COMS22202: 2015/16

Language Engineering

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Question 1

The semantic function $\mathcal{S}_{\mathbf{ds}}$ is a *total* function as it maps each and every statement S to some state transformer **g**.

The state transformer **g**, however, is a *partial* function as it is undefined on some states (i.e. those which would cause the program to loop infinitely) .

Question 3

Proove there is at most lfp of a function f wrt. partial order $\leq$ on a set X:

Let a and b denote any two lfps of f wrt $\leq$ on X

Recall that

0. $fix(f)=\{x\in X \mid f(x) = x\}$

Since a is an lfp

1. $a\in fix(f)$
2. $a\leq x$ for all $x\in fix(f)$

Since b is an lfp

3. $b\in fix(f)$
4. $b\leq x$ for all $x\in fix(f)$

Since $\leq$ is a partial order (i.e. reflexive, transitive and antisymmetric)

5. $x\leq x$ for all $x\in X$
6. if $x\leq y$ and $y\leq z$ then $x\leq z$ for all $\{x,y,z\}\subseteq X$
7. if $x\leq y$ and $y\leq x$ then $x=y$ for all $\{x,y\}\subseteq X$

Hence

8. $a\leq b$	using 3 to set $x=b$ in 2
9. $b\leq a$	using 1 to set $x=a$ in 4
10. $a=b$	using 8 and 9 to set $x=a$ and $y=b$ in 7

Question 4

Let f = $(\text{square} \circ \text{half} \circ \text{inc}) = \lambda x.((x+1)/2)^2$

Then $\text{fix } f$ = $\{x \mid x = ((x+1)/2)^2\} = \{x \mid 4x = x^2+2x+1\} = \{x \mid x^2-2x+1=0\}$

= $\{2/2+\sqrt{(4-4)}/2 \,,\, 2/2-\sqrt{(4-4)}/2\}$

= $\{1\}$

Thus $\text{lfp } f$ = 1 (wrt any partial order)

Discussion 5(a)

[[while ¬(x=0) do skip]] = FIX F where

$$
\begin{aligned}
F\ g\ s &= \text{cond}\ (\mathcal{B}[[\neg(x=0)]],\ g \circ \mathcal{S}_{ds}[[\text{skip}]],\ \text{id})\ s \\
&= \text{cond}\ (\mathcal{B}[[\neg(x=0)]],\ g \circ \text{id},\ \text{id})\ s \\
&= \text{cond}\ (\mathcal{B}[[\neg(x=0)]],\ g,\ \text{id})\ s \\
&= \begin{cases} g\ s & \text{if } \mathcal{B}[[\neg(x=0)]]\ s = \text{tt} \\ \text{id}\ s & \text{otherwise} \end{cases} \\
&= \begin{cases} g\ s & \text{if } \mathcal{B}[[x=0]]\ s = \text{ff} \\ s & \text{otherwise} \end{cases} \\
&= \begin{cases} g\ s & \text{if } \mathcal{A}[[x]]\ s \neq \mathcal{A}[[0]]\ s \\ s & \text{otherwise} \end{cases} \\
&= \begin{cases} g\ s & \text{if } s\ x \neq \mathcal{N}[[0]] \\ s & \text{otherwise} \end{cases} \\
&= \begin{cases} g\ s & \text{if } s\ x \neq 0 \\ s & \text{otherwise} \end{cases}
\end{aligned}
$$

Discussion 5(a)

$$\mathbf{fix\ F} = \left\{ g \in \mathbf{State} \hookrightarrow \mathbf{State} \;\middle|\; g\ \mathbf{s} = \begin{cases} g\ \mathbf{s} & \mathbf{if\ s\ x \neq 0} \\ \mathbf{s} & \mathbf{if\ s\ x = 0} \end{cases} \quad \mathbf{for\ all\ s \in State} \right\}$$

$$= \left\{ g \in \mathbf{State} \hookrightarrow \mathbf{State} \;\middle|\; g\ \mathbf{s} = \mathbf{s} \quad \mathbf{if\ s\ x = 0} \quad \mathbf{for\ all\ s \in State} \right\}$$

(as g **s** = g **s** is trivially satisfied in case that x is non-zero)

Each of the following is a fixpoint of $\mathbf{F_1}$

g = id

g s = s if s x=0 and s_0 otherwise (where s_0 maps every variable to 0)

g s = s if s x=0 and <u>undef</u> otherwise

Only the last of these is least with respect to definedness (subset inclusion)