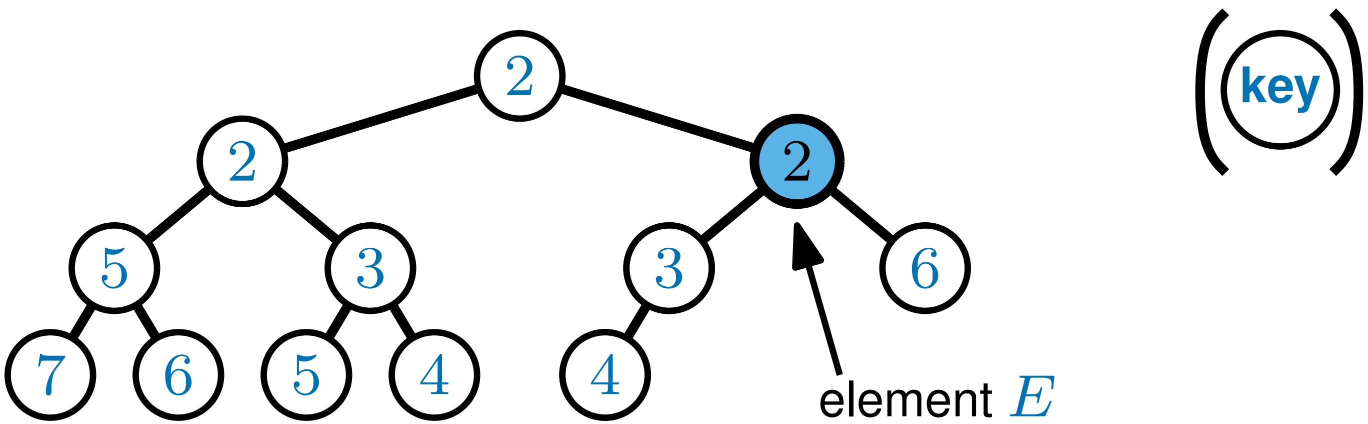

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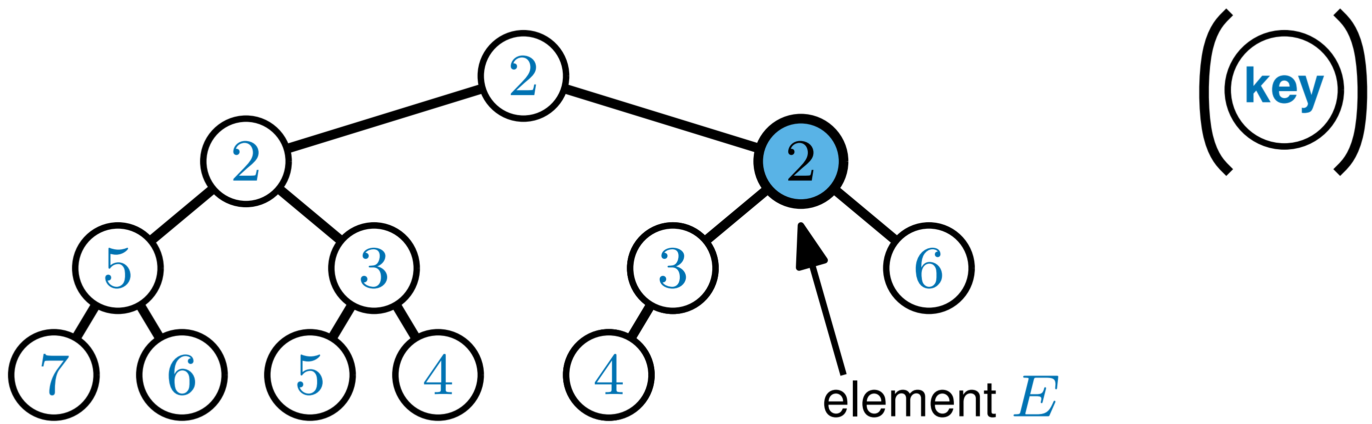

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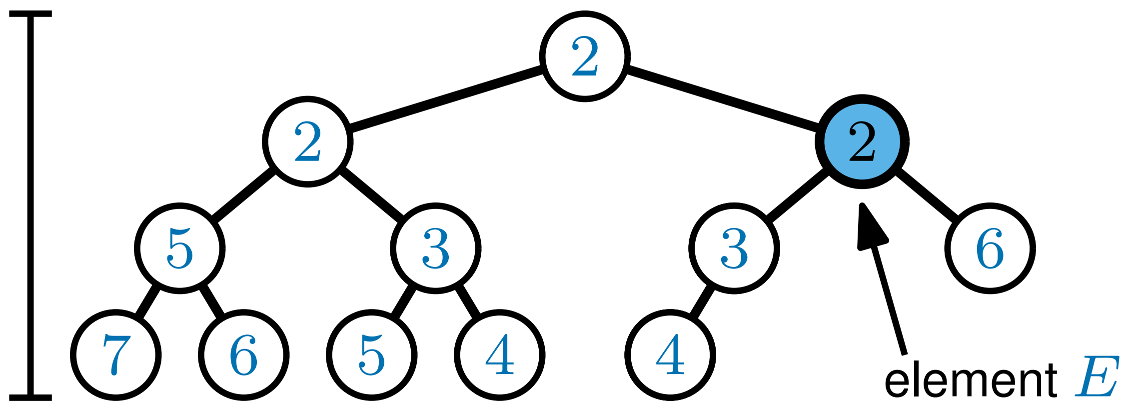

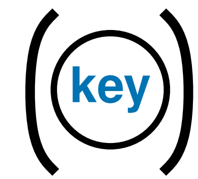

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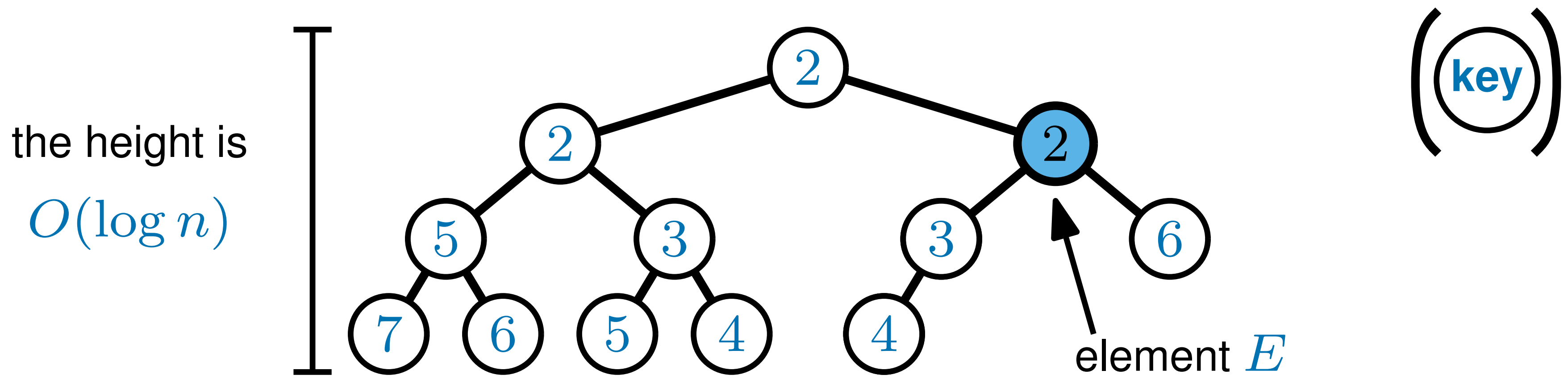

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The height of the tree is $O(\log n)$ *so there are* $O(\log n)$ *swaps*

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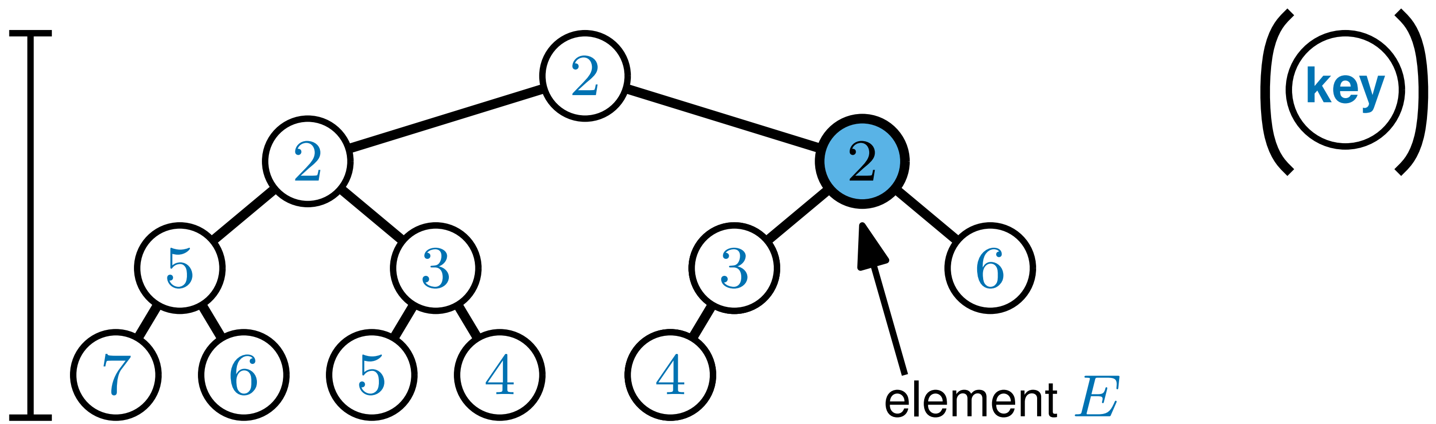

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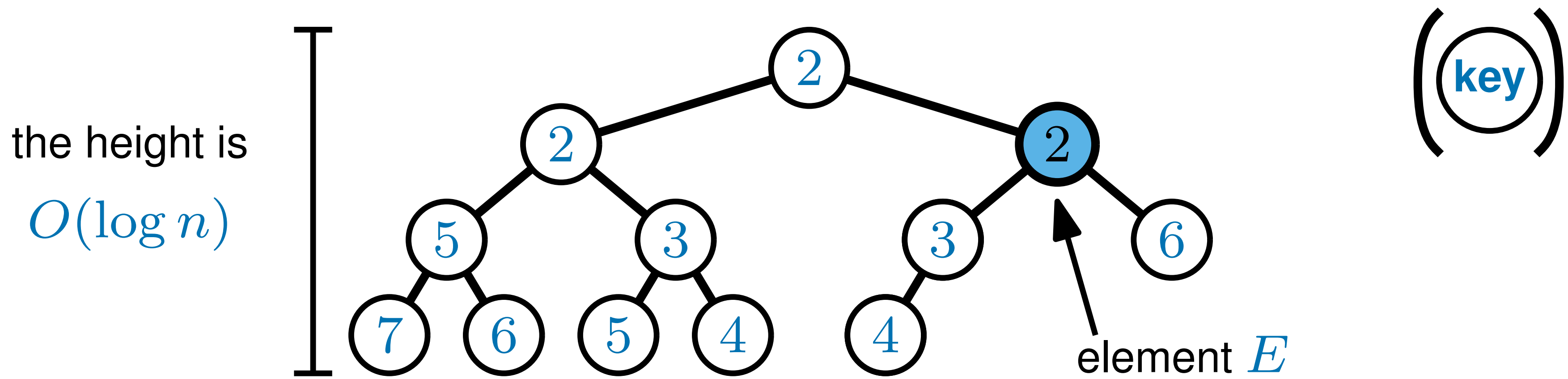

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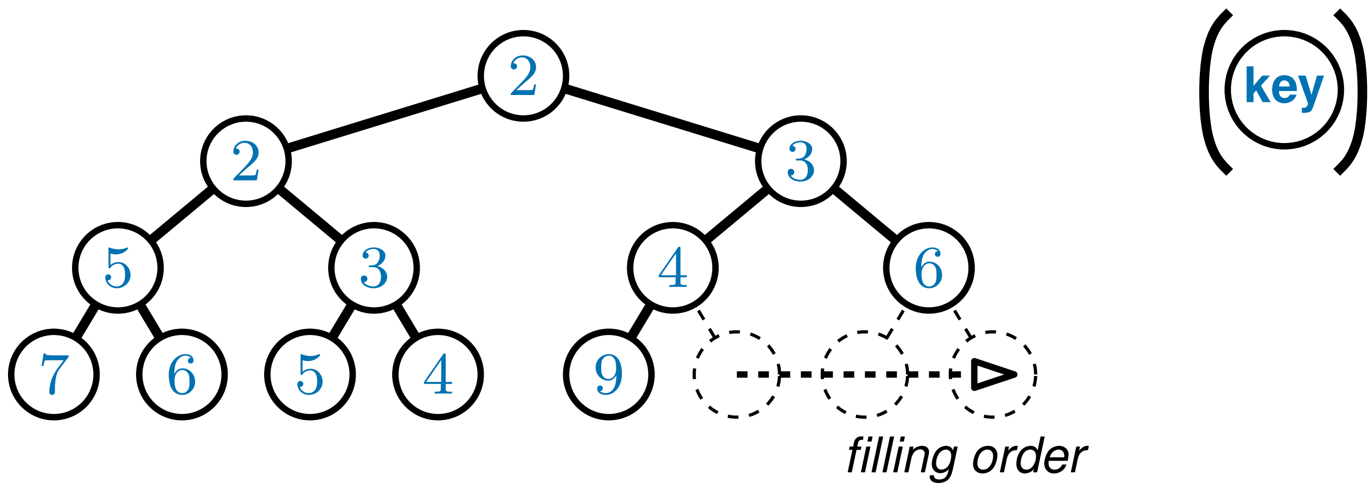
INSERT with a Binary Heap

$\textsc{Insert}(x, k)$ - inserts x with x.key $= k$



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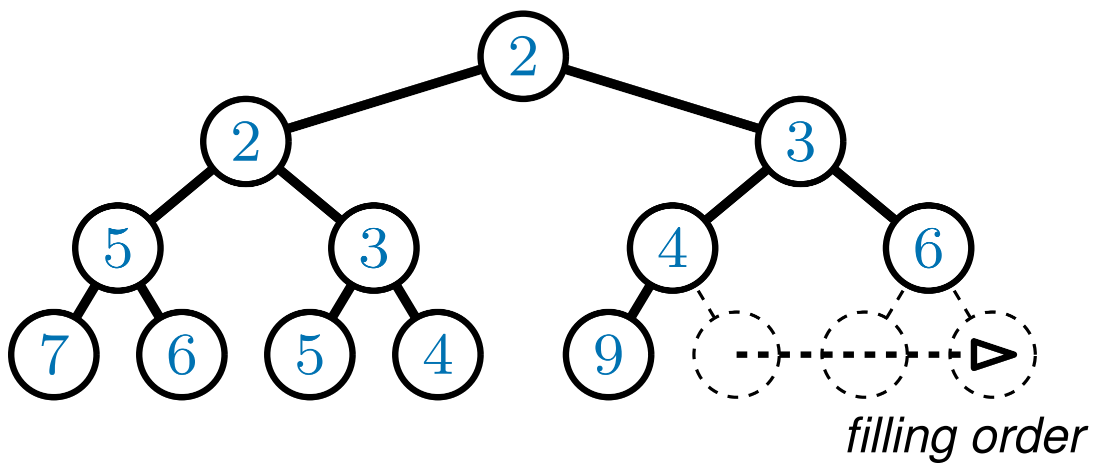
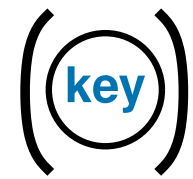
INSERT$(F, 1)$





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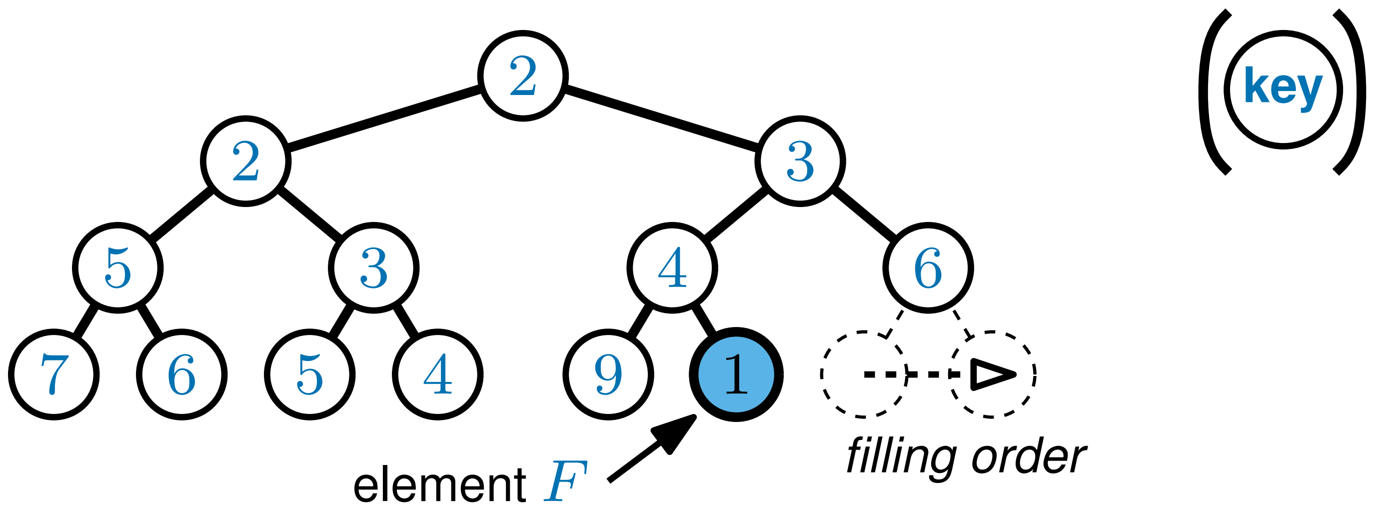

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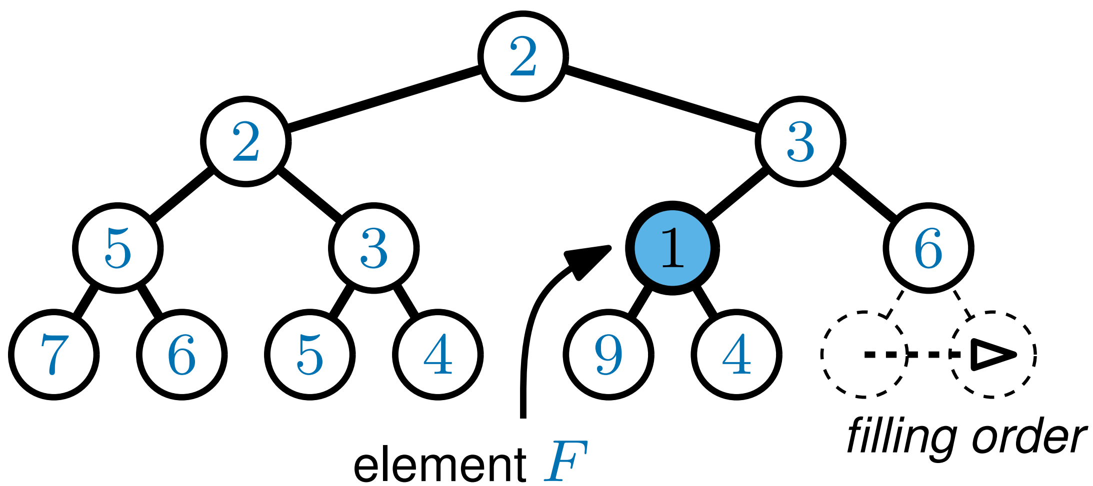

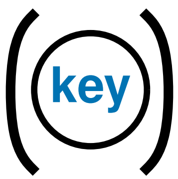

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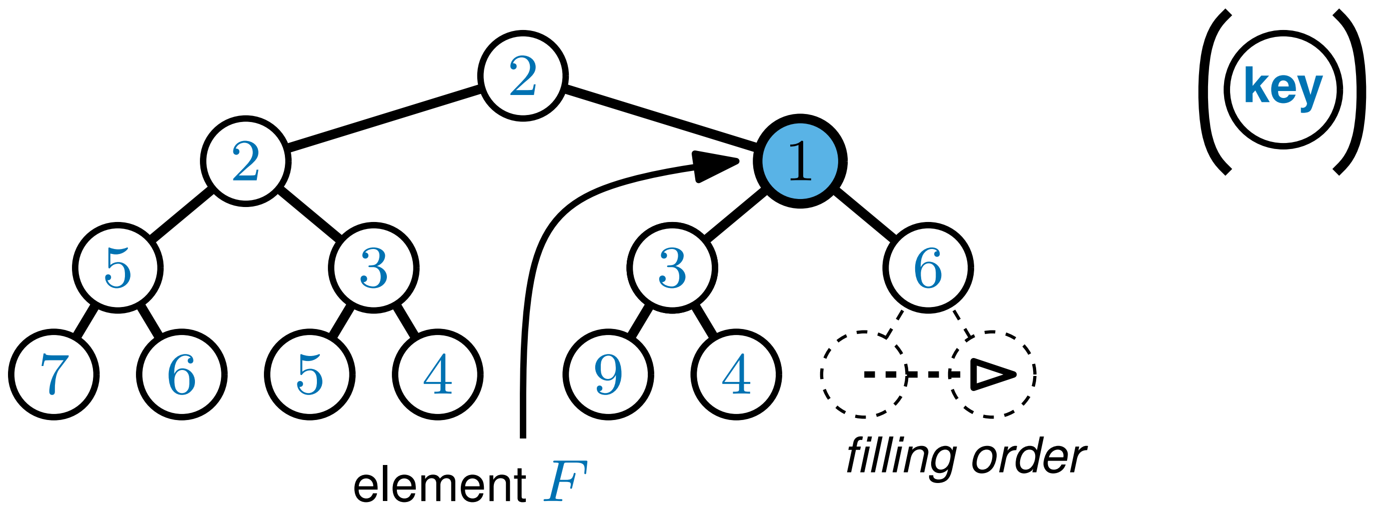

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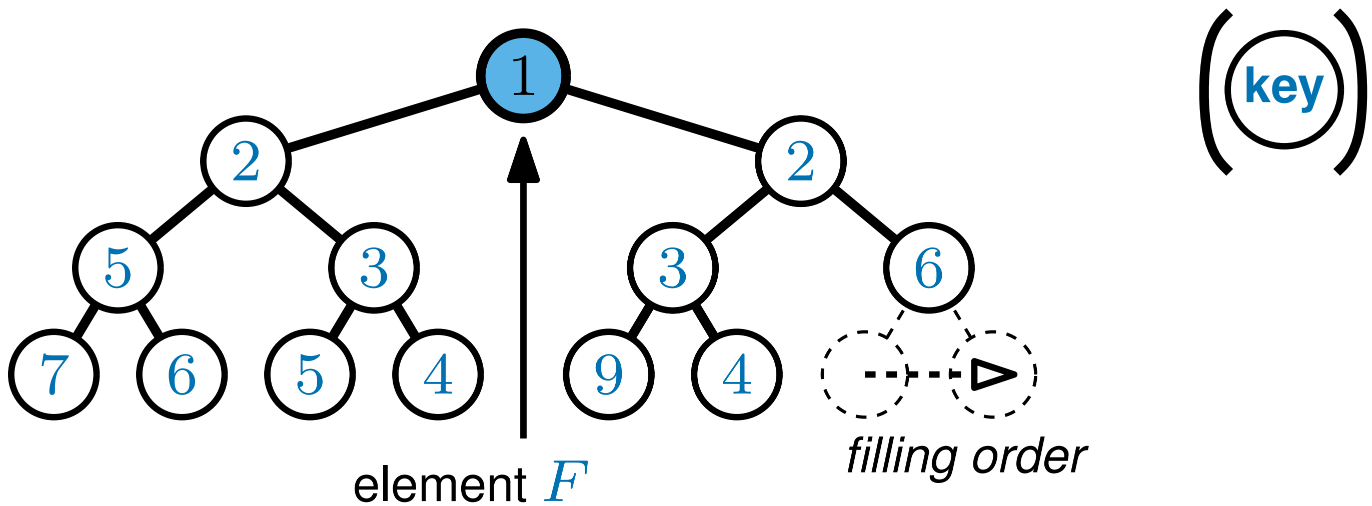

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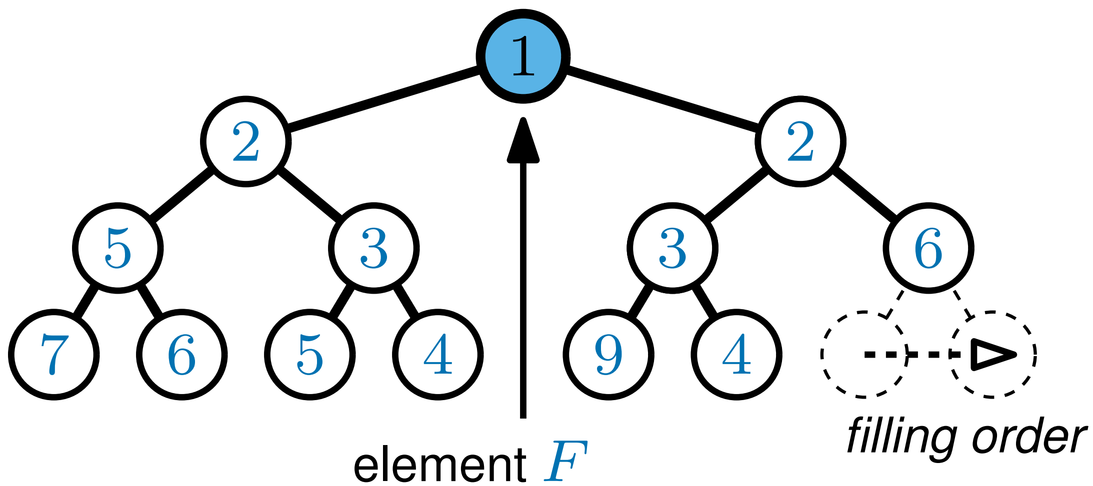

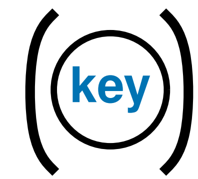

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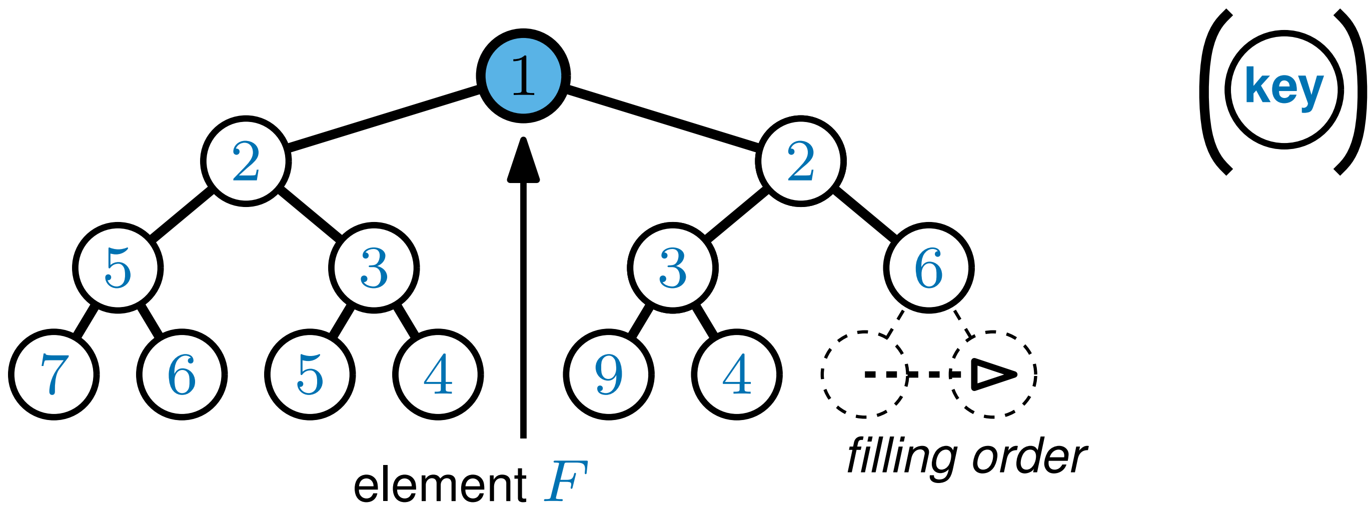

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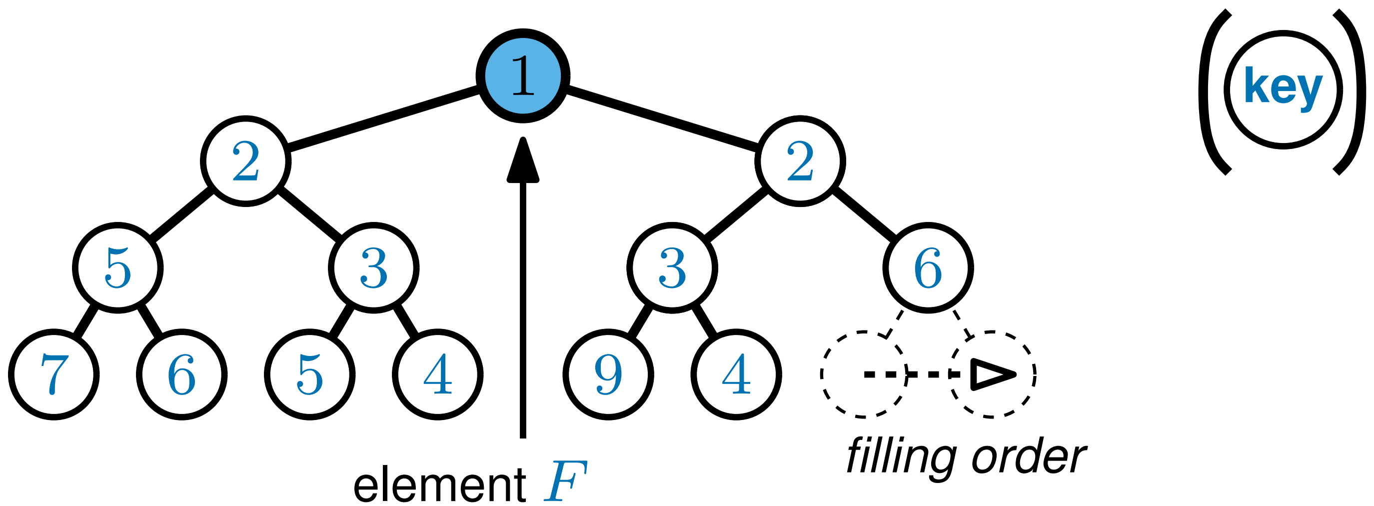
$\textsc{Insert}(F, 1)$



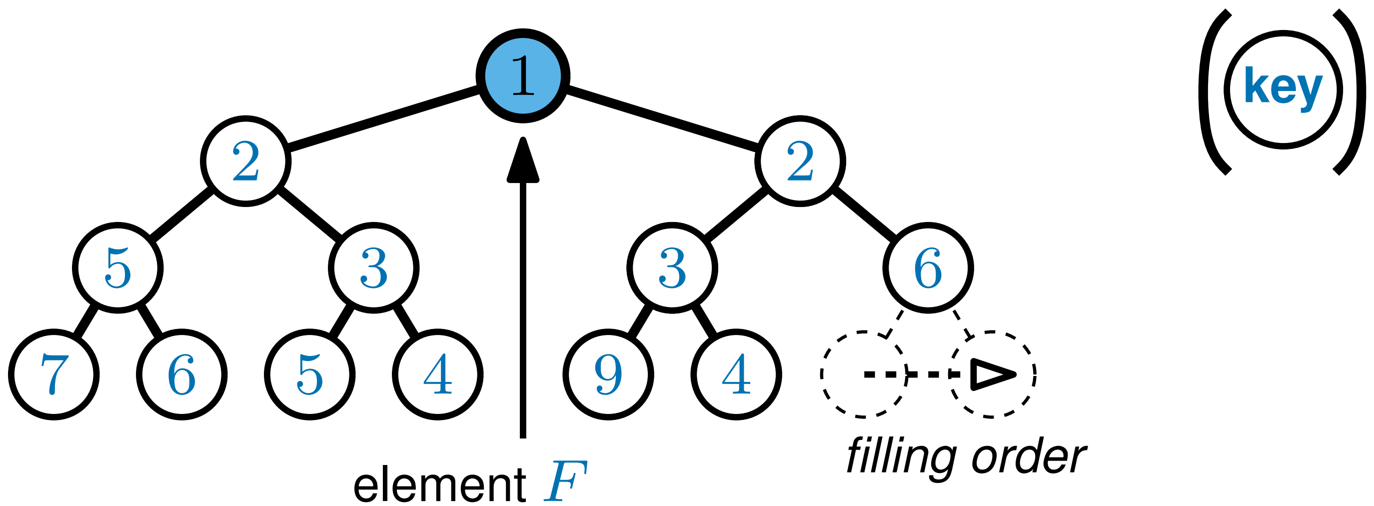
Step 1: Put element x in the next free slot ← $O(1)$ time

Step 2: Run $\textsc{DecreaseKey}$(x,k). ← $O(\log n)$ time

Overall this takes $O(\log n)$ time

INSERT with a Binary Heap

INSERT(x, k) - inserts x with x.key $= k$



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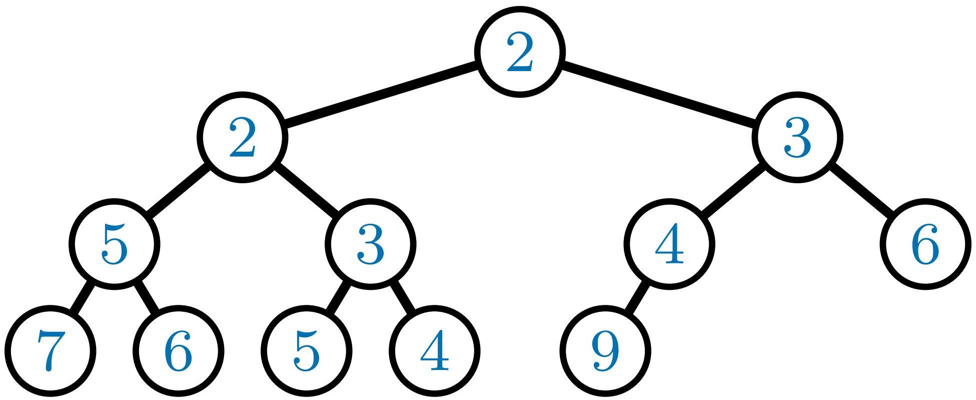
ExtractMin with a Binary Heap

ExtractMin() - removes and returns the element with the smallest key



Step 1: Extract the element at the root

by definition, it is the minimum

Step 2: Move the rightmost element in the bottom level to the root

(call this element y)

Step 3: While y.key is larger than one of its children's: *(stop if y becomes a leaf)*

swap y with the child with the *smaller key*

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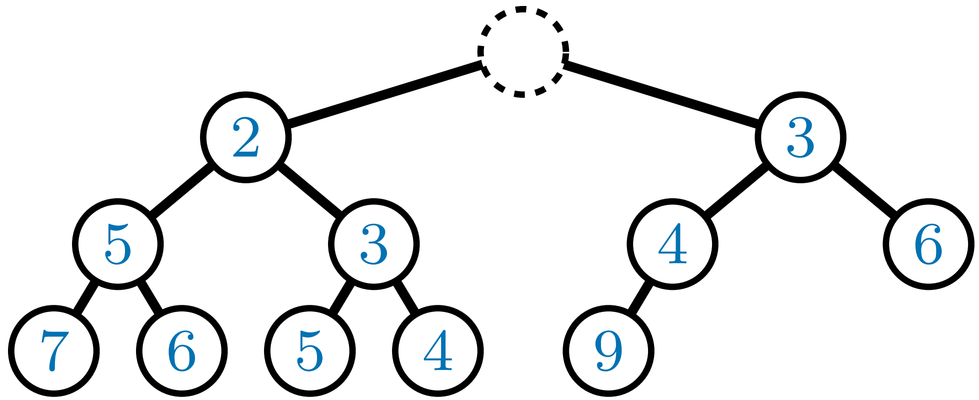

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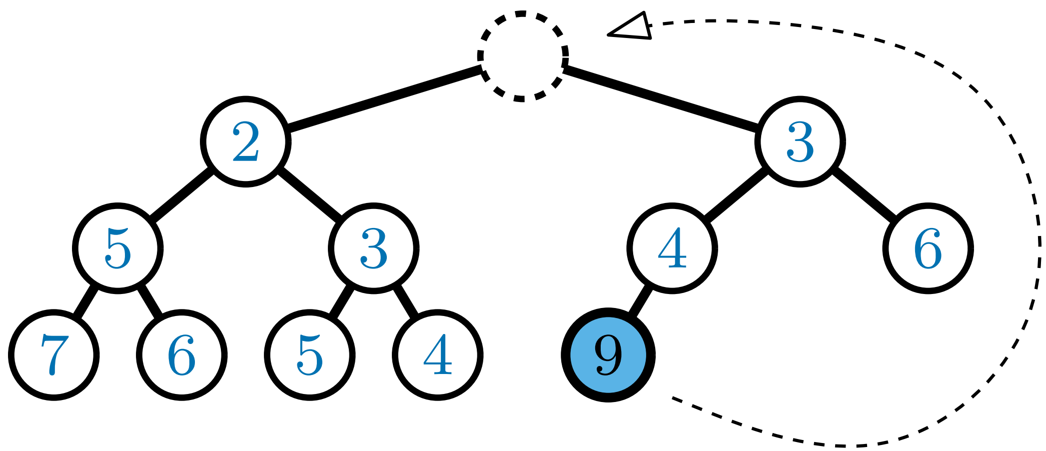

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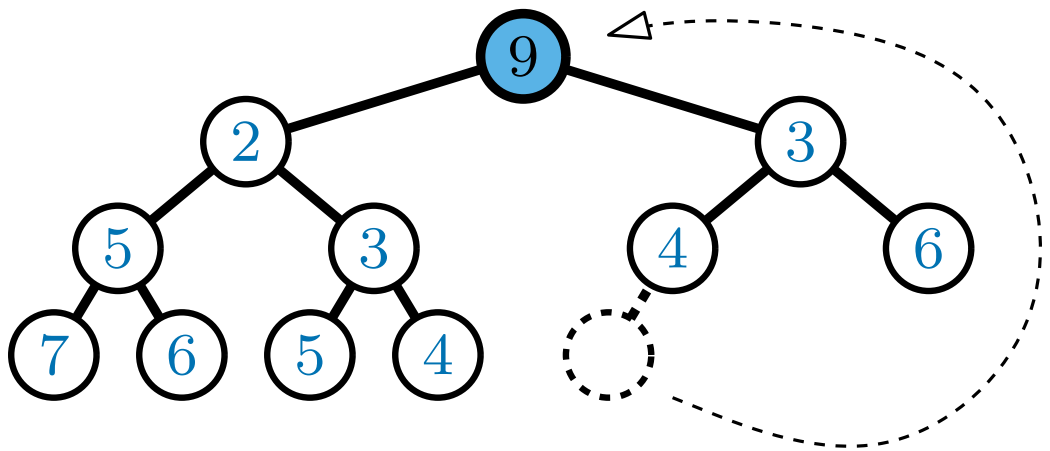

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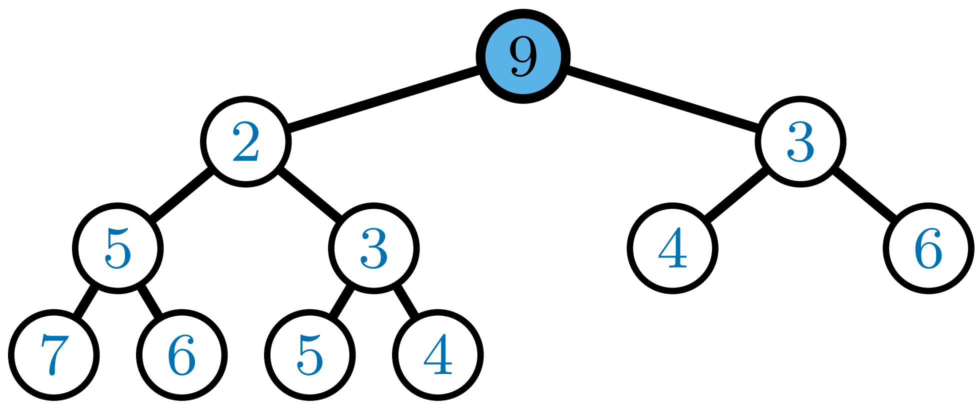

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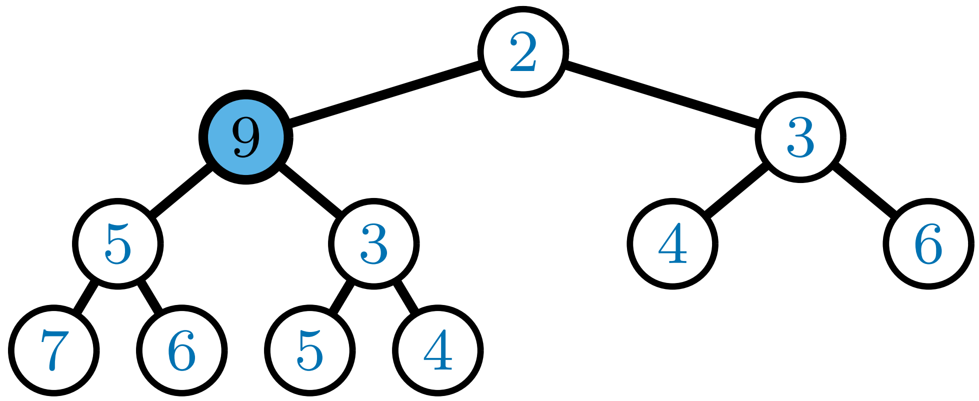

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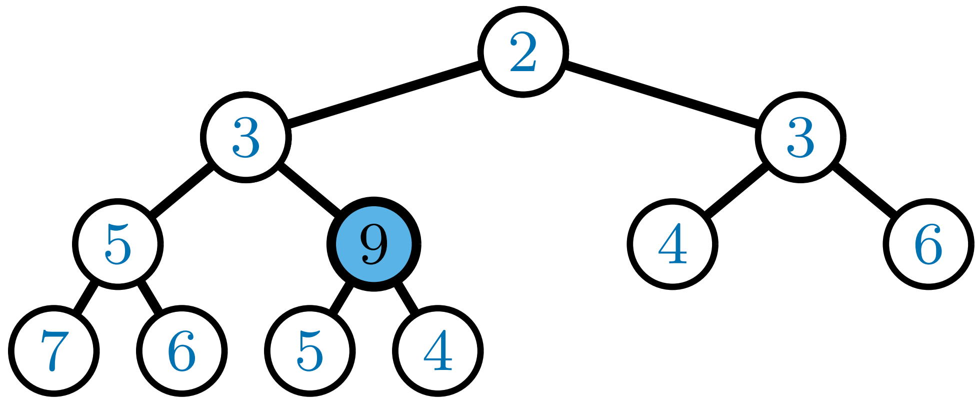

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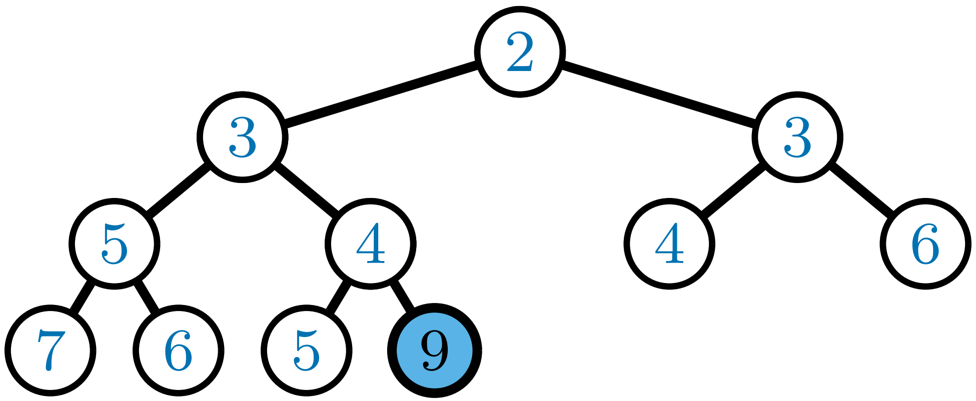

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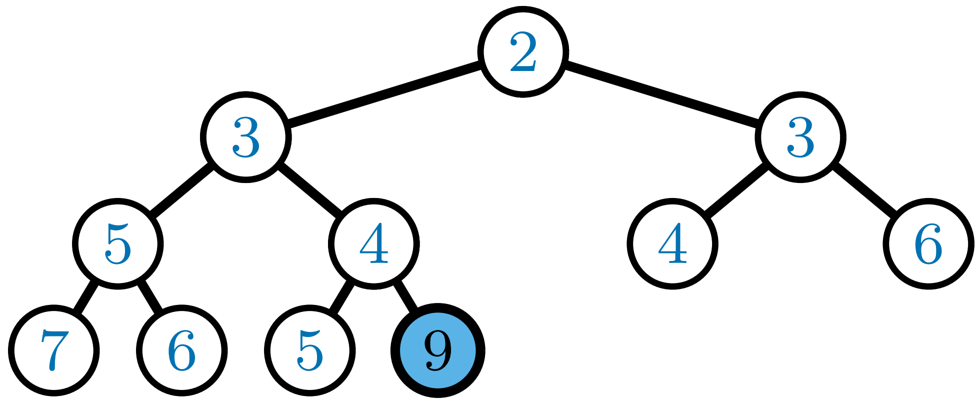

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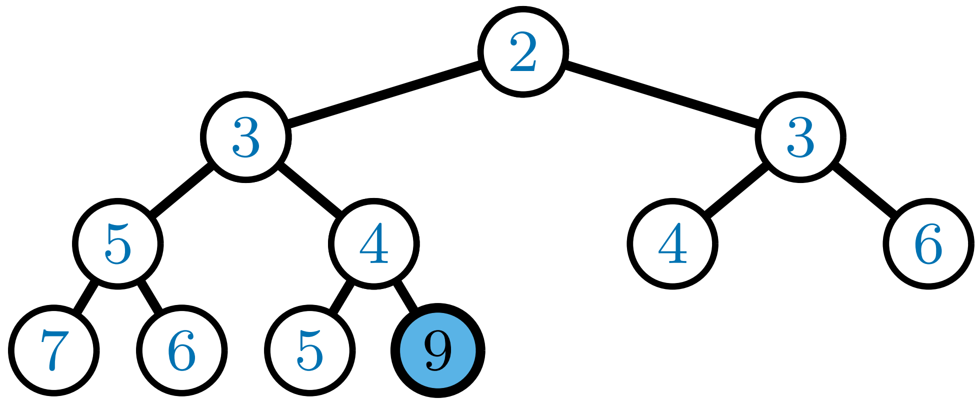

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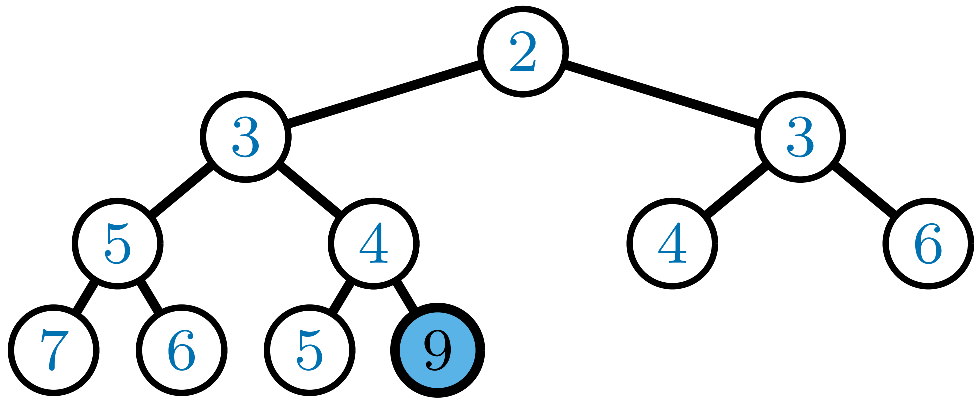

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Each swap takes

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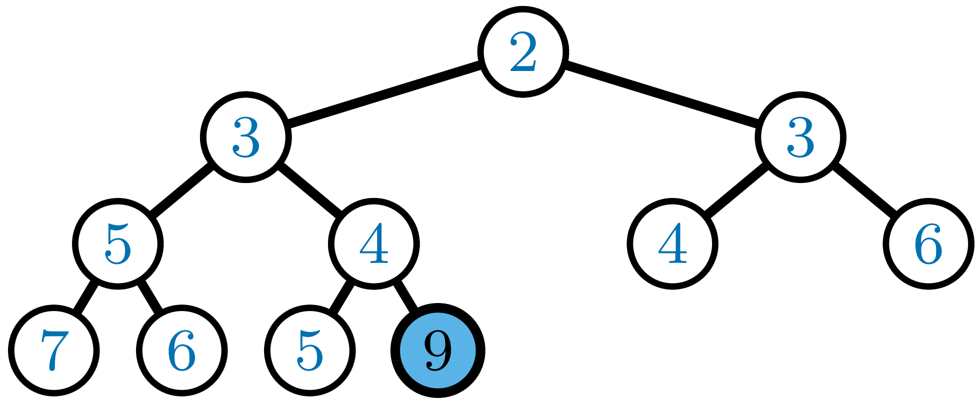

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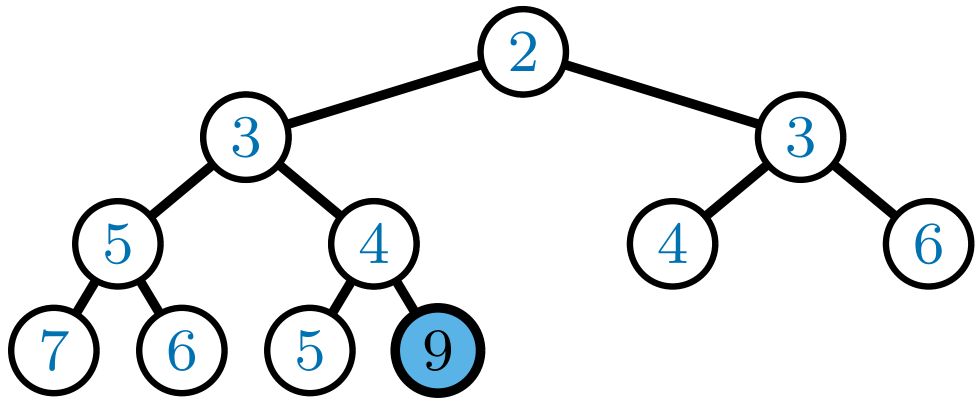
The height of the tree is $O(\log n)$ so there are $O(\log n)$ swaps (again)

ExtractMin with a Binary Heap

ExtractMin() - removes and returns the element with the smallest key



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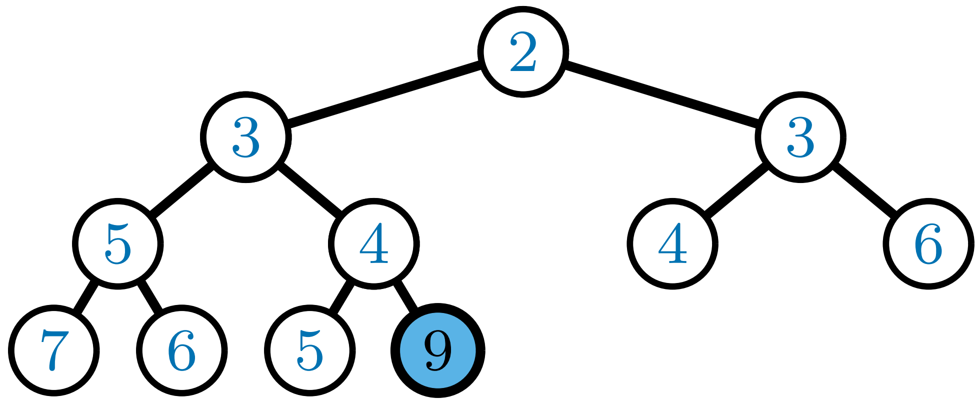

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Overall this takes $O(\log n)$ time

Priority queue Summary

We have seen three different **priority queue** implementations each supporting the following operations:

INSERT(x, k) - inserts x with x.key $= k$

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	INSERT	DECREASEKEY	EXTRACTMIN
Unsorted Linked List	$O(1)$	$O(n)$	$O(n)$
Sorted Linked List	$O(n)$	$O(n)$	$O(1)$
Binary Heap	$O(\log n)$	$O(\log n)$	$O(\log n)$

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What happened to that assumption?

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	$\textsc{Insert}$	$\textsc{DecreaseKey}$	$\textsc{ExtractMin}$
Unsorted Linked List	$O(1)$	$O(n)$	$O(n)$
Sorted Linked List	$O(n)$	$O(n)$	$O(1)$
Binary Heap	$O(\log n)$	$O(\log n)$	$O(\log n)$

What happened to that assumption?

Assumption we can find the location of any element x in the Heap in $O(1)$ time

That pesky **assumption**. . .

That pesky **assumption**...

Old Assumption we can find the location of any element x in the Heap in $O(1)$ time

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Previously we said that. . .

Each element x has an associated value called its key - x.key

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New (more reasonable) Assumption

Each element x also has an associated (unique) positive integer **ID** - x.ID $\leqslant N$

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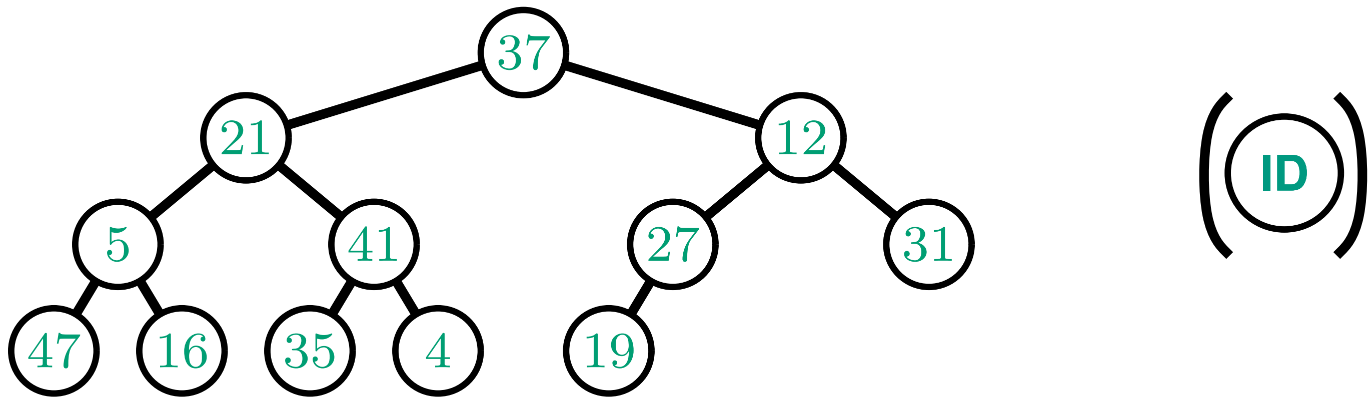

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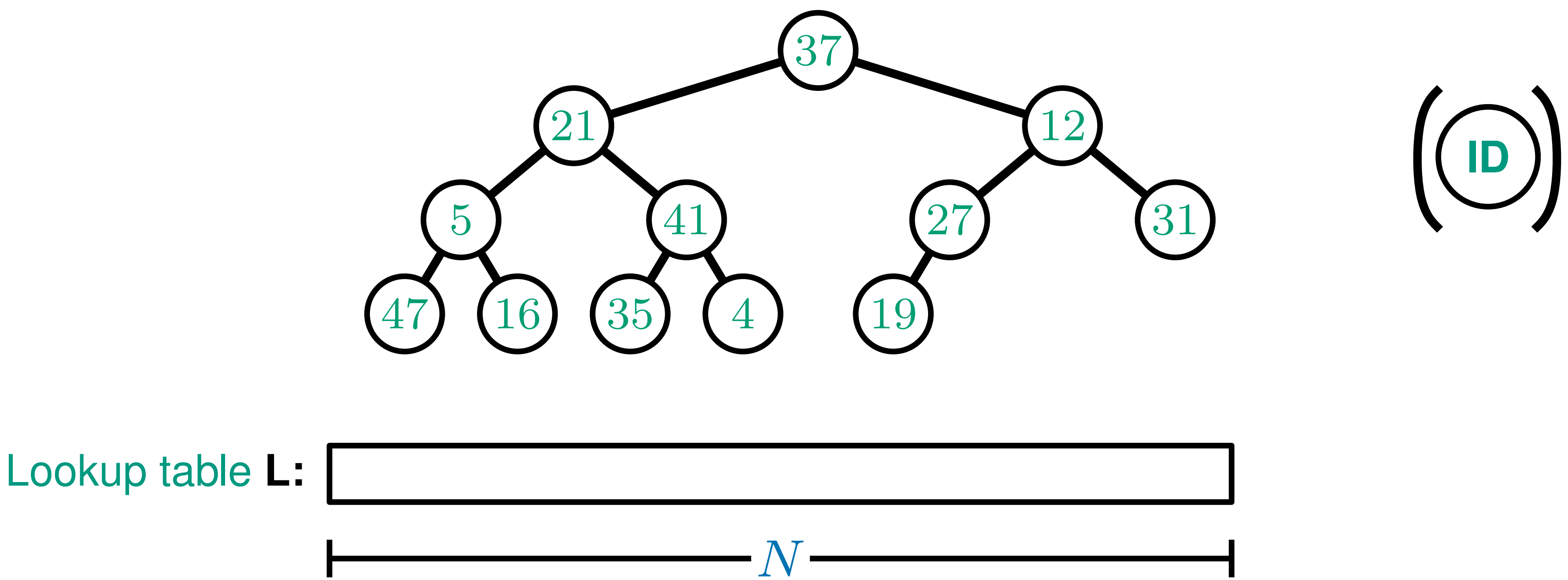

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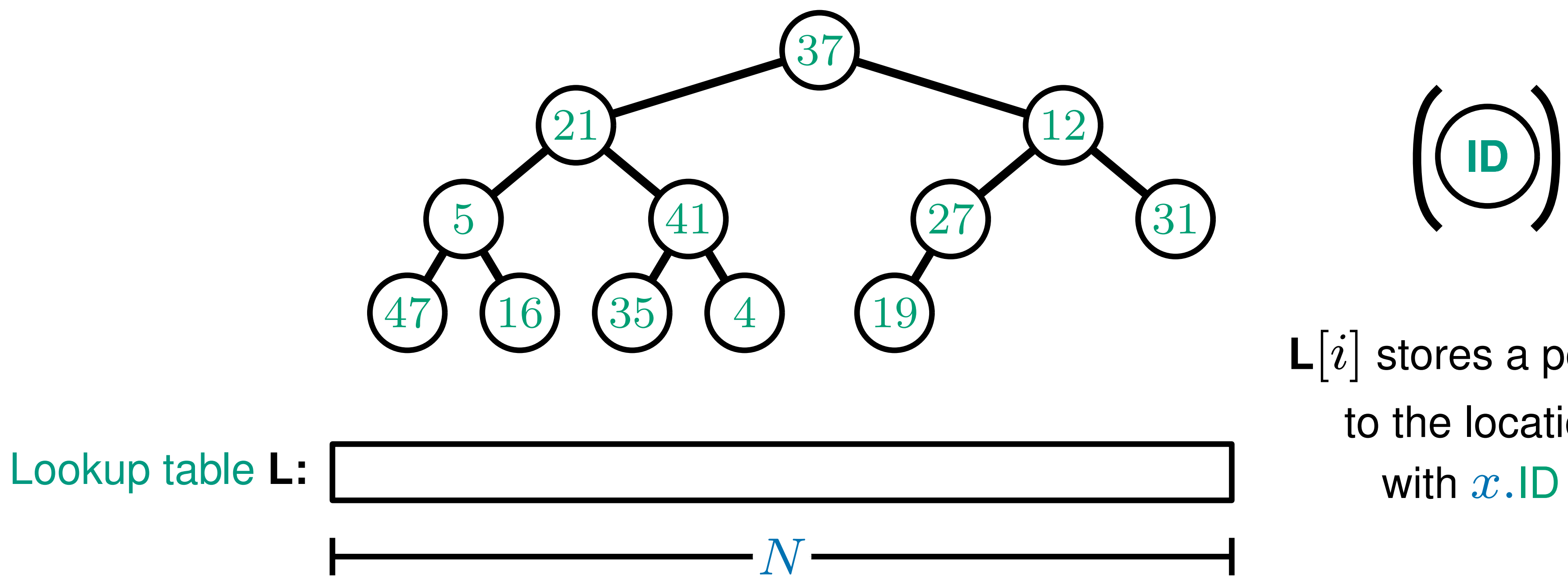

L$[i]$ stores a pointer

to the location of x

with $x.\text{ID} = i$

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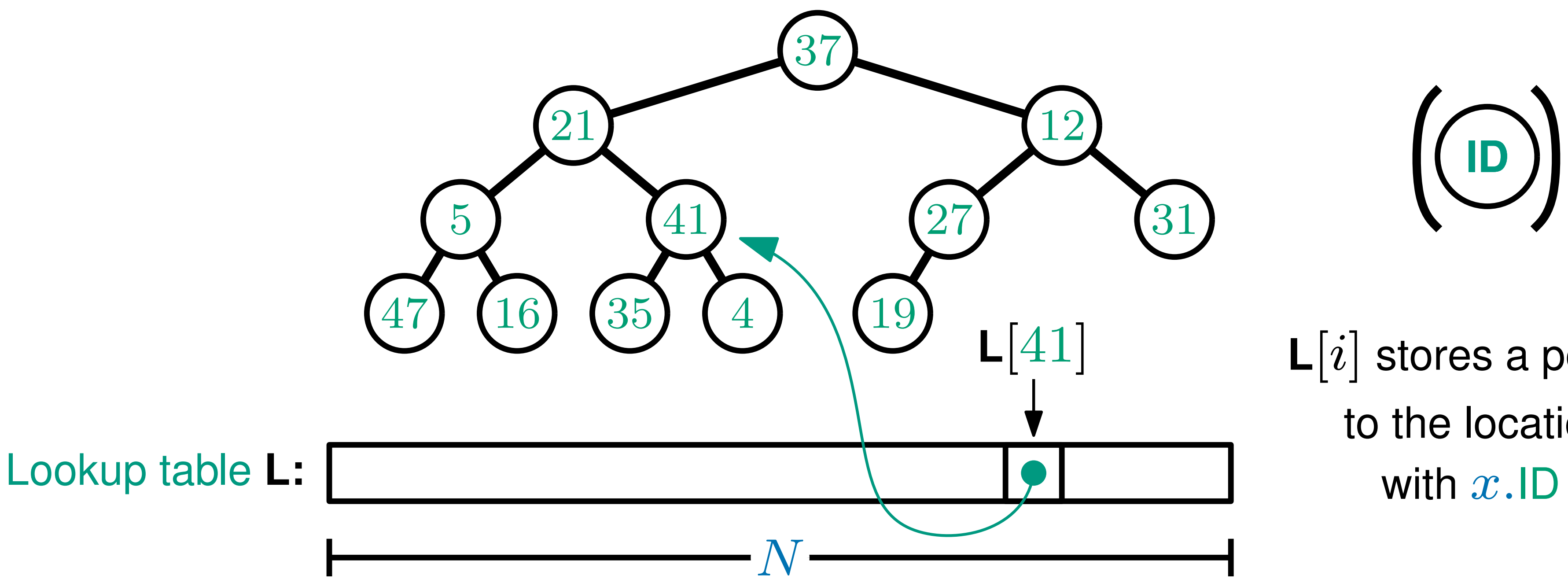

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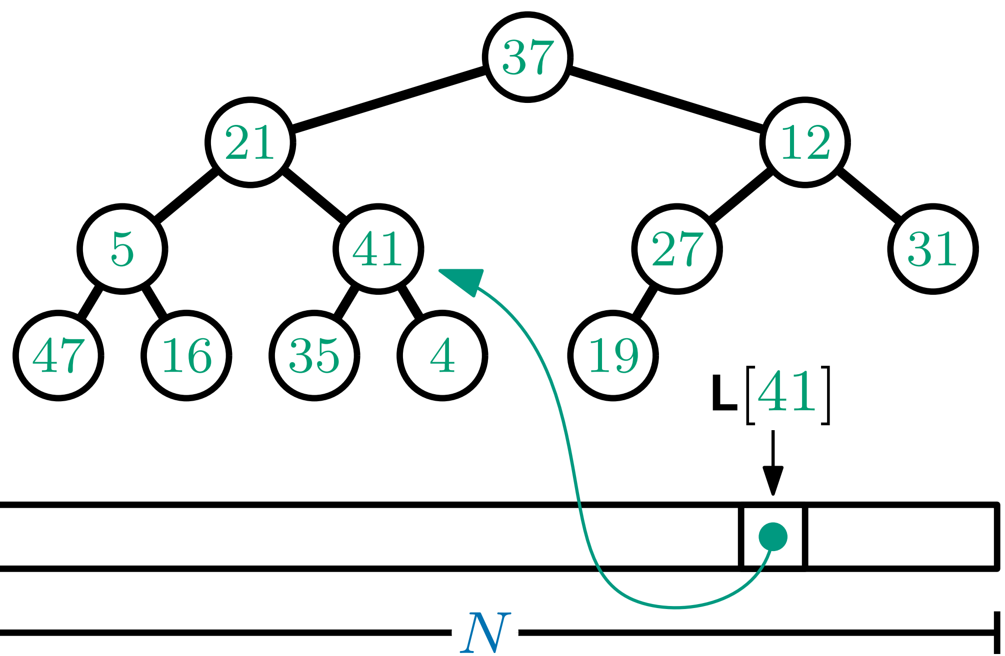

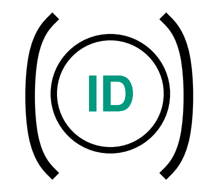

Lookup table **L:**

L$[i]$ stores a pointer to the location of x with $x.\text{ID} = i$

Whenever we move an element we update **L** in $O(1)$ time

That pesky **assumption**. . .

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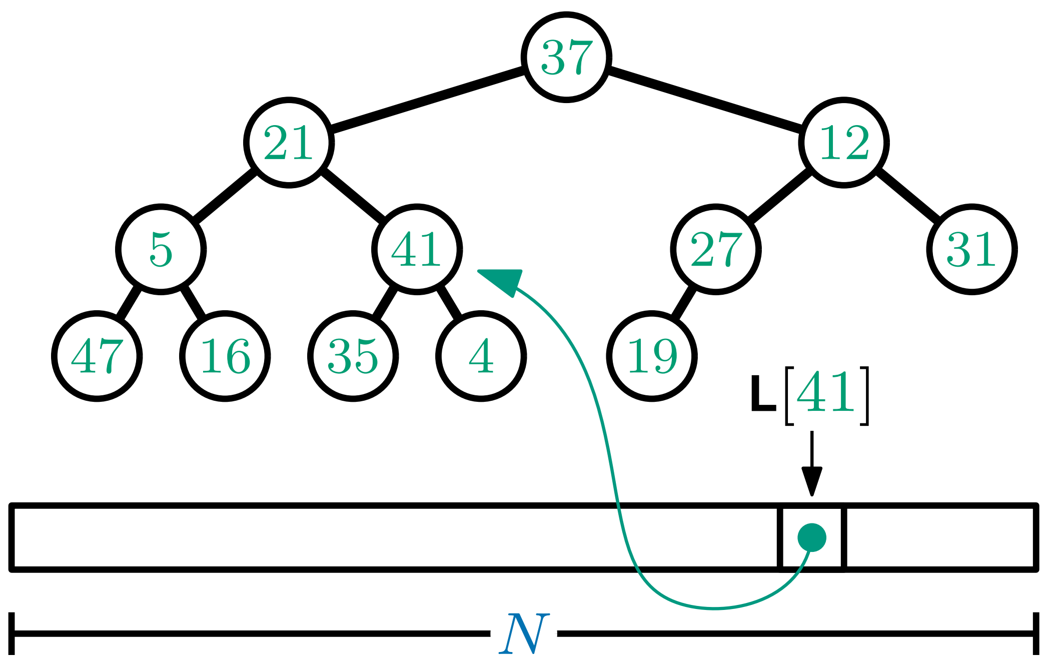

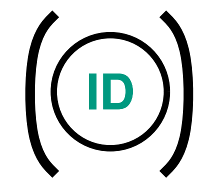

Lookup table **L:**

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Whenever we move an element we update **L** in $O(1)$ time

Finding element x takes $O(1)$ time as required

Priority queue Summary

We have seen three different **priority queue** implementations each supporting the following operations:

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EXTRACTMIN$()$ - removes and returns the element with the smallest key

	INSERT	DECREASEKEY	EXTRACTMIN	SPACE
Unsorted Linked List	$O(1)$	$O(n)$	$O(n)$	$O(n)$
Sorted Linked List	$O(n)$	$O(n)$	$O(1)$	$O(n)$
Binary Heap	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(N)$

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Unsorted Linked List	$O(1)$	$O(n)$	$O(n)$	$O(n)$
Sorted Linked List	$O(n)$	$O(n)$	$O(1)$	$O(n)$
Binary Heap	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(N)$

Spoiler: for the shortest path problem, $N = O(n)$

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Binary Heap	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(N)$

Is this the best possible?

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	Insert	DecreaseKey	ExtractMin	Space
Unsorted Linked List	$O(1)$	$O(n)$	$O(n)$	$O(n)$
Sorted Linked List	$O(n)$	$O(n)$	$O(1)$	$O(n)$
Binary Heap	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(N)$

Is this the best possible? *actually, no :)*

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Sorted Linked List	$O(n)$	$O(n)$	$O(1)$	$O(n)$
Binary Heap	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(N)$

Is this the best possible? *actually, no :)*

Fibonacci Heap	$O(1)$	$O(1)$	$O(\log n)$	$O(n)$

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Unsorted Linked List	$O(1)$	$O(n)$	$O(n)$	$O(n)$
Sorted Linked List	$O(n)$	$O(n)$	$O(1)$	$O(n)$
Binary Heap	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(N)$

Is this the best possible? *actually, no :)*

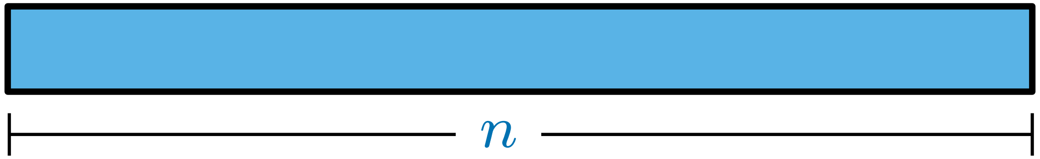
Fibonacci Heap	$O(1)$	$O(1)$	$O(\log n)$	$O(n)$

. . . but Fibonacci Heaps are complicated, amortised and have large hidden constants

One more thing. . .

Take an array of elements of length n

$\mathbf{A}$



Insert every element into a priority queue:

ExtractMin from the priority queue n times

and put the elements in A' in the order they come out

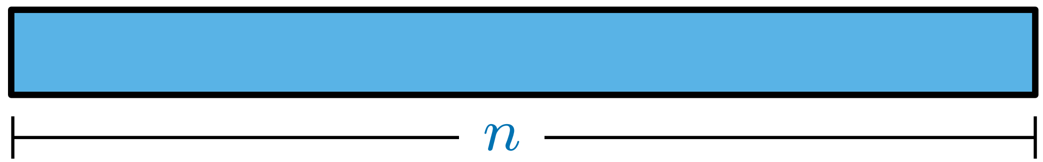
$\mathbf{A}'$

what is $\mathbf{A}'$?

One more thing. . .

Take an array of elements of length n

$\mathbf{A}$



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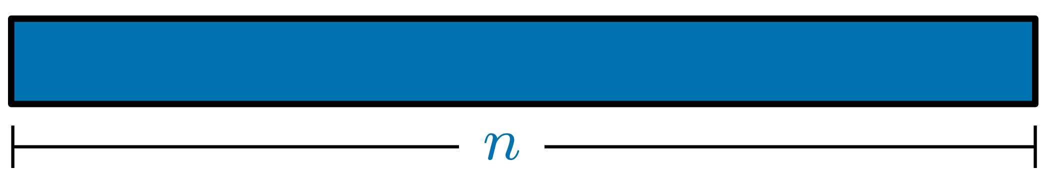

what is $\mathbf{A}'$? *it's A in sorted order*

One more thing...

Take an array of elements of length n

$\mathbf{A}$ — n

INSERT every element into a priority queue:

If you implement the priority queue as a Binary Heap

You can use this to sort in $O(n \log n)$ time

EXTRACTMIN from the priority queue n times

and put the elements in A' in the order they come out

$\mathbf{A}'$ — n

what is $\mathbf{A}'$? *it's A in sorted order*

HeapSort

Take an array of elements of length n

$\mathbf{A}$

n

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If you implement the priority queue as a Binary Heap

You can use this to sort in $O(n \log n)$ time

EXTRACTMIN from the priority queue n times

and put the elements in A' in the order they come out

$\mathbf{A}'$

n

what is $\mathbf{A}'$? *it's A in sorted order*

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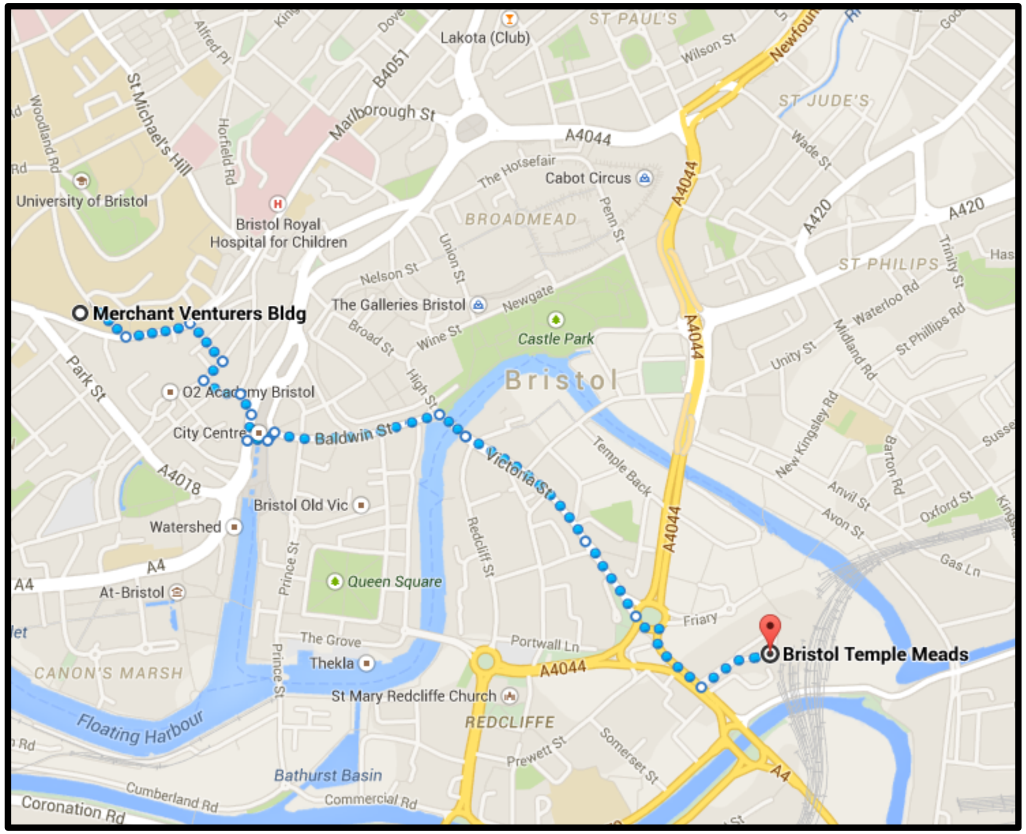
End of part one

Part two

Dijkstra’s Algorithm

In today's lectures we'll be discussing the **single source shortest paths** problem
in a weighted, directed graph. . .



The shortest path from MVB to Temple Meads
(according to Google Maps)

In particular we'll be interested in **Dijkstra's Algorithm**

which is based on an abstract data structure called a priority queue
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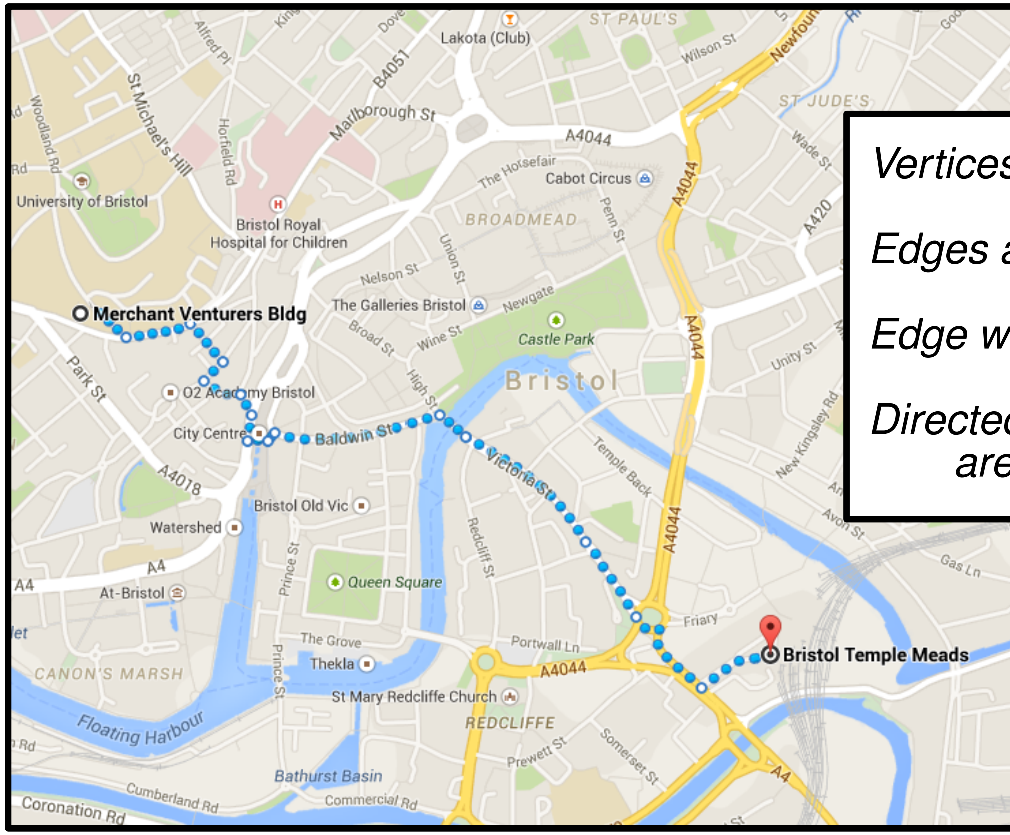


Vertices are junctions

Edges are roads

Edge weights are in miles

Directed edges
are one-way streets

The shortest path from MVB to Temple Meads
(according to Google Maps)

In particular we'll be interested in **Dijkstra's Algorithm**

which is based on an abstract data structure called a priority queue
. . . which can be efficiently implemented as a binary heap

Post-lunch Priority Queue refresher

A **priority queue**, Q stores a set of distinct elements

Each element x has an associated value called its key - x.key

A priority queue supports the following operations:

$\textsc{Insert}(x, k)$ - inserts x with x.key $= k$

$\textsc{DecreaseKey}(x, k)$ - decreases the value of x.key to k

where $k < x.key$

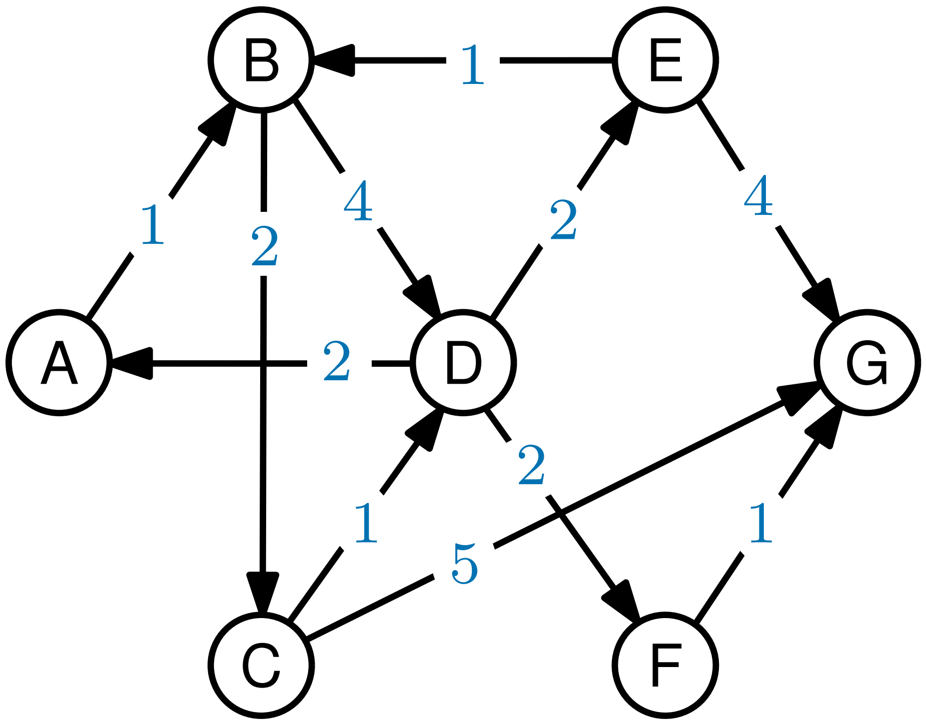
$\textsc{ExtractMin}()$ - removes and returns the element with the smallest key

(ties are broken arbitrarily)

Single source shortest paths

Dijkstra's Algorithm solves the **single source shortest paths** problem

in a **weighted**, directed graph...



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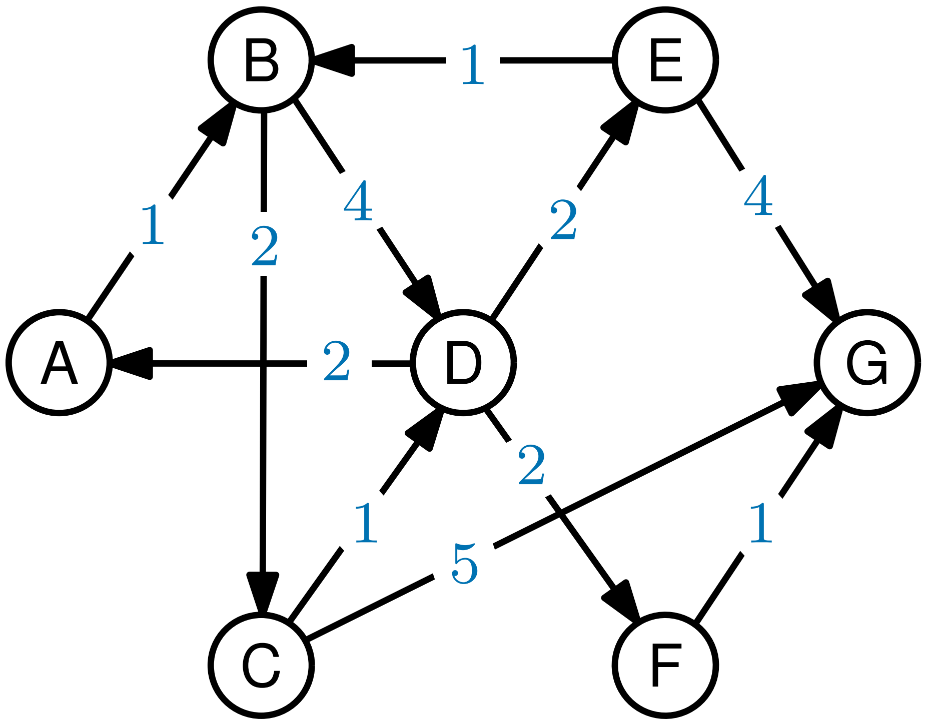

It finds the shortest path from

a given *source* vertex

to every other vertex

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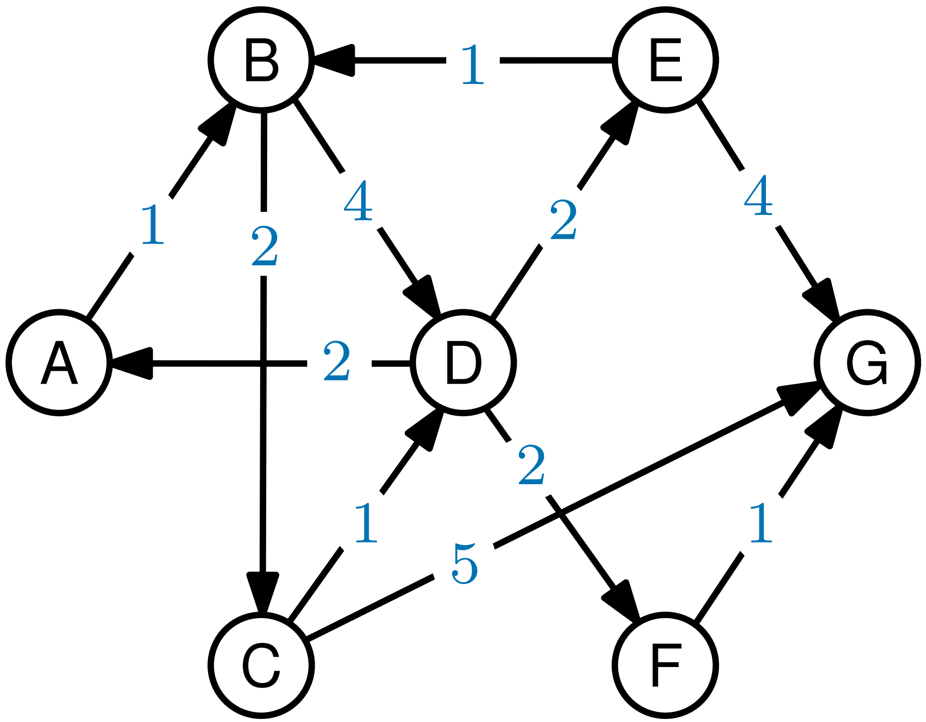

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The weights have to be non-negative

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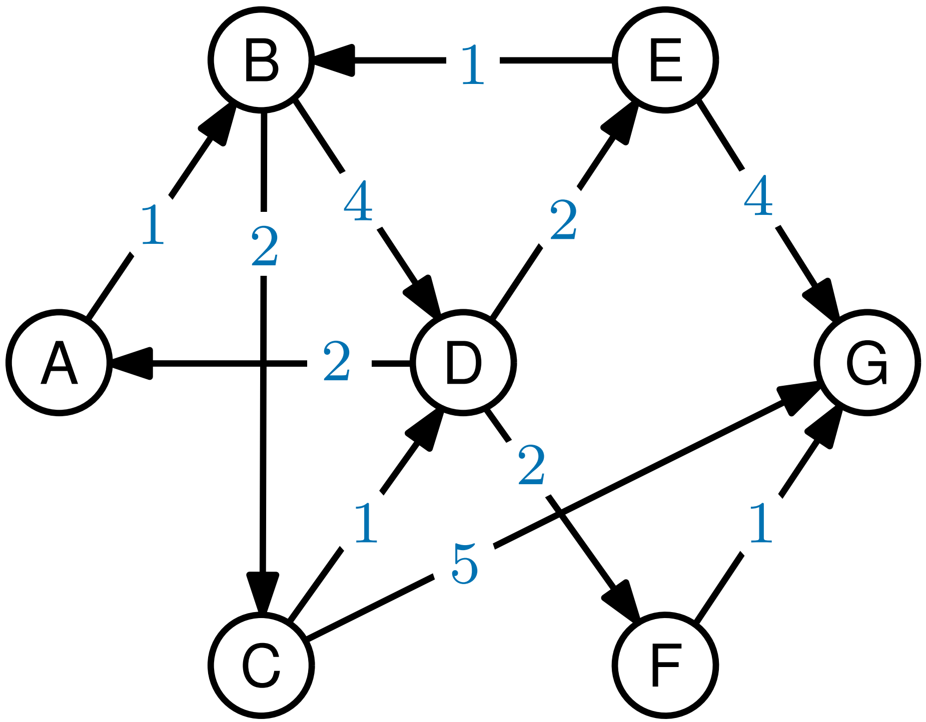

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The graph is stored as an **Adjacency List**

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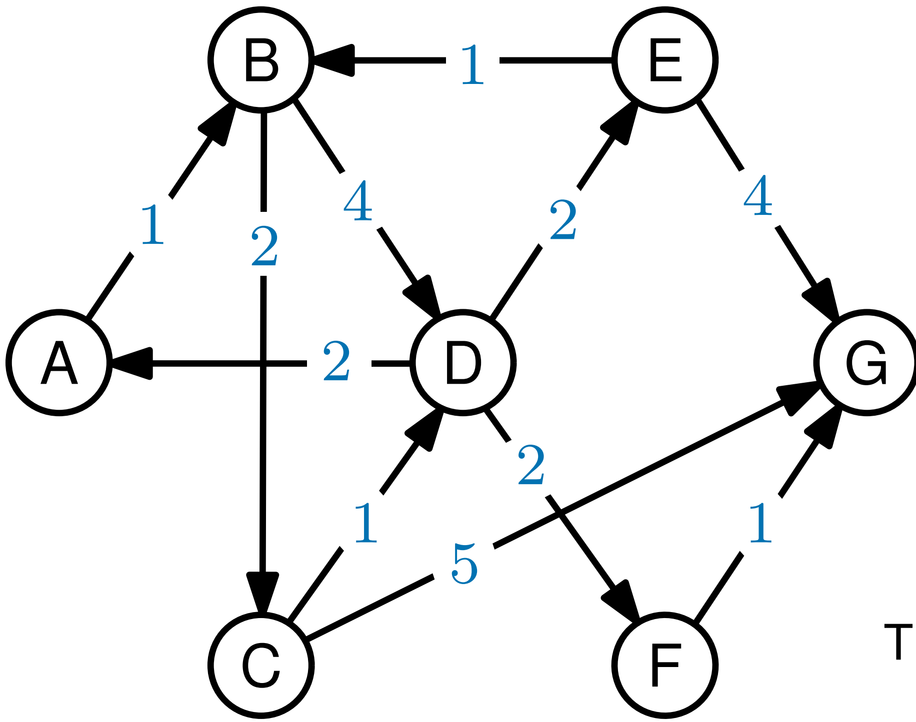

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The time complexity will depend on
how efficient the priority queue used is

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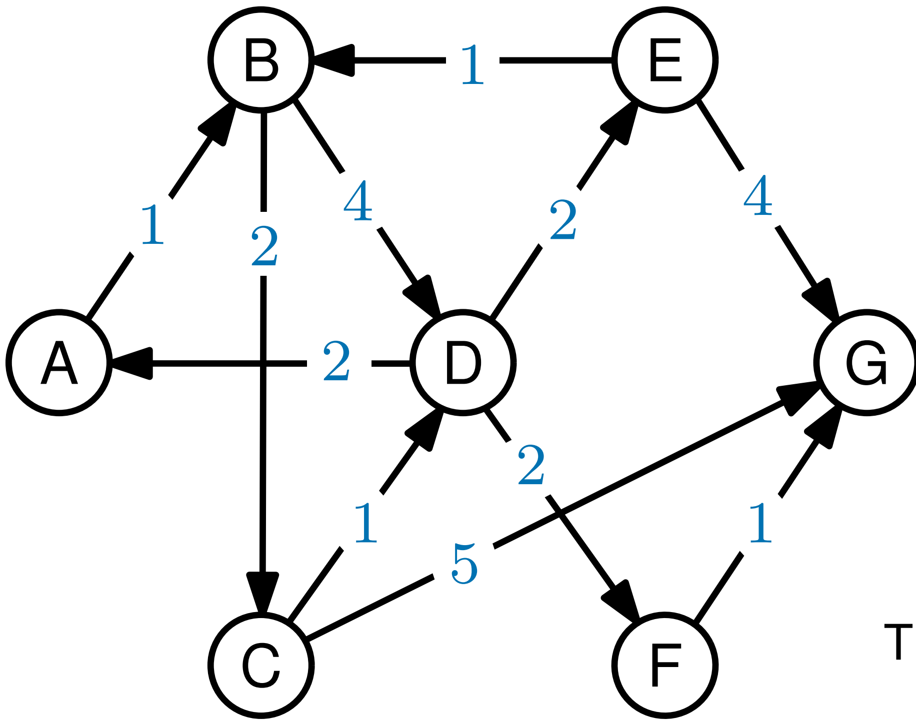

It finds the shortest path from

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The weights have to be non-negative

The graph is stored as an **Adjacency List**

The time complexity will depend on how efficient the priority queue used is

Remember from Monday's lecture that in **unweighted**, directed graphs, Breadth First Search solves this problem in $O(|V| + |E|)$ time

$|V|$ is the number of vertices and $|E|$ is the number of edges

Dijkstra's algorithm

We assume that we have a priority queue, supporting
INSERT, DECREASEKEY and EXTRACTMIN

DIJKSTRA(s)

```
For all v, set dist(v) = ∞
set dist(s) = 0
For each v, Do INSERT(v, dist(v))
While the queue is not empty
   u = EXTRACTMIN()
   For every edge (u, v) ∈ E
      If dist(v) > dist(u) + weight(u, v)
         dist(v) = dist(u) + weight(u, v)
         DECREASEKEY(v, dist(v))
```

$(u, v) \in E$ iff there is an edge *from* u *to* v

$\texttt{weight}(u, v)$ is the weight of the edge from u to v

$\texttt{dist}(v)$ is the length of the best path between s and v, *found so far*

Claim when Dijkstra's algorithm terminates,

for each vertex v, $\texttt{dist}(v)$ is the distance between s and v

$\textsc{Dijkstra}(s)$

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```

We're going to simulate

$\textsc{Dijkstra}(A)$

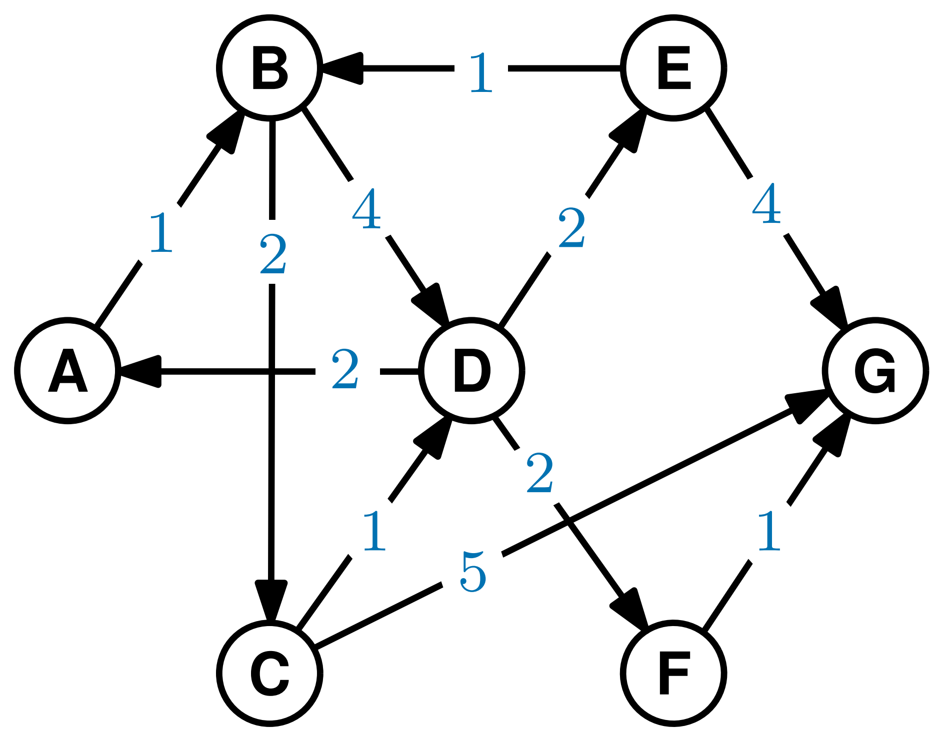
i.e. $s = A$

	A	B	C	D	E	F	G
dist:							

$\texttt{dist}(v)$ is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

$v.\text{key} = \texttt{dist}(v)$



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	A	B	C	D	E	F	G
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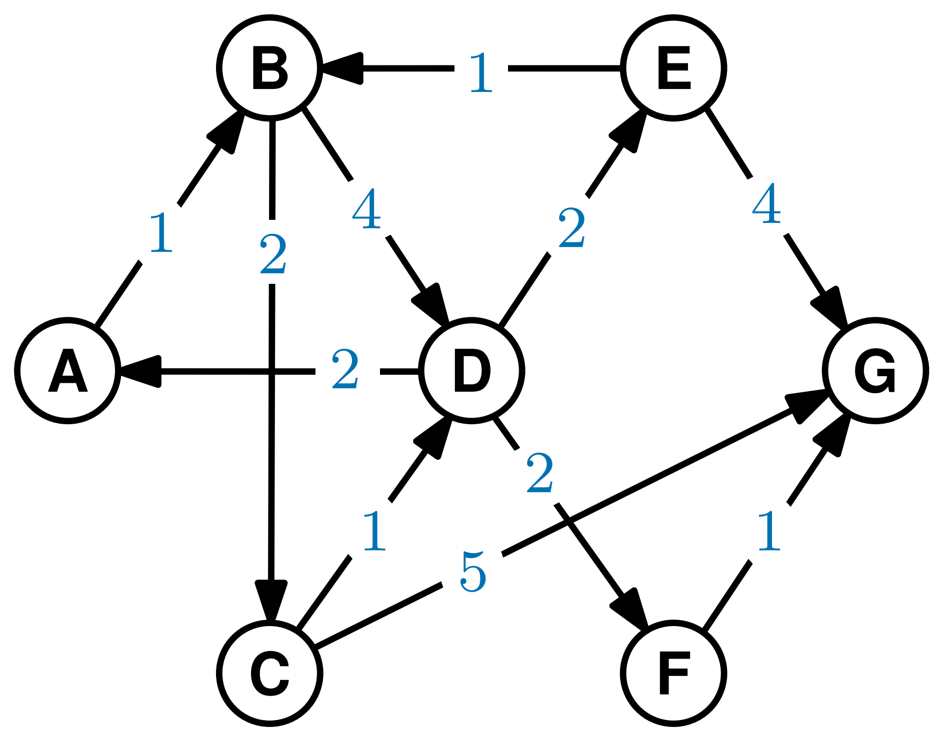

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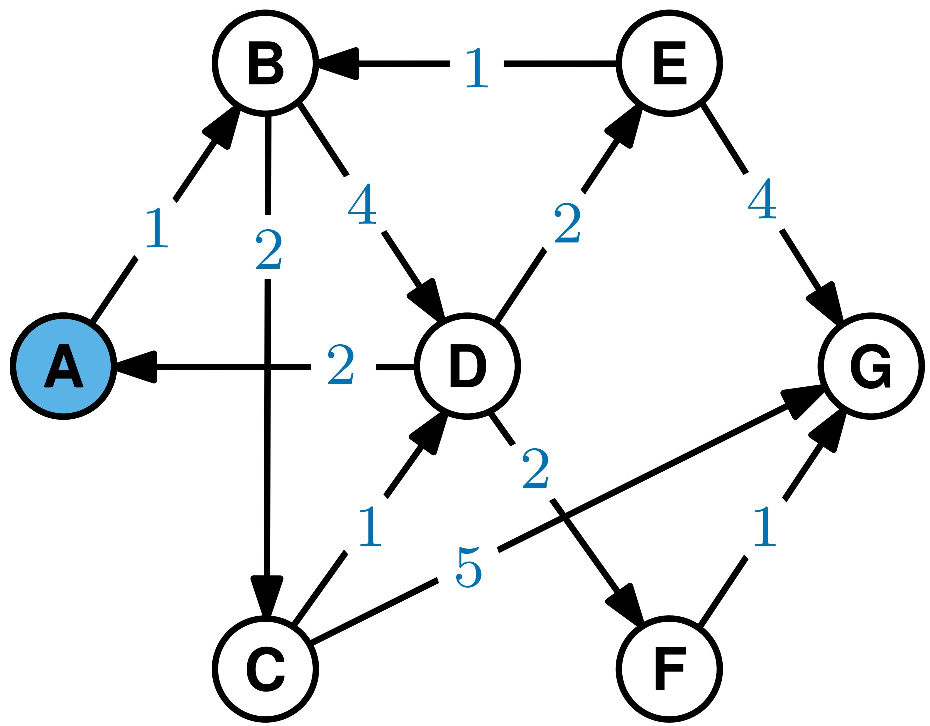

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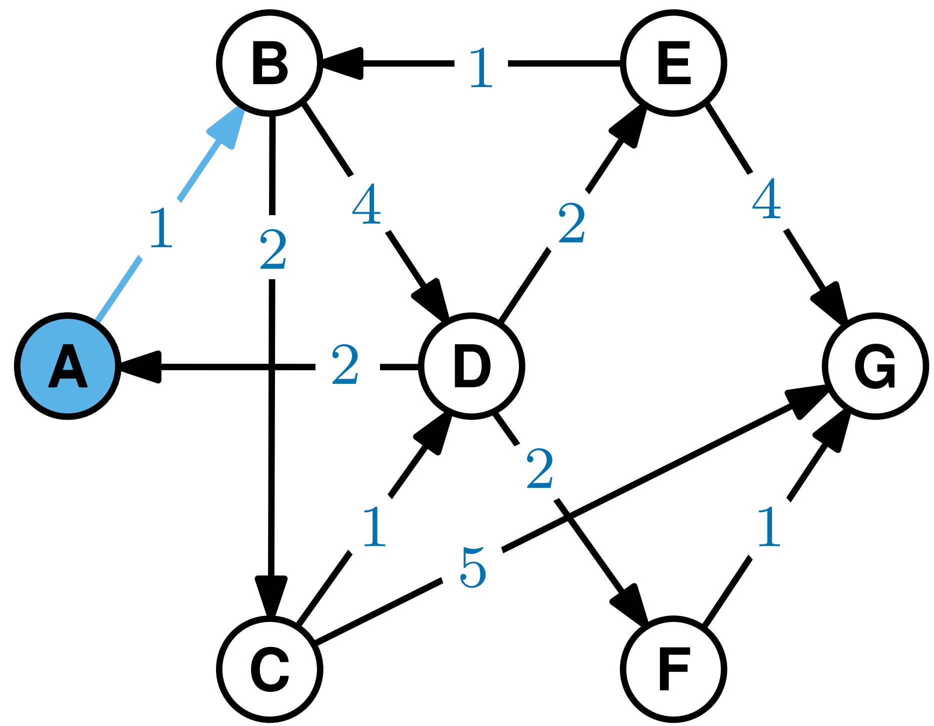
DIJKSTRA(A)

i.e. s = A

new path to **B** = $0 + 1 = 1$

	A	B	C	D	E	F	G
dist:	0	∞	∞	∞	∞	∞	∞



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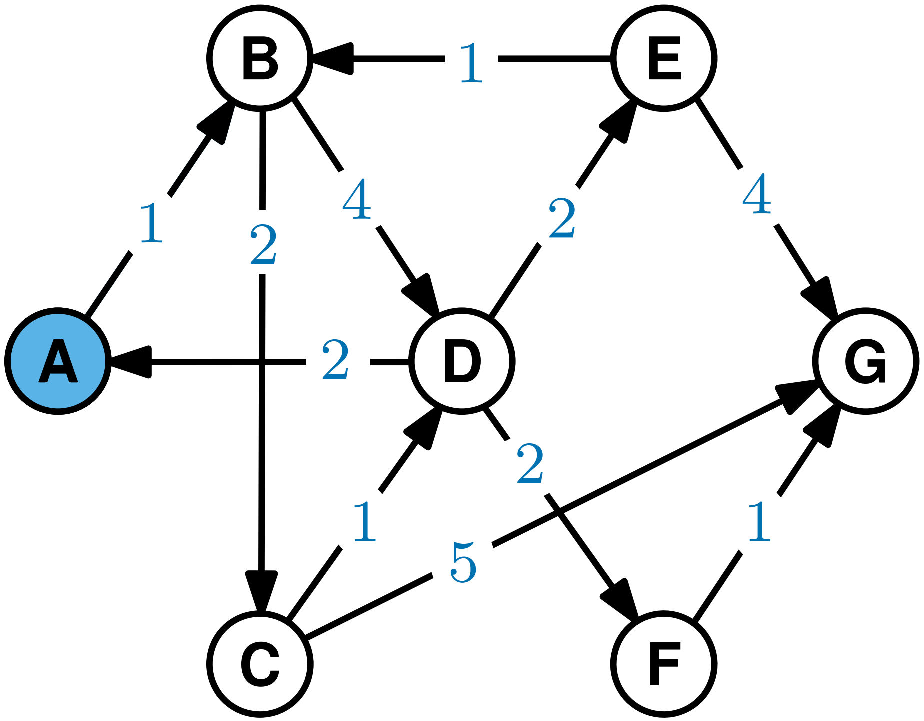
new path to **B** = $0 + 1 = 1$

	A	B	C	D	E	F	G
dist:	0	1	∞	∞	∞	∞	∞



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We're going to simulate DIJKSTRA(A)

i.e. $s = A$

this is called relaxing edge (u, v)

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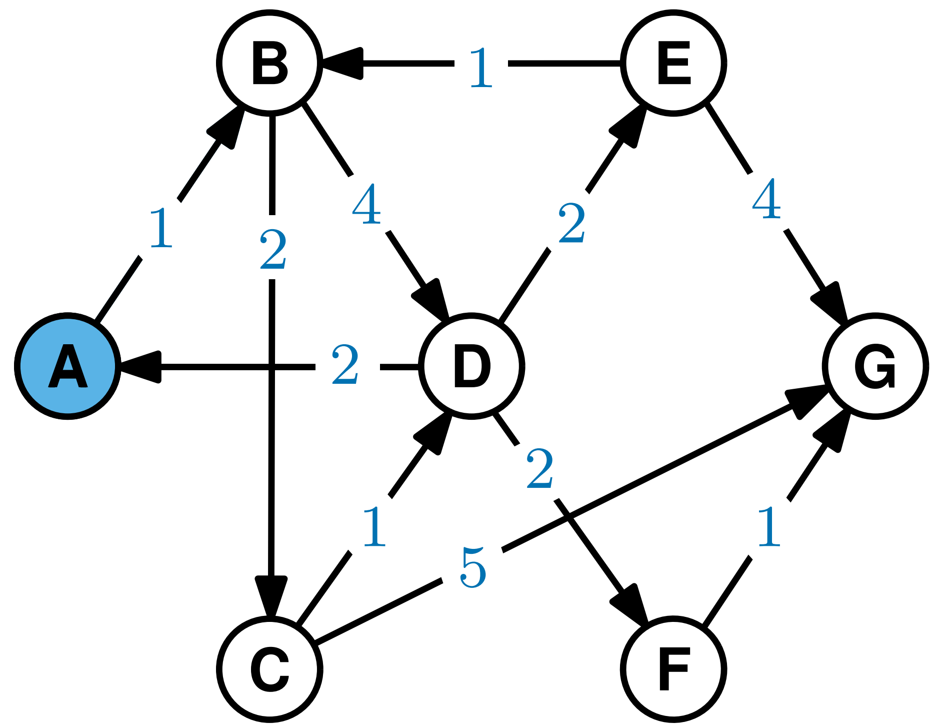

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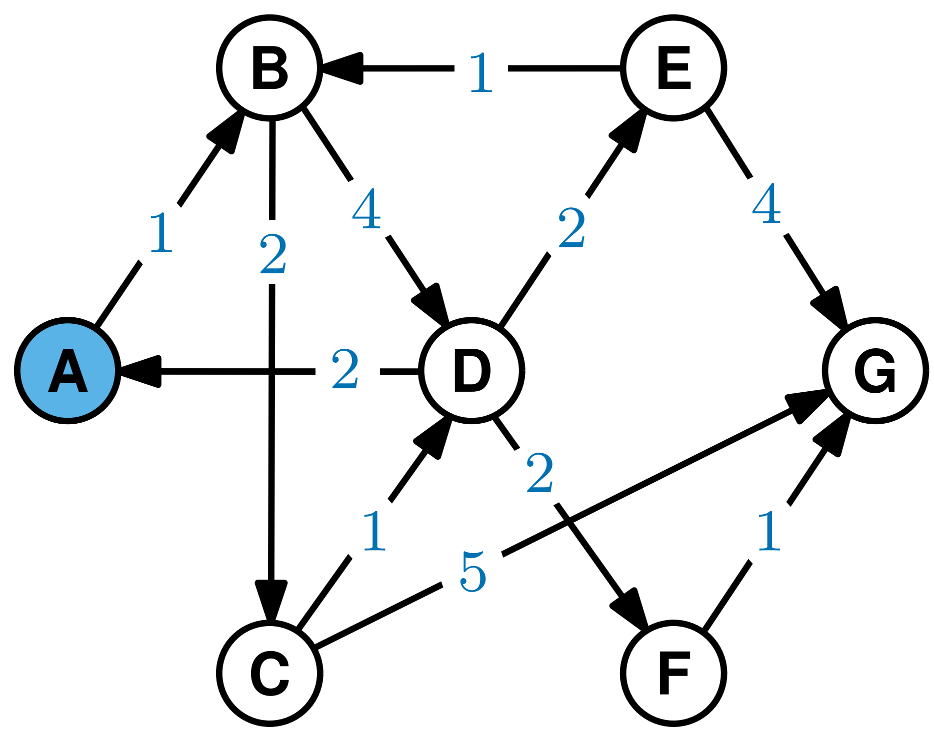

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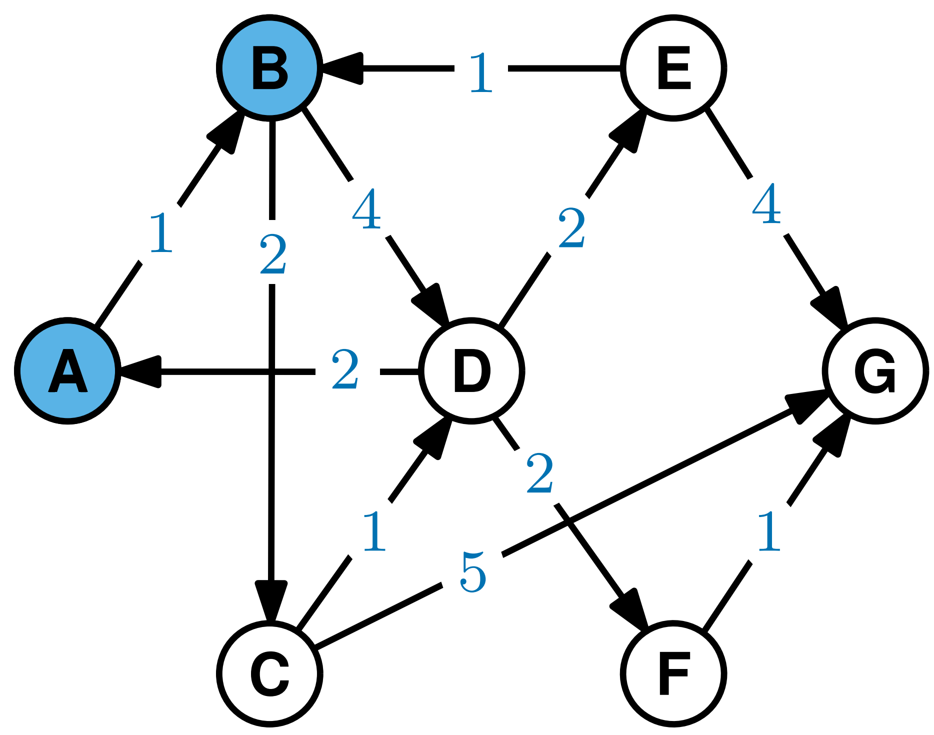

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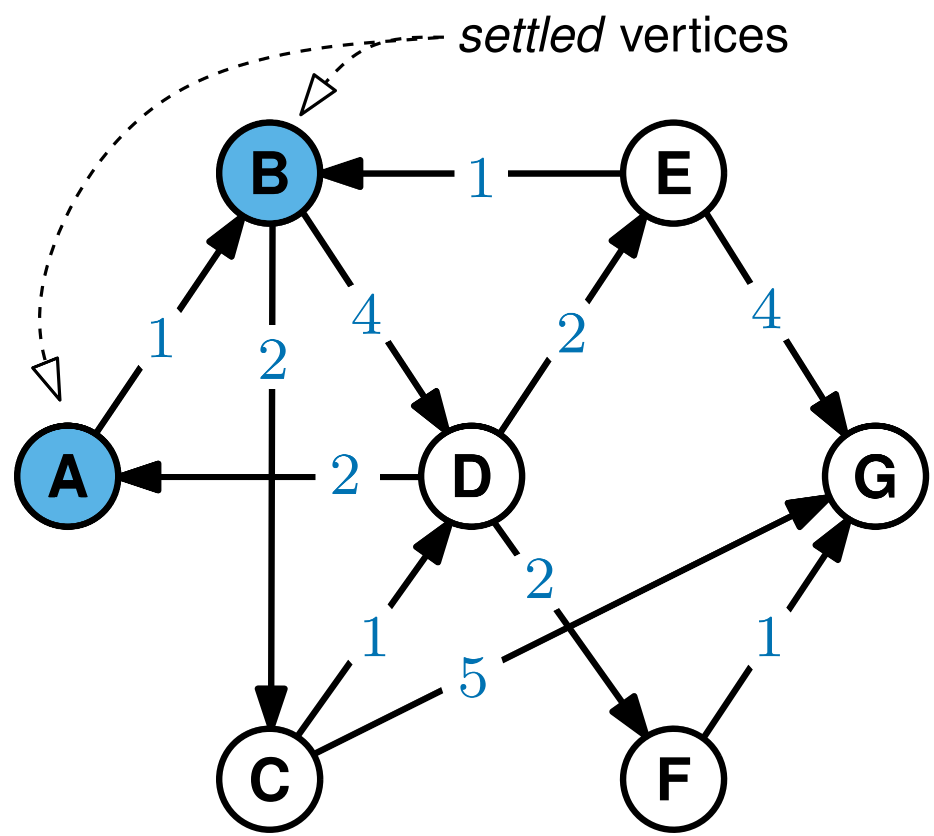

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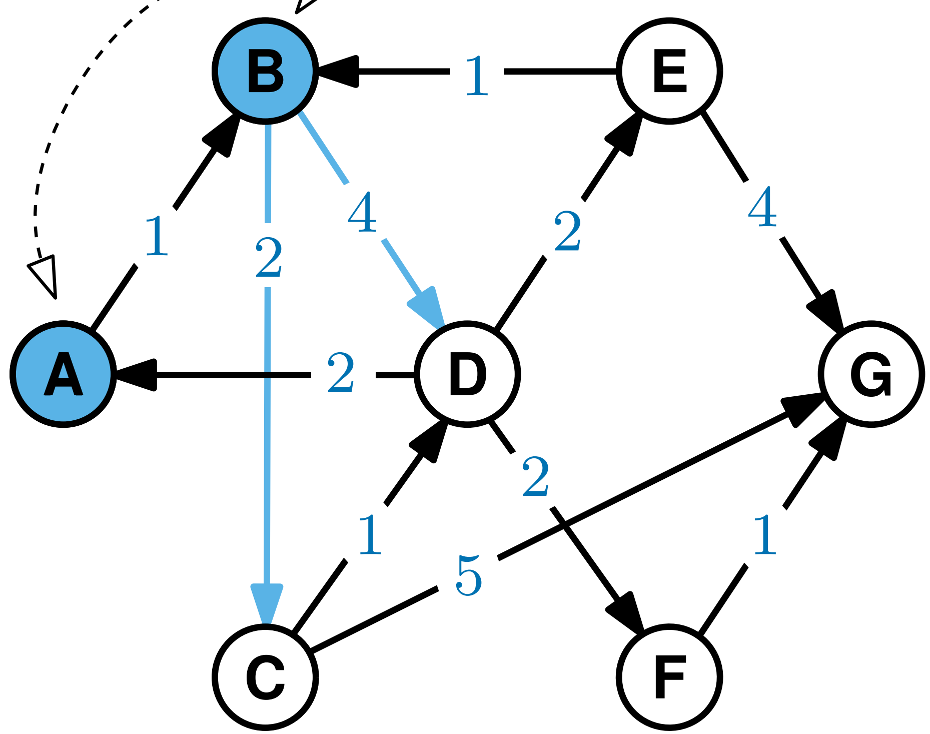
We're going to simulate DIJKSTRA(A)

i.e. $s = $ A

this is called relaxing edge (u, v)

settled vertices



	A	B	C	D	E	F	G
dist:	0	1	∞	∞	∞	∞	∞

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i.e. s = A

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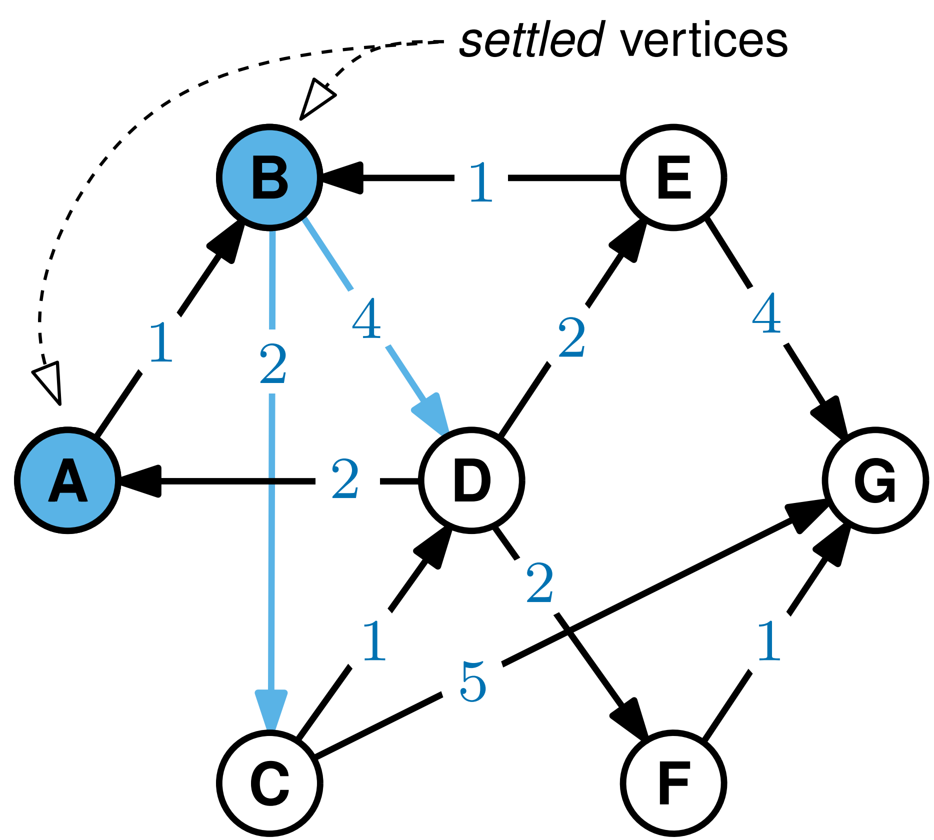

new path to **C** = $1 + 2 = 3$

	A	B	C	D	E	F	G
dist:	0	1	∞	∞	∞	∞	∞

$\texttt{dist}(v)$ is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

v.key $= \texttt{dist}(v)$

```
DIJKSTRA(s)
```

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We're going to simulate DIJKSTRA(A)

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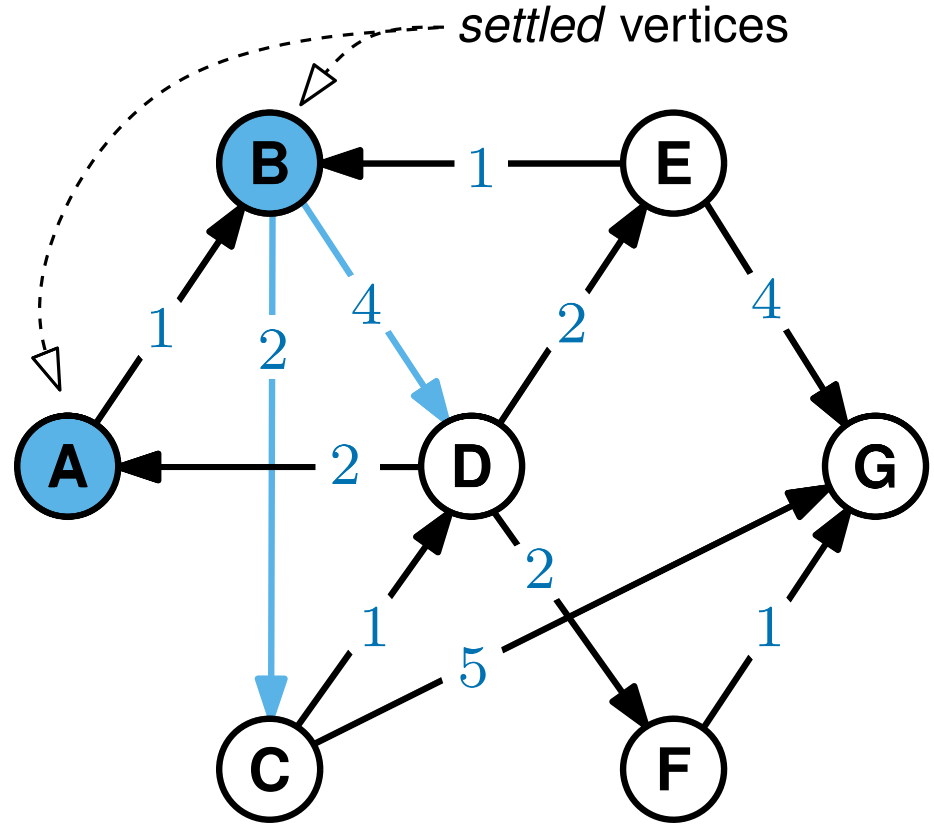
new path to **C** = $1 + 2 = 3$

	A	B	C	D	E	F	G
dist:	0	1	3	∞	∞	∞	∞



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```

We're going to simulate DIJKSTRA(A)

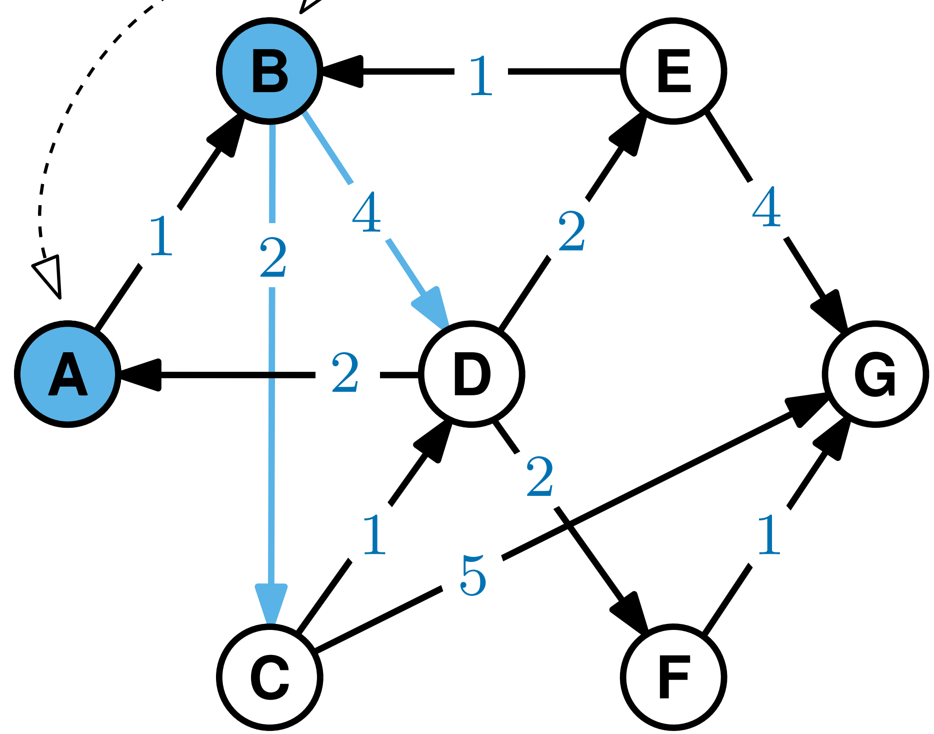
i.e. s = A

this is called relaxing edge (u, v)

new path to **D** = $1 + 4 = 5$

	A	B	C	D	E	F	G
dist:	0	1	3	∞	∞	∞	∞

settled vertices



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```

We're going to simulate DIJKSTRA(A)

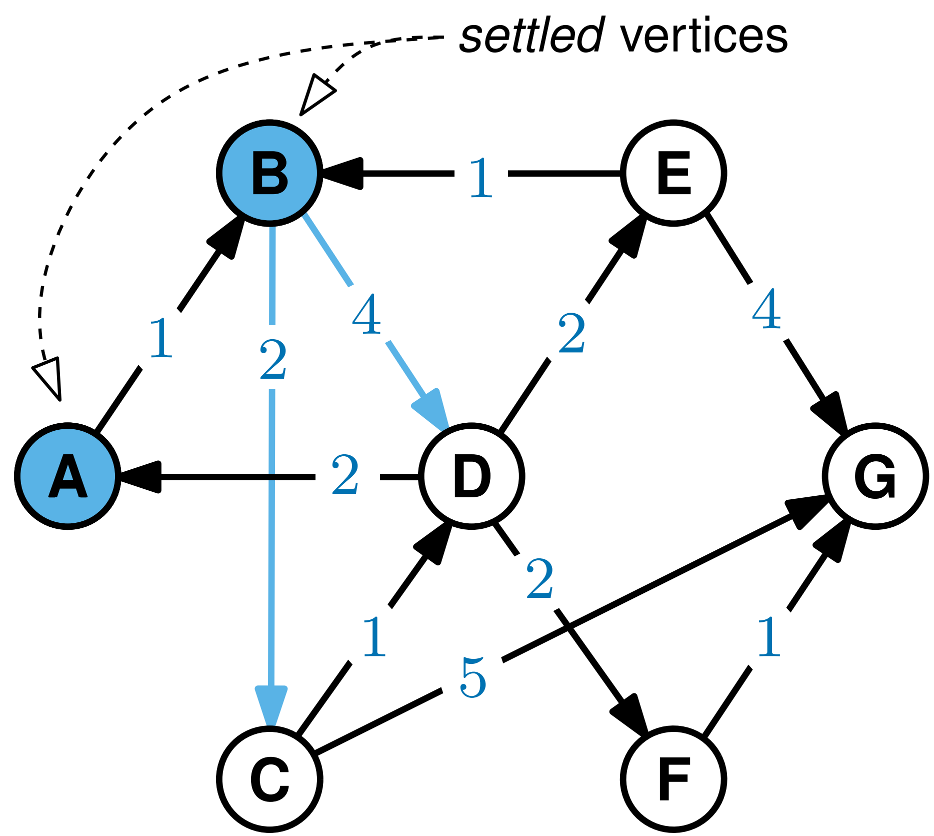
i.e. $s = \text{A}$

this is called relaxing edge (u, v)

new path to **D** = $1 + 4 = 5$

dist:

A	B	C	D	E	F	G
0	1	3	5	∞	∞	∞



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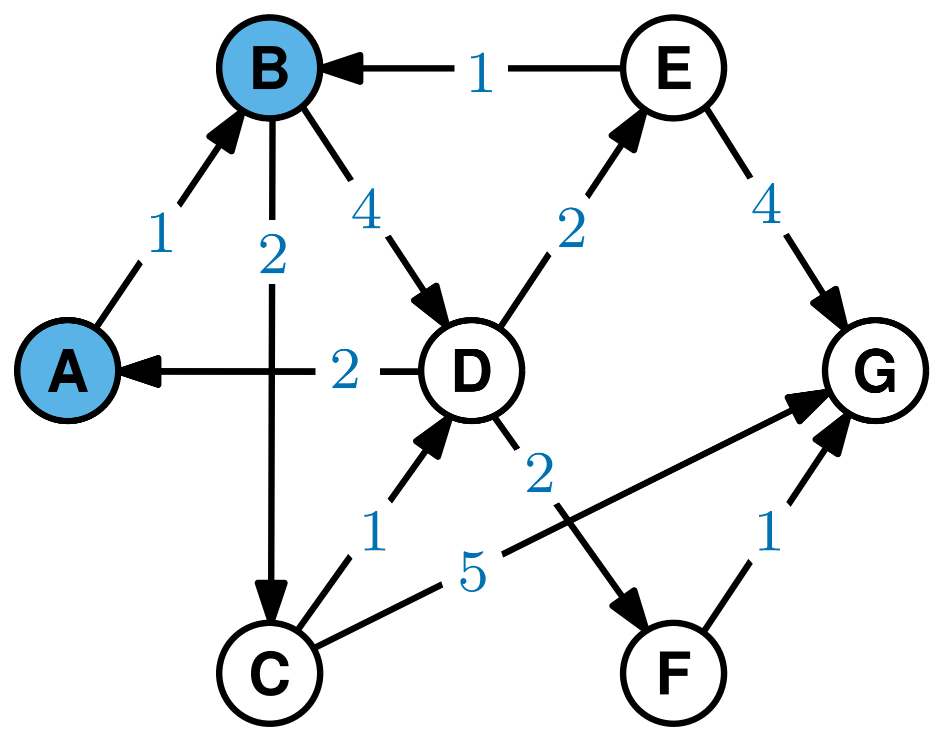
We're going to simulate DIJKSTRA(A)

i.e. s = A

this is called relaxing edge (u, v)

	A	B	C	D	E	F	G
dist:	0	1	3	5	∞	∞	∞



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	A	B	C	D	E	F	G
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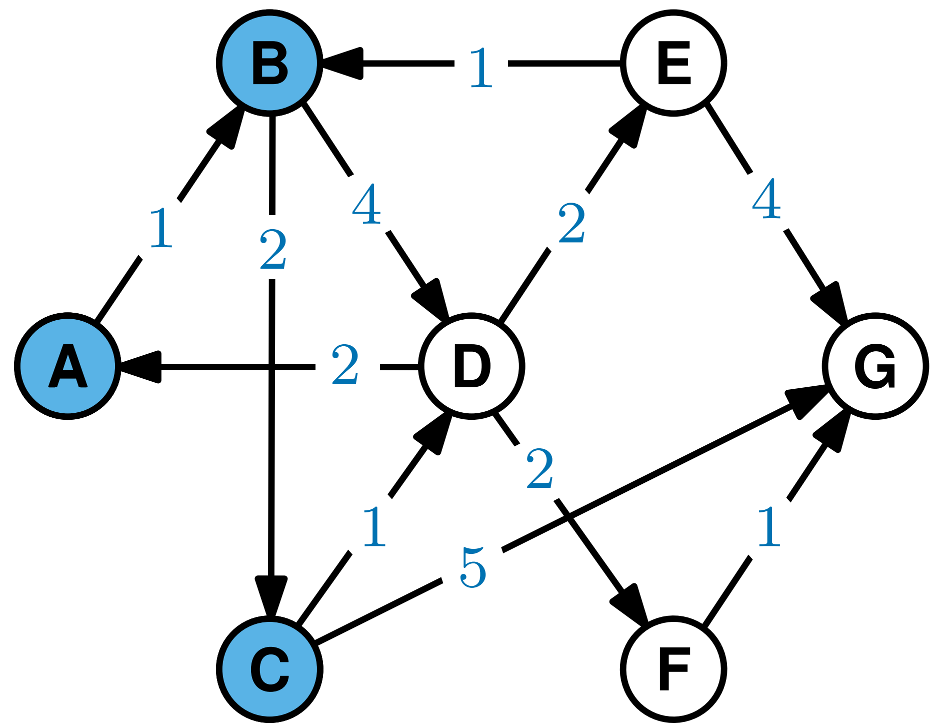

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set dist(s) = 0
For each v, Do INSERT(v, dist(v))
While the queue is not empty
   u = EXTRACTMIN()
   For every edge (u, v) ∈ E
      If dist(v) > dist(u) + weight(u, v)
         dist(v) = dist(u) + weight(u, v)
         DECREASEKEY(v, dist(v))
```

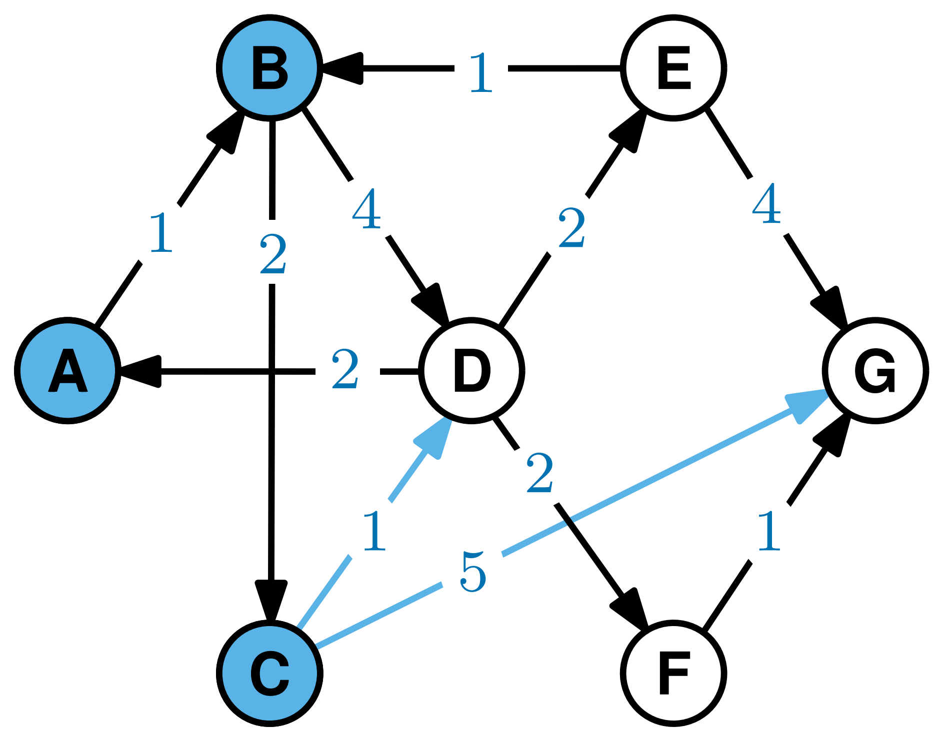
this is called relaxing edge (u, v)

We're going to simulate DIJKSTRA(A)

i.e. s = A

dist:

A	B	C	D	E	F	G
0	1	3	5	∞	∞	∞



dist(v) is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

v.key = dist(v)

$\textsc{Dijkstra}(s)$

```
For all v, set dist(v) = ∞
set dist(s) = 0
For each v, Do INSERT(v, dist(v))
While the queue is not empty
  u = EXTRACTMIN()
  For every edge (u, v) ∈ E
    If dist(v) > dist(u) + weight(u, v)
      dist(v) = dist(u) + weight(u, v)
      DECREASEKEY(v, dist(v))
```

We're going to simulate

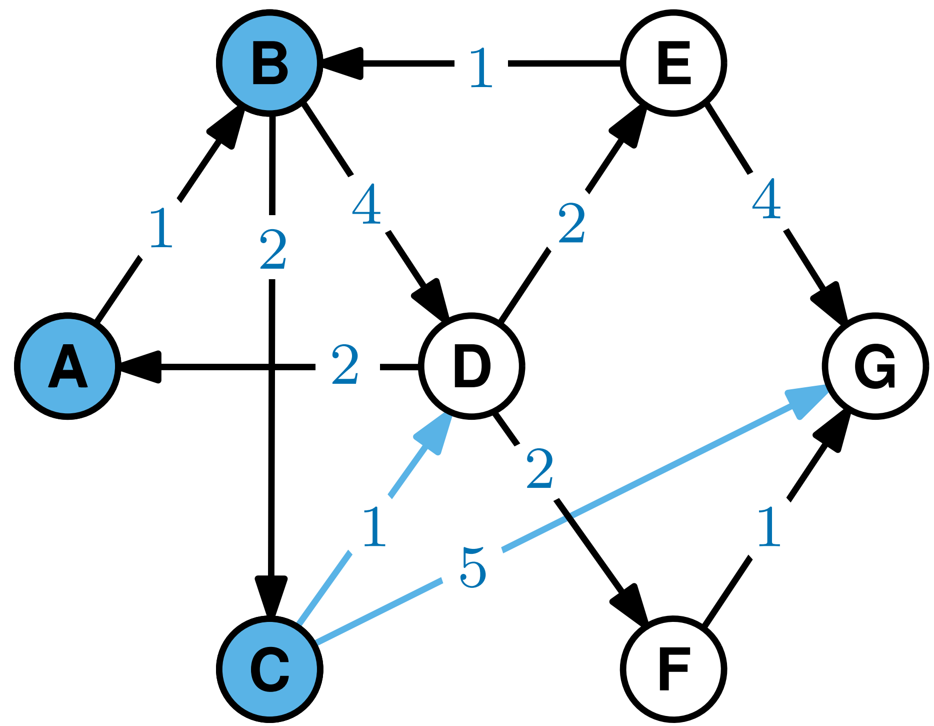
$\textsc{Dijkstra}(\text{A})$

i.e. $s = \text{A}$

this is called relaxing edge (u, v)

new path to **D** = $3 + 1 = 4$

	A	B	C	D	E	F	G
dist:	0	1	3	5	∞	∞	∞



$\texttt{dist}(v)$ is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

$v.\text{key} = \texttt{dist}(v)$

DIJKSTRA(s)

For all v, set $\texttt{dist}(v) = \infty$
set $\texttt{dist}(s) = 0$
For each v, Do INSERT(v, $\texttt{dist}(v)$)
While the queue is not empty
  u = EXTRACTMIN()
  For every edge $(u, v) \in E$
    If $\texttt{dist}(v) > \texttt{dist}(u) + \texttt{weight}(u, v)$
      $\texttt{dist}(v) = \texttt{dist}(u) + \texttt{weight}(u, v)$
      DECREASEKEY(v, $\texttt{dist}(v)$)

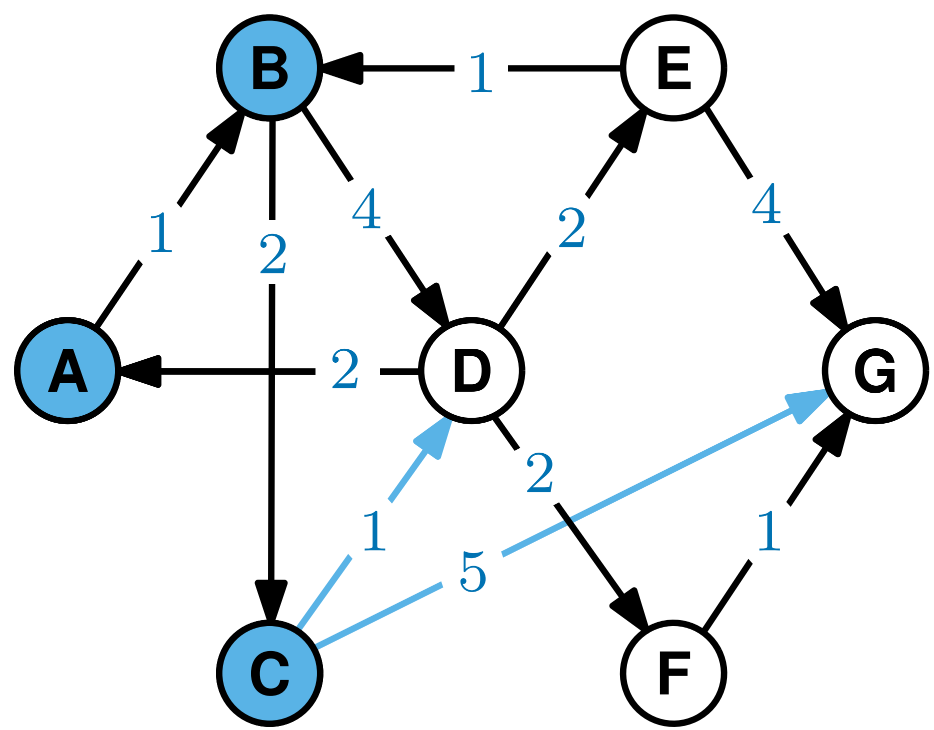
this is called relaxing edge (u, v)

We're going to simulate DIJKSTRA(A)

i.e. s = A

new path to **D** = $3 + 1 = 4$

	A	B	C	D	E	F	G
dist:	0	1	3	4	∞	∞	∞



$\texttt{dist}(v)$ is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

v.key $= \texttt{dist}(v)$

DIJKSTRA(s)

```
For all v, set dist(v) = ∞
set dist(s) = 0
For each v, Do INSERT(v, dist(v))
While the queue is not empty
  u = EXTRACTMIN()
  For every edge (u, v) ∈ E
    If dist(v) > dist(u) + weight(u, v)
      dist(v) = dist(u) + weight(u, v)
      DECREASEKEY(v, dist(v))
```

We're going to simulate

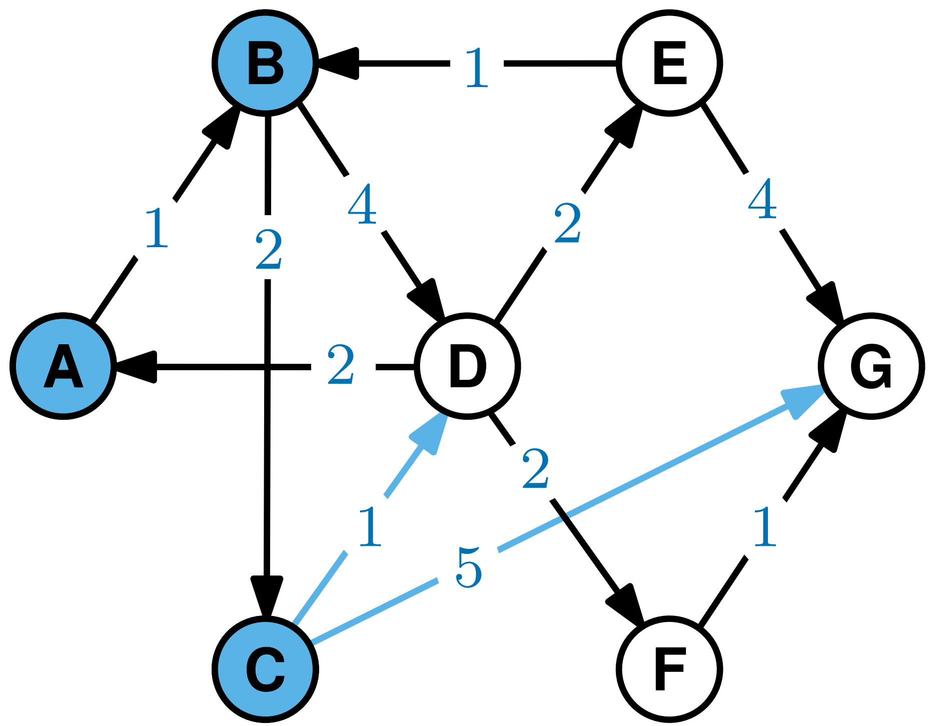
DIJKSTRA(A)

i.e. s = A

this is called relaxing edge (u, v)

new path to **G** = $3 + 5 = 8$

	A	B	C	D	E	F	G
dist:	0	1	3	4	∞	∞	∞



$\mathtt{dist}(v)$ is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

v.key $= \mathtt{dist}(v)$

DIJKSTRA(s)

```
For all v, set dist(v) = ∞
set dist(s) = 0
For each v, Do INSERT(v, dist(v))
While the queue is not empty
   u = EXTRACTMIN()
   For every edge (u, v) ∈ E
      If dist(v) > dist(u) + weight(u, v)
         dist(v) = dist(u) + weight(u, v)
         DECREASEKEY(v, dist(v))
```

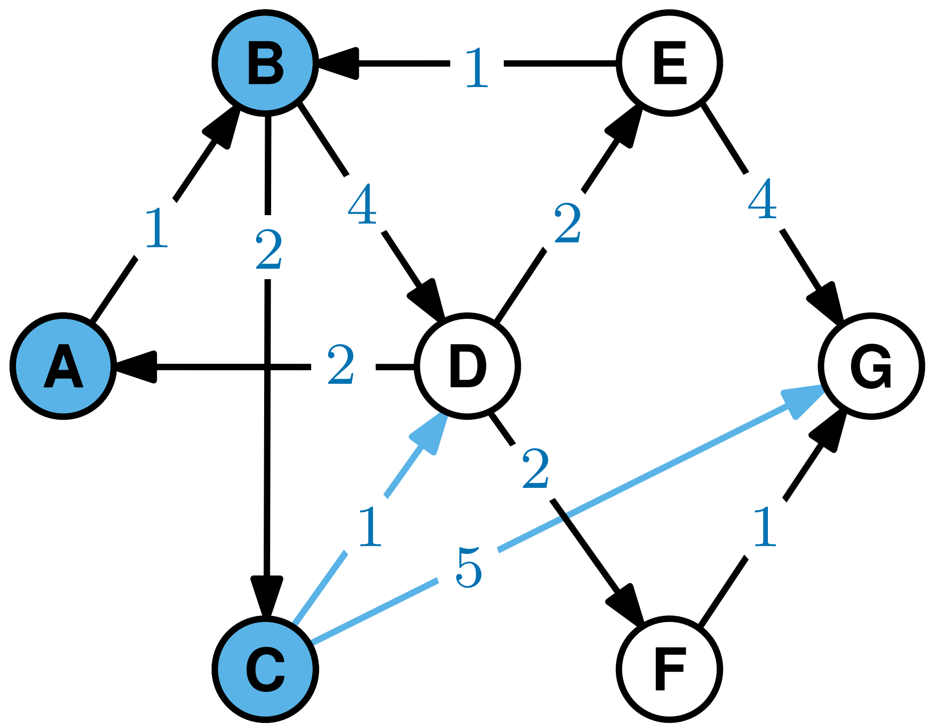
We're going to simulate DIJKSTRA(A)

i.e. s = A

this is called relaxing edge (u, v)

new path to **G** = $3 + 5 = 8$

	A	B	C	D	E	F	G
dist:	0	1	3	4	∞	∞	8



$\mathtt{dist}(v)$ is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

v.key $= \mathtt{dist}(v)$

$\textsc{Dijkstra}(s)$

```
For all v, set dist(v) = ∞
set dist(s) = 0
For each v, Do INSERT(v, dist(v))
While the queue is not empty
   u = EXTRACTMIN()
   For every edge (u, v) ∈ E
      If dist(v) > dist(u) + weight(u, v)
         dist(v) = dist(u) + weight(u, v)
         DECREASEKEY(v, dist(v))
```

this is called relaxing edge (u, v)

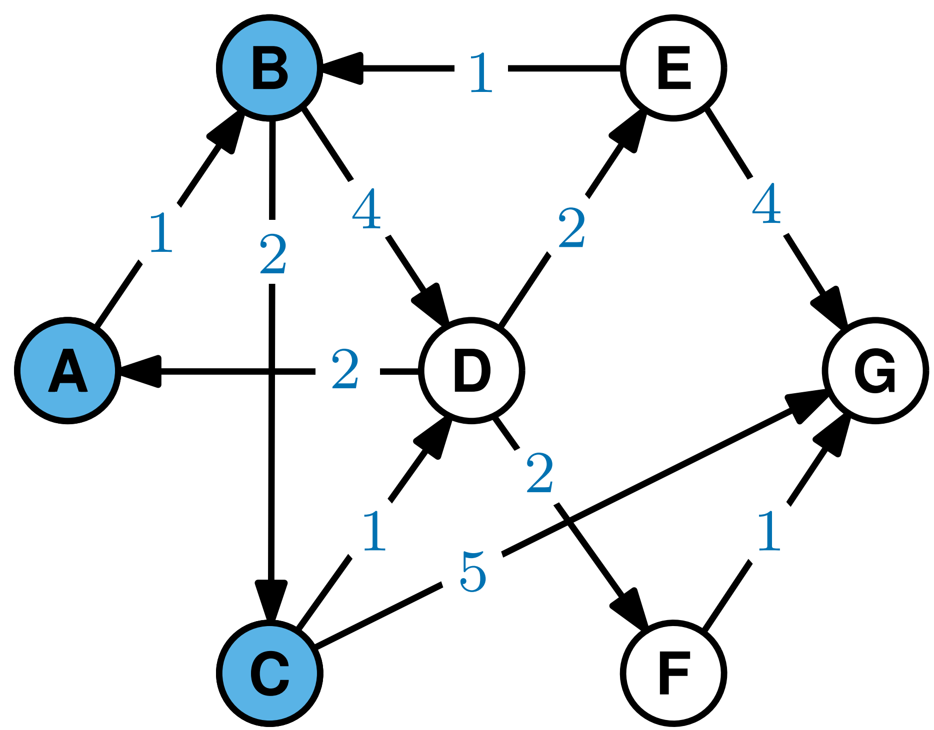
We're going to simulate

$\textsc{Dijkstra}(A)$

i.e. $s = A$

dist:

A	B	C	D	E	F	G
0	1	3	4	∞	∞	8



$\texttt{dist}(v)$ is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

$v.\text{key} = \texttt{dist}(v)$

$\textsc{Dijkstra}(s)$

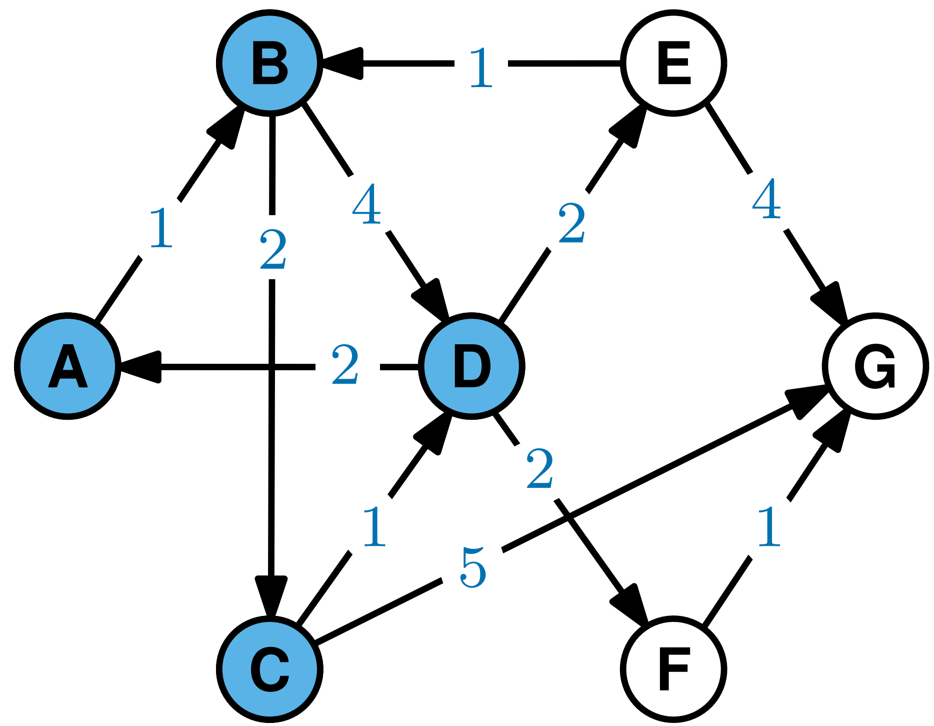
```
For all v, set dist(v) = ∞
set dist(s) = 0
For each v, Do INSERT(v, dist(v))
While the queue is not empty
   u = EXTRACTMIN()
   For every edge (u, v) ∈ E
      If dist(v) > dist(u) + weight(u, v)
         dist(v) = dist(u) + weight(u, v)
         DECREASEKEY(v, dist(v))
```

We're going to simulate $\textsc{Dijkstra}(A)$

i.e. $s =$ A

this is called relaxing edge (u, v)

	A	B	C	D	E	F	G
dist:	0	1	3	4	∞	∞	8



$\texttt{dist}(v)$ is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

$v.\text{key} = \texttt{dist}(v)$

$\text{DIJKSTRA}(s)$

```
For all v, set dist(v) = ∞
set dist(s) = 0
For each v, Do INSERT(v, dist(v))
While the queue is not empty
   u = EXTRACTMIN()
   For every edge (u, v) ∈ E
      If dist(v) > dist(u) + weight(u, v)
         dist(v) = dist(u) + weight(u, v)
         DECREASEKEY(v, dist(v))
```

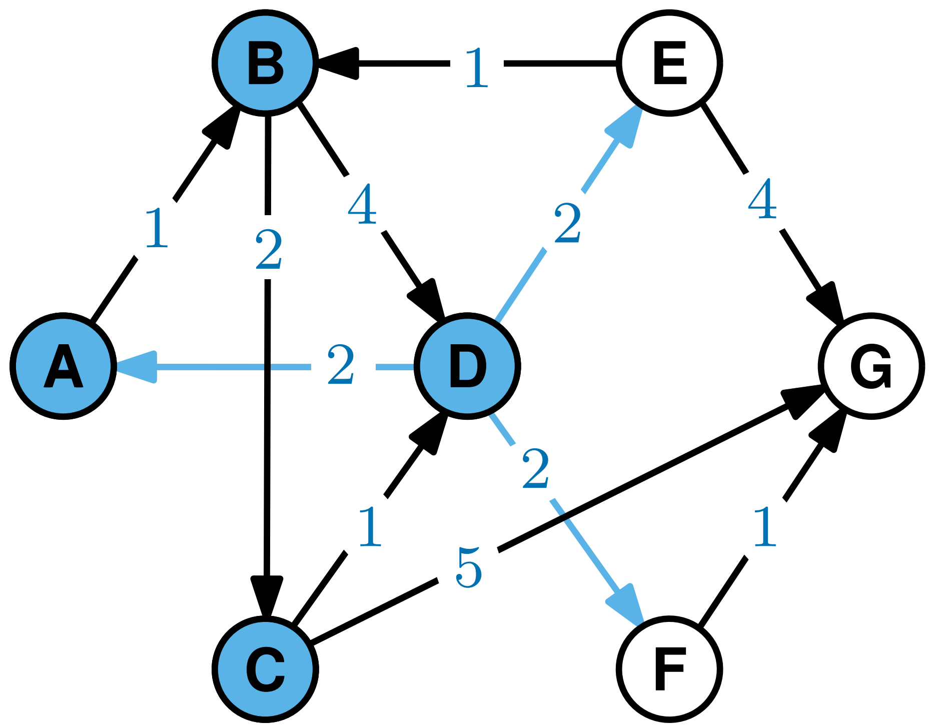
We're going to simulate

$\text{DIJKSTRA}(A)$

i.e. $s = A$

this is called relaxing edge (u, v)

	A	B	C	D	E	F	G
dist:	0	1	3	4	∞	∞	8



$\mathtt{dist}(v)$ is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

$v.\text{key} = \mathtt{dist}(v)$

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DIJKSTRA(s)

For all v, set $\texttt{dist}(v) = \infty$
set $\texttt{dist}(s) = 0$
For each v, Do $\textsc{Insert}(v, \texttt{dist}(v))$
While the queue is not empty
  $u = \textsc{ExtractMin}()$
  For every edge $(u, v) \in E$
    If $\texttt{dist}(v) > \texttt{dist}(u) + \texttt{weight}(u, v)$
      $\texttt{dist}(v) = \texttt{dist}(u) + \texttt{weight}(u, v)$
      $\textsc{DecreaseKey}(v, \texttt{dist}(v))$

We're going to simulate DIJKSTRA(A)

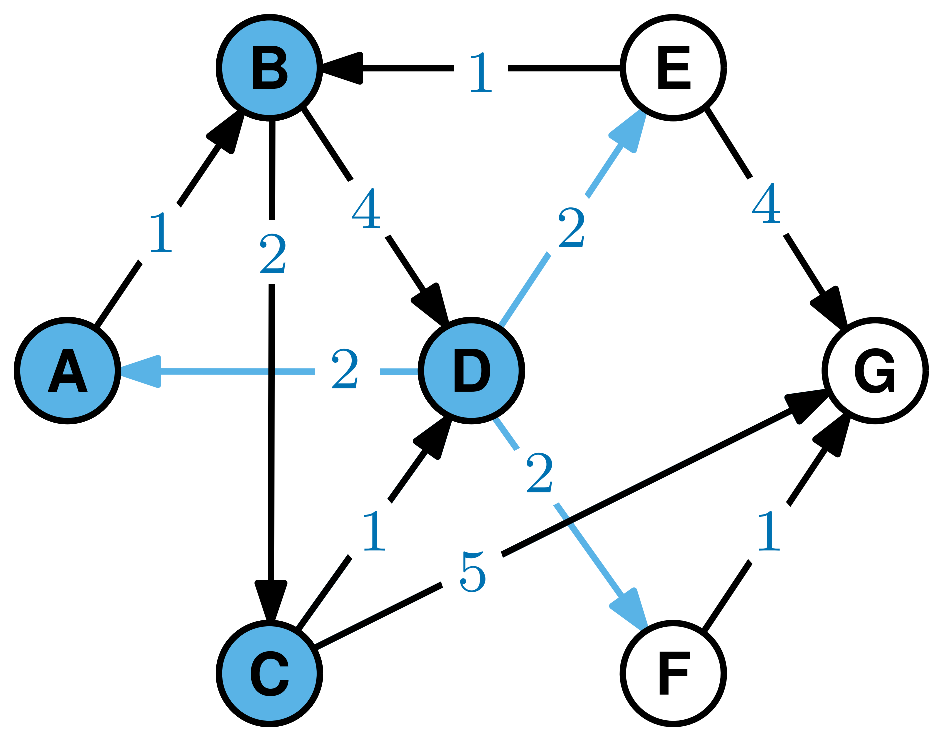
i.e. s = A

this is called relaxing edge (u, v)

new path to **A** = $0 + 2 = 2$

dist:

A	B	C	D	E	F	G
0	1	3	4	∞	∞	8



$\texttt{dist}(v)$ is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

v.key $= \texttt{dist}(v)$

DIJKSTRA(s)

```
For all v, set dist(v) = ∞
set dist(s) = 0
For each v, Do INSERT(v, dist(v))
While the queue is not empty
   u = EXTRACTMIN()
   For every edge (u, v) ∈ E
      If dist(v) > dist(u) + weight(u, v)
         dist(v) = dist(u) + weight(u, v)
         DECREASEKEY(v, dist(v))
```

this is called relaxing edge (u, v)

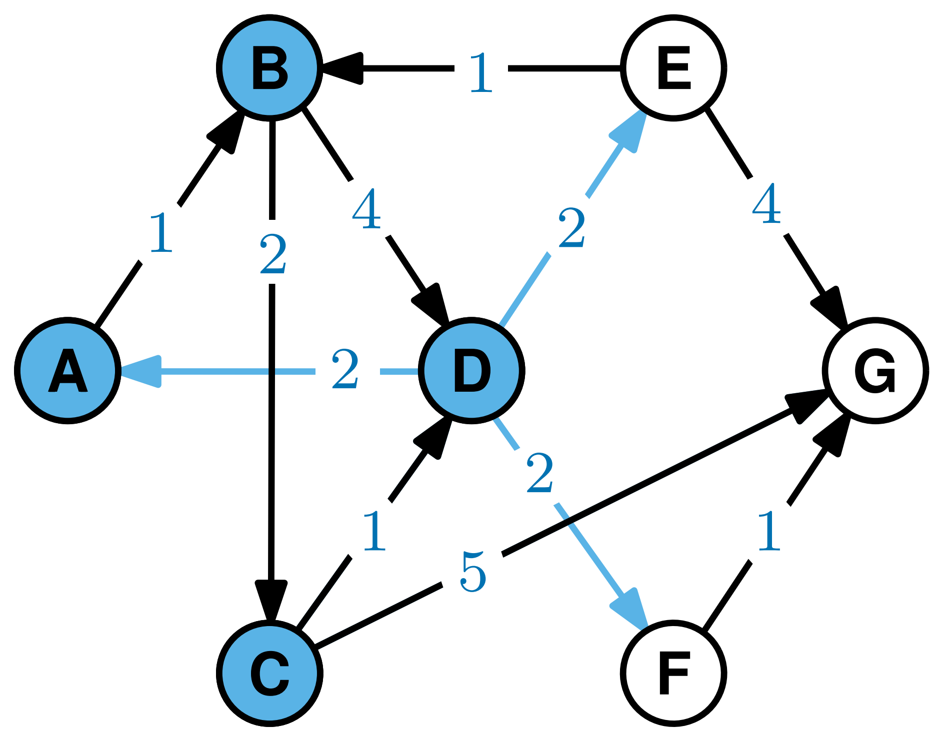
We're going to simulate

DIJKSTRA(A)

i.e. s = A

new path to **E** = $4 + 2 = 6$

	A	B	C	D	E	F	G
dist:	0	1	3	4	∞	∞	8



$\texttt{dist}(v)$ is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

v.key $= \texttt{dist}(v)$

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DIJKSTRA(s)

```
For all v, set dist(v) = ∞
set dist(s) = 0
For each v, Do INSERT(v, dist(v))
While the queue is not empty
  u = EXTRACTMIN()
  For every edge (u, v) ∈ E
    If dist(v) > dist(u) + weight(u, v)
      dist(v) = dist(u) + weight(u, v)
      DECREASEKEY(v, dist(v))
```

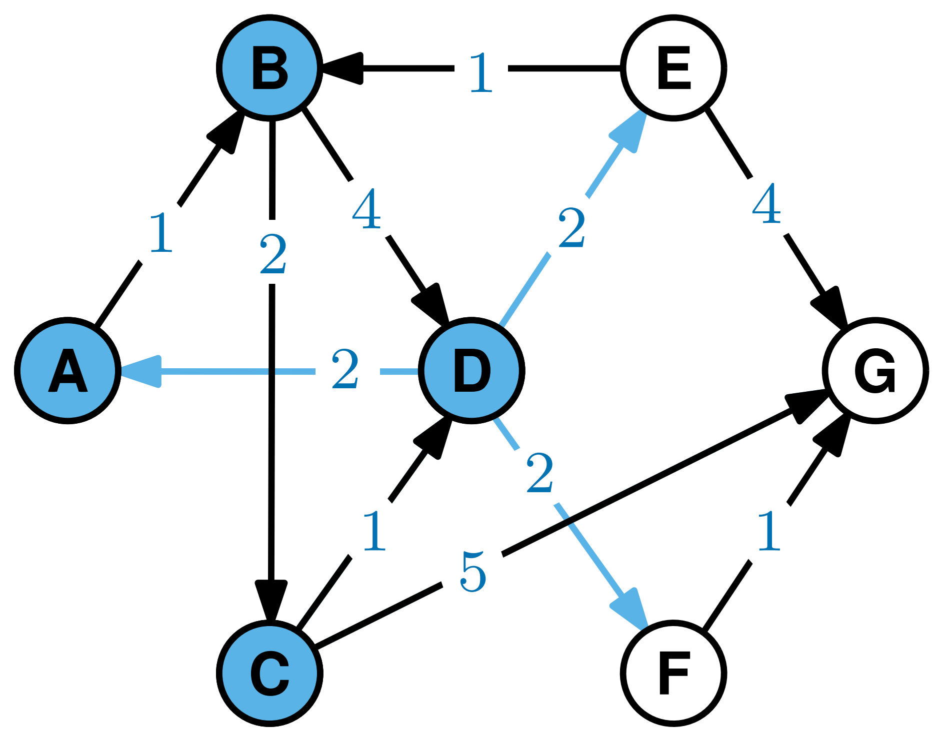
We're going to simulate DIJKSTRA(A)

i.e. $s = $ A

this is called relaxing edge (u, v)

new path to **E** = $4 + 2 = 6$

	A	B	C	D	E	F	G
dist:	0	1	3	4	6	∞	8



dist(v) is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

v.key = dist(v)

DIJKSTRA(s)

```
For all v, set dist(v) = ∞
set dist(s) = 0
For each v, Do INSERT(v, dist(v))
While the queue is not empty
  u = EXTRACTMIN()
  For every edge (u, v) ∈ E
    If dist(v) > dist(u) + weight(u, v)
      dist(v) = dist(u) + weight(u, v)
      DECREASEKEY(v, dist(v))
```

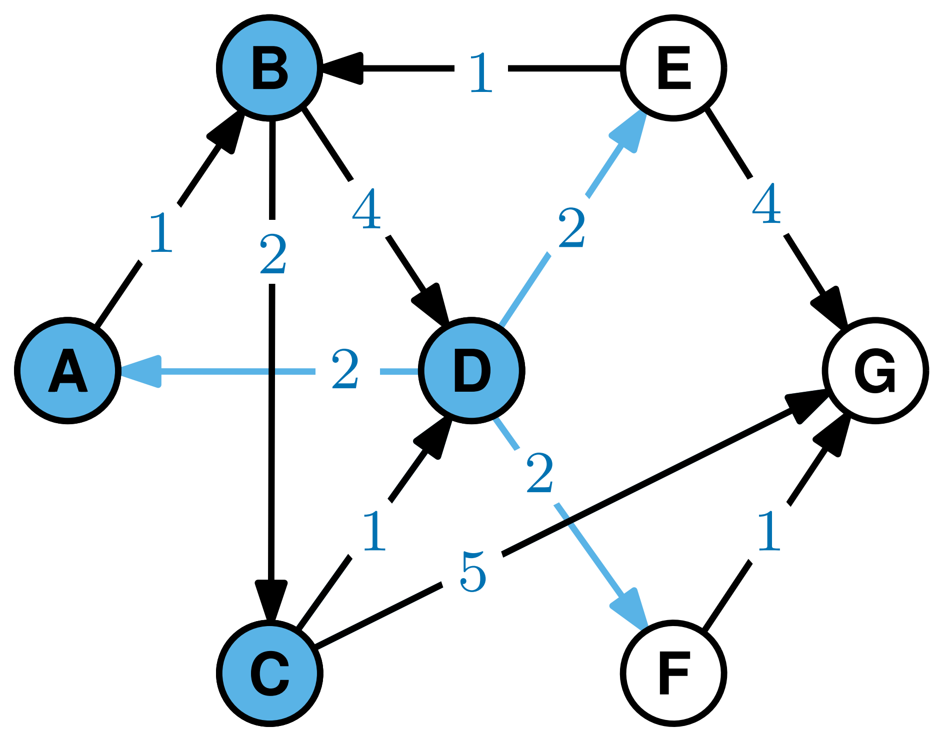
We're going to simulate DIJKSTRA(A)

i.e. s = A

this is called relaxing edge (u, v)

new path to **F** = $4 + 2 = 6$

	A	B	C	D	E	F	G
dist:	0	1	3	4	6	∞	8



$\texttt{dist}(v)$ is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

$v.\text{key} = \texttt{dist}(v)$

DIJKSTRA(s)

For all v, set $\texttt{dist}(v) = \infty$
set $\texttt{dist}(s) = 0$
For each v, Do $\textsc{Insert}(v, \texttt{dist}(v))$
While the queue is not empty
  $u = \textsc{ExtractMin}()$
  For every edge $(u, v) \in E$
    If $\texttt{dist}(v) > \texttt{dist}(u) + \texttt{weight}(u, v)$
      $\texttt{dist}(v) = \texttt{dist}(u) + \texttt{weight}(u, v)$
      $\textsc{DecreaseKey}(v, \texttt{dist}(v))$

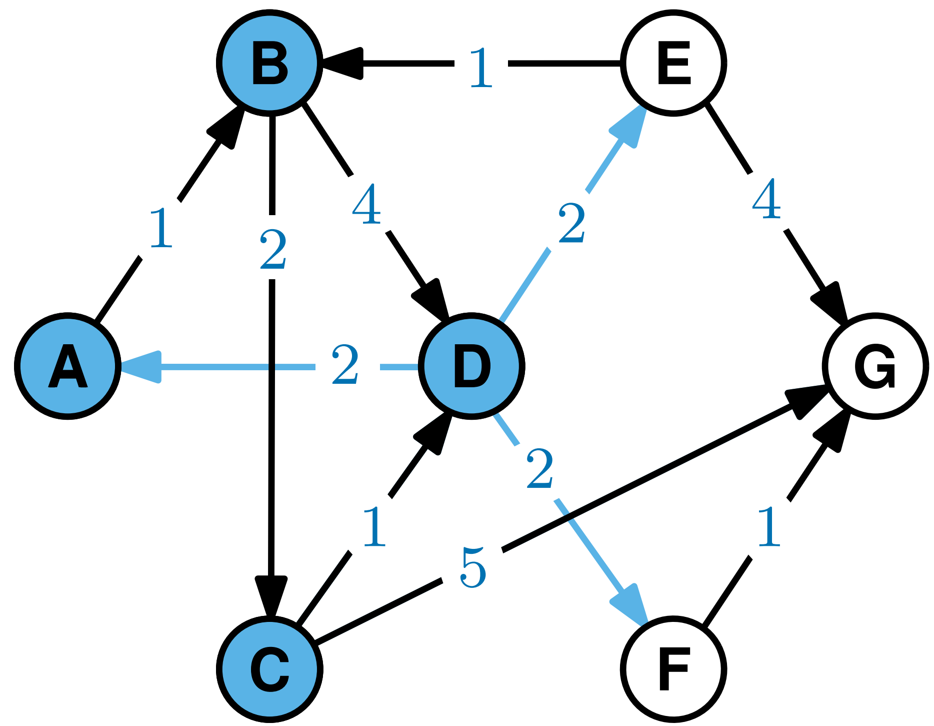
We're going to simulate DIJKSTRA(A)

i.e. s = A

this is called relaxing edge (u, v)

new path to **F** = $4 + 2 = 6$

	A	B	C	D	E	F	G
dist:	0	1	3	4	6	6	8



$\texttt{dist}(v)$ is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

v.key $= \texttt{dist}(v)$

$\textsc{Dijkstra}(s)$

```
For all v, set dist(v) = ∞
set dist(s) = 0
For each v, Do INSERT(v, dist(v))
While the queue is not empty
   u = EXTRACTMIN()
   For every edge (u, v) ∈ E
      If dist(v) > dist(u) + weight(u, v)
         dist(v) = dist(u) + weight(u, v)
         DECREASEKEY(v, dist(v))
```

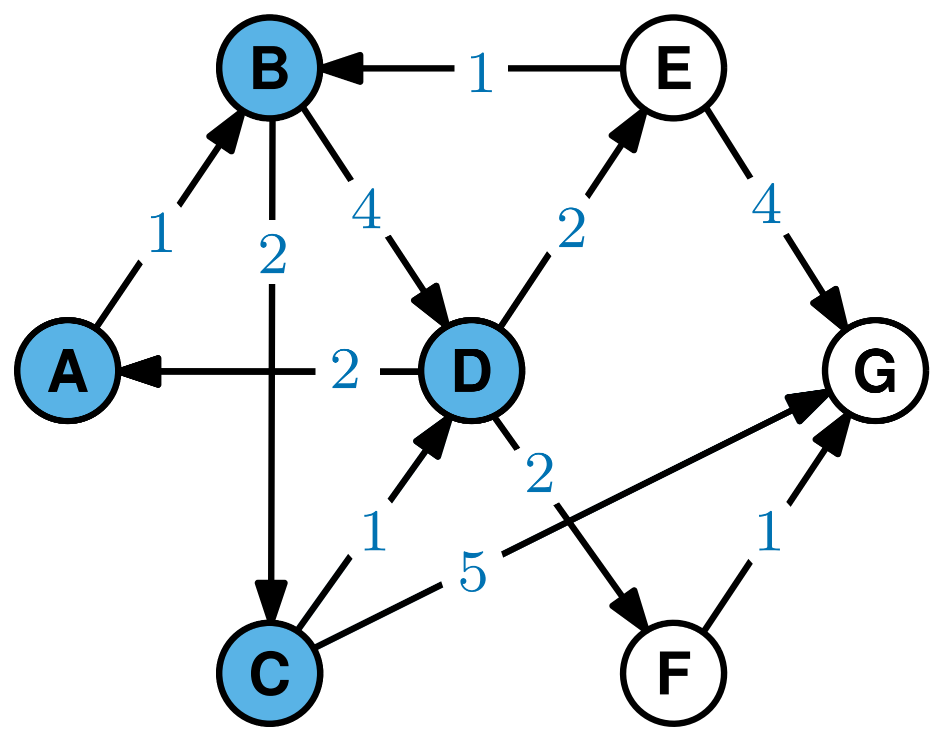
We're going to simulate $\textsc{Dijkstra}(\text{A})$

i.e. $s =$ A

this is called relaxing edge (u, v)

dist:

A	B	C	D	E	F	G
0	1	3	4	6	6	8



dist(v) is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

$$v.\text{key} = \texttt{dist}(v)$$

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DIJKSTRA(s)

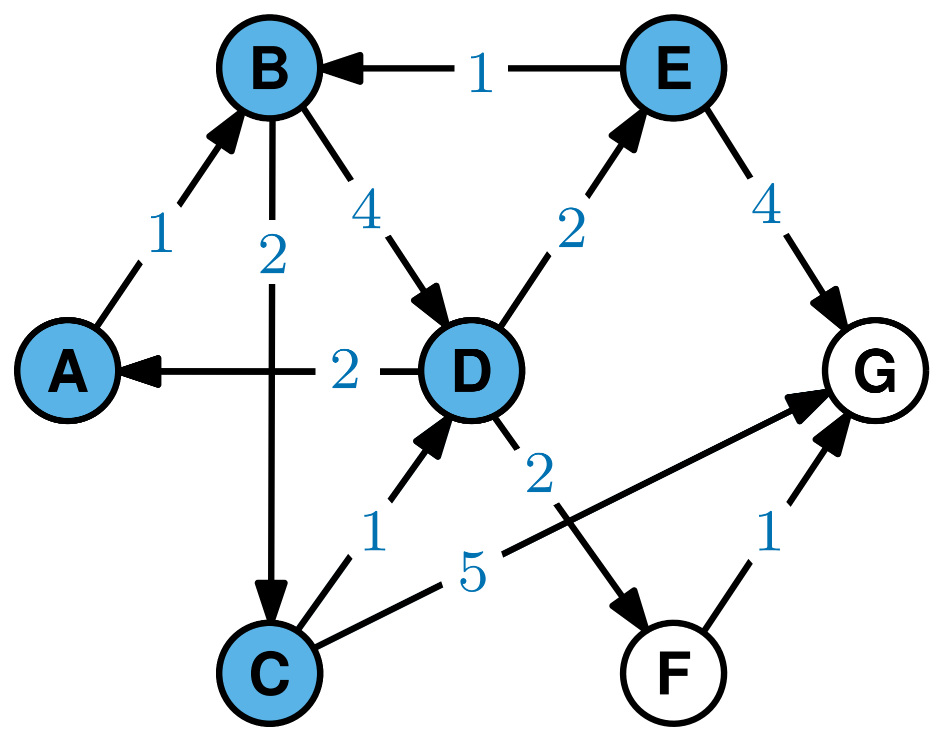
For all v, set $\texttt{dist}(v) = \infty$
set $\texttt{dist}(s) = 0$
For each v, Do INSERT$(v, \texttt{dist}(v))$
While the queue is not empty
  $u = $ EXTRACTMIN()
  For every edge $(u, v) \in E$
    If $\texttt{dist}(v) > \texttt{dist}(u) + \texttt{weight}(u, v)$
      $\texttt{dist}(v) = \texttt{dist}(u) + \texttt{weight}(u, v)$
      DECREASEKEY$(v, \texttt{dist}(v))$

We're going to simulate DIJKSTRA(A)

i.e. $s =$ A

this is called relaxing edge (u, v)

	A	B	C	D	E	F	G
dist:	0	1	3	4	6	6	8



$\texttt{dist}(v)$ is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

v.key $= \texttt{dist}(v)$

DIJKSTRA(s)

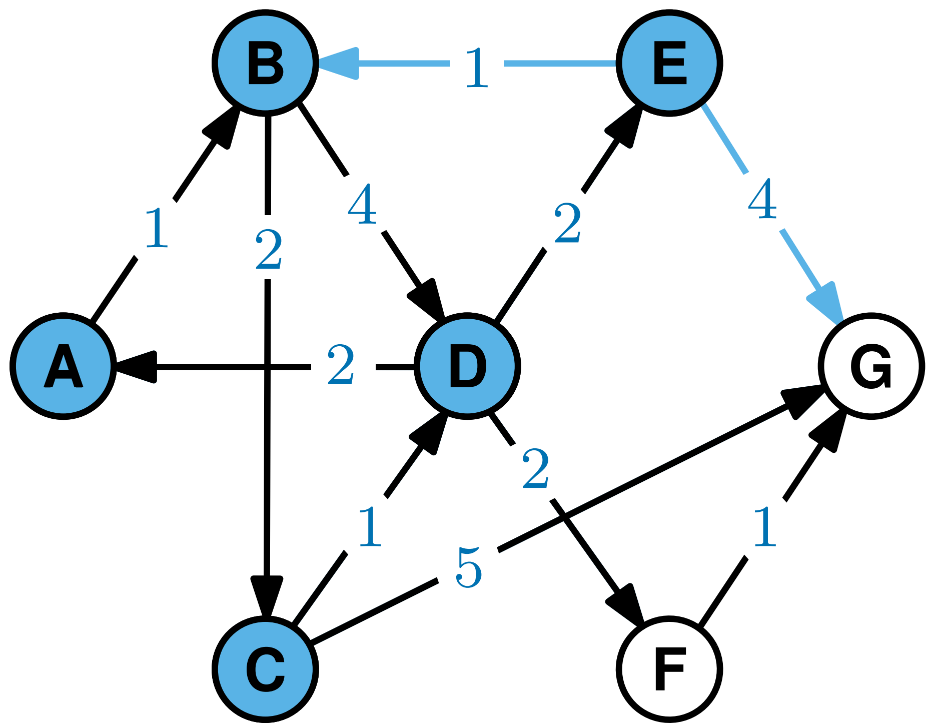
```
For all v, set dist(v) = ∞
set dist(s) = 0
For each v, Do INSERT(v, dist(v))
While the queue is not empty
   u = EXTRACTMIN()
   For every edge (u, v) ∈ E
      If dist(v) > dist(u) + weight(u, v)
         dist(v) = dist(u) + weight(u, v)
         DECREASEKEY(v, dist(v))
```

We're going to simulate DIJKSTRA(A)

i.e. s = A

this is called relaxing edge (u, v)

	A	B	C	D	E	F	G
dist:	0	1	3	4	6	6	8



$\texttt{dist}(v)$ is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

$v.\text{key} = \texttt{dist}(v)$

DIJKSTRA(s)

For all v, set $\texttt{dist}(v) = \infty$
set $\texttt{dist}(s) = 0$
For each v, Do $\textsc{Insert}(v, \texttt{dist}(v))$
While the queue is not empty
  $u = \textsc{ExtractMin}()$
  For every edge $(u, v) \in E$
    If $\texttt{dist}(v) > \texttt{dist}(u) + \texttt{weight}(u, v)$
      $\texttt{dist}(v) = \texttt{dist}(u) + \texttt{weight}(u, v)$
      $\textsc{DecreaseKey}(v, \texttt{dist}(v))$

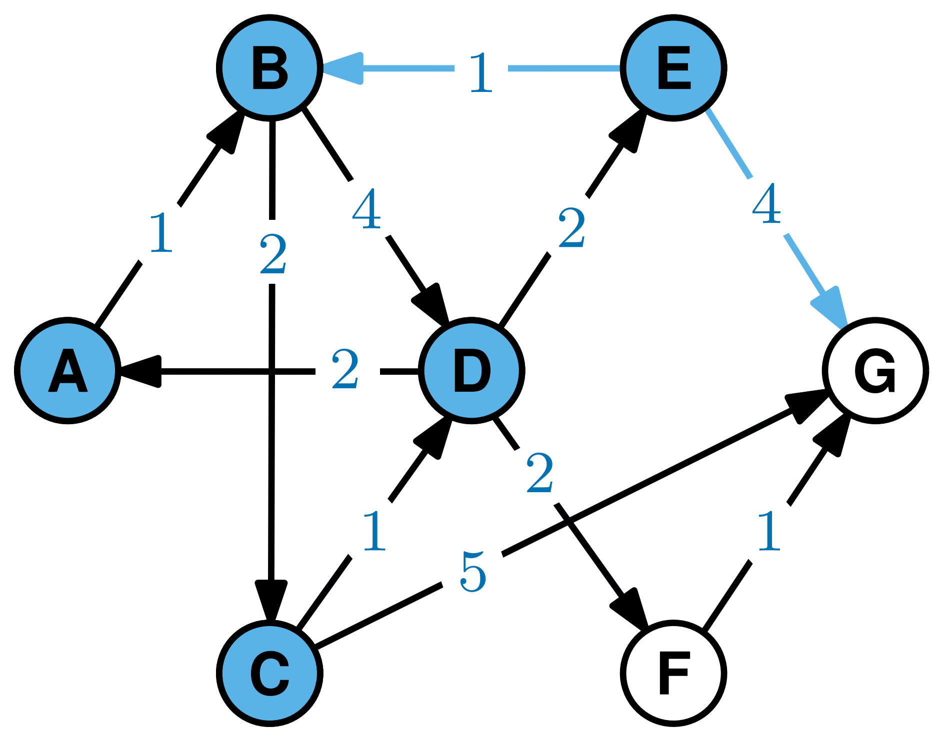
We're going to simulate DIJKSTRA(A)

i.e. s = A

this is called relaxing edge (u, v)

new path to **B** = $6 + 1 = 7$

	A	B	C	D	E	F	G
dist:	0	1	3	4	6	6	8



$\texttt{dist}(v)$ is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

v.key $= \texttt{dist}(v)$

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DIJKSTRA(s)

For all v, set dist$(v) = \infty$
set dist$(s) = 0$
For each v, Do INSERT$(v,$ dist$(v))$
While the queue is not empty
  $u =$ EXTRACTMIN()
  For every edge $(u, v) \in E$
    If dist$(v) >$ dist$(u) +$ weight(u, v)
      dist$(v) =$ dist$(u) +$ weight(u, v)
      DECREASEKEY$(v,$ dist$(v))$

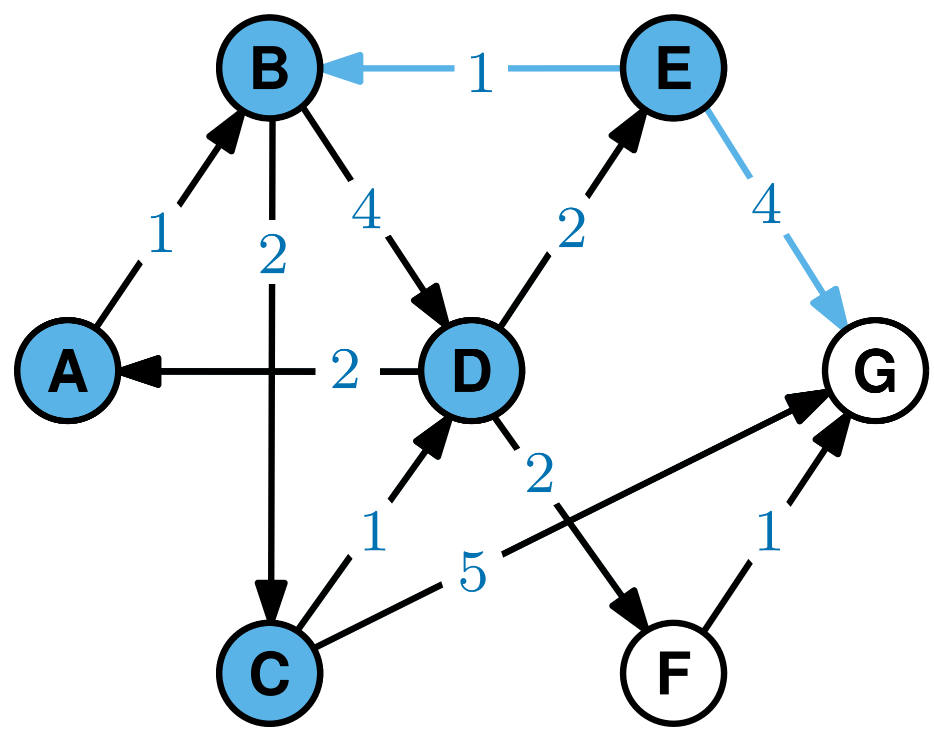
We're going to simulate DIJKSTRA(A)

i.e. $s =$ A

this is called relaxing edge (u, v)

new path to **G** = $6 + 4 = 10$

	A	B	C	D	E	F	G
dist:	0	1	3	4	6	6	8



dist(v) is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

v.key $=$ dist(v)

DIJKSTRA(s)

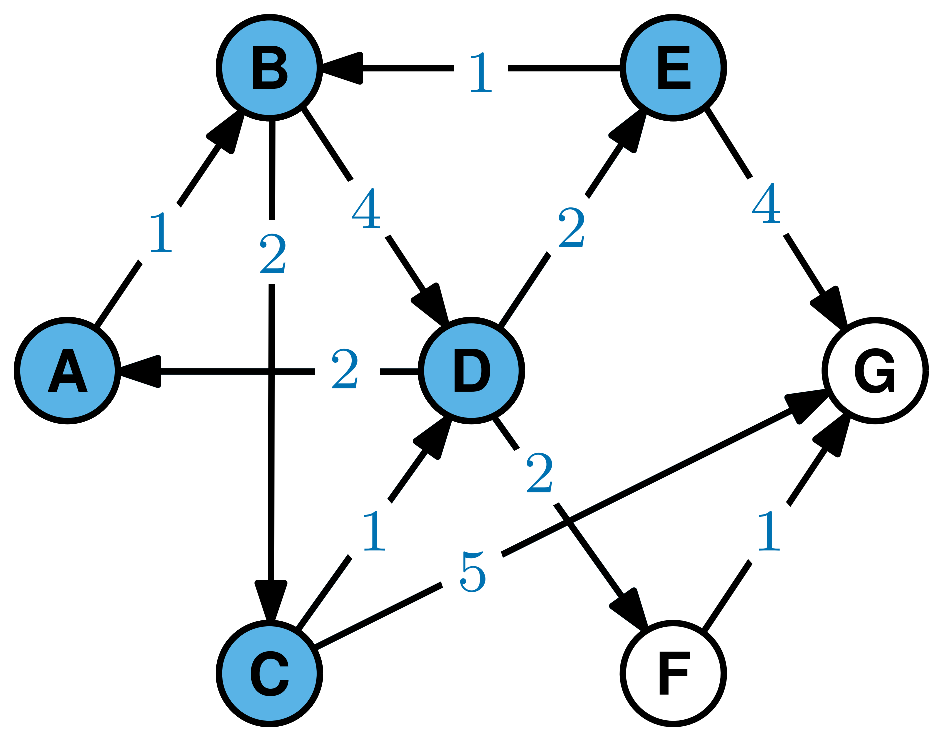
For all v, set $\texttt{dist}(v) = \infty$
set $\texttt{dist}(s) = 0$
For each v, Do $\textsc{Insert}(v, \texttt{dist}(v))$
While the queue is not empty
  $u = \textsc{ExtractMin}()$
  For every edge $(u, v) \in E$
    If $\texttt{dist}(v) > \texttt{dist}(u) + \texttt{weight}(u, v)$
      $\texttt{dist}(v) = \texttt{dist}(u) + \texttt{weight}(u, v)$
      $\textsc{DecreaseKey}(v, \texttt{dist}(v))$

We're going to simulate DIJKSTRA(A)

i.e. s = A

this is called relaxing edge (u, v)

	A	B	C	D	E	F	G
dist:	0	1	3	4	6	6	8



$\texttt{dist}(v)$ is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

$v.\text{key} = \texttt{dist}(v)$

Dijkstra(s)

```
For all v, set dist(v) = ∞
set dist(s) = 0
For each v, Do Insert(v, dist(v))
While the queue is not empty
   u = ExtractMin()
   For every edge (u, v) ∈ E
      If dist(v) > dist(u) + weight(u, v)
         dist(v) = dist(u) + weight(u, v)
         DecreaseKey(v, dist(v))
```

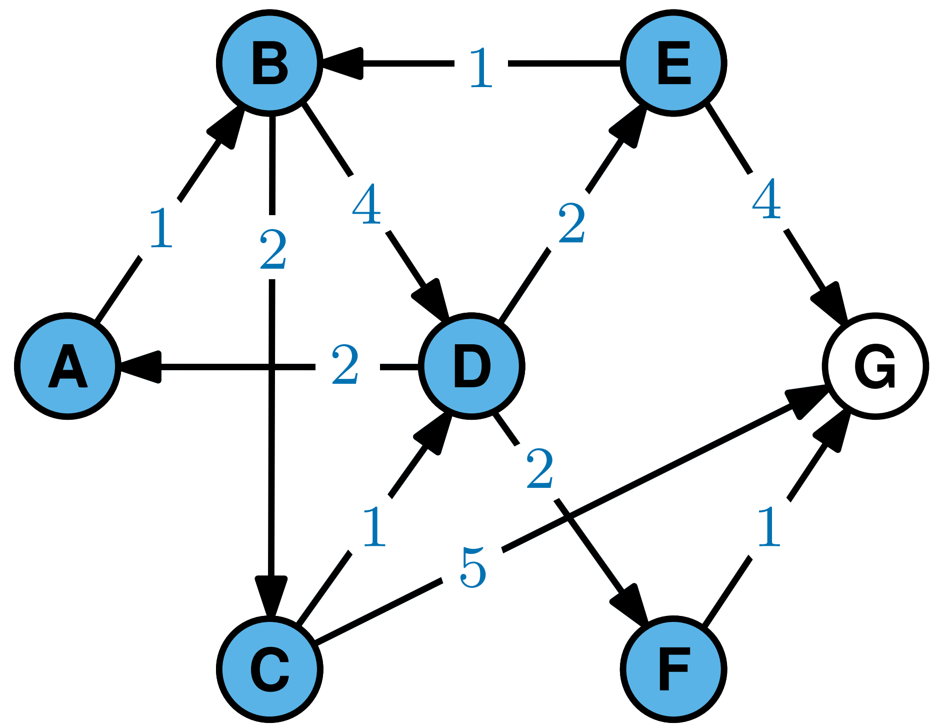
We're going to simulate

Dijkstra(A)

i.e. s = A

this is called relaxing edge (u, v)

	A	B	C	D	E	F	G
dist:	0	1	3	4	6	6	8



dist(v) is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

v.key = dist(v)

DIJKSTRA(s)

```
For all v, set dist(v) = ∞
set dist(s) = 0
For each v, Do INSERT(v, dist(v))
While the queue is not empty
  u = EXTRACTMIN()
  For every edge (u, v) ∈ E
    If dist(v) > dist(u) + weight(u, v)
      dist(v) = dist(u) + weight(u, v)
      DECREASEKEY(v, dist(v))
```

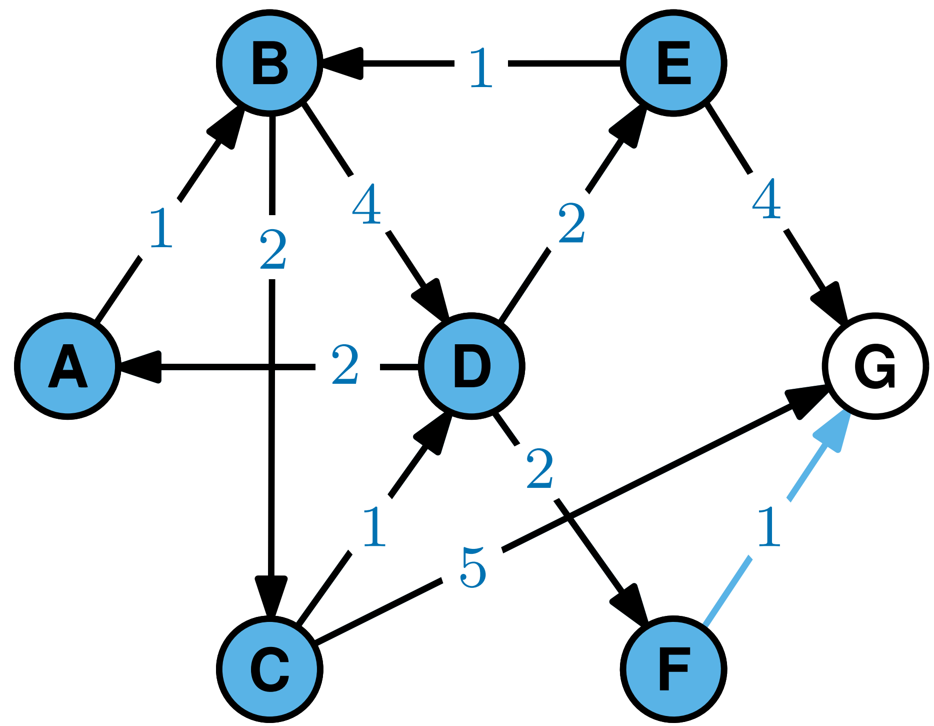
We're going to simulate

DIJKSTRA(A)

i.e. s = A

this is called relaxing edge (u, v)

	A	B	C	D	E	F	G
dist:	0	1	3	4	6	6	8



$\texttt{dist}(v)$ is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

v.key $= \texttt{dist}(v)$

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DIJKSTRA(s)

For all v, set $\texttt{dist}(v) = \infty$
set $\texttt{dist}(s) = 0$
For each v, Do $\textsc{Insert}(v, \texttt{dist}(v))$
While the queue is not empty
  $u = \textsc{ExtractMin}()$
  For every edge $(u, v) \in E$
    If $\texttt{dist}(v) > \texttt{dist}(u) + \texttt{weight}(u, v)$
      $\texttt{dist}(v) = \texttt{dist}(u) + \texttt{weight}(u, v)$
      $\textsc{DecreaseKey}(v, \texttt{dist}(v))$

We're going to simulate

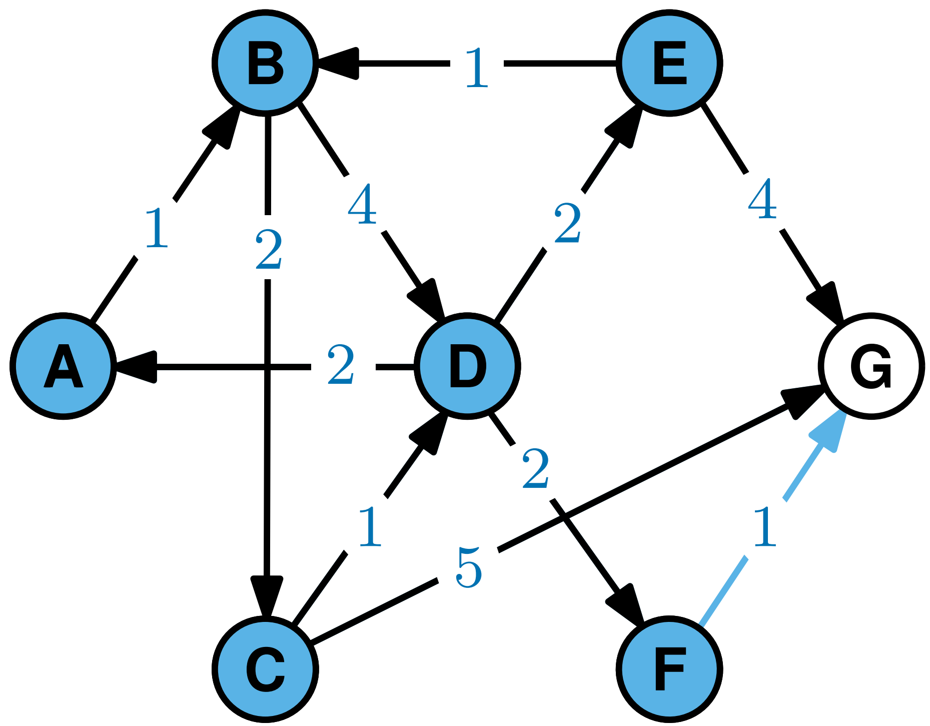
DIJKSTRA(A)

i.e. s = A

this is called relaxing edge (u, v)

new path to **G** = $6 + 1 = 7$

	A	B	C	D	E	F	G
dist:	0	1	3	4	6	6	8



$\texttt{dist}(v)$ is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

$v.\text{key} = \texttt{dist}(v)$

DIJKSTRA(s)

For all v, set $\texttt{dist}(v) = \infty$
set $\texttt{dist}(s) = 0$
For each v, Do $\textsc{Insert}(v, \texttt{dist}(v))$
While the queue is not empty
  $u = \textsc{ExtractMin}()$
  For every edge $(u, v) \in E$
    If $\texttt{dist}(v) > \texttt{dist}(u) + \texttt{weight}(u, v)$
      $\texttt{dist}(v) = \texttt{dist}(u) + \texttt{weight}(u, v)$
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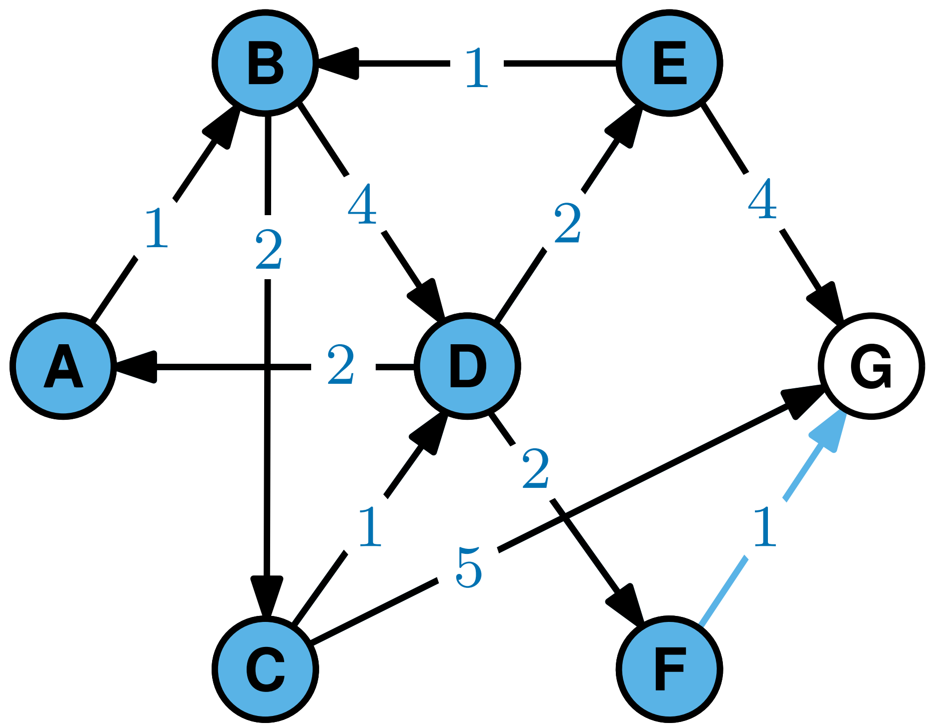

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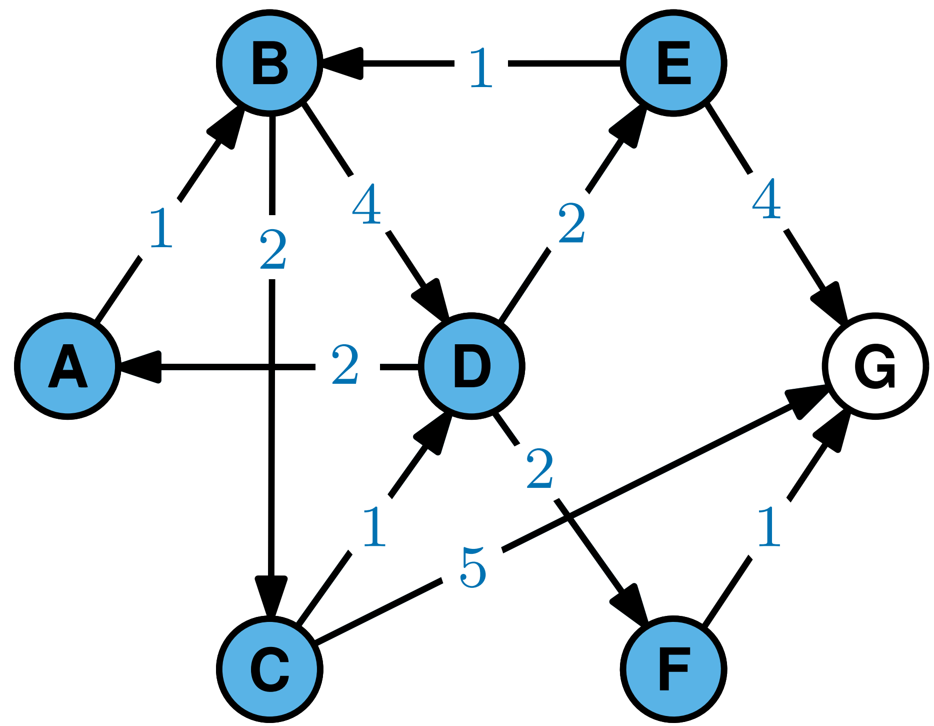
this is called relaxing edge (u, v)

We're going to simulate DIJKSTRA(A)

i.e. s = A

dist:

A	B	C	D	E	F	G
0	1	3	4	6	6	7



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at all times, for each vertex v,

v.key = $\texttt{dist}(v)$

$\textsc{Dijkstra}(s)$

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```

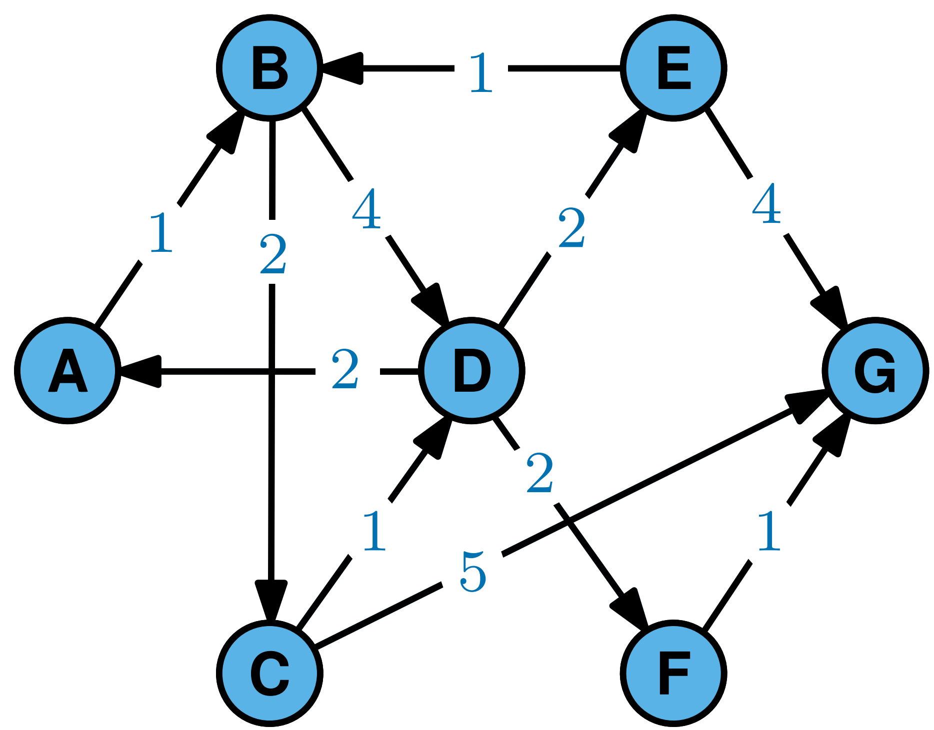
We're going to simulate

$\textsc{Dijkstra}(A)$

i.e. $s = A$

this is called relaxing edge (u, v)

	A	B	C	D	E	F	G
dist:	0	1	3	4	6	6	7



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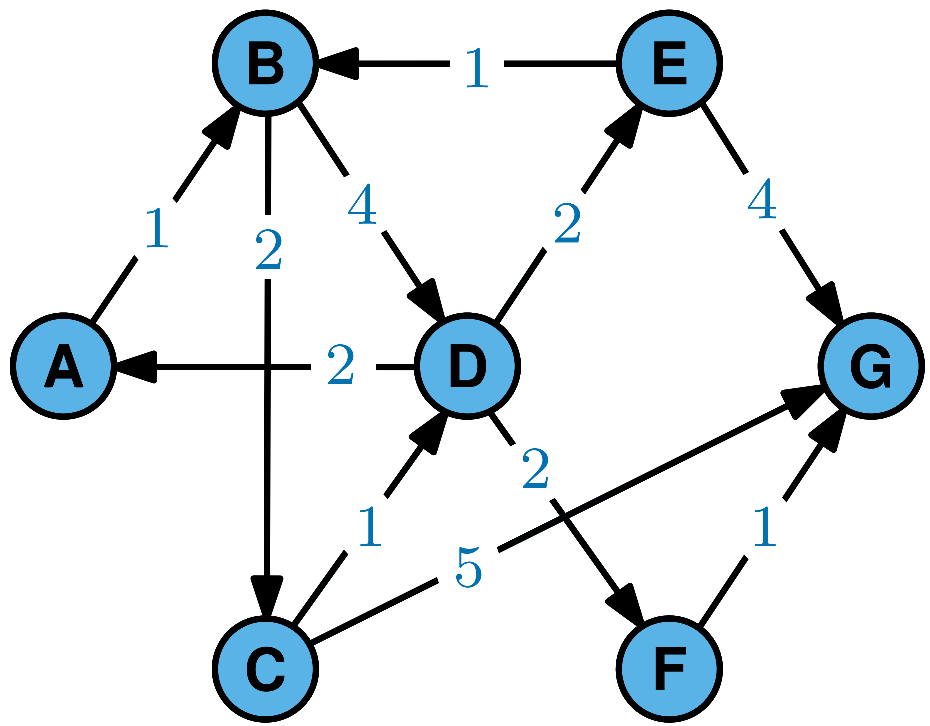
We're going to simulate DIJKSTRA(A)

i.e. s = A

this is called relaxing edge (u, v)

shortest paths from s = A:

	A	B	C	D	E	F	G
dist:	0	1	3	4	6	6	7



$\texttt{dist}(v)$ is the length of the shortest path between s and v, *found so far*

at all times, for each vertex v,

$v.\text{key} = \texttt{dist}(v)$

Proof of Correctness

Claim when Dijkstra's algorithm terminates, for each vertex v, $\mathtt{dist}(v) = \delta(s, v)$

where $\delta(s, v)$ is the *true* distance between s and v

DIJKSTRA(s)

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      dist(v) = dist(u) + weight(u, v)
      DECREASEKEY(v, dist(v))
```

Observation At all times, $\mathtt{dist}(v)$ is the length of *some* path from s to v

(unless $\mathtt{dist}(v) = \infty$)

Therefore, for each vertex v, $\delta(s, v) \leqslant \mathtt{dist}(v)$

Proof of Correctness

Claim when Dijkstra's algorithm terminates, for each vertex v, $\texttt{dist}(v) = \delta(s, v)$

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Further, observe that after a vertex v is EXTRACTED,

$\texttt{dist}(v)$ certainly doesn't increase

Proof of Correctness

Claim when Dijkstra's algorithm terminates, for each vertex v, $\mathtt{dist}(v) = \delta(s, v)$

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      DECREASEKEY(v, dist(v))
```

Further, observe that after a vertex v is EXTRACTED,

$\mathtt{dist}(v)$ certainly doesn't increase

So we focus on proving that for all v,

when vertex v is EXTRACTED, $\mathtt{dist}(v) = \delta(s, v)$

Proof of Correctness

the true distance
between s and v

Lemma Whenever a vertex v is EXTRACTED, $\texttt{dist}(v) = \delta(s, v)$

Proof

Proof of Correctness

the true distance *between s and v*

Lemma Whenever a vertex v is EXTRACTED, $\mathtt{dist}(v) = \delta(s, v)$

Proof

To derive a contradiction, we let v be the first vertex to be EXTRACTED with $\mathtt{dist}(v) \neq \delta(s, v)$

Proof of Correctness

the true distance between s and v

Lemma Whenever a vertex v is EXTRACTED, $\mathtt{dist}(v) = \delta(s, v)$

Proof

To derive a contradiction, we let v be the first vertex to be EXTRACTED with $\mathtt{dist}(v) \neq \delta(s, v)$

v cannot be the source, s because $\mathtt{dist}(s) = 0 = \delta(s, s)$

Proof of Correctness

the true distance between s and v

Lemma Whenever a vertex v is EXTRACTED, $\texttt{dist}(v) = \delta(s, v)$

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There must be a path from s to v, otherwise

$$\texttt{dist}(v) = \infty = \delta(s, v)$$

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(Dijkstra doesn't find paths that aren't there)

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Consider the point in the algorithm immediately before v is EXTRACTED

Proof of Correctness

the true distance between s and v

Lemma Whenever a vertex v is EXTRACTED, $\mathtt{dist}(v) = \delta(s, v)$

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In particular consider a shortest path from s to v:

Proof of Correctness

the true distance between s and v

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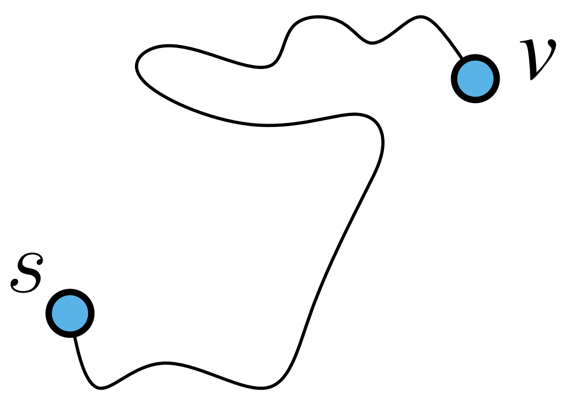
Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



Proof of Correctness

the true distance between s and v

Lemma Whenever a vertex v is EXTRACTED, $\texttt{dist}(v) = \delta(s, v)$

Proof

To derive a contradiction, we let v be the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

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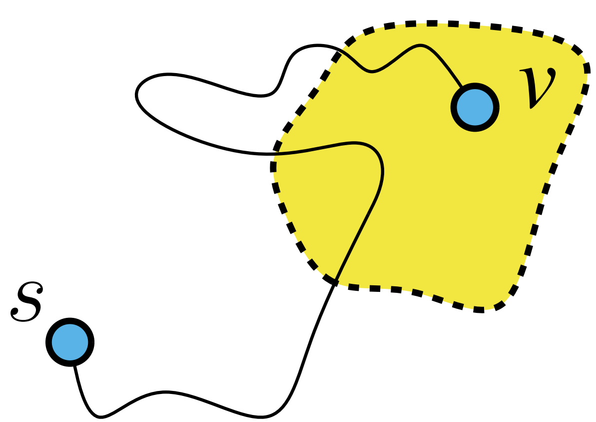

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the true distance between s and v

Lemma Whenever a vertex v is EXTRACTED, $\texttt{dist}(v) = \delta(s, v)$

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There must be a path from s to v, otherwise

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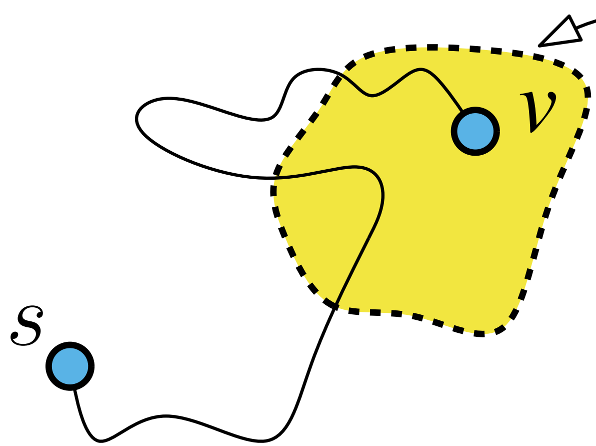

these are the vertices still in the queue

Proof of Correctness

the true distance between s and v

Lemma Whenever a vertex v is EXTRACTED, $\texttt{dist}(v) = \delta(s, v)$

Proof

To derive a contradiction, we let v be the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

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There must be a path from s to v, otherwise

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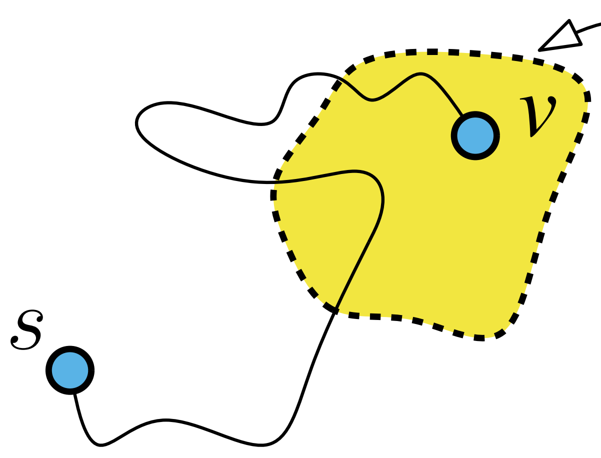


these are the vertices still in the queue

(v is in but s isn't)

Proof of Correctness

the true distance between s and v

Lemma Whenever a vertex v is EXTRACTED, $\texttt{dist}(v) = \delta(s, v)$

Proof

To derive a contradiction, we let v be the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

v cannot be the source, s because $\texttt{dist}(s) = 0 = \delta(s, s)$

(this is in the algorithm description)

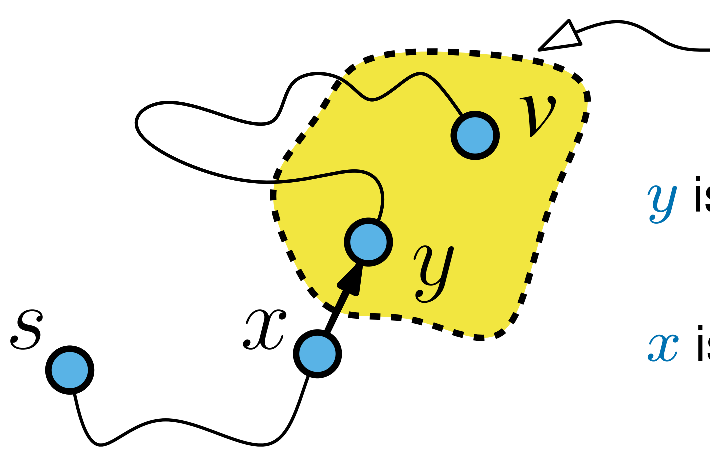
There must be a path from s to v, otherwise

$$\texttt{dist}(v) = \infty = \delta(s, v)$$

(Dijkstra doesn't find paths that aren't there)

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



these are the vertices still in the queue

(v is in but s isn't)

y is the first vertex on the path that is in the queue

(there must be one because v is in the queue but s isn't)

x is the vertex before y on the path

(it's definitely not in the queue)

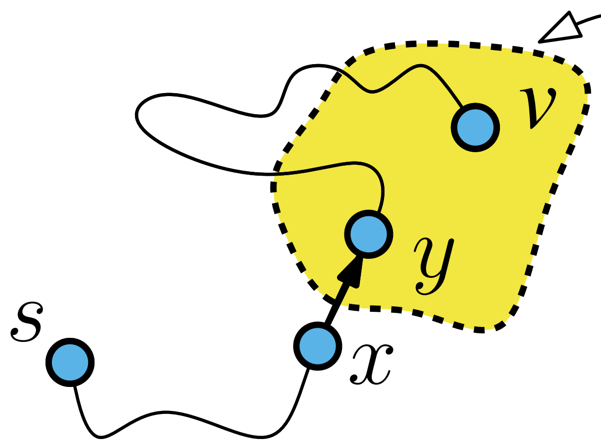
Proof of Correctness

the true distance between s and v

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



these are the vertices still in the queue

y is the first vertex on the path that is in the queue

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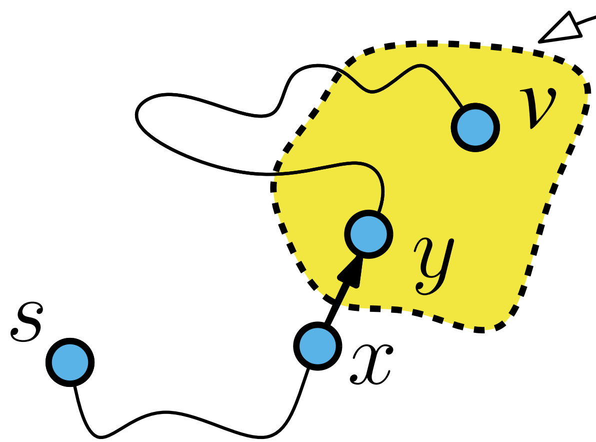
Proof of Correctness

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s,v)$

the true distance between s and v

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



these are the vertices still in the queue

y is the first vertex on the path that is in the queue

x is the vertex before y on the path

The path shown from s to y is a shortest path

Proof of Correctness

the true distance between s and v

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

Consider the point in the algorithm immediately before v is EXTRACTED

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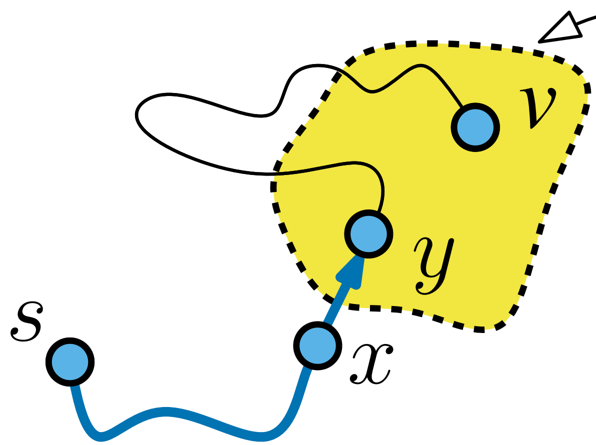

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The path shown from s to y is a shortest path

Proof of Correctness

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

the true distance between s and v

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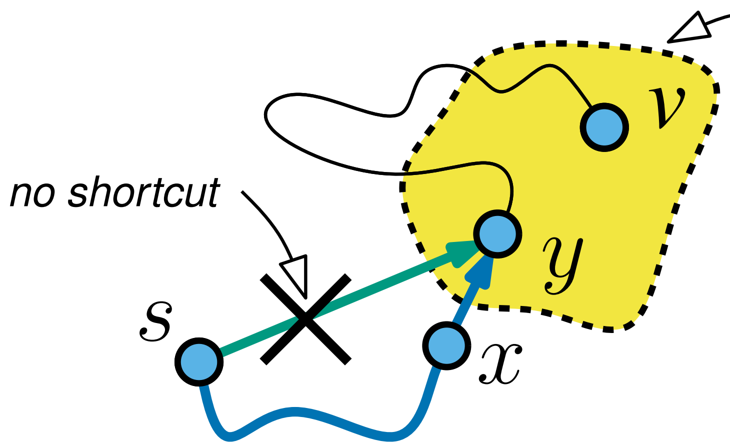

these are the vertices still in the queue

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The path shown from s to y is a shortest path

Proof of Correctness

the true distance between s and v

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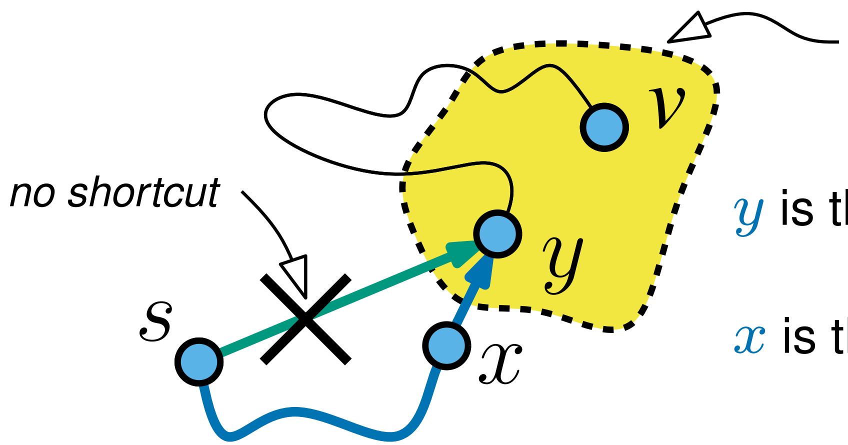
In particular consider a shortest path from s to v:



these are the vertices still in the queue

y is the first vertex on the path that is in the queue

x is the vertex before y on the path

The path shown from s to y is a shortest path *(otherwise, the path to v isn't shortest)*

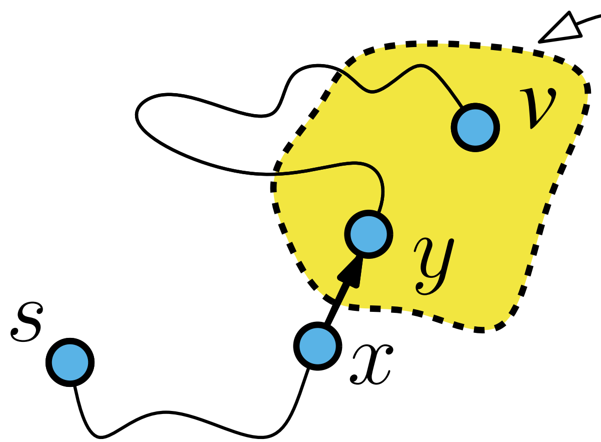
Proof of Correctness

the true distance between s and v

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



these are the vertices still in the queue

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The path shown from s to y is a shortest path *(otherwise, the path to v isn't shortest)*

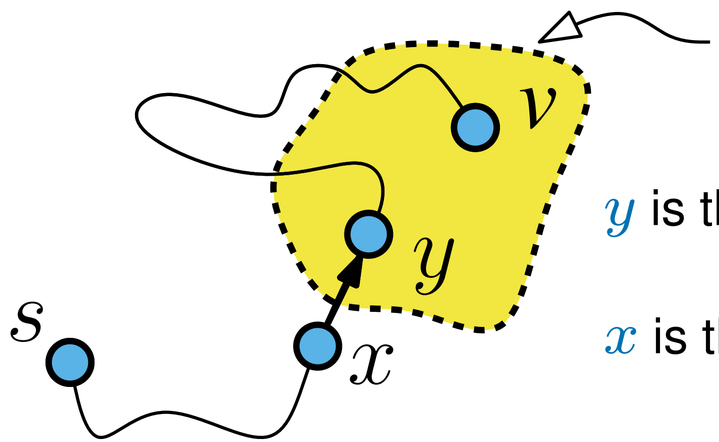
Proof of Correctness

the true distance between s and v

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



these are the vertices still in the queue

y is the first vertex on the path that is in the queue

x is the vertex before y on the path

The path shown from s to y is a shortest path *(otherwise, the path to v isn't shortest)*

therefore, $\delta(s, y) \leqslant \delta(s, v)$

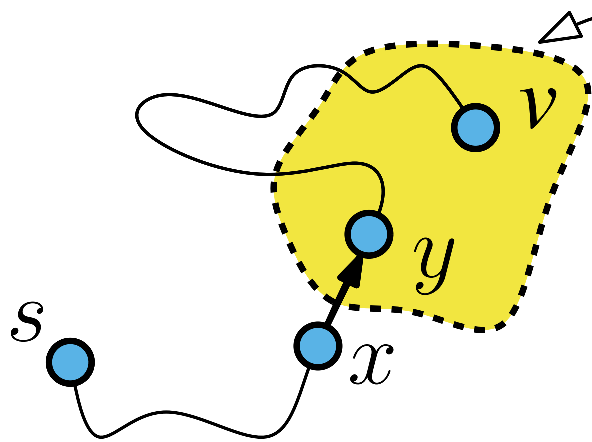
Proof of Correctness

the true distance between s and v

v is the first vertex to be EXTRACTED with $\mathtt{dist}(v) \neq \delta(s, v)$

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



these are the vertices still in the queue

y is the first vertex on the path that is in the queue

x is the vertex before y on the path

The path shown from s to y is a shortest path *(otherwise, the path to v isn't shortest)*

therefore, $\delta(s, y) \leqslant \delta(s, v)$

The vertex x is EXTRACTED from the queue *before* v

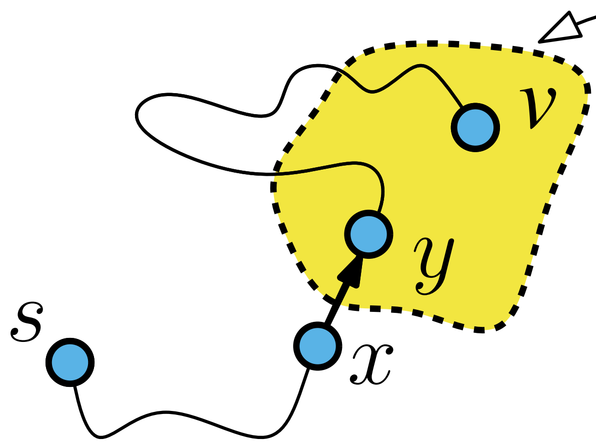
Proof of Correctness

the true distance between s and v

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



these are the vertices still in the queue

y is the first vertex on the path that is in the queue

x is the vertex before y on the path

The path shown from s to y is a shortest path *(otherwise, the path to v isn't shortest)*

therefore, $\delta(s, y) \leqslant \delta(s, v)$

The vertex x is EXTRACTED from the queue *before* v

therefore $\texttt{dist}(x) = \delta(s, x)$

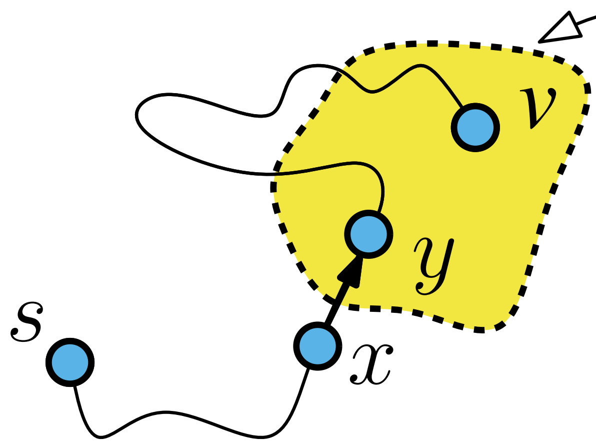
Proof of Correctness

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s,v)$

the true distance
between s and v

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



these are the vertices still in the queue

y is the first vertex on the path that is in the queue

x is the vertex before y on the path

The path shown from s to y is a shortest path *(otherwise, the path to v isn't shortest)*

therefore, $\delta(s,y) \leqslant \delta(s,v)$

The vertex x is EXTRACTED from the queue *before* v

therefore $\texttt{dist}(x) = \delta(s,x)$ *(v is the first vertex EXTRACTED with the wrong distance)*

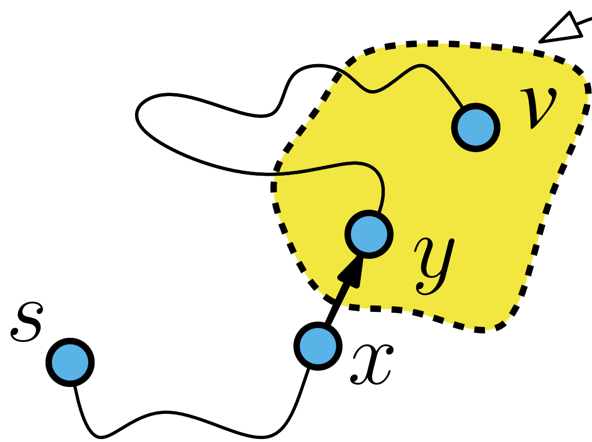
Proof of Correctness

the true distance between s and v

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s,v)$

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



these are the vertices still in the queue

y is the first vertex on the path that is in the queue

x is the vertex before y on the path

The path shown from s to y is a shortest path *(otherwise, the path to v isn't shortest)*

therefore, $\delta(s,y) \leqslant \delta(s,v)$

The vertex x is EXTRACTED from the queue *before* v

therefore $\texttt{dist}(x) = \delta(s,x)$ *(v is the first vertex EXTRACTED with the wrong distance)*

Further, when x was EXTRACTED we relaxed edge (x,y)

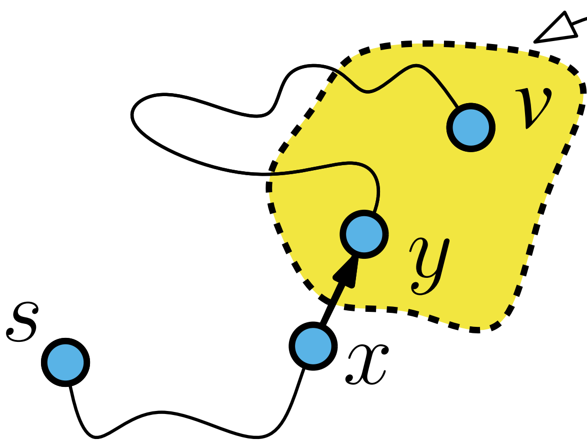
Proof of Correctness

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

the true distance between s and v

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



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therefore, $\delta(s, y) \leqslant \delta(s, v)$

The vertex x is EXTRACTED from the queue *before* v

therefore $\texttt{dist}(x) = \delta(s, x)$ *(v is the first vertex EXTRACTED with the wrong distance)*

Further, when x was EXTRACTED we relaxed edge (x, y)

therefore $\texttt{dist}(y) \leqslant \texttt{dist}(x) + \texttt{weight}(x, y)$

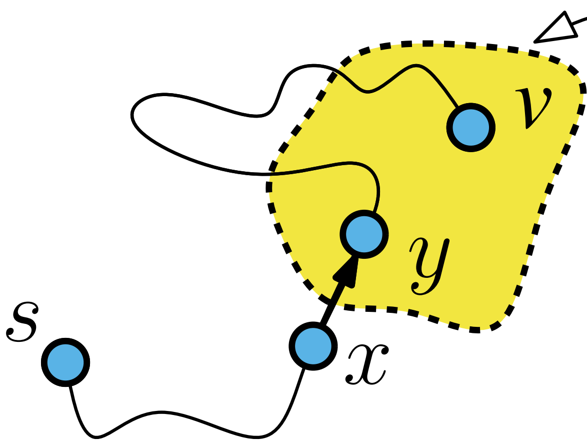
Proof of Correctness

v is the first vertex to be EXTRACTED with $\mathtt{dist}(v) \neq \delta(s, v)$

the true distance between s and v

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



these are the vertices still in the queue

y is the first vertex on the path that is in the queue

x is the vertex before y on the path

The path shown from s to y is a shortest path *(otherwise, the path to v isn't shortest)*

therefore, $\delta(s, y) \leqslant \delta(s, v)$

The vertex x is EXTRACTED from the queue *before* v

therefore $\mathtt{dist}(x) = \delta(s, x)$ *(v is the first vertex EXTRACTED with the wrong distance)*

Further, when x was EXTRACTED we relaxed edge (x, y)

therefore $\mathtt{dist}(y) \leqslant \mathtt{dist}(x) + \mathtt{weight}(x, y)$

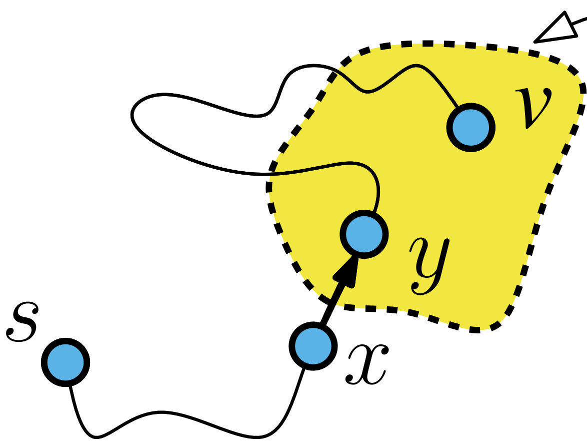
Proof of Correctness

v is the first vertex to be Extracted with $\texttt{dist}(v) \neq \delta(s, v)$

the true distance between s and v

Consider the point in the algorithm immediately before v is Extracted

In particular consider a shortest path from s to v:



these are the vertices still in the queue

y is the first vertex on the path that is in the queue

x is the vertex before y on the path

The path shown from s to y is a shortest path *(otherwise, the path to v isn't shortest)*

therefore, $\delta(s, y) \leqslant \delta(s, v)$

The vertex x is Extracted from the queue *before* v

therefore $\texttt{dist}(x) = \delta(s, x)$ *(v is the first vertex Extracted with the wrong distance)*

Further, when x was Extracted we relaxed edge (x, y)

therefore $\texttt{dist}(y) \leqslant \delta(s, x) + \texttt{weight}(x, y)$

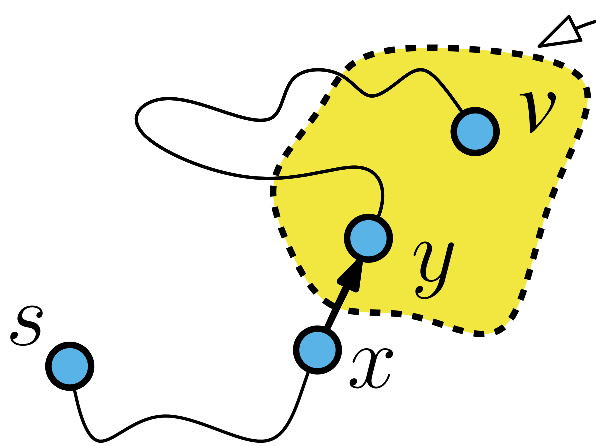
Proof of Correctness

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

the true distance between s and v

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



these are the vertices still in the queue

y is the first vertex on the path that is in the queue

x is the vertex before y on the path

The path shown from s to y is a shortest path *(otherwise, the path to v isn't shortest)*

therefore, $\delta(s, y) \leqslant \delta(s, v)$

The vertex x is EXTRACTED from the queue *before* v

therefore $\texttt{dist}(x) = \delta(s, x)$ *(v is the first vertex EXTRACTED with the wrong distance)*

Further, when x was EXTRACTED we relaxed edge (x, y)

therefore $\texttt{dist}(y) \leqslant \delta(s, x) + \texttt{weight}(x, y)$

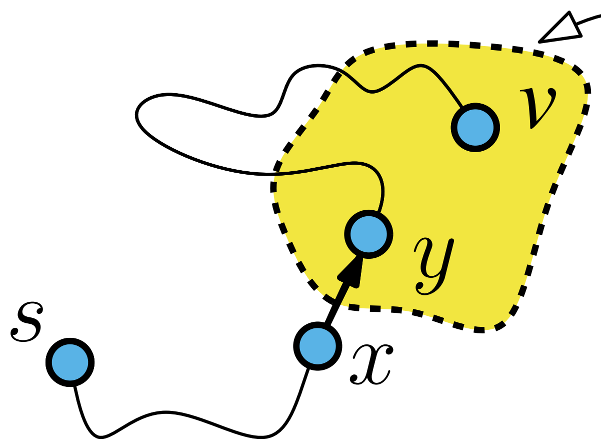
Proof of Correctness

the true distance between s and v

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



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therefore, $\delta(s, y) \leqslant \delta(s, v)$

The vertex x is EXTRACTED from the queue *before* v

therefore $\texttt{dist}(x) = \delta(s, x)$ *(v is the first vertex EXTRACTED with the wrong distance)*

Further, when x was EXTRACTED we relaxed edge (x, y)

therefore $\texttt{dist}(y) \leqslant \delta(s, x) + \texttt{weight}(x, y) = \delta(s, y)$

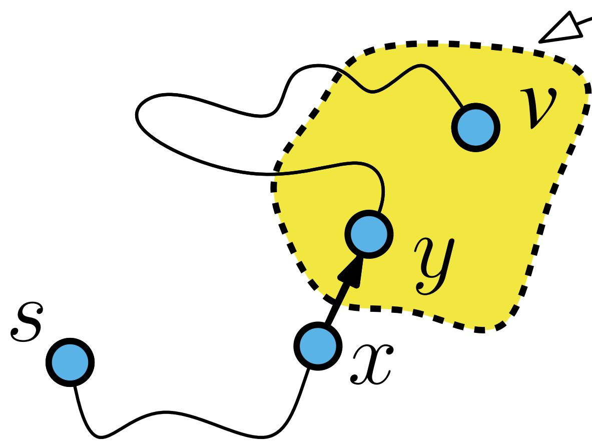
Proof of Correctness

the true distance between s and v

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s,v)$

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



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therefore, $\delta(s,y) \leqslant \delta(s,v)$

The vertex x is EXTRACTED from the queue *before* v

therefore $\texttt{dist}(x) = \delta(s,x)$ *(v is the first vertex EXTRACTED with the wrong distance)*

Further, when x was EXTRACTED we relaxed edge (x,y)

therefore $\texttt{dist}(y) \leqslant \delta(s,x) + \texttt{weight}(x,y) = \delta(s,y)$ *(the path shown is shortest)*

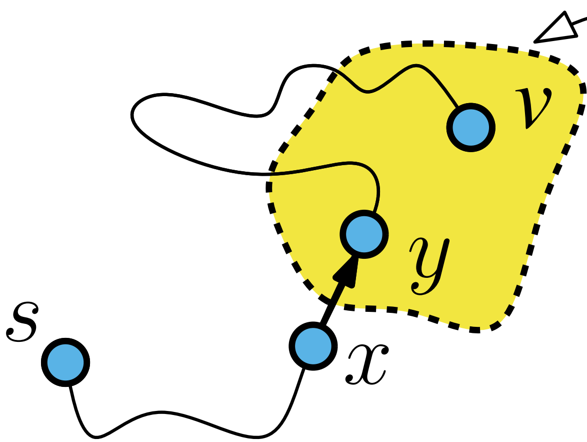
Proof of Correctness

the true distance
between s and v

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s,v)$

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



these are the vertices still in the queue

y is the first vertex on the path that is in the queue

x is the vertex before y on the path

We have shown that: $\texttt{dist}(y) \leqslant \delta(s,y)$ and $\delta(s,y) \leqslant \delta(s,v)$

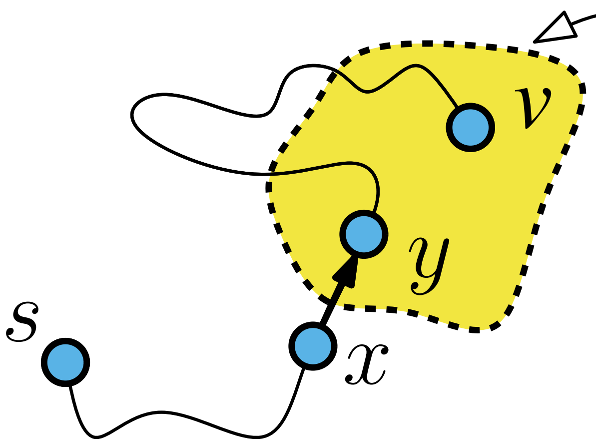
Proof of Correctness

the true distance between s and v

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



these are the vertices still in the queue

y is the first vertex on the path that is in the queue

x is the vertex before y on the path

We have shown that: $\texttt{dist}(y) \leqslant \delta(s, y)$ and $\delta(s, y) \leqslant \delta(s, v)$

We are almost there :)

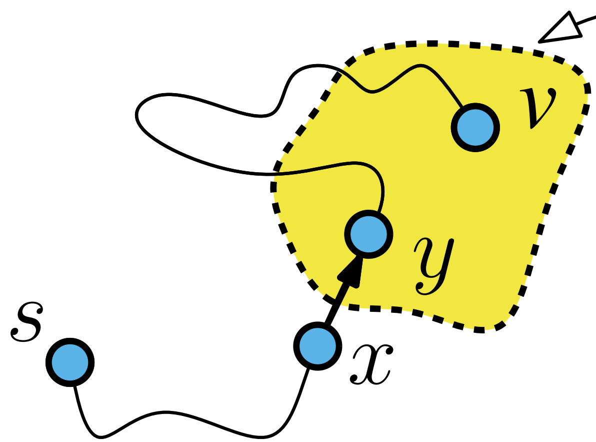
Proof of Correctness

the true *distance between* s *and* v

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



these are the vertices still in the queue

y is the first vertex on the path that is in the queue

x is the vertex before y on the path

We have shown that: $\texttt{dist}(y) \leqslant \delta(s, y)$ and $\delta(s, y) \leqslant \delta(s, v)$

We are almost there :)

Finally, we have that $\texttt{dist}(v) \leqslant \texttt{dist}(y)$

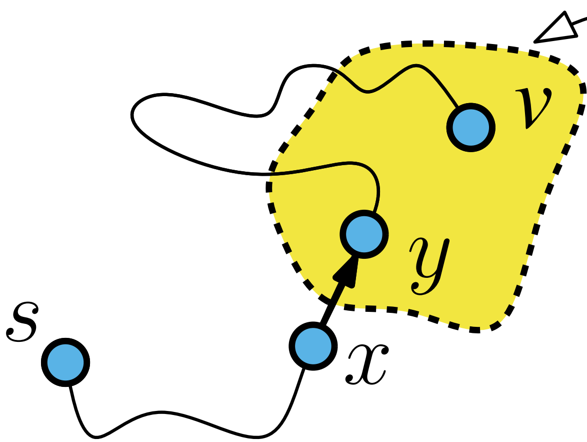
Proof of Correctness

the true distance between s and v

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



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y is the first vertex on the path that is in the queue

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We have shown that: $\texttt{dist}(y) \leqslant \delta(s, y)$ and $\delta(s, y) \leqslant \delta(s, v)$

We are almost there :)

Finally, we have that $\texttt{dist}(v) \leqslant \texttt{dist}(y)$

because v is EXTRACTED next *(so has the minimum $\texttt{dist}$)*

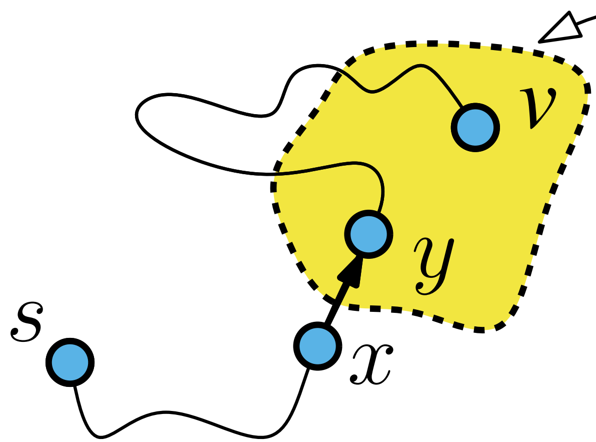
Proof of Correctness

the true distance between s and v

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s,v)$

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



these are the vertices still in the queue

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x is the vertex before y on the path

We have shown that: $\texttt{dist}(y) \leqslant \delta(s,y)$ and $\delta(s,y) \leqslant \delta(s,v)$

We are almost there :)

Finally, we have that $\texttt{dist}(v) \leqslant \texttt{dist}(y)$

because v is EXTRACTED next *(so has the minimum* $\texttt{dist}$*)*

Putting it all together, $\texttt{dist}(v) \leqslant \texttt{dist}(y) \leqslant \delta(s,y) \leqslant \delta(s,v)$

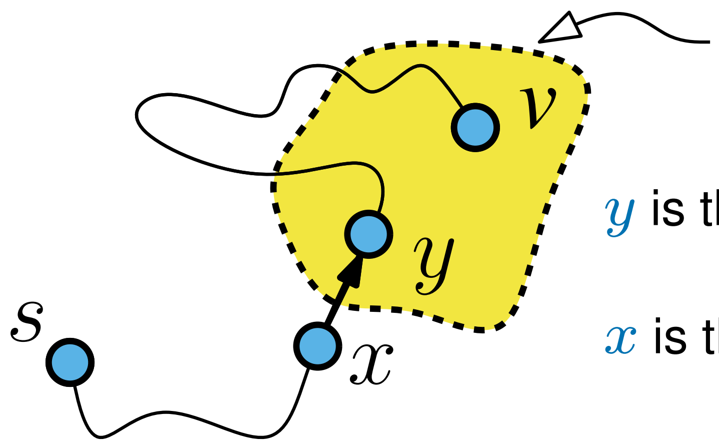
Proof of Correctness

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

the true distance between s and v

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



these are the vertices still in the queue

y is the first vertex on the path that is in the queue

x is the vertex before y on the path

We have shown that: $\texttt{dist}(y) \leqslant \delta(s, y)$ and $\delta(s, y) \leqslant \delta(s, v)$

We are almost there :)

Finally, we have that $\texttt{dist}(v) \leqslant \texttt{dist}(y)$

because v is EXTRACTED next *(so has the minimum* $\texttt{dist}$*)*

Putting it all together, $\texttt{dist}(v) \leqslant \texttt{dist}(y) \leqslant \delta(s, y) \leqslant \delta(s, v)$

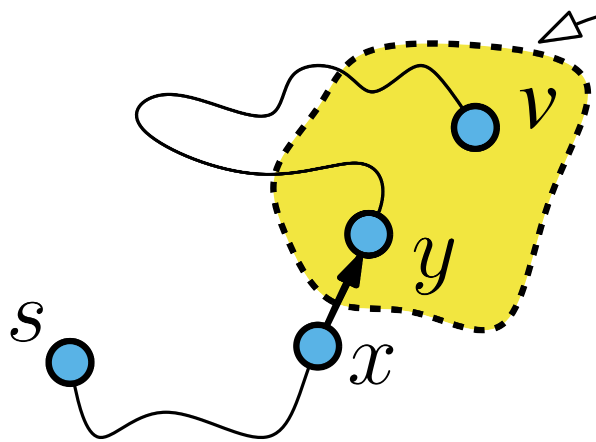
Proof of Correctness

the true distance between s and v

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



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We are almost there :)

Finally, we have that $\texttt{dist}(v) \leqslant \texttt{dist}(y)$

because v is EXTRACTED next *(so has the minimum* $\texttt{dist}$*)*

Putting it all together, $\texttt{dist}(v) \leqslant \texttt{dist}(y) \leqslant \delta(s, y) \leqslant \delta(s, v)$

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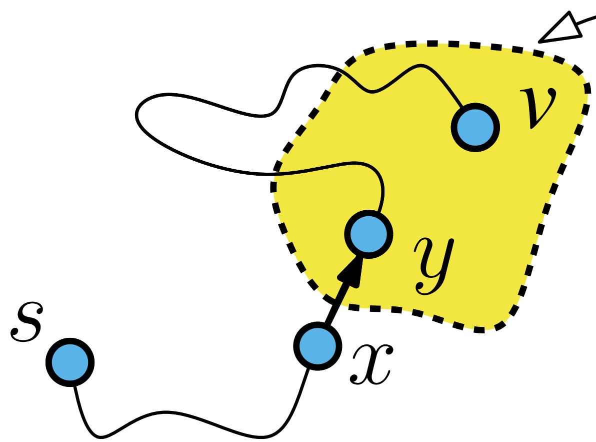
Proof of Correctness

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

the true distance between s and v

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



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We are almost there :)

Finally, we have that $\texttt{dist}(v) \leqslant \texttt{dist}(y)$

because v is EXTRACTED next *(so has the minimum $\texttt{dist}$)*

Putting it all together, $\texttt{dist}(v) \leqslant \texttt{dist}(y) \leqslant \delta(s, y) \leqslant \delta(s, v)$

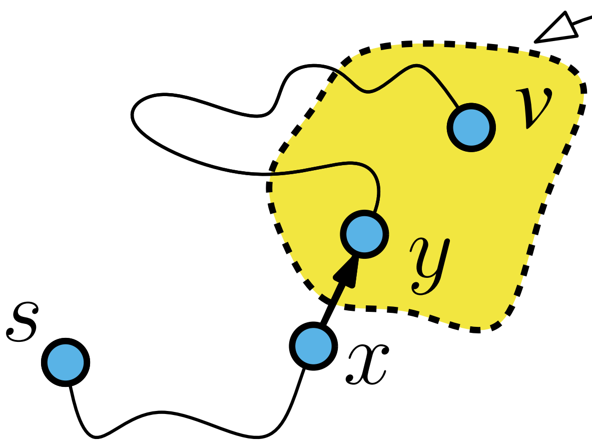
Proof of Correctness

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

the true distance between s and v

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



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y is the first vertex on the path that is in the queue

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We have shown that: $\texttt{dist}(y) \leqslant \delta(s, y)$ and $\delta(s, y) \leqslant \delta(s, v)$

We are almost there :)

Finally, we have that $\texttt{dist}(v) \leqslant \texttt{dist}(y)$

because v is EXTRACTED next *(so has the minimum* $\texttt{dist}$*)*

Putting it all together, $\texttt{dist}(v) \leqslant \texttt{dist}(y) \leqslant \delta(s, y) \leqslant \delta(s, v)$

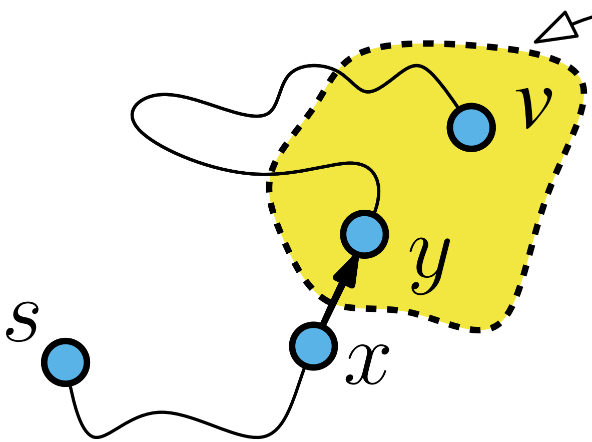
Proof of Correctness

the true distance between s and v

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



these are the vertices still in the queue

y is the first vertex on the path that is in the queue

x is the vertex before y on the path

We have shown that: $\texttt{dist}(y) \leqslant \delta(s, y)$ and $\delta(s, y) \leqslant \delta(s, v)$

We are almost there :)

Finally, we have that $\texttt{dist}(v) \leqslant \texttt{dist}(y)$

because v is EXTRACTED next *(so has the minimum* $\texttt{dist}$*)*

Putting it all together, $\texttt{dist}(v)$ $\leqslant \delta(s, v)$

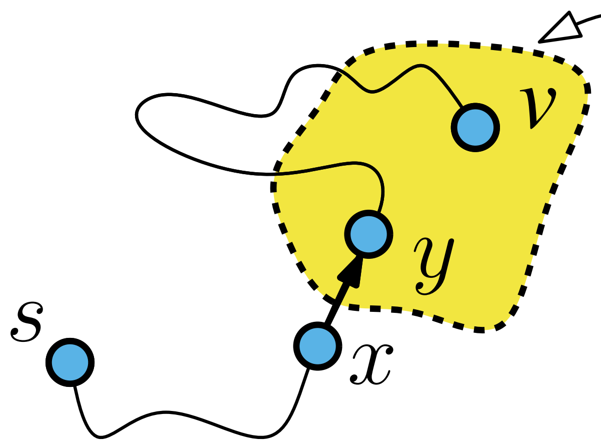
Proof of Correctness

the true *distance* *between s and v*

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



these are the vertices still in the queue

y is the first vertex on the path that is in the queue

x is the vertex before y on the path

We have shown that: $\texttt{dist}(y) \leqslant \delta(s, y)$ and $\delta(s, y) \leqslant \delta(s, v)$

We are almost there :)

Finally, we have that $\texttt{dist}(v) \leqslant \texttt{dist}(y)$

because v is EXTRACTED next *(so has the minimum* $\texttt{dist}$*)*

Putting it all together, $\texttt{dist}(v) \leqslant \delta(s, v)$

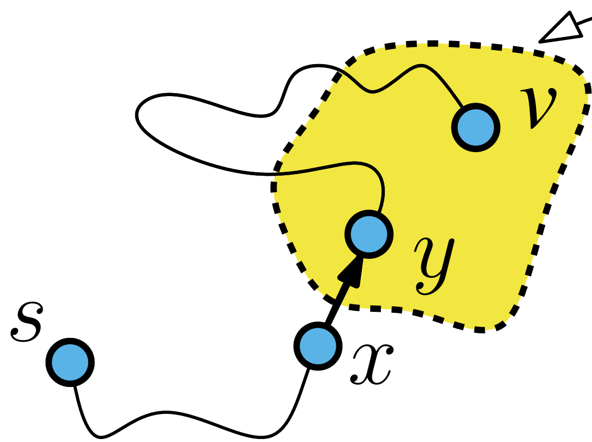
Proof of Correctness

the true distance *between s and v*

v is the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

Consider the point in the algorithm immediately before v is EXTRACTED

In particular consider a shortest path from s to v:



these are the vertices still in the queue

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x is the vertex before y on the path

Summary

we assumed that v was the first vertex to be EXTRACTED with $\texttt{dist}(v) \neq \delta(s, v)$

we proved that $\texttt{dist}(v) \leqslant \delta(s, v)$

however, we also have that, $\texttt{dist}(v) \geqslant \delta(s, v)$ *(Dijkstra only finds actual paths)*

so we have that $\texttt{dist}(v) = \delta(s, v)$ **Contradiction!** there is no such v

(in other words all distances are correct)

Time Complexity

The time complexity of Dijkstra's algorithm depends on the time complexities of INSERT, DECREASEKEY and EXTRACTMIN supported by the queue

DIJKSTRA(s)

```
For all v, set dist(v) = ∞
set dist(s) = 0
For each v, Do INSERT(v, dist(v))
While the queue is not empty
  u = EXTRACTMIN()
  For every edge (u, v) ∈ E
    If dist(v) > dist(u) + weight(u, v)
      dist(v) = dist(u) + weight(u, v)
      DECREASEKEY(v, dist(v))
```

Time Complexity

The time complexity of Dijkstra's algorithm depends on the time complexities of INSERT,DECREASEKEY and EXTRACTMIN supported by the queue

DIJKSTRA(s)

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```

$O(|V|)$ time

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         DECREASEKEY(v, dist(v))
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$O(|V|)$ time

(*we store* dist *in an array of length* $|V|$)

Time Complexity

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         dist(v) = dist(u) + weight(u, v)
         DECREASEKEY(v, dist(v))
```

$O(|V|)$ time

(*we store* dist *in an array of length* $|V|$)

$O(1)$ time

Time Complexity

The time complexity of Dijkstra's algorithm depends on the time complexities of INSERT, DECREASEKEY and EXTRACTMIN supported by the queue

DIJKSTRA(s)

```
For all v, set dist(v) = ∞
set dist(s) = 0
For each v, Do INSERT(v, dist(v))
While the queue is not empty
   u = EXTRACTMIN()
   For every edge (u, v) ∈ E
      If dist(v) > dist(u) + weight(u, v)
         dist(v) = dist(u) + weight(u, v)
         DECREASEKEY(v, dist(v))
```

$O(|V|)$ time

(*we store* `dist` *in an array of length* $|V|$)

$O(1)$ time

$O(|V| \cdot T_{\text{INSERT}})$ time

Time Complexity

The time complexity of Dijkstra's algorithm depends on the time complexities of
INSERT, DECREASEKEY and EXTRACTMIN
supported by the queue

DIJKSTRA(s)

```
For all v, set dist(v) = ∞
set dist(s) = 0
For each v, Do INSERT(v, dist(v))
While the queue is not empty
   u = EXTRACTMIN()
   For every edge (u, v) ∈ E
      If dist(v) > dist(u) + weight(u, v)
         dist(v) = dist(u) + weight(u, v)
         DECREASEKEY(v, dist(v))
```

$O(|V| \cdot T_{\text{INSERT}})$ time

for the setup

Time Complexity

The time complexity of Dijkstra's algorithm depends on the time complexities of INSERT,DECREASEKEY and EXTRACTMIN supported by the queue

DIJKSTRA(s)

```
For all v, set dist(v) = ∞
set dist(s) = 0
For each v, Do INSERT(v, dist(v))
While the queue is not empty
   u = EXTRACTMIN()
   For every edge (u, v) ∈ E
      If dist(v) > dist(u) + weight(u, v)
         dist(v) = dist(u) + weight(u, v)
         DECREASEKEY(v, dist(v))
```

$O(|V| \cdot T_{\text{INSERT}})$ time *for the setup*

$O(T_{\text{EXTRACTMIN}})$ time per iteration

Time Complexity

The time complexity of Dijkstra's algorithm depends on the time complexities of
INSERT, DECREASEKEY and EXTRACTMIN
supported by the queue

DIJKSTRA(s)

For all v, set $\texttt{dist}(v) = \infty$
set $\texttt{dist}(s) = 0$
For each v, Do INSERT$(v, \texttt{dist}(v))$
While the queue is not empty
  $u = $ EXTRACTMIN$()$
  For every edge $(u, v) \in E$
    If $\texttt{dist}(v) > \texttt{dist}(u) + \texttt{weight}(u, v)$
      $\texttt{dist}(v) = \texttt{dist}(u) + \texttt{weight}(u, v)$
      DECREASEKEY$(v, \texttt{dist}(v))$

$O(|V| \cdot T_{\text{INSERT}})$ time

for the setup

$O(T_{\text{EXTRACTMIN}})$ time per iteration

relaxing an edge takes
$O(T_{\text{DECREASEKEY}})$ time

Time Complexity

The time complexity of Dijkstra's algorithm depends on the time complexities of
INSERT, DECREASEKEY and EXTRACTMIN
supported by the queue

DIJKSTRA(s)

```
For all v, set dist(v) = ∞
set dist(s) = 0
For each v, Do INSERT(v, dist(v))
While the queue is not empty
   u = EXTRACTMIN()
   For every edge (u, v) ∈ E
      If dist(v) > dist(u) + weight(u, v)
         dist(v) = dist(u) + weight(u, v)
         DECREASEKEY(v, dist(v))
```

$O(|V| \cdot T_{\text{INSERT}})$ time

for the setup

$O(T_{\text{EXTRACTMIN}})$ time per iteration

relaxing an edge takes
$O(T_{\text{DECREASEKEY}})$ time

We do $O(|V|)$ iterations of the while loop
and relax each edge at most once...

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$$O(|V| \cdot T_{\text{INSERT}} + |V| \cdot T_{\text{EXTRACTMIN}} + |E| \cdot T_{\text{DECREASEKEY}}) \text{ time}$$

Summary

The time complexity of Dijkstra's algorithm depends on the time complexities of INSERT, DECREASEKEY and EXTRACTMIN supported by the queue

Recall from last lecture, the complexities of some priority queues:

	INSERT	DECREASEKEY	EXTRACTMIN
Unsorted Linked List	$O(1)$	$O(n)$	$O(n)$
Sorted Linked List	$O(n)$	$O(n)$	$O(1)$
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remember that for the Binary Heap n to find an element in $O(1)$ time
we needed each element to have an **ID** $\leqslant N \ldots$ *here* $N = |V|$

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Dijkstra's algorithm solves the single source shortest path algorithm
on weighted, directed graphs. . .
with **non-negative** edge weights