JEE Main 2023 April 15 Shift 1

EE24BTECH11002 - Agamjot Singh

1) Let S be the set of all values of λ, for which the shortest distance between the lines $\frac{x-\lambda}{0} = \frac{y-3}{4} = \frac{z+6}{1}$ and $\frac{x+\lambda}{3} = \frac{y}{-4} = \frac{z-6}{0}$ is 13. Then $8\left|\sum_{\lambda\in S} \lambda\right|$ is equal to
   a) 302
   b) 306
   c) 304
   d) 308

2) Let S be the set of all (λ, μ) for which the vectors $\lambda\mathbf{i} - \mathbf{j} + \mathbf{k}, \mathbf{i} + 2\mathbf{j} + \mu\mathbf{k}$ and $3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$, where $\lambda - \mu = 5$, are coplanar, then $\sum_{(\lambda,\mu)\in S} 80\left(\lambda^2 + \mu^2\right)$ is equal to
   a) 2130
   b) 2210
   c) 2290
   d) 2370

3) Let the foot of perpendicular of the point $P\begin{pmatrix}3\\-2\\-9\end{pmatrix}$ on the plane passing through the points $\begin{pmatrix}-1\\-2\\-3\end{pmatrix}, \begin{pmatrix}9\\3\\4\end{pmatrix}, \begin{pmatrix}9\\-2\\1\end{pmatrix}$ be $Q\begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix}$. Then the distance of Q from the origin is
   a) $\sqrt{29}$
   b) $\sqrt{38}$
   c) $\sqrt{42}$
   d) $\sqrt{35}$

4) If the set $\left[Re\left(\frac{z-\bar{z}+z\bar{z}}{2-3\bar{z}+5\bar{z}}\right) : z \in \mathbb{C}, Re(z) = 3\right]$ is equal to the interval $(\alpha\ ,\ \beta]$, then $24(\beta - \alpha)$ is equal to
   a) 36
   b) 27
   c) 30
   d) 42

5) Let $x = x(y)$ be the solution of the differential equation $2(y+2)\log_e(y+2)\,dx + (x + 4 - 2log_e(y+2))\,dy = 0, y > -1$ with $x\left(e^4 - 2\right) = 1$. Then $x\left(e^9 - 2\right)$ is equal to
   a) $\frac{4}{9}$
   b) $\frac{32}{9}$
   c) $\frac{10}{3}$
   d) 3

6) If $\int_0^1 \frac{1}{(5+2x-2x^2)\left(1+e^{(2-4x)}\right)}\,dx = \frac{1}{\alpha}\log_e\left(\frac{\alpha+1}{\beta}\right), \alpha, \beta > 0$, then $\alpha^4 - \beta^4$ is equal to
   a) 19
   b) -21
   c) 21
   d) 0

7) The number of common tangents, to the circles $x^2 + y^2 - 18x - 15y + 131 = 0$ and $x^2 + y^2 - 6x - 6y - 7 = 0$, is
   a) 4
   b) 1
   c) 3
   d) 2

8) Let $ABCD$ be a quadrilateral. If E and F are the mid points of the diagonals AC and BD respectively and $(\mathbf{AB} - \mathbf{BC}) + (\mathbf{AD} - \mathbf{DC}) = k\mathbf{FE}$, then k is equal to
   a) 4
   b) 2
   c) -2
   d) -4

9) Let $\left(a + bx + cx^2\right)^{10} = \sum_{i=0}^{20} p_i x^i, a, b, c \in \mathbb{N}$. If $p_1 = 20$ and $p_2 = 210$, then $2(a + b + c)$ is equal to
   a) 8

b) 12
c) 6
d) 15

10) Let $[x]$ denote the greatest integer function and $f(x) = \max\{1 + x + [x], 2 + x, x + 2[x]\}, 0 \le x \le 2$. Let m be the number of poitns in $[0, 2]$, where f is not continuous and n be the number of points in $(0, 2)$, where f is not differentiable. Then $(m + n)^2 + 2$ is equal to
a) 6
b) 3
c) 2
d) 11

11) A bag contains 6 white and 4 black balls. A die is rolled once and the number of ball equal to the number obtained on the die are drawn from the bag at random. The probability that all the balls drawn are white is
a) $\frac{1}{4}$
b) $\frac{9}{50}$
c) $\frac{11}{50}$
d) $\frac{1}{5}$

12) If the domain of the function $f(x) = \log_e\left(4x^2 + 11x + 6\right) + \sin^{-1}(4x + 3) + \cos^{-1}\frac{10x+6}{3}$ is $(\alpha, \beta]$, then $36|\alpha + \beta|$ is equal to
a) 72
b) 63
c) 45
d) 54

13) Let the determinant of a square matrix $\mathbf{A}$ of order m be $m - n$, where m and n satisfy $4m + n = 22$ and $17m + 4n = 93$. If $det(n\ adj(adj(m\mathbf{A}))) = 3^a 5^b 6^c$, then $a + b + c$ is equal to
a) 101
b) 84
c) 109
d) 96

14) The mean and standard deviation for 10 observations are 20 and 8 respectively. Later on, it was observed that one observation was recorded as 50 instead of 40. Then the correct variance is
a) 14
b) 11
c) 12
d) 13

15) If (α, β) is the orthocenter of the triangle ABC with vertices $A\begin{pmatrix}3\\-7\end{pmatrix}, B\begin{pmatrix}-1\\2\end{pmatrix}$ and $C\begin{pmatrix}4\\5\end{pmatrix}$, then $9\alpha - 6\beta + 60$ is equal to
a) 30
b) 35
c) 40
d) 25