

Give a direct characterisation of the denotational semantics of this program:

`while  $\neg(x=1)$  do ( $y:=y*x$ ;  $x:=x-1$ )`

with respect to the standard denotational semantics of **While** statements:

$$S_{ds}[[x := a]]s = s[x \mapsto \mathcal{A}[[a]]s]$$

$$S_{ds}[[\text{skip}]] = \text{id}$$

$$S_{ds}[[S_1; S_2]] = S_{ds}[[S_2]] \circ S_{ds}[[S_1]]$$

$$S_{ds}[[\text{if } b \text{ then } S_1 \text{ else } S_2]] = \text{cond}(\mathcal{B}[[b]], S_{ds}[[S_1]], S_{ds}[[S_2]])$$

$$S_{ds}[[\text{while } b \text{ do } S]] = \text{FIX } F \text{ where } Fg = \text{cond}(\mathcal{B}[[b]], g \circ S_{ds}[[S]], \text{id})$$

where

$$\text{cond}(b, c, d)x = \begin{cases} c(x) & \text{if } b(x) = \text{tt} \\ d(x) & \text{otherwise} \end{cases}$$

along with the usual semantics of Boolean and arithmetic expressions.