Documentation: Assignment 10

Abhinav Gupta – 150123001

6 April 2017

Q 1 Generate 10 sample paths for the standard Brownian Motion in the time interval [0, 5] using the recursion.W(0)=0, Mean=0, Sigma=1.

$$W(t_{i+1}) = W(t_i) + \sqrt{t_{i+1} - t_i}.Z_{i+1}$$

Code: R

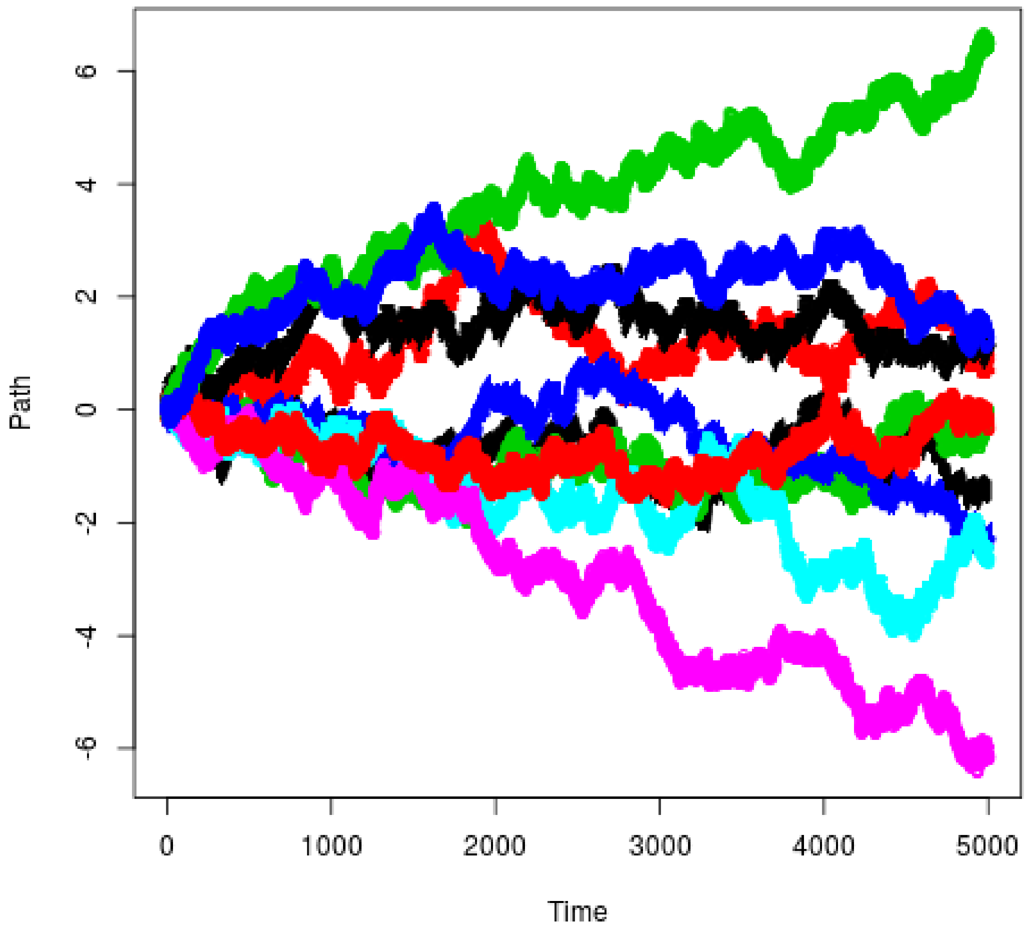
```
paths<-10
count<-5000
interval<-5/count
main_sample<-matrix(0,nrow=(count+1), ncol=paths)
png("question1.png")
for(i in 1:paths){
    main_sample[1,i]<-0
    z_sample<-rnorm(count+1)
    for(j in 2:(count+1)){
        main_sample[j,i]<-main_sample[j-1,i]+((interval)^.5)*z_sample[j]
    }
}
cat("E[W(2)] = ",mean(main_sample[2001,]),"\n")
cat("E[W(5)] = ",mean(main_sample[5001,]),"\n")
matplot(main_sample, xlab="Time", ylab="Path")
```

Output:

E[W(2)] = -0.6676345

E[W(5)] = -1.040181

Graph:



(a) *X1*

Q 2 Generate 10 sample paths for the standard Brownian Motion in the time interval [0, 5] using the recursion. X(0)=5,Mean=0.06, Sigma=0.3.

$$X(t_{i+1}) = X(t_i) + \mu(t_{i+1} - t_i) + \sigma\sqrt{t_{i+1} - t_i}.Z_{i+1}$$

Code: R

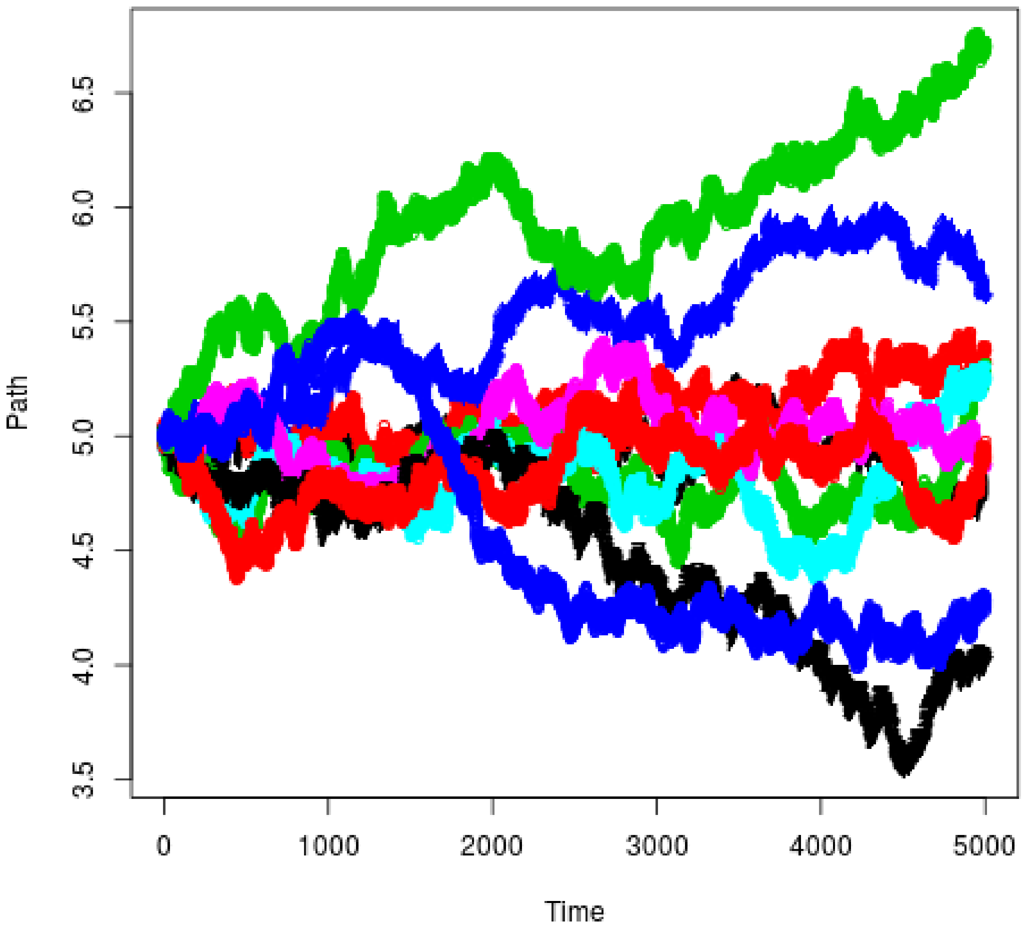
```
paths<-10
count<-5000
interval<-5/count
sd<-0.3
mean<-0.06
main_sample<-matrix(0,nrow=(count+1), ncol=paths)
png("question2.png")
for(i in 1:paths){
    main_sample[1,i]<-5
    z_sample<-rnorm(count+1)
    for(j in 2:(count+1)){
        main_sample[j,i]<-main_sample[j-1,i]+mean*(interval)+((interval)^.5)*z_sample[j]*sd
    }
}
cat("E[W(2)] = ",mean(main_sample[2001,]),"\n")
cat("E[W(5)] = ",mean(main_sample[5001,]),"\n")
matplot(main_sample,xlab="Time",ylab="Path")
```

Output:

E[W(2)] = 4.725716

E[W(5)] = 4.984472

Graph:



(b) *X1*

Q 3 The Euler approximated recursion with time dependent mean and sigma.Y(0)=5.

$$Y(t_{i+1}) = Y(t_i) + \mu(t_i)(t_{i+1} - t_i) + \sigma(t_i)\sqrt{t_{i+1} - t_i}.Z_{i+1}$$

Code: R

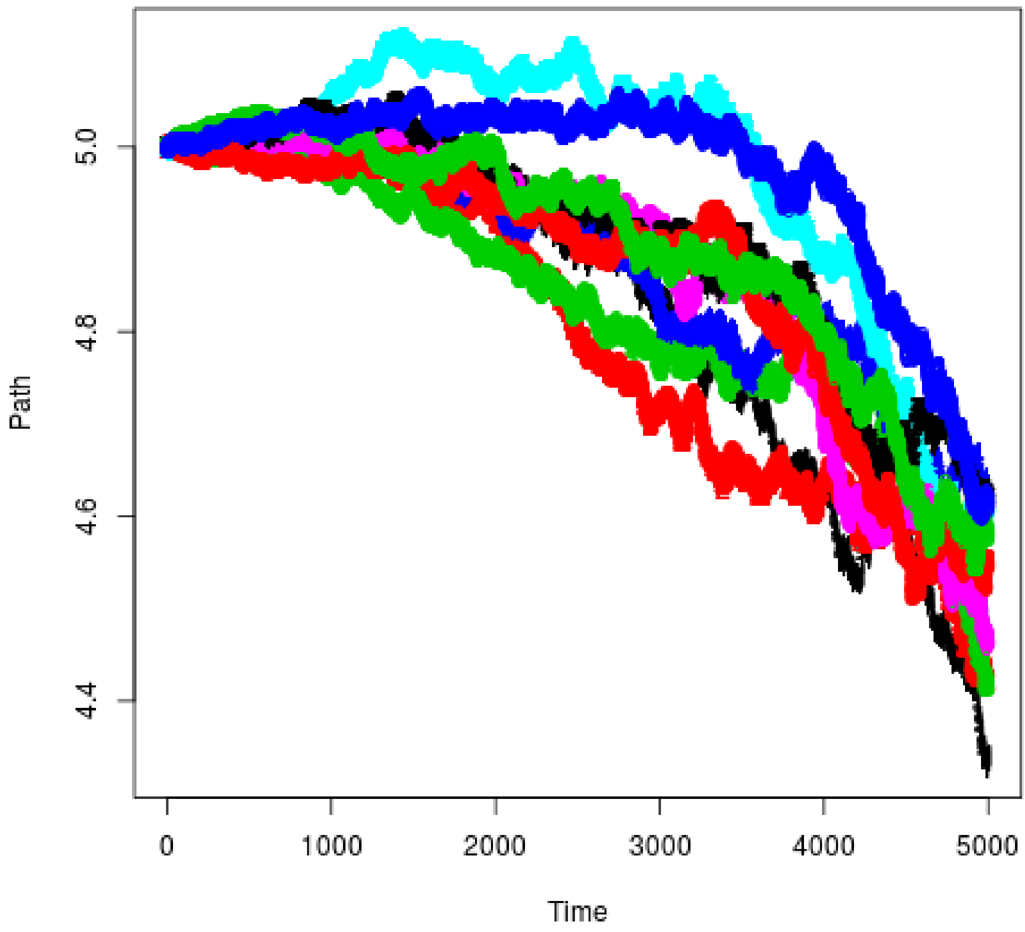
```
paths<-10
count<-5000
interval<-5/count
sigma<-function(x){
    return (0.012+0.0138*x+0.00125*x*x)
}
mean1<-function(x){
    return (0.0325-0.05*x)
}
main_sample<-matrix(0,nrow=(count+1), ncol=paths)
png("question3.png")
for(i in 1:paths){
    main_sample[1,i]<-5
    z_sample<-rnorm(count+1)
    for(j in 2:(count+1)){
        main_sample[j,i]<-main_sample[j-1,i]+mean1((j-2)*(interval))*interval+((
            interval)^.5)*z_sample[j]*sigma((j-2)*interval)
    }
}
cat("E[W(2)] = ",mean(main_sample[2001,]),"\n")
cat("E[W(5)] = ",mean(main_sample[5001,]),"\n")
matplot(main_sample,xlab="Time",ylab="Path")
```

Output:

E[W(2)] = 4.960855

E[W(5)] = 4.633403

Graph:



(c) *X1*