

1.8.19

EE24BTECH11002 - Agamjot Singh

Question:

Find the points on X axis which are at a distance of $2\sqrt{5}$ from the point $(7, -4)$. How many such points are there?

Solution:

Let $\mathbf{A} \begin{pmatrix} 7 \\ -4 \end{pmatrix}$ and $\mathbf{B} \begin{pmatrix} x \\ y \end{pmatrix}$ be the desired point on the X axis.

The L_p norm, written as $\|x\|_p$, is defined as

$$\|x\|_p = \sqrt[p]{\sum_{i=1}^n |x_i|^p}, p \geq 1 \quad (1)$$

If \mathbf{B} is at a distance of $2\sqrt{5}$ from point \mathbf{A} .

$$\|\mathbf{B} - \mathbf{A}\|_p = 2\sqrt{5} \quad (2)$$

$$\Rightarrow \sqrt[p]{|x - 7|^p + |y + 4|^p} = 2\sqrt{5} \quad (3)$$

$$(4)$$

Taking $y = 0$, as \mathbf{B} lies on the X axis.

$$\sqrt[p]{|x - 7|^p + |4|^p} = 2\sqrt{5} \quad (5)$$

$$|x - 7|^p + (4)^p = (2\sqrt{5})^p \quad (6)$$

$$|x - 7| = \sqrt[p]{(2\sqrt{5})^p - (4)^p} \quad (7)$$

$$x = 7 \pm \sqrt[p]{(2\sqrt{5})^p - (4)^p} \quad (8)$$

By solving equation (7) with $y = 0$, we get,

$$\mathbf{B} = \begin{pmatrix} 7 \pm \sqrt[p]{(2\sqrt{5})^p - (4)^p} \\ 0 \end{pmatrix}, p \geq 1 \quad (9)$$

There are infinitely many such points as p varies from 1 to inf.

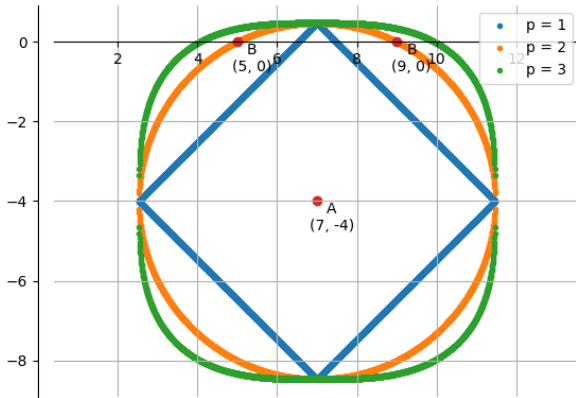


Fig. 0: Graph representing the locus of \mathbf{B} , taking $p = 1, 2, 3$