

1.3.4

EE24BTECH11002 - Agamjot Singh

Question:

If $\mathbf{A}(1, 3)$, $\mathbf{B}(-1, 2)$, $\mathbf{C}(2, 5)$ and $\mathbf{D}(x, 4)$ are the vertices of a parallelogram $ABCD$, then the value of x is

Solution: Let \mathbf{D} be some $\begin{pmatrix} x \\ y \end{pmatrix}$. By parallelogram law of addition,

$$\mathbf{BA} + \mathbf{BC} = \mathbf{BD} \quad (1)$$

$$\mathbf{BA} = \mathbf{A} - \mathbf{B} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad (2)$$

$$\mathbf{BC} = \mathbf{C} - \mathbf{B} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \quad (3)$$

$$\mathbf{BD} = \mathbf{D} - \mathbf{B} = \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} x+1 \\ y-2 \end{pmatrix} \quad (4)$$

$$\text{By equation (1), you get} \quad (5)$$

$$\begin{pmatrix} x+1 \\ y-2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \quad (6)$$

$$x = 4, y = 6 \quad (7)$$

By equation (7), we can see that no such \mathbf{D} in the form $\begin{pmatrix} x \\ 4 \end{pmatrix}$ exists.

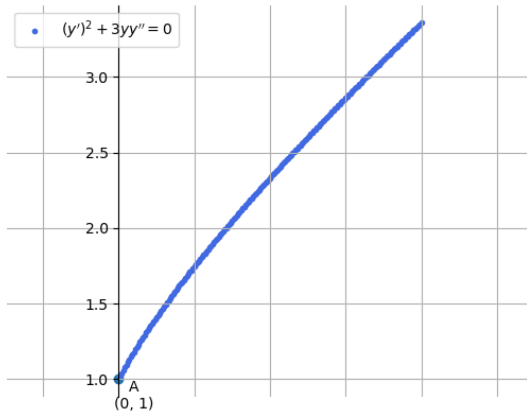


Fig. 0: Quadrilateral ABCD formed with given equations