COMS10003 Work Sheet 20

Linear Algebra: Vector Spaces, Span and Basis

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1. Define the set of vectors which are spanned by the following set of vectors

$$\{(2,-1,0,3),(1,3,-1,2),(1,-1,1,-2)\}$$

2. Determine whether each of the following sets of vectors are dependent or independent. Hence determine the dimension of the space spanned by each set.

   (a) $\{(-1,2,2.4),(1.25,-2.5,-3)\}$
   (b) $\{(1,-1,2),(2,-1,-2)\}$
   (c) $\{(1,0,-1,1),(0,2,-1,1),(-2,4,0,0),(1,-4,1,-1\}$

3. Show that the coordinates of a vector projected onto a subspace are given by the projection of the vector onto each vector of an orthonormal basis for the subspace.

4. Determine the projection of the vector $\mathbf{v}=(2,-1,3,-2)$ onto the subspace spanned by the orthogonal vectors $(1,1,1,0)$, $(0,-1,1,1)$, $(-1,1,0,1)$

5. Extend the following sets of vectors to be orthogonal bases for 2-D and 3-D subspaces of $\mathcal{R}^3$ and $\mathcal{R}^4$, respectively. How many such subspaces are there in each case?

   (a) $\{(1,-1,2)\}$
   (b) $\{(1,1,-1,-1),(2,-1,1,0)\}$

6. For the basis sets and vectors below, determine an orthogonal basis for the subspace spanned by the basis set $\mathcal{B}$ (keeping $\mathbf{v}_1$ in the basis) and hence determine the projection of the vector onto the subspace. Compute the distance between the vector and the subspace and confirm that the error vector between the projection and the vector is orthogonal to the projection.

   (a) Basis set: $\mathcal{B}=\{\mathbf{v}_1=(1,1,1),\mathbf{v}_2=(1,0,1)\}$ and vector $v=(1,1,4)$.
   (b) Basis set: $\mathcal{B}=\{\mathbf{v}_1=(1,1,0,1),\mathbf{v}_2=(2,-1,1,0)\}$ and vector $v=(-1,-1,2,-1)$.

7. *Given two vectors $\mathbf{u}=\frac{1}{\sqrt{3}}(1,-1,1)$ and $\mathbf{v}=\frac{1}{\sqrt{6}}(-1,1,2)$, answer the following:

   (a) Show that the vectors are unit vectors and that they are orthogonal, i.e. that they are orthonormal
   (b) Determine the projections of the vectors $\mathbf{w}=(3,-3,0)$ and $\mathbf{z}=(0,1,3)$ onto the subspace spanned by $\mathbf{u}$ and $\mathbf{v}$.
   (c) Which of the two vectors lies within the subspace? Explain how you arrived at your answer. Use the other vector and its projection to determine a vector which is orthogonal to the subspace. Show that it is orthogonal to the subspace.

   *This question is taken from the 2013-14 exam.