

AE 2008 Q52-68

EE24BTECH11002 - Agamjot Singh

37) Let

$$\tau_1 = \{G \subseteq \mathbb{R} : G \text{ is finite or } \mathbb{R} \setminus G \text{ is finite}\}$$

and

$$\tau_2 = \{G \subseteq \mathbb{R} : G \text{ is countable or } \mathbb{R} \setminus G \text{ is countable}\}$$

Then

- a) neither τ_1 nor τ_2 is a topology on \mathbb{R}
 - b) τ_1 is a topology on \mathbb{R} but τ_2 is not a topology on \mathbb{R}
 - c) τ_2 is a topology on \mathbb{R} but τ_1 is not a topology on \mathbb{R}
 - d) both τ_1 nor τ_2 are topologies on \mathbb{R}
- 38) Which one of the following ideals of the ring $\mathbb{Z}[i]$ of Gaussian integers is NOT maximal?
- a) $\langle 1 + i \rangle$
 - b) $\langle 1 - i \rangle$
 - c) $\langle 2 + i \rangle$
 - d) $\langle 3 + i \rangle$
- 39) If $Z(G)$ denotes the centre of a group G , then the order of the quotient group $G/Z(G)$ cannot be
- a) 4
 - b) 6
 - c) 15
 - d) 25
- 40) Let $\text{Aut}(G)$ denote the group of automorphisms of a group G . Which one of the following is NOT a cyclic group?
- a) $\text{Aut}(\mathbb{Z}_4)$
 - b) $\text{Aut}(\mathbb{Z}_6)$
 - c) $\text{Aut}(\mathbb{Z}_8)$
 - d) $\text{Aut}(\mathbb{Z}_{10})$
- 41) Let X be a non-negative integer valued random variable with $E(X^2) = 3$ and $E(X) = 1$. Then $\sum_{i=1}^{\infty} iP(X \geq i) =$
- a) 1
 - b) 2
 - c) 3
 - d) 4
- 42) Let X be a random variable with probability density function $f \in \{f_0, f_1\}$, where

$$f_0(x) = \begin{cases} 2x & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

and

$$f_1(x) = \begin{cases} 3x^2 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

For testing the null hypothesis $H_0: f \equiv f_0$ against the alternative hypothesis $H_1: f \equiv f_1$ at level of significance $\alpha = 0.19$, the power of the most powerful test is

- a) 0.729
 - b) 0.271
 - c) 0.615
 - d) 0.385
- 43) Let X and Y be independent and identically distributed $U(0, 1)$ random variables. Then $P\left(Y < \left(X - \frac{1}{2}\right)^2\right) =$
- a) $\frac{1}{12}$
 - b) $\frac{1}{4}$

- c) $\frac{1}{\omega}$
 d) $\frac{\omega}{3}$

44) Let X and Y be Banach spaces and let $T: X \rightarrow Y$ be a linear map. Consider the statements:

P : If $x_n \rightarrow x$ in X then $Tx_n \rightarrow Tx$ in Y

Q : If $x_n \rightarrow x$ in X then $Tx_n \rightarrow y$ in Y then $Tx = y$

- a) P implies Q and Q implies P
 b) P implies Q but Q does not imply P
 c) Q implies P but P does not imply Q
 d) neither P implies Q nor Q implies P

45) If $y(x) = x$ is a solution of the differential equation $y'' - \left(\frac{2}{x^2} + \frac{1}{x}\right)(xy' - y) = 0$, $0 < x < \infty$, then its general solution is

- a) $(\alpha + \beta e^{-2x})x$
 b) $(\alpha + \beta e^{2x})x$
 c) $\alpha x + \beta e^x$
 d) $(\alpha e^x + \beta)x$

46) Let $P_n(x)$ be the Legendre polynomial of degree n such that $P_n(1) = 1$, $n = 1, 2, \dots$. If

$$\int_{-1}^1 \left(\sum_{j=1}^n \sqrt{j(2j+1)} P_j(x) \right)^2 dx = 20$$

then $n =$

- a) 2
 b) 3
 c) 4
 d) 5

47) The integral surface satisfying the equation $y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y} = x^2 + y^2$ and passing through the curve $x = 1 - t, y = 1 + t, z = 1 + t^2$ is

- a) $z = xy + \frac{1}{2}(x^2 - y^2)^2$
 b) $z = xy + \frac{1}{4}(x^2 - y^2)^2$
 c) $z = xy + \frac{1}{8}(x^2 - y^2)^2$
 d) $z = xy + \frac{1}{16}(x^2 - y^2)^2$

48) For the diffusion problem $u_{xx} = u_t$ ($0 < x < \pi, t > 0$), $u(0, t) = 0, u(\pi, t) = 0$ and $u(x, 0) = 3 \sin 2x$, the solution is given by

- a) $3e^{-t} \sin 2x$
 b) $3e^{-4t} \sin 2x$
 c) $3e^{-9t} \sin 2x$
 d) $3e^{-2t} \sin 2x$