9.1.3

EE24BTECH11002 - Agamjot Singh

Question:
Solve the differential equation:

$$\left(\frac{dy}{dx}\right)^4 + 3y\frac{d^2y}{dx^2} = 0 \tag{1}$$

Solution:
Theoritical solution: TODO
Computational Solution: Euler's method
By the first principle of derivative,

$$y'(x) = \lim_{h\to 0} \frac{y(x+h) - y(x)}{h} \tag{2}$$

$$y(x+h) = y(x) + h\left(y'(x)\right), h \to 0 \tag{3}$$

For a m^{th} order differential equation,
Let $y_1 = y$, $y_2 = y'$, $y_3 = y''\ldots$, $y_m = y^{m-1}$, then we obtain the system

$$\begin{pmatrix} y_1' \\ y_2' \\ \vdots \\ y_{m-1}' \\ y_m' \end{pmatrix} = \begin{pmatrix} y_2 \\ y_3 \\ \vdots \\ y_m \\ f(x, y_1, y_2, \ldots, y_m) \end{pmatrix} \tag{4}$$

Here, f is described by the given differential equation. The initial conditions $y_1(x_0) = K_1$, $y_2(x_0) = K_2$, $\ldots$, $y_m(x_0) = K_m$.
Using Euler's method (first principle of derivative),

$$\begin{pmatrix} y_1(x+h) \\ y_2(x+h) \\ \vdots \\ y_m(x+h) \end{pmatrix} = \begin{pmatrix} y_1(x) + hy_2(x) \\ y_2(x) + hy_3(x) \\ \vdots \\ y_m(x) + hf(x, y_1, y_2 \ldots y_m) \end{pmatrix} \tag{5}$$

The given differential equation can be represented as,

$$(y')^4 + 3yy'' = 0 \tag{6}$$

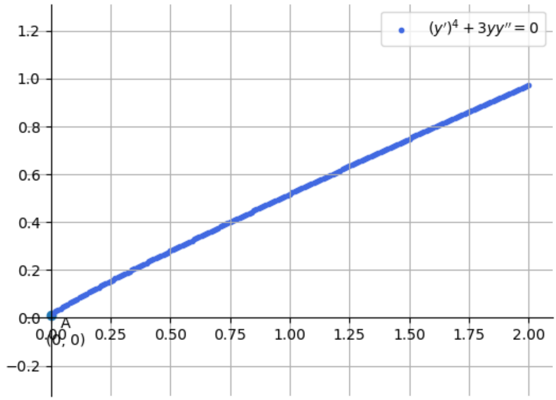

Fig. 0: Quadrilateral ABCD formed with given equations