10.5.3.9

EE24BTECH11002 - Agamjot Singh

Question:
If $\mathbf{A}\,(1,3)$, $\mathbf{B}\,(-1,2)$, $\mathbf{C}\,(2,5)$ and $\mathbf{D}\,(x,4)$ are the vertices of a parallelogram $ABCD$, then the value of x is
Solution: By definition of parallelogram, we say that the line **AD** is parallel to the line **BC**. Therefore, the slopes of these lines must be equal.

The direction vector of **PQ** is defined as

$$\mathbf{m} = \mathbf{Q} - \mathbf{P} = k\begin{pmatrix}1\\ m\end{pmatrix}$$

where m is the slope of **PQ**. We also say that

$$\mathbf{m} \equiv \begin{pmatrix}1\\ m\end{pmatrix}$$

Direction vector of $\mathbf{m} = \mathbf{BC}$ is given as

$$\mathbf{m} = \mathbf{C} - \mathbf{B} \tag{1}$$

$$\mathbf{m} = \begin{pmatrix}2\\ 5\end{pmatrix} - \begin{pmatrix}-1\\ 2\end{pmatrix} = \begin{pmatrix}3\\ 3\end{pmatrix} \tag{2}$$

$$\mathbf{m} \equiv \begin{pmatrix}1\\ 1\end{pmatrix} \tag{3}$$

$$m = 1 \tag{4}$$

So, the slope of **BC** is 1, which is equal to the slope of **AD**.
Direction vector of $\mathbf{m_1} = \mathbf{AD}$ is given as

$$\mathbf{m_1} = \mathbf{D} - \mathbf{A} \tag{5}$$

$$\mathbf{m_1} = \begin{pmatrix}x\\ 4\end{pmatrix} - \begin{pmatrix}1\\ 3\end{pmatrix} = \begin{pmatrix}x-1\\ 1\end{pmatrix} \tag{6}$$

$$\mathbf{m_1} \equiv \begin{pmatrix}1\\ \frac{1}{x-1}\end{pmatrix} \tag{7}$$

By equation (7), we get the slope of **AD** is $\frac{1}{x-1}$.
By equation (4), we get

$$\frac{1}{x-1} = 1 \tag{8}$$

$$x = 2 \tag{9}$$

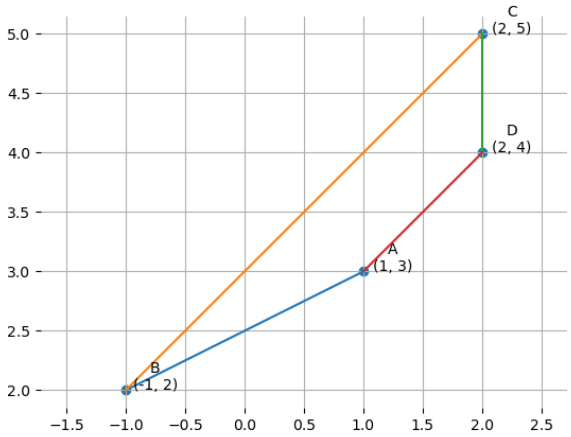

Fig. 0: Quadrilateral ABCD formed with given equations

As we can see in the graph, the given quadrilateral formed with x found in equation (9) is not a parallelogram. Therefore, we conclude that there is no x for which $ABCD$ is a parallelogram.