ARE212: Section 05

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This section will briefly walk through two general concepts, one econometric and one `R` oriented. We will examine the characteristics of generalized least squares (GLS), and specifically the efficiency gains from feasible generalized least squares (FGLS). This is the econometric concepts. We will then recreate the graphs from Figures 2.6 and 2.7, roughly, in the notes using `ggplot2` a very popular, very powerful graphing package in `R`. This is part is optional, especially since it is only a very brief treatment of the package — there is a lot more to learn.

Let $x \sim U(0, 2000)$ and $\epsilon \sim N(0, (x/1000)^2)$. The underlying population in (2.102) is given as

$$y_i = \alpha + x_i\beta + \epsilon,$$

where $\alpha = 0.5$ and $\beta = 1.5$. The objective is to plot the simulated sampling distribution of the OLS estimator applied to $B = 10,000$ draws, each of size $n = 1000$. First, let's generate the sample data for one draw.

```
n <- 1000
x <- runif(n, min=0, max=2000)
eps <- rnorm(n, 0, sqrt((x/1000)^2))
y <- 0.5 + x*1.5 + eps
```

Now we can calculate the standard OLS parameter vector $[\hat{\alpha} \;\; \hat{\beta}]'$ by noting that $\mathbf{X}$ is just the x vector bound to a column of ones. We will only examine $\hat{\beta}$ for this section, rather than both parameters.

```
X <- cbind(1, x)
params <- solve(t(X) %*% X) %*% t(X) %*% y
beta <- params[2]
print(beta)
```

```
 [1] 1.499982
```

Let's package this into a function, called `rnd.beta`, so that we can collect the OLS parameter for an arbitrary number of random samples, noting that n is a constant so we may as well keep it out of the function so that 1000 is not reassigned thousands of times to n.

```
rnd.beta <- function(i) {
  # the argument ‘i‘ is not used within the function, but rather to
  # index the function call; useful for =apply= functions
  x <- runif(n)
  eps <- rnorm(n, 0, sqrt(x/10))
  y <- 0.5 + x*1.5 + eps
  X <- cbind(1, x)
  params <- solve(t(X) %*% X) %*% t(X) %*% y
  beta <- params[2]
  return(beta)
}
```

Since there aren't any supplied arguments, the function will return an estimated $\hat{\beta}$ from a a different random sample for each call:

```
rnd.beta()
rnd.beta()
```

```
 [1] 1.475412
 [1] 1.507417
```

This is convenient for bootstrapping without loops, but rather applying the function to a list of effective indices.[1] Now replicating the process for B draws is straightforward:

```
B <- 1000
beta.vec <- sapply(1:B, rnd.beta)
mean(beta.vec)
sd(beta.vec)
```

```
 [1] 1.500669
 [1] 0.02426108
```

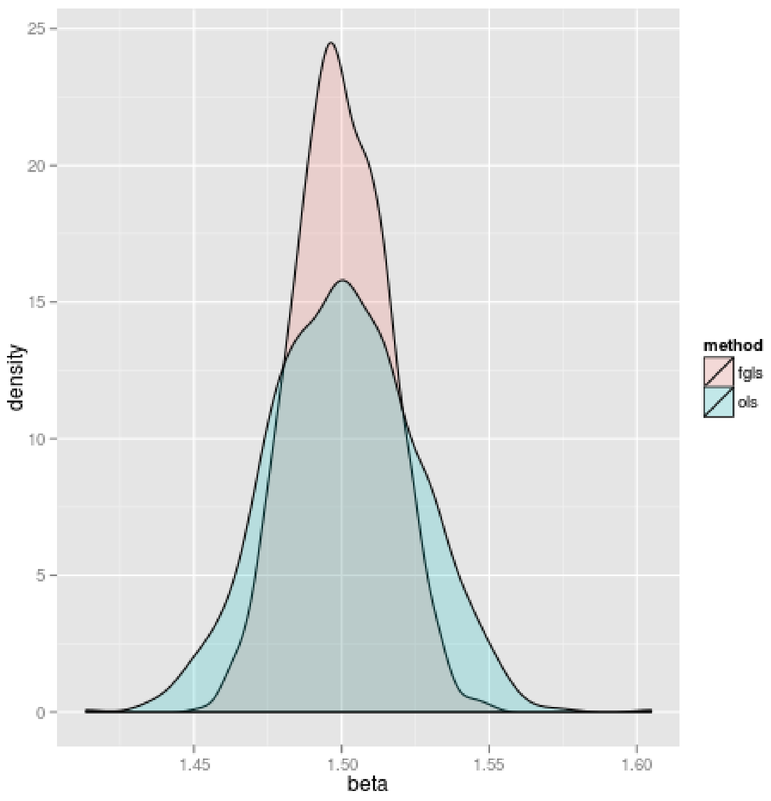
Alright. Looking good. The average of the simulated sample is much closer to β than any individual call of `rnd.beta`, suggesting that the distribution of the simulated parameters will be appropriately centered. Now, let's create another, similar function that returns the FGLS estimates.

```
rnd.fgls.beta <- function(i) {
  x <- runif(n)
  eps <- rnorm(n, 0, sqrt(x/10))
  y <- 0.5 + x*1.5 + eps
  X <- cbind(1, x)
  C <- diag(1/sqrt(sapply(x,function(x){x/10})))
```

[1]This is much more comfortable for me, with a background in functional programming. There is some inherent value, however, in keeping code compact by mapping across indices rather than incrementing an index within a `for` loop; the code is more readable and less prone to typos.

```
  y.wt <- C %*% y
  X.wt <- C %*% X
  b.fgls <- solve(t(X.wt) %*% X.wt) %*% t(X.wt) %*% y.wt
  beta <- b.fgls[2]
  return(beta)
}
  fgls.beta.vec <- sapply(1:B, rnd.fgls.beta)

library(ggplot2)
ols <- cbind(beta.vec, 0)
fgls <- cbind(fgls.beta.vec, 1)
data <- data.frame(rbind(ols, fgls))
names(data) <- c("beta", "mt")
data$method <- "ols"
data$method[data$mt==1] <- "fgls"
m <- ggplot(data, aes(x=beta, ..density.., fill=method))
m + geom_density(alpha=0.2)
```


density
beta
method
fgls
ols