1 The Pumping Lemma ($\star\star$)

Prove that the following languages are not regular.

1. $\{\mathtt{0}^n\mathtt{1}^n\mathtt{2}^n \mid n \geq 0\}$
2. $\{www \mid w \in \{\mathtt{a}, \mathtt{b}\}^*\}$
3. $\{\mathtt{a}^{2^n} | n \geq 0\}$ i.e. a string of '`a`'s whose length is a power of two.

For proving that the following languages are not regular, it may be useful to recall that the class of regular languages is closed under union, intersection and complement. What does this mean for proving that a language is *not* regular?

4. $\{\mathtt{0}^n\mathtt{1}^m\mathtt{0}^n \mid m, n \geq 0\}$
5. $\{\mathtt{0}^m\mathtt{1}^n \mid m \neq n\}$
6. $\{w \mid w \in \{\mathtt{0}, \mathtt{1}\}^* \text{ is not a palindrome}\}$ (A palindrome is a word that reads the same forwards and backwards, like `radar`.)
7. $\{wtw \mid w, t \in \{\mathtt{0}, \mathtt{1}\}^+\}$

Here are some pumping-lemma exercises that require a little bit of extra thought to get started.

8. Over $\Sigma = \{\mathtt{1}, \mathtt{\#}\}$, show that

$$\{w \mid w = x_1\mathtt{\#}x_2\mathtt{\#}\cdots\mathtt{\#}x_k \text{ for } k \geq 0, \text{ for all } i : x_i \in \{\mathtt{1}\}^* \text{ and } x_i \neq x_j \text{ for } i \neq j\}$$

   is not a regular language.

9. Over $\Sigma = \{\mathtt{0}, \mathtt{1}, \mathtt{+}, \mathtt{=}\}$, show that

$$ADD = \{x\mathtt{=}y\mathtt{+}z \mid x, y, z \text{ are binary integers and the sum is correct}\}$$

   is not regular.

2 Regular or not? ($\star\star$)

Here are two similar-looking languages.

1. $L_1 = \{\mathtt{1}^k y \mid k \geq 1, y \in \{\mathtt{0}, \mathtt{1}\}^* \text{ and } y \text{ contains at least } k\ \mathtt{1}\text{s}\}$
2. $L_2 = \{\mathtt{1}^k y \mid k \geq 1, y \in \{\mathtt{0}, \mathtt{1}\}^* \text{ and } y \text{ contains at } \textit{most } k\ \mathtt{1}\text{s}\}$

Show that L_1 *is* regular but L_2 is not.

3 When Pumping Fails $(\star\star\star)$

Consider the language

$$L = \{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$$

1. Show that L acts like a regular language in the pumping lemma: give a pumping length p and show that F satisfies the three conditions of the lemma for this value of p.

2. Show that L is not regular. (You have just shown in (1) that you cannot use the pumping lemma directly to do this!)

3. Explain why parts (1) and (2) above do not contradict the pumping lemma.