# MatGeo Assignment Presentation

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# Outline

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# Problem

# Problem Statement

Sketch the region $(x, 0) : y = \sqrt{4 - x^2}$ and $x$-axis. Find the area of the region using integration.

# Solution

## General Equation of Circle and Variable Description

The general equation of circle is given by

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{3.1}$$

| Variable | Description |
|---|---|
| $\mathbf{O}$ | Center of the circle |
| $r$ | Radius of the circle |
| $\mathbf{u}$ | $-\mathbf{O}$ |
| $f$ | $\|\mathbf{u}\|^2 - r^2$ |
| $D$ | Region for which the area has to be found |
| $A$ | Area of the region |

# Equation of given circle as a general equation in matrix form

The given curve $y = \sqrt{4 - x^2}$ is that of a semicircle, since $y \geq 0$.
The equation of the curve can be written as

$$x^2 + y^2 - 4 = 0, y \geq 0 \tag{3.2}$$

By comparing with equation (3.1), the parameters of the circle are

$$\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -4 \implies r = 2, \mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3.3}$$

# Expressing boundary of $D$ in parametric form

Boundary of $D$ is the semicircle of radius $r$, which we can parameterize (in counter clock-wise orientation) using

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} r\cos t \\ r\sin t \end{pmatrix}, 0 \leq t \leq \pi \tag{3.4}$$

# Using Green's theorem to find area using line integeral

**Green's Theorem:**

Let $C$ be a curve in the plane, and $D$ be the region bounded by it. If $L$ and $M$ are the functions of $(x, y)$ defined on an open region containing $D$, then

$$\oint_C (L\,dx + M\,dy) = \int\int_D \left( \frac{\partial M}{\partial x} - \frac{\partial L}{\partial y} \right) dA \qquad (3.5)$$

In this case, to find the area of the region $D$, we substitute $L = -\frac{y}{2}$ and $M = \frac{x}{2}$, then

$$\frac{1}{2}\oint_C (x\,dy - y\,dx) = \int\int_D dA = \text{area of region } D \qquad (3.6)$$

# Calculating the area using line integral

By Green's Theorem (3.6),

$$\text{area of D} = A = \int\int dA \tag{3.7}$$

$$= \frac{1}{2}\oint_C x\,dy - y\,dx \tag{3.8}$$

$$= \frac{1}{2}\int_0^{\pi} r^2\left(\cos^2 t + \sin^2 t\right)\,dt \tag{3.9}$$

$$= \frac{r^2}{2}\int_0^{\pi} dt \tag{3.10}$$

$$= \frac{\pi r^2}{2} \tag{3.11}$$

$$= 2\pi \tag{3.12}$$

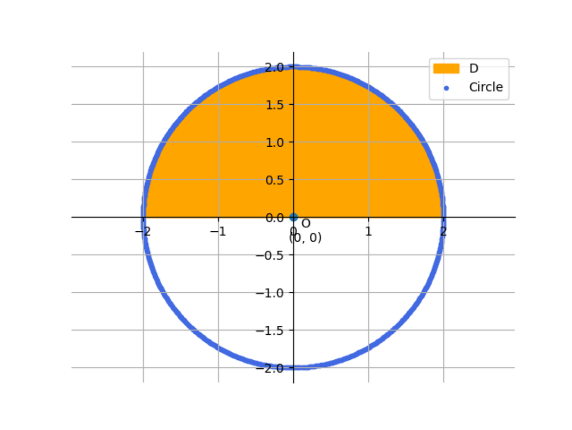


**Figure 1:** Shaded area representing area of region given

## C Code

C function for getting '$n$' number of points to graph a circle of radius $r$ and center $(x, y)$:

```
float **circleGet(int n, float x, float y, float r) {
    float **pts = (float **) malloc(sizeof(float *) * n);
    float theta = 0;
    for(int i = 0; i < n; i++){
        pts[i] = (float *) malloc(sizeof(float) * 2);
        pts[i][0] = x + r*cos(theta);
        pts[i][1] = y + r*sin(theta);
        theta += 2*PI/n;
    }
    return pts;
}
```

# Python Code

The python code for generating the graph can be found at:

https://github.com/agamjotsingh1/EE1030/blob/main/matgeo_questions/q9/codes/graph.py