COMS10003 Work Sheet 11

Probability I

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Introduction

This worksheet is about Probability. Much of it you may have met before. But, beware - being blasé about probability is likely to lead to disappointment. So concentration and care is needed at all times!

Some of the questions have been taken, or adapted, from my bookshelf books:

Probability, Random Variables and Stochastic Processes by A.Papoulis, McGraw-Hill
Linear Algebra and Probability for Computer Science Applications by E.Davis, CRC Press
The Mathematics of Games by J.D Beasley, 2006.

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1. Warm up questions. Compute the probabilities of the following :
   (a) Rolling 3 or greater with a fair dice
   (b) Rolling an even number with a fair dice
   (c) Getting at least two heads when flipping a fair coin three times
   (d) Getting at least two heads when flipping a coin three times but the coin is biased so that heads are 3 times more likely than tails.
   (e) Obtaining a total score of at least 5 when three fair dice are thrown.
   (f) Not obtaining a double when two fair dice are thrown.
   (g) Obtaining three doubles when two fair dice are thrown three times
   (h) Obtaining a number less than 10 when multiplying the numbers obtained when two fair dice are thrown.

2. The Monty Hall problem, as discussed in the lecture, is as follows. In a game show, there are 3 doors. Behind two of the doors is a goat and behind one door is a car. The contestant, who wishes to win the car, is asked to select a door. The show host - Monty Hall - who knows which door hides the car - then opens one of the other doors to reveal a goat. The contestant is then asked whether they wish to stick with their original selection or whether they want to switch to the other unopened door. Convince yourselves that the contestant has twice as much chance of winning the car if they decide to switch their selection.

3. Balls in a bag. A bag contains four red balls and nine black balls. If we pick a ball from the bag (without looking!) and replace it before picking another, it is called 'with replacement'. Compute the probabilities of the following

   (a) Picking a red ball followed by a black ball, with replacement
   (b) As above but without replacement
   (c) Picking exactly one red ball if three balls are picked, with replacement
   (d) As above but without replacement
   (e) Picking a red ball second when picking two balls, with replacement
   (f) As above but without replacement

4. You are given 100 marbles, 50 white and 50 black, and two identical bowls. You can place the marbles in the bowls in any way you wish. You are then blindfolded and the bowls are moved around so you don't know which is which. You are then asked to pick a marble from a bowl. How would you distribute the marbles between the bowls to maximise your chances of picking a white marble? What is the probability? Oh, by the way, the marbles are mixed up in the bowls before you pick one and you're not allowed feel how many marbles are in each bowl!

5. In the card game bridge, four players play in teams of two. Each player is dealt a hand of thirteen cards. If a player has a hand containing only four spades, what is the probability that her partner also has only four spades? NB a pack of cards has 52 cards, 4 suits and 13 cards in each suit.

6. Distributions of suits in a hand, i.e. 4-3-3-3 is four of one suit and 3 each of the other suits. Which is more likely, 4-3-3-3 or 4-4-3-2?

7. Consider a sample space with 6 outcomes $\Omega = \{a, b, c, d, e, f\}$. The outcomes have the following probabilities:

$$P(a) = 0.05 \quad P(b) = 0.1 \quad P(c) = 0.05 \quad P(d) = 0.1 \quad P(e) = 0.3 \quad P(f) = 0.4$$

   Consider the events $X = \{a, b, c\}$, $Y = \{b, d, e\}$ and $Z = \{b, e, f\}$. Compute the following probabilities:

   (a) $P(X \cap Y)$
   (b) $P(X|Z)$
   (c) $P(Y|X \cap Z)$
   (d) $P(X \cap Y|Z)$

8. If the events A and B are independent, show that A' and B are independent and also that A' and B' are independent.

9. Trains X and Y arrive at a station at random between 8am and 8.20am. Train X stops for 4 minutes and train Y for 5 minutes. The trains arrive independently of each other. Determine the probability of the following:

   (a) That train X arrives before train Y;

   (b) That the trains meet at the station;

   (c) Assuming they do meet, that train X arrived before train Y.

   Hint: Consider an outcome to be the arrival times of each train and consider the sample space as a square with sides of length 20 minutes.

10. Finally, as we demonstrated in the lecture, the number of experiments we perform to compute probabilities does matter. Consider the following. A desease D has a mild M and a severe S form. Two treatments are available - A and B. Data is collected about the success of each treatment as follows: 87 people with condition M received treatment A and for 81 it was effective; 270 people with condition M received treatment B and for 234 it was effective; 263 people with condition S received treatment A and for 192 it was effective; 80 people with condition S received treatment B and for 55 it was effective. Compute the probability that each treatment will be effective for each condition and then compute the probability that each treatment will be effective for both forms of the condition. You should notice something odd. What? This is known as Simpson's paradox and the above example is based on a real life example for treating kidney stones - see e.g. http://en.wikipedia.org/wiki/Simpson's_paradox.