Worksheet 6 - Fibonacci - Part 2

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Week 6

This worksheet continues using Fibonacci numbers to explore the concepts in the theory part of the course. It asks you to consider a generalised form of the Fibonacci series in which the first two starting values are explicitly specified. You will write and compare two implementations of this series using naive recursion and tail recursion, respectively; and you will do some inductive proofs.

Define the n'th number in the *generalised* Fibonacci series starting with a, b (a, b, $a+b$, $a+2b$, $2a+3b$, $3a+5b$, $5a+8b$, $8a+13b$, $\ldots$) as follows:

$$g(n,a,b) = \begin{cases} a & \text{if} \quad n=0 \\ b & \text{if} \quad n=1 \\ g(n-1,a,b)+g(n-2,a,b) & \text{if} \quad n \geq 2 \end{cases}$$

1. Translate the above definition directly into a recursive function with the prototype `int g(int n, int a, int b)` that returns the n'th number in the series starting a, b.

2. Prove (by induction) that $g(n, 0, 1) = f(n)$ for all $n \geq 0$ where $f(n)$ is the standard definition of Fibonacci numbers given last week.

3. Explain why the function `g` is *not* tail recursive.

4. Write a tail recursive function `int h(int n, int a, int b)` that gives the n'th number in the series starting a, b. **Hint:** use the fact the n'th number in the series starting a, b is the $n-1$'th number in the series starting b, $a+b$, etc.