Assignment 2

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Section - B JEE Main/AIEEE

20) Let a, b, c be such that $b(a+c) \neq 0$ if

$$\begin{vmatrix} a & a+1 & a-1 \\ b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} +$$

$$\begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}\,a & (-1)^{n+1}\,b & (-1)^{n}\,c \end{vmatrix} = 0$$

, then the value of n is: (2009)

a) any even integer
b) any odd integer
c) any integer
d) zero

21) The number of 3×3 non-singular matrices with four entries as 1 and all other entries as 0, is (2010)

a) 5
b) 6
c) atleast 7
d) less than 4

22) Let $\mathbf{A}$ be a 2×2 matrix with non-zero entries and let $\mathbf{A}^2 = \mathbf{I}$, where $\mathbf{I}$ is 2×2 identity matrix. Define
$Tr(\mathbf{A})$- sum of diagonal elements of $\mathbf{A}$ and
$|\mathbf{A}|$ - determinant of matrix $\mathbf{A}$.
Statement - 1: $Tr(\mathbf{A}) = 0$.
Statement - 2: $|\mathbf{A}| = 1$

(2010)

a) Statement - 1 is true, Statement - 2 is true; Statement - 2 is **not** a correct explanation for Statement-1.
b) Statement - 1 is true, Statement - 2 is false.
c) Statement - 1 is false, Statement - 2 is true.
d) Statement - 1 is true, Statement - 2 is true; Statement - 2 is a correct explanation for Statement-1.

23) Consider the system of linear equations;

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 3 \\ 2x_1 + 3x_2 + x_3 &= 3 \\ 3x_1 + 5x_2 + 2x_3 &= 1 \end{aligned}$$

(2010)

a) exactly 3 solutions
b) a unique solution
c) no solution
d) infinite number of solutions

24) The number of values of k for which the linear equations $4x + ky + 2z = 0$, $kx + 4y + z = 0$ and $2x + 2y + z = 0$ possess a non zero solution is (2011)

a) 2
b) 1
c) zero
d) 3

25) Let $\mathbf{A}$ and $\mathbf{B}$ be two symmetrix matrices of order 3.
Statement - 1: $\mathbf{A}(\mathbf{BA})$ and $(\mathbf{AB})\mathbf{A}$ are symmetric matrices.
Statement - 2: $\mathbf{AB}$ is symmetric matrix if matrix multiplication of $\mathbf{A}$ with $\mathbf{B}$ is commutative.

a) Statement - 1 is true, Statement - 2 is true; Statement - 2 is **not** a correct explanation for Statement-1.
b) Statement - 1 is true, Statement - 2 is false.
c) Statement - 1 is false, Statement - 2 is true.
d) Statement - 1 is true, Statement - 2 is true; Statement - 2 is a correct explanation for Statement-1.

26) Let

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$$

If $\mathbf{u_1}$ and $\mathbf{u_2}$ are column matrices such that

$$\mathbf{Au_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

and

$$\mathbf{Au_2} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

, then $\mathbf{u_1} + \mathbf{u_2}$ is equal to: (2012)

a)

$$\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

b)

$$\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$$

c)

$$\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$$

d)

$$\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$

27) Let $\mathbf{P}$ and $\mathbf{Q}$ be 3×3 matrices $\mathbf{P} \neq \mathbf{Q}$. If $\mathbf{P}^3 = \mathbf{Q}^3$ and $\mathbf{P}^2\mathbf{Q} = \mathbf{Q}^2\mathbf{P}$ then determinant of $\left(\mathbf{P}^2 + \mathbf{Q}^2\right)$ is equal to (2012)

a) -2
b) 1
c) 0
d) -1

28) If

$$\mathbf{P} = \begin{pmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{pmatrix}$$

is the adjoint of a 3×3 matrix $\mathbf{A}$ and $|\mathbf{A}| = 4$, then α is equal to: $(JEEM2014)$

a) 4
b) 11
c) 5
d) 0

29) If $\alpha, \beta \neq 0$ and $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$$

$= K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$, then K is equal to

$(JEEM2014)$

a) 1
b) -1
c) $\alpha\beta$
d) $\frac{1}{\alpha\beta}$

30) If $\mathbf{A}$ is a 3×3 non-singular matrix such that $\mathbf{AA}' = \mathbf{A}'\mathbf{A}$ and $\mathbf{B} = \mathbf{A}^{-1}\mathbf{A}'$, then $\mathbf{BB}'$ equals: $(JEEM2014)$

a) $\mathbf{B}^{-1}$
b) $\left(\mathbf{B}^{-1}\right)'$
c) $\mathbf{I} + \mathbf{B}$
d) $\mathbf{I}$

31) The set of all values of λ for which the system of linear equations:

$$\begin{aligned} 2x_1 - 2x_2 + x_3 &= \lambda x_1 \\ 2x_1 - 3x_2 + 2x_3 &= \lambda x_2 \\ -x_1 + 2x_2 &= \lambda x_3 \end{aligned}$$

has a non-trivial solution

$(JEEM2015)$

a) contains two elements
b) contains more than two elements
c) is an empty set
d) is a singleton

32) If

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{pmatrix}$$

is a matrix satisfying the equation $\mathbf{AA}^T = 9\mathbf{I}$, where $\mathbf{I}$ is 3×3 identity matrix, then the ordered part (a, b) is equal to: $(JEEM2015)$

a) $(2, 1)$
b) $(-2, -1)$
c) $(2, -1)$
d) $(-2, 1)$

33) The system of linear equations

$$\begin{aligned} x + \lambda y - z &= 0 \\ \lambda x - y - z &= 0 \\ x + y - \lambda z &= 0 \end{aligned}$$

has a non-trivial solution for:

$(JEEM2016)$

a) exactly two values of λ
b) exactly three values of λ
c) inifinitely many values of λ
d) exactly one value of λ

34) If

$$\mathbf{A} = \begin{pmatrix} 5a & -b \\ 3 & 2 \end{pmatrix}$$

and $\mathbf{A}\mathrm{adj}(\mathbf{A}) = \mathbf{AA}^T$, then $5a + b$ is equal to: $(JEEM2016)$

a) 4
b) 13
c) -1
d) 5

35) Let k be an integer such that triangle with vertices $(k, -3k)$, $(5, k)$, $(-k, 2)$ has area 28 sq. units. Then the orthocentre of this triangle is at the point: $(JEEM2017)$

a) $\left(2, \frac{1}{2}\right)$
b) $\left(2, \frac{-1}{2}\right)$
c) $\left(1, \frac{3}{4}\right)$
d) $\left(1, \frac{-3}{4}\right)$

36) Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$$

, then k is equal to: $(JEEM2017)$

(a) 1
(b) $-z$
(c) z
(d) -1