

## Extras 6 - Fibonacci - Part II

Oliver Ray

Week 6

If you find the worksheet easy, or finish quickly, you might like to try these:

1. By observing that the standard Fibonacci series can be naturally continued to the left  $(\dots, -3, 2, -1, 1, 0, 1, 1, 2, 3, \dots)$  write a function `int d(int n)` that returns the  $n$ 'th Fibonacci number for all  $n$  (both positive and negative).
2. Prove (using the matrix characterisation of Fibonacci numbers given last week) that the following holds for all  $n > 1$ :

$$f(n) = \begin{cases} 2f(\frac{n}{2})f(\frac{n}{2} + 1) - f^2(\frac{n}{2}) & \text{if } n \text{ is even} \\ f^2(\frac{n-1}{2}) + f^2(\frac{n-1}{2} + 1) & \text{if } n \text{ is odd} \end{cases}$$

**Hint:** use the fact that  $M^{2m} = M^m M^m$  for any square matrix  $M$ .