10.4.2.6

EE24BTECH11002 - Agamjot Singh

Question:
A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was 90, find the number of articles produced and the cost of each article.
Solution:
Let the number of articles produced in a day be $x \geq 0$, then the above question can the following quadratic equation will be formed.

$$(3 + 2x)\,x = 90 \tag{1}$$
$$2x^2 + 3x - 90 = 0 \tag{2}$$

Theoretically, it can easily be solved using the quadratic formula,

$$x = \frac{-3 \pm \sqrt{729}}{4} = 6, -\frac{30}{4} \tag{3}$$

As we take $x \geq 0$, $x = 6$ is the solution.
Now we use the **Newton-Raphson method** to computationally find the roots.
Let

$$f(x) = 2x^2 + 3x - 90 \tag{4}$$
$$\implies f'(x) = 4x + 3 \tag{5}$$

The difference equation by the Newton-Raphson method is given by,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)},\ f'(x_n) \neq 0 \tag{6}$$

If we, at any point, encouter a situation in which $f'(x_n) = 0$, it implies that our inital guess (x_0) lies betweeen the roots or the equation has no roots.
Substituting $f(x)$ and $f'(x)$ in the difference equation, we get,

$$x_{n+1} = x_n - \left(\frac{2(x_n)^2 + 3x_n - 90}{4x_n + 3}\right) \tag{7}$$
$$x_{n+1} = \frac{2(x_n)^2 + 90}{4x_n + 3},\ x_n \neq -\frac{3}{4} \tag{8}$$

Taking initial guess $(x_0) = 8$, we get the root as $x = 6.000000476837158$. **Matrix Method:**
Frobenius **companion matrix** for a polynomial p of the form,

$$p(x) = c_0 + c_1 x + c_2 x^2 + \cdots + c_{n-1} x^{n-1} + x_n \tag{9}$$

is given by

$$C(p) = \begin{pmatrix} 0 & 0 & \dots & 0 & -c_0 \\ 1 & 0 & \dots & 0 & -c_1 \\ 0 & 1 & \dots & 0 & -c_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & -c_{n-1} \end{pmatrix} \tag{10}$$

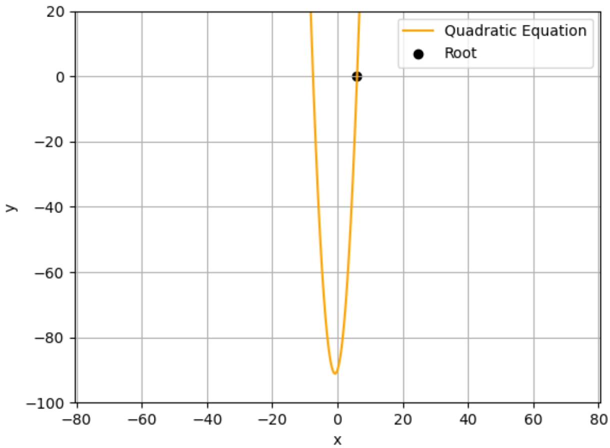

Fig. 0: Objective Function with the minimum point