Assignment 2

EE24BTECH11002 - Agamjot Singh

20) Let a, b, c be such that $b(a+c) \neq 0$ if $\begin{vmatrix} a & a+1 & a-1 \\ b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$, then the value of n is:

    (a) any even integer (b) any odd integer

    (c) any integer (d) zero

21) The number of 3×3 non-singular matrices with four entries as 1 and all othe entries as 0, is

    (a) 5 (b) 6

    (c) atleast 7 (d) less than 4

22) Let A be a 2×2 matrix with non-zero entries and let $A^2 = I$, where I is 2×2 identity matrix. Define
    Tr(A) - sum of diagonal elements of A and
    $|A|$ - determinant of matrix A.
    Statement - 1: $\text{Tr}(A) = 0$.
    Statement - 2: $|A| = 1$
    (a) Statement - 1 is true, Statement - 2 is true; Statement - 2 is **not** a correct explanation for Statement-1.
    (b) Statement - 1 is true, Statement - 2 is false.
    (c) Statement - 1 is false, Statement - 2 is true.
    (d) Statement - 1 is true, Statement - 2 is true; Statement - 2 is a correct explanation for Statement-1.

23) Consider the system of linear equations;

$$x_1 + 2x_2 + x_3 = 3$$
$$2x_1 + 3x_2 + x_3 = 3$$
$$3x_1 + 5x_2 + 2x_3 = 1$$

    (a) exactly 3 solutions
    (b) a unique solution
    (c) no solution
    (d) infinite number of solutions

24) The number of values of k for which the linear equations $4x + ky + 2z = 0$, $kx + 4y + z = 0$ and $2x + 2y + z = 0$ posses a non zero solution is (2011)

    (a) 2 (b) 1 (c) zero (d) 3

25) Let A and B be two symmetrix matrices of order 3.
    Statement - 1: $A(BA)$ and $(AB)A$ are symmetric matrices.
    Statement - 2: AB is symmetric matrix if matrix multiplication of A with B is commutative.
    (a) Statement - 1 is true, Statement - 2 is true; Statement - 2 is **not** a correct explanation for Statement-1.
    (b) Statement - 1 is true, Statement - 2 is false.
    (c) Statement - 1 is false, Statement - 2 is true.
    (d) Statement - 1 is true, Statement - 2 is true; Statement - 2 is a correct explanation for Statement-1.

26) Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$ If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to: (2012)

    (a) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ (c) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ (d) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

27) Let P and Q be 3×3 matrices $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$ then determinant of $\left(P^2 + Q^2\right)$ is equal to (2012)

    (a) -2 (b) 1 (c) 0 (d) -1

28) If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equa l to: $(JEEM2014)$

(a) 4 (b) 11 (c) 5 (d) 0

29) If $\alpha, \beta, \neq 0$, and $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$$

$= K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$, then K is equal to

$(JEEM2014)$

(a) 1 (b) -1 (c) $\alpha\beta$ (d) $\frac{1}{\alpha\beta}$

30) If A is a 3×3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' equals: $(JEEM2014)$

(a) B^{-1} (b) $\left(B^{-1}\right)'$ (c) $I + B$ (d) I

31) The set of all values of λ for which the system of linear equations:

$$2x_1 - 2x_2 + x_3 = \lambda x_1$$
$$2x_1 - 3x_2 + 2x_3 = \lambda x_2$$
$$-x_1 - 2x_2 = \lambda x_3$$

has a non-trivial solution

$(JEEM2015)$

(a) contains two elements
(b) contains more than two elements
(c) is an empty set
(d) is a singleton

32) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3×3 identity matrix, then the ordered part (a, b) is equal to: $(JEEM2015)$

(a) $(2, 1)$ (b) $(-2, -1)$

(c) $(2, -1)$ (d) $(-2, 1)$

33) The system of linear equations

$$x + \lambda y - z = 0$$
$$\lambda x - y - z = 0$$
$$x + y - \lambda z = 0$$

has a non-trivial solution for:

$(JEEM2016)$

(a) exactly two values of λ
(b) exactly three values of λ
(c) inifinitely many values of λ
(d) exactly one value of λ

34) If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A\,adj A = AA^T$, then $5a+b$ is equal to: $(JEEM2016)$

(a) 4 (b) 13

(c) -1 (d) 5

35) Let k be an integer such that triangle with vertices $(k, -3k)$, $(5, k)$, $(-k, 2)$ has area 28 sq. units. Then the orthocentre of this triangle is at the point: $(JEEM2017)$

(a) $\left(2, \frac{1}{2}\right)$ (b) $\left(2, \frac{-1}{2}\right)$

(c) $\left(1, \frac{3}{4}\right)$ (d) $\left(1, \frac{-3}{4}\right)$

36) Let ω be a complex number such that $2\omega+1 = z$ where $z = \sqrt{-3}$. If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to: $(JEEM2017)$

(a) 1 (b) $-z$

(c) z (d) -1