Documentation: Assignment 8

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Q 1 Use the following Monte Carlo estimator to approximate the expected value

$$I = E(exp(\sqrt{U}))$$

Code: R

```
for(i in 2:5){
    sample<-runif(10^i)
    sample<-exp(sample^(0.5))
    mean<-mean(sample)
    variance<-var(sample)
    alpha<-.05
    value<-1-alpha/2
    left<-mean-(variance^.5)*(qnorm(value,0,1))/(10^(i/2))
    right<-mean+(variance^.5)*(qnorm(value,0,1))/(10^(i/2))
    cat("95% confidence interval for M = ",10^i," is (",left,",",right,") .\n")
}
```

Output:

95% confidence interval for M = 100 is (1.922472 , 2.111564) .

95% confidence interval for M = 1000 is (1.983088 , 2.036677) .

95% confidence interval for M = 10000 is (1.988161 , 2.005463) .

95% confidence interval for M = 1e+05 is (1.997964 , 2.00342) .

Q 2 Repeat the above exercise using antithetic variates via the following estimator and calculate the percentage of variance reduction:

Code: R

```
for(i in 2:5){
   sample<-runif(10^i)
   sample1<-runif(10^i)
   sample1<-exp(sample^(0.5))
   sample<-(exp(sample^(0.5))+exp((1-sample)^(0.5)))/2
   mean<-mean(sample)
   mean1<-mean(sample1)
   variance<-var(sample)
   variance1<-var(sample1)
   alpha<-.05
   value<-1-alpha/2
   left<-mean1-(variance1^.5)*(qnorm(value,0,1))/(10^(i/2))
   right<-mean1+(variance1^.5)*(qnorm(value,0,1))/(10^(i/2))
   cat("95% confidence interval for M = ",10^i," is (",left,",",right,") .\n")
   cat("Variance Reduction Percentage: ",(variance1/2-variance)*100/(variance1/2),"\
       n")
}
```

Output:

95% confidence interval for M = 100 is (1.911808 , 2.084074) .

Variance Reduction Percentage: 98.96747

95% confidence interval for M = 1000 is (1.963686 , 2.019227) .

Variance Reduction Percentage: 99.01007

95% confidence interval for M = 10000 is (1.992793 , 2.010024) .

Variance Reduction Percentage: 98.89845

95% confidence interval for M = 1e+05 is (1.996319 , 2.001781) .

Variance Reduction Percentage: 98.88595

Q 3 Use $\sqrt{U}$ to construct control variate estimate and repeat the above exercise. Calculate the percentage of variance reduction

Code: R

```
for(i in 2:5){
   u<-runif(10^i)
   sample<-exp(u^(0.5))
   y<-u^.5
   mean_y<-mean(y)
   variance_y<-var(y)
   mean<-mean(sample)
   variance<-var(sample)
   alpha<-.05
   value<-1-alpha/2
   c<-(-cov(sample,y))/variance_y
   variance_control_variates<-var(sample+c*(y-mean_y))
   left<-mean-(variance^.5)*(qnorm(value,0,1))/(10^(i/2))
   right<-mean+(variance^.5)*(qnorm(value,0,1))/(10^(i/2))
   cat("95% confidence interval for M = ",10^i," is (",left,",",right,") .\n")
   cat("Variance Reduction Percentage: ",(variance - variance_control_variates)*100/
       (variance),"\n")
}
```

Output:

95% confidence interval for M = 100 is (1.897786 , 2.072987) .

Variance Reduction Percentage: 98.67375

95% confidence interval for M = 1000 is (1.952612 , 2.008801) .

Variance Reduction Percentage: 98.61719

95% confidence interval for M = 10000 is (1.98517 , 2.002491) .

Variance Reduction Percentage: 98.62091

95% confidence interval for M = 1e+05 is (1.995684 , 2.001159) .

Variance Reduction Percentage: 98.59625