Project for Lecture 6 & 7

Group #2 of Cohort #2

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The Excel file `lecture6p.xlsx` contains daily market data for Microsoft, Intel, Southwest, McDonald's, and Johnson & Johnson from 12/29/1989 to 9/28/2018, obtained from Yahoo Finance. The file also includes a daily risk-free rate time series from Kenneth French's Data Library. For this problem set, you should calculate time series of weekly returns. Re-use the mean-variance frontier for the Intel-Microsoft combination from the previous assignment.

1. Add remaining stocks to the mix. Compute the mean-variance frontier and plot it on the same chart with the one from the previous question. Indicate the minimum-variance portfolio and the efficient frontier. How do they compare to the ones in the previous question?

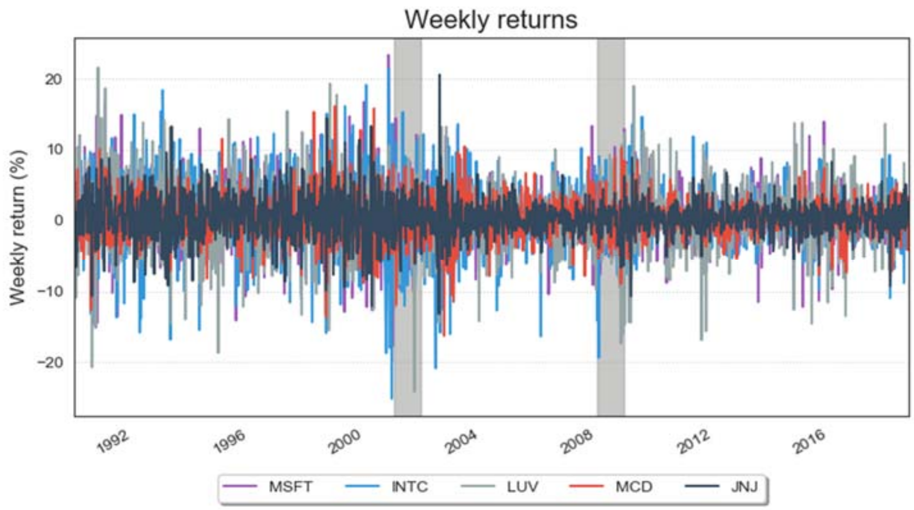
2. Add the riskless asset and construct the tangent portfolio for the Intel-Microsoft case. Next, construct the tangent portfolio for the full set of stocks. Compare the Sharpe ratios of the two tangent portfolios.

3. Assume your risk aversion is $A = 5$. What your optimal mix of assets?

Solution:

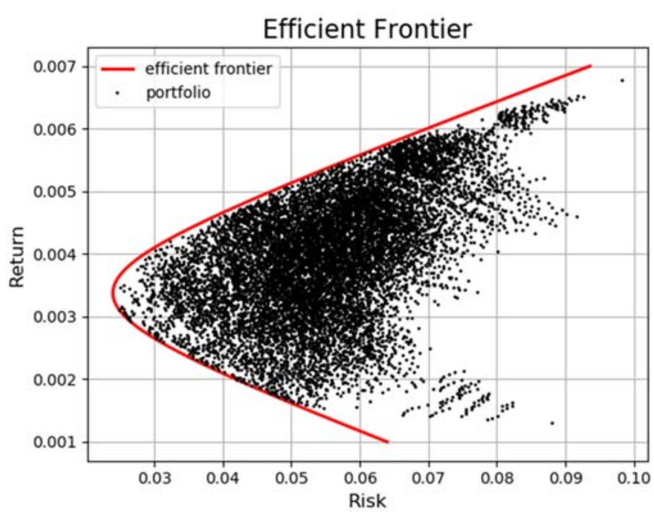
In order to include the effect of dividends on the stock returns, we should use the adjusted stock prices for computation.

The time series of weekly returns are shown below



	Weekly returns	Standard deviation
MSFT	0.46%	4.30%
INTC	0.43%	5.17%
LUV	0.40%	4.80%
MCD	0.29%	3.21%
JNJ	0.32%	2.92%

The mean-variance efficient frontier can be computed as



The minimum-variance portfolio (MVP) can be found as

MVP: $E(R_r) = 0.34\%$, $\sigma(R_r) = 2.40\%$

The minimum-variance portfolio is the solution to the problem

$$\min_{\mathbf{x}} \mathbf{x}^{\mathrm{T}}\mathbf{V}\mathbf{x} \quad \text{s.t.} \quad \mathbf{1}^{\mathrm{T}}\mathbf{x} = 1$$

where $\mathbf{V}$ represents the covariance matrix of stock returns and $\mathbf{x}$ denotes the portfolio weights. The optimal solution can be derived as

$$\mathbf{x} = \frac{\mathbf{V}^{-1}\mathbf{1}}{\mathbf{1}^{\mathrm{T}}\mathbf{V}^{-1}\mathbf{1}}$$

which can be calculated as (0.113, 0.020, 0.057, 0.337, 0.472) for (MSFT, INTC, LUV, MCD, JNJ).

The portfolio weekly return and standard deviation can be computed as

$$R_{\mathrm{MVP}} = \mathbf{x}^{\mathrm{T}} r = 0.34\% \quad \text{and} \quad \sigma_{\mathrm{MVP}} = \mathbf{x}^{\mathrm{T}}\mathbf{V}\mathbf{x} = 2.40\%$$

which is consistent with our estimation from efficient frontier.

Tangent portfolio can be found by locating the maximum Sharpe ratio on efficient frontier

$$SR_{\max} = 0.126$$

	Portfolio weights
MSFT	0.2656
INTC	0.0417
LUV	0.1099
MCD	0.1772
JNJ	0.4056

Assume mean-variance utility

$$U(R_{t+1}) = E_t(R_{t+1}) - \frac{A}{2}\sigma_t(R_{t+1})^2$$

Suppose the risky investment on stocks has weight of w in the total investment. In the optimum portfolio, the portion of risky investment can be calculated as

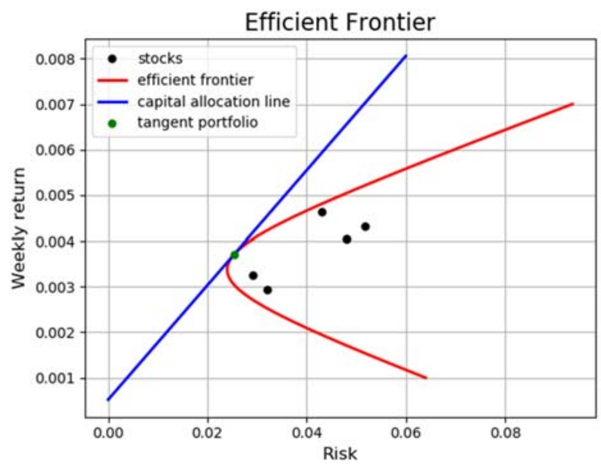
$$w_t^* = \frac{E_t(R_{r,t+1}) - R_{f,t}}{A \cdot \sigma_t^2(R_{r,t+1})} = \frac{0.0036999 - 0.0005177}{5 \times 0.0253214^2} = 0.9926$$

Therefore, we have

$$E_t^*(R_{t+1}) = w_t^*\left(E_t(R_{r,t+1}) - R_{f,t}\right) + R_{f,t} = 0.368\%$$

$$\sigma_t^*(R_{t+1}) = \left|w_t^*\right| \sigma_t(R_{r,t+1}) = 2.513\%$$

$$U^*(R_{t+1}) = E_t^*(R_{t+1}) - \frac{A}{2}\sigma_t^*(R_{t+1})^2 = 0.002097$$



The optimum weights on both risky assets and riskless asset can be summarized as

	Portfolio weights
MSFT	0.2636
INTC	0.0414
LUV	0.1091
MCD	0.1758
JNJ	0.4026
Rf	0.0074