Problems for Lecture 5 & 6

Group #2 of Cohort #2

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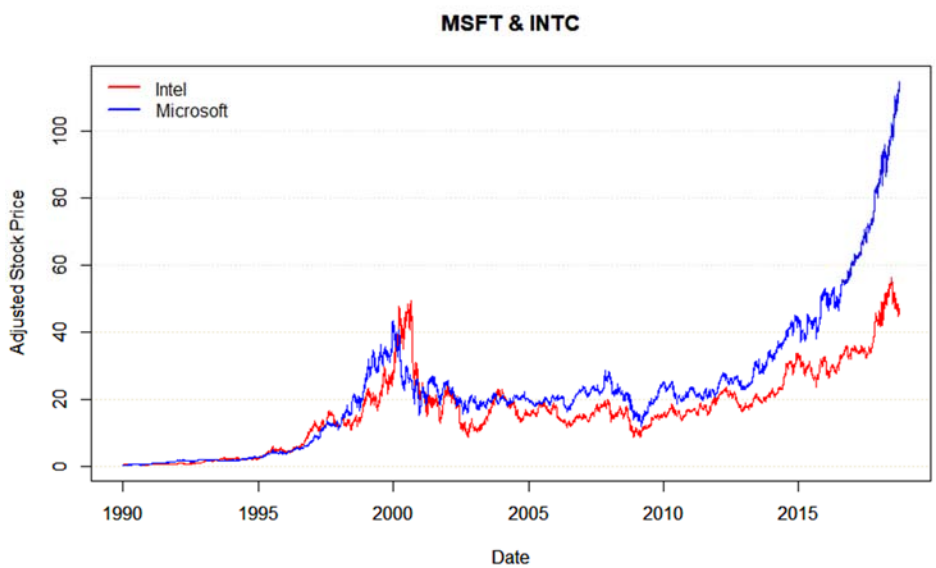
Download the Intel and Microsoft data (12/29/1989 – 09/28/2018) from Yahoo Finance.

Construct weekly simple total returns from the price data. Your returns should include dividends. Compute the mean and standard deviation of the returns. Next, annualize the mean and volatility.

In order to include the effect of dividends on the weekly returns, we should use the adjusted stock prices for computation.



The mean and standard deviation of the weekly returns can be computed as

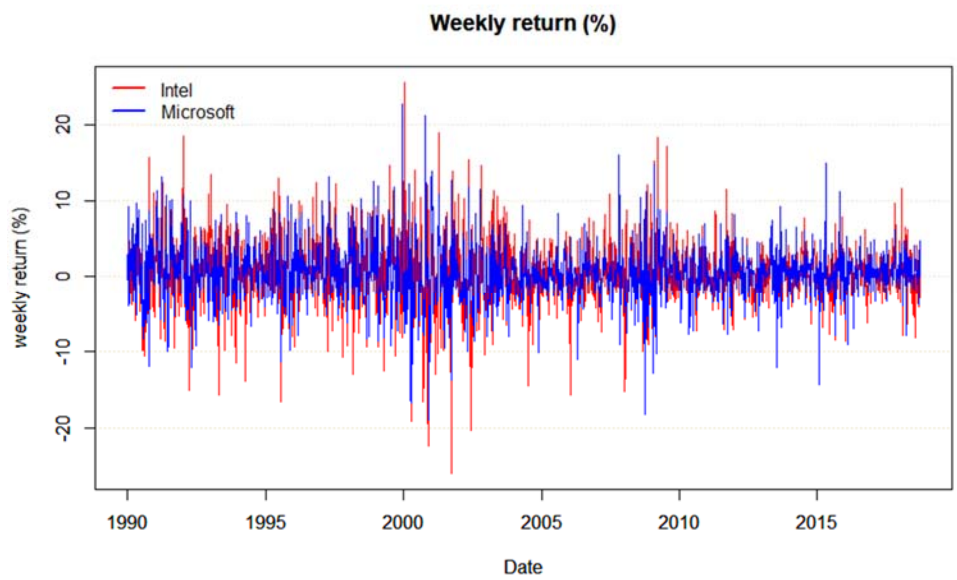
INTC: $E(R_r) = 0.416\%$, $\sigma(R_r) = 4.925\%$

MSFT: $E(R_r) = 0.456\%$, $\sigma(R_r) = 4.130\%$

The annualized mean and standard deviation returns can be easily obtained as

INTC: $E(R_r) = 0.416\% \times 52 = 21.67\%$, $\sigma(R_r) = 4.925\% \times \sqrt{52} = 35.51\%$

MSFT: $E(R_r) = 0.456\% \times 52 = 23.71\%$, $\sigma(R_r) = 4.130\% \times \sqrt{52} = 29.78\%$



Assume that the annualized risk-free rate is 1% and that your coefficient of risk aversion, A, is equal to 4. How will you allocate your capital between the risk-free asset and Intel? How about the risk-free asset and Microsoft?

Assume

$$U(R_{t+1}) = E_t(R_{t+1}) - \frac{A}{2}\sigma_t(R_{t+1})^2$$

Suppose the risky investment on stocks has weight of w in the total investment. In the optimum portfolio, the portion of risky investment can be calculated as

$$w_t^* = \frac{E_t(R_{r,t+1}) - R_{f,t}}{A \cdot \sigma_t^2(R_{r,t+1})}$$

Based on the historical stock data (12/29/1989 – 09/28/2018), we have

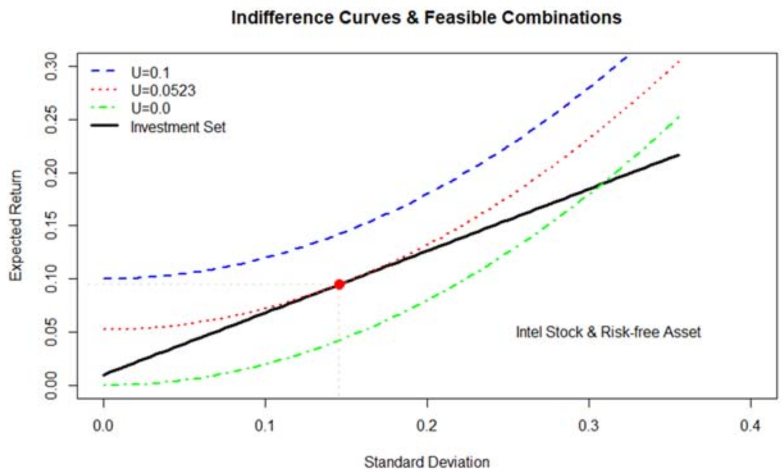
- **Risk-free asset and Intel**: $E_t(R_{r,t+1}) = 21.67\%$, $\sigma_t(R_{r,t+1}) = 35.51\%$

$$w_t^* = \frac{E_t(R_{r,t+1}) - R_{f,t}}{A \cdot \sigma_t^2(R_{r,t+1})} = \frac{0.2167 - 0.01}{4 \times 0.3551^2} = 0.41$$

$$E_t^*(R_{t+1}) = w_t^*\left(E_t(R_{r,t+1}) - R_{f,t}\right) + R_{f,t} = 0.41 \times (0.2167 - 0.01) + 0.01 = 9.47\%$$

$$\sigma_t^*(R_{t+1}) = \left|w_t^*\right| \sigma_t(R_{r,t+1}) = 0.41 \times 0.3551 = 14.55\%$$

$$U^*(R_{t+1}) = E_t^*(R_{t+1}) - \frac{A}{2}\sigma_t^*(R_{t+1})^2 = 0.0947 - \frac{4}{2}\times 0.1455^2 = 0.0523$$



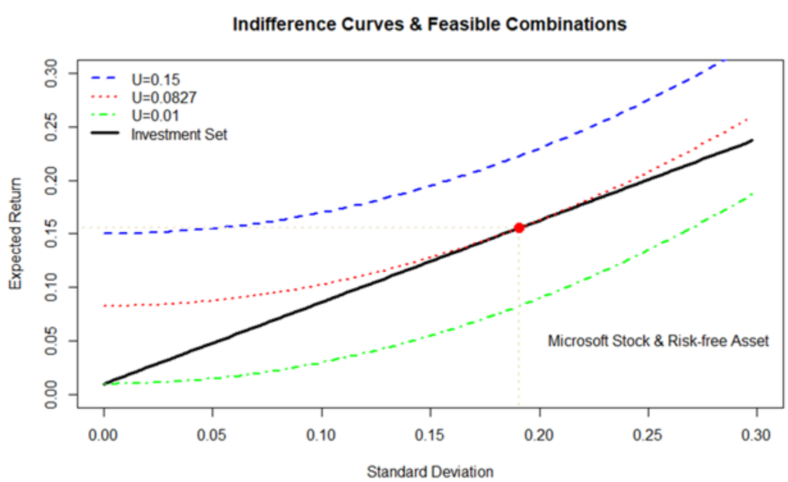
- **Risk-free asset and Microsoft**: $E_t(R_{r,t+1}) = 23.71\%$, $\sigma_t(R_{r,t+1}) = 29.78\%$

$$w_t^* = \frac{E_t(R_{r,t+1}) - R_{f,t}}{A\cdot\sigma_t^2(R_{r,t+1})} = \frac{0.2371 - 0.01}{4\times 0.2978^2} = 0.64$$

$$E_t^*(R_{t+1}) = w_t^*\left(E_t(R_{r,t+1}) - R_{f,t}\right) + R_{f,t} = 0.64\times(0.2371 - 0.01) + 0.01 = 15.54\%$$

$$\sigma_t^*(R_{t+1}) = \left|w_t^*\right|\sigma_t(R_{r,t+1}) = 0.64\times 0.2978 = 19.07\%$$

$$U^*(R_{t+1}) = E_t^*(R_{t+1}) - \frac{A}{2}\sigma_t^*(R_{t+1})^2 = 0.1554 - \frac{4}{2}\times 0.1907^2 = 0.0827$$



If you could only choose to allocate between the risk-free asset and either Intel or Microsoft, which would you choose and why?

According to the utility theory of portfolio investment, the investor should choose the portfolio that delivers the highest utility. For the case of optimum investment, we can clearly see that the portfolio of risk-free asset and Microsoft produces a much higher utility than the portfolio of risk-free asset and Intel (0.0827 > 0.0523). Therefore, we would like to choose **risk-free asset and Microsoft** as our investment portfolio.

Construct the mean-variance frontier for the Intel-Microsoft combination. Indicate the minimum-variance portfolio and the efficient frontier (the efficient frontier is a set of expected returns - risks that you would want to consider investing in).

The expected return of a portfolio with two risky stocks is calculated as

$$E(R) = w_1 E(R_1) + w_2 E(R_2)$$

And the standard deviation is given by

$$\sigma(R) = \sqrt{w_1^2 \sigma_{R_1}^2 + w_2^2 \sigma_{R_2}^2 + 2 w_1 w_2 \rho_{R_1 R_2} \sigma_{R_1} \sigma_{R_2}}$$

The mean-variance frontier for the Intel-Microsoft combination portfolio can be easily constructed, and the minimum-variance portfolio can be found at

$$w_1^* = 0.33, \; E_t^*(R_{t+1}) = 23.04\%, \; \sigma_t^*(R_{t+1}) = 27.78\%$$

which indicates that we invest the portion of 33% in Intel stock and 67% in Microsoft stock.

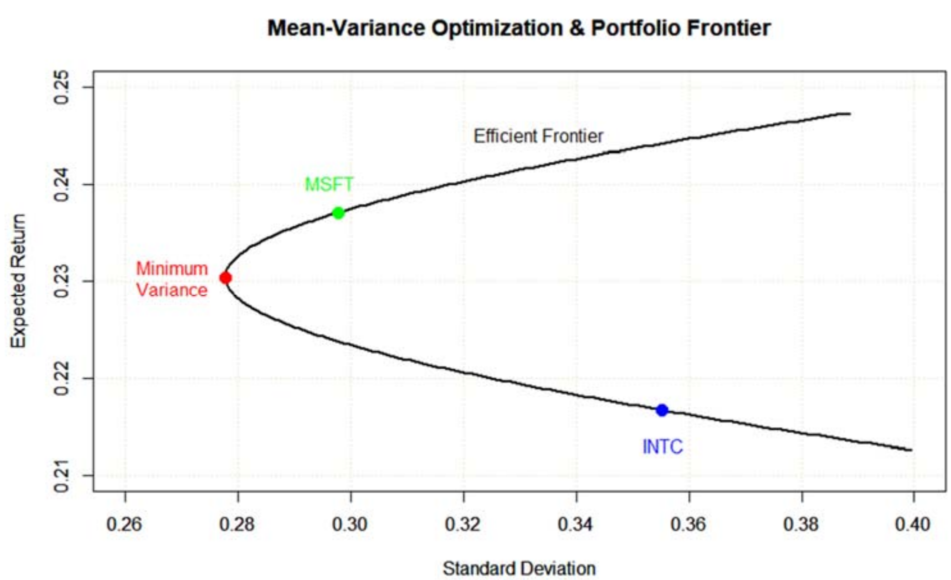


Figure. Portfolio Frontier with Short Sales: Intel and Microsoft