COMS21202 – Symbols, Patterns and Signals

Problem Sheet: Representations and Transformations

1 – Using *sin(2πnx)*, demonstrate the concept of superposition as follows:

(a) first plot three sine functions over the range ±3 in steps of 0.1 using *n={1/4,1,2}*. Note, plots should appear in the same graph to give a better sense of what is happening.

(b) Now plot in a different colour the sum of all the sines above.

(c) Add more sine functions over the same range and repeat step (b).

Answer (Matlab):

(a) First define the range, say x = [-3:0.1:3]
The sine function plot over the specified range with *n=1/4* is then "plot(sin(2*pi*x*1/4))"
Hold the plot. Now plot again for the other values of *n.*

(b) Add the sines from (a) and plot the new function using 'r' as a parameter of the plot function to draw in red. See *help plot* if unsure of the syntax.

Answer (Python):

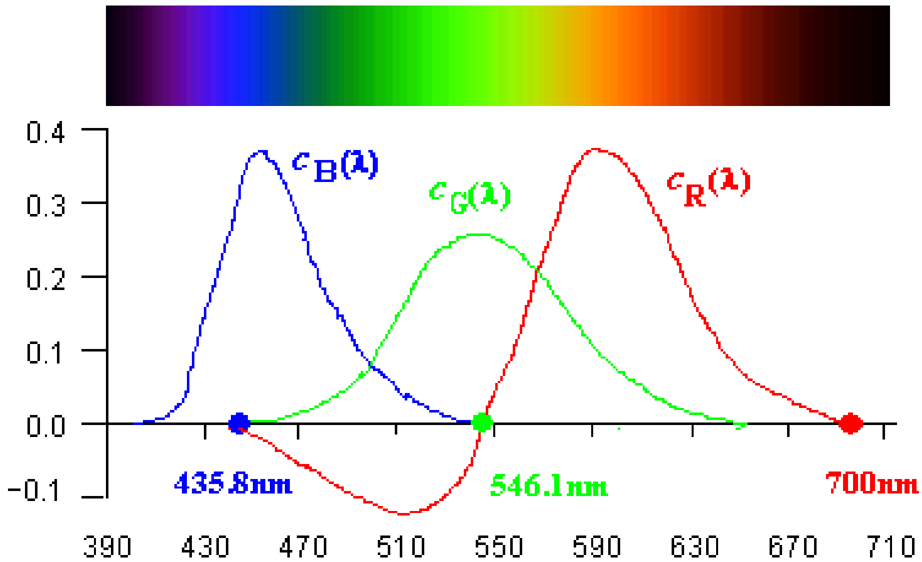
(a) Define the range, say: np.arange(-3, 3.1, 0.1)
The sine function plot over the specified range with n=1/4 is then
"plt.plot(x, np.sin(2*np.pi*x*1/4))"
Now plot again for the other values of n on the same plot by using the same plot object.

(b) Add the sines from (a) and plot the new function using 'r' as a colour parameter of the plot function to draw in red. See help plt.plot if unsure of the syntax.

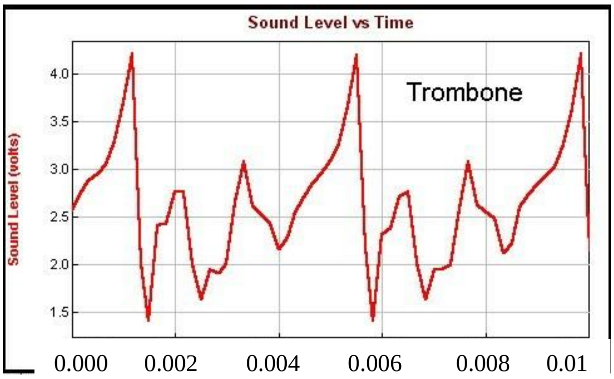
2 – What is White Light? Illustrate your answer with an approximate graph.

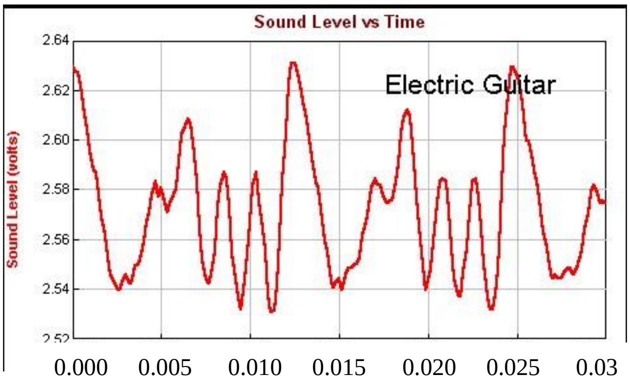
Answer:

White light is made up of a linear combination of variable wavelengths of each component colour (i.e. R,G, and B). Many (but not all) other colours can be induced by some linear combination of these three components, e.g. in digital terms, to get the colour *turquoise* you might mix 0.25*R+0.88*G+0.79*B. 'By superimposing all of them in equal amounts we get a spectral profile with energy distributed more or less uniformly over the whole visible spectrum, so it is perceived as white light.' Source: *http://www.mathpages.com/home/kmath578/kmath578.htm*



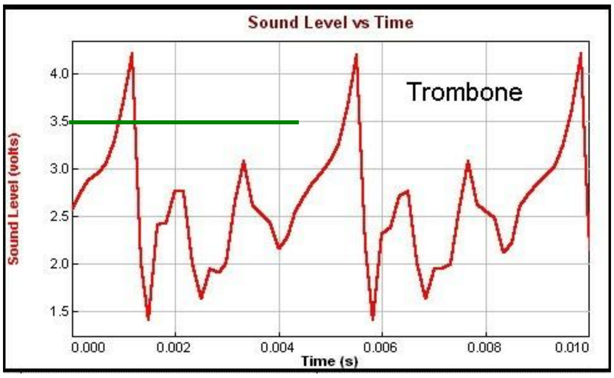
3 – The graphs below display the amplitude of the sound wave for a Trombone and an Electric Guitar as a function of time. The y-axis is the amplitude axis and the x-axis is the time axis. Notice that each one is plotted over a different length of time.

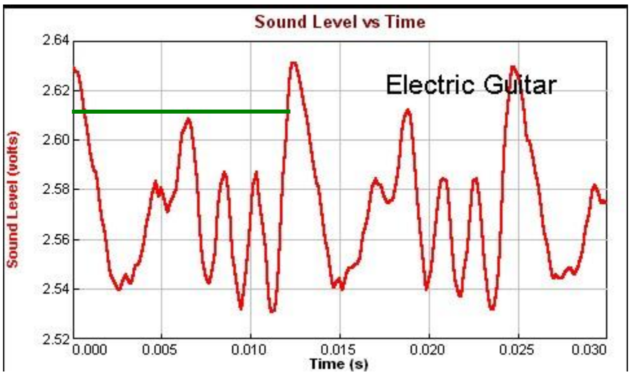




(a) Mark the period of the signal for each instrument.
(b) Approximately, how many periods are shown in these graphs for each instrument?
(c) Approximately, what is the peak amplitude in each case?
(d) Approximately, what is the frequency given the signal period in each case?
(e) Which signal contains higher frequency information? Why?

Answer:





(a) Marked in Green in the diagram above, about 0.0045 and 0.012 respectively.
(b) In both cases around 2 and a bit.
(c) Trombone: about 4.2 EG: about 2.63
(d) $f = 1/T$ so $1/0.0045 = 222.2$ and $1/0.012 = 83.3$ respectively.
(e) The Trombone as it cycles more frequently than the EG over the same time period.

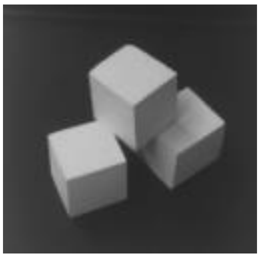
4 – How would low pass filtering be achieved using the Fourier domain? In your answer describe what is meant by Cut-off Frequency.

Answer:

Low pass filtering can be achieved by removing higher frequency information in the Fourier space, i.e. by retaining and letting lower frequencies pass through a filtering operation. Example filters are the ideal low pass filter and the Butterworth low pass filter. Some filter types have an abrupt cut-off point above which no higher frequencies are passed through, while others, like a Butterworth or Gaussian based filters, are more smoothly varying and do not have an abrupt cut-off point.

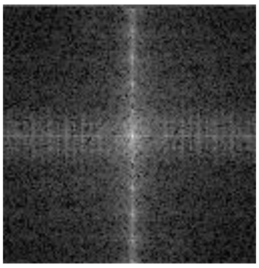
5 – Consider the two images (Sugar and Bricks) on the left. Identify which of the Fourier spaces (FS1 and FS2) on the right belongs to which image and explain clearly why.

Sugar

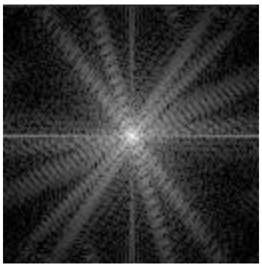


Bricks

FS1



FS2



Answer:

FS2 belongs to the sugar blocks image and FS1 belongs to the brick image. The high magnitude frequencies in FS1 are for the Brick image as they clearly signify the presence of very strong horizontal and vertical lines in that image. The angled lines in the sugar blocks image result in the strong non-horizontal and non-vertical directional lines in FS2.

6 – The following gene sequence contains significant frequencies. Design two different symbolic encodings and in each case apply your encoding to extract some of these frequencies.

ACAGAGATACAGAGATACAG......

Answer:

A=1, G=C=T=0 ➔ *10101010101010101... so period is 2, f=1/2*
A=1,G=2,C=3,T=4 ➔ *12131314121313141213... so period is 8, f=1/8*

7 – Calculate the result of the convolution $A*B$ in each of the examples below by hand.

(i) $A = \begin{pmatrix} 1 & 2 & 1 \end{pmatrix}$ $\quad B = \begin{pmatrix} 2 & 2 & 3 & 3 & 2 \end{pmatrix}$

(ii) $A = \begin{pmatrix} 1 & 1 & 3 & 1 & 1 \end{pmatrix}$ $\quad B = \begin{pmatrix} 3 & 3 & -2 & -1 & 0 & 1 & 2 & 3 & 3 \end{pmatrix}$

(iii) $A = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix}$ $\quad B = \begin{pmatrix} 0 & 5 & 5 & 5 & 0 \\ 0 & 5 & 10 & 5 & 0 \\ 0 & 10 & 10 & 10 & 0 \\ 0 & 5 & 10 & 5 & 0 \\ 0 & 5 & 5 & 5 & 0 \end{pmatrix}$

Now verify your result using the *conv* family of functions in Matlab. Use *help conv* to determine what convention Matlab uses when convolving at the border points.

Answer:

H is the result by hand using the convention seen in the lecture. ***M*** is the result using Matlab and the convention used by *conv* (which does not normalise and leaves it to the user).

(i) *Without normalisation*

$H = \begin{pmatrix} 9 & 11 & 11 \end{pmatrix}$ $\qquad M = \begin{pmatrix} 2 & 6 & 9 & 11 & 11 & 7 & 2 \end{pmatrix}$

With normalisation

$H = \frac{1}{4}\begin{pmatrix} 9 & 11 & 11 \end{pmatrix}$ $\qquad M = \begin{pmatrix} 2 & 6 & 9 & 11 & 11 & 7 & 2 \end{pmatrix}$

$M = M * 1/4$

(ii) Normalisation factor is $\frac{1}{7}$

$$H = (-1 \quad -1 \quad 0 \quad 7 \quad 13) \qquad M = (3 \quad 6 \quad 10 \quad 9 \quad -1 \quad -1 \quad 0 \quad 7 \quad 13 \quad 14 \quad 11 \quad 5 \quad 2)$$

(iii) Normalisation factor is $\frac{1}{8}$

$$H = \begin{pmatrix} -35 & 0 & 35 \\ -40 & 0 & 40 \\ -35 & 0 & 35 \end{pmatrix} \qquad M = \begin{pmatrix} 0 & -5 & -5 & 0 & 5 & 5 & 0 \\ 0 & -15 & -20 & 0 & 20 & 15 & 0 \\ 0 & -25 & -35 & 0 & 35 & 25 & 0 \\ 0 & -30 & -40 & 0 & 40 & 30 & 0 \\ 0 & -25 & -35 & 0 & 35 & 25 & 0 \\ 0 & -15 & -20 & 0 & 20 & 15 & 0 \\ 0 & -5 & -5 & 0 & 5 & 5 & 0 \end{pmatrix}$$

8 – Convolution in the spatial domain is equivalent to multiplication in the frequency domain, i.e. $f*g = FG$ (see Convolution lecture).

(a) Write a Matlab program that demonstrates $f*g = FG$ using any NxN image f and a $5x5$ averaging filter g consisting of all 1s. (Hint: you will need to pad g with zeros to make it the same size as the image before using fft2 in Matlab).
(b) Use Matlab's clock command to time how convolution in the spatial domain compares with multiplication in the Fourier domain.

Answer (Matlab):

(a) See file mc.m on the unit www page for the solution. The filter is created and then placed in the top-left part of a zero matrix of the same size as the image. This is the same effect as creating a filter and padding it with zeros till it is the same size as the image. The multiplication is then applied on an element-by-element basis of the two matrices.
(b) Use the *clock* and *etime* Matlab functions. First get the clock value before the main convolution or multiplication command. Then compute the elapsed time using the old clock value and a new clock reading AFTER the operation. Use *help etime* if unsure.

Answer (Python):

(a) See file mc.py on the unit www page for the solution.
(b) Use the *time()* Python function (form *time* library). First get the time value before the main convolution or multiplication command. Then compute the elapsed time using the old clock value and a new clock reading AFTER the operation, e.g.

```
import time
start = time.time()
...
end = time.time()
print(end - start)
```

9 – Determine the coordinates of point $S=(x_s,y_s)$ on the view plane as the perspective projection of point P=(44,75,50) in the world coordinate system. Assume the view plane is parallel to the XY plane and positioned along the Z-axis at the origin. The COP point is at C=(0,0,-25). Give the answer correct to 2 decimal places as well as rounded to integer screen coordinates.

Answer:

Using similar triangles, $x_s / d = x / z \qquad y_s / d = y / z$

$x_s = x.d / z = 44 * 25 / 75 = 14.66$ or 15

$y_s = y.d / z = 75 * 25 / 75 = 25$

10 - a) In Figure 1, the top row shows an original image of a Clown and its Fourier space after an FFT operation. The rest of the Figure shows four pairs of images, labelled (*A*,*B*,*C*,*D*). In each case one image is a filter mask and the other is the result of applying the mask to the Clown's Fourier space. What effect does each filter have on the frequencies in the Fourier space?

Answer:

A is a crosshair filter which removes a narrow range of horizontal and vertical frequencies except for those in the central area of the FS which correspond to the lowest frequencies. B is a ring-like filter that will remove the lowest frequencies at the centre and the high frequency content in the image. C is an elliptical filter that removes much of the high frequencies in the image only preserving some mid range frequencies in the 135 degrees direction. The low frequencies near the centre are also mostly kept too. D is also a cross-hair filter but this time it rejects all frequencies except those in a narrow range in the horizontal and vertical directions, plus a small square range of frequencies at the centre of the FS (i.e. at the lowest frequencies).

b) The four images in Figure 2, labelled (*W*,*X*,*Y*,*Z*) represent, *in an arbitraryorder*, the inverse FFT of the Fourier spaces in (*A*,*B*,*C*,*D*) in Figure 1. Correctly state which inverse FFT image corresponds to which filtered Fourier image, and why.

Answer:

A corresponds to Z because one sees little smoothing in the non-horizontal and non-vertical information in the image, and with many of the low frequency vertical lines preserved in the background. Other vertical and horizontal parts are smoothed and fuzzy. B corresponds to Y for the principle reason that the central FS frequency has been removed and so the IFFT image looks very dark. The image is heavily smoothed as most of the image high frequencies are filtered out. C corresponds to W as one can see heavy removal of the high frequencies but somewhat less so than in Y, and the main low frequencies are also preserved however the general smoothing is not uniform but directional, i.e. the elliptical shape of the filter does give the same uniform smoothing had it been circular. D corresponds to X where one can observe some low frequecy information that is preserved due to the central region of the cross hair filter and many horizontal and vertical lines due to the preservation those frequencies by the shape of the filter.