General information

Problems are graded with stars according to difficulty:

⋆ These problems just ask if you have understood a definition or concept. If yes, you should be able to give a short answer (often just one or two lines) without too much thought.

⋆⋆ Problems that require some more thought and/or a more detailed answer, for example a proof.

⋆⋆⋆ Challenging problems that require a solid understanding of the ideas involved, combination of concepts from different parts of the course and/or more involved proofs.
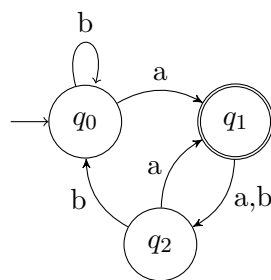
1 Set Theory refresher (⋆)

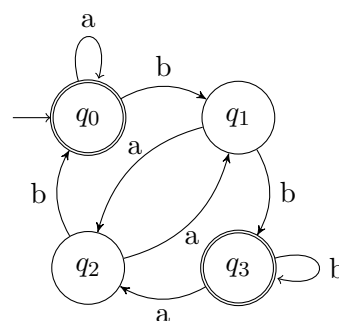Let $A = \emptyset$, $B = \{a, b\}$ and $C = \{1, 2, 3\}$. What are:

1. $A \cup B$

2. $\mathcal{P}(A)$

3. $\mathcal{P}(C)$

4. $B \times C$

5. $A \times C$

6. (⋆⋆) Is it true that for sets X, Y we have $\mathcal{P}(X \cup Y) = \mathcal{P}(X) \cup \mathcal{P}(Y)$ in general? Prove or disprove.

2 DFA state diagrams (⋆)

Answer the following questions for the state diagrams of DFAs M_1 and M_2 below.

M_1 M_2

1. What is the start state?

2. What is the set of accept states?

3. What sequence of states does the machine go through on input `aabb`?

4. Does the machine accept the string `aabb`?

5. Does the machine accept the string ε?

3 Formal descriptions ($\star$)

1. Give the formal descriptions of the machines in the problem above.

2. Machine M has the following formal description:

$$M = (\{q_1, q_2, q_3, q_4, q_5\}, \{\mathtt{u}, \mathtt{d}\}, \delta, q_3, \{q_3\})$$

where δ is given by the following table. Give the state diagram of M.

	u	d
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_2	q_4
q_4	q_3	q_5
q_5	q_4	q_5

3. Give examples of strings that are accepted by M. What is the shortest such string? What is the shortest string that is not accepted by M?

4 Diagrams for simple languages ($\star$)

Give a DFA (state diagram) for each of the following languages over the alphabet $\Sigma = \{a, b\}$.

1. $\{w \mid w$ has an even length$\}$

2. $\{w \mid w$ has at exactly two '`a`'s$\}$

3. $\{w \mid w$ has at least three '`a`'s$\}$

4. $\{w \mid w$ has an even number of '`a`'s$\}$

5. $\{w \mid w$ has an odd number of '`b`'s$\}$