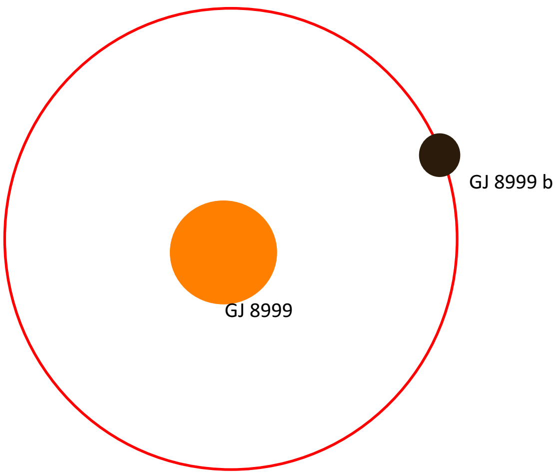
Question 1 – Exoplanet Characterization

In this question, you will estimate the mass and radius of a planet from its radial velocity and transit data.

A mysterious new (and fake!) planet, GJ 8999 b, has been detected orbiting the M dwarf GJ 8999. GJ 8999 is a *very* small star, with a mass of $0.2M_{\odot}$ and a radius of $0.2R_{\odot}$. (If you haven't seen those symbols before, $M_{\odot}$ and $R_{\odot}$ are the mass and radius of the Sun, respectively.)

The cunning astronomer you are, you have been measuring transit and radial velocity data of this star to figure out the planet's mass and radius of this planet, so you can publish a paper on the system! Let's characterize this planet now.

a) What is the inclination of GJ 8999 b?



Answer: i ~ 90 degrees
Explanation: In the question, it is mentioned that transit method had been used which is possible
if an only if the inclination of the planet is 90 degrees or very close to 90 degrees.

b) New transit data from the Transiting Exoplanet Survey Satellite (TESS) has come in, and it very much looks like we have some exoplanet transits! A plot of the flux from the full 28-day observation period of TESS is shown here, as well as a plot that is zoomed into a single transit.

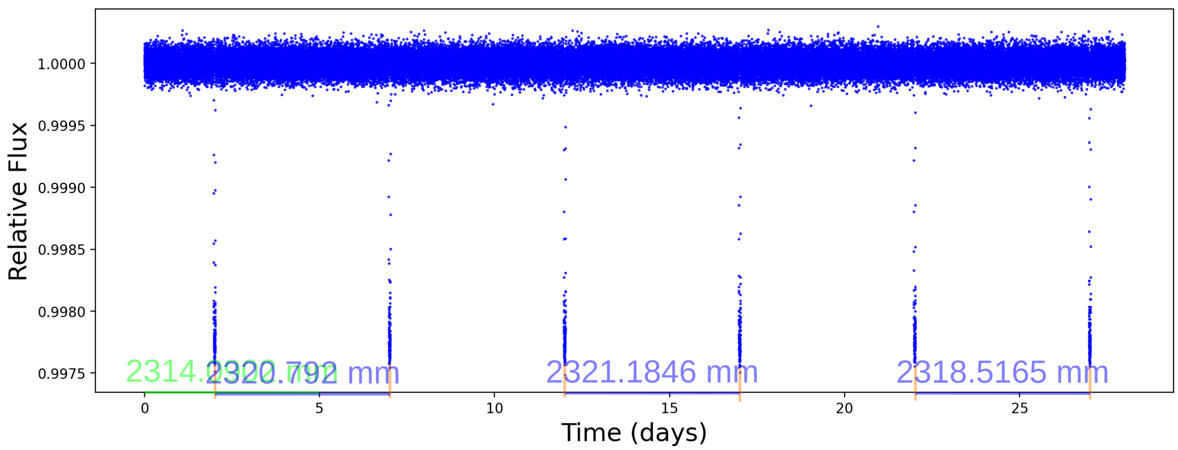


Figure 1: A plot of the flux of GJ 8999 over time over a 28-day period.

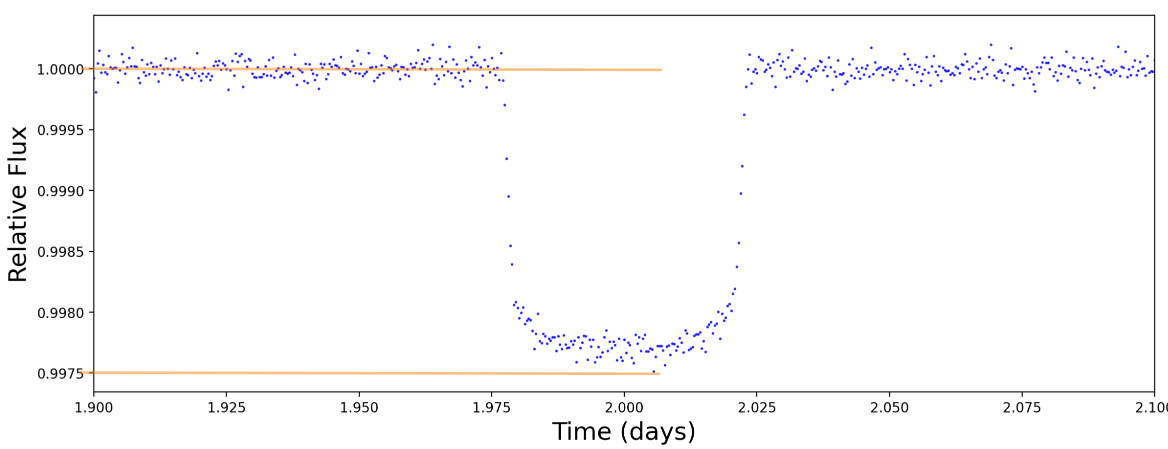


Figure 2: A plot of the flux of GJ 8999 over time, zoomed into a single exoplanet transit.

What is the period of this exoplanet?

From the above graph,
5 days == 2314.0902 mm
And,
T1 = 2320.792 mm == 5.014 days
T2 = 2321.1846 mm == 5.015 days
T3 = 2318.5165 mm == 5.009 days
so,
T_avg = 5.013 days = 5 days 18 minutes 54 seconds (roughly)

c) What is the radius of this planet?

Transit depth = Z = (fractional decrease in stellar flux) = (1 - 0.9975)/(1.0000) x 100% = 0.25%
We know,
Z= (R_p/R_s)^2
or, R_p = R_s x sqrt(Z) = 0.2 R x sqrt(0.0025) = 0.2 R x 0.05 = 0.01 R

d) Luckily for us, we have gotten some radial velocity data to figure out this planet's mass, too. This data, taken over a period of 30 days, measures the star's Doppler shift as it moves back and forth due to the planet's gravity.

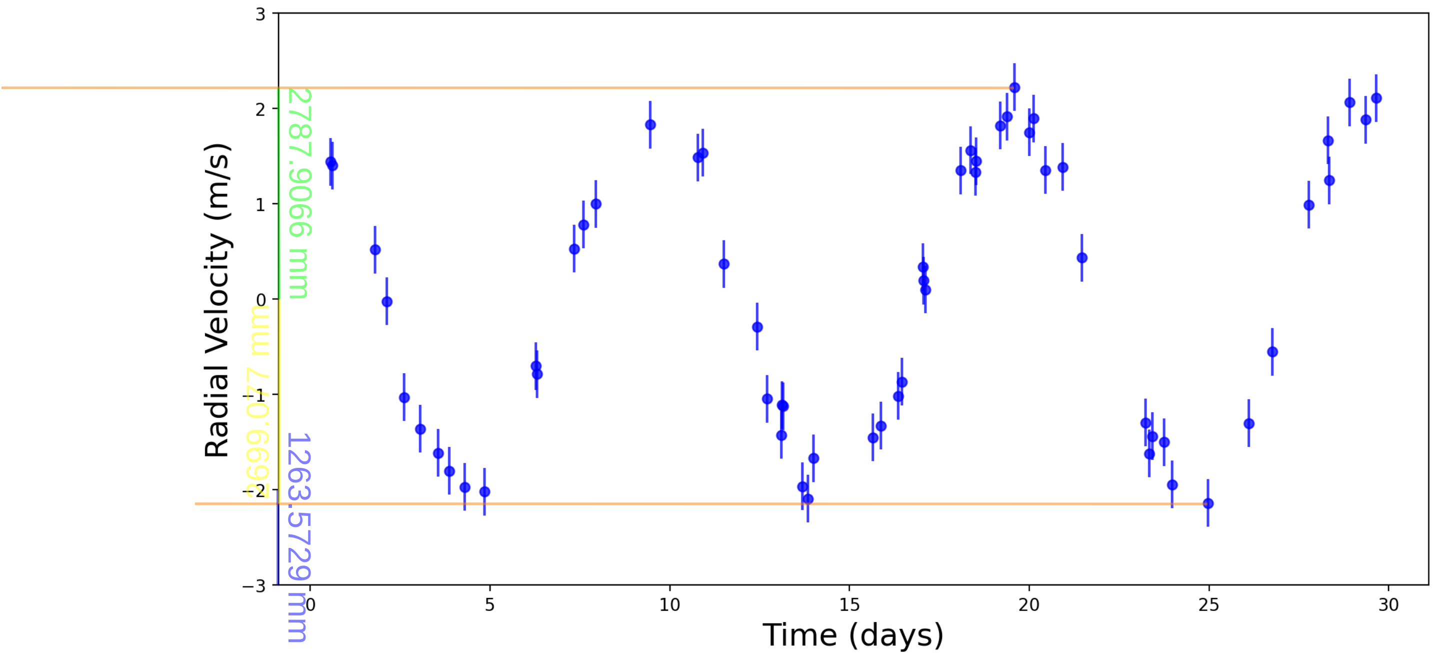


Figure 3: A plot of the radial velocity of GJ 8999 over time.

What is the semi-amplitude K of this planetary signal?

In the graph,
1 m/s == 1263.5729 mm

and,
v_max == 2787.9066 mm == 2.206 m/s
v_min == -2699.077 mm == -2.136 m/s

so,
semi-amplitude, K = (v_max - v_min)/2 = (2.206 - (-2.136))/2 = 2.171 m/s

e) What is the mass of this planet?

$$K = M_p \sin i \left(\frac{2\pi G}{P M_s^2}\right)^{1/3}$$

$$\Rightarrow M_p = \frac{K}{\sin i \cdot \left(\frac{2\pi G}{P M_s^2}\right)^{1/3}}$$

where,

$K = 2.171$ m/s

$i = 90°$

$G = 6.67 \times 10^{-11}\ m^3/Kg s^2$

$P = 5$ days 18 minutes 54 seconds

$= 433134$ s

$M = 0.2 M_\odot = 0.2 \times 2 \times 10^{30}$ Kg

on solving,

we get :... (long calcn :))

$M_p \approxeq 1.187 \times 10^{25}$ Kg

$$\frac{m/s}{\left(\frac{m^3}{Kg s^2} \cdot \frac{1}{s \cdot Kg^2}\right)^{1/3}} = \frac{\cancel{m}}{\cancel{s}} \cdot \frac{Kg \cancel{s}}{\cancel{m}} = Kg$$

$$M = \frac{(2.171)}{\sin(90)\left(\frac{(2 \cdot \pi \cdot 6.67430 \cdot 10^{-11})}{433134 \cdot (0.2 \cdot 1.988416 \cdot 10^{30})^2}\right)^{\frac{1}{3}}} = 1.186761771 \times 10^{25}$$

f) So, now that we've found the mass and radius of our planet, let's try to figure out what it's made of!

The following plot shows (very rough) 'mass-radius curves' of rocky exoplanets of different compositions. A planet lying on a given curve has a mass and radius consistent with being made of the corresponding composition.

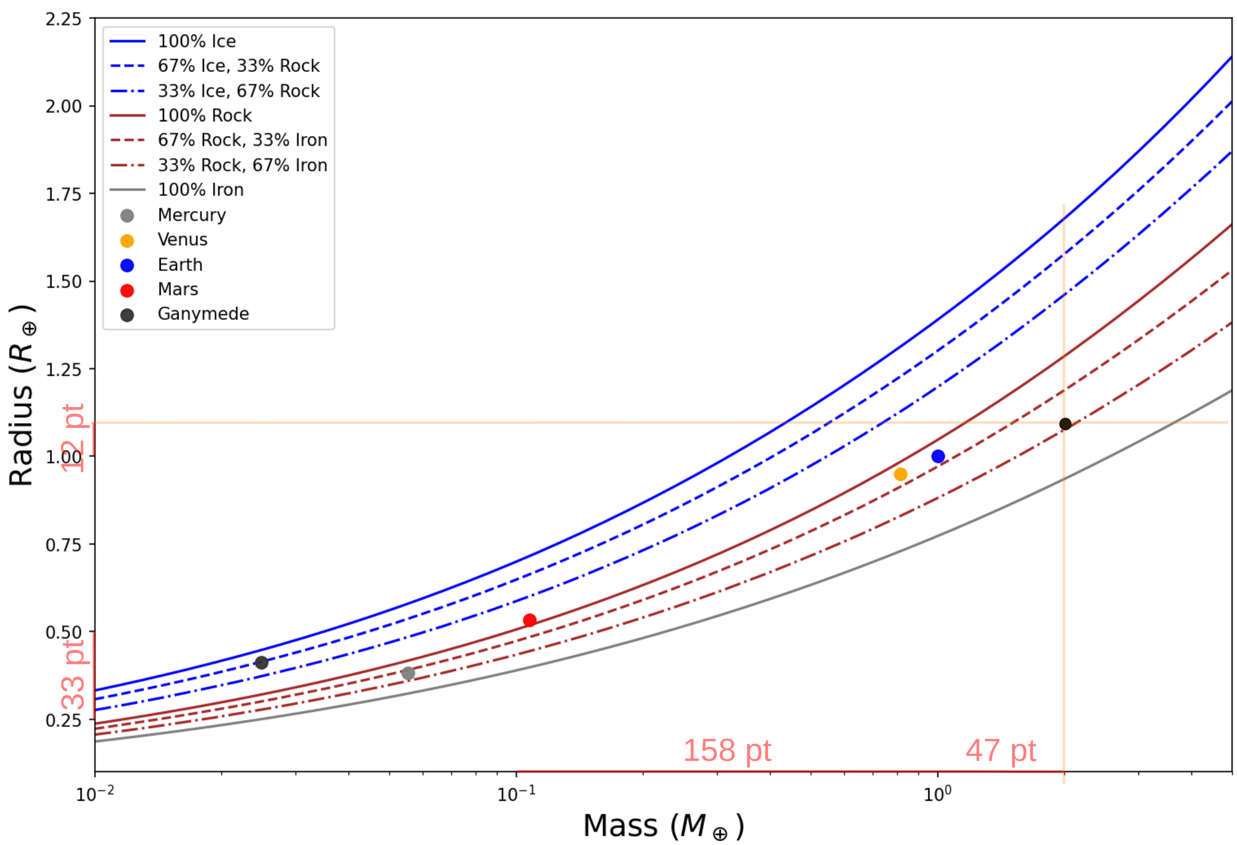


Figure 4: A plot showing the mass-radius curves for different exoplanet compositions.

The five rocky planets (plus Ganymede) are all shown on the plot as well. For example, Earth lies very near the '67% rock, 33% iron' curve, and Earth's composition IS indeed about 67% rock and 33% iron.

With this in mind, what is the composition of GJ 8999 b?

From earlier, R_p = 0.01 R_sun = 0.01 x 6.957 x 10^8 m = 1.092 R_earth
And, M_p = 1.186761771 x 10^25 kg = 1.99 M_earth ~ 10^(0.299) M_earth

In the graph, 0.25 R_earth == 33 pt so, 0.092 R_earth ~ 12 pt ;1 unit in the x-axis == 158 pt so, 0.299 units ~ 47 pt

From the graph, it's clear that the composition of the planet is very near to 33% Rock, 67% Iron.

THANK YOU