MFE 408 Fixed Income Markets

Homework 6

Group #9 of Cohort #2

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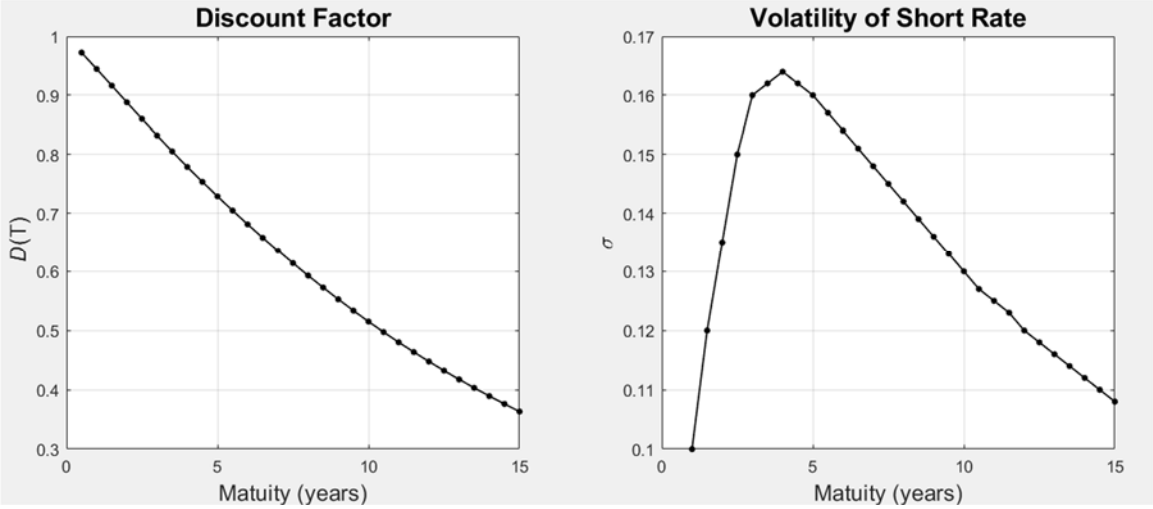
Problem 1 Black-Derman-Toy Model



r

30x30 double

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.0566	0	0	0	0	0	0	0	0	0	0	0
2	0.0633	0.0549	0	0	0	0	0	0	0	0	0	0
3	0.0726	0.0612	0.0517	0	0	0	0	0	0	0	0	0
4	0.0841	0.0695	0.0574	0.0474	0	0	0	0	0	0	0	0
5	0.0983	0.0795	0.0643	0.0520	0.0421	0	0	0	0	0	0	0
6	0.1142	0.0910	0.0726	0.0579	0.0462	0.0368	0	0	0	0	0	0
7	0.1291	0.1027	0.0817	0.0649	0.0516	0.0411	0.0327	0	0	0	0	0
8	0.1460	0.1158	0.0918	0.0728	0.0578	0.0458	0.0363	0.0288	0	0	0	0
9	0.1625	0.1292	0.1028	0.0817	0.0650	0.0517	0.0411	0.0327	0.0260	0	0	0
10	0.1808	0.1442	0.1150	0.0917	0.0731	0.0583	0.0465	0.0371	0.0296	0.0236	0	0
11	0.2001	0.1602	0.1283	0.1028	0.0823	0.0659	0.0528	0.0423	0.0339	0.0271	0.0217	0
12	0.2206	0.1774	0.1427	0.1148	0.0923	0.0743	0.0597	0.0480	0.0386	0.0311	0.0250	0.0201
13	0.2421	0.1956	0.1580	0.1276	0.1031	0.0832	0.0672	0.0543	0.0439	0.0354	0.0286	0.0231
14	0.2644	0.2145	0.1740	0.1411	0.1145	0.0928	0.0753	0.0611	0.0496	0.0402	0.0326	0.0264
15	0.2873	0.2340	0.1906	0.1553	0.1265	0.1030	0.0839	0.0684	0.0557	0.0454	0.0370	0.0301
16	0.3106	0.2541	0.2079	0.1700	0.1391	0.1138	0.0931	0.0762	0.0623	0.0510	0.0417	0.0341

From the properties of binomial distribution, the probability of j upward and $n - j$ downward movements at time step n is given by

$$\text{prob}(n, j) = \frac{n!}{j!(n-j)!}\left(\frac{1}{2}\right)^n$$

Thus, the expected value for r at each time step is expressed as the probability weighted interest rate

$$\mathbb{E}[r_n] = \sum_{j=0}^{n} \text{prob}(n, j) \cdot r(n, j)$$

where $r(n,j)$ represents all the possible short rates at time step n in the BDT tree.

The discount factor can then be obtained from the expected short rates as

$$D(T_n) = \frac{1}{\prod_{i=1}^{n}(1 + 0.5 r_i)}$$

The spot rate can be calculated from term structure data as

$$D(T) = \frac{1}{(1 + r/2)^{2T}} \Rightarrow r = \left[D(T)^{-\frac{1}{2T}} - 1 \right] \times 2$$

(Note: r is different from short rates r_i or r_n or $r(n, j)$.)

The comparison between expected r values and calculated spot rates is shown in figure below