Inverse problems for the Helmholtz equation with first order terms. Applications to acoustic tomography of moving fluid

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- A moving fluid in a bounded domain $D \subset \mathbb{R}^d$, $d = 2$, is characterized by sound speed $c = c(x)$, density $\rho = \rho(x)$, velocity $\mathbf{v} = \mathbf{v}(x)$ and absorption $\alpha = \omega^{\zeta(x)}\alpha_0(x)$
- There are acoustic transducers on ∂D. A transducer produces time-harmonic acoustic waves which are scattered by the fluid. Scattered acoustic waves are recorded by other transducers.

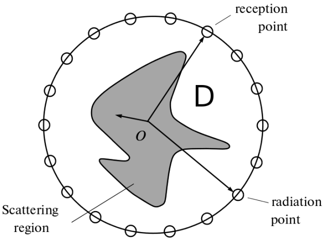


image: (Burov et al. '13)

Acoustic tomography problem. Given this data, recover fluid parameters.

Main applications in ocean tomography (*determine the ocean temperature and heat transferring currents*) and in medical diagnostics (*determine scalar inhomogeneities and the blood flow*)

Acoustic tomography of moving fluid

Figure: Acoustic tomograph (Acoustics Department, MSU). Rotating array of 26 transducers mimicking a 256 transducer array.

- Typical frequencies in ocean: 10Hz-1MHz (<10Hz: propagation is not possible; >1MHz: quick absorbtion)
- Most commonly used tranceducers emit at a single frequency (around 2MHz) to minimise technical difficulties

Acoustic tomography of moving fluid

$$L_\omega = -\Delta - 2i\left(\frac{\omega \mathbf{v}}{c^2} + \frac{i}{2}\nabla \ln \rho\right) \cdot \nabla - \frac{\omega^2}{c^2} - 2i\omega\frac{\alpha}{c}$$

- Data from point sources (spherical wave scattering data):

$$\mathfrak{D}(\Omega, X, Y) = \{G_\omega(x, y) \colon \omega \in \Omega,\ x \in X,\ y \in Y\},$$
$$G_\omega \text{ radiating Green function for } L_\omega,$$
$$\Omega \subset \mathbb{R}_{\geq 0},\ X \subset \partial D,\ Y \subset \partial D$$

Acoustic tomography problem: Given $\mathcal{D}(\Omega, X, Y)$, find c, $\mathbf{v}$, $\nabla\rho$ and α in D

Roussef-Winters '94, Rychagov-Ermert '96, Henkin-Novikov '88, Burov et al. '13: particular cases of the model

Rumyantseva-Shurup (Acoustical Physics, to app.): equation in general form

Solving the acoustic tomography problem

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
\boxed{\text{data from point sources}} \arrow[r, Rightarrow, "\text{acoustic tomography problem}"] \arrow[d] & \boxed{\text{fluid parameters}} \\
\boxed{\text{D-to-N map(s)}} \arrow[r, "\color{red}\text{inverse Dirichlet-to-Neumann problem}"] & \boxed{\text{gauge class(es) of magn. Schrödinger operators}} \arrow[u]
\end{tikzcd}
$$

Inverse DN problem for the magnetic Schrödinger operator

- Magnetic Schrödinger operator in bounded domain $D \subset \mathbb{R}^{d \geq 2}$

$$L_{A,q} = -(\nabla + iA(x))^2 + q(x), \quad x \in D$$

- Dirichlet-to-Neumann map $\Lambda_{A,q} f = \nu \cdot (\nabla + iA)\psi|_{\partial D}$, where

$$\begin{cases} L_{A,q}\psi = E\psi & \text{in } D, \quad (E \text{ not a DE for } L_{A,q} \text{ in } D) \\ \psi|_{\partial D} = f \end{cases}$$

- Gauge invariance: $\Lambda_{A+\nabla\varphi,q} = \Lambda_{A,q}$, $\varphi|_{\partial D} = 1$

The inverse DN problem: given $\Lambda_{A,q}$ (at fixed E), recover A, q modulo a GT

Problems of this type go back to Gelfand (PICM'54)

Inverse DN problem for the magnetic Schrödinger operator

Sun '93, Nakamura-Sun-Uhlmann '95, Eskin-Ralston '95, Guillarmou-Tzou '11, Immanuilov-Uhlmann-Yamamoto '12, Krupchyk-Uhlmann '14: $F = \operatorname{curl} A$ and q are determined by $\Lambda_{A,q}$

Brown-Salo (AA'06): tangential part of $A|_{\partial D}$ is determined by $\Lambda_{A,q}$

Salo (CPDE'06): reconstruction algorithm, $d \geq 3$

Agaltsov-R. Novikov (JMP'14), Agaltsov (JIIP'15): reconstruction algorithm, $d = 2$

Shurup-Rumyantseva (Acoustical Physics, to app.): implementation and numerical verification of *Agaltsov-R. Novikov (JMP'14)*

Solving the acoustic tomography problem

$$
\begin{tikzcd}
\boxed{\text{data from point sources}} \arrow[rr, Rightarrow, "\text{acoustic tomography problem}"] \arrow[d, "\text{second Green formula}"] & & \boxed{\text{fluid parameters}} \\
\boxed{\text{D-to-N map(s)}} \arrow[rr, "\text{inverse Dirichlet-to-Neumann problem}"] & & \boxed{\text{gauge class(es) of magn. Schrödinger operators}} \arrow[u]
\end{tikzcd}
$$

Acoustic tomography: reduction to the IDN problem

Dyson (PR'49), Schwinger (PNAS'51): formulas relating Green functions in QFTs (Dyson and Schwinger-Dyson equations)

Berezanski (TMMO'58): formulas relating data from point sources and plane wave scattering data (amplitude and wavefunction)

R. Novikov (FAA'88): formulas relating plane wave scattering data to Dirichlet-to-Neumann maps

Possible integral equation :

$$G^0_\omega(x,y) - G_\omega(x,y) = \int\limits_{\partial D}\int\limits_{\partial D} G^0_\omega(x,z)(\Lambda_\omega - \Lambda^0_\omega)(z,w)G_\omega(w,y)\,dy\,dw$$

where G^0_ω, Λ^0_ω correspond to $\mathbf{v} = 0$, $\rho = 0$, $c = c_0$, $\alpha_0 = 0$

(see, e.g., Nachman (AM'88) for the case of Schrödinger operator and Laplacian)

Acoustic tomography: reduction to the IDN problem

The second resolvent identity:

- $\mathcal{A}$, $\mathcal{B}$ invertible operators: $\mathcal{B} - \mathcal{A} = \mathcal{B}(\mathcal{A}^{-1} - \mathcal{B}^{-1})\mathcal{A}$
- Apply it to $\mathcal{A} = (L_\omega - i0)^{-1}$, $\mathcal{B} = (L_\omega^0 - i0)^{-1}$:

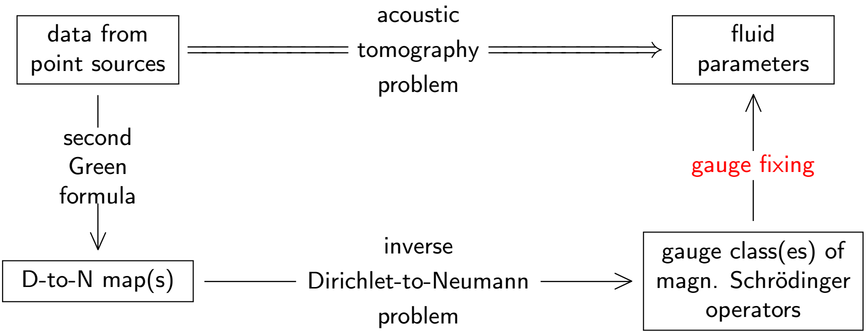
$$G_\omega^0(x,y) - G_\omega(x,y) = \int_D G_\omega^0(x,z)(L_\omega - L_\omega^0)_z G_\omega(z,y)\,dz$$

Passing to the boundary:

- Let v: $L_\omega^0 v = 0$ in D, $v|_{\partial D} = \mathcal{G}_\omega f$ and let $u = \mathcal{G}_\omega f$
- The second Green identity applied to $G_\omega^0(x,\cdot)$ and $u - v$:

$$\int_{\partial D} G_\omega^0(x,y)\frac{\partial(u-v)}{\partial \nu_y}dy - u(x) + v(x) = \int_D G_\omega^0(x,z)(L_\omega - L_\omega^0)_z u(z)\,dz$$

Solving the acoustic tomography problem



Gauge fixing

$$L_\omega = -(\nabla + iA_\omega(x))^2 + q_\omega(x),$$

$$\begin{array}{l|l} A_\omega = \frac{\omega \mathbf{v}}{c^2} + \frac{i}{2}\nabla \ln \rho & f_1 = \rho^{\frac{1}{2}} \Delta \rho^{-\frac{1}{2}} \\ q_\omega = f_1 - \omega^2 f_2 + i\omega f_3 - 2i\omega^{1+\zeta}\alpha_0 & f_2 = \frac{1}{c^2} + \frac{\mathbf{v}}{c^2}\frac{\mathbf{v}}{c^2} \\ & f_3 = \nabla \cdot \left(\frac{\mathbf{v}}{c^2}\right) - \frac{\mathbf{v}}{c^2} \cdot \nabla \ln \rho \end{array}$$

- Λ_ω at fixed ω determines curl $A_\omega = \omega \operatorname{curl} \frac{\mathbf{v}}{c^2}$ and q_ω
- $\Lambda_{\omega_1}, \Lambda_{\omega_2} \implies$ linear algebraic equations for f_1, f_2:

$$\operatorname{Re} q_{\omega_1} = f_1 - \omega_1^2 f_2,$$
$$\operatorname{Re} q_{\omega_2} = f_1 - \omega_2^2 f_2$$

Gauge fixing

- $\Lambda_{\omega_1}, \Lambda_{\omega_2}, \Lambda_{\omega_3} \implies$ equations for f_3, ζ, α_0 (in general, transcendental):

$$\begin{aligned} \omega_1^{-1} \operatorname{Im} q_{\omega_1} &= f_3 - 2\omega_1^{\zeta}\alpha_0, \\ \omega_2^{-1} \operatorname{Im} q_{\omega_2} &= f_3 - 2\omega_2^{\zeta}\alpha_0, \\ \omega_3^{-1} \operatorname{Im} q_{\omega_1} &= f_3 - 2\omega_3^{\zeta}\alpha_0, \end{aligned}$$

- As a corollary,

$$\{x \colon \alpha_0(x) = 0\} = \{x \colon \omega_3^{-1} \operatorname{Im} q_{\omega_3} = \omega_1^{-1} \operatorname{Im} q_{\omega_1}\},$$

$$\frac{\omega_2^{-1} \operatorname{Im} q_{\omega_2} - \omega_1^{-1} \operatorname{Im} q_{\omega_1}}{\omega_3^{-1} \operatorname{Im} q_{\omega_3} - \omega_1^{-1} \operatorname{Im} q_{\omega_1}} = \frac{\left(\frac{\omega_2}{\omega_1}\right)^{\zeta} - 1}{\left(\frac{\omega_3}{\omega_1}\right)^{\zeta} - 1}, \quad \text{where } \alpha_0 \neq 0$$

$$f_3 = \frac{\frac{\omega_1^{\zeta}}{\omega_2} \operatorname{Im} q_{\omega_2} - \frac{\omega_2^{\zeta}}{\omega_1} \operatorname{Im} q_{\omega_1}}{\omega_1^{\zeta} - \omega_2^{\zeta}}, \quad \text{where } \alpha_0 \neq 0$$

Gauge fixing

$$\frac{\omega_2^{-1}\,\mathrm{Im}\,q_{\omega_2} - \omega_1^{-1}\,\mathrm{Im}\,q_{\omega_1}}{\omega_3^{-1}\,\mathrm{Im}\,q_{\omega_3} - \omega_1^{-1}\,\mathrm{Im}\,q_{\omega_1}} = \frac{\left(\frac{\omega_2}{\omega_1}\right)^{\zeta} - 1}{\left(\frac{\omega_3}{\omega_1}\right)^{\zeta} - 1}, \quad \text{where } \alpha_0 \neq 0$$

- If $\omega_1 < \omega_2 < \omega_3$, the RHS is strictly decreasing in $\zeta \in (0, +\infty)$
- In medical diagnostics $\zeta \in [0.2, 2]$
- If $\frac{\omega_2}{\omega_1} = \left(\frac{\omega_3}{\omega_1}\right)^2$, then there is an explicit formula for ζ, since

$$\frac{\left(\frac{\omega_2}{\omega_1}\right)^{\zeta} - 1}{\left(\frac{\omega_3}{\omega_1}\right)^{\zeta} - 1} = \left(\frac{\omega_3}{\omega_1}\right)^{\zeta} + 1$$

- Use many frequencies to increase stability

Gauge fixing

$$L_\omega = -(\nabla + iA_\omega(x))^2 + q_\omega(x),$$

$$\begin{aligned} A_\omega &= \tfrac{\omega \mathbf{v}}{c^2} + \tfrac{i}{2}\nabla \ln \rho \\ q_\omega &= f_1 - \omega^2 f_2 + i\omega f_3 - 2i\omega^{1+\zeta}\alpha_0 \end{aligned} \quad \left| \quad \begin{aligned} f_1 &= \rho^{\frac{1}{2}}\Delta\rho^{-\frac{1}{2}} \\ f_2 &= \tfrac{1}{c^2} + \tfrac{\mathbf{v}}{c^2}\tfrac{\mathbf{v}}{c^2} \\ f_3 &= \nabla\cdot\left(\tfrac{\mathbf{v}}{c^2}\right) - \tfrac{\mathbf{v}}{c^2}\cdot\nabla\ln\rho \end{aligned} \right.$$

- After determining $F = \operatorname{curl} \frac{\mathbf{v}}{c^2}$, f_1, f_2, f_3, α_0, ζ solve direct problems for PDEs to determine ρ and $\mathbf{u} = \frac{\mathbf{v}}{c^2}$

$$\Delta\rho^{-\frac{1}{2}} = f_1\rho^{-\frac{1}{2}}, \quad \rho|_{\partial D} \text{ is given},$$

$$\begin{cases} \nabla\cdot\mathbf{u} - \mathbf{u}\cdot\nabla\ln\rho = f_3, \\ \operatorname{curl}\mathbf{u} = F, \quad \mathbf{u}|_{\partial D} = 0 \end{cases}$$

- Finally, $c = (f_2 - |\mathbf{u}|^2)^{-2}$, $\mathbf{v} = c^2\cdot\mathbf{u}$

Gauge fixing

Agaltsov (BSM'15): If $\rho \equiv \rho_0$, $\alpha_0 \equiv 0$, then Λ_ω at fixed ω uniquely determines $\mathbf{v}$, c; design of reconstruction algorithm in this case

Agaltsov-R. Novikov (JIIP'15) ($\alpha = \omega^{\zeta(x)}\alpha_0(x)$):

I. $\alpha_0 \equiv 0 \implies \Lambda_\omega$ at 2 ω's determines $\mathbf{v}$, c, ρ.

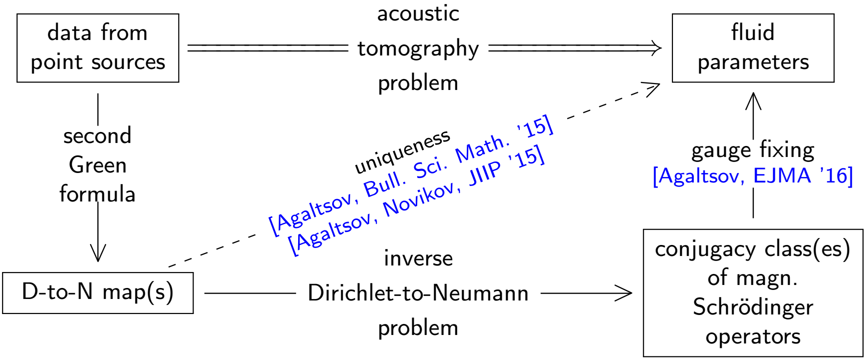
II. $\zeta \neq 0 \implies \Lambda_\omega$ at 3 ω's determines $\mathbf{v}$, c, ρ, ζ, α_0

III. Explicit examples of non-distinguishable fluids when $\zeta \equiv 0$

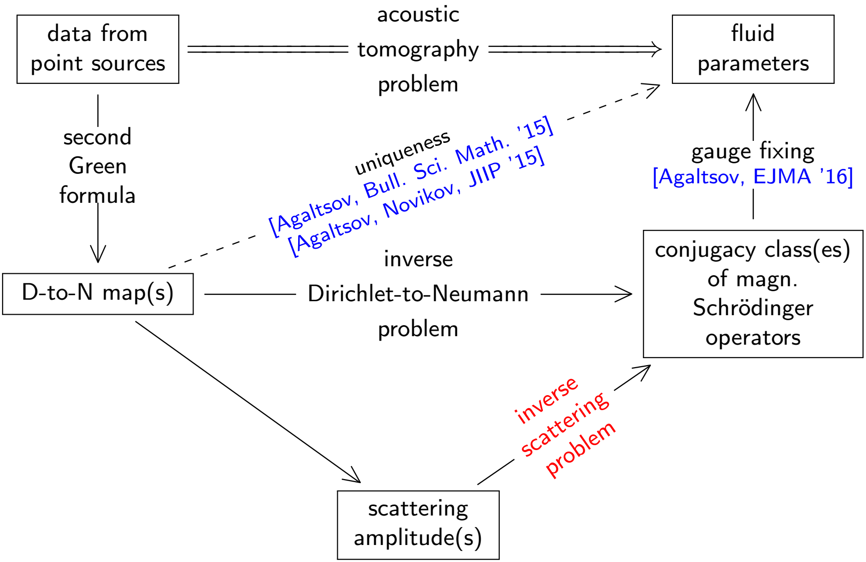
Agaltsov (EJMA'16): design of reconstruction algorithms for the cases of *Agaltsov-R. Novikov (JIIP'15)*

Shurup-Rumyantseva (Acoustical Physics, to app.): numerical implementation, verification, study and improvement of stability

Solving the acoustic tomography problem



Solving the acoustic tomography problem



$$-(\nabla + iA(x))^2\psi + q(x)\psi = E\psi, \quad x \in \mathbb{R}^d,\ E > 0,\ d \geq 2,$$
$$\operatorname{supp} A,\ \operatorname{supp} q \subset D$$

- Classical scattering solutions $\psi^+(\cdot, k)$, $k \in \mathbb{R}^d$, $k^2 = E$:

$$\psi^+(x,k) = e^{ikx} + \begin{pmatrix}\text{universal}\\ \text{spherical}\\ \text{wave}\end{pmatrix} \cdot f_{A,q}(k, \tfrac{|k|}{|x|}x) \cdot (1 + o(1)), \quad |x| \to +\infty$$

- Scattering data $f_{A,q}(k,l)$, k, $l \in \mathbb{R}^d$, $k^2 = l^2 = E$
- Gauge invariance: $f_{A+\nabla\varphi,q} = f_{A,q}$ for $\varphi \overset{\text{fast}}{\to} 0$ at ∞

Direct Scattering Problem: given A, q, find $f_{A,q}$

Inverse Scattering Problem: given $f_{A,q}$ (at fixed E), recover A, q modulo a gauge transform

The direct and inverse scattering problem

- Charged spin-0 particle in a magnetic field:

$$-\frac{1}{2m}\left(\hbar\nabla - iqA(r)\right)^2\psi + q\phi(r)\psi = E\psi$$

- $|f(k,l)|^2$ is proportional to **probability density** of scattering in direction l for a particle emmited in direction k
- **phase shift**: $\frac{\partial \arg f(k,l)}{\partial |k|}$ time delay due to interaction

Lippman-Schwinger (PR'50): equations of type

$$\psi^+(x,k) = e^{ikx} + \int_D G^+(x-y,k)(L_{A,q} - L_{0,0})\psi^+(x,k)\,dx$$

- Eigenfunctions (corr. to E) of $L_{0,0}$ are e^{ikx}, $k^2 = E$
- Search for eigenfunctions $\psi(x,k) = e^{ikx} + \alpha(x,k)$ of $L_{A,q}$:

$$0 = (L_{A,q} - E)\psi(x,k) = (L_{A,q} - L_{0,0})\psi(x,k) + (L_{0,0} - E)\alpha(x,k)$$
$$\text{yielding} \quad \alpha(x,k) = (E - L_{0,0} + i0)^{-1}(L_{A,q} - L_{0,0})\psi^+(x,k)$$

- In 3d $G^+(x,y) = -\frac{1}{2\pi}\frac{e^{i|k|\cdot|x-y|}}{|x-y|}$ (use momentum represent.)
- Since $|x-y| = |x| - \frac{x}{|x|}\cdot y + O(1/|x|)$ at ∞, we have

$$\psi^+(x,k) = e^{ikx} - \frac{1}{2\pi}\frac{e^{i|k||x|}}{|x|}\int e^{-i\frac{|k|}{|x|}x\cdot y}(L_{A,q} - L_{0,0})\psi^+(y,k)\,dy + O(\tfrac{1}{|x|^2}),$$
$$\text{yielding} \quad f_{A,q}(k,l) = (2\pi)^{-3}\int e^{-il\cdot x}(L_{A,q} - L_{0,0})\psi^+(y,k)\,dy$$

The inverse scattering: Born approximation

Consider the case $A = 0$ for simplicity

$$\psi^+(x,k) = e^{ikx} + \int G^+(x-y,k)q(y)\psi^+(y,k)\,dy,$$
$$f_{0,q}(k,l) = (2\pi)^{-d}\int e^{-ilx}q(y)\psi^+(y,k)\,dy$$

- If $q = O(\varepsilon)$, then $\psi^+ = e^{ikx} + O(\varepsilon)$, $f(k,l) = \widehat{q}(k-l) + O(\varepsilon)$
- Faddeev (VLU'56): $f(k,l) = \widehat{q}(k-l) + O(E^{-\frac{1}{2}})$
- Note that for any $p : |p| \le 2\sqrt{E}$ there exist k, l: $k - l = p$, $k^2 = l^2 = E$ (expect to recover $\widehat{q}(p)$, $|p| \le 2\sqrt{E}$, from f at E)

The inverse scattering problem with $A \neq 0$

R. Novikov-Henkin '87, Eskin-Ralston '95, Xiaosheng '05, Päivärinta-Salo-Uhlmann '12: uniqueness

Shiota '85, Henkin-Novikov '88, Arians '97, Nicoleau '97: Born-type reconstruction in high energy limit

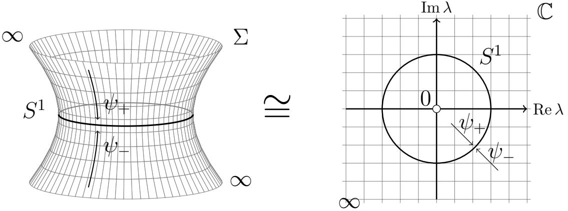
Weder-Yafaev '05, Salo '06: reconstruction at fixed E for $d \geq 3$

Agaltsov-R. Novikov (JMP'14): reconstruction at fixed E, $d = 2$; based on *Faddeev '65*, RHM: *Manakov '81*, *Grinevich-Manakov '86*, *R. Novikov '86, '92, '99*; develops *R. Novikov '92* and generalizes *R. Novikov '99* to $A \neq 0$

Shurup-Rumyantseva (Acoustical Physics, to app.): numerical implementation, verification, stability and convergence study of *Agaltsov-R. Novikov (JMP'14)*

Riemann-Hilbert problem approach to IS in 2d

Faddeev (SPD'66): generalized scattering solutions $\psi(x,k)$, $k \in \Sigma$,
$\Sigma = \left\{k \in \mathbb{C}^2 \setminus \mathbb{R}^2 : \, k_1^2 + k_2^2 = E\right\} \cong \mathbb{C} \setminus (S^1 \cup 0)$; $\mu = e^{-ikx}\psi$ bnd



Grinevich-Manakov (FAA'86): $\bar\partial\mu(\lambda) = r(\lambda)\mu(-1/\bar\lambda)$ in $\mathbb{C} \setminus S^1$

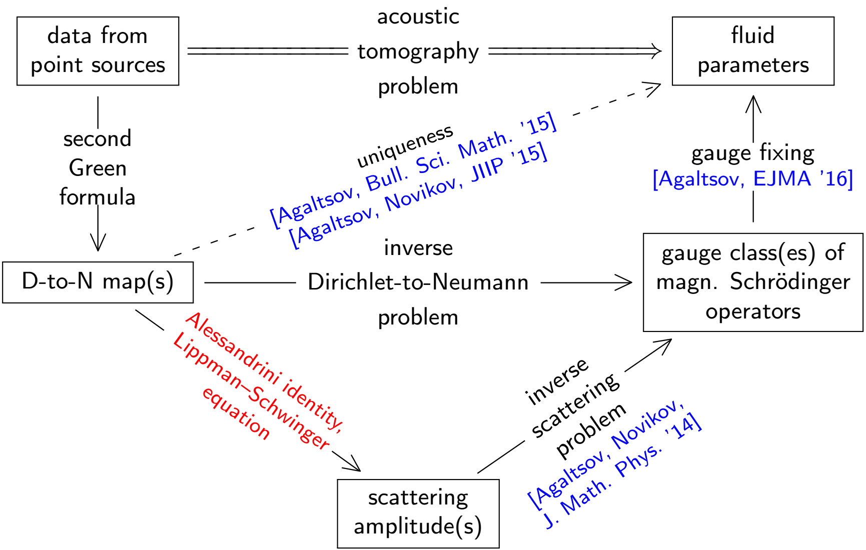
Riemann-Hilbert-Manakov problem: find $\mu \in C(\mathbb{C} \setminus S^1)$ such that
$\bar\partial\mu = 0$ in $\mathbb{C} \setminus S^1$, $\mu(\infty) = 1$ and $\mu_+ - \mu_- = K\mu_+$ (non-loc)

Grinevich-R. Novikov (FAA'85), (DAN'86): solution of this RHM for two types of symmetries of K (relevant to $A = 0$ and to the self-adjoint case)

R. Novikov (TMF'86): finding K from the scattering amplitude

R. Novikov (JFA'92), (PSIM'99): reconstruction algorithm by posing $r = 0$ and estimates ($A = 0$); error $O(E^{-\frac{n-2}{2}})$ vs $O(E^{-\frac{n-2}{2n}})$ in Born app. (n-smt.)

Solving the acoustic tomography problem



Reduction of the IDN problem to the ISP

Novikov (FAA'88):

$$\psi^+(x,k) = e^{ikx} + \int_{\partial D} G^+(x-y,k)\big(\Lambda_{0,q}-\Lambda_{0,0}\big)\psi^+(x,k)\,dx,$$

$$f_{0,q}(k,l) = (2\pi)^{-d}\int_{\partial D} e^{-ilx}\big(\Lambda_{0,q}-\Lambda_{0,0}\big)\psi^+(x,k)\,dx$$

Eskin-Ralston (IPWP'97): non-zero vector potential A: $\Lambda_{A,q}-\Lambda_{0,0}$

R. Novikov (IP'05): non-zero background potential q_0: $\Lambda_{0,q}-\Lambda_{0,q_0}$

Agaltsov (JIIP'15): non-zero vector potential A + background potential q_0 (as in $2D$ acoustic tomography); gauge-covariant Schrödinger operator:

$$L_{A,Q} = -\sum\nolimits_{k=1}^{d}\big(\tfrac{\partial}{\partial x_k}+iA_k(x)\big)^2 + Q(x),$$

$A_j(x)$, $Q(x)\in M_n(\mathbb{C})$ and non-zero background potentials arising in mode tomography of moving ocean, see *Baykov-Burov-Sergeev (ECUA'96)*; aka Schrödinger operator in an external Yang-Mills field

Reduction of the IDN problem to the ISP

Easiest approach: LS equation + NA identity

Lippmann-Schwinger (PR'50): equations of type (for $A = 0$)

$$\psi^+(x,k) = e^{ikx} + \int_D G^+(x-y,k)q(y)\psi^+(y,k)\,dy,$$
$$f_{0,q}(k,l) = (2\pi)^{-d}\int_D e^{-il\cdot y}q(y)\psi^+(y,k)\,dy$$

R. Novikov (FAA'88) ($q_2 = 0$), Alessandrini (AA'88) ($q_2 \neq 0$): if $-\Delta u_1 + q_1 u_1 = Eu_1$, $-\Delta u_2 + q_2 u_2 = Eu_2$, then

$$\int_D v_1(q_2 - q_1)v_2\,dx = \int_{\partial D} v_1(\Lambda_{0,q_2} - \Lambda_{0,q_1})v_2 dx$$

Combination:

$$\psi^+(x,k) = e^{ikx} + \int_{\partial D} G^+(x-y,k)q(y)\psi^+(y,k)\,dy$$

And in a similar way for the scattering amplitude

Reduction of a 3d problem to a multi-channel 2d problem

$$\begin{aligned} &-\Delta\psi + q(x,y,z)\psi = 0, \\ &\psi|_{z=-H} = 0,\ \psi|_{z=0} = 0, \end{aligned} \quad \text{where } (x,y) \in D,\ z \in [-H,0]$$

Mode representation: $\psi = \sum_j \psi_j(x,y)\phi_j(z)$,

$$\begin{cases} -\phi_j'' = \lambda_j\phi_j & \text{in } [-H,0], \\ \phi_j(-H) = \phi_j(0) = 0, \end{cases}$$

$$-\Delta_{x,y}\psi_i + \lambda_i\psi_i + \sum_j Q_{ij}(x,y)\psi_j = 0, \quad i \geq 1,$$

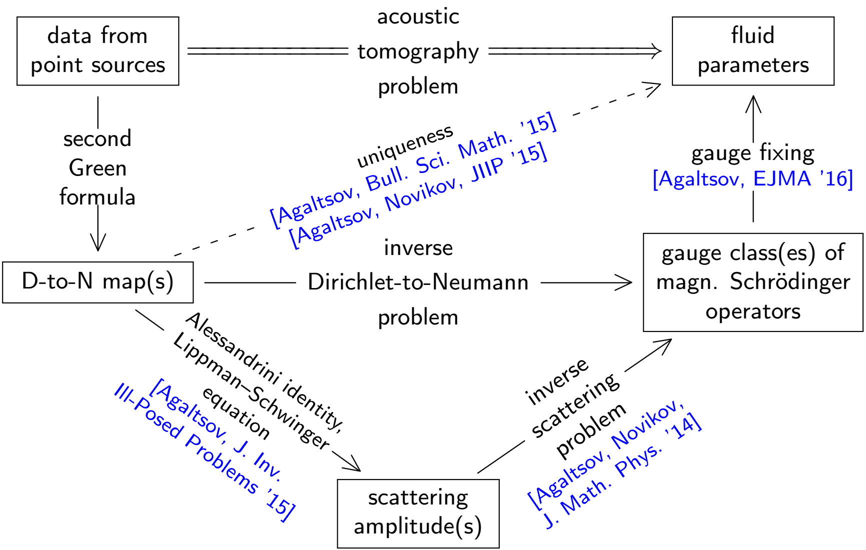
$$Q_{ij}(x,y) = \int_{-H}^{0} \overline{\psi_i(z)}q(x,y,z)\phi_j(z)\,dz$$

see, e.g., Novikov-Santacesaria (BSM'11)

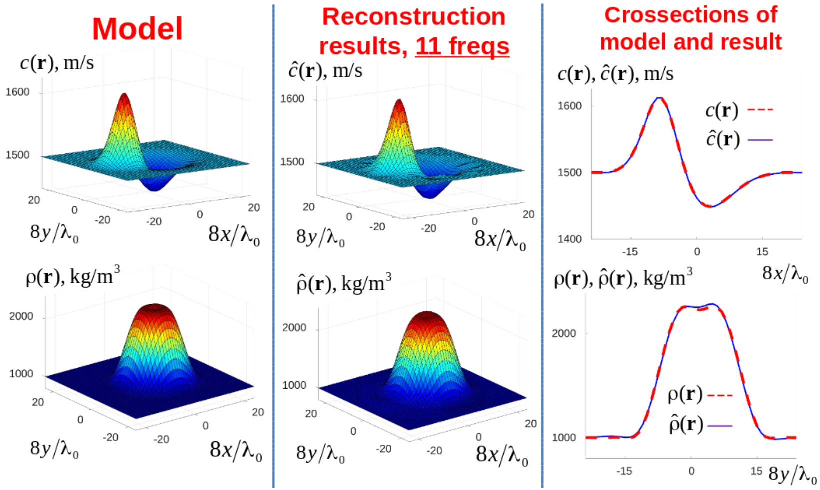
<u>Baykov-Burov-Sergeev (ECUA'96)</u>: fluid flows; application to mode tomography of moving ocean

<u>Novikov-Santacesaria (BSM'11)</u>: analog for the DN maps

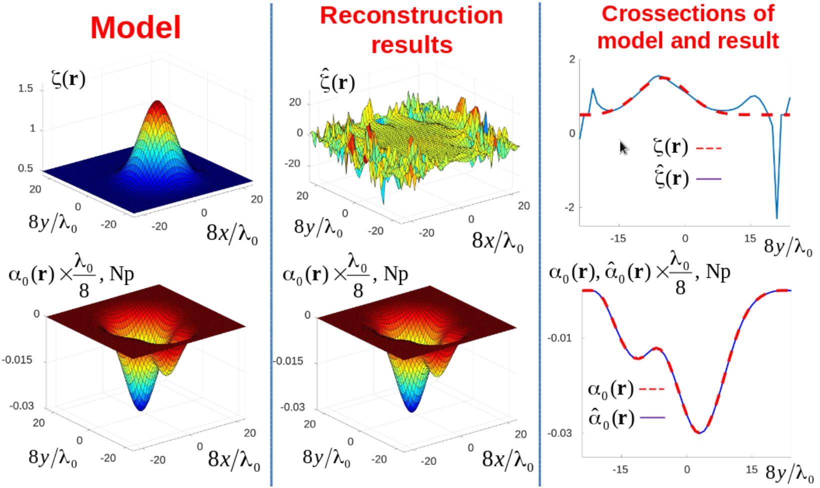
Solving the acoustic tomography problem



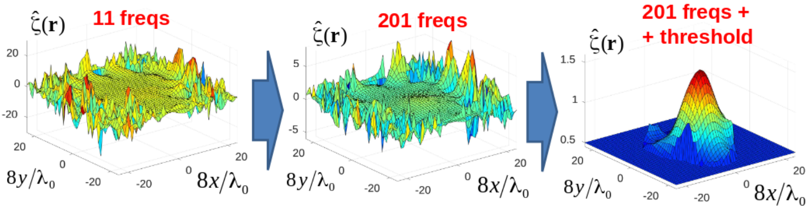
Normally distributed 3% noise



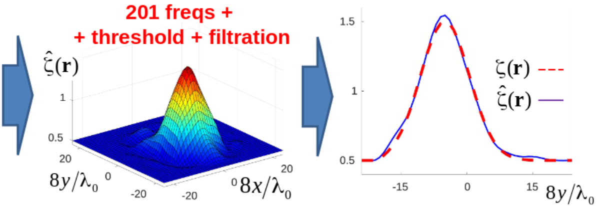
Acoustic tomography: numerics by Shurup-Rumyantseva

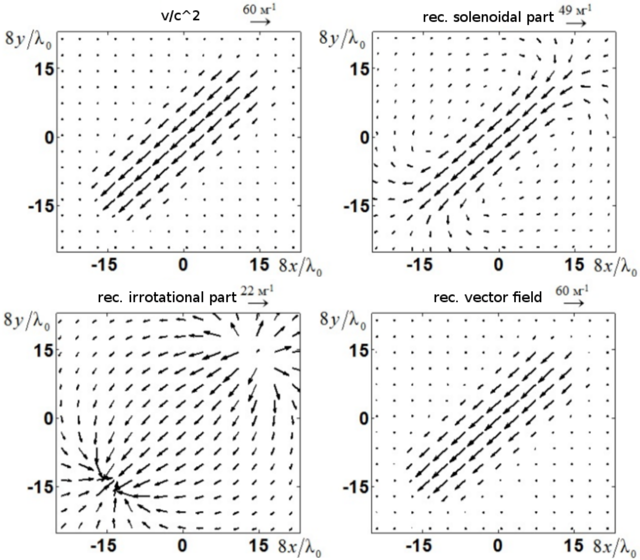


Acoustic tomography: numerics by Shurup-Rumyantseva



Filtration:
- space-spectrum,
- background values.




v/c^2
60 м⁻¹
rec. solenoidal part
49 м⁻¹
rec. irrotational part
22 м⁻¹
rec. vector field
60 м⁻¹
$8y/\lambda_0$
$8x/\lambda_0$
15
0
-15

Thank you!

List of publications

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II. Agaltsov A. D., Novikov R. G., *Uniqueness and non-uniqueness in acoustic tomography of moving fluid*, JIIP 24 (3), 2016, 333-340
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