JEE Main 2021 March 16 Shift 2

EE24BTECH11002 - Agamjot Singh

16) Let C_1 be the curve obtained by the solution of the differential equation $2xy\frac{dy}{dx} = y^2 - x^2, x > 0$. Let curve C_2 be the solution of $\frac{2xy}{x^2-y^2} = \frac{dy}{dx}$. If both the curves pass through $(1, 1)$, then the area enclosed by the curves C_1 and C_2 is equal to
   a) $\frac{\pi}{2} - 1$
   b) $\frac{\pi}{4} + 1$
   c) $\pi - 1$
   d) $\pi + 1$
17) Let $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$. If $\mathbf{r} \times \mathbf{a} = \mathbf{b} \times \mathbf{r}$, $\mathbf{r} \cdot (\alpha\mathbf{i} + 2\mathbf{j} + \mathbf{k}) = 3$ and $\mathbf{r} \cdot (2\mathbf{i} + 5\mathbf{j} - \alpha\mathbf{k}) = -1, \alpha \in \mathbb{R}$, then the value of $\alpha + |r|^2 =$
   a) 11
   b) 15
   c) 9
   d) 13
18) Let $P(x) = x^2 + bx + c$ be a quadratic polynomial with real coefficients such that $\int_0^1 P(x)\, dx$ and $P(x)$ leaves remainder 5 when divided by $(x - 2)$. Then the value of $9(b + c)$ is equal to
   a) 7
   b) 11
   c) 15
   d) 9
19) If the points of intersections of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = 4b, b > 4$ lie on the curve $y^2 = 3x^2$, then b is equal to
   a) 5
   b) 6
   c) 12
   d) 10
20) Let $A = \{2, 3, 4, 5, \dots, 30\}$ and '$\simeq$' be an equivalence relation on $A \times A$, defined by $(a, b) \simeq (c, d)$, if and only if $ad = bc$. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair $(4, 3)$ is equal to
   a) 7
   b) 5
   c) 6
   d) 8
21) Let $\mathbf{c}$ be a vector perpendicular to the vectors $\mathbf{a} = \mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$. If $\mathbf{c} \cdot (\mathbf{i} + \mathbf{j} + 3\mathbf{k}) = 8$, then the value of $\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$ is equal to
22) In $\triangle ABC$, the lengths of the sides AC and AB are 12 cm and 5 cm, respectively. If the area of $\triangle ABC$ is 30 cm^2 and R and r are respectively the radii of the cirumcircle and incircle of $\triangle ABC$, then the value of $2R + r$ (in cm) is equal to
23) Consider the statistics of two sets of observations as follows

	Size	Mean	Variance
Observation 1	10	2	2
Observation 2	n	3	1

   If the variance of the combined set of these two observations is $\frac{17}{9}$, then the value of n is equal to
24) Let $S_n(x) = \log_{a^{\frac{1}{2}}} x + \log_{a^{\frac{1}{3}}} x + \log_{a^{\frac{1}{6}}} x + \log_{a^{\frac{1}{11}}} x + \log_{a^{\frac{1}{18}}} x + \log_{a^{\frac{1}{27}}} x + \dots$ upto n-terms, where $a > 1$. If $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$, then the value of a is equal to
25) Let n be a positive integer. Let $A = \sum_{k=0}^{n} (-1)^k\, {}^nC_k \left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k\right]$. If $63A = 1 - \frac{1}{2^{30}}$, then n is equal to
26) Let $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} x + a & x < 0 \\ |x - 1| & x \geq 0 \end{cases}$$

and

$$g(x) = \begin{cases} x+1 & x<0 \\ (x-1)^2+b & x \geq 0 \end{cases}$$

where a, b are non-negative real numbers. If $(g \circ f)(x)$ is continuous for all $x \in \mathbb{R}$, then $a+b$ is equal to

27) If the distance of the point $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ from the plane $x+2y-3z+10=0$ measured parallel to the line, $\frac{x-1}{3} = \frac{2-y}{m} = \frac{z+3}{1}$ is $\frac{\sqrt{7}}{2}$, then the value of $|m|$ is equal to

28) Let $\frac{1}{16}$, a and b be in G.P. and $\frac{1}{a}$, $\frac{1}{b}$, 6 be in A.P., where $a, b > 0$. Then $72(a+b)$ is equal to

29) Let

$$\mathbf{A} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

and

$$\mathbf{B} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

be two 2×1 matrices with real entries such that $\mathbf{A} = \mathbf{XB}$, where

$$\mathbf{X} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & -1 \\ 1 & k \end{pmatrix}$$

and $k \in \mathbb{R}$. If $\left(a_1^2 + a_2^2\right) = \frac{2}{3}\left(b_1^2 + b_2^2\right)$ and $\left(k^2+1\right) b_2^2 \neq -2 b_1 b_2$, then the value of k is

30) For any real numbers α, β, γ and δ, if

$$\int \frac{\left(x^2-1\right) + \tan^{-1}\left(\frac{x^2+1}{x}\right)}{\left(x^4+3x^2+1\right)\tan^{-1}\left(\frac{x^2+1}{x}\right)}\, dx = \alpha \log_e\left(\tan^{-1}\left(\frac{x^2+1}{x}\right)\right) + \beta \tan^{-1}\left(\frac{\gamma\left(x^2-1\right)}{x}\right) + \delta \tan^{-1}\left(\frac{x^2+1}{x}\right) + C$$

where C is an arbitary constant, then the value of $10(\alpha + \beta\gamma + \delta)$ is equal to