COMS21103: Data Structures and Algorithms

Problem Sheet - Week 7

1. **String Matching**

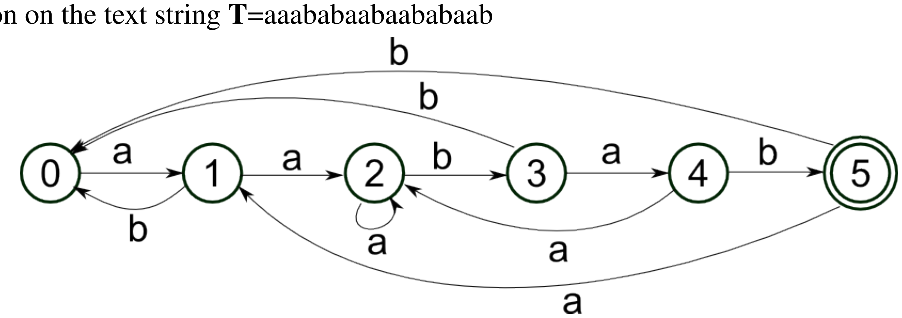
(a) For the Knuth-Morris-Pratt algorithm, compute the prefix function π for the pattern ababbabbabbababbaba

the prefix table is [0, 0, 1, 2, 0, 1, 2, 0, 1, 2, 3, 4, 5, 6, 7, 1]

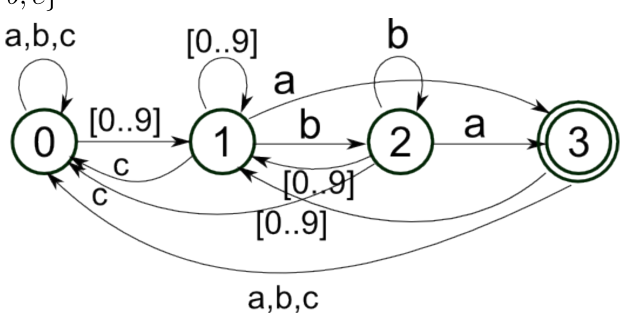
(b) Give a linear-time algorithm to determine whether a text **T** contains a cyclic rotation of another string **P**.
For example:
T = This is the arc de triumph
P = car
Finds the matched cyclic rotation "arc"

Hint: build on top of KMP pseudo code. match to any letter in the pattern. iterate, and cycle back to start of pattern when end of pattern is reached. A match is found when the number of matched letters equals the length of the pattern m. Notice that this doesnt work with repetition within the pattern, how can you solve that?

(c) Construct the string-matching automaton for the pattern **P**=aabab and illustrate its operation on the text string **T**=aaababaabaababaab



(d) Construct the string-matching automaton for the pattern P=$[0..9]^+b^*a$ over the alphabet $\Sigma = \{0..9, a, b, c\}$



2. **Linear Programming**

(a) From the slides (Slide 45), solve the linear program using $SIMPLEX$:

$$\begin{array}{ll} \text{maximise} & 18x_1 + 12.5x_2 \\ \text{subject to} & x_1 + x_2 \leq 20 \\ & x_1 \leq 12 \\ & x_2 \leq 16 \\ & x_1, x_2 \geq 0 \end{array}$$

Slack form:

$$\boxed{\begin{array}{lllllll} z = & & & 18x_1 & + & 12.5x_2 \\ x_3 = & \textit{20} & - & x_1 & - & x_2 \\ x_4 = & \textit{12} & - & x_1 & & \\ x_5 = & \textit{16} & & & - & x_2 \end{array}}$$

Initial Solution $(x_1, x_2, x_3, x_4) = (0, 0, 20, 12, 16)$
Replace x_1 *with* x_4

$$\boxed{\begin{array}{lllllll} z = & & & 18(12 - x_4) & + & 12.5x_2 \\ x_3 = & \textit{20} & - & (12 - x_4) & - & x_2 \\ x_1 = & \textit{12} & - & x_4 & & \\ x_5 = & \textit{16} & & & - & x_2 \end{array}}$$

And Simplify:

$$\boxed{\begin{array}{lllllll} z = & \textit{216} & - & 18x_4 & + & 12.5x_2 \\ x_3 = & \textit{8} & + & x_4 & - & x_2 \\ x_1 = & \textit{12} & - & x_4 & & \\ x_5 = & \textit{16} & & & - & x_2 \end{array}}$$

Current solution $(x_1, x_2, x_3, x_4, x_5) = (12, 0, 8, 0, 16)$
Replace x_2 *with* x_3

$$\boxed{\begin{array}{lllllll} z = & \textit{216} & - & 18x_4 & + & 12.5(8 + x_4 - x_3) \\ x_2 = & \textit{8} & + & x_4 & - & x_3 \\ x_1 = & \textit{12} & - & x_4 & & \\ x_5 = & \textit{16} & & & - & (8 + x_4 - x_3) \end{array}}$$

And Simplify:

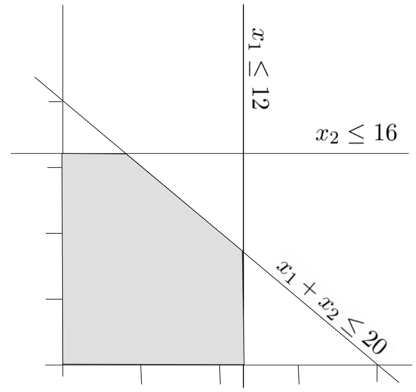
$$\boxed{\begin{array}{lllllll} z = & \textit{316} & - & 5.5x_4 & - & 12.5x_3 \\ x_2 = & \textit{8} & + & x_4 & - & x_3 \\ x_1 = & \textit{12} & - & x_4 & & \\ x_5 = & \textit{8} & - & x_4 & + & x_3 \end{array}}$$

Final solution $(x_1, x_2, x_3, x_4, x_5) = (12, 8, 0, 0, 8)$ *and maximum value of objective function is 316*

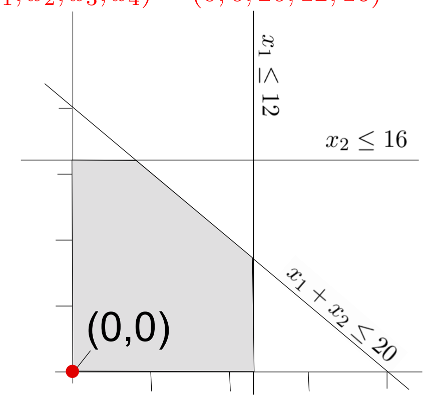
(b) On a 2D graph, show how your solution for the linear program in (a) corresponds to vertices of the convex hull of the space of feasible solutions,

i. For each of the constraints, show the space of feasible solutions
ii. Show the space of feasible solutions of the Linear Program
iii. Is the linear program infeasible? Is it unbounded?
iv. Trace the solution in (a) on the graph

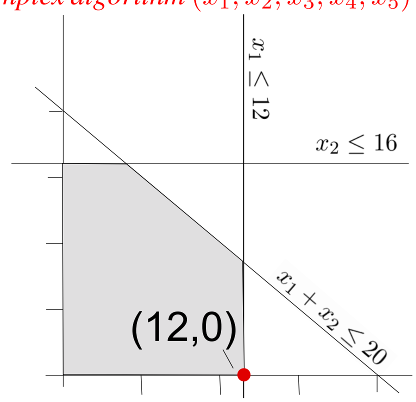
Solution is feasible space (shaded area below)



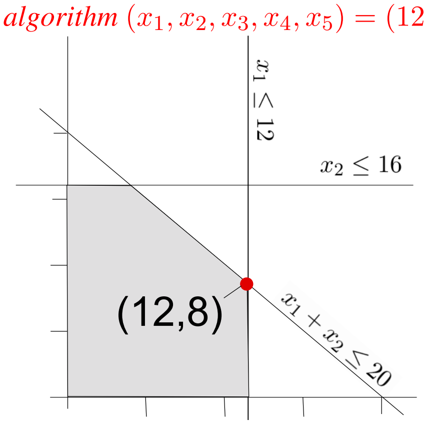
Initial solution is feasible $(x_1, x_2, x_3, x_4) = (0, 0, 20, 12, 16)$



Next vertex/iteration of the Simplex algorithm $(x_1, x_2, x_3, x_4, x_5) = (12, 0, 8, 0, 16)$



Final solution of the simplex algorithm $(x_1, x_2, x_3, x_4, x_5) = (12, 8, 0, 0, 8)$



The figures show the vertices traversed in solving (a). The space of solutions is bounded and the linear program is feasible.

(c) Solve the following linear program using $SIMPLEX$:

$$
\begin{array}{ll}
\text{maximise} & x_1 + 3x_2 \\
\text{subject to} & -x_1 + x_2 \leq -1 \\
& -x_1 - x_2 \leq -3 \\
& -x_1 + 4x_2 \leq 2 \\
& x_1, x_2 \geq 0
\end{array}
$$

Slack form:

$$
\boxed{\begin{array}{lccccc}
z = & & & x_1 & + & 3x_2 \\
x_3 = & \textit{-1} & + & x_1 & - & x_2 \\
x_4 = & \textit{-3} & + & x_1 & + & x_2 \\
x_5 = & \textit{2} & + & x_1 & - & 4x_2
\end{array}}
$$

Initial Solution $(x_1, x_2, x_3, x_4) = (0, 0, -1, -3, 2)$ *is infeasible, Initialise-Simplex is needed*

Form L_{aux}

$$
\boxed{\begin{array}{lcccccccc}
z = & - & x_0 & & & & & & \\
x_3 = & \textit{-1} & + & x_0 & + & x_1 & - & x_2 \\
x_4 = & \textit{-3} & + & x_0 & + & x_1 & + & x_2 \\
x_5 = & \textit{2} & + & x_0 & + & x_1 & - & 4x_2
\end{array}}
$$

Replace x_0 *with* x_4

$$
\boxed{\begin{array}{lcccccccc}
z = & \textit{-3} & - & x_4 & + & x_1 & + & x_2 \\
x_3 = & \textit{2} & + & x_4 & & & - & 2x_2 \\
x_0 = & \textit{3} & + & x_4 & - & x_1 & - & x_2 \\
x_5 = & \textit{5} & + & x_4 & & & - & 5x_2
\end{array}}
$$

Final Solution $(x_0, x_1, x_2, x_3, x_4) = (3, 0, 0, 2, 0, 5)$*, but* x_0 *is basic*

Replace x_0 *with* x_1

$$
\boxed{\begin{array}{lcccccccc}
z = & & - & x_0 & & & & \\
x_3 = & \textit{2} & + & x_4 & & & - & 2x_2 \\
x_1 = & \textit{3} & + & x_4 & - & x_0 & - & x_2 \\
x_5 = & \textit{5} & + & x_4 & & & - & 5x_2
\end{array}}
$$

Current Solution $(x_0, x_1, x_2, x_3, x_4) = (0, 3, 0, 2, 0, 5)$

Reformulate L

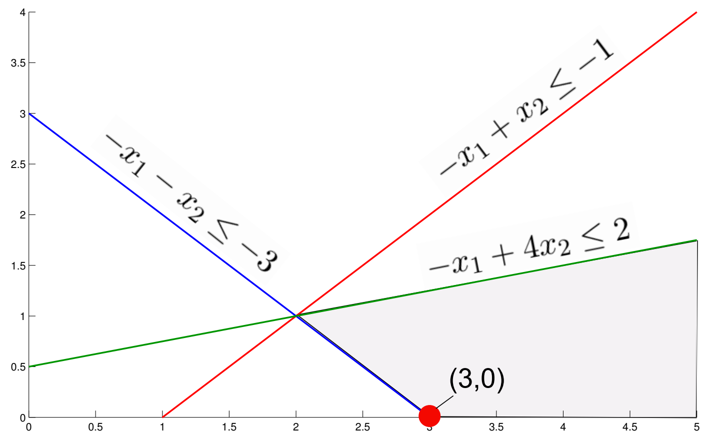
$$
\boxed{\begin{array}{lcccccc}
z = & \textit{3} & + & x_4 & + & 2x_2 \\
x_3 = & \textit{2} & + & x_4 & - & 2x_2 \\
x_1 = & \textit{3} & + & x_4 & - & x_2 \\
x_5 = & \textit{5} & + & x_4 & - & 5x_2
\end{array}}
$$

Current Solution $(x_1, x_2, x_3, x_4) = (3, 0, 2, 0, 5)$

We now know that the linear program is indeed "feasible" because we have a solution that is feasible.

Replace x_4 *with ?? - Notice that we can increase* x_4 *to infinity without breaking any constraint*

Linear program is "unbounded".



$-x_1 - x_2 \leq -3$
$-x_1 + x_2 \leq -1$
$-x_1 + 4x_2 \leq 2$
(3,0)