Essay 1
Social Choice and Proof

MATH 100: The Mathematics of Democracy
Fall 2013
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Description

By this point in the course, you have seen six interesting social choice procedures (Condorcet, Plurality, Borda count, Hare, Sequential Pairs, and Dictatorship) and five "desirable properties" those procedures might have (Always-a-winner, Condorcet winner criterion, Pareto efficiency, monotonicity, and independence of irrelevant alternatives). From each combination of a procedure and a property, Taylor and Pacelli have obtained a proposition, either in the positive ("Procedure A has property X") or the negative ("Procedure A does *not* have property X").

Now it's your turn.

Design a social choice procedure which is not one of the six already given. Be sure that your description is completely unambiguous—I should be able to compute the result of any election, with any number of candidates and any number of voters, using your procedure. In a few hundred words, explain your motivations. What advantages might your procedure have over others? Do you hope to avoid any pitfalls or paradoxes with it?

Once you have described your procedure, the real mathematical work begins. Consider the five "desirable properties". For each one, *prove* that your procedure does or does not satisfy the property. These proofs should have the same character as those in §1.5–1.6 of the text.

Finally, consider the outcome of your proofs. Of course, the Impossibility Theorem implies that at least one property will not be satisfied, but you may be missing others. Is this an acceptable loss? If so, explain why the missing properties are not as important as the ones your procedure does satisfy. If not, discuss which properties you would sacrifice to recover the missing ones and how you might achieve that goal by modifying your system.

Requirements

Your paper must be double-spaced with a standard 12 pt font and 1 in margins. If submitted on paper (as opposed to electronically), it must be stapled and should be double-sided.

It should take approximately 1000–1500 words to complete the requirements of this assignment, but there is no length requirement.

This assignment involves mathematical proof. To a mathematician, "proof" means something very specific and powerful: it is a rigorous, airtight, logical argument for the truth of a statement. The five proofs you will write are the true mathematical content of this assignment. The examples in the text should serve as a good model, but if you have any questions about how to approach these proofs, please get in touch with me or the Writing Assistant!

Drafts

Prior to your final submission, you will prepare and receive feedback on a draft version. During the week prior to the draft deadline, **you are required to schedule a meeting with the Writing Assistant**. At this meeting, you will show him your written procedure description and at least one proof, which he will review for the quality and clarity of your writing and logical arguments. You will then turn in a draft version to me which includes your description and *two* proofs. I will review these over the weekend and return them; you will then submit a revised and extended version with the description, all five proofs, and your reflective discussion at the end of that week.

Deadlines

4 October Procedure description and at least two proofs due. This draft will be returned with comments; you may elect to write up more than two proofs to get more feedback from me.

11 October Final paper due.