Inverse Scattering Without Phase Information

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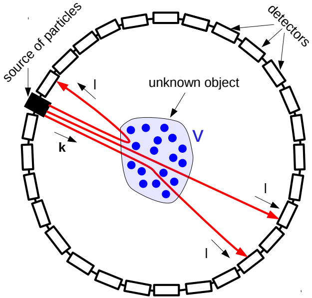
Joint work with T. Hohage and R. G. Novikov

Malta – May 24, 2018

Elastic scattering by a macroscopic object



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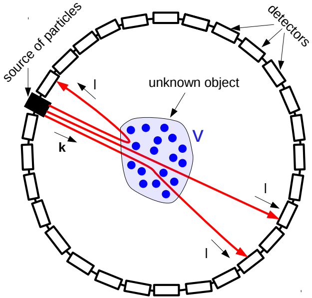

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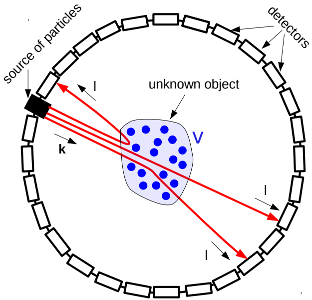

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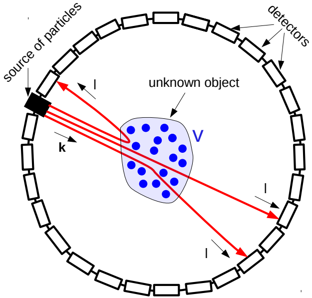

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- **Similar problems:** Klibanov ('14: uniq. in 3d), Klibanov-Romanov ('16: recons.), Romanov-Yamamoto('18)

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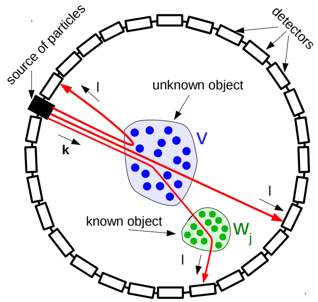

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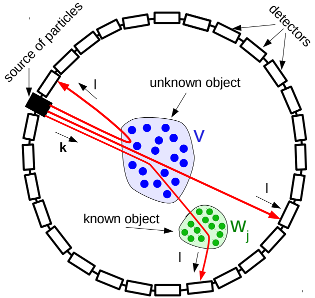

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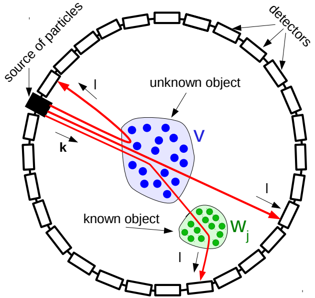

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- **Similar idea in 1d:** Aktosun-Sacks (IP'98)

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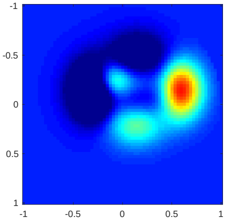
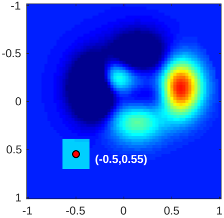
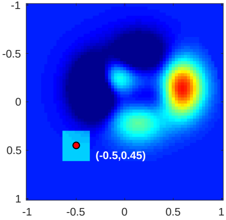
Unknown potential and background potentials:



v



$v + w_1$



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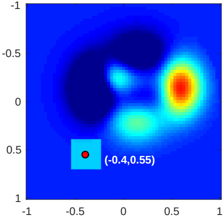

$v + w_3$

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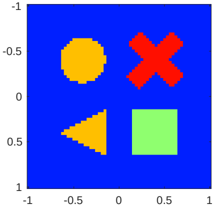
10^8	23	1.8	1.6
10^9	22	1.3	0.83

Nonsmooth potential:



Exact potential

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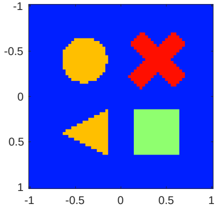

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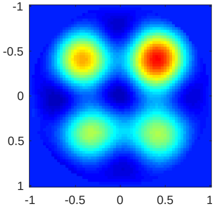

Born approximation

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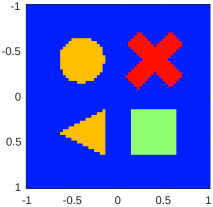

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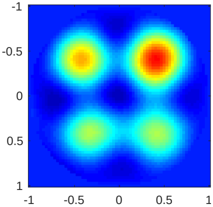

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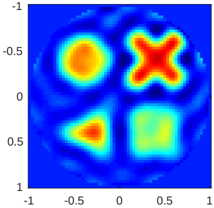

Our method

Our method vs NewtonCG: Pros and Cons

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stopping rule	no	yes

IV. Implementation

- Born approximation formula

$$\widehat{v}(p) = f(\mathbf{k}, \mathbf{l}) + O(E^{-\frac{1}{2}}), \quad p = \mathbf{k} - \mathbf{l}$$

$$(\mathbf{k}, \mathbf{l}) \in T_E \;\Rightarrow\; p \in B_{2\sqrt{E}}$$

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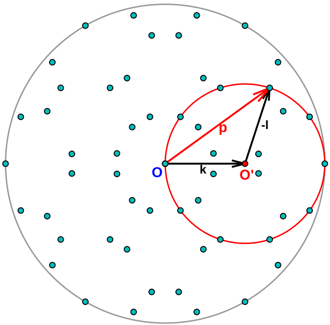

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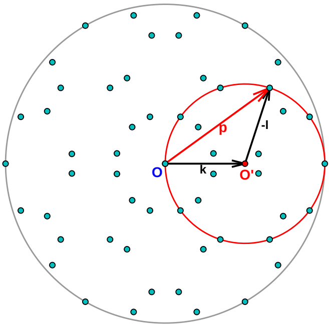

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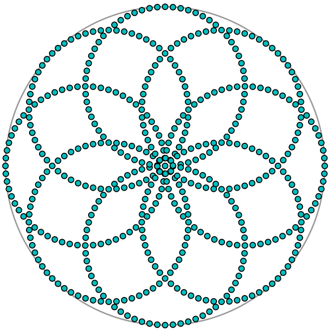
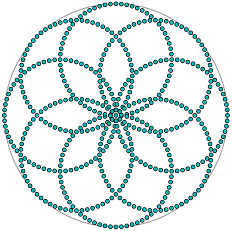


$n_1 = 8,\ n_2 = 64$



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$$v(x) = \int_{\mathbb{R}^2} e^{-ipx} \widehat{v}(p) dp$$

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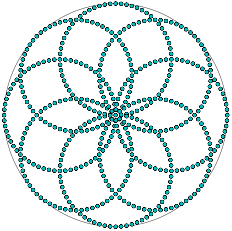

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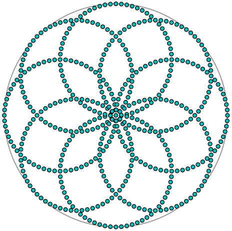

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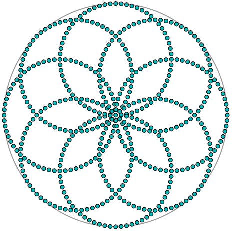

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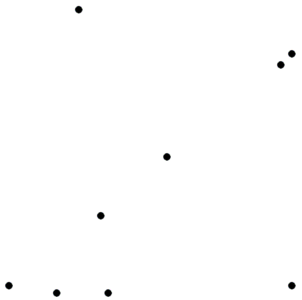
Question: How to subdivide $B_{2\sqrt{E}}$ in cells with nodes at $\mathcal{E}_{n_1,n_2}$?

Voronoi diagram:



/wiki/Voronoi_diagram

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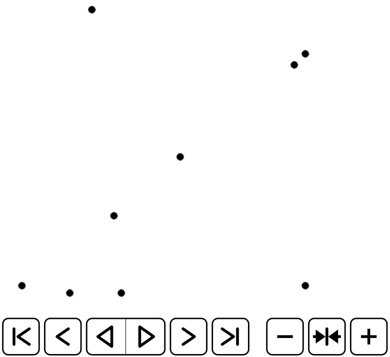

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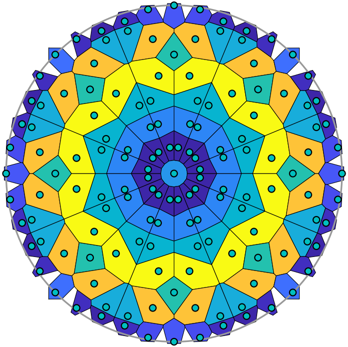

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Limitations:

- Convergence at small E

References

[1] A. D. Agaltsov, T. Hohage, and R. G. Novikov. "An iterative approach to monochromatic phaseless inverse scattering". In: (). in preparation.

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