MFE 408 Fixed Income Markets

Homework 7

Group #9 of Cohort #2

Group members: ***Chong Cao***, Arjun Atul Deshpande, Mengqi Gong, Yufan Wei

Problem 1 String Market Model

1. Simulate the evolution of the string of discount bond prices out to 10 years.

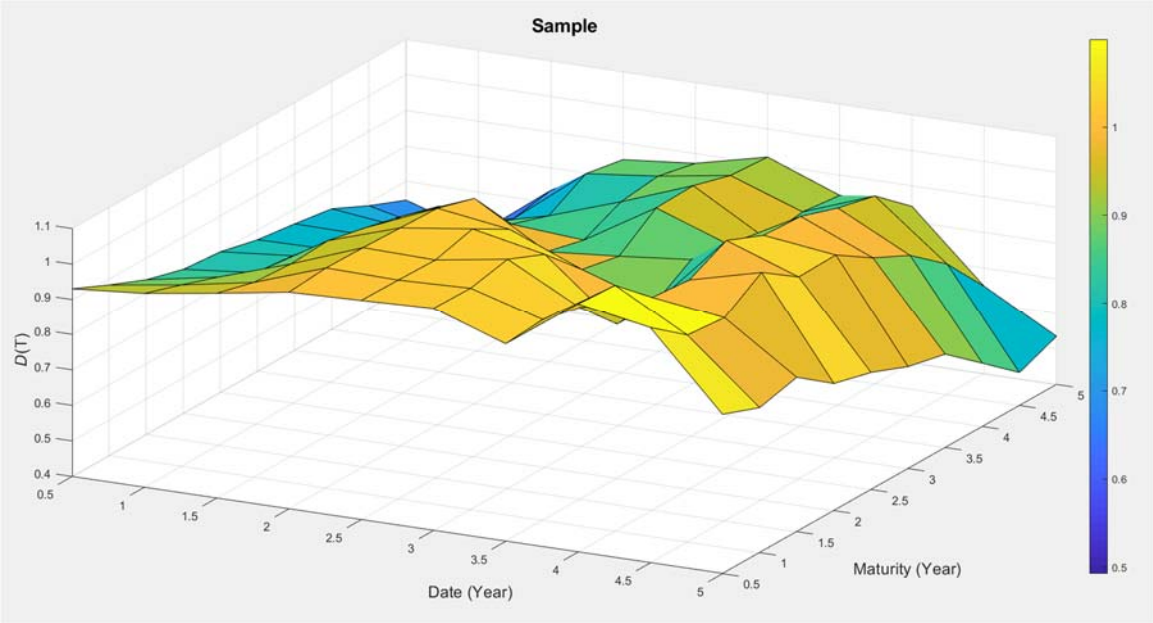
Solution:

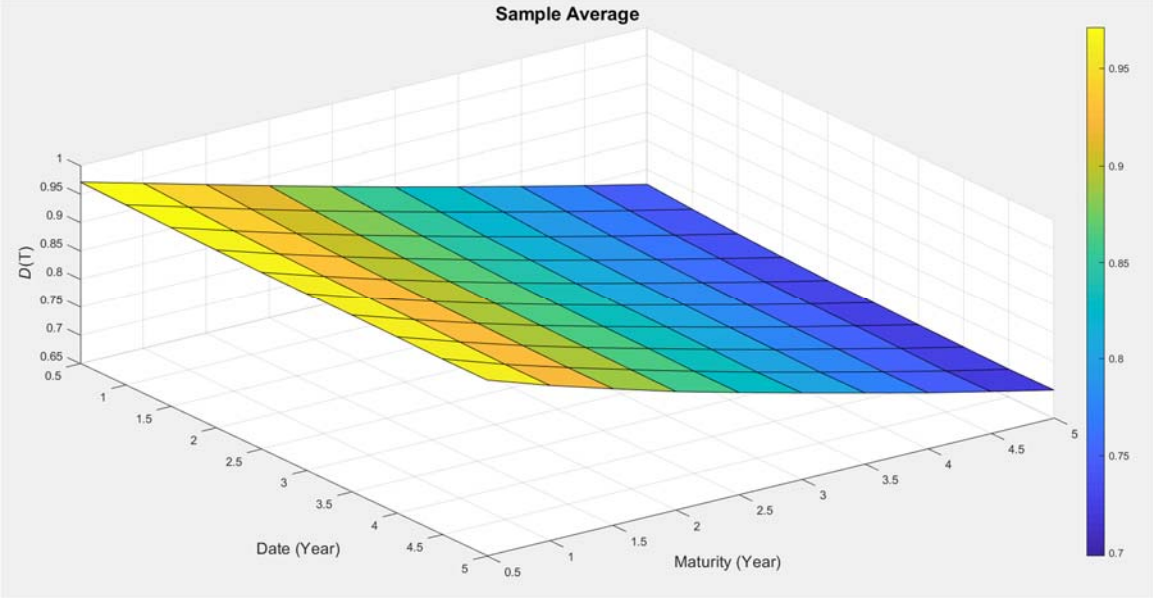
10,000 simulation has been conducted for the Monte Carlo simulation, the sample average discount factors are shown below.

DD

20x20 double

1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	0	0	0	0	0
0.9713	1	0	0	0	0	0	0	0	0	0	0
0.9423	0.9690	1	0	0	0	0	0	0	0	0	0
0.9130	0.9380	0.9666	1	0	0	0	0	0	0	0	0
0.8839	0.9072	0.9337	0.9646	1	0	0	0	0	0	0	0
0.8555	0.8774	0.9020	0.9306	0.9628	1	0	0	0	0	0	0
0.8279	0.8490	0.8722	0.8987	0.9284	0.9622	1	0	0	0	0	0
0.8011	0.8213	0.8434	0.8684	0.8959	0.9271	0.9617	1	0	0	0	0
0.7748	0.7945	0.8155	0.8396	0.8653	0.8944	0.9263	0.9607	1	0	0	0
0.7493	0.7681	0.7884	0.8116	0.8361	0.8634	0.8929	0.9245	0.9594	1	0	0
0.7245	0.7428	0.7624	0.7848	0.8082	0.8344	0.8620	0.8912	0.9227	0.9597	1	0
0.7003	0.7181	0.7371	0.7588	0.7814	0.8065	0.8331	0.8606	0.8900	0.9231	0.9599	1
0.6769	0.6942	0.7127	0.7336	0.7557	0.7801	0.8056	0.8319	0.8593	0.8904	0.9237	0.9630
0.6539	0.6705	0.6884	0.7086	0.7298	0.7535	0.7781	0.8034	0.8296	0.8584	0.8907	0.9222
0.6317	0.6478	0.6650	0.6846	0.7052	0.7279	0.7517	0.7761	0.8010	0.8298	0.8597	0.8894
0.6100	0.6256	0.6423	0.6613	0.6814	0.7034	0.7266	0.7503	0.7741	0.8020	0.8318	0.8551





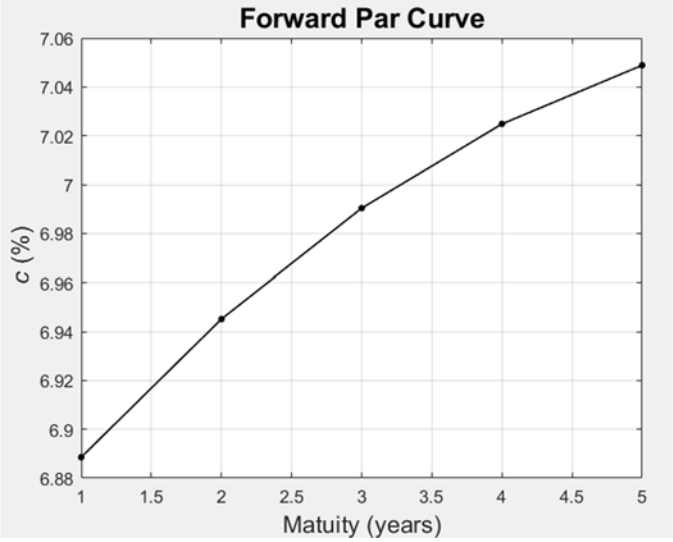
2. Given the initial term structure, solve for the forward par rates for 1, 2, 3, 4, and 5 year semiannual coupon bonds 5 years forward. By construction, each of these bonds has a current forward price of 100.

Solution:

For forward-starting par bond, par rate equals coupon rate

$$c = 2\left[\frac{100D(N) - 100D(N+M)}{\sum_{i=1}^{2M} D(N + i/2)}\right]$$

where $N = 10$ periods (5 years forward) and D is the initial term structure.



3. Now use the string model to solve for the futures price of a contract expiring in 5 years, where any of these bonds (with coupon rates equal to the forward par rates solved for in the previous question) are deliverable at the expiration of the contract (conversion factor is 1.000 for all five bonds). Note that the short has the option of which bond to deliver. How much is the delivery option worth?

Solution:

Using the sample average discount factors obtained from Monte Carlo Simulation, we have the coupon bond value calculated as

$$P = \begin{bmatrix} 98.79 & 98.45 & 98.60 & 98.67 & 98.85 \end{bmatrix}$$

Since the short side has the option of which bond to deliver, the delivery option is worth \$98.45.