Additional puzzles

1. **Partitioned regression**: Generate a 100×4 matrix $\mathbf{X}$ *including* a column of ones for the intercept. Additionally, generate a vector $\mathbf{y}$ according to the generating process:

$$y_i = 1 + x_{1i} + 2x_{2i} + 3x_{3i} + \epsilon_i,$$

where $\epsilon_i \sim N(0, 1)$. Let $\mathbf{Q}$ be the first three columns of $\mathbf{X}$ and let $\mathbf{N}$ be the final column. In addition, let

$$\begin{aligned} \hat{\gamma}_1 &= (\mathbf{Q}'\mathbf{Q})^{-1}\mathbf{Q}'\mathbf{y} \quad \text{and} \quad \mathbf{f} = \mathbf{y} - \mathbf{Q}\hat{\gamma}_1 \\ \hat{\gamma}_2 &= (\mathbf{Q}'\mathbf{Q})^{-1}\mathbf{Q}'\mathbf{N} \quad \text{and} \quad \mathbf{g} = \mathbf{N} - \mathbf{Q}\hat{\gamma}_2 \\ \hat{\gamma}_3 &= \mathbf{f} \cdot \mathbf{g}/||\mathbf{g}||^2 \quad \text{and} \quad \mathbf{e} = \mathbf{f} - \mathbf{g}\hat{\gamma}_3 \end{aligned}$$

Show that $\hat{\beta} = [\hat{\gamma}_1 - \hat{\gamma}_2\hat{\gamma}_3 \quad \hat{\gamma}_3]$. Note that the total dimension of $\hat{\beta}$ is 4.

Answer:

```
X <- cbind(1, randomMat(100, 3))
e <- rnorm(100)

beta <- c(1, 1, 2, 3)
y <- X %*% beta + e

Q <- X[, 1:3]
N <- X[, 4]
gamma.1 <- solve(t(Q) %*% Q) %*% t(Q) %*% y
gamma.2 <- solve(t(Q) %*% Q) %*% t(Q) %*% N
f <- y - Q %*% gamma.1
g <- N - Q %*% gamma.2
gamma.3 <- as.numeric(crossprod(f,g)/crossprod(g,g))
e <- f - g * gamma.3

(b <- c(gamma.1 - gamma.2 * gamma.3, gamma.3))
```

```
Error in cbind(1, randomMat(100, 3)) :
  could not find function "randomMat"
Error: object 'X' not found
Error: object 'X' not found
Error: object 'X' not found
Error in t(Q) : object 'Q' not found
Error in t(Q) : object 'Q' not found
Error: object 'y' not found
Error: object 'N' not found
Error in crossprod(f, g) : object 'f' not found
Error: object 'f' not found
Error: object 'gamma.1' not found
```