COMS21103: Data Structures and Algorithms

Stable Roommate Problem

Irving's algorithm: References Irving, Robert W. (1985), "An efficient algorithm for the stable roommates problem", Journal of Algorithms 6 (4): 577595

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Pseudocode
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```
while there are unmatched people do
    Let i be the smallest value such that a_i is unmatched
    a_i proposes to his favourite roommate a_i who has not rejected him previously
    if a_i has not received a proposal before then
        a_j accepts a_i
    else
        if a_i prefers a_i over his current match a_k then
            a_i accepts a_i
            a_k rejects a_i
        else
            a_j rejects a_i
        end
    end
for all accepted proposals (a_i, a_j) do
    reject all (a_i, a_k) where a_i prefers a_i over a_k
for all cycles (p_1,...p_n) and associated second preferences (q_1,...,q_n) such that:
  q_i is the second preference of p_i
  p_{i+1} is the last preference of q_i
  p_n \in \{p_1, .., p_{n-1}\} do
   for i = 1..n - 1 do
        q_i rejects p_{i+1}
    end
end
```

No stable matching exists if at any point a person is rejected by everyone else

Ex.

```
A : D F B E C
B : F C E A D
C : D E A F B
D : B F E A C
E : D B C F A
F : E A D B C
```

solution

- 1. Loop 1
 - (a) $A \rightarrow D$ (accept)
 - (b) $B \rightarrow F$ (accept)
 - (c) $C \rightarrow D$ (reject)
 - A: DFBEC
 - B : F C E A D
 - C : E A F B
 - D: BFEA
 - E: DBCFA
 - F : E A D B C
 - (d) $C \rightarrow E$ (accept)
 - (e) $D \rightarrow B$ (accept)
 - (f) $E \rightarrow D$ (accept) $-A \rightarrow D$ (reject)
 - A : F B E C
 - B : F C E A D
 - C : E A F B
 - D: BFE
 - E: DBCFA
 - F : E A D B C
 - (g) $A \rightarrow F$ (accept) $-B \rightarrow F$ (reject)
 - A: FBEC
 - B : C E A D
 - C : E A F B
 - D: BFE
 - E: DBCFA
 - F : E A D C
 - $\text{(h) } B \to C \text{ (accept)}$
 - (i) $F \rightarrow E$ (reject)
 - A: FBEC
 - B : C E A D
 - C : E A F B
 - D: BFE
 - E: DBCA
 - F: A D C
 - $(j) \ F \to A \ (accept)$
- 2. Loop 2
 - (a) $C \rightarrow E E \rightarrow A$ (reject)
 - A : F B C
 - B : C E A D
 - C : E A F B
 - D: BFE
 - E: DBC
 - F : A D C
 - (b) $D \rightarrow B$ nothing
 - (c) $E \rightarrow D$ nothing

- (d) $A \rightarrow F F \rightarrow D$ (reject) and $F \rightarrow C$ (reject)
 - A: FBC
 - B : C E A D
 - C : E A B
 - D : B E
 - E : D B C
 - F : A
- (e) $B \to C$ nothing
- (f) $F \rightarrow A A \rightarrow B$ (reject) and $A \rightarrow C$ (reject)
 - A : F
 - B : C E D
 - C : E B
 - D: BE
 - E : D B C
 - F : A
- 3. Loop 3
 - (a) First circular
 - p_i : B C D C
 - q_i : E B E
 - i. $E \rightarrow C$ (reject)
 - A : F
 - B : C E D
 - C : B
 - D : B E
 - E : D B
 - F : A
 - ii. $B \rightarrow D$ (reject)
 - A : F
 - B : C E
 - C : B
 - D : E
 - E : D B
 - F : A
 - (b) Second circular
 - p_i : B B
 - q_i : E
 - $E \rightarrow B$ (reject)
 - A : F
 - B : C
 - $C \quad : \quad B$
 - D : E
 - $E \quad : \quad D$
 - F : A
 - (c) No more circulars
- 4. Final stable pairs: $\{(A,F), (B,C), (D,E)\}$