Simple example: suppose you're trying to figure out the wage effect of being married, being left-handed, and being married and left-handed. You would would want to run the following model:

$$w_i = \alpha + \beta_1 D_i^1 + \beta_2 D_i^2 + \beta_3 D_i^1 \times D_i^2 + \varepsilon_i$$

Where w_i is your wage, $D_i^1 = 1$ if the person is married, and $D_i^2 = 1$ if the person is left-handed. The easiest way to think about the interpretation of your mode is by considering considering what happens to the model about when you set the dummy variables to 1 or 0.

For example, suppose you wanted to get the average wage for un-married, right-handed people. This is the omitted category because $D_i^1 = D_i^2 = 0$. Plugging those two into our model gives that $w_i = \alpha$. So the average wage is just α.

Next suppose you were interested in the average wage for married ($D_i^1 = 1$), right-handed people ($D_i^2 = 0$). Now we get that $w_i = \alpha + \beta_1$. Similarly, the average wage for unmarried ($D_i^1 = 0$), left-handed ($D_i^2 = 1$) people is $w_i = \alpha + \beta_2$.

Finally the average wage for married ($D_i^1 = 1$), left-handed ($D_i^2 = 1$) people is just the sum of all three: $w_i = \alpha + \beta_1 + \beta_2 + \beta_3$.

In table form, this can be summarized as:

Description	D_i^1	D_i^2	Average wage
Un-married, right-handed	0	0	α
Married, right-handed	1	0	$\alpha + \beta_1$
Unmarried, left-handed	0	1	$\alpha + \beta_2$
Married, left-handed	1	1	$\alpha + \beta_1 + \beta_2 + \beta_3$

Interpretation
But what about interpreting the coefficients α and β_j themselves? Following the discussion above, the following is hopefully logical:

- α: The average wage for the omitted category (unmarried, right-handed)

- β_1: The difference in average wage for married vs. people who are not

- β_2: The difference in average wage for left-handed vs. people who are not

- β_3: The difference in average wage for married AND left-handed people vs. people who are not

The last one is the head-scratcher. The idea of the coefficients is that they're the effect of "turning on" the variable they're attached to. That doesn't have a lot of real-world meaning in some context — it is difficult to "turn on" being left handed — but it might be the easiest way to think about it.