Machine Learning for
Combinatorial Optimization
COMPETITION 2021

NIPS 2021 ML4CO Dual Task 1st Solution

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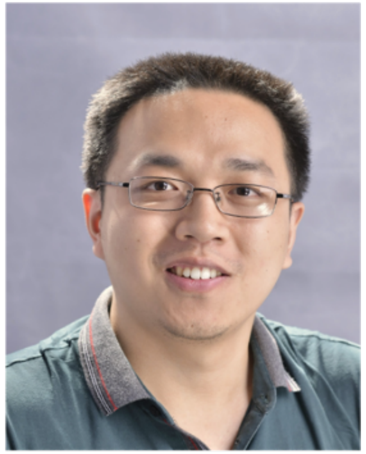


Results

Dual task

team	item_placement			load_balancing			anonymous		
	dual integral	(cum. reward)	rank	dual integral	(cum. reward)	rank	dual integral	(cum. reward)	rank
Nuri	2307.39	(6684.00)	1	6178.82	(630787.18)	6	3770677.01	(27810782.42)	1
EI-OROAS	2321.09	(6670.30)	2	5221.69	(631744.31)	1	4423016.69	(27158442.74)	4
EFPP	2503.86	(6487.53)	3	5600.98	(631365.02)	3	5241194.97	(26340264.47)	13
KAIST_OSI	2794.83	(6196.56)	7	5555.42	(631410.58)	2	4955048.57	(26626410.86)	9
qqy	2614.16	(6377.23)	6	6408.69	(630557.31)	11	4359960.40	(27221499.03)	2
DaShun	4172.32	(4819.07)	14	6067.75	(630898.25)	4	4430033.29	(27151426.15)	5
lxj24	2547.84	(6443.55)	4	6812.76	(630153.24)	19	4529137.95	(27052321.48)	7
null_	4326.88	(4664.51)	16	6667.65	(630298.35)	14	4397369.92	(27184089.51)	3
Superfly	2967.19	(6024.20)	9	6219.04	(630746.96)	9	5208108.44	(26373350.99)	10

	Item Placement	Load Balancing	Anonymous
Baseline	4937.8	624043.6	30965031.6
Our Model	**7561.6 (1st)**	**624928.9 (6th)**	**32898846.0 (1st)**

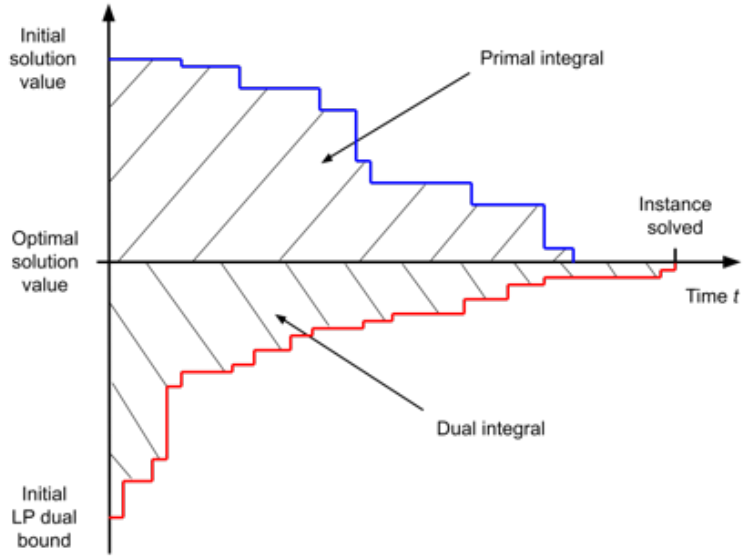
Performance Comparison on Validation Dataset

Overview

- Background
- Different Methods on Benchmarks
  - "Anonymous"
  - "Item Placement"
  - "Load Balancing"

Background: Dual Task VS Primal Task

- Primal Task: Produce feasible solutions, in order to minimize the **primal integral** over time
- Dual Task: Select branching variables, in order to minimize the **dual integral** over time
- Making branching decisions has received little theoretical understanding to this day

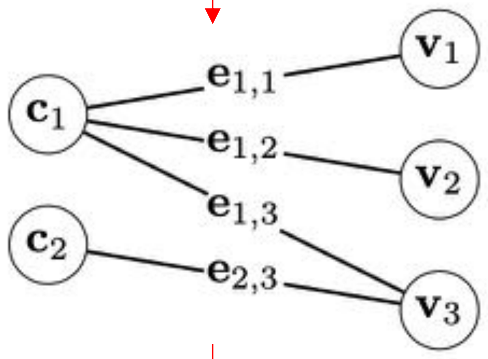


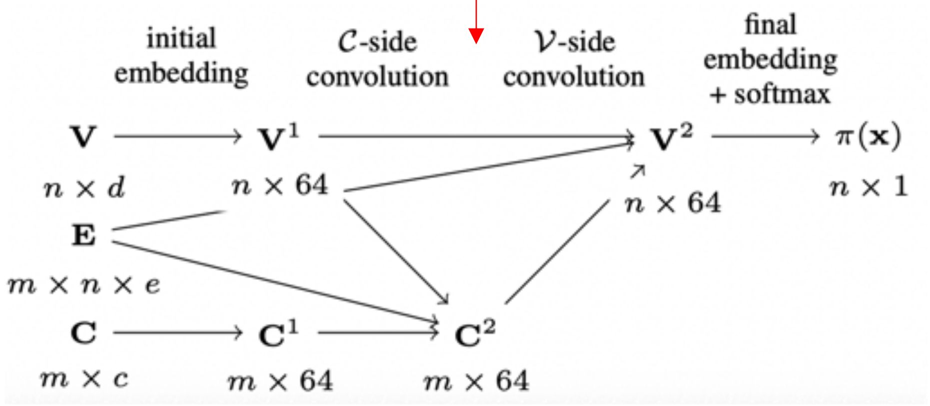
Background: Baseline Method

- Data Collection
  - Interact with CO solver SCIP and use Strong Branching as expert knowledge
- Training
  - Convert CO expressions to bipartite graphs
  - Parametrize variable selection policy as GCNN
  - Calculate cross entropy loss between GNN’ s node selection policy and expert knowledge of Strong Branching

X1 + X2 + X3 <= 5

X3 <= 2

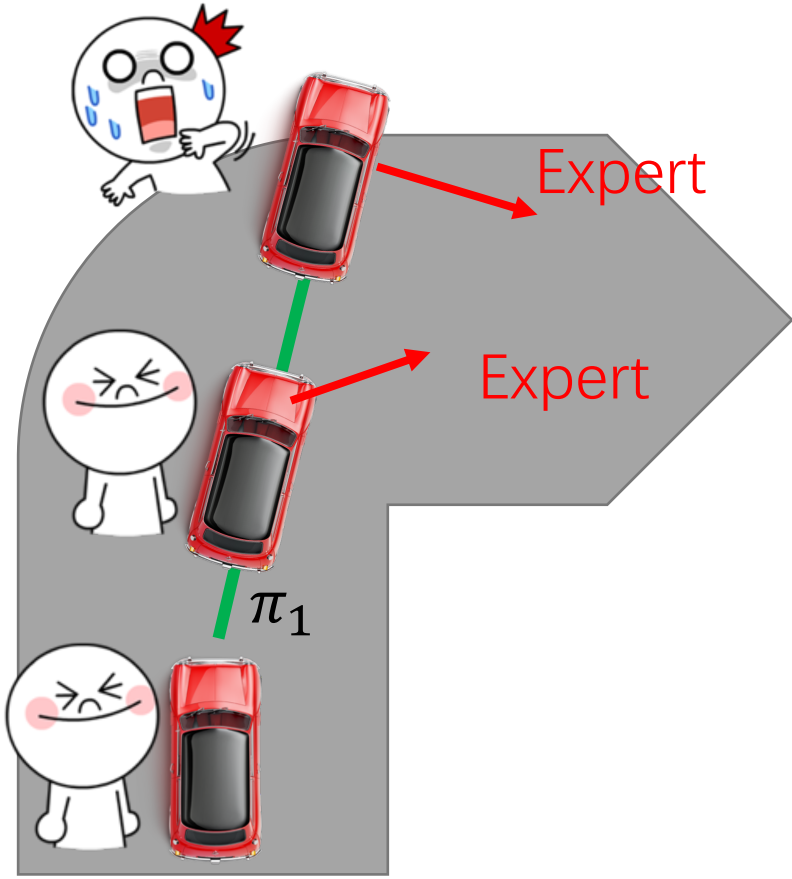




Exact Combinatorial Optimization with Graph Convolutional Neural Networks (NIPS 2019)

Improvement on "Anonymous"

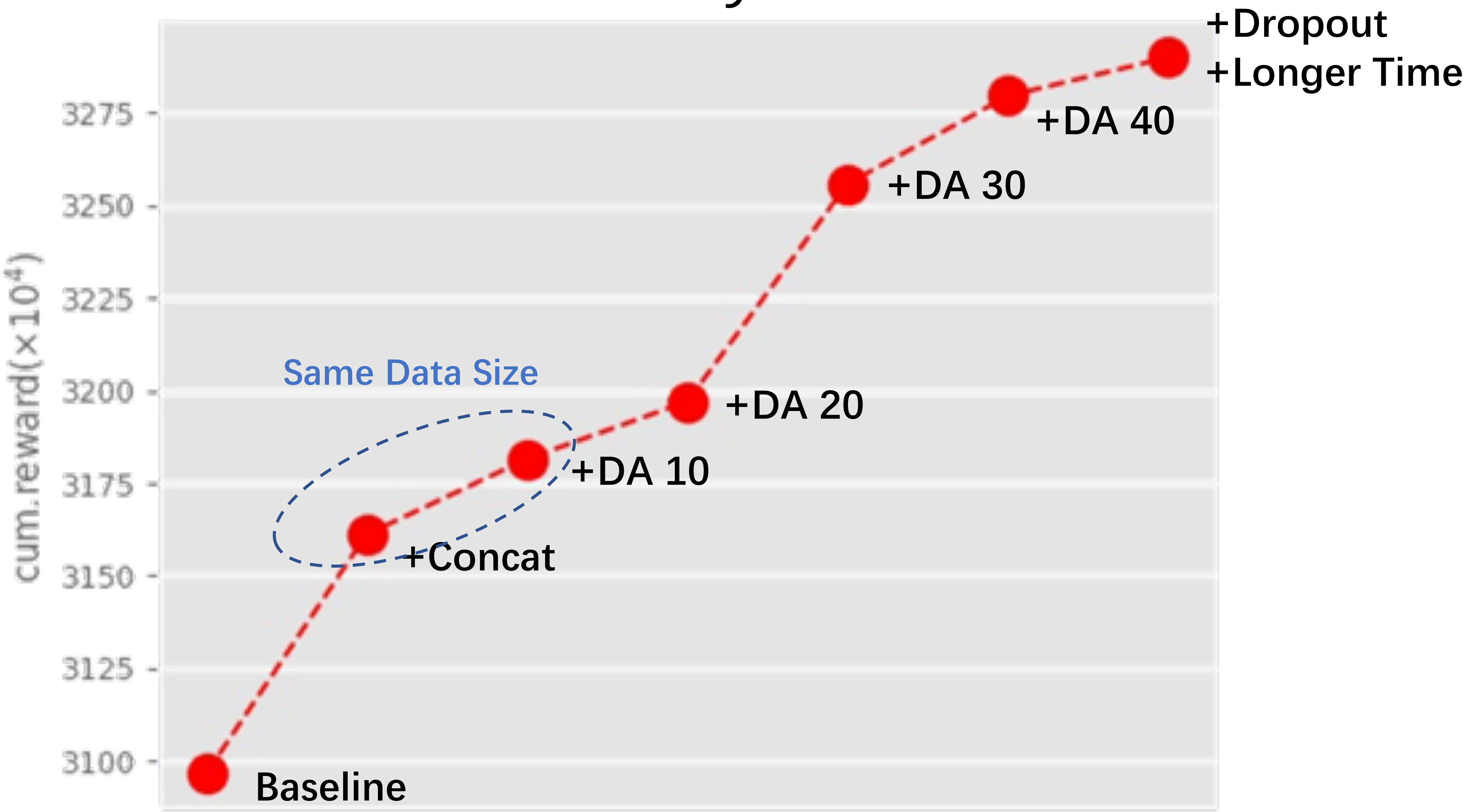
- Built upon Baseline Method
- Training only on "expert" data loses diversity
  - Use DAgger [1]
  - Initialize a random model π_0 and an empty dataset D
  - Iteratively collect new data and train new models
- Extra Tricks
  - Longer time limit when collecting: 15min → 20min
  - Summation to Concatenation:
    - R = f(g(L+R+Edge), R) → R = f(g(concat(L, R, Edge)),R)
  - Dropout
    - Add dropout layers (p=0.2) at embedding layers and final output layer



Demonstration of Dataset Aggregation [2]

[1] A Reduction of Imitation Learning and Structured Prediction to No-Regret Online Learning (AISTATS 2011)
[2] Imitation Learning (Hongyi Li, Machine Learning 2021 Spring Course)

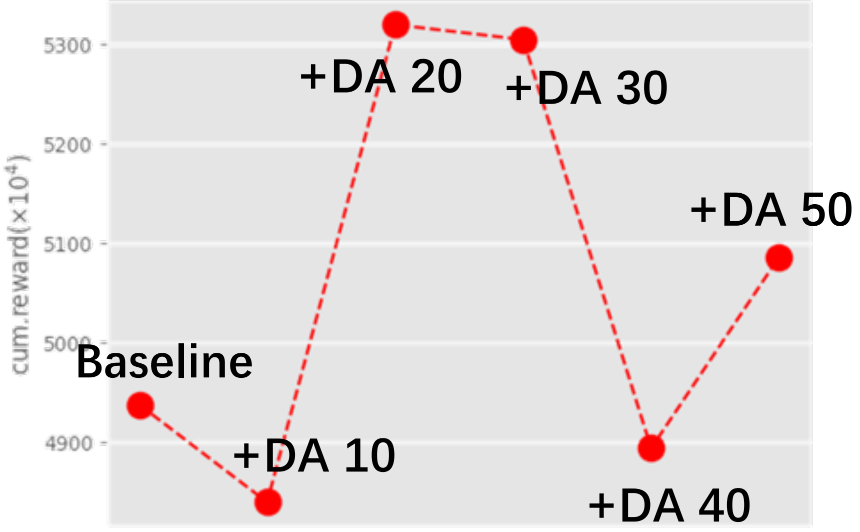
Improvement on "Anonymous"



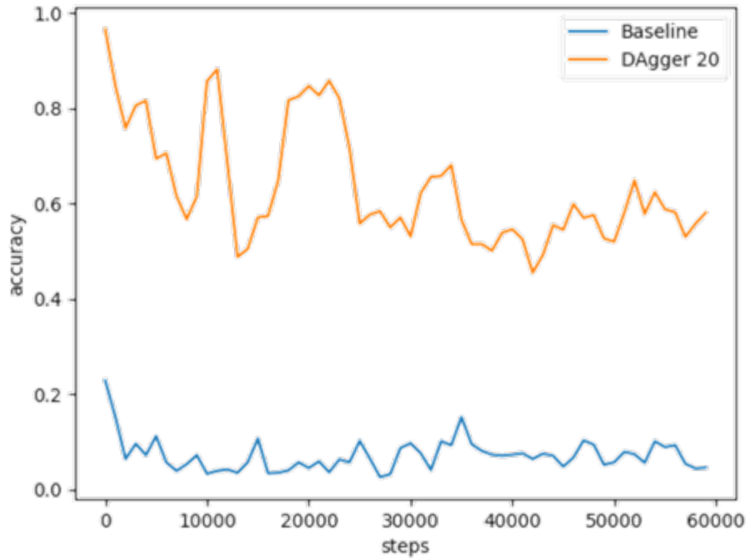
Cum. Reward of Anonymous in Validation Dataset

But, "Item Placement" is Different...

- DAgger improves accuracy **significantly**, but only improves rewards **slightly**



Cum. Reward of Item Placement in Validation Dataset

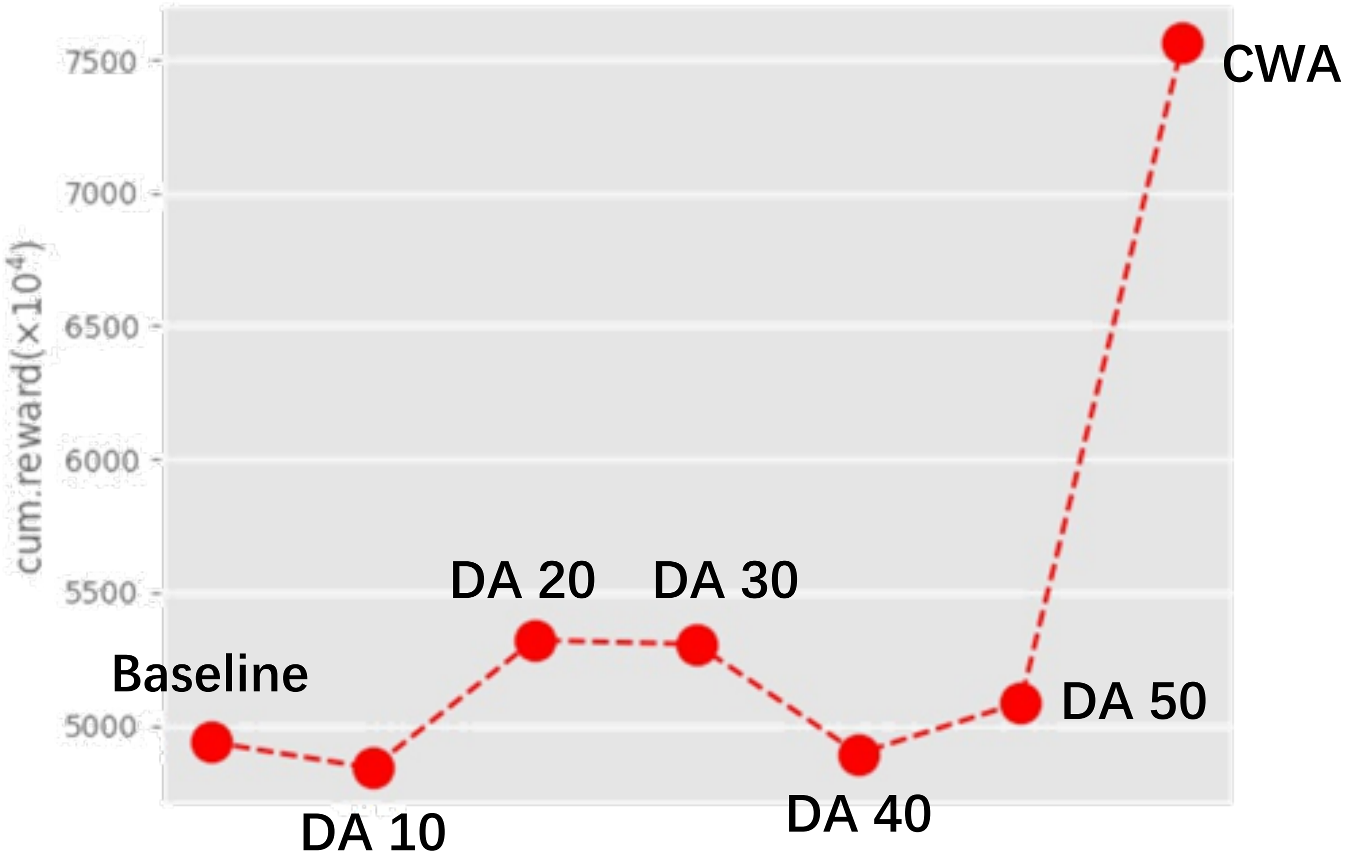


Accuracy When Interacting With Solver

	Accuracy When Interacting	Cum. Reward
Baseline	0.072	4937.8
DAgger 20	**0.635**	**5319.8**

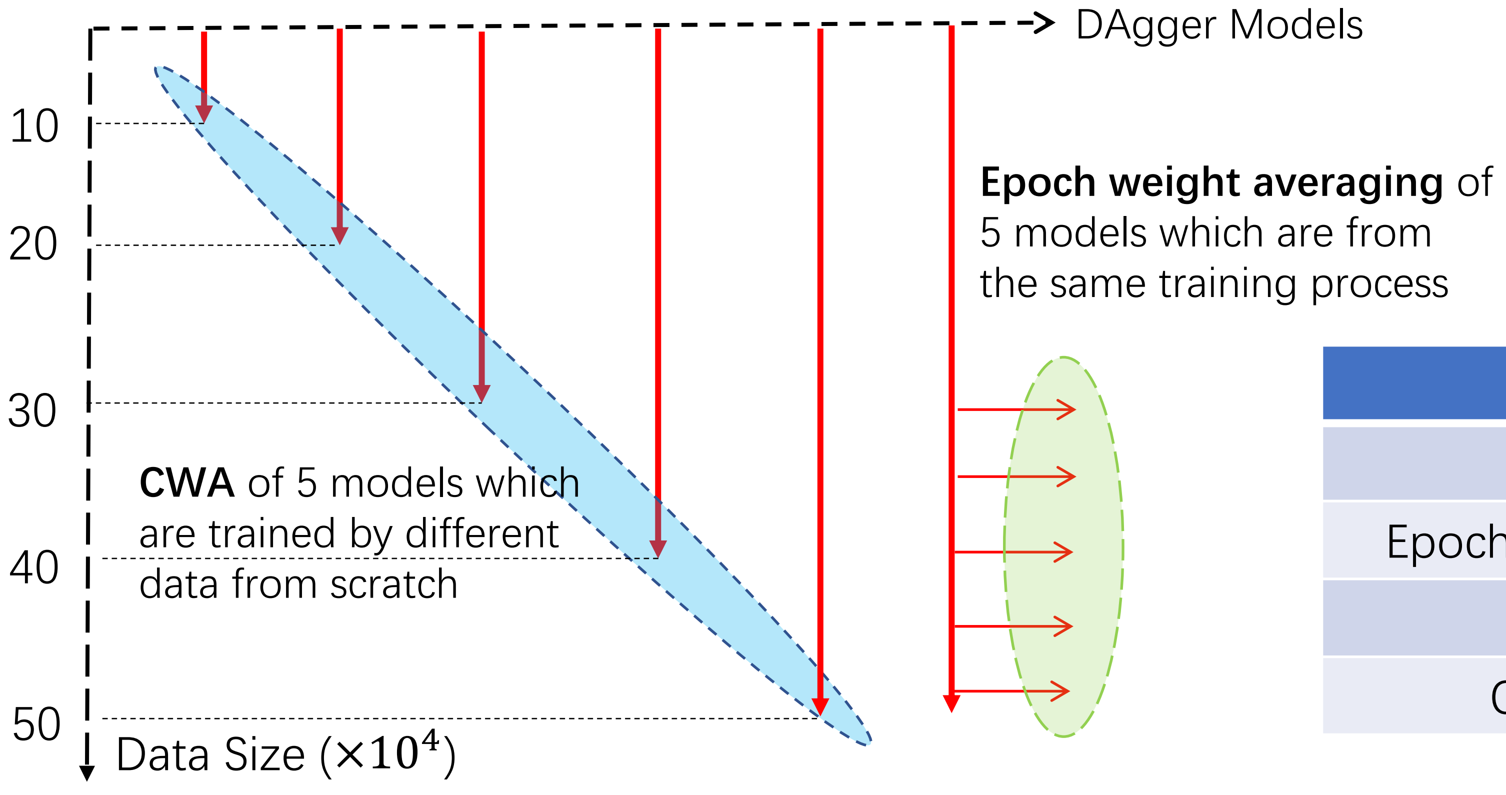
CWA: Cross-Model Weight Average

- CWA performs far more better than all previous trained models



CWA: Cross-Model Weight Averaging

- CWA: For models trained by different data from scratch with parameters $(\theta_0, \theta_1, \ldots, \theta_{n-1})$, build a new model π_{avg} with parameters $\theta_{avg} = \sum_{i=0}^{n-1} \theta_i / n$



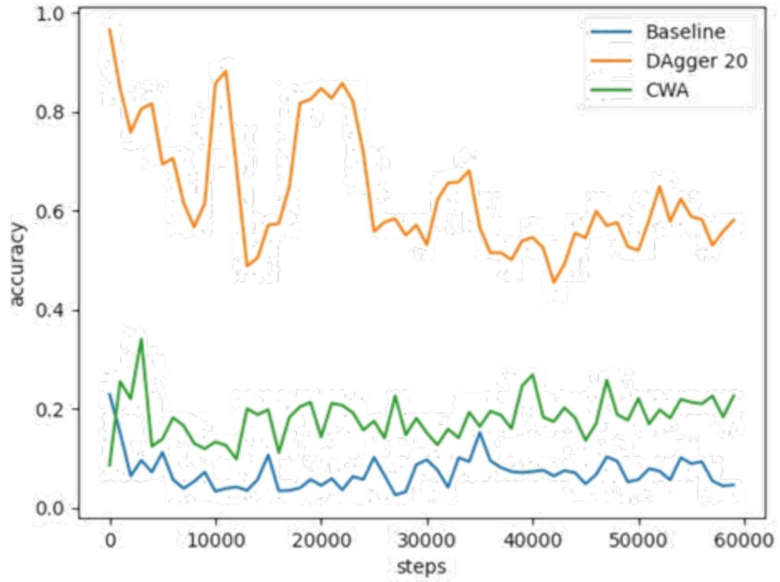
CWA VS Epoch Weight Average

	Cum. Reward
Baseline	4937.8
Epoch Weight Average	4759.1
CWA (5)	6850.7
CWA (Top 3)	**7561.6**

CWA: Cross-Model Weight Averaging

- CWA improves accuracy **slightly**, but improves rewards **significantly**

	Accuracy When Interacting	Cum. Reward
Baseline	0.072	4937.8
DAgger 20	**0.635**	5319.8
CWA (Top 3)	0.182	**7561.6**



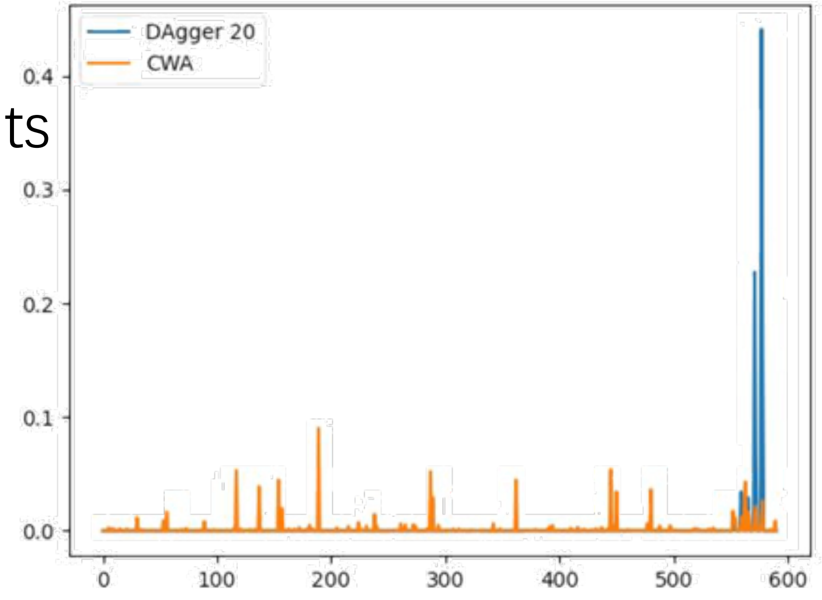
Accuracy When Interacting With Solver

CWA Works, but Why?

- ***We don' t know (yet)...***
- Test CWA on unseen dataset (collected by baseline method)
  - has lower accuracy but still has lower loss
  - produces more high probability selectable points

	Top 1	Top 3	Top 5	Loss	Cum. Reward
Model 0	**0.850**	**0.957**	**0.981**	7.38	5304.9
Model 1	0.797	0.917	0.966	9.50	5319.8
Model 2	0.795	0.916	0.961	6.18	5237.5
CWA	0.721	0.822	0.870	**2.96**	**7561.6**

Performance of models before and after weight averaging



Probabilities of Selecting Different Variables

Strong Branching Fails on "Load Balancing"?

- Models trained by baseline method and its modifications can not surpass random strategy.
- Strong Branching seems to fail

	Top 1	Top 3	Top 5	Cum. Reward
Baseline	**0.456**	**0.729**	**0.820**	624043.6
Random	0.013	0.034	0.052	**624928.9**

Performance Comparison of Baseline and Random

Conclusion

- Best performing methods have diverged on different benchmarks
    - "Anonymous": DAgger without CWA works
    - "Item Placement": DAgger with CWA rocks
    - "Load Balancing": Random strategy is the best we can get