Astrometry basics and interpretation of astrometric data

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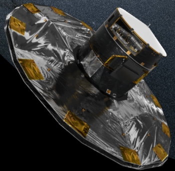


ESA/Gaia/DPAC

Scope of this lecture

- Astrometry developed through vector formulation
- Aimed at use and interpretation of modern astrometric catalogue data
- Topics not treated
  - astrometric measurements and data processing
  - earth orientation (nutation, precession, etc)
  - relativistic light deflection
  - aberration
  - details of coordinate systems and coordinate transformations
  - time scales

Some references

- Gaia online documentation, chapter 3, section 3.1, 3.3.3
- *A Practical Relativistic Model for Microarcsecond Astrometry in Space*: Klioner, 2003, AJ 125, p. 1580
- *The astrometric core solution for the Gaia mission. Overview of models, algorithms, and software implementation*: Lindegren et al., 2012, A&A 538, A78
- *Astrometry for Astrophysics*: Methods, Models, and Applications, 2013, ed. W.F. van Altena, Cambridge University Press (chapters 4, 5, 7)

Astrometry Basics

Vector notation used here

$$\boldsymbol{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \tag{1}$$

$\boldsymbol{a}'$: transpose of vector $\boldsymbol{a}$

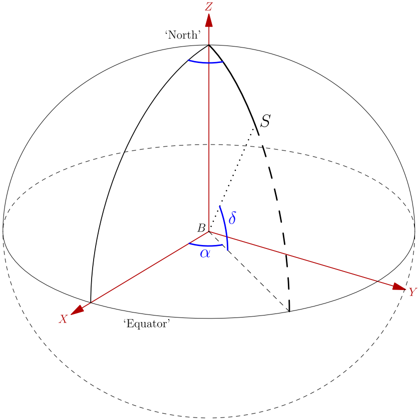
Inner product and cross-product

$$\boldsymbol{a}'\boldsymbol{b} = a_x b_x + a_y b_y + a_z b_z \quad \text{and} \quad \boldsymbol{a} \times \boldsymbol{b} = \begin{bmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{bmatrix} \tag{2}$$

$$\text{normalization: } |\boldsymbol{a}| = \sqrt{\boldsymbol{a}'\boldsymbol{a}} = (a_x^2 + a_y^2 + a_z^2)^{1/2}$$

$$\langle \boldsymbol{a} \rangle = \boldsymbol{a}|\boldsymbol{a}|^{-1} \tag{3}$$

Celestial coordinates and reference systems

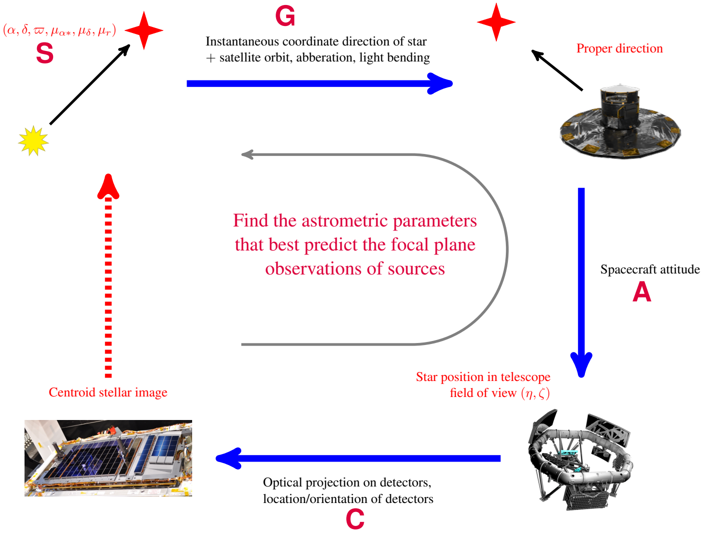


- Celestial positions (α, δ) of sources S are referred to an idealized coordinate system, the International Celestial Reference System (ICRS)
  - origin at the solar system barycentre
- Kinematically defined with *fixed axis directions* with respect to distant matter in the universe
  - principle plane close to mean equator at J2000.0, with origin close to dynamical equinox at J2000.0
  - there is no precession/nutation of the axes!
  - i.e., transforming source coordinates to a different epoch only involves the proper motion!
- Practical realization is the International Celestial Reference Frame (ICRF)
  - defined through precise coordinates of extragalactic objects
  - $\sim$ 4500 sources for radio frame (ICRF3), $\sim$ 550 000 sources for optical frame (Gaia-CRF2)
  - frames aligned through overlapping sources: limited by differences in physical origin of radio/optical emission

Astrometry: observational process

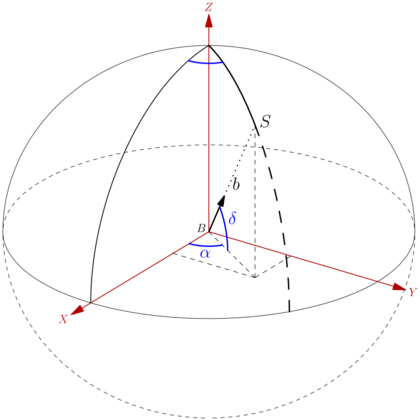
- Astrometric measurements consist of repeated measurements of the directions to sources on the sky
- The celestial directions have to be *modelled* to extract the astrometric parameters (position, parallax, proper motion, etc) of interest
- The modelling includes:
  - source model
  - light propagation model
  - observer's orbit (for aberration correction, parallax scaling, and solar system objects)
  - observer's orientation (spacecraft attitude for Gaia)
- Light propagation and aberration are ignored in the following
- Solar system object astrometry is not treated

Schematic of Gaia observational process



This is an illustration of the practical challenges of astrometry with Gaia. The details behind this schematic not treated in the following slides. Image credits: ESA/Airbus DS.

Modelling source positions



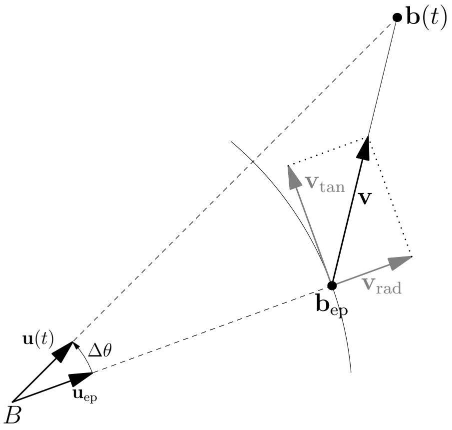
- Source directions are modelled in the Barycentric Celestial Reference System (BCRS)
  - includes parallax, aberration, and relativistic light deflection for apparent directions
  - BCRS is a system of space-time coordinates for the solar system in the framework of General Relativity with a specific metric tensor
  - aligned with ICRS
  - associated time scale is Barycentric Coordinate Time (TCB)

Barycentric source coordinates

- Source S at distance b from solar system barycentre (B)
- Barycentric coordinates of S: $\boldsymbol{b} = b\boldsymbol{u}$, with $\boldsymbol{u}$ the direction from B to S

$$\boldsymbol{u} = \begin{bmatrix} \cos\delta\cos\alpha \\ \cos\delta\sin\alpha \\ \sin\delta \end{bmatrix} \tag{4}$$

Modelling source motion I



Standard model for stellar motion:

$$\boldsymbol{b}(t) = \boldsymbol{b}_{\mathrm{ep}} + (t - t_{\mathrm{ep}})\boldsymbol{v}\,, \quad \boldsymbol{u}(t) = \langle \boldsymbol{b}_{\mathrm{ep}} + (t - t_{\mathrm{ep}})\boldsymbol{v} \rangle \qquad (5)$$

Proper motion and (astrometric!) radial velocity:

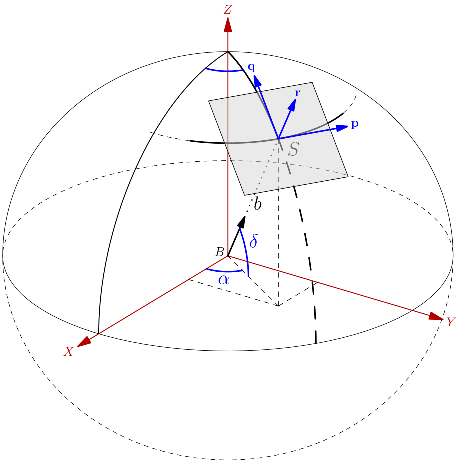
$$\boldsymbol{\mu}(t) = \frac{d\boldsymbol{u}}{dt}\,, \quad v_{\mathrm{rad}}(t) = \frac{db}{dt} \qquad (6)$$

Note the time dependence of $\boldsymbol{\mu}$ and $v_{\mathrm{rad}} \longrightarrow$ 'perspective acceleration' for nearby fast moving stars. To good approximation:

$$\mu = \frac{v_{\mathrm{tan}}}{b} \quad v_{\mathrm{rad}} = \boldsymbol{v}'\boldsymbol{u} \qquad (7)$$

Astrometric and spectroscopic radial velocity are not the same: see Lindegren & Dravins (2003)

Modelling source motion II



'Normal triad' $\begin{bmatrix}\boldsymbol{p} & \boldsymbol{q} & \boldsymbol{r}\end{bmatrix}$:

$$\boldsymbol{r} = \boldsymbol{u}_{\text{ep}}\,, \quad \boldsymbol{p} = \langle \boldsymbol{z} \times \boldsymbol{r} \rangle\,, \quad \boldsymbol{q} = \boldsymbol{r} \times \boldsymbol{p} \tag{8}$$

$$\begin{bmatrix}\boldsymbol{p} & \boldsymbol{q} & \boldsymbol{r}\end{bmatrix} = \begin{bmatrix} -\sin\alpha & -\sin\delta\cos\alpha & \cos\delta\cos\alpha \\ \cos\alpha & -\sin\delta\sin\alpha & \cos\delta\sin\alpha \\ 0 & \cos\delta & \sin\delta \end{bmatrix} \tag{9}$$

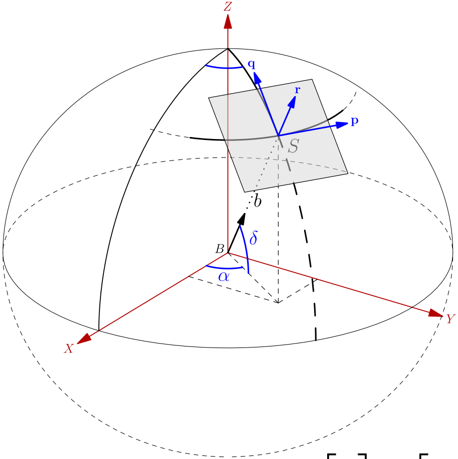
$$\frac{\partial\boldsymbol{r}}{\partial\alpha} = \boldsymbol{p}\cos\delta\,, \quad \frac{\partial\boldsymbol{r}}{\partial\delta} = \boldsymbol{q}\,, \quad d\boldsymbol{r} = \boldsymbol{p}\,d\alpha\cos\delta + \boldsymbol{q}\,d\delta \tag{10}$$

$$\mu_\alpha = \frac{d\alpha}{dt}\,, \quad \mu_{\alpha*} = \frac{d\alpha}{dt}\cos\delta\,, \quad \mu_\delta = \frac{d\delta}{dt} \tag{11}$$

$\mu_{\alpha*}$ and μ_δ can be seen as projections of $d\boldsymbol{r}/dt$ on $\boldsymbol{p}$ and $\boldsymbol{q}$. In general $\boldsymbol{u}(t)$ is different from $\boldsymbol{r}$, so express $\boldsymbol{\mu}$ with respect to fixed triad:

$$\boldsymbol{\mu} = \frac{d\boldsymbol{u}}{dt} = \boldsymbol{p}\mu_{\alpha*} + \boldsymbol{q}\mu_\delta \tag{12}$$

Modelling source motion III



Source velocity in terms of $\begin{bmatrix}\boldsymbol{p} & \boldsymbol{q} & \boldsymbol{r}\end{bmatrix}$:

$$\boldsymbol{v} = \boldsymbol{v}_{\mathrm{tan}} + \boldsymbol{r}v_{\mathrm{rad}} = \boldsymbol{\mu}A_v b + \boldsymbol{r}v_{\mathrm{rad}}$$

$$\boldsymbol{v} = \boldsymbol{p}\mu_{\alpha*}A_v/\varpi + \boldsymbol{q}\mu_\delta A_v/\varpi + \boldsymbol{r}v_{\mathrm{rad}}\,, \tag{13}$$

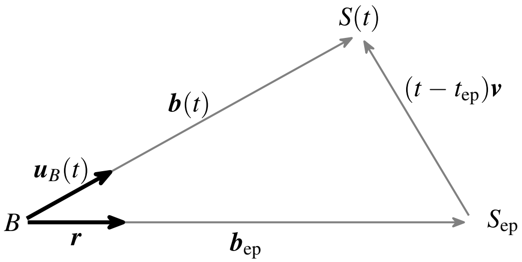
with $A_v = 4.74047\ldots$ km yr s^{-1} and $\varpi = 1/b$.

Eq. (13) is a rotation:

$$\boldsymbol{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} -\sin\alpha & -\sin\delta\cos\alpha & \cos\delta\cos\alpha \\ \cos\alpha & -\sin\delta\sin\alpha & \cos\delta\sin\alpha \\ 0 & \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} \mu_{\alpha*}A_v/\varpi \\ \mu_\delta A_v/\varpi \\ v_{\mathrm{rad}} \end{bmatrix} \tag{14}$$

Source motion including parallax

Relative to solar system barycentre B

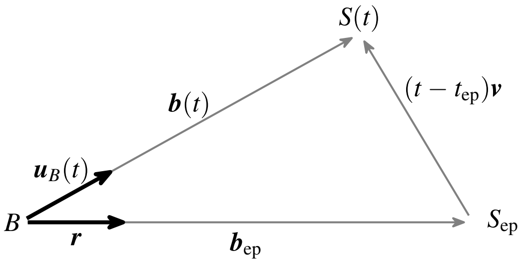


$$\boldsymbol{u}_B(t) = \langle \boldsymbol{b}_{\mathrm{ep}} + (t - t_{\mathrm{ep}})\boldsymbol{v} \rangle$$

$$\boldsymbol{r} = \boldsymbol{u}_{B,\mathrm{ep}} = \boldsymbol{b}_{\mathrm{ep}}/b$$

Source motion including parallax

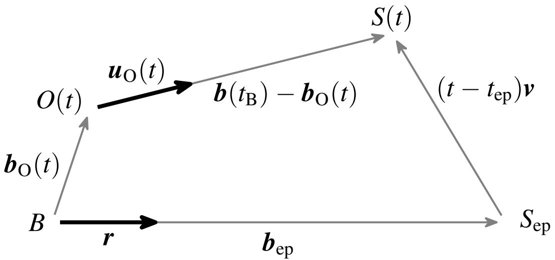
Relative to solar system barycentre B



$$\boldsymbol{u}_B(t) = \langle \boldsymbol{b}_{\rm ep} + (t - t_{\rm ep})\boldsymbol{v} \rangle$$

$$\boldsymbol{r} = \boldsymbol{u}_{B,\rm ep} = \boldsymbol{b}_{\rm ep}/b$$

Relative to observer O



$$\boldsymbol{u}_{\rm O}(t) = \langle \boldsymbol{b}_{\rm ep} + (t_{\rm B} - t_{\rm ep})\boldsymbol{v} - \boldsymbol{b}_{\rm O}(t) \rangle$$

t: time of observation

$t_{\rm B}$: barycentric time corresponding to the observed source direction

$$t_{\rm B} = t + \boldsymbol{r}'\boldsymbol{b}_{\rm O}(t)/c$$

Astrometric source model

Source model in terms of astrometric parameters

Only coordinate directions are modelled here; light bending and aberration not included

$$\boldsymbol{u}_{\mathrm{O}}(t) = \langle b \times [\boldsymbol{u}_{B,\mathrm{ep}} + (t_{\mathrm{B}} - t_{\mathrm{ep}})\boldsymbol{v}/b - \boldsymbol{b}_{\mathrm{O}}(t)/b]\rangle$$

$$\boldsymbol{u}_{\mathrm{O}}(t) = \langle \boldsymbol{r} + (t_{\mathrm{B}} - t_{\mathrm{ep}})(\boldsymbol{p}\mu_{\alpha*} + \boldsymbol{q}\mu_{\delta} + \boldsymbol{r}\mu_r) - \boldsymbol{b}_{\mathrm{O}}(t)\varpi/A_{\mathrm{u}}\rangle \tag{15}$$

$\mu_r = v_{\mathrm{rad}}\varpi/A_v$ is the ‘radial proper motion’ which accounts for perspective acceleration. All angles here in radians, distances in AU, time in Julian years.

Simplified astrometric equations

Assume we have access directly to measurements $(\Delta\alpha(t), \Delta\delta(t))$ with respect to some known reference position $(\alpha(t_{\mathrm{ep}}), \delta(t_{\mathrm{ep}}))$

$$\begin{aligned} \Delta\alpha*(t) = \Delta\alpha(t)\cos\delta(t) = \boldsymbol{p}'\boldsymbol{u}_{\mathrm{O}}(t) &\approx (t_{\mathrm{B}} - t_{\mathrm{ep}})\mu_{\alpha*} - \varpi\boldsymbol{p}'\boldsymbol{b}_{\mathrm{O}}(t)/A_{\mathrm{u}} \\ \Delta\delta(t) = \boldsymbol{q}'\boldsymbol{u}_{\mathrm{O}}(t) &\approx (t_{\mathrm{B}} - t_{\mathrm{ep}})\mu_{\delta} - \varpi\boldsymbol{q}'\boldsymbol{b}_{\mathrm{O}}(t)/A_{\mathrm{u}} \end{aligned} \tag{16}$$

Use and interpretation of astrometric data

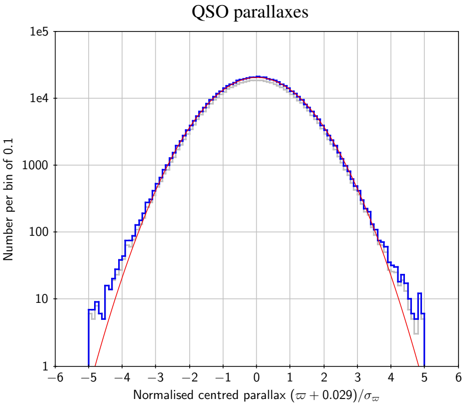
Remarks

The following material was developed with the analysis of Gaia catalogue data in mind. However, most of the considerations are not exclusive to Gaia catalogue data but apply to data analysis in general.

The presentation slides largely follow the paper by Luri et al. (2018), 'Gaia Data Release 2: Using Gaia Parallaxes', `https://doi.org/10.1051/0004-6361/201832964`.

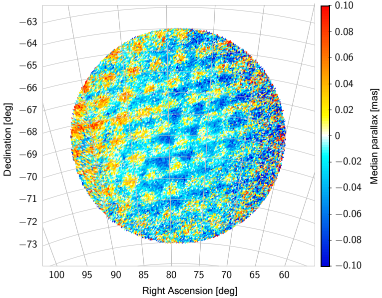
Corresponding Python/R notebooks at:
`https://github.com/agabrown/astrometry-inference-tutorials`

Gaia DR2 astrometry: uncertainties and systematic errors



- Uncertainties are nearly Gaussian
  - NOTE: uncertainties on the astrometric parameters are correlated
- Dependencies on celestial position, magnitude, colour
- Systematic errors are present
  - non-zero mean of Gaussian uncertainty
  - dependencies on celestial position, magnitude, colour
  - spatially correlated

Median parallax LMC region



Images: Lindegren et al. (2018)

Correlated uncertainties

Covariance matrix for a measured k-dimensional vector of quantities $\boldsymbol{x}$:

$$\mathbf{C}_{ij} = E[(x_i - \mu_i)(x_j - \mu_j)] = \rho_i^j \sigma_i \sigma_j \qquad \mu_i = E(x_i)$$

Distribution of $\boldsymbol{x}$ around mean $\boldsymbol{\mu}$ can be treated as multivariate normal:

$$p(\boldsymbol{x}|\boldsymbol{\mu}, \mathbf{C}) = \mathcal{N}_k(\boldsymbol{\mu}, \mathbf{C}) = \frac{1}{\sqrt{(2\pi)^k \det(\mathbf{C})}} \exp\left(-\frac{1}{2}(\boldsymbol{x} - \boldsymbol{\mu})' \mathbf{C}^{-1} (\boldsymbol{x} - \boldsymbol{\mu})\right)$$

Transformation:

$$\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x}) \quad \longrightarrow \quad \mathbf{C}_{\boldsymbol{y}} = \mathbf{J}_f \mathbf{C}_{\boldsymbol{x}} \mathbf{J}_f' \qquad \mathbf{J}_{ij} = \frac{\partial f_i}{\partial x_j}$$

Example:

$$\begin{pmatrix} \mu_{l*} \\ \mu_b \end{pmatrix} = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} \mu_{\alpha*} \\ \mu_\delta \end{pmatrix} \qquad \mathbf{C}_{lb} = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} \sigma^2_{\mu_{\alpha*}} & \rho^{\mu_\delta}_{\mu_{\alpha*}} \sigma_{\mu_{\alpha*}} \sigma_{\mu_\delta} \\ \rho^{\mu_\delta}_{\mu_{\alpha*}} \sigma_{\mu_{\alpha*}} \sigma_{\mu_\delta} & \sigma^2_{\mu_\delta} \end{pmatrix} \begin{pmatrix} c & -s \\ s & c \end{pmatrix}$$

$$c = c(\alpha, \delta), s = s(\alpha, \delta)$$

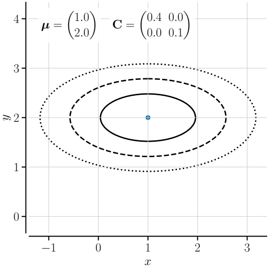
Correlated uncertainties

Example

Measurements of x and y, uncertainties are uncorrelated ($\sigma_{xy} = 0$):

$$\boldsymbol{\mu}_{xy} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \mathbf{C}_{xy} = \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix} = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.1 \end{pmatrix}$$

$$p(x, y) = \frac{1}{0.4\pi} \exp\left(-\frac{1}{2} \left(\frac{(x-1)^2}{0.4} + \frac{(y-2)^2}{0.1} \right) \right)$$

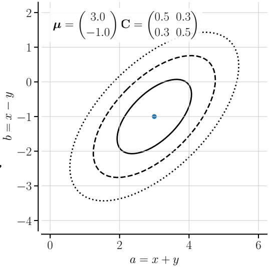


New quantities $a = x + y$, $b = x - y$:

$$\mathbf{J} = \begin{pmatrix} \partial a/\partial x & \partial a/\partial y \\ \partial b/\partial x & \partial b/\partial y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\boldsymbol{\mu}_{ab} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \quad \mathbf{C}_{ab} = \begin{pmatrix} \sigma_x^2 + \sigma_y^2 & \sigma_x^2 - \sigma_y^2 \\ \sigma_x^2 - \sigma_y^2 & \sigma_x^2 + \sigma_y^2 \end{pmatrix} = \begin{pmatrix} 0.5 & 0.3 \\ 0.3 & 0.5 \end{pmatrix} \quad (\rho = 0.6)$$

$$p(a, b) = \frac{1}{0.8\pi} \exp\left(-\frac{1}{1.28} \left(\frac{(a-3)^2}{0.5} + \frac{(b+1)^2}{0.5} - \frac{1.2(a-3)(b+1)}{0.5} \right) \right)$$



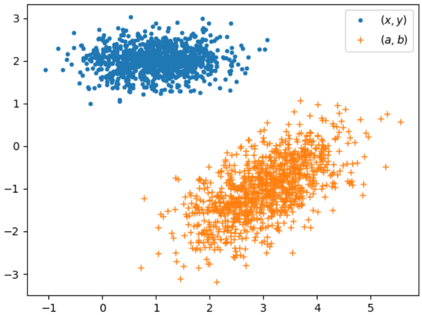
Correlated uncertainties

Simple Python simulation of material on previous slide

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm

x = norm.rvs(loc=1, scale=np.sqrt(0.4), size=1000)
y = norm.rvs(loc=2, scale=np.sqrt(0.1), size=1000)
a = x+y
b = x-y

plt.plot(x, y, '.', label=r'$(x,y)$')
plt.plot(a, b, '+', label=r'$(a,b)$')
plt.legend()
plt.show()
```

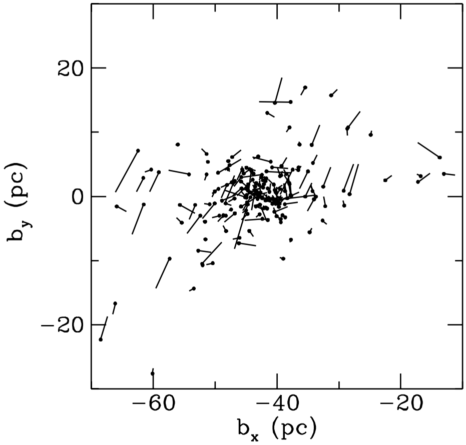


Correlated uncertainties

Account for covariances in your data analysis when:

- propagating uncertainties on subsets and/or linear combinations of astrometric parameters
- estimating model parameters: χ^2-fitting, maximum likelihood, Bayesian inference, etc
- sampling the astrometric uncertainties in some Monte Carlo procedure
  - usually better to sample in the astrometric parameters before transforming to, e.g., phase space quantities

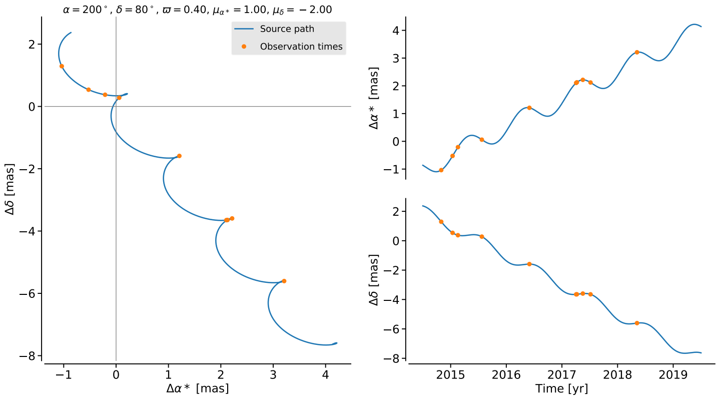
Correlated uncertainties



Hyades velocity residuals suggest shear/rotation in cluster. Is it real?

- Transformation from astrometric parameters to Cartesian positions/velocities will introduce correlations in the propagated uncertainties
  - even in the absence of correlations in the observables, as speed = proper motion/parallax
- In this case similar proper motion vectors for the cluster members lead to apparent correlations between velocity residuals and positions
  - See Brown et al. (1997, arXiv:astro-ph/9707040v1) for details
- Keep correlated uncertainties in mind when interpreting your data!

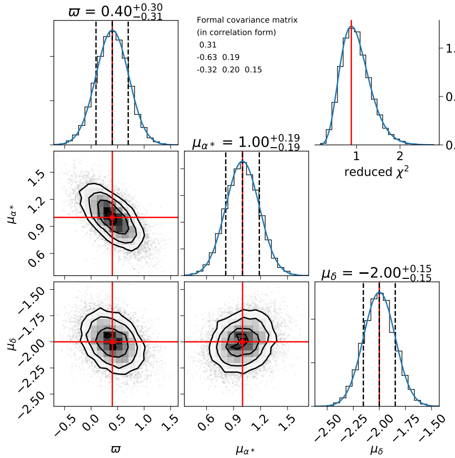
What's with the negative parallaxes?



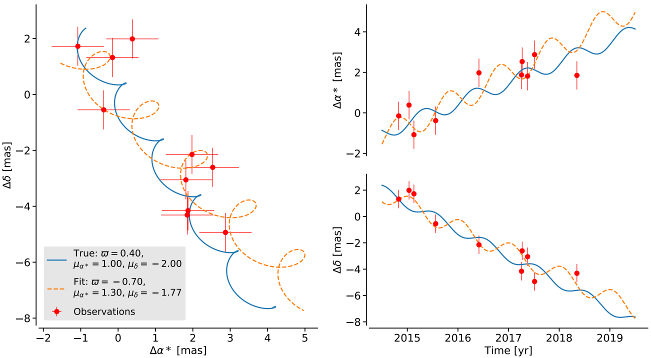
- Source motion on sky from model in Eq. (15)
- Simulated observations at ten epochs spread over 5 years
- Use Eq. (16) to set up a linear system of equations to solve for the astrometric parameters

Simulated solutions for parallax and proper motion

10 000 simulated solutions for different noise realizations



Solution with most negative parallax



Conclusions on negative parallaxes

- Negative (or zero) parallaxes are an expected outcome in the presence of observational uncertainties comparable in value to the parallax itself
- A negative parallax is a perfectly legitimate *measured* value of some true (positive) parallax
  - loosely speaking the epoch astrometry is modelled with the observer going the 'wrong way around the sun'
- Given a correct model for the astrometric observations and normally distributed measurement uncertainties (with zero mean), a measured parallax is an *unbiased* estimate of the true parallax according to

$$p(\varpi \mid \varpi_{\mathrm{true}}) = \frac{1}{\sigma_\varpi\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\varpi - \varpi_{\mathrm{true}}}{\sigma_\varpi}\right)^2\right)$$

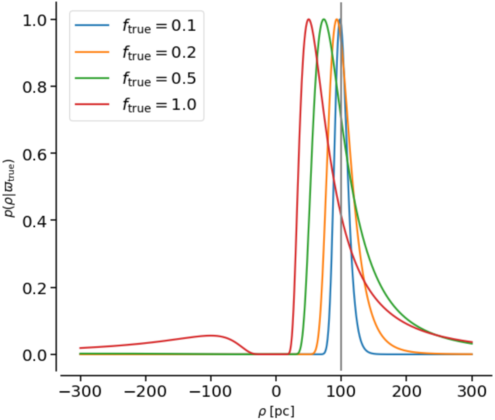
Responsible use of parallax information

Why can't I invert the parallax?

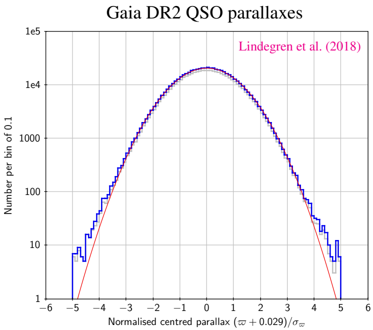
Naive estimate for distance $\rho = 1/\varpi$

$$p(\rho \mid \varpi_{\mathrm{true}}) = \frac{1}{\rho^2 \sigma_\varpi \sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{1/\rho - \varpi_{\mathrm{true}}}{\sigma_\varpi}\right)^2\right)$$

- PDF of ρ has nonphysical negative tail
- Mode moves away from true value of parallax as $f_{\mathrm{true}} = \sigma_\varpi / \varpi_{\mathrm{true}}$ increases
- Expectation value and variance are undefined
- PDF expressed in terms of *unknown* value of ϖ_{true}
- Statements above also hold for small relative uncertainties



Okay, so I keep only positive parallaxes with small uncertainties?



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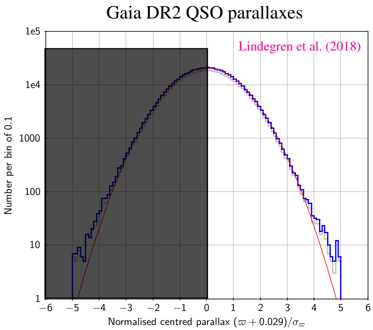

After discarding negative parallaxes
average QSO parallax is 0.8 mas (!)

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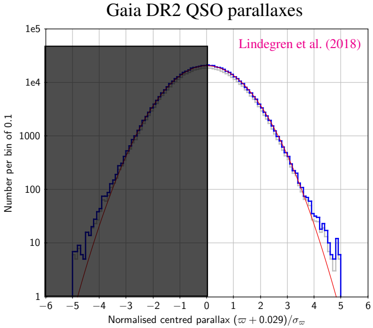

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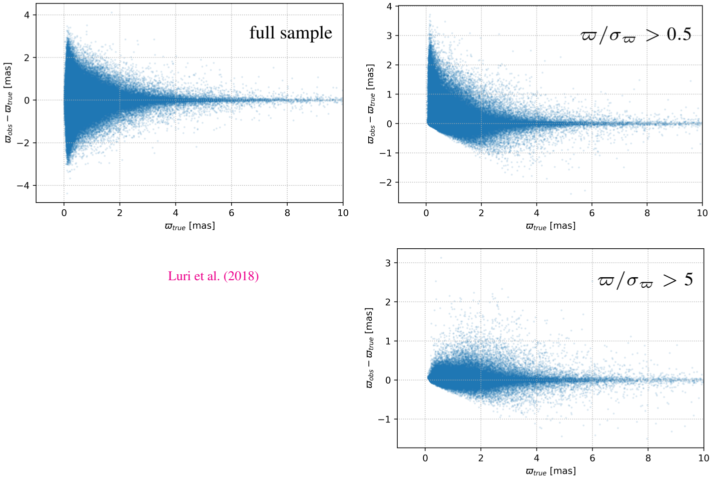

Truncation on the data values distorts the underlying sample and will bias the interpretation

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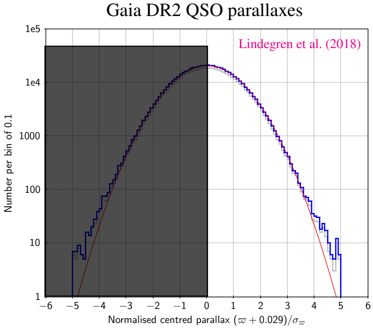

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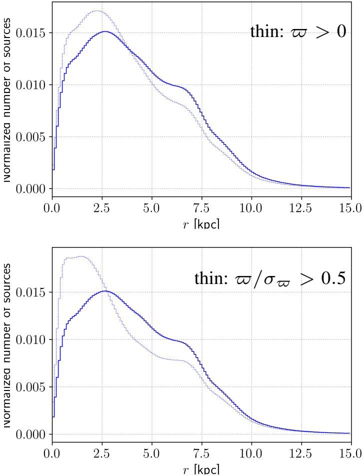
Luri et al. (2018)



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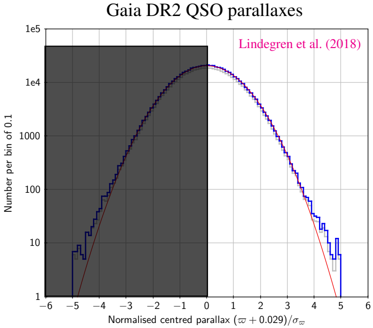
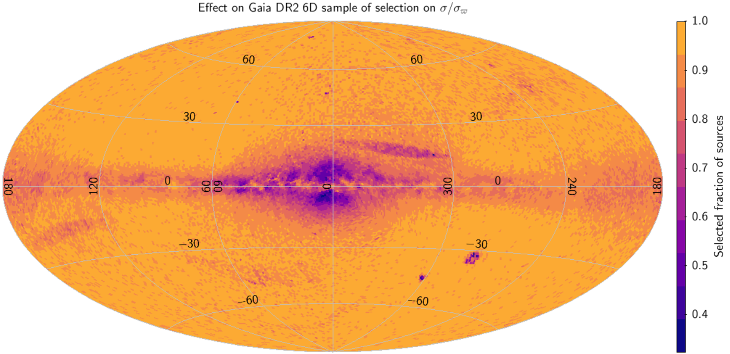


After discarding negative parallaxes average QSO parallax is 0.8 mas (!)



Truncation on the data values distorts the underlying sample and will bias the interpretation

So what should I do?

- Treat the derivation of quantities or model parameters from the astrometric data as an inference problem
- Where possible formulate the problem in the data space
  - data uncertainties well understood
  - easier handling of covariances in measured quantities
  - quantities to be inferred are parameters in ‘forward model’
- Use all relevant information
  - proper motions, magnitudes, colours, all contain distance information
- Account for data selection, survey completeness
- Bayesian analysis naturally fits above points
  - Use proper priors (such that posterior is normalized) that represent the information you already have
- Maximum likelihood as alternative is fine when you have large amounts of data or very precise measurements
- For *initial exploration* of the problem it is okay to select the ‘best data’
  - beware sample truncation effects!
- Is the *distance* really of interest to the question you are trying to answer?

Basic example: distance from single parallax measurement

Detaisl in Bailer-Jones (2015, arXiv:1507.02105)

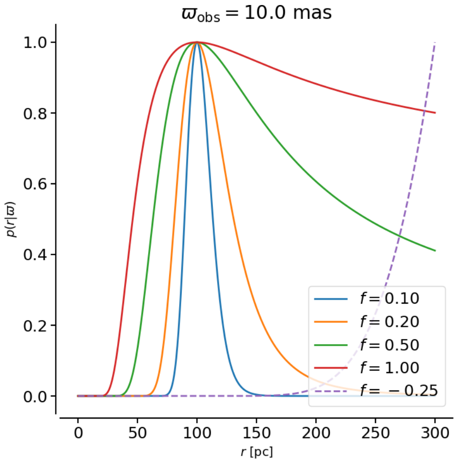
Single source, only ϖ and σ_ϖ known, wish to infer distance r

$$p(r \mid \varpi, \sigma_\varpi) = \frac{1}{Z} p(\varpi \mid r, \sigma_\varpi) p(r)$$

Minimal prior

$p(r) =$ constant for $r > 0$, $p(r) = 0$ for $r \leq 0$

- correctly excludes negative distances, plausible posterior for negative parallaxes
- leads to not-normalizable posterior (no expectation value, variance, etc)
- assumes nonphysical space density that drops as $1/r^2$ from position of the Sun

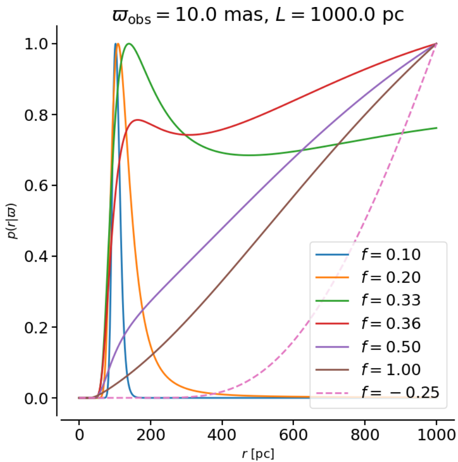


Basic example: distance from single parallax measurement

Currently popular is the exponentially decreasing space density (EDSD) prior (Bailer-Jones 2015):

$$p(r) = \begin{cases} \frac{1}{2L^3} r^2 e^{-r/L} & r > 0 \\ 0 & r \leq 0 \end{cases}$$

- Uniform space density around sun with exponential cut-off
  - normalized posterior
  - posterior transitions from likelihood to prior as $f = \sigma_\varpi / \varpi$ increases
- Do not use blindly!
  - note multiple modes for certain values of f
  - at least calibrate value of L for your problem
  - preferably use different prior, based on knowledge about the space density of sources in your sample
  - NOTE: Bailer-Jones et al. (2018) include -29 μas zero point in distance estimates



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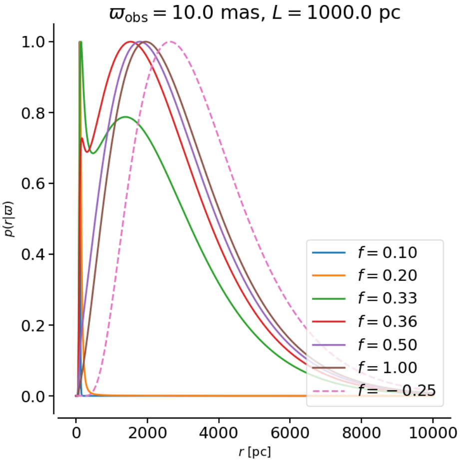
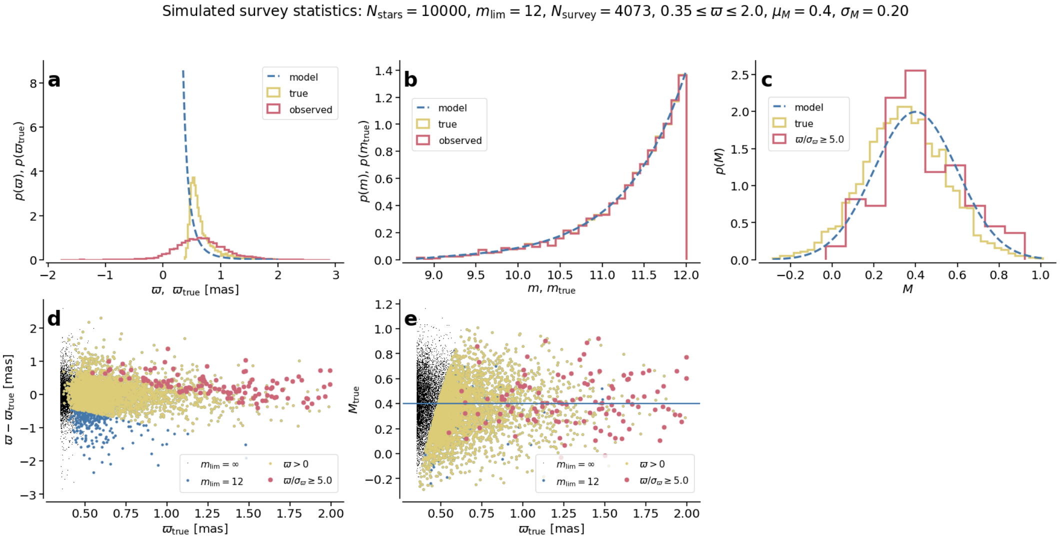


Luminosity calibration

- Single class of stars observed to obtain parallaxes $\{\varpi_k\}$ and apparent magnitudes $\{m_k\}$
- What is the mean absolute magnitude μ_M and its standard deviation σ_M?
  - small σ_M might make this class interesting as standard candle
- Distribution of absolute magnitudes is $M \sim \mathcal{N}(\mu_M, \sigma_M)$
- Stars have uniform space density between known minimum (r_L) maximum (r_H) distances: $r \sim r^2$
- Survey is magnitude limited $m \leq m_\mathrm{lim}$
- Observational errors in ϖ and m vary with apparent brightness and include 'calibration floor' at the bright end
- Python notebooks at `https://github.com/agabrown/astrometry-inference-tutorials/tree/master/luminosity-calibration`

Luminosity calibration

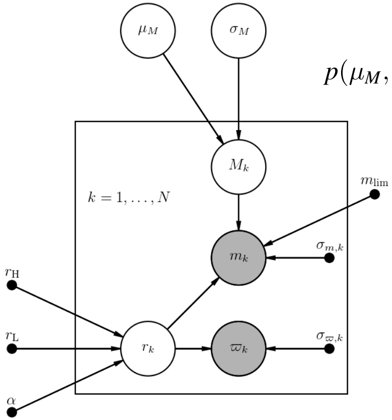


Luminosity calibration

- Simulated survey on previous page is meant to resemble a sample of red-clump stars in the Gaia DR1 (TGAS) catalogue
- Survey limit selects against distant and/or intrinsically faint stars
  - resulting sample is too bright with respect to underlying population
- Selecting on ϖ/σ_ϖ creates a sample of stars for which the distance tends to be underestimated
  - may partially compensate survey limit effect
  - this would be a weak argument to put forward to defend a luminosity calibration based on the 'best' parallaxes…
- Inference approach to luminosity calibration allows to overcome these problems

Luminosity calibration

Build hierarchical Bayesian model

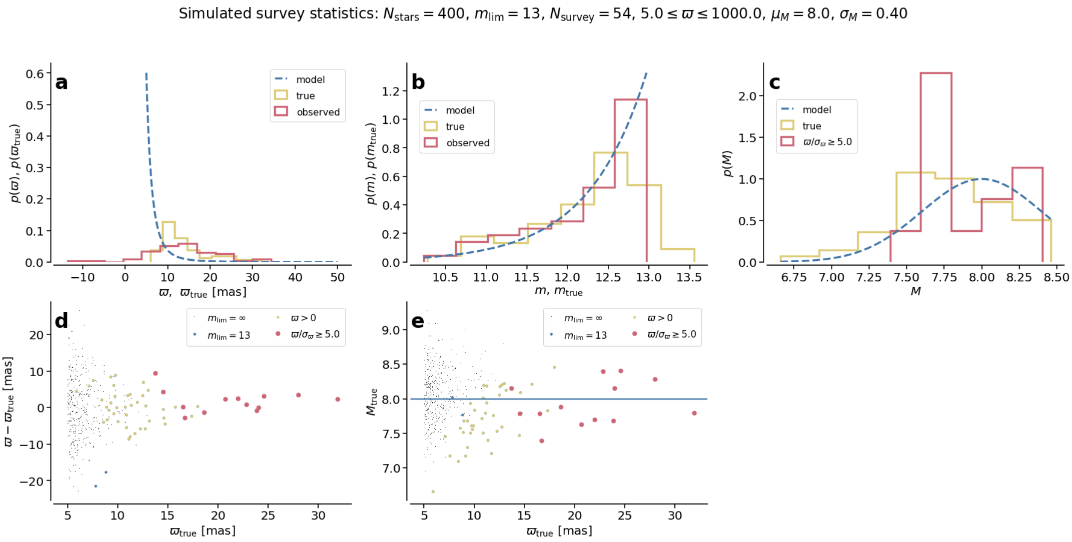


$$\text{posterior} = \text{prior} \times \text{likelihood}$$

$$\begin{aligned}
p(\mu_M, \sigma_M, \boldsymbol{r}, \boldsymbol{M} \mid D) &= p(\mu_M, \sigma_M, \boldsymbol{r}, \boldsymbol{M}) \times p(D \mid \boldsymbol{r}, \boldsymbol{M}, \mu_M, \sigma_M) \\
&= p(\mu_M, \sigma_M) \times p(\boldsymbol{r}, \boldsymbol{M} \mid \mu_M, \sigma_M) \times p(D \mid \boldsymbol{r}, \boldsymbol{M}) \\
&= p(\mu_M) p(\sigma_M) \times p(D \mid \boldsymbol{r}, \boldsymbol{M}) \times p(\boldsymbol{r}, \boldsymbol{M} \mid \mu_M, \sigma_M) \\
&\propto p(\mu_M) p(\sigma_M) \times \prod_k \mathcal{N}(\varpi_k \mid \varpi_{\mathrm{true},k}, \sigma_{\varpi,k}) \times \\
&\quad \mathcal{N}(m_k \mid m_{\mathrm{true},k}, \sigma_{m,k}) \times \frac{3r_k^2}{A} \times \mathcal{N}(M_k \mid \mu_M, \sigma_M) \, .
\end{aligned}$$

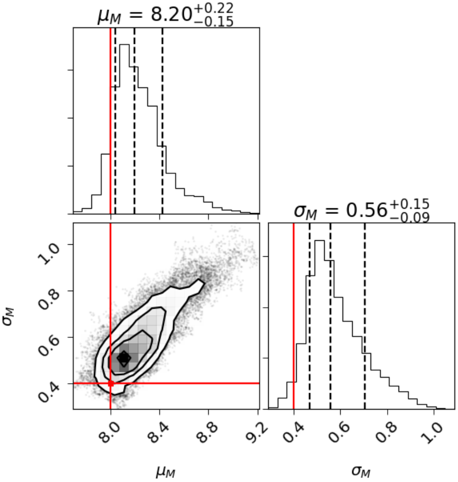
'hierarchical' $\neq$ 'magic'; just think about how you would simulate the data from the underlying model

Luminosity calibration: simulated example

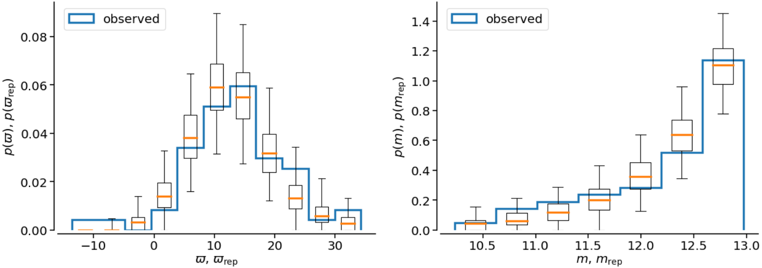


Luminosity calibration: simulated example

- Account for magnitude limit through modification of posterior with selection function $\mathcal{S}(\boldsymbol{m})$
  - requires renormalizing posterior
- 'Solve' for μ_M, σ_M through MCMC sampling, treat $\boldsymbol{r}$ and $\boldsymbol{M}$ as 'nuisance' parameters
- All parallaxes are used, including negative and 'bad' parallaxes
- Survey limit leads to correlation between μ_M and σ_M: narrow distribution requires intrinsically bright stars that enter survey, wide distribution contains a bright tail that enters the survey



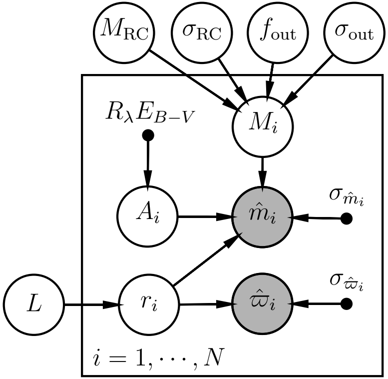
Luminosity calibration: simulated example

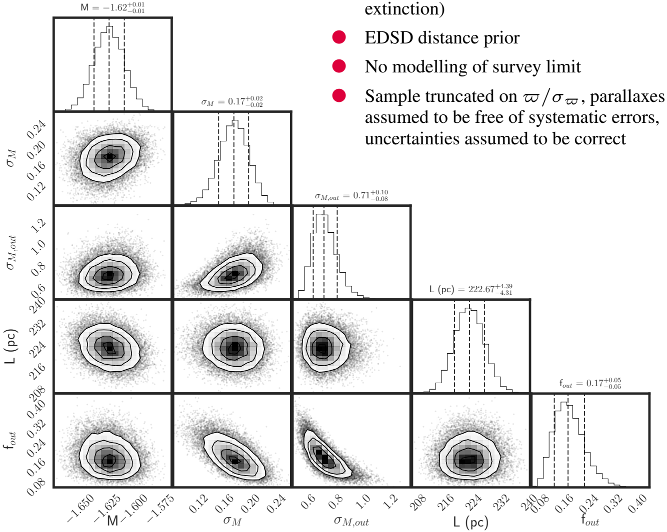


- Check model against data by simulating the observations for samples from the possible model parameters
- In practice model correctness can only be checked by how well it predicts the data
    - do not fall into trap of tuning your algorithm to get close to the simulated 'truth'

Luminosity calibration: real-life example

Hawkins et al., MNRAS, 2017 (arXiv:1705.08988)

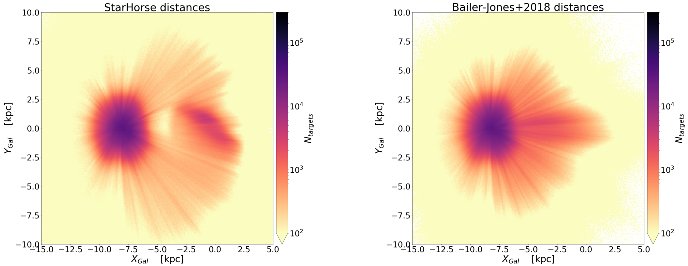




- More complex model (outlier population, extinction)
- EDSD distance prior
- No modelling of survey limit
- Sample truncated on ϖ / σ_ϖ, parallaxes assumed to be free of systematic errors, uncertainties assumed to be correct

Further examples of inference from Gaia DR2 and other data

- Bailer-Jones et al. (2018): Estimating Distance from Parallaxes. IV. Distances to 1.33 Billion Stars in Gaia Data Release 2
- Sanders & Das (2018): Isochrone ages for $\sim$ 3 million stars with the second Gaia data release
- McMillan (2018): Simple Distance Estimates for Gaia DR2 Stars with Radial Velocities
- Mints (2018): Isochrone fitting in the Gaia era. II. Distances, ages and masses from UniDAM using Gaia DR2 data
- Anders et al. (2019): Photo-astrometric distances, extinctions, and astrophysical parameters for Gaia DR2 stars brighter than $G = 18$



Anders et al. (2019)