1.3.4

EE24BTECH11002 - Agamjot Singh

Question:

If $\mathbf{A}(1,3)$, $\mathbf{B}(-1,2)$, $\mathbf{C}(2,5)$ and $\mathbf{D}(x,4)$ are the vertices of a parallelogram $ABCD$, then the value of x is

Solution: Let $\mathbf{D}$ be some $\begin{pmatrix} x \\ y \end{pmatrix}$. By parallelogram law of addition,

$$\mathbf{BA} + \mathbf{BC} = \mathbf{BD} \tag{1}$$

$$\mathbf{BA} = \mathbf{A} - \mathbf{B} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{2}$$

$$\mathbf{BC} = \mathbf{C} - \mathbf{B} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \tag{3}$$

$$\mathbf{BD} = \mathbf{D} - \mathbf{B} = \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} x+1 \\ y-2 \end{pmatrix} \tag{4}$$

$$\text{By equation (1), you get} \tag{5}$$

$$\begin{pmatrix} x+1 \\ y-2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \tag{6}$$

$$x = 4, y = 6 \tag{7}$$

By equation (7), we can see that no such $\mathbf{D}$ in the form $\begin{pmatrix} x \\ 4 \end{pmatrix}$ exists.

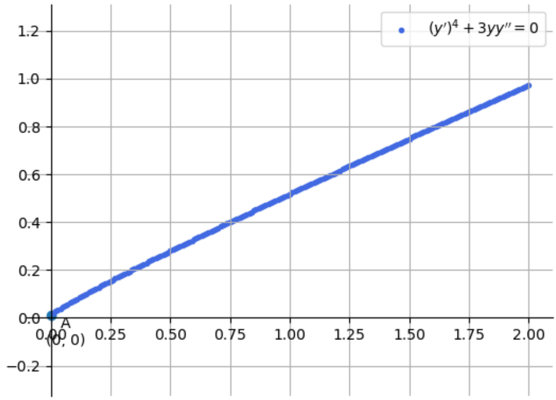

Fig. 0: Quadrilateral ABCD formed with given equations