Documentation: Assignment 11

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Q 1 The process $S(t)$ is a GBM with drift parameter μ , volatility parameter σ, and initial value $S(0)$ if

$$S(t) = S(0)exp([\mu - \sigma^2/2]t + \sigma W(t))$$

if where $W(t)$ is a standard BM. As with the case of a BM, we have a simple recursive procedure to simulate a GBM at $0 = t_0 < t_1 < ...t_n$ as

$$S(t_{i+1}) = S(t_i)exp([\mu - \sigma^2/2](t_{i+1} - t_i) + \sigma\sqrt{t_{i+1} - t_i}Z_{i+1})$$

where $Z_1, Z_2, Z_3, ..., Z_n$ are independent $N(0,1)$ variates. In the interval [0,5], taking both positive and negative values for μ and for at least two different values of σ^2, simulate and plot at least 10 sample paths of the GBM (taking sufficiently large number of sample points for each path). Also, by generating a large number of sample paths, compare the actual and simulated distributions of $S(5)$.Calculate expectation and variance of $S(5)$ and match it with the theoretical values.

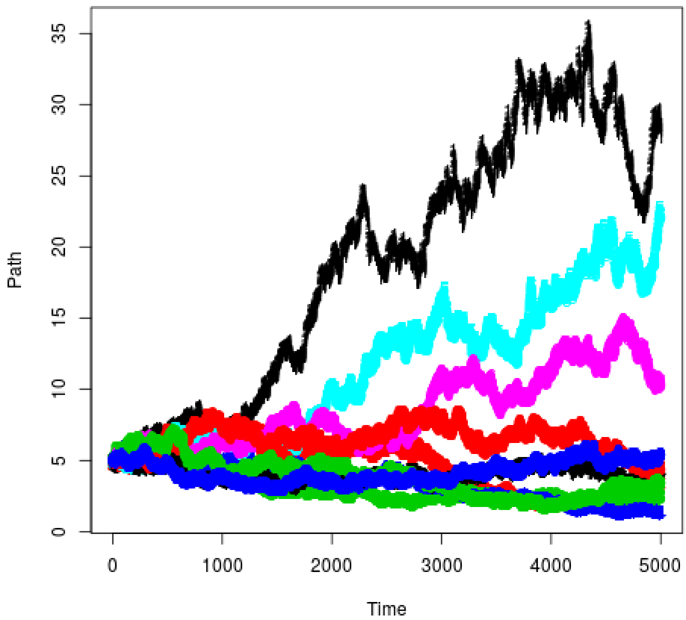
Code without S(5) values and plotting graphs: R

```
paths<-10
count<-5000
interval<-5/count
sd<-c(0.3,0.4)
mean<-c(-0.06,0.06)

for(x in sd){
for(y in mean){
main_sample<-matrix(0,nrow=(count+1), ncol=paths)
for(i in 1:paths){
main_sample[1,i]<-5
for(j in 2:(count+1)){
main_sample[j,i]<-main_sample[j-1,i]*exp((y-(x^2)/2)*interval+x*rnorm(1)*(interval)^.5)
}
}
string<-paste("question1-mean=",toString(y),"variance=",toString(x^2),".png",sep="")
png(string)
matplot(main_sample,xlab="Time",ylab="Path")
}
}
```

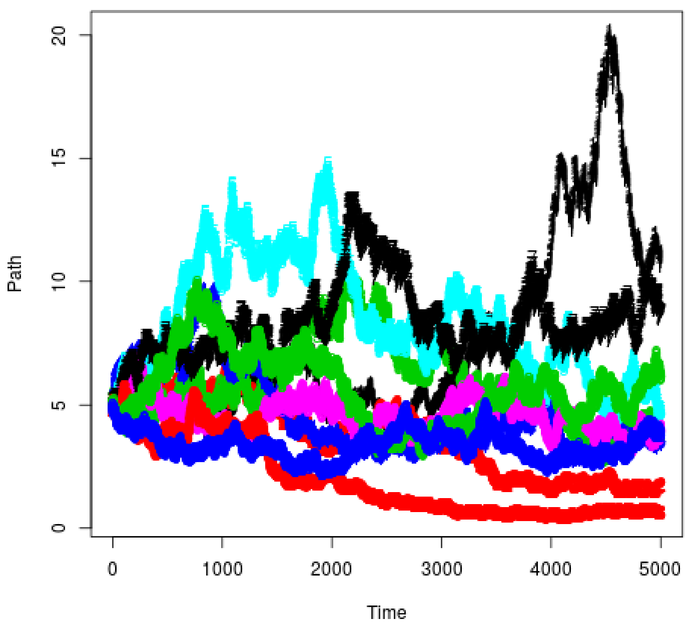
Graph:

Mean= 0.06 and Variance= 0.09



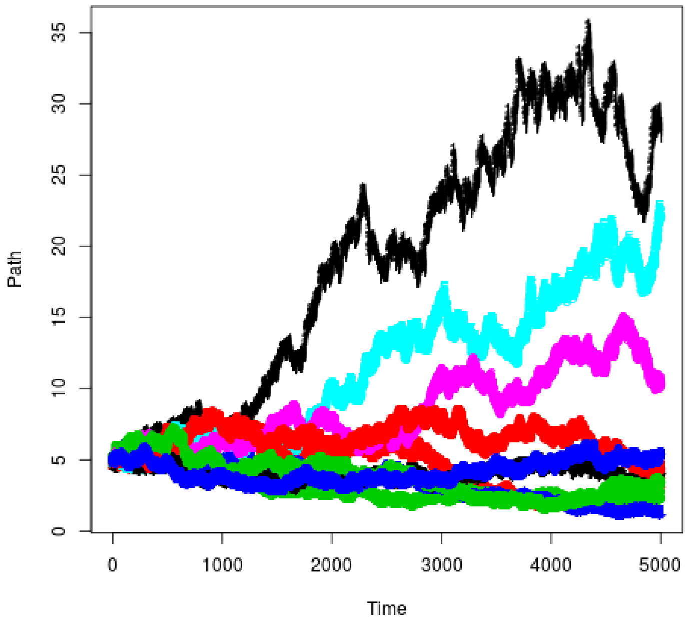
(a) *X1*

Mean= -0.06 and Variance= 0.09



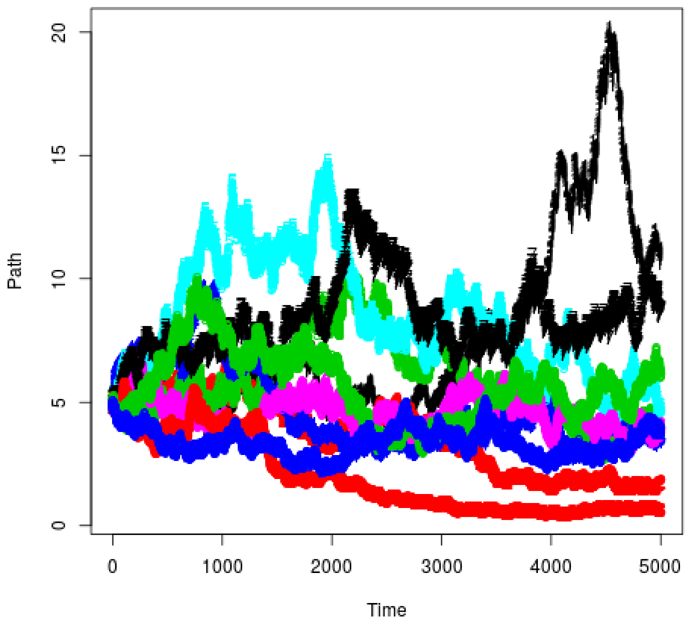
(b) *X1*

Mean= 0.06 and Variance= 0.16



(c) *X1*

Mean= -0.06 and Variance= 0.16



(d) *X1*

Code with expected and observed value of S(5): R

```
paths<-1000
count<-5000
interval<-5/count
sd<-c(0.3,0.4)
mean<-c(-0.06,0.06)
exp_mean<-function(s0,mean){
    return(s0*exp(mean*5))
}
exp_variance<-function(s0,mean,sd){
    return((s0^2)*exp(2*mean*5)*(exp(5*sd^2)-1))
}
for(x in sd){
for(y in mean){
main_sample<-matrix(0,nrow=(count+1), ncol=paths)
for(i in 1:paths){
main_sample[1,i]<-5
for(j in 2:(count+1)){
main_sample[j,i]<-main_sample[j-1,i]*exp((y-(x^2)/2)*interval+x*rnorm(1)*(interval
    )^.5)
}
}
cat("Mean is: ",y,"\n")
cat("Variance is: ",x,"\n")
cat("Theoretical Expectation of S(5): ",exp_mean(5,y),"\n")
cat("Observed Expectation of S(5): ",mean(main_sample[5001,]),"\n")
cat("Theoretical Variance of S(5): ",exp_variance(5,y,x),"\n")
cat("Observed Variance of S(5): ",var(main_sample[5001,]),"\n")
}
cat("\n")
}
```

Output:

Mean is: -0.06

Variance is: 0.3

Theoretical Expectation of S(5): 3.704091

Observed Expectation of S(5): 3.748786

Theoretical Variance of S(5): 7.797409

Observed Variance of S(5): 8.629581

Mean is: 0.06

Variance is: 0.3

Theoretical Expectation of S(5): 6.749294

Observed Expectation of S(5): 6.876775

Theoretical Variance of S(5): 25.88831

Observed Variance of S(5): 27.23867

Mean is: -0.06

Variance is: 0.4

Theoretical Expectation of S(5): 3.704091

Observed Expectation of S(5): 3.419998

Theoretical Variance of S(5): 16.81478

Observed Variance of S(5): 11.90325

Mean is: 0.06

Variance is: 0.4

Theoretical Expectation of S(5): 6.749294

Observed Expectation of S(5): 6.781992

Theoretical Variance of S(5): 55.82703

Observed Variance of S(5): 58.36891