Data Structures and Algorithms – COMS21103

2015/2016

Bloom Filters

Benjamin Sach

(based on slides by Ashley Montanaro)

University of BRISTOL

Introduction

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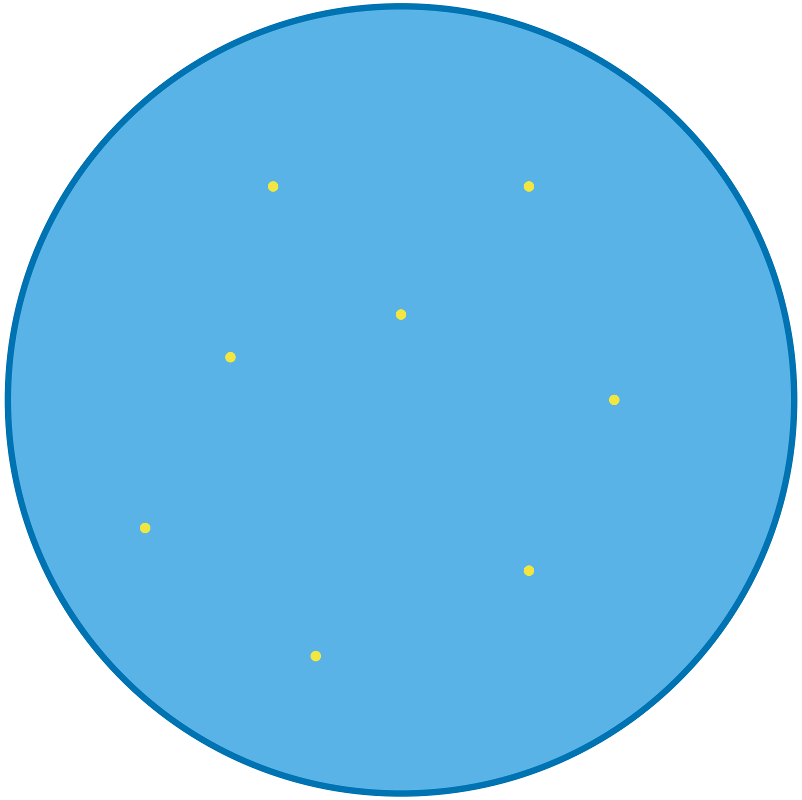

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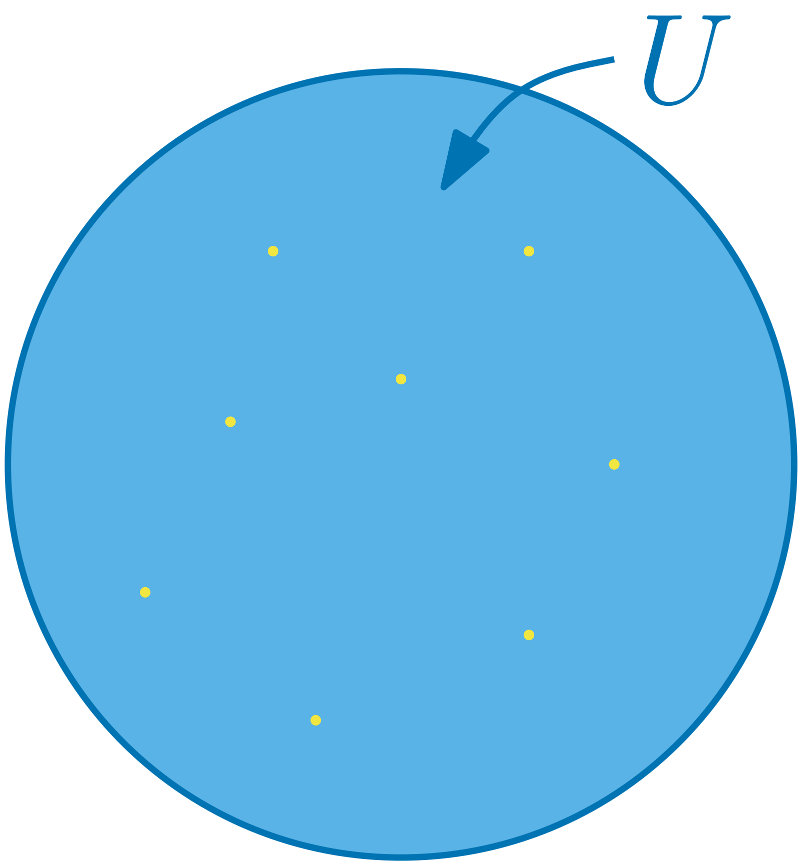

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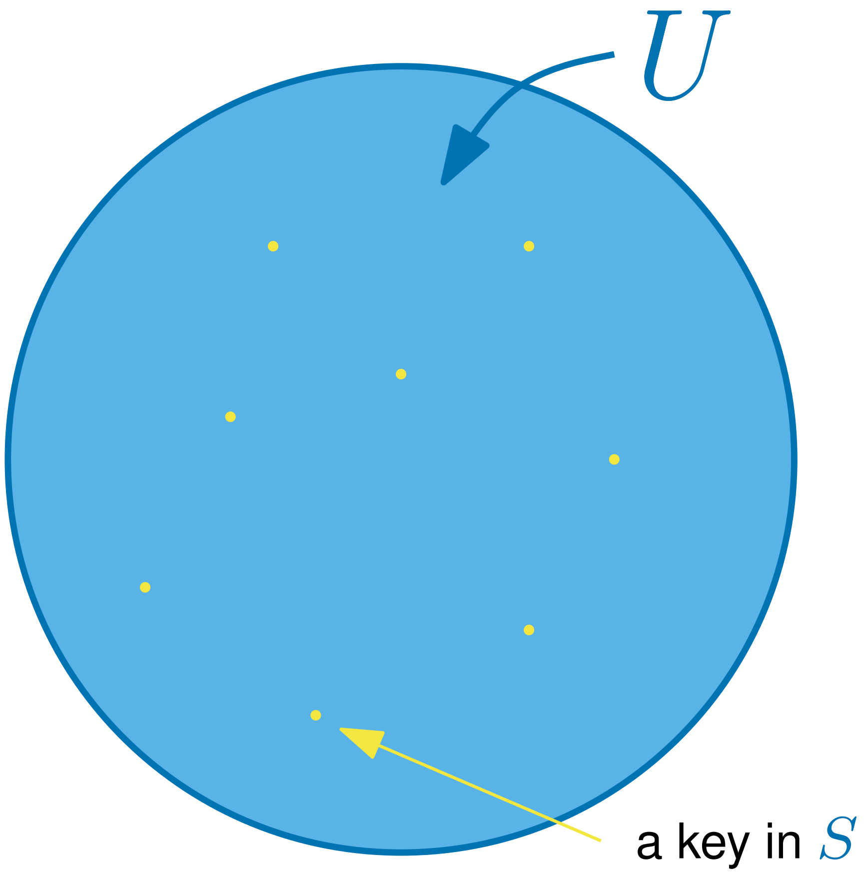

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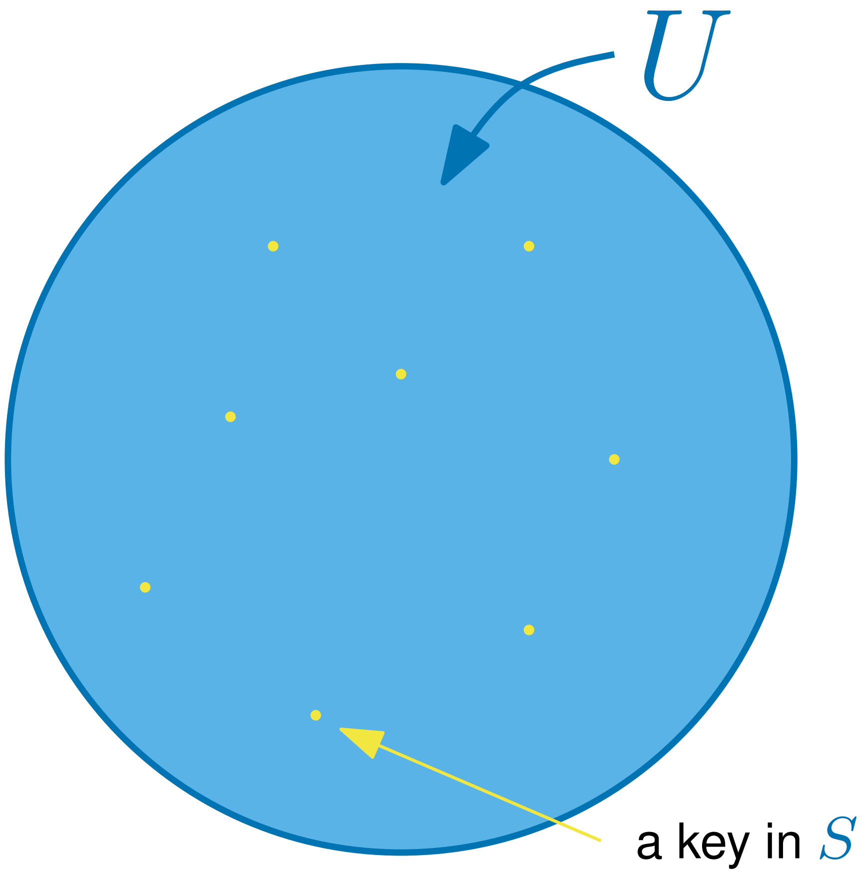

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Important: You cannot ask "which keys are in S?", only "is this key in S?"

Example and Motivation

Imagine you are attempting to build a **blacklist** of unsafe URLs that users should not visit

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we'll come back to this at the end

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It certainly isn't suitable for the application we have seen

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We now maintain a *much shorter* bit string B of some length $m < |U|$

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we will incorrectly output 'yes'

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Important: *h is chosen before any operations happen and never changes*

For every key $k \in U$, the value of $h(k)$ is chosen independently and uniformly at random:

that is, the probability that $h(k) = j$ is $\frac{1}{m}$ for all j between 1 and m

(each position is equally likely)

What is the probability of an error?

Assume we have already INSERTED n keys into the structure

Further, we have just called

MEMBER(k) for some key k **not** in S

(which will check whether $B[h(k)] = 1$)

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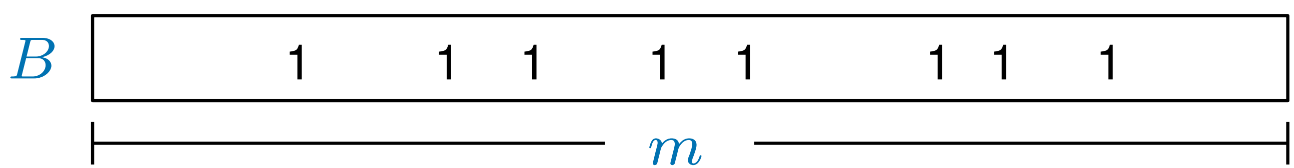

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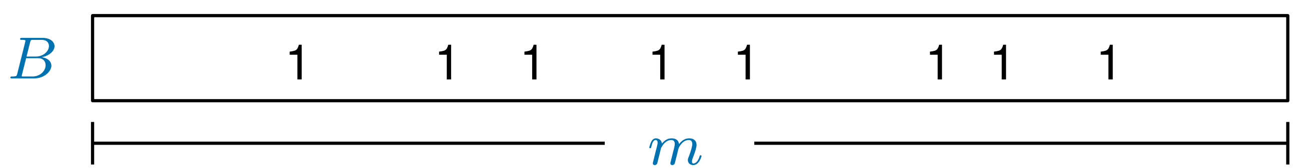

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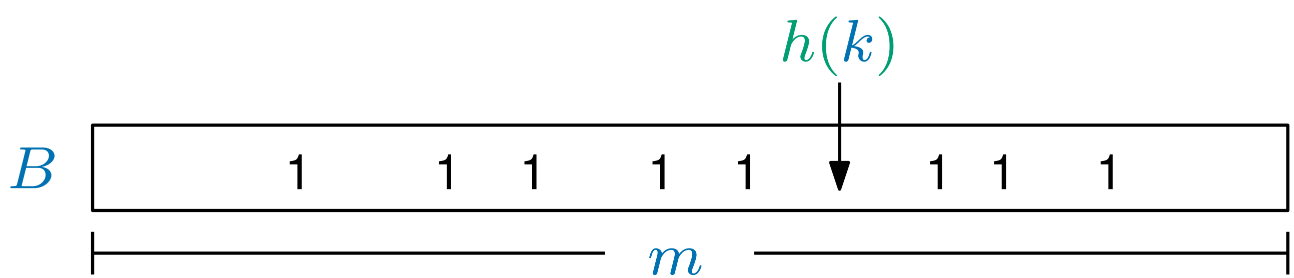

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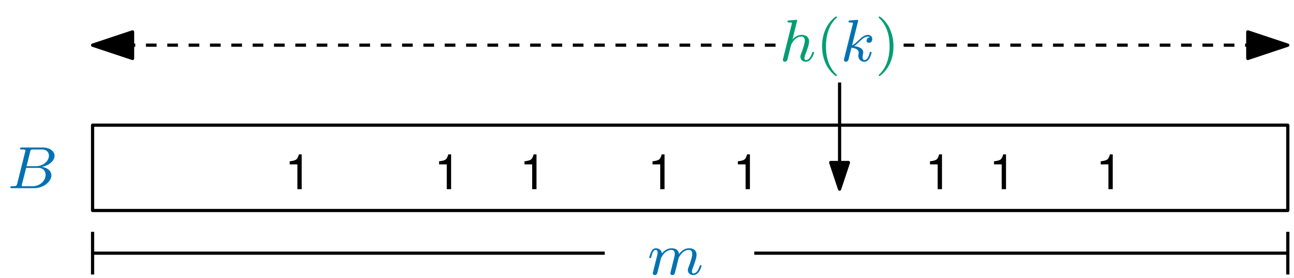

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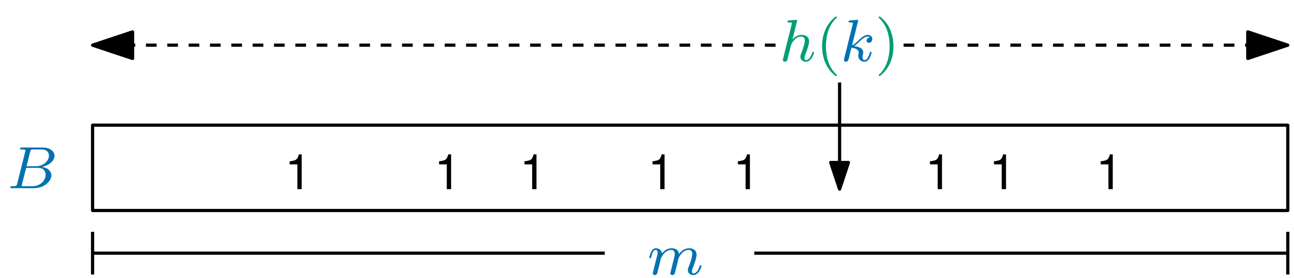

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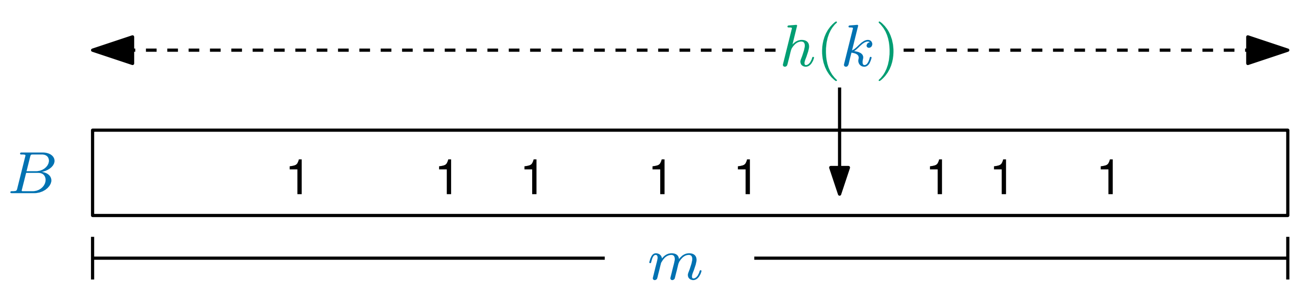

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If we choose $m = 100n$ then we get a failure probability of at most 1%

Approach 2: build a hash table

We have developed a *randomised* data structure for storing a set S
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Why use a Bloom filter then?

we will get *much better* space usage for the same probability

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We still maintain a bit string B of some length $m < |U|$

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but what is the probability of a wrong answer?

What is the probability of an error?

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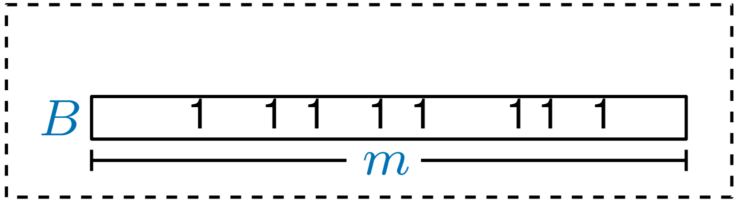

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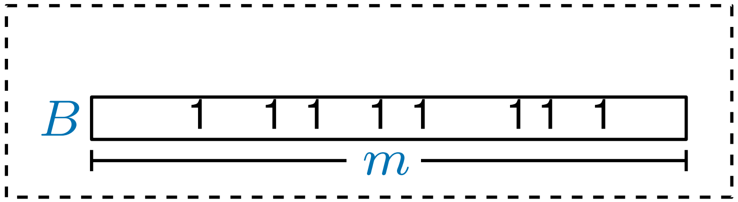

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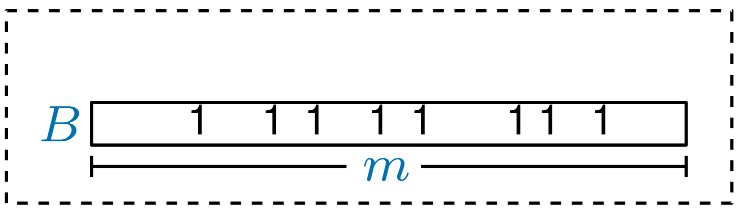

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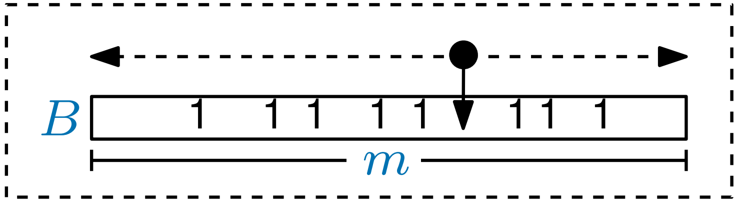

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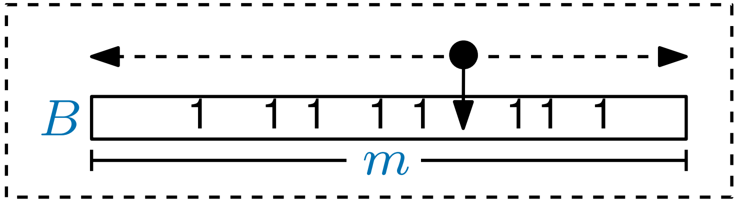
at most nr bits of B are set to 1

(each INSERT *sets at most r bits to 1)*



So the fraction of bits set to 1 is at most $\dfrac{nr}{m}$

so the probability that a randomly chosen bit is 1 is at most $\dfrac{nr}{m}$

so the probability that r randomly chosen bits all equal 1 is at most $\left(\dfrac{nr}{m}\right)^r$

What is the probability of an error?

Assume we have already INSERTED n keys into the bloom filter

Further, we have just called MEMBER(k) for some key k **not** in S

this will check whether $B[h_i(k)] = 1$ for all $j = 1, 2, \ldots r$

This is the same as checking whether r randomly chosen bits of B all equal 1

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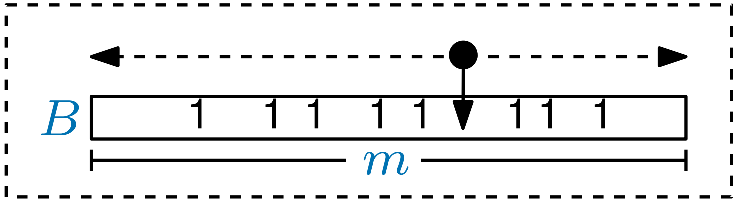

(do this independently r times)

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This is much better than the $100n$ bits we needed with a single hash function

to achieve the same probability

Bloom filter summary

A **Bloom filter** is a *randomised* data structure for storing a set S
which supports two operations, each in $O(1)$ time

The $\textsc{Insert}(k)$ operation inserts the key k from U into S
(it never does this incorrectly)

In a bloom filter, the $\textsc{Member}(k)$ operation
always returns 'yes' if $k \in S$

however, if k is not in S
there is a small chance, ϵ, that it will still say 'yes'

We have seen that if $\epsilon = 0.01$ (1%) the the space used is $m \approx 12.52n$ bits
when storing up to n keys

By impoving the analysis, one can show that only $\approx 1.44 \log_2(1/\epsilon)$ bits are needed
($\approx 9.57n$ bits when $\epsilon = 0.01$)

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One way of doing this for integer keys (see CLRS 11.3.3) is the following:

For each i:

1. Pick a prime number $p > |U|$.
2. Pick random integers $a \in \{1, \ldots, p-1\}$, $b \in \{0, \ldots, p-1\}$.
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Nevertheless, in practice hash functions like this are very effective.

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