COMS10003 MMCS : Linear Algebra

Introduction, vectors and the dot product

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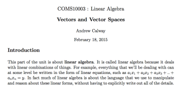
Reminder : Resources

Your notes from lectures

$2x+1=1$

$x=0$

My notes – unit web page



WWW – huge amount of resources online

Lecture slides – unit web page

Workshop problem sheets

Textbooks – many in the library

Wikipedia : Linear Algebra

Linear algebra is the branch of mathematics concerning vector spaces and **linear** mappings between such spaces. It includes the study of lines, planes, and subspaces, but is also concerned with properties common to all vector spaces.

Umm .... ? ?

Linear Algebra

Deals with systems of linear equations, e.g.

$$y_i = a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n$$

Linear algebra concerns the language and representations that we use to manipulate and reason about these linear systems.

Applications

All over Computer Science

Cryptography

Graphics

Machine learning

Data compression

Computer vision

Quantum computing

Search engines

Document analysis

Robotics

Speech recognition

Algorithms

MMCS Linear Algebra - Topics

1. Introduction, vectors and the dot product
2. Vector spaces, span and basis
3. Matrices and linear transformations
4. Linear systems and inverting matrices
5. Eigenvalues and eigenvectors

Vectors and the Dot Product

- Vectors - properties and examples
- Magnitude and direction
- Vector addition
- Multiplying by scalars
- The dot product
- Examples of vectors

Vectors

a - scalar

$\mathbf{u} = (3,2)$ $\quad \mathbf{v} = (2,-1.5,2.1)$

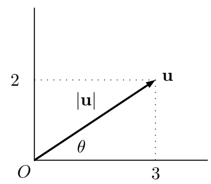
components

$\mathbf{w} = (0.5,-0.5,-0.5,0.5)$

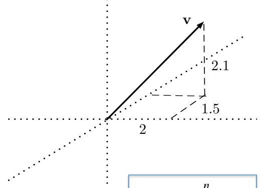
$\mathbf{y} = (y_1, y_2, \ldots, y_n)$ $\quad$ n-dimensional vector

$\mathbf{y} \in \Re^n$ $\longrightarrow$ n-D real coordinate space

Magnitude and Direction



$|\mathbf{v}|^2 = 10.66$

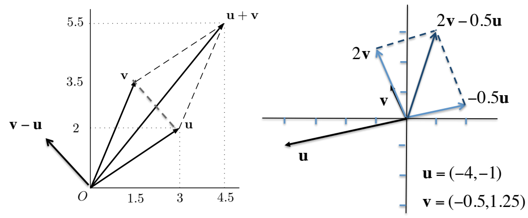


$|\mathbf{u}|^2 = u_1^2 + u_2^2 = 13$

$\theta = \arctan(2/3)$

$|\mathbf{u}|^2 = \sum_{i=1}^{n} u_i^2$

Addition and Scalar Multiplication



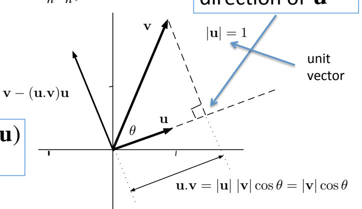
$\mathbf{v} + \mathbf{u} = (v_1 + u_1, v_2 + u_2, \ldots, v_n + u_n)$ $\quad a\mathbf{v} = (av_1, av_2, \ldots, av_n)$

Dot (or Scalar) Product

$\mathbf{u} \,.\, \mathbf{v} = (u_1 v_1 + u_2 v_2 + \ldots + u_n v_n)$

Projection onto direction of $\mathbf{u}$

$\mathbf{u} \,.\, \mathbf{v} = |u||v| \cos\theta$



$\mathbf{v} \,.\, \mathbf{u} = \underset{a}{\arg\min}\, d(\mathbf{v}, a\mathbf{u})$

$\mathbf{u} \,.\, \mathbf{v} = 0$ $\Rightarrow$ **orthogonal vectors**

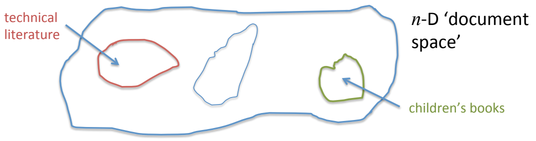
Vectors in Reality

- Classical examples of vectors are 2-D and 3-D physical `things' with magnitude and direction, e.g. force and velocity.
- But they need not be ....
- We can use vectors to represent anything and with any number of dimensions
- Often harder to visualize but really useful
- Enables principles of linear algebra to be used

Documents as Vectors

 $\longrightarrow$ $\mathbf{w} = (w_1, w_2, \ldots, w_n)$

w_i $\longrightarrow$ Number of occurrences of word i



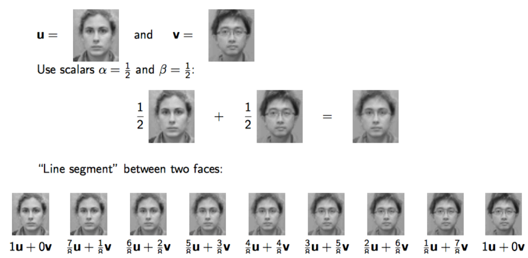
Vectors in Cryptography

- *Plaintext can be represented as a vector*
- *Associate letters with numbers:*

C A L W A Y $\longrightarrow$ $(3,1,12,23,1,25)$

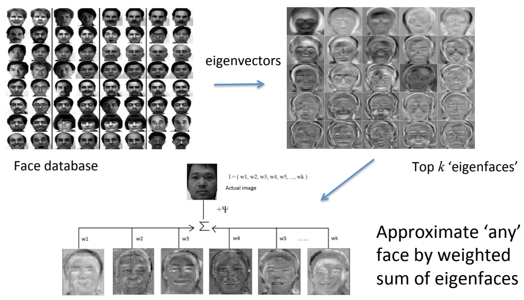
- *Manipulation using linear algebra can then be used to encrypt and decrypt*
- *Vectors defined over finite fields, i.e. not* $\Re^n$

Images as Vectors



Ref: Coding the Matrix – Klein 2013

Eigenfaces



Approximate 'any' face by weighted sum of eigenfaces