Documentation: Assignment 5

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Q 1 Generate 1000 standard normal variates using standard Double-exponential distribution by acceptance-rejection method. Calculate the necessary constant c, where

$$f(x)/g(x) <= c$$

$$f(x) and g(x)$$

are the pdfs of standard normal and standard Double-exponential distribution respectively. Calculate the the oretical and simulated acceptance probability. How do you justify your generated random numbers are correct ? Provide as many verification as you can.

Code: R

```
count<-0
gInv<-function(u){
    if(u<1/2){
        return (log(2*u))
    }
    else{
        return (-1*log(2*(1-u)))
    }
}
c=sqrt(2*exp(1)/pi)
f<-function(x){
    return (sqrt(1/(2*pi))*exp(-1*x*x/2))
}
g<-function(x){
    return ((1/2)*exp(-1*abs(x)))
}
vector<-c()
total<-0
while(1){
u1<-runif(1)
u2<-runif(1)
x<-gInv(u1)
if(u2<=(f(x)/(c*g(x)))) {
cat("Sample Values are: ",x)
cat("\n")
count<-count+1
vector[count]<-x
}
```

```
total<-total+1
if(count==1000)
break
}
cat("\nSimulated Acceptance: ",count/total)
cat("\nTheoretical Acceptance: ",1/c)
cat("\nMean: ",mean(vector))
cat("\nVariance: ",var(vector))
png("question1.png")
hist(vector, breaks=50 , col="light cyan",plot=TRUE)
cat("\n")
```

Output:

Simulated Acceptance: 0.7745933

Theoretical Acceptance: 0.7601735

Mean: 0.04431985

Variance: 0.9399264

Observation:

$$g(x) = 1/2 * e^{(-|x|)} \tag{1}$$

$$f(x) = \sqrt{1/2\pi} * e^{(-x^2/2)} \tag{2}$$

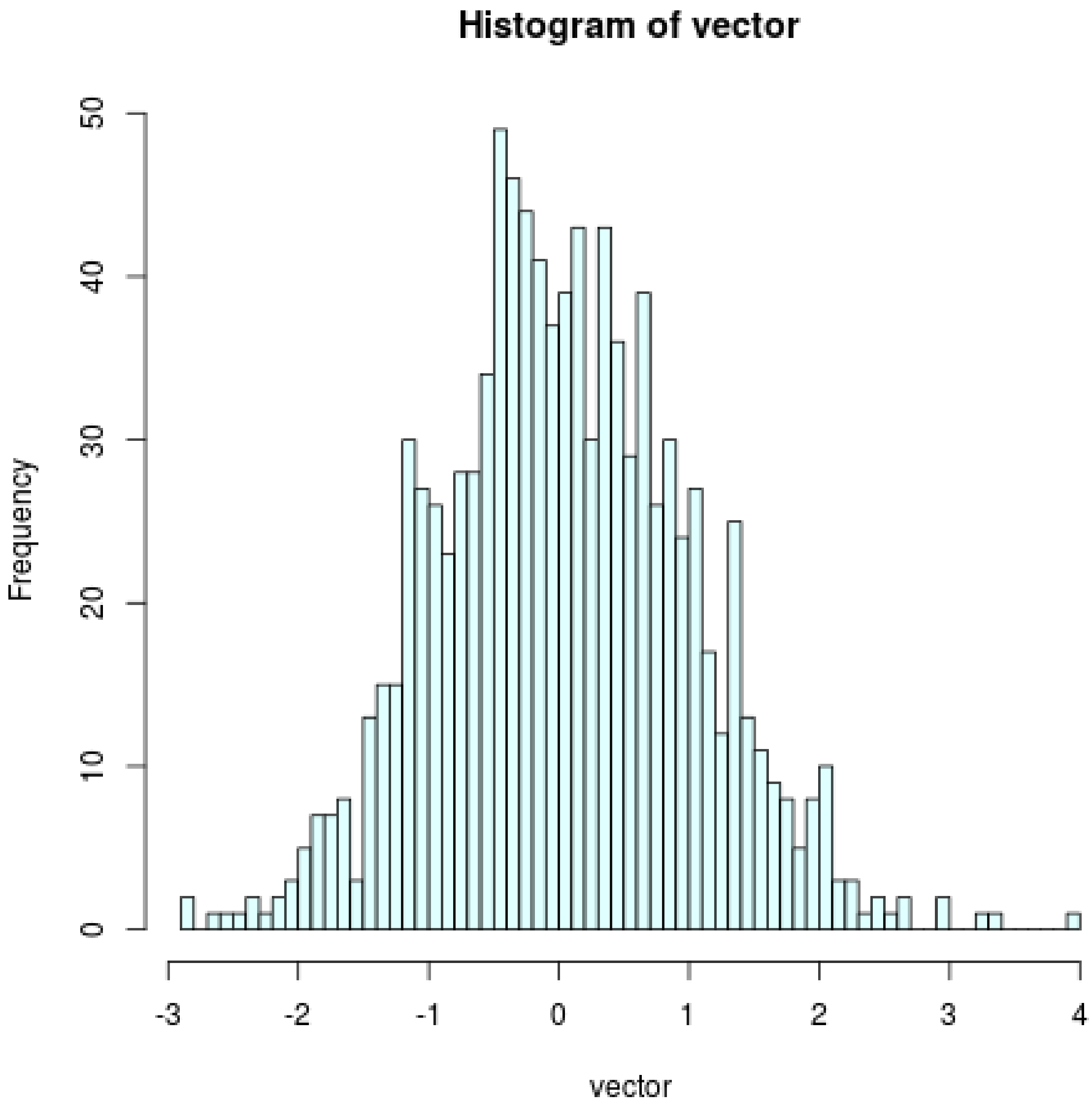
$$f(x)/g(x) = \sqrt{2/\pi} e^{(-x^2/2+|x|)} <= \sqrt{2e/\pi} = constant \tag{3}$$

- Simulated acceptance probability: 0.7745933
- Theoretical acceptance probability: 0.7601735
- **Justification of why my random numbers are correct**: As my mean and variance are closer to the standard normal function's respective attributes and also graph is similar to it. So my output is near to correct distribution.

  Mean: 0.04431985

  Variance: 0.9399264

  Graph:



(a) *using R*

Q 2 Do the same exercise for generating random numbers from half-standard normal distribution using exponential distribution with mean 1 by acceptance-rejection method.

Code: R

```
count<-0
gInv<-function(u){
    return(-1*log(1-u))
}
c=sqrt(2*exp(1)/pi)
f<-function(x){
```

```
    return (2*sqrt(1/(2*pi))*exp(-1*x*x/2))
}
g<-function(x){
    return (exp(-1*x))
}
vector<-c()
total<-0
while(1){
u1<-runif(1)
u2<-runif(1)
x<-gInv(u1)
if(u2<=(f(x)/(c*g(x)))) {
cat("Sample Values are: ",x)
cat("\n")
count<-count+1
vector[count]<-x
}
total<-total+1
if(count==10000)
break
}
cat("\nSimulated Acceptance: ",count/total)
cat("\nTheoretical Acceptance: ",1/c)
cat("\nMean: ",mean(vector))
cat("\nVariance: ",var(vector))
png("question2.png")
hist(vector, breaks=50 , col="light cyan",plot=TRUE)
cat("\n")
```

Output:

Simulated Acceptance: 0.7628347

Theoretical Acceptance: 0.7601735

Mean: 0.8043221

Variance: 0.367382

Observation:

$$g(x) = e^{(-x)} \tag{4}$$

$$f(x) = 2\sqrt{1/2\pi} * e^{(-x^2/2)} \tag{5}$$

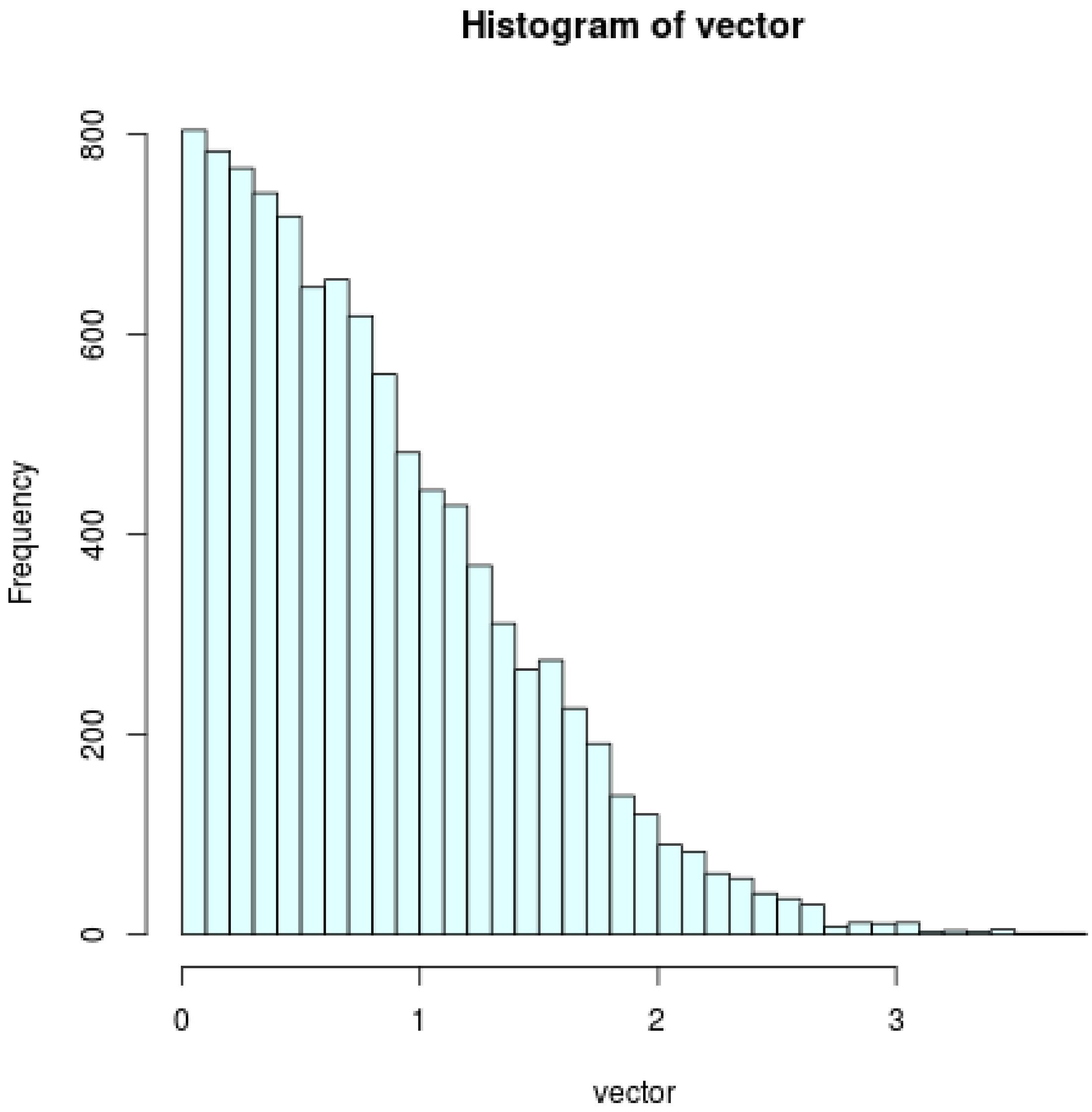
$$f(x)/g(x) <= \sqrt{2e/\pi} = constant \tag{6}$$

- Simulated acceptance probability: 0.7628347
- Theoretical acceptance probability: 0.7601735
- **Justification of why my random numbers are correct**: As my mean and variance are closer to the standard half normal function's respective attributes and also graph is similar to it. So my output is near to correct distribution.

  Mean: 0.8043221

  Variance: 0.367382

  Graph:



(b) *Using R*

Q 3 Consider the following discrete distribution.

a) Generate 10 random numbers from the above probability mas s function using usual procedure (inverse transform) of generating random number from discrete distribution defined on finite number of points. Calculate mean and varianc e of the generated numbers.

b) Generate 10 random numbers from the same probability mass function by acceptance-rejection principle. Calculate mean and variance of the gen erated numbers

Code for part "A": Inverse Transform

```
cdf<-c(0.05,0.30,0.75,0.90,1.0)
values<-c(1,2,3,4,5)
samples<-c()
count<-0
while(count!=10){
    u<-runif(1)
    for(i in 1:5){
        if(u<=cdf[i]){
            count<-count+1
            samples[count]=values[i]
            cat("\nSample Values are: ",samples[count])
            break
        }
    }
}
cat("\nMean: ",mean(samples))
cat("\nVariance: ",var(samples))
cat("\n")
```

Output of Inverse Transform generated from 10 values:

Mean: 3.1

Variance: 1.433333

Code for part "B": Acceptance Rejection

```
pdf<-c(0.05,0.25,0.45,0.15,0.10)
values<-c(1,2,3,4,5)
samples<-c()
N<-5
count<-0
max<-0
gInv<-function(u){
    return(floor(N*u)+1)
}
for(i in 1:N){
    if(max<=(pdf[i]*N)) {
        max<-pdf[i]*N
    }
}
f<-function(x){
```

```
    return (pdf[x])
}
g<-function(){
    return (1/N)
}
while(1){
u1<-runif(1)
u2<-runif(1)
x<-gInv(u1)
if(u2<=(f(x)/(max*g()))) {
cat("Sample Values are: ",x)
cat("\n")
count<-count+1
samples[count]<-x
}
if(count==10)
break
}
cat("\nMean: ",mean(samples))
cat("\nVariance: ",var(samples))
cat("\n")
```

Output of Acceptance Rejection generated from 10 values:

Mean: 3.2

Variance: 0.8444444