

1.6.7

EE24BTECH11002 - Agamjot Singh

Question:

The value of m which makes the points $(0, 0)$, $(2m, -4)$, and $(3, 6)$ collinear, is

Solution:

The points **A**, **B** and **C** are collinear if

$$(\mathbf{AB}) = k(\mathbf{AC}) \quad (1)$$

$$(\mathbf{B} - \mathbf{A}) = k(\mathbf{C} - \mathbf{A}) \quad (2)$$

$$(\mathbf{B} - \mathbf{A}) - k(\mathbf{C} - \mathbf{A}) = 0 \quad (3)$$

Collinearity matrix is given by

$$(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A})^T \quad (4)$$

Equation (3) is basically doing row reduction on the collinearity matrix given in equation (4), and if the rank of the matrix is 1, we find some real k that satisfies equation (3).

Let the points be **A** $(0, 0)$, **B** $(3, 6)$ and **C** $(2m, -4)$. The collinearity matrix is given by

$$(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A})^T = \begin{pmatrix} 3 & 6 \\ 2m & -4 \end{pmatrix} \quad (5)$$

$$\xrightarrow{R_1 = \frac{R_1}{2}} \begin{pmatrix} 1 & 2 \\ 2m & -4 \end{pmatrix} \xrightarrow{R_2 = R_2 - (2m)R_1} \begin{pmatrix} 1 & 2 \\ 0 & -4 - 4m \end{pmatrix} \quad (6)$$

For the points to be collinear, the rank of this matrix has to be one.

$$-4 - 4m = 0 \quad (7)$$

$$m = -1 \quad (8)$$

So, the point **C** is given by

$$\mathbf{C} = \begin{pmatrix} -2 \\ -4 \end{pmatrix} \quad (9)$$

The line joining **A**, **B** and **C** is given by

$$y = 2x \quad (10)$$

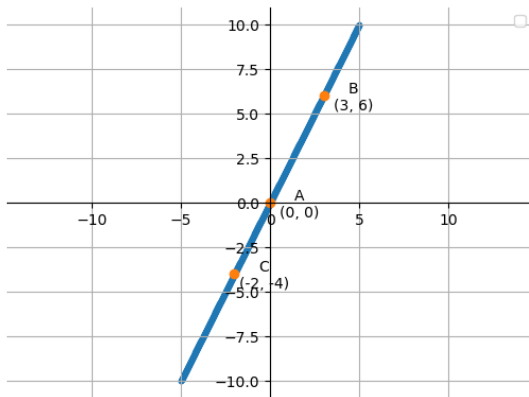


Fig. 0: Line containing points **A**, **B** and **C**