

Documentation: Assignment 10

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Q 1 Generate 10 sample paths for the standard Brownian Motion in the time interval $[0, 5]$ using the recursion. $W(0)=0$, Mean=0, Sigma=1.

$$W(t_{i+1}) = W(t_i) + \sqrt{t_{i+1} - t_i} \cdot Z_{i+1}$$

Code: R

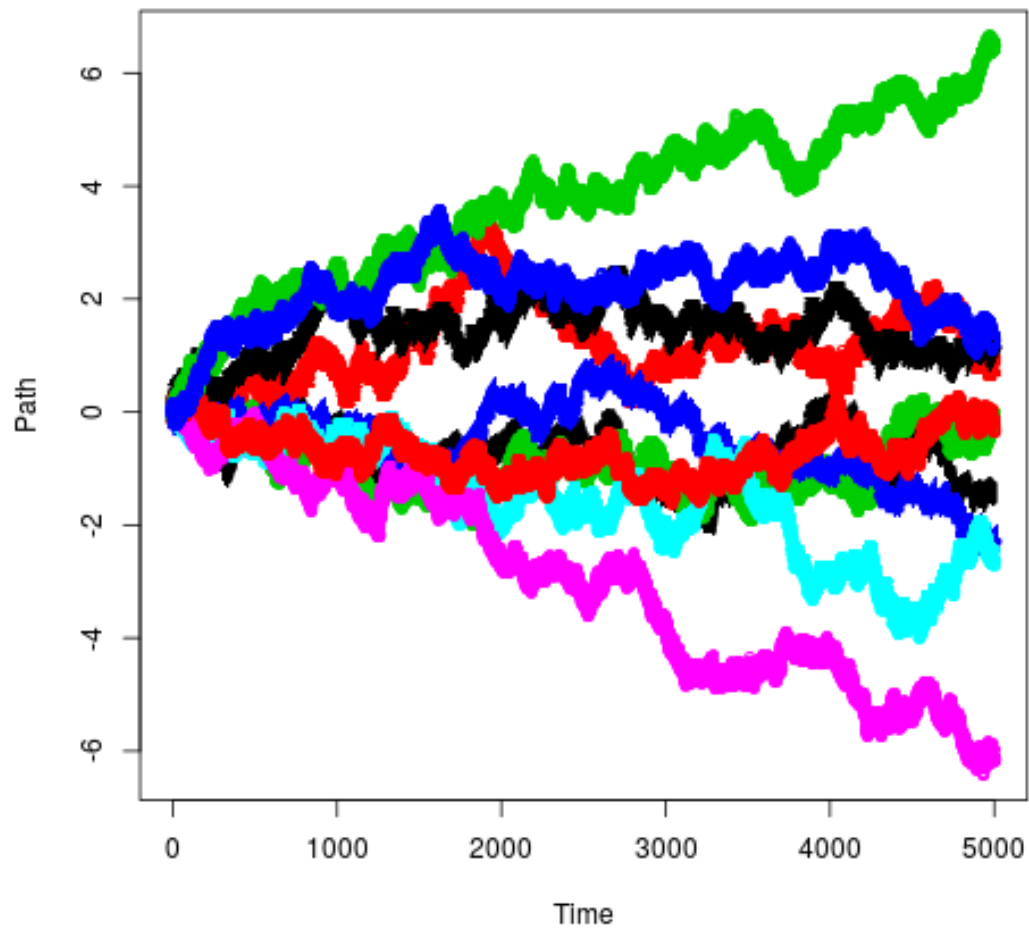
```
1 paths<-10
2 count<-5000
3 interval<-5/count
4 main_sample<-matrix(0,nrow=(count+1), ncol=paths)
5 png("question1.png")
6 for(i in 1:paths){
7   main_sample[1,i]<-0
8   z_sample<-rnorm(count+1)
9   for(j in 2:(count+1)){
10     main_sample[j,i]<-main_sample[j-1,i]+((interval)^.5)*z_sample[j]
11   }
12 }
13 cat("E[W(2)] = ",mean(main_sample[2001,]),"\n")
14 cat("E[W(5)] = ",mean(main_sample[5001,]),"\n")
15 matplot(main_sample,xlab="Time",ylab="Path")
```

Output:

$E[W(2)] = -0.6676345$

$E[W(5)] = -1.040181$

Graph:



(a) X_1

Q 2 Generate 10 sample paths for the standard Brownian Motion in the time interval [0, 5] using the recursion. $X(0)=5$, Mean=0.06, Sigma=0.3.

$$X(t_{i+1}) = X(t_i) + \mu(t_{i+1} - t_i) + \sigma \sqrt{t_{i+1} - t_i} \cdot Z_{i+1}$$

Code: R

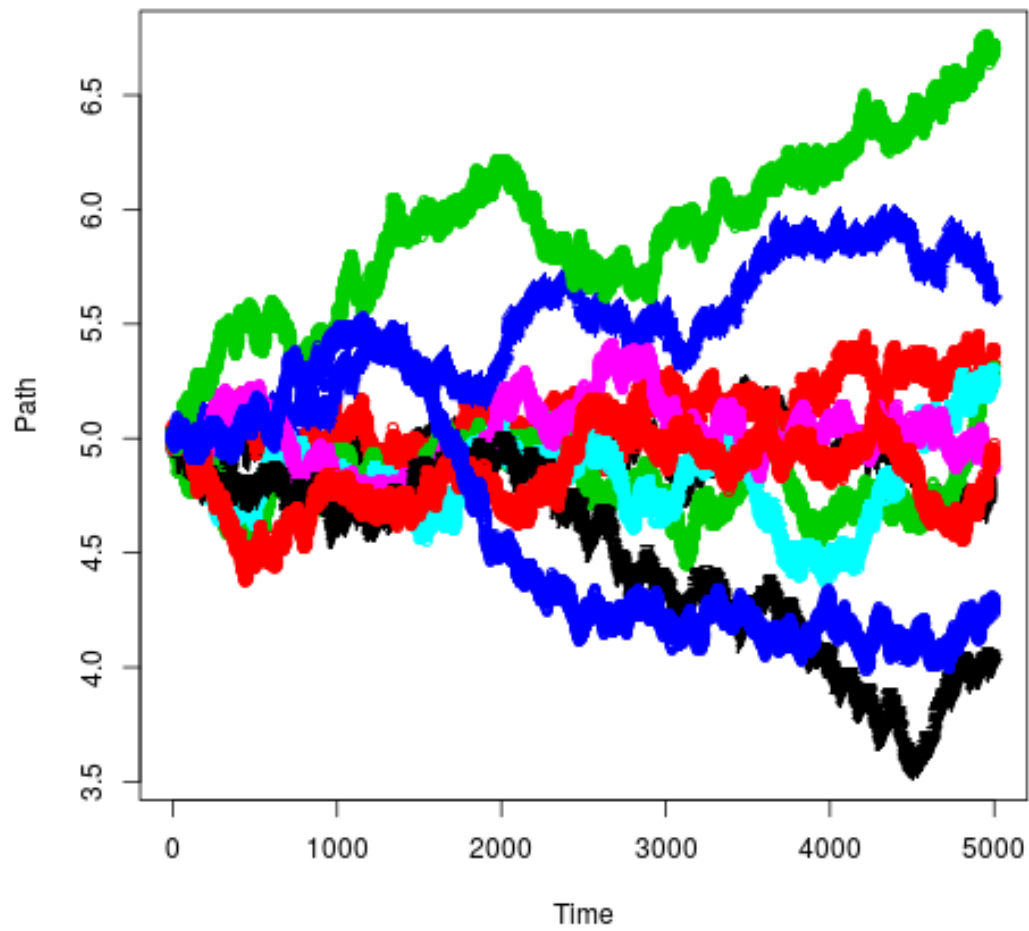
```
1 paths<-10
2 count<-5000
3 interval<-5/count
4 sd<-0.3
5 mean<-0.06
6 main_sample<-matrix(0,nrow=(count+1), ncol=paths)
7 png("question2.png")
8 for(i in 1:paths){
9   main_sample[1,i]<-5
10  z_sample<-rnorm(count+1)
11  for(j in 2:(count+1)){
12    main_sample[j,i]<-main_sample[j-1,i]+mean*(interval)+((interval)^.5)*z_sample[
      j]*sd
13  }
14 }
15 cat("E[W(2)] = ",mean(main_sample[2001,]),"\n")
16 cat("E[W(5)] = ",mean(main_sample[5001,]),"\n")
17 matplot(main_sample,xlab="Time",ylab="Path")
```

Output:

E[W(2)] = 4.725716

E[W(5)] = 4.984472

Graph:



(b) $X1$

Q 3 The Euler approximated recursion with time dependent mean and sigma. $Y(0)=5$.

$$Y(t_{i+1}) = Y(t_i) + \mu(t_i)(t_{i+1} - t_i) + \sigma(t_i)\sqrt{t_{i+1} - t_i} \cdot Z_{i+1}$$

Code: R

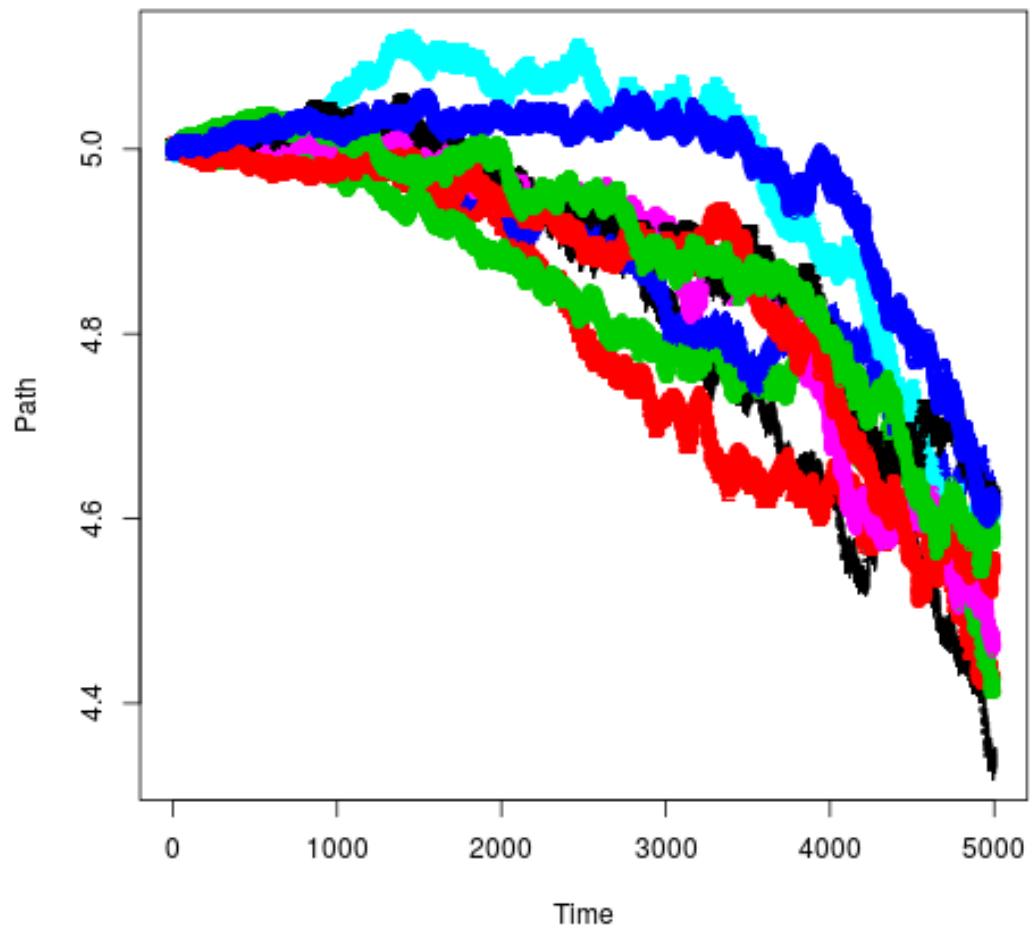
```
1 paths<-10
2 count<-5000
3 interval<-5/count
4 sigma<-function(x){
5   return (0.012+0.0138*x+0.00125*x*x)
6 }
7 mean1<-function(x){
8   return (0.0325-0.05*x)
9 }
10 main_sample<-matrix(0,nrow=(count+1), ncol=paths)
11 png("question3.png")
12 for(i in 1:paths){
13   main_sample[1,i]<-5
14   z_sample<-rnorm(count+1)
15   for(j in 2:(count+1)){
16     main_sample[j,i]<-main_sample[j-1,i]+mean1((j-2)*interval)*interval+((
17       interval)^.5)*z_sample[j]*sigma((j-2)*interval)
18   }
19 }
19 cat("E[W(2)] = ",mean(main_sample[2001,]),"\n")
20 cat("E[W(5)] = ",mean(main_sample[5001,]),"\n")
21 matplot(main_sample,xlab="Time",ylab="Path")
```

Output:

$E[W(2)] = 4.960855$

$E[W(5)] = 4.633403$

Graph:



(c) $X1$