Non-Linear Systems: Logistic Maps and Chaos

The logistic map is a model of population growth that exhibits many different types of behavior, dependingon the value of a few constants. The equation then, for somepopulation X_n+1 after an arbitrary time step, starting with population X_n is:

X_n+1 = r*X_n*(1 - X_n)

Experiment Report by Group 10A

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References Used

- Explanation of Logistic maps by Veritasium
- The Feigenbaum Constant (4.669) - Numberphile

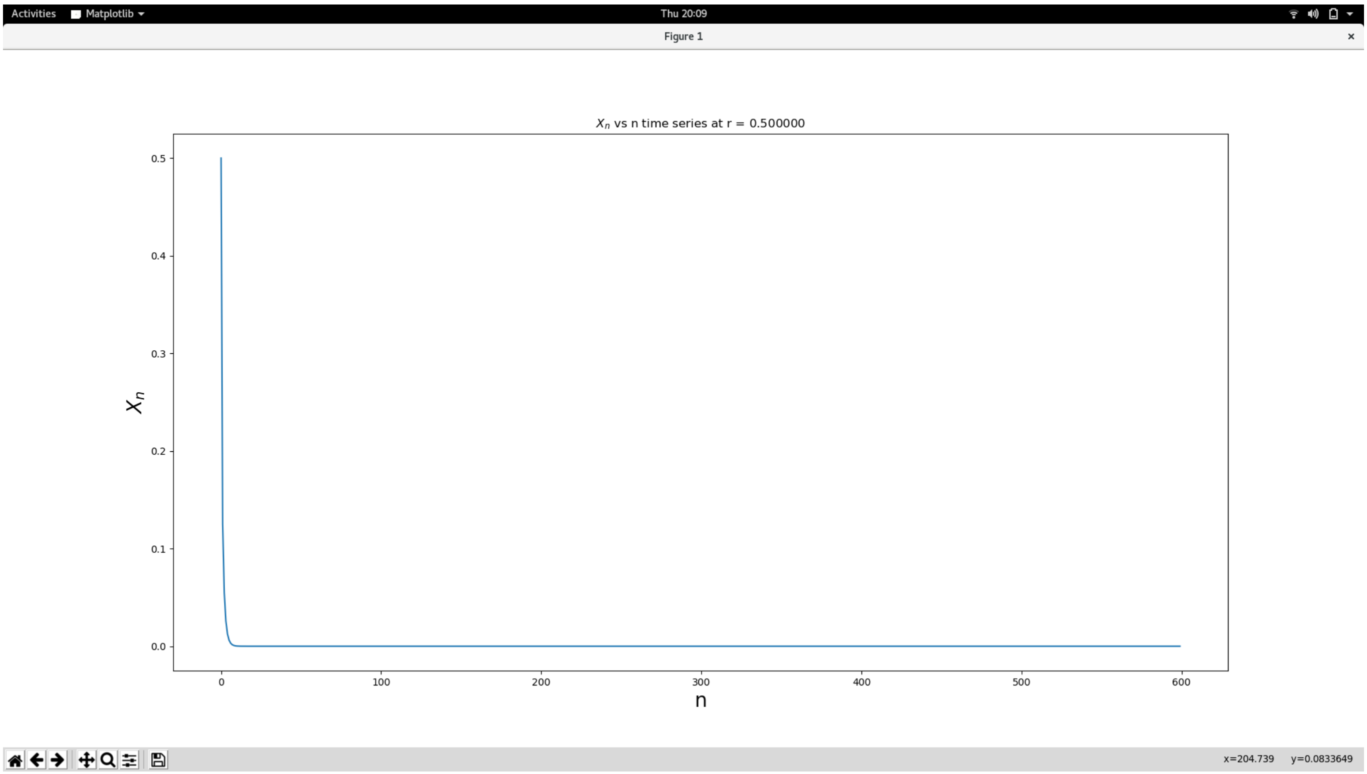
Time Series Plots

Code used to generate the time series plots:

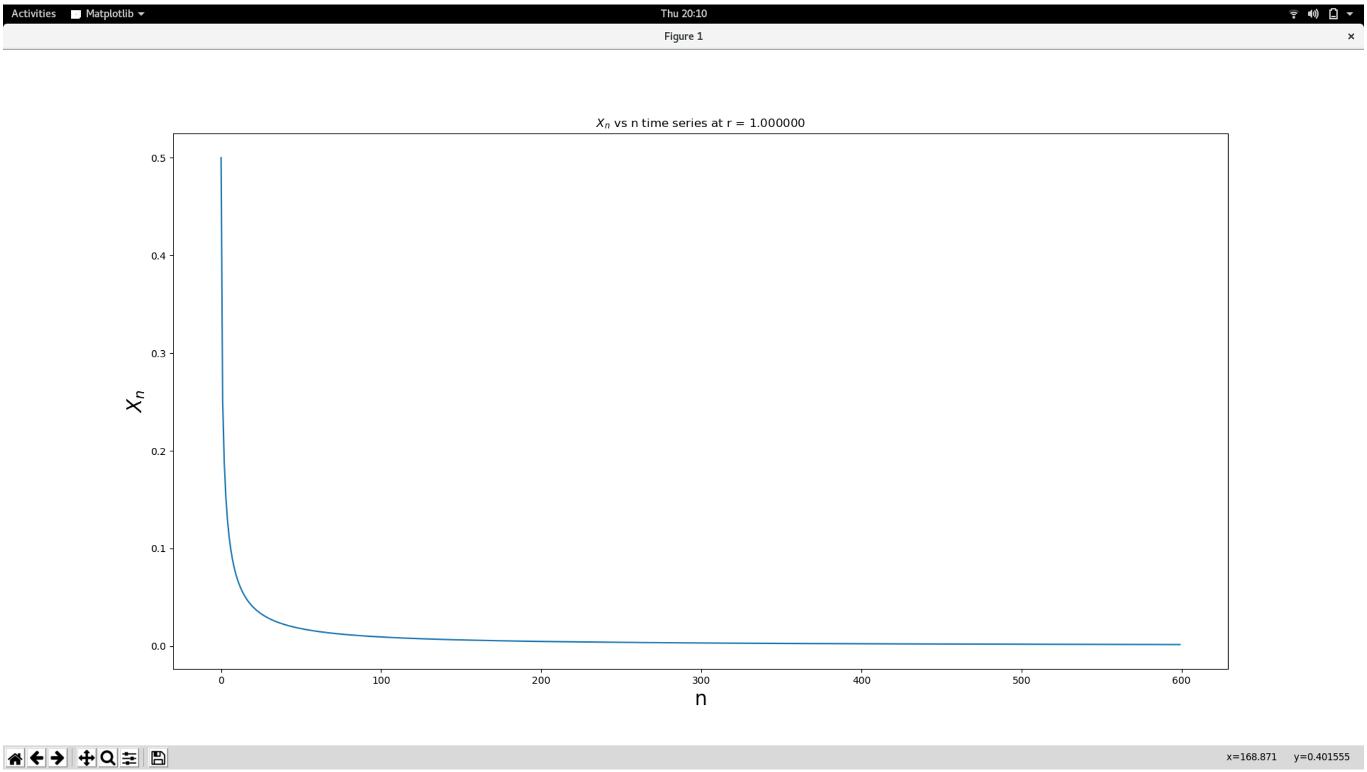
```
import numpy as np
import matplotlib.pyplot as plt

def fun(r,x):
    return r*x*(1-x)

r_value = 2.85
init_value = 0.5
max_n = 600
xn_values = []
xn_values.append(init_value)
for i in range(1,max_n):
    xn_values.append(fun(r_value,xn_values[i-1]))

plt.xlabel('n', fontsize=20)
plt.ylabel(r'$X_n$', fontsize=20)
plt.title(r'$X_n$ vs n time series at r = %f'%(r_value))
plt.plot(range(0,max_n),xn_values)
plt.savefig('timeseries%f.png'%(r_value))
plt.show()
```

Corresponding graphs with r values ranging from 1 to 4

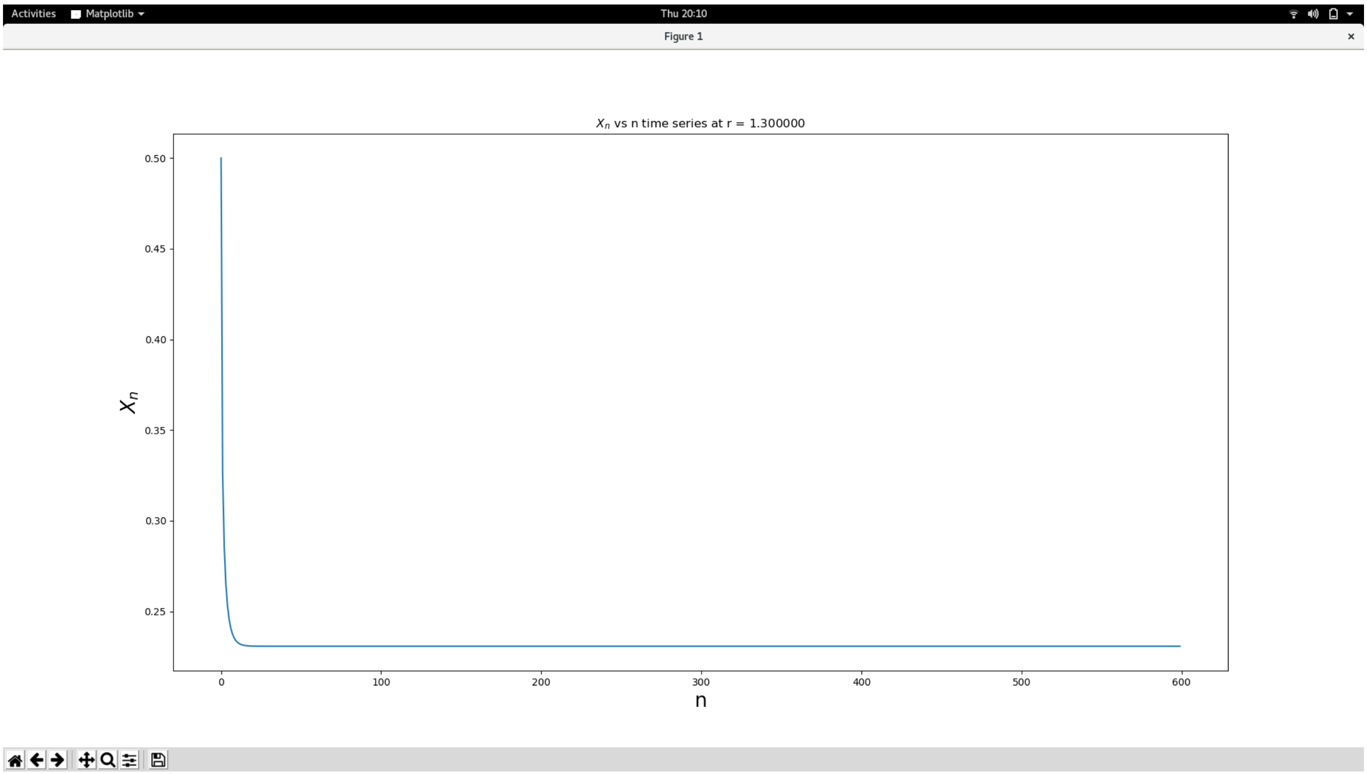
Graph1 (Done by Abhigna)



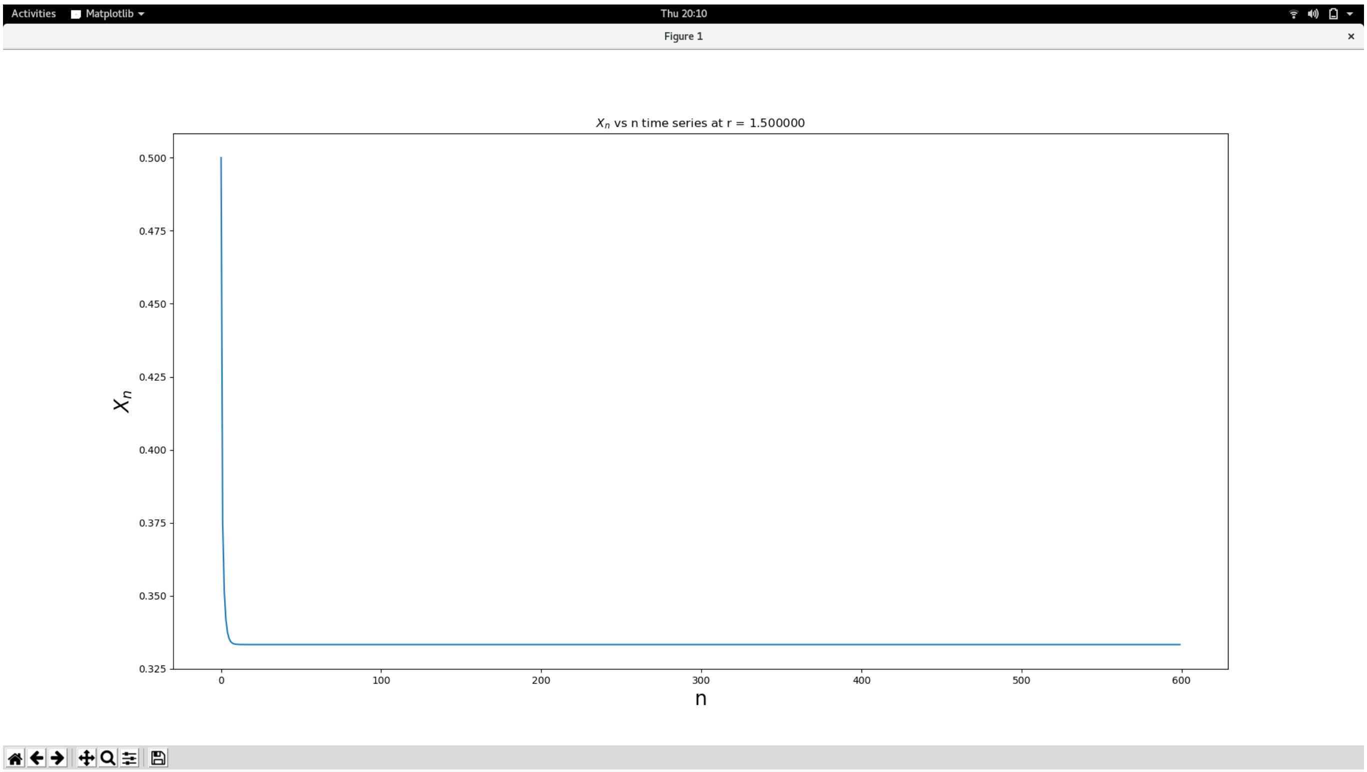
Graph2 (Done by Abhigna)



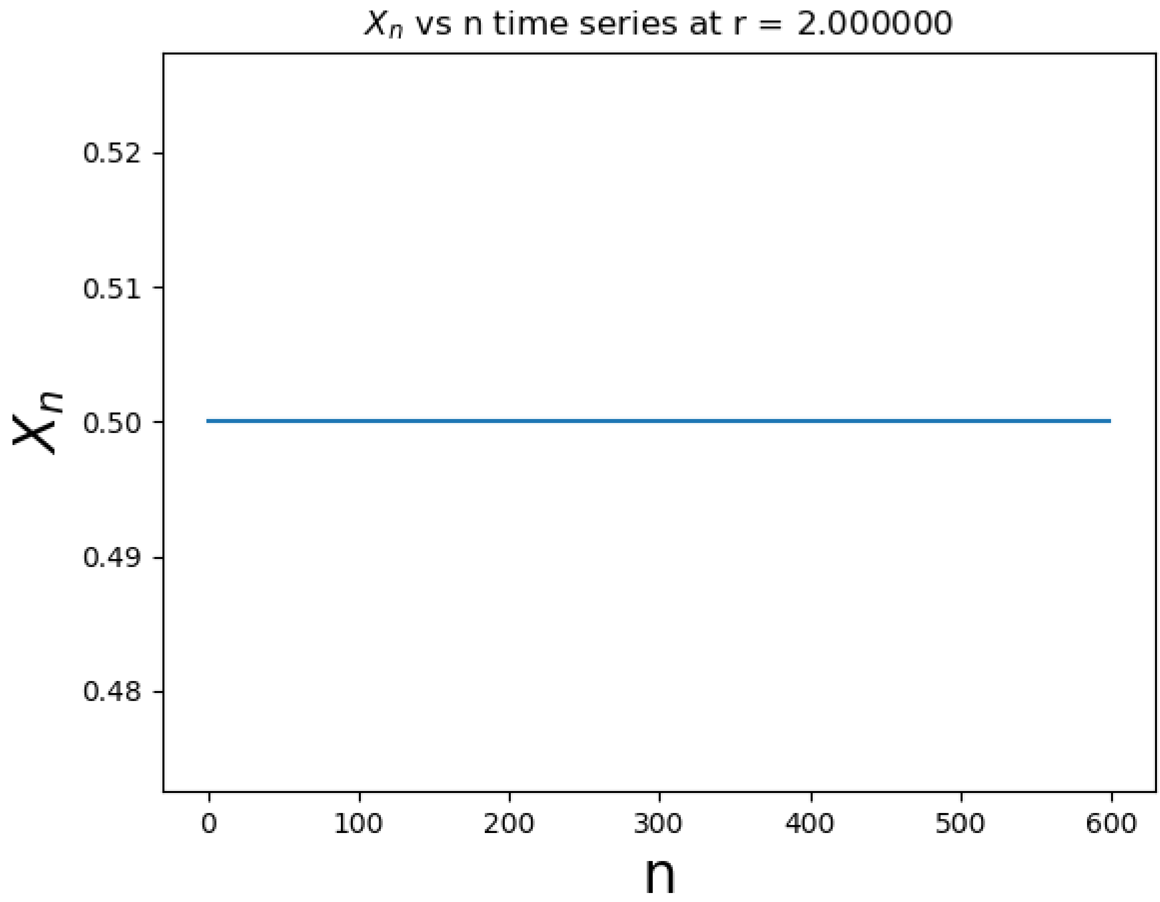
Graph3 (Done by Abhigna)



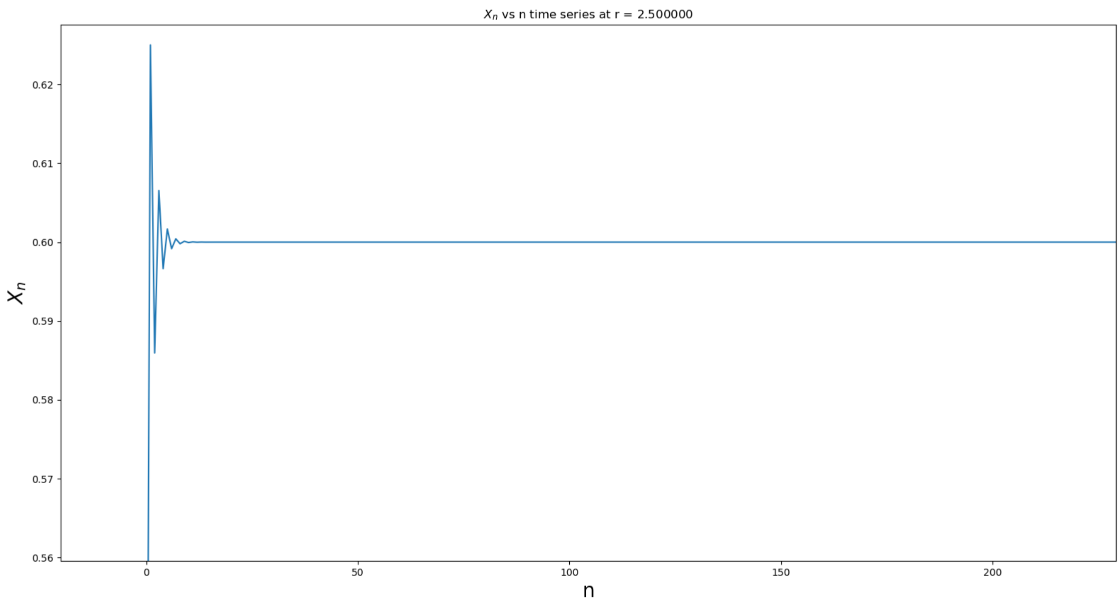
Graph4 (Done by Abhigna)



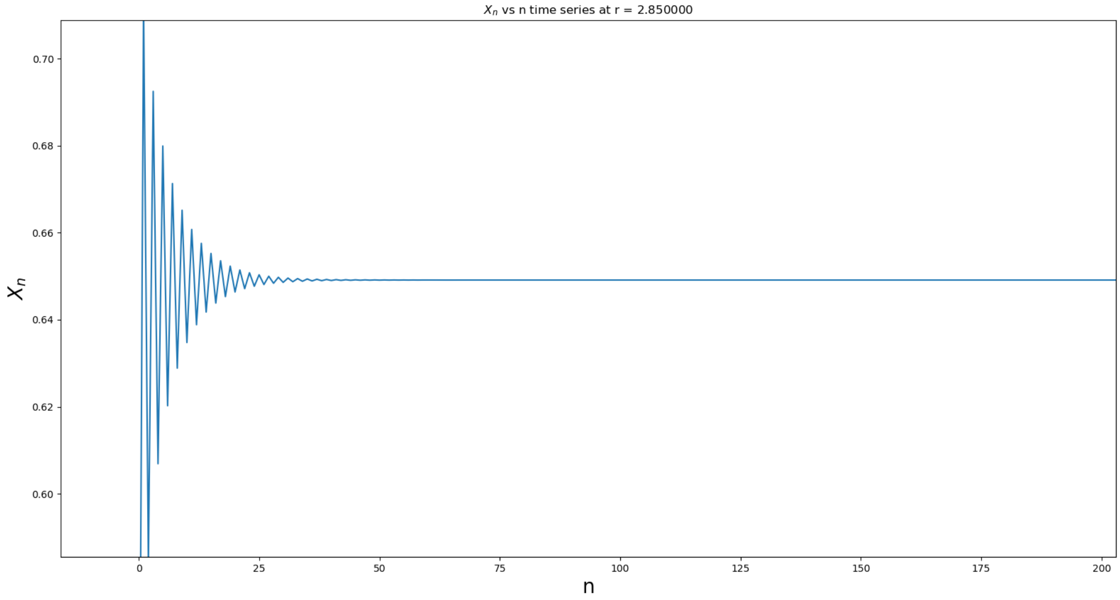
Graph5 (Done by Agam)



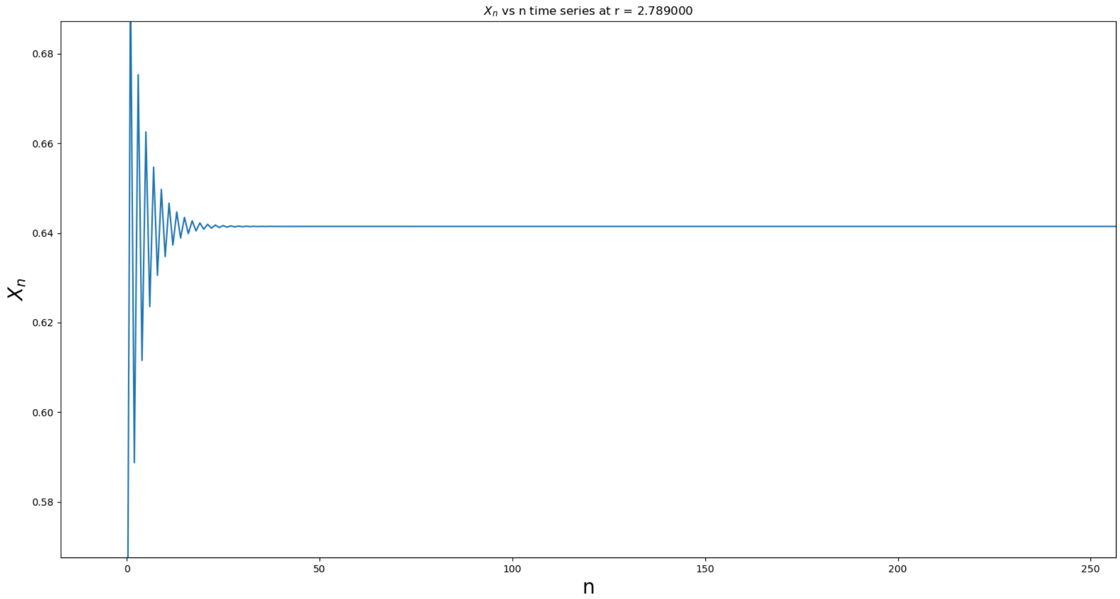
Graph6 (Done by Agam)



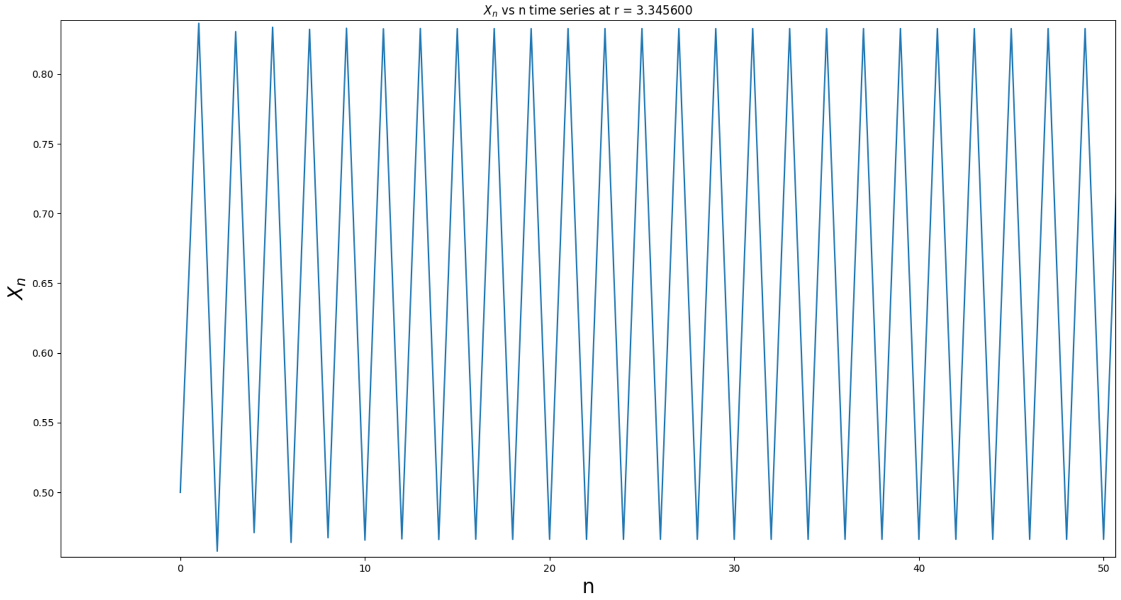
Graph7 (Done by Agam)



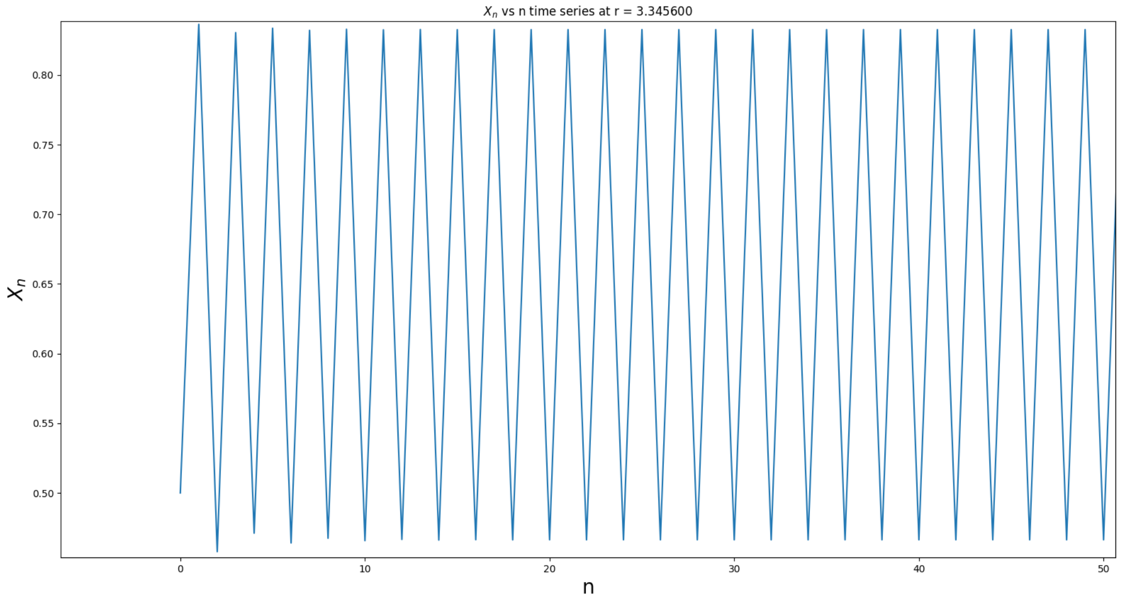
Graph8 (Done by Agam)



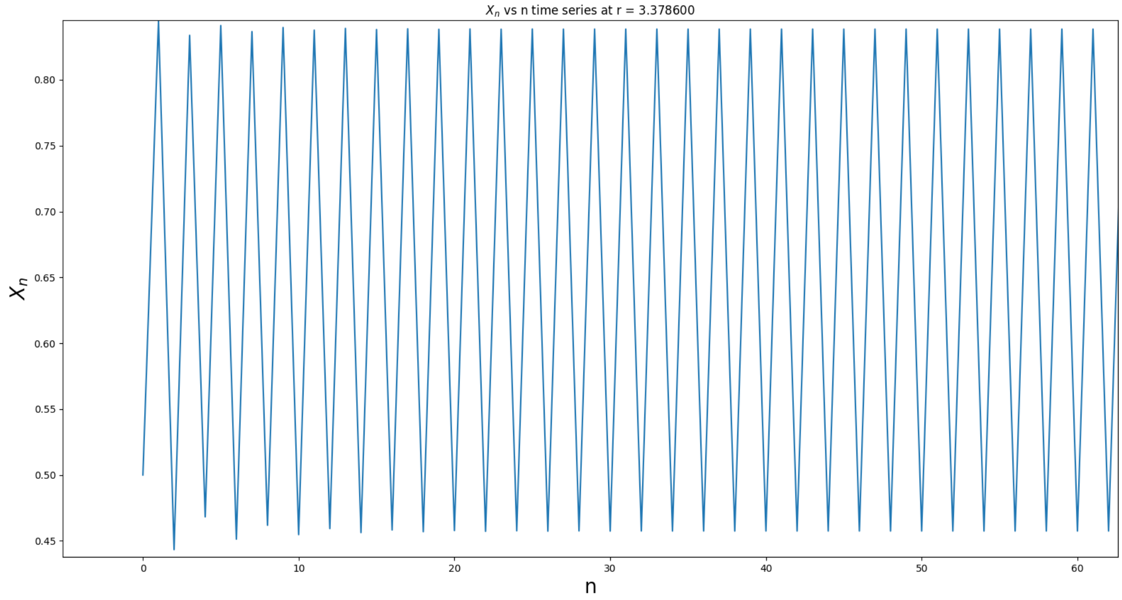
Graph9 (Done by Kashif)



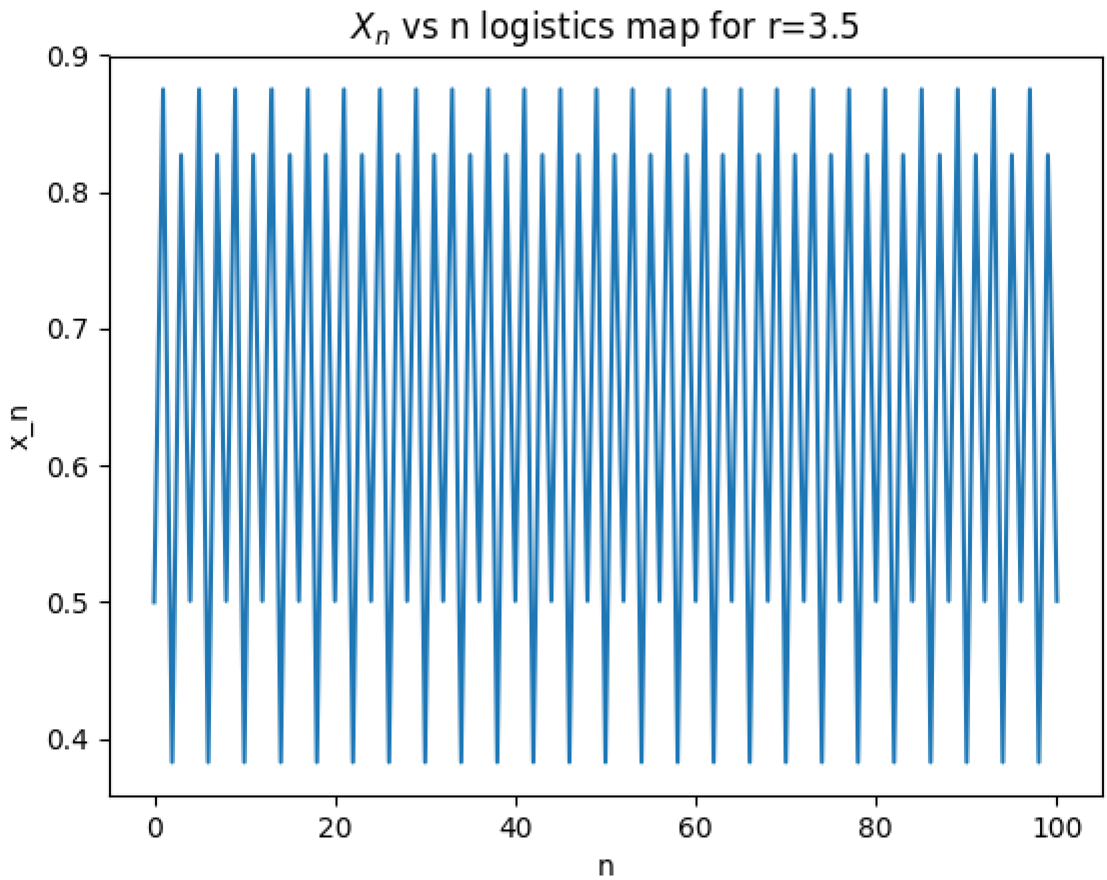
Graph10 (Done by Kashif)



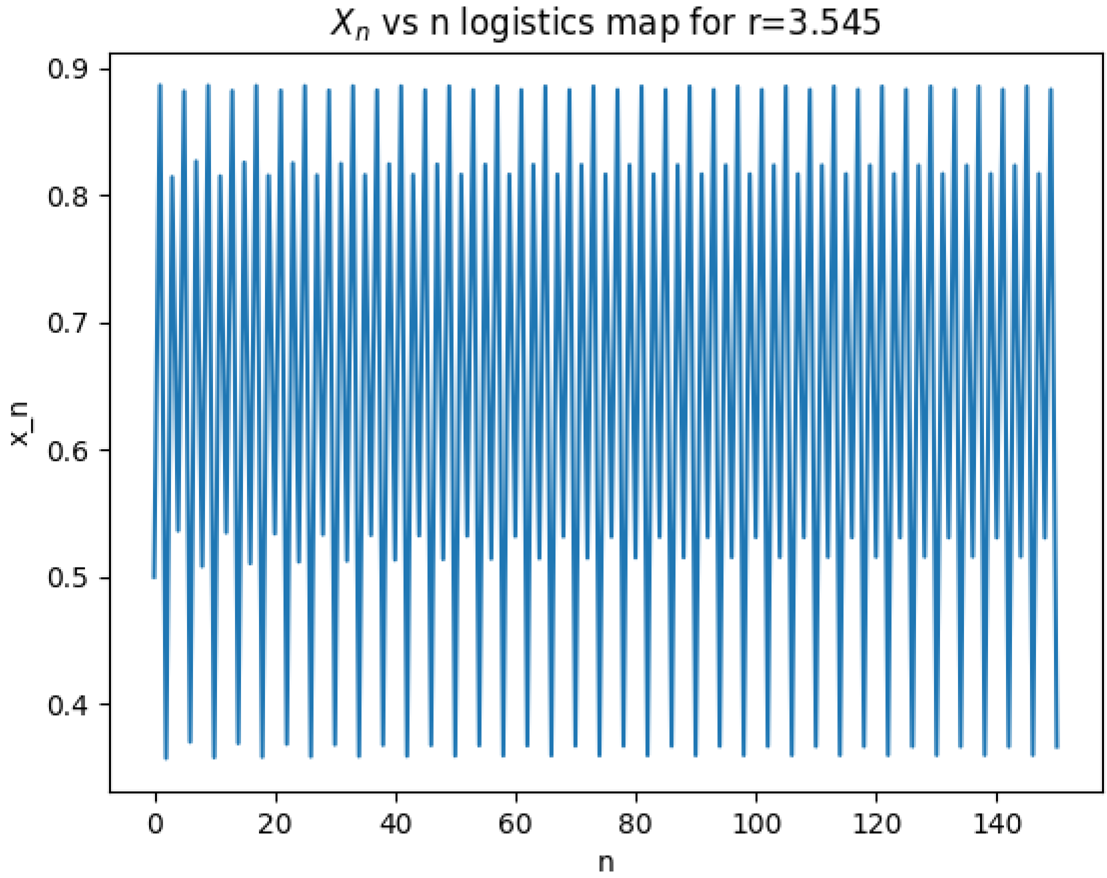
Graph11 (Done by Kashif)



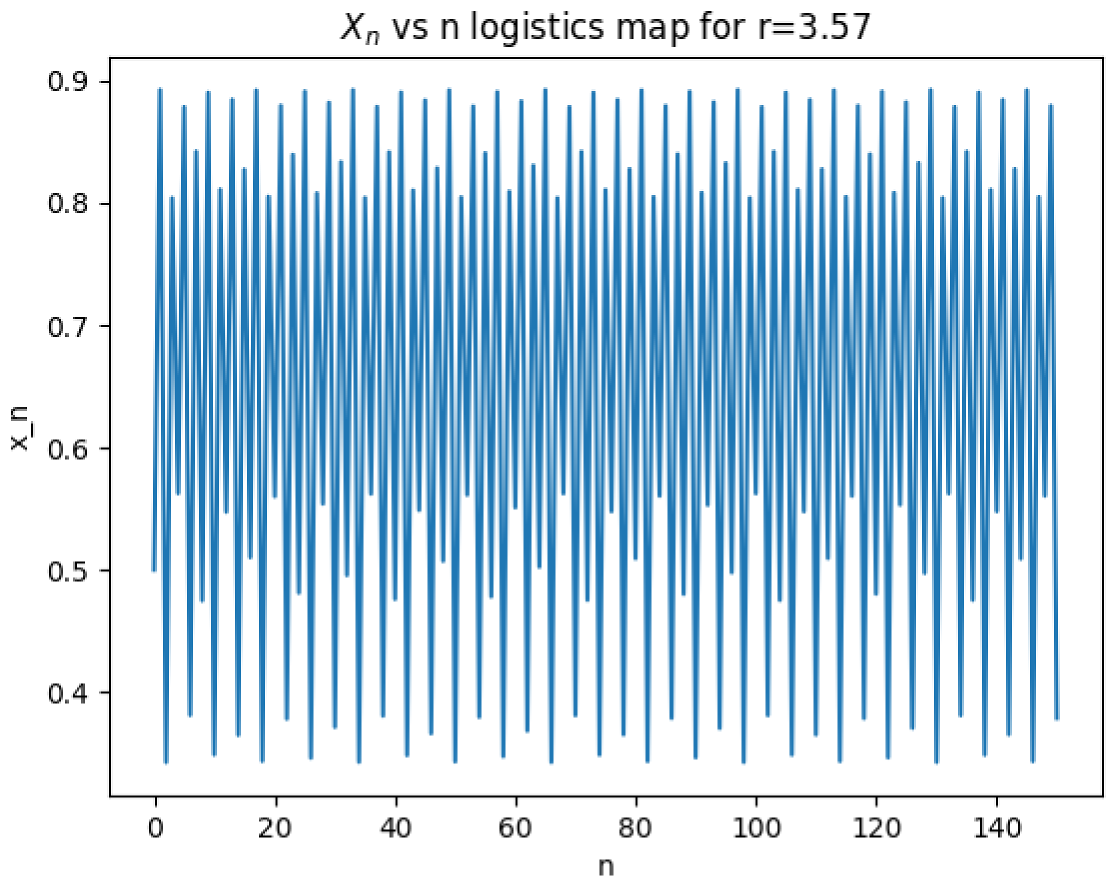
Graph12 (Done by Aarushi)



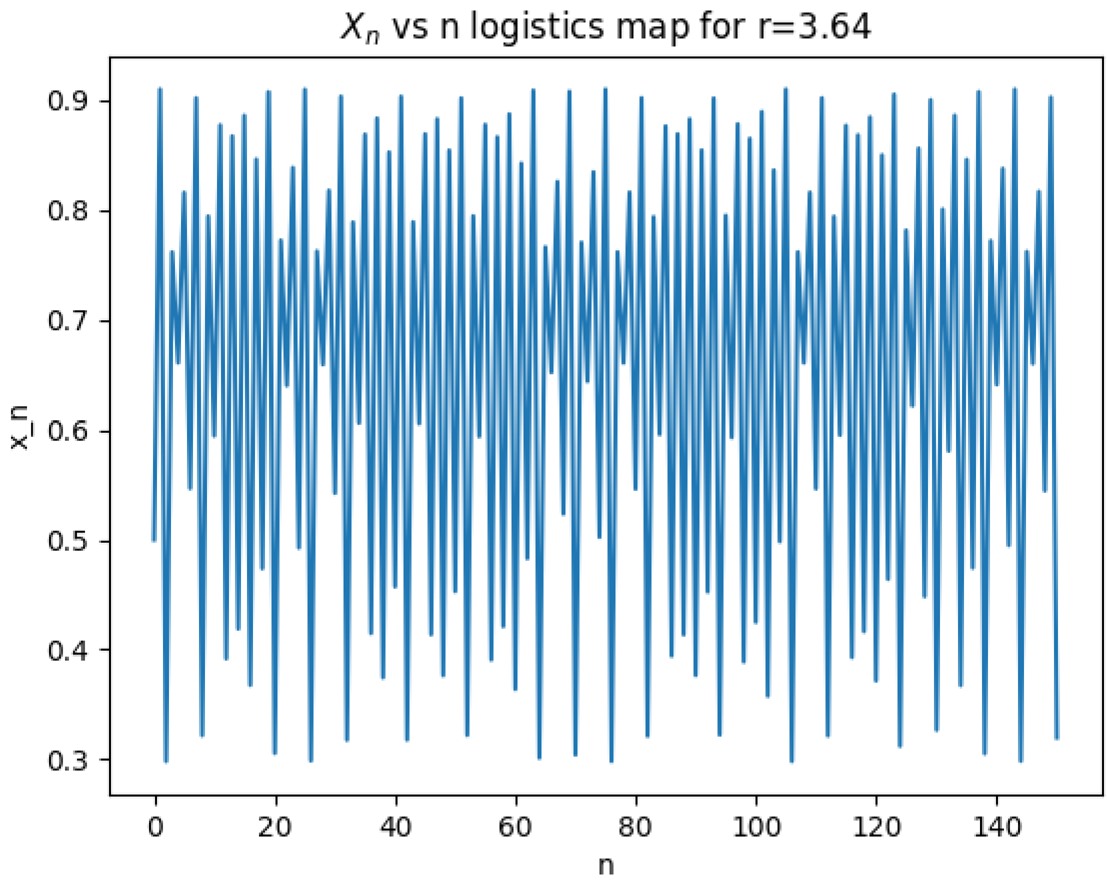
Graph13 (Done by Aarushi)



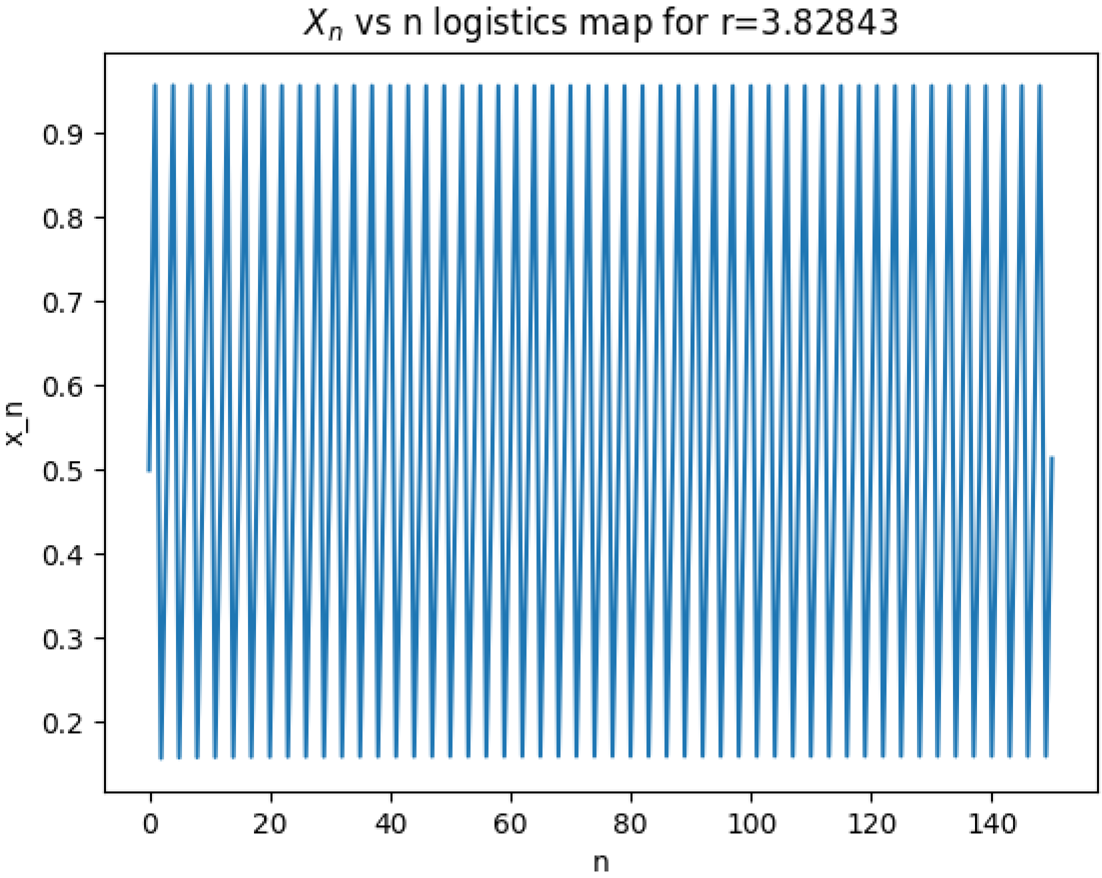
Graph14 (Done by Aarushi)



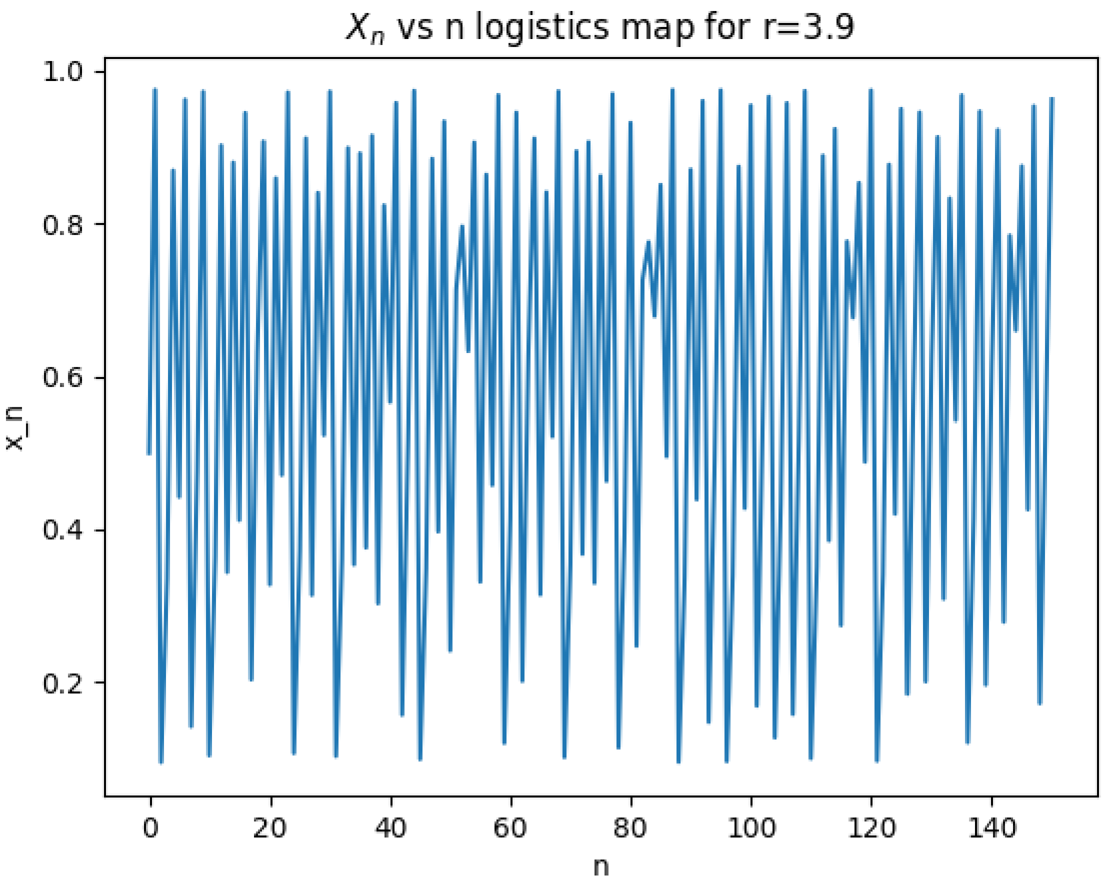
Graph15 (Done by Aarushi)



Graph16 (Done by Aarushi)



Graph17 (Done by Aarushi)



Conclusions Drawn from this

By Aarushi

Values	Observation
<= 1	Population Will Die i.e. X_n goes to 0
1.5	Gives a Constant Value
2	Gives a Constant Value
2.5	Becomes constant after fluctuating for a while
2.789	Becomes constant after fluctuating for a while
2.85	Becomes constant after fluctuating for a while
3	The rate of convergence is less than linear
3.34560	Oscillates between 2 values : Period 2
3.5	Oscillates between 4 values : Period doubles
3.545	Oscillates between 8 values : Again Period has doubles from before
3.57	It is chaotic, Can't find repeating values periodically
3.64	It is Chaotic
3.82843	should've Oscillated between 3 values but didn't happen
3.9	Chaotic
4	Chaotic

> One fine point that we noticed is, after 3.57, a smal change in the value of r can make the system change suddenly from chaotic to stable and vice versa.

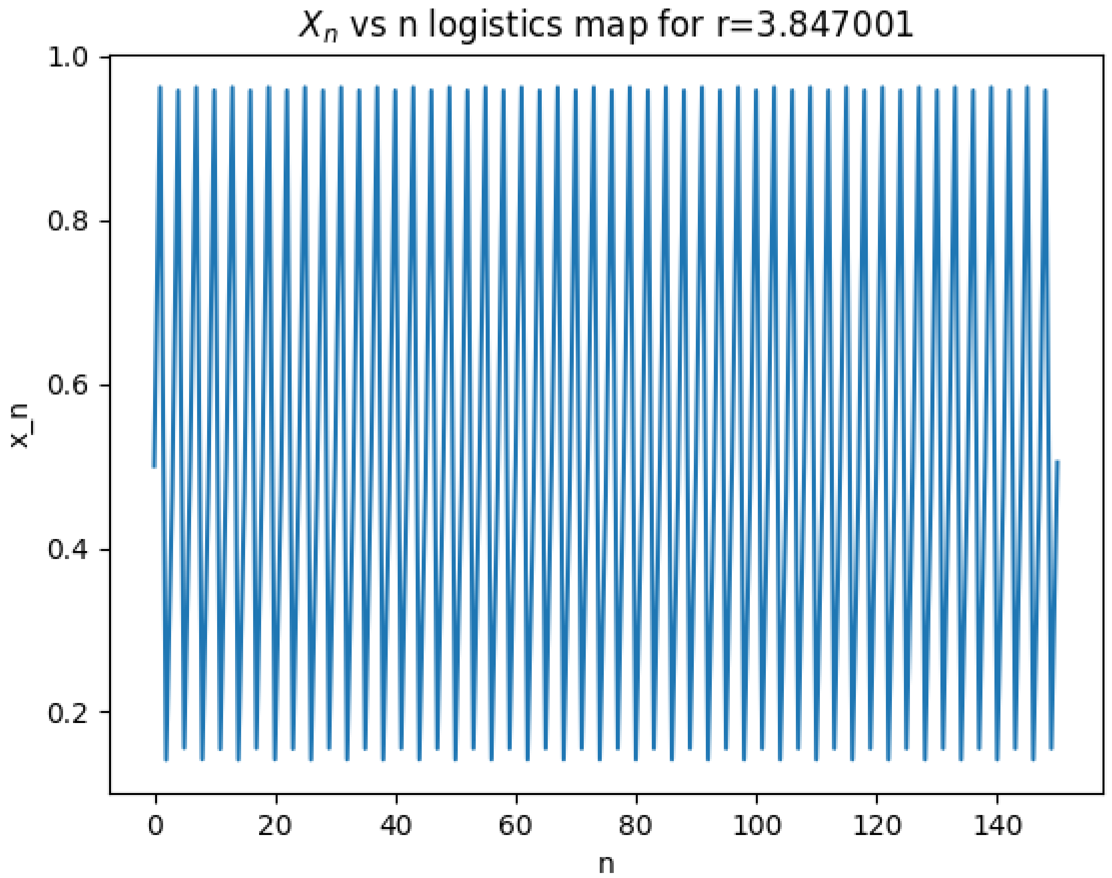
Time-Series Graphs made from Value of r using r = 3.847XXX

Code used by Aarushi

```
import numpy as np
from matplotlib import pyplot as plt

r = 3.847001

x = np.arange(0,151)

y = [0.5]

for i in range(150):
    g = y[-1]
    y.append(r*g*(1-g))
```

```
plt.xlabel('n', fontsize=10)
plt.ylabel('x_n', fontsize=10)
plt.title(r'$X_n$ vs n logistics map for r=3.847001')
plt.plot(x,y)
plt.show()
```

Graph by Aarushi Roll number IMT2018001

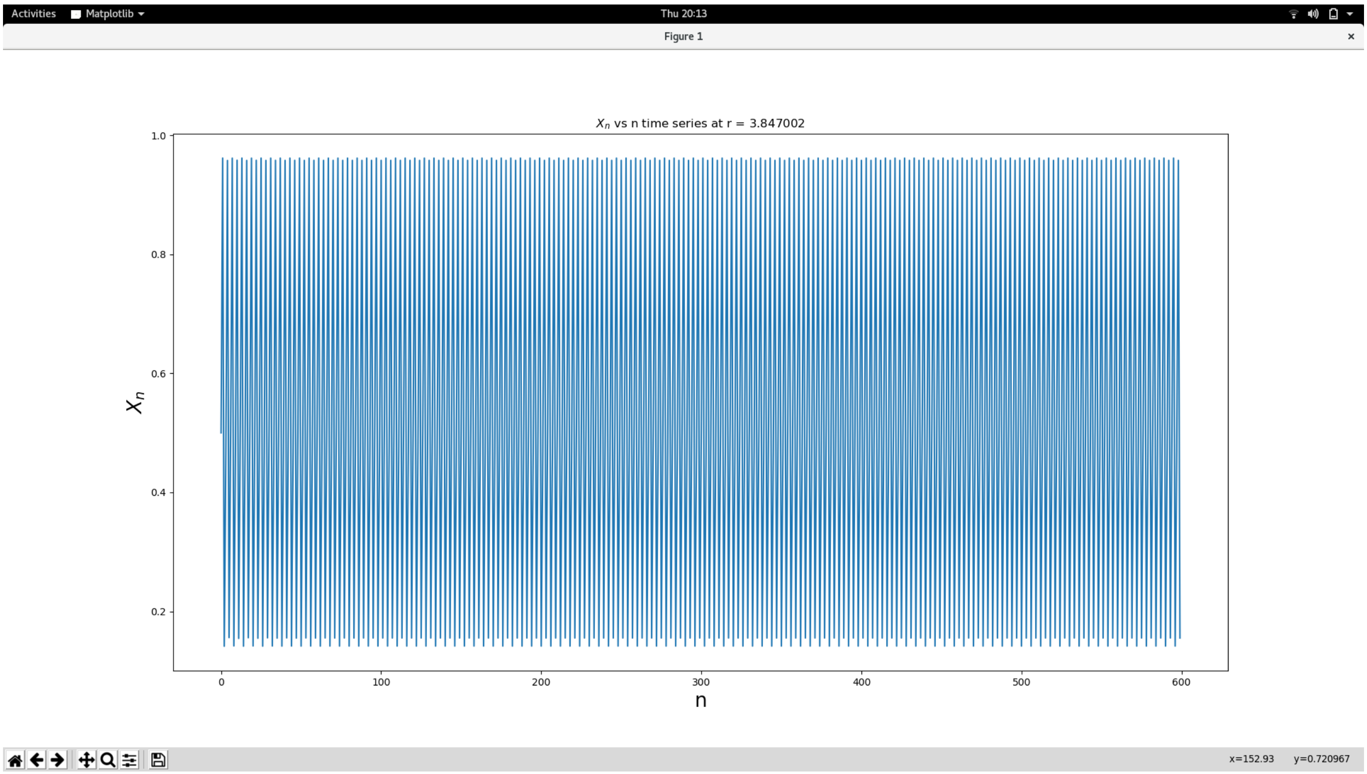


Code used by Abhigna

```
import numpy as np
import matplotlib.pyplot as plt

x0=0.5
r=3.847002
X=[]
Y=[]
X.append(0)
Y.append(x0)
count=0

for n in range (1,600):
    X.append(n)
    Y.append(r*Y[count]*(1-Y[count]))
    count=count+1
```

```
plt.xlabel('n',fontsize=20)
plt.ylabel('$X_n$',fontsize=20)
plt.title('$X_n$ vs n')
plt.plot(X, Y)
plt.show()
```

Graph by Abhigna Roll number IMT2018002

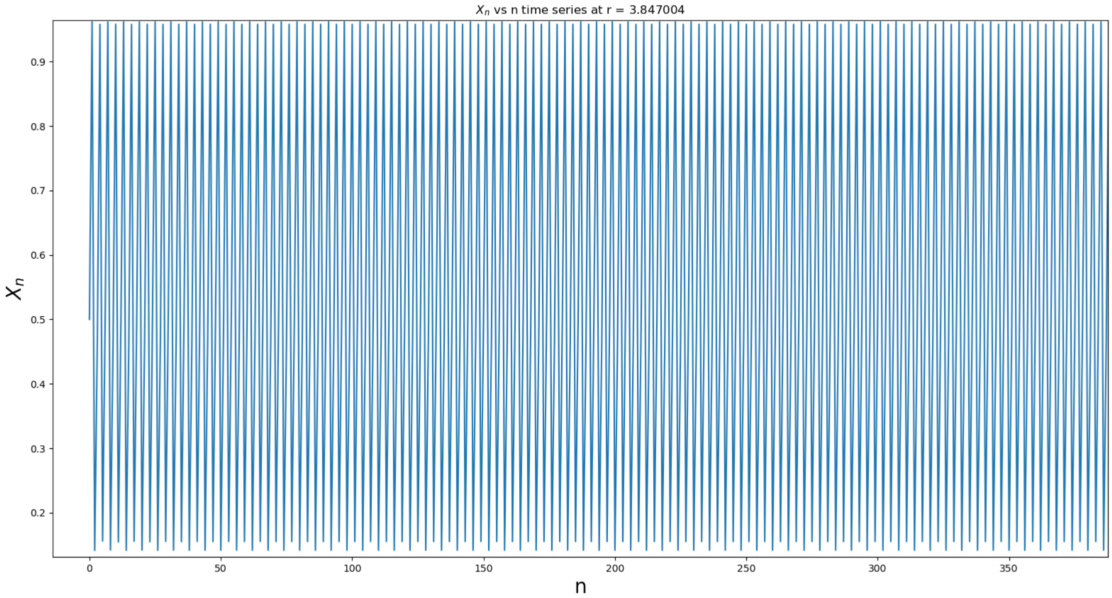


Code by Agam

```
import numpy as np
import matplotlib.pyplot as plt

def fun(r,x):
    return r*x*(1-x)

r_value = 2.85
init_value = 0.5
max_n = 600
xn_values = []
xn_values.append(init_value)
for i in range(1,max_n):
    xn_values.append(fun(r_value,xn_values[i-1]))

plt.xlabel('n', fontsize=20)
plt.ylabel(r'$X_n$', fontsize=20)
plt.title(r'$X_n$ vs n time series at r = %f'%(r_value))
plt.plot(range(0,max_n),xn_values)
```

```
plt.savefig('timeseries%f.png'%(r_value))
plt.show()
```

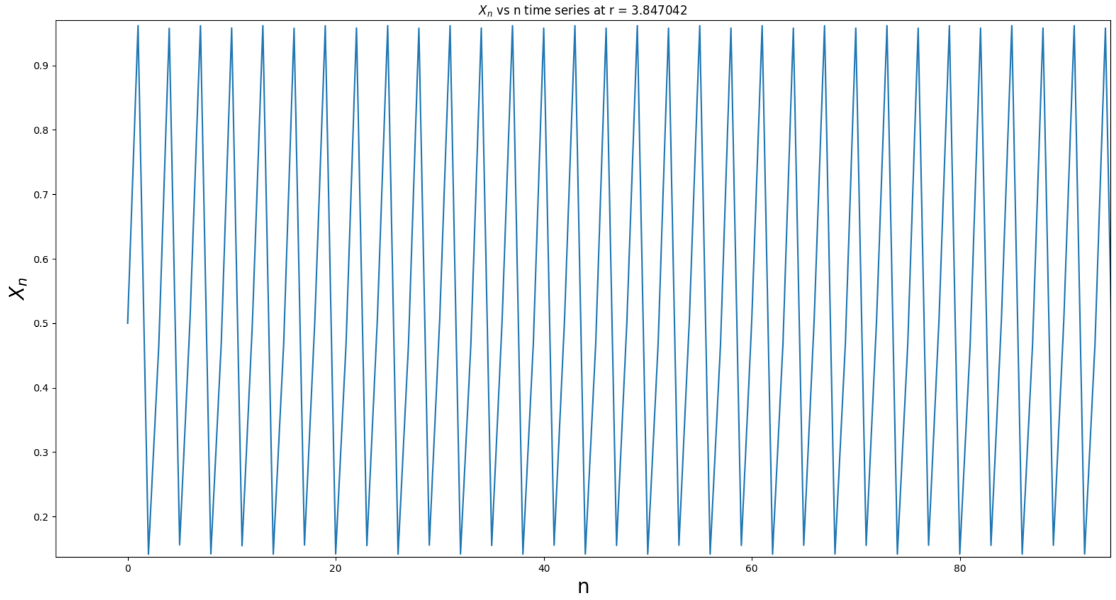
Graph by Agam Roll number IMT2018004



Code by Kashif

```
import numpy as np
import matplotlib.pyplot as plt


r =  3.847042
X = []
X.append(0.5)
for i in range(1,800):
    X.append(r*X[i-1]*(1-X[i-1]))

plt.xlabel('n')
plt.ylabel(r'$X_n$')
plt.show()
```

Graph by Kashif Roll number IMT2018042



Bifurcation Diagrams

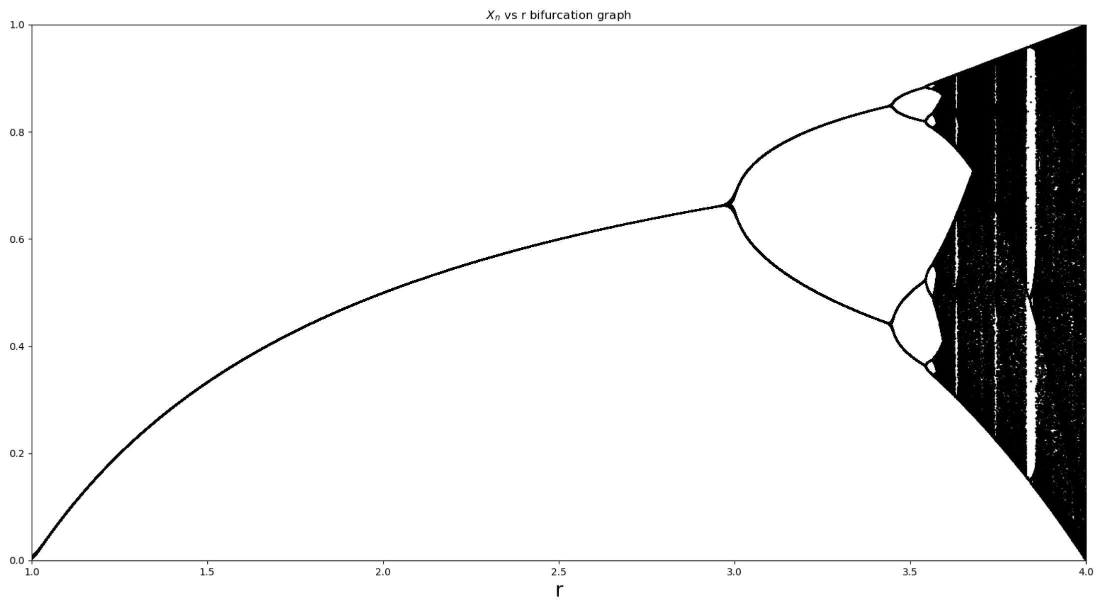
A bifucation is a period-doubling, a change from an N-point attractor to a 2N-point attractor, which occurs when the control parameter is changed. A Bifurcation Diagram is a visual summary of the succession of period-doubling produced as r increases. The next figure shows the bifurcation diagram of the logistic map, r along the x-axis. For each value of r the system is first allowed to settle down and then the successive values of x are plotted for a few hundred iterations.

Code 1 used (Written by Agam Kashyap)

```
import numpy as np
import matplotlib.pyplot as plt

def fun(r,x):
    return r*x*(1-x)

def calc(n1,n2,nr,init_value):

    r_values = np.linspace(nr,4,1000)
    for r in r_values:
        alt = []
        xn_values = []
        alt.append(init_value)
        for i in range(1,n1):
            alt.append(fun(r,alt[i-1]))
        xn_values.append(alt[n1-1])
        for i in range(1, n2-n1):
            xn_values.append(fun(r,xn_values[i-1]))
        xn_values = np.array(xn_values)
        r_arr = xn_values*0.0 + r
        plt.plot(r_arr,xn_values,'ko',markersize=1)
```

```
r_min = 1
plt.xlabel('r', fontsize=20)
plt.axis([r_min,4.0,0,1.0])
plt.title(r'$X_n$ vs r bifurcation graph')
calc(100,200,r_min,0.05)
plt.show()
```

Bifurcation Graph from Code 1



Feigenbaum Constants

The first Feigenbaum constant is the limiting ratio of each bifurcation interval to the next between every period doubling, of a one-parameter map

x_i+1 = f(x_i)

where f(x) is a function parameterized by the bifurcation parameter a.

It is given by the limit

δ = $\displaystyle \lim_{n \to \infty} g(x)$ = 4.669201609....

where g(x) = $\frac{a_{n-1} - a_{n-2}}{a_n - a_{n-1}}$

where a_n are discrete values of a at the n-th period doubling.

Observations (Done by Abhigna and Kashif)

r1	r2	r3	g(x)
3.002	3.44912	3.53708	5.083219645

r1	r2	r3	g(x)
3.002	3.44912	3.54197	4.815508885
3.002	3.44668	3.54197	4.666596705
3.002	3.44668	3.53708	4.919026549

Average value of Feigenbaum constant = 4.871087945999999

Report Made by Agam