9.6.9

EE24BTECH11002 - Agamjot Singh

Question:
Solve the differential equation:

$$\frac{dy}{dx} + \sqrt{\frac{1 - y^2}{1 - x^2}} = 0 \tag{1}$$

Theoritical solution: The given equation is a linear ordinary differential equation.

$$\frac{dy}{dx} = -\sqrt{\frac{1 - y^2}{1 - x^2}} \tag{2}$$

$$\frac{dy}{\sqrt{1 - y^2}} = -\frac{dx}{\sqrt{1 - x^2}} \tag{3}$$

$$\tag{4}$$

Integrating on both sides, we get,

$$\int \frac{dy}{\sqrt{1 - y^2}} = \int -\frac{dx}{\sqrt{1 - x^2}} \tag{5}$$

$$\sin^{-1} y = \sin^{-1} x + C, \text{ where } C \text{ is the constant of integration} \tag{6}$$

Computational Solution: Euler's method
By the first principle of derivative,

$$y'(x) = \lim_{h \to 0} \frac{y(x + h) - y(x)}{h} \tag{7}$$

$$y(x + h) = y(x) + h\left(y'(x)\right), \; h \to 0 \tag{8}$$

Expressing this system in an iterative format (by method of finite differences),

$$y(x_{n+1}) = y(x_n) + hy'(x_n) \tag{9}$$

$$y_{n+1} = y_n + hy'(x_n) \tag{10}$$

$$x_{n+1} = x_n + h \tag{11}$$

Substituting the value of $y'(x)$, we get,

$$y_{n+1} = y_n + h\left(-\sqrt{\frac{1 - y^2}{1 - x^2}}\right) \tag{12}$$

Iteratively plotting the above system taking intial conditions as,

$$x_0 = -0.5 \;,\; y_{1,0} = \sin\left(1 + \frac{\pi}{6}\right) \tag{13}$$

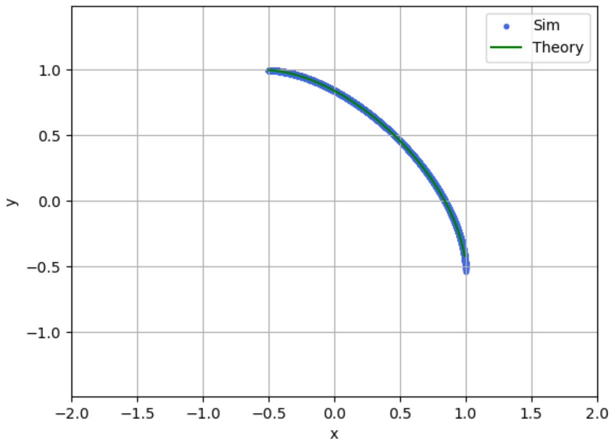

Fig. 0: Computational solution for $y' + \sqrt{\frac{1-y^2}{1-x^2}} = 0$