Contents

Documentation- **Newton-Raphson Method**

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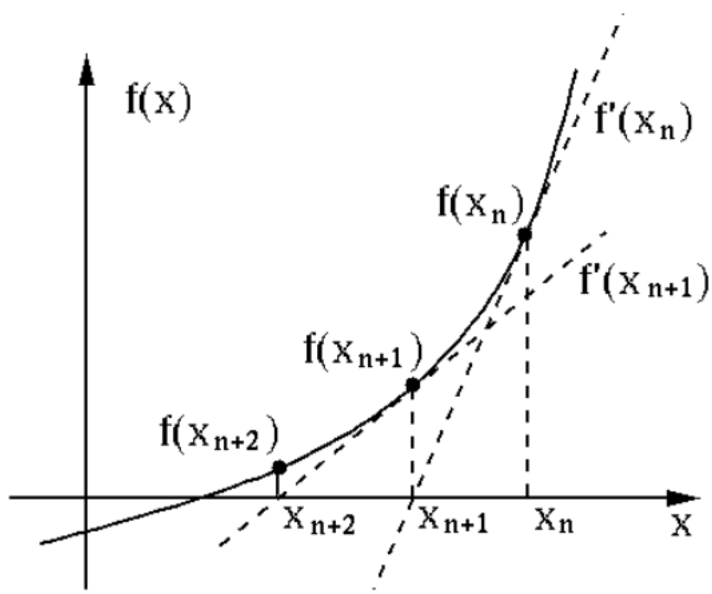
1 Theory

1.1 Short Summary

The Newton-Raphson Method is an iterative process for solving the root of the equation $f(x) = 0$.According to the method, starting with an initial guess of x_0 ,apply the iterative formula

$$x_{n+1} = x_n - f(x_n)/f'(x_n)$$

where f denotes the derivative of the function. The iteration stops until you arrive at an acceptable limit $|x_{n+1} - x_n| < \epsilon$, where ϵ is some pre-specified tolerance value.



1.2 Convergence of Newton-Raphson Method

The tangent line approximation is—*an approximation*. Let's try to get a handle on the error. Imagine a particle travelling in a straight line, and let $f(x)$ be its position at time x. Then $f'(x)$ is the velocity at time x. If the acceleration of the particle were always 0, then the change in position from time x_0 to time $x_0 + h$ would be $hf'(x_0)$. So the position at time $x_0 + h$ would be $f(x_0) + hf'(x_0)$—-note that this is the tangent line approximation, which we can also think of as the zero-acceleration approximation.

If the velocity varies in the time from x_0 to x_0+h, that is, if the acceleration is not 0, then in general the tangent line approximation will not correctly predict the displacement at time $x_0 + h$. And the bigger the acceleration, the bigger the error. It can be shown that if f is twice differentiable then the error in the tangent line approximation is $(1/2)h^2 f''(c)$ for some c between x_0 and x_0+h. In particular, if $|f''(x)|$ is large between x_0 and x_0+h, then the error in the tangent line approximation is large. Thus we can expect large second derivatives to be bad for the Newton Method.

These informal considerations can be turned into positive theorems about the behaviour of the error in the Newton Method. For example, if $|f''(x)/f'(x)|$ is not too large near r, and we start with an x_0 close enough to r, the Newton Method converges very fast to r. (Naturally, the theorem gives "not too large," "close enough," and "very fast" precise meanings.) The study of the behaviour of the Newton Method is part of a large and important area of mathematics called Numerical Analysis.

1.3 Where Newton's method fails

- ***Initial guess is a critical point of f(x) :*** the definition of the Newton iteration function is

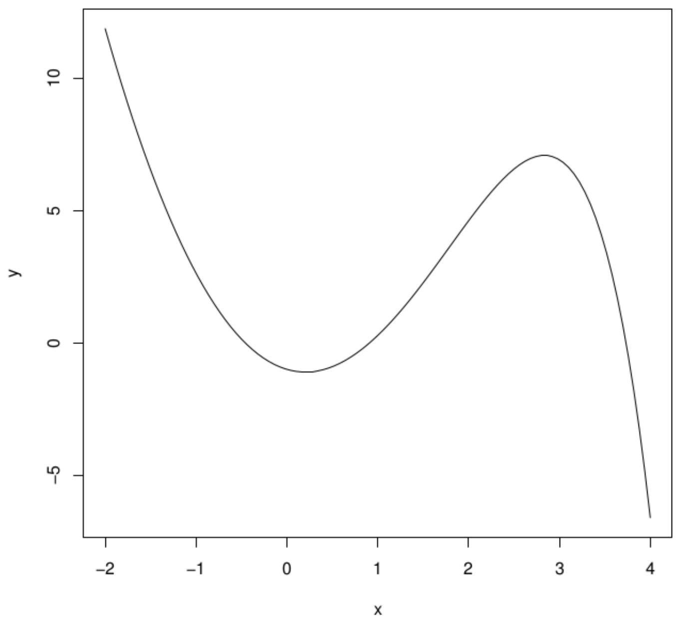
  $$N(x) = x - f(x)/f'(x)$$

  From this definition we see that $N(x)$will not exist if $f'(x) = 0$. If we chose an initial point where $f'(x) = 0$, then Newton's method will fail to converge to a root. Similarly if $f'(x_n) = 0$ for some iteration x_n, then Newton's method will also fail to converge to a root.

- ***No root to find :*** Another way in which Newton's method will fail to converge to the root of a function is if there is no root.

- ***Periodic Cycle :*** A third way in which Newton's method will fail to converge is if the initial guess or an iteration coincides with a cycle (i.e. a set of points occur repeatedly while approximating).

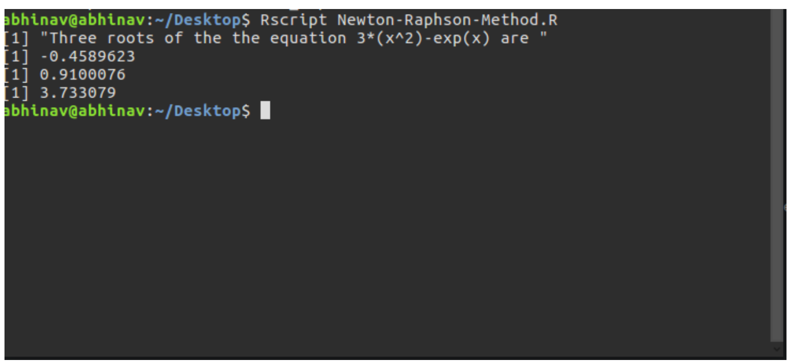
2 Why my answer is reasonable ??



- While choosing the initial point to start the approximation, I considered the graph of the function and chose such a integer point that was close to the root. As a result I considered a pair of consecutive integers for reference between whom the root was graphically existing **(function had opposite signs on those integer points)**.Then I found the point among them on which the function derivative was large **(this was done to rectify slow movement of the approximations towards root)**.As there were 3 roots i choose three pair of such integer points for initialisation.Finally to stop the process at a certain point, tolerance was used and by the help of Newton-raphson method we reached to a approximate root of the equation.

3 Code and Output

```r
"Three roots of the the equation 3*(x^2)-exp(x) are "
# Given Function

f <- function(x){3*(x^2)-exp(x)}
tolerance=1e-5

curve(x^2, from=1, to=50, , xlab="x", ylab="y")

# finding derivative

diff<-function(x){6*x-exp(x)}

# Function to calculate the approximate root of the given equation within the tolerance

newton<-function(f,diff,x0,tolerance){
    x1=0
    while (TRUE) {
    # Applying Newton-Raphson Method to approximate following points
    x1 = (x0 - (f(x0) / diff(x0)))
    # To check that the root is within tolerance
    if (abs(x1 - x0) < tolerance) break
    x0 = x1
    }
    return (x1)
}

# Setting initial point to start approximation. Point with higher derivative is chosen to reach a root quickly. The two integer points
# are chosen as functionchanges it's sign between them.

#3 roots

if(diff(-1)>=diff(0)){
    print(newton(f,diff,-1,tolerance))
}else
    print(newton(f,diff,0,tolerance))

if(diff(0)>=diff(1)){
    print(newton(f,diff,0,tolerance))
}else
    print(newton(f,diff,1,tolerance))

if(diff(3)>=diff(4)){
    print(newton(f,diff,3,tolerance))
}else
    print(newton(f,diff,4,tolerance))
```

```
abhinav@abhinav:~/Desktop$ Rscript Newton-Raphson-Method.R
[1] "Three roots of the the equation 3*(x^2)-exp(x) are "
[1] -0.4589623
[1] 0.9100076
[1] 3.733079
abhinav@abhinav:~/Desktop$
```