9.1.3

EE24BTECH11002 - Agamjot Singh

Question:
Solve the differential equation:

$$x\frac{dy}{dx} + y - x + xy\cot x = 0 \ , \ x \neq 0 \tag{1}$$

$$\tag{2}$$

Theoritical solution: The given equation is a linear ordinary differential equation.

$$\frac{dy}{dx} + \frac{y}{x} - 1 + y\cot x = 0 \tag{3}$$

$$\frac{dy}{dx} + y\left(\frac{1}{x} + \cot x\right) = 1 \tag{4}$$

The integrating factor is given by

$$e^{\int\left(\frac{1}{x} + \cot x\right)dx} = e^{\log(x\sin x)} \tag{5}$$

$$= x\sin x \tag{6}$$

Multiplying on both sides, we get,

$$x\sin x\frac{dy}{dx} + y\left(\sin x + x\cos x\right) = x\sin x \tag{7}$$

$$d\left(yx\sin x\right) = x\sin x\, dx \tag{8}$$

Integrating on both sides, we get,

$$yx\sin x = \int x\sin x\, dx \tag{9}$$

$$= -x\cos x + \sin x + C, \text{ where } C \text{ is the constant of integration} \tag{10}$$

$$\implies y = y\left(x\right) = -\cot x + \frac{1}{x} + \frac{C}{x\sin x} \tag{11}$$

Computational Solution: Euler's method
By the first principle of derivative,

$$y'\left(x\right) = \lim_{h\to 0}\frac{y\left(x+h\right) - y\left(x\right)}{h} \tag{12}$$

$$y\left(x+h\right) = y\left(x\right) + h\left(y'\left(x\right)\right),\ h \to 0 \tag{13}$$

Expressing this system in an iterative format (by method of finite differences),

$$y(x_{n+1}) = y(x_n) + hy'(x_n) \tag{14}$$

$$y_{n+1} = y_n + hy'(x_n) \tag{15}$$

$$x_{n+1} = x_n + h \tag{16}$$

Substituting the value of $y'(x)$, we get,

$$y_{n+1} = y_n + h\left(1 - y_n\left(\frac{1}{x_n} + \cot x_n\right)\right) \tag{17}$$

For iteratively plotting the above system, we only take 3 intervals as the value tends to infinity at infinitely many points, thus we take 3 initial conditions,

$$(x_0)_1 = 0.21 \;,\; (y_{1,0})_1 = 22 \tag{18}$$

$$(x_0)_2 = 3.2 \;,\; (y_{1,0})_2 = -20 \tag{19}$$

$$(x_0)_3 = -3.1 \;,\; (y_{1,0})_3 = -20 \tag{20}$$

we get the following plot.

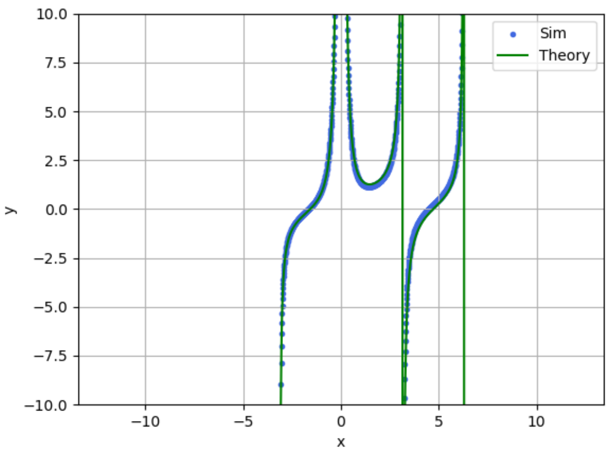

Fig. 0: Computational solution for $xy' + y - x + xy\cot x = 0$