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Mapping Equity and Quality in Mathematics Education

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Chapter 18
Foundational Mathematics: A Neglected Opportunity

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Early childhood education represents a powerful opportunity to address equity issues in mathematics education in the United States. This is true in two distinct respects. First, there are persistent differences in mathematics achievement between the United States and many other developed nations (National Research Council 1989, 1990; Schoenfeld 1992). Tellingly, these differences appear as early as preschool, where children from other developed and developing countries outperform their American counterparts on such beginning mathematics concepts as number words and early addition (e.g., Ginsburg et al. 1997; Starkey et al. 1999). Second, there is clear evidence of a national mathematics "achievement gap" among U.S. students of different demographic backgrounds, which appears as early as kindergarten (Entwisle and Alexander 1990; Griffin et al. 1994; Jordan et al. 1992). This gap disproportionately favors children from higher socio-economic status backgrounds (Denton and McPhee 2009), indicating that children who lack economic means have an additional disadvantage in terms of readiness to learn primary-level mathematics. Because this gap persists and often widens throughout schooling (National Research Council 2009), early childhood is the best time to eliminate it, creating subsequent equitable access to mathematical learning in elementary school.

Given that approximately 80% of U.S. preschool-age children are in some form of out-of-home care, there is ample opportunity to address mathematics through the early care and education system (Jacobson et al. 2007). While intervention that begins early has effects that extend into later years (Bowman et al. 2001; Clements et al. 2004), early intervention specifically focused on mathematics has positive effects on student learning that generalize beyond it (Fuson et al. 1997). The effectiveness of such intervention points out a dearth of quality teaching that, if remedied, can provide more equitable preparation for elementary school among and for U.S. students.

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Unfortunately, mathematics education for preschool children has been neglected in the United States, and for many years. This is in contrast to early literacy, an educational topic that has received a wealth of attention over the past two decades. So successful have been the efforts of the early literacy community to explain itself that "Today…early childhood literacy is regarded as the single best investment for enabling children to develop skills that will likely benefit them for a lifetime," (Dickinson and Neuman 2006, p. 1). This disparity of attention paid to early literacy versus early mathematics is seen in conference programs, and evidenced in the teaching practices of early childhood teachers. For example, at a recent meeting of the National Association for the Education of Young Children (NAEYC 2008), a conference focused almost entirely on preschool education, there were only 23 presentations on mathematics, compared to 65 on literacy. Further, a recent study of early childhood classrooms in Chicago, Illinois, showed that 90% were likely to conduct literacy-related activities on any given day, while only 21% were observed to conduct mathematics activities (Chicago Program Evaluation Project 2008).

Neglect of early mathematics also occurs among researchers in mathematics education. At the 2008 Research Pre-session of the Annual Meeting of the National Council of Teachers of Mathematics (NCTM), only three of 106 presentations focused specifically on the mathematics education and learning of children before first grade. The Chicago Public Schools, the third largest public school system in the United States, provides math coaches to classroom teachers in grades Kindergarten to 12, but no such provision is made for pre-k teachers and the children they teach. Clearly, early mathematics education for three- to five-year-olds has not yet become part of U.S. mainstream education (Ginsburg et al. 2008), and while quality mathematics teaching is the focus of current reform efforts in elementary and secondary schools (see, e.g., Hill et al. 2005), there is little to no discussion of its relevance to early childhood education.

We argue that a re-definition of early mathematics as *foundational mathematics* may further both quality and equity agendas in the United States. As described in greater detail below, foundational mathematics is that mathematical thinking that can develop prior to primary school. It anticipates arithmetic and does not rely upon the use of secondary symbol systems, such as written numerals, but is clearly mathematical, and susceptible to teaching intervention. Specifically, we propose that the reconceptualization of early mathematics as foundational mathematics: (1) distinguishes it from the mathematics that is more appropriate for elementary school, (2) names this mathematics, legitimizing it as content that can be both understood and taught, and (3) suggests its important role in preparing learners for more advanced thinking.

To advance this case, we begin by describing how in the United States both the early childhood and mathematics education communities have come to neglect early mathematics. We define foundational mathematics at length, endeavoring to make clear its centrality to equitable access to mathematical learning, and delineating how its common acceptance might address those misconceptions that have kept it from being taught. Finally, we report on a proven intervention designed to help early childhood teachers understand foundational mathematics, thereby improving the quality of their teaching.

Early Childhood Education: Why Does It Neglect Mathematics?

There are two related reasons why early education has managed to ignore the importance of improving its teaching of mathematics for so long. First, there is a belief that young children are not ready to engage mathematical thinking before primary school, an idea often (misguidedly) supported through reference to "developmentally appropriate practice," the dominant paradigm for early care and education in the United States. Second, there is a distinct lack of both good mathematical knowledge and mathematical confidence among early childhood teachers, who like all teachers, tend to teach best what they know best. While each of these causes has an unfortunate tendency to feed and fortify the other, each also has roots in different socio-historical events and understandings.

Developmentally Appropriate Practice

In the United States in the early 1980s, there was a convergence between middle-class families who hoped preschool could give their children a "leg up" on academic achievement, and those concerned with promoting educational equity by improving the preparation of children from socio-economically disadvantaged families (Golbeck 2001). As a result of this convergence, more state and local governments included preschool in the public schools (Spodek and Brown 1993), and emphasized early formal academic instruction, based on the idea that it was an enriching experience for young children (Gersten and Carnine 1984; Veras 1975). Direct instruction, drill, and worksheets focused on imparting basic skills tended to be part of this effort to enhance the education of young children, as the typical first grade curriculum was simply "pushed down." This approach to preschool was met with something of a backlash within the decade. Elkind (1987), for example, argued that children's predilection to learn by play and exploration was inappropriately squelched by a curricular focus on early academic achievement. Other child development experts joined the debate and echoed his sentiments (see, e.g., Kagan and Zigler 1987; Sigel 1987), and in 1987, the NAEYC issued its Position Statement on Developmentally Appropriate Practices (DAP) (Bredekamp 1987).

Essentially, the Position Statement on DAP supported a developmentally universalist, child-centered approach to teaching. While it was not tied to any specific curricula, it advocated a set of principles NAEYC felt should govern the education of all young children. Specifically, it recommended that teaching be responsive to the needs and capabilities of individual children, emphasizing teacher observation and support over assessment and the introduction of skills. Though not without controversy, the influence of DAP on early childhood practice in the United States and elsewhere has since been particularly pervasive and far-reaching (Bowman et al. 2001).

In part due to this almost-universal emphasis on child-led teaching at the preschool level, there are many early childhood teachers who firmly believe the study of mathematics before elementary school is developmentally inappropriate. Not unlike many of their elementary counterparts, preschool teachers often think that memorization of facts is the key element of mathematical learning (Sarama and DiBiase 2004). There is also evidence that the math-related knowledge and teaching strategies of early childhood teachers, scant though they may be, are focused on modeling the use of procedures to solve problems and ensuring correct implementation of computational skills (Copley 2004, p. 403). While the larger mathematics education community has made a lot of headway in "redefining mathematics as a dynamic discipline full of opportunity for inquiry and discovery," the early childhood community has not yet gotten the message (Feiler 2004, p. 399). Given their ideas, skill sets, and beliefs about mathematics, it is easy to see how early childhood teachers would think it developmentally inappropriate to teach it to preschoolers.

Lack of Content Knowledge and Confidence

Early childhood teachers' sense that mathematics is not an appropriate topic for preschoolers is further encouraged by their belief that it is not an appropriate topic for themselves (Ginsburg et al. 2006). Most early childhood teachers, unlike their peers at the elementary school level, have received no training in teaching mathematics, even if they have a bachelor's degree in early childhood education (Copple 2004; Ginsburg et al. 2006). This educational lack is both compounded by and compounds a lack of confidence in their mathematical abilities among early childhood teachers, many of whom were counseled into teaching at the younger grades so they could "escape" mathematics (Andews 2009, personal communication). The joint position statement on preschool math by the NAEYC and the NCTM (NAEYC 2005) supports this view, noting that the general lack of knowledge and preparation contributes directly to poor math-related attitudes and a lack of confidence among many early childhood teachers.

While lack of confidence does not, in itself, prevent a teacher from teaching math, it appears to feed an unfortunate tendency to avoid math in the classroom. A recent contribution to preschool math teacher training goes so far as to devote a section to lack of confidence, noting "Math Anxiety—You Can Handle It" (Smith 2001, p. 2), and when surveyed (Carpenter et al. 1988), both pre- and in-service teachers in early childhood classrooms expressed great reluctance to teach mathematics, making comments like "I don't do math." In the United States, we have unwittingly assembled a cadre of early childhood teachers who are less knowledgeable and more fearful of mathematics, on average, than the general population. It is no wonder they are less likely to teach it than might be helpful.

It is also true that this lack of knowledge and confidence on the part of early childhood teachers has the added detriment of diluting the quality of mathematics that is taught. Research shows that when knowledge and confidence are weak, teachers understandably tend to rely on text, and to present content as a collection of unchanging and not highly connected facts (Brophy 1991). Only a solid and nuanced understanding of content allows a teacher to anticipate how thinking about it develops, implement curricula effectively, and most importantly, flexibly capitalize on those moments in classroom life when a key concept can be clarified or reiterated (National Research Council 2009). Describing early childhood educators, Copley notes "to them, mathematics is a difficult subject to teach and one area that they often ignore except for counting and simple arithmetic" (2004, p. 402). There is good reason to assume that the lack of mathematics knowledge among preschool teachers has not only influenced the field to avoid and ignore mathematics as a topic, but has ensured that most of the math teaching that occurs is mediocre, at best.

Mathematics Education: Why Does It Neglect Early Childhood?

Over the last century, the dominant paradigm for analyzing elementary math content and advocating reform in its teaching in the western world has expressed itself as a dichotomy between math *concepts*, on the one hand, and math *procedures* on the other. In this orientation, *conceptual mathematical knowledge* is richly connected mathematical thinking that embodies information about relationships between things, such as the idea that any set of items can be counted, or that adding and subtracting are inverse operations and therefore can "undo" one another. *Procedural mathematical knowledge*, on the other hand, is meant to include the forms, rules, and procedures that make it possible to complete mathematical tasks. Procedures are generally sequential lists of actions that produce desired outcomes, such as "perform operations within parentheses first" or "when multiplying by 10, move the decimal one place to the right."

While mathematics education reformers spent much of the twentieth century debating whether to emphasize concepts over procedures or vice versa (see, e.g., Bruner 1960; Gagné 1977; McLellan and Dewey 1895; Thorndike 1922; Wheeler 1939), in the early 1980s Resnick and Ford suggested that *relationships between* concepts and procedures might be a better place for educational focus. This suggestion was buoyed by the work of Giyoo Hatano, who, in 1982, opined that the goal of education—whatever the subject matter—should be to foster *adaptive expertise* among students (Hatano 1982). As he defined it, adaptive expertise is "the ability to apply meaningfully learned procedures flexibly and creatively" (Hatano 2003, p. xi). In this construction, teachers should prepare students who can use their existing knowledge to *create* effective procedures that solve problems they have not encountered before. Hatano makes the point that conceptual knowledge gives "meaning to each step of the skill and provide(s) criteria for selection among alternative possibilities for each step within the procedures" (1982, p. 15). Elementary math education reformers latched onto this notion of adaptive expertise as a worthwhile goal, and have advocated teaching methods meant to prevent a child from learning a mathematical procedure with no connection to mathematical meaning, or concepts (see, e.g., Baroody and Coslick 1998; Baroody and Dowker 2003; Clements et al. 2004).

This emphasis on avoiding disconnection rests on the proposition that the mathematics to be learned employs written symbols: namely, numerals and mathematical notation. Disconnection between procedures and concepts is only possible when mathematical procedures are not *implicitly* meaningful, as occurs when written notation is in use. Because of its focus on this problem, the concept-procedure literature includes an unspoken assumption (e.g., Hiebert 1986) that all the important mathematics occurs at the point when the thinker begins to use written symbols to describe and manipulate it. Any mathematical thinking that occurs prior to writing is generally not considered. Hiebert and Lefevre (1986) comment, "connections between conceptual and procedural knowledge still are in place as children enter school," (p. 19), thereby dismissing the preschool period as a time for teaching interventions. Their assumption seems to be that the mathematical thinking that develops before school does so in a manner that is beyond intervention; differences in the educational experiences of the very young are not acknowledged by this argument as significant. This idea about preschool mathematics, combined with the math education literature's characterization of good teaching as imparting new written procedures in ways that are meaningfully connected to concepts, has distracted math educators from a serious consideration of the mathematics teaching that is possible prior to the use of written symbols by its students.

Why Should We Call Early Mathematics Foundational?

While it is undoubtedly true that the use of numerals, notation, and arithmetic propels mathematical thinking into levels of complexity far beyond what humans would be capable of without them, it is also true that mathematical thought can and does occur prior to their use, and that it is highly abstract. In fact, the number system, which many consider the earliest mathematics, rests on the understanding of relational abstractions, and does so prior to the use of any written notation (Dehaene 1997; Wiese 2003).

Take the idea of "three." To illustrate three, we must ask "What is the commonality among three dogs, three houses, and three pencils?" Their *threeness* is a quality of the sets, but not their members. Three, and all number names, are relational abstractions, since they describe similarities *between collections.* To see the threeness of a single collection, one must see past the qualities of the things themselves, and see only quantity. While number is best defined in the relationships between things, it exists mainly in the organized thinking of human beings as an abstract idea (Wiese 2003). This is true of shape as well—"square-ness" is a common characteristic of a cracker and a cocktail napkin, not dependent upon size, and existing only in the imagined two-dimensional world. It is defined by the relationships among its parts—sides and angles. So the two mathematical topics we tend to consider most basic to the mathematical domain—number and shape—are both highly abstract and relatively complex. The concept-procedure literature, by focusing attention on keeping written procedures meaningful, has missed the important mathematics that develops as children see and verbally name quantitative ideas in the world. This thinking—that occurs well before kindergarten—is the raw material that must be elaborated upon in order to construct a meaningful understanding of numerals and notation. It is therefore foundational to the mathematical learning that ought to proceed in elementary school.

While the concept-procedure literature dismisses early childhood as an important time for mathematics teaching, it also provides rich descriptions of the very mathematics we have termed "foundational" in its portrait of the predilections and abilities of toddlers (children between the ages of about 12 and 30 months). For example, Sinclair and Sinclair describe a two-year-old placing a spoon in each teacup as constructing a working concept of one-to-one correspondence (1986). Similarly, when a toddler sorts a set of animal figurines into "families," she is said to be exploring ideas of similarity, difference, and classification. In these depictions, foundational mathematical knowledge is exhibited through action, and has not yet been separated from the world of concrete objects. Sinclair and Sinclair (1986) comment "the young child cannot do without actual experience when logico-mathematical knowledge is in its beginnings" (p. 63).

This literature misses the fact that three- to five-year-olds, unlike their younger counterparts, can be somewhat sophisticated discussants, and are generally interested in actively using language and their burgeoning pictoral representation skills to get their versions of reality "right" (see, e.g., Bodrova and Leong 1996; Lindfors 1991; McLane and McNamee 1990; Paley 1988). Preschoolers are capable of and derive fairly complex benefits from talking about and otherwise elaborating upon their mathematical ideas about and actions taken upon objects. As mathematical actions and ideas are named, language acts to explicitly connect them to a larger world of social convention, while also separating them out as distinctly meaningful—a binding process that helps children sort and organize their thinking (Gentner and Loewenstein 2002). This elaboration of thought through the flexible use of language—one of the very earliest and most powerful symbolic systems to develop—has a profound impact on the development of mathematical thinking that occurs before elementary school, providing a base upon which symbolic numeracy and operations can later be built.

Clements (2004) notes differences between "the intuitive, implicit, conceptual foundation for later mathematics" and the subsequent elaboration that produces something more like conventional math knowledge (p. 11). He calls the process by which the toddler's embedded and foundational ideas become the preschooler's more elaborated and explicit ones "mathematization," noting that without it, children have "fewer chances…to connect their informal experiences to later school experiences in mathematics" (pp. 11–12). Sophian (1999) is specific about mathematization's effects, noting that children's very early conceptual knowledge is inconsistent, in that it is "in evidence at one moment and yet may not be at the next" (p. 17). She goes on to suggest that assisting children to broaden the application of their budding generalizations and make the knowledge they have begun to construct more explicit and transferable is an important preschool math teaching goal. It is the preschooler's growing facility with language and other primary symbol systems that makes such mathematization possible; it is also this key developmental shift and the clearly mathematical thinking it fosters that the mathematics education literature has so far neglected.

Copley (2000) agrees that there is an important shift in mathematical thinking during this period, and adds to this idea, noting "Early childhood educators say that children learn by doing. The statement is true, but it represents only part of the picture. In reality a child learns by doing, talking, reflecting, discussing, observing, investigating, listening, and reasoning" (p. 29). By contending that the early conceptual mathematical constructions of young children might benefit from social interaction with experienced and knowledgeable others, this idea emphasizes the role of teaching. While it is true that children at this stage of development do not need teachers to "re-connect" procedures to the concepts they refer to (as is needed when arithmetic procedures are learned in elementary school math), preschoolers do need assistance establishing and consolidating initial connections between their budding ideas and more generalized concepts. This developmentally informed perspective on mathematical thinking in early childhood makes the relevance of children's experiences clear.

Robust findings about differences in the home language experiences of young children suggest the problem: children from homes with fewer economic resources learn fewer words, have fewer experiences with words in interaction with other persons, and acquire a vocabulary of words more slowly (see Hart and Risley 1995). Given the key role of language in the development of foundational mathematical thinking, it is unsurprising that socio-economic status and math achievements tend to rise and fall together at the preschool level (Denton and McPhee 2009). Children from lower-resourced family environments need enriched early education mathematics experiences if they are to catch up with their peers. Average teaching is clearly not enough to ensure they are equitably prepared for mathematics in elementary school.

Because there is generally no training in mathematics provided to early childhood teachers (Copley 2004), they tend to rely on their own understandings to teach math (Sarama and DiBiase 2004). Unfortunately, foundational mathematics is invisible to most adults. As a means of efficiency, adults have learned not to check that five pennies are still five pennies whether arranged in a circle or in a line. Similarly, early childhood teachers have experienced the fact that a single set of objects can be sorted in more than one way so many times, that have generally forgotten they once did not know it. Young children are just discovering these truths, and learning the formal mathematics of elementary school is dependent upon their becoming explicit and transferable. It is only when these foundational ideas are clear enough that preschoolers can talk about them (or at least listen with understanding) that they can begin to represent them with secondary symbol systems, such as numerals, and learn to act upon them in ever more sophisticated ways. Since early childhood teachers generally miss the existence of foundational mathematical thinking among their students, they lack an explicit awareness of the concepts their teaching ought to help build.

Improving Teachers' Understandings of Foundational Mathematics

To help promote preschool teachers' understanding of the content of foundational mathematics and how it develops in the thinking of young children, we launched the Early Mathematics Education Project in 2007. Designed primarily for preschool teachers, this program includes workshops, on-site coaching, and the use of videotape to promote reflective practice among teachers. Instructors are Erikson faculty, and coaches are experienced preschool teachers, who participate in content training alongside their teachers. Emphasis is placed on addressing both teacher confidence and teacher knowledge, and specially designed adult learning tasks help teachers "see" the foundational mathematics. Finally, teachers are explicitly instructed in the use of math-related language, a powerful tool for helping young children solidify and build upon their mathematical thinking in preparation for the use of written notation to come. Key elements of the program, which has a demonstrated impact on children's mathematical learning, are described below.

Key Program Elements

Addressing confidence. Lack of math-related confidence among preschool teachers was explicitly considered in the design of the professional development. Since most early childhood teachers feel comfortable with stories as a part of their curriculum, children's storybooks are used throughout. Each workshop's activities—both the adult learning experiences and the recommended activities for children—are built out of rich children's literature, and teachers are provided with storybooks to support their teaching. In addition, the professional development is ongoing, with five day-long meetings held once every other month throughout the school year. This design allows the assignment of preschool teaching tasks between sessions and the opportunity to discuss their implementation at the next meeting. As importantly, the extended time allows each cohort of teachers to become familiar with one another, creating a safe environment for the expression of frustration as well as accomplishment. Coaching, too, is meant to encourage real changes in practice. Our coaching process emphasizes teacher strengths, and is designed to help teachers learn to be reflective. By making clear the coach's consultative—rather than supervisory—role, this coaching process helps underconfident teachers use the coach as a non-threatening source of support. Finally, teachers are explicitly engaged as partners in research. Their insights as they implement activities in their classrooms are actively sought by our instructors, and they are encouraged to present their own work to their peers as a means of sharing expertise. In addition to generating new ideas and solidifying understanding, these assumptions of professionalism among the teachers boost their sense of their own competence and promote their active attempts to improve practice.

Big ideas. Recognizing that early childhood teachers lacked knowledge of foundational mathematics and its development among young children, our team developed lists of Big Ideas within each of the content strands identified by the NCTM. Each Big Idea is meant to represent important conceptual material, central to the content area, that elaborates and solidifies the mathematical experiences and thinking of young children between the ages of three and five years. For example, a Big Idea in the content strand of measurement would be that "every object has many different attributes that can be measured, such as length or weight." While three- and four-year-olds can experience this truth on their own, the program operates on the assumption that structured activities and teacher language that make it explicit have a "mathematizing" effect on young children's thinking (see Table 18.1 for sample Big Ideas).

Adult learning tasks. To help teachers recognize and integrate both foundational mathematics and the Big Ideas, activities were constructed that would highlight them while being complex enough to engage adult learners. For example, when studying geometry, teachers are asked to describe a shape without using its name, forcing them to notice and name shape attributes, such as number of sides and size of angles. Similarly, in the study of algebraic thinking, teachers are challenged to name as many different sets of objects as they can find in the story "Goldilocks and the Three Bears." These sets are further analyzed to discover size and sequence patterns among them. Subsequently, teachers are provided with a sample lesson for children in which a large set of concrete objects of three sizes, such as spoons, mittens, and toothbrushes, is sorted in two distinct ways: first by object type, and then by size, so that "each bear can have his or her own things." In this way, adult

Table 18.1 Sample big ideas by mathematics content strand

Content strand	Big idea text
Algebra	The same collection can be sorted in different ways
Number and operations	A collection can be made larger by adding items to it, and made smaller by taking items from it
Measurement	Many different attributes can be measured, even when measuring a single object
Geometry	Two- and three-dimensional shapes can be used to represent and understand the world around us
Data analysis and probability	How data are gathered and organized depends upon the question they address

learning activities were designed to help teachers construct their own understanding of foundational mathematics, and then to suggest ways to emphasize and build upon it in the teaching of young children.

Math-related language. There is interesting evidence that mathematical learning during the preschool year is affected by language input from teachers. In each of three studies (Ehrlich 2007; Klibanoff et al. 2006; McCray 2008), researchers found a significant, positive relationship between the amount of math-related language by teachers and the growth of conventional mathematics knowledge among preschoolers in their classrooms from fall to spring. That is, the more preschool teachers talked about math in their classrooms, the greater the gains in math knowledge made by their students. Incorporating these findings, the workshops and coaching sessions emphasize both awareness and use of math-related verbalization. Teachers learn how to use "mathematizing" language to describe children's daily activities during transition, snack, dramatic play, and outdoor time. Additional emphasis is placed on asking questions that encourage children to describe their thinking using mathematical language. While the findings behind these language-related practices are correlational as opposed to causal, they strongly support the theory that what teachers say and what they invite children to say can help preschoolers solidify their undifferentiated mathematical thinking and link it to conventionally named concepts. For this reason, the professional development urges teachers to talk with children about foundational mathematics, pointing it out when it occurs and urging students to describe how they understand it.

Results of the Intervention

During the 2008–2009 school year, we conducted program evaluation in 28 preschool classrooms in a large urban public school system. The mathematics achievement scores of 236 children from socio-economically disadvantaged backgrounds were gathered once in the fall and once in the spring. The results demonstrated that compared to students whose teachers did not participate in our training program, students with program-participating teachers showed significantly greater growth on both the Child Math Assessment (CMA, Klein and Starkey 2006) and the Applied Problems Subtest (#10) of the Woodcock Johnson III (WJ-III, Woodcock et al. 2001) over the course of the school year. Using Hierarchical Linear Modeling, we controlled for Time 1 scores, since children in our participating classrooms were significantly ahead of their non-participating counterparts in the fall. While average scores increased in both groups, analysis of WJ-III Age Estimate scores attributes 2.74 additional months of growth in mathematics learning to the intervention ($p < 008$). Compared to students whose teachers did not participate in our program, children with program-participating teachers learned almost three months more material in the same amount of classroom time. Teachers, too, report high levels of satisfaction with the training, and credit it with making positive changes in their mathematics teaching.

Conclusion

In sum, recognition and understanding of foundational mathematics, its abstract nature, its role in preschool teaching, its susceptibility to intervention, and its relationship to elementary school mathematics directly addresses the misconceptions of both early childhood educators and researchers in mathematics education. First, it makes clear that there is mathematical thinking and learning that precedes the use of written symbols, that this thinking is based in experience with objects and space, and that it is well placed in a child-centered, developmentally appropriate curriculum. The tendency of some educators to "push-down" the subject matter intended for first grade is thereby pre-empted. Second, describing foundational mathematics helps define the specific kind of mathematical knowledge that preschool teachers need to be more effective supporters of children's mathematical development. It makes clear that their need for more content knowledge cannot be addressed by requiring higher-level mathematics classes, and instead suggests a new type of mathematics class for teachers, focused on numerical and spatial abstraction that occurs prior to the use of written symbols. Finally, by emphasizing the role of a solid understanding of foundational mathematics as a necessary precursor to the development of school-based, conventional mathematics, this argument draws attention to the importance of preschool teaching for lifelong learning in math. Specifically, acknowledgment of foundational mathematics makes clear the role of language as a mechanism for identifying and solidifying its concepts, and highlights the need for adults to help young children find and use the words that will be central in the understanding of written numerals, notation, arithmetic, and algorithms to come.

If mathematics educators become more aware of both the distinct nature and centrality of foundational mathematics as a basis for preparation for elementary school, perhaps they will exhort and encourage early childhood educators to take their charge seriously as it relates to math, sharing their own findings and concerns. If early childhood educators recognize that there is developmentally appropriate mathematics content for their students, and that exposure to this content can make or break the capability of some children to understand the mathematics of elementary school, they may be inspired to learn about it and teach it. The general public must also be educated, so that policymakers and funders will be impelled to support and expand these changes in our understanding. By establishing recognition of foundational mathematics, and helping the public in the United States to understand its relevance, early childhood and mathematics educators will have taken a vital first step in enhancing opportunities for truly equitable mathematics learning.

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