

10.3.3.1.3

EE24BTECH11002 - Agamjot Singh

Question:

Solve the following pair of linear equations,

$$3x - y = 3 \quad (1)$$

$$9x - 3y = 9 \quad (2)$$

Solution:

Let

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (3)$$

Expressing the system in matrix form,

$$\begin{pmatrix} 3 & -1 \\ 9 & -3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 \\ 9 \end{pmatrix} \quad (4)$$

$$\text{which is of the form } \mathbf{Ax} = \mathbf{b} \quad (5)$$

Any non-singular matrix A can be expressed as a product of an upper triangular matrix U and a lower triangular matrix L , such that

$$A = LU \quad (6)$$

$$\Rightarrow LU\mathbf{x} = \mathbf{b} \quad (7)$$

U is determined by row reducing A using a pivot,

$$\begin{pmatrix} 3 & -1 \\ 9 & -3 \end{pmatrix} \xrightarrow{R_2 \rightarrow R_2 - 3R_1} \begin{pmatrix} 3 & -1 \\ 0 & 0 \end{pmatrix} \quad (8)$$

Let

$$L = \begin{pmatrix} 1 & 0 \\ l & 1 \end{pmatrix} \quad (9)$$

l is the multiplier used to zero out a_{21} in A .

$$L = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \quad (10)$$

We see that there is a zero on the diagonal of the upper triangular matrix U which implies that A is singular and hence the system has either zero or infinitely many solutions.

Let $\mathbf{y} = U\mathbf{x}$,

$$L\mathbf{y} = \mathbf{b} \quad (11)$$

After we find \mathbf{y} , we find \mathbf{x} using the following equation,

$$U\mathbf{x} = \mathbf{y} \quad (12)$$

Applying forward substitution on equation (11), we get,

$$\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 9 \end{pmatrix} \quad (13)$$

$$\Rightarrow \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad (14)$$

Substituting \mathbf{y} in equation (12), we get,

$$\begin{pmatrix} 3 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad (15)$$

$$\Rightarrow 0(x) + 0(y) = 0 \quad (16)$$

This shows that the equation has infinitely many solutions.

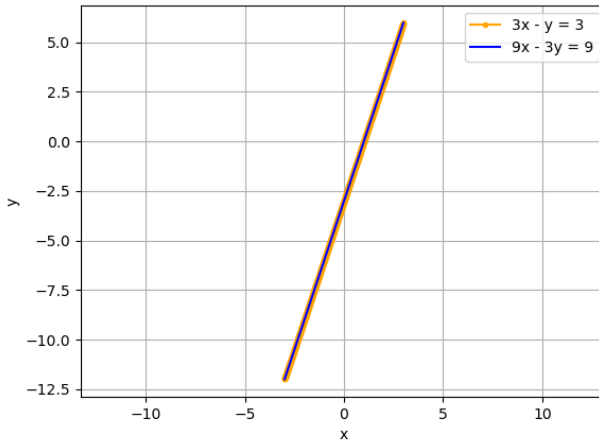


Fig. 0: Plotting the two lines, which come out as parallel