MA 2009 Q37-48

EE24BTECH11002 - Agamjot Singh

1) Let

$$\tau_1 = \{G \subseteq \mathbb{R} \colon G \text{ is finite or } \mathbb{R} \backslash G \text{ is finite}\}$$

and

$$\tau_2 = \{G \subseteq \mathbb{R} \colon G \text{ is countable or } \mathbb{R} \backslash G \text{ is countable}\}$$

Then (2009-MA)

a) neither τ_1 nor τ_2 is a topology on $\mathbb{R}$
b) τ_1 is a topology on $\mathbb{R}$ but τ_2 is not a topology on $\mathbb{R}$
c) τ_2 is a topology on $\mathbb{R}$ but τ_1 is not a topology on $\mathbb{R}$
d) both τ_1 nor τ_2 are topologies on $\mathbb{R}$

2) Which one of the following ideals of the ring $\mathbb{Z}[i]$ of Gaussian integers is NOT maximal? (2009-MA)

a) $\langle 1+i \rangle$
b) $\langle 1-i \rangle$
c) $\langle 2+i \rangle$
d) $\langle 3+i \rangle$

3) If $Z(G)$ denotes the centre of a group G, then the order of the quotient group $G/Z(G)$ cannot be (2009-MA)

a) 4
b) 6
c) 15
d) 25

4) Let $Aut(G)$ denote the group of automorphisms of a group G. Whice one of the following is NOT a cyclic group? (2009-MA)

a) $Aut(\mathbb{Z}_4)$
b) $Aut(\mathbb{Z}_6)$
c) $Aut(\mathbb{Z}_8)$
d) $Aut(\mathbb{Z}_{10})$

5) Let X be a non-negative integer valued random variable with $E\left(X^2\right) = 3$ and $E(X) = 1$. Then $\sum_{i=1}^{\infty} iP(X \geq i) =$ (2009-MA)

a) 1
b) 2
c) 3
d) 4

6) Let X be a random variable with probability density function $f \in \{f_0, f_1\}$, where

$$f_0(x) = \begin{cases} 2x & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

and

$$f_1(x) = \begin{cases} 3x^2 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

For testing the null hypothesis $H_0 \colon f \equiv f_0$ against the alternative hypothesis $H_1 \colon f \equiv f_1$ at level of significance $\alpha = 0.19$, the power of the most powerful test is

(2009-MA)

a) 0.729
b) 0.271
c) 0.615
d) 0.385

7) Let X and Y be independent and identically distributed $U(0,1)$ random variables. Then $P\left(Y < \left(X - \frac{1}{2}\right)^2\right) =$ (2009-MA)

a) $\frac{1}{12}$
b) $\frac{1}{4}$
c) $\frac{1}{3}$
d) $\frac{2}{3}$

8) Let X and Y be Banach spaces and let $T\colon X \to Y$ be a linear map. Consider the statements:

$$P\text{: If } x_n \to x \text{ in } X \text{ then } Tx_n \to Tx \text{ in } Y$$
$$Q\text{: If } x_n \to x \text{ in } X \text{ then } Tx_n \to y \text{ in } Y \text{ then } Tx = y$$

(2009-MA)

a) P implies Q and Q implies P
b) P implies Q but Q does not imply P
c) Q implies P but P does not imply Q
d) neither P implies Q nor Q implies P

9) If $y(x) = x$ is a solution of the differential equation $y'' - \left(\frac{2}{x^2} + \frac{1}{x}\right)(xy' - y) = 0$, $0 < x < \infty$, then its general solution is (2009-MA)

a) $\left(\alpha + \beta e^{-2x}\right)x$
b) $\left(\alpha + \beta e^{2x}\right)x$
c) $\alpha x + \beta e^{x}$
d) $(\alpha e^{x} + \beta)\,x$

10) Let $P_n(x)$ be the Legendre polynomial of degree n such that $P_n(1) = 1$, $n = 1, 2, \ldots$ If

$$\int_{-1}^{1}\left(\sum_{j=1}^{n}\sqrt{j(2j+1)}P_j(x)\right)^2 dx = 20$$

then $n =$ (2009-MA)

a) 2
b) 3
c) 4
d) 5

11) The integral surface satsifying the equation $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = x^2 + y^2$ and passing through the curve $x = 1 - t, y = 1 + t, z = 1 + t^2$ is (2009-MA)

a) $z = xy + \frac{1}{2}\left(x^2 - y^2\right)^2$
b) $z = xy + \frac{1}{4}\left(x^2 - y^2\right)^2$
c) $z = xy + \frac{1}{8}\left(x^2 - y^2\right)^2$
d) $z = xy + \frac{1}{16}\left(x^2 - y^2\right)^2$

12) For the diffusion problem $u_{xx} = u_i\,(0 < x < \pi, t > 0)\,, u(0,t) = 0, u(\pi,t) = 0$ and $u(x,0) = 3\sin 2x$, the solution is given by (2009-MA)

a) $3e^{-t}\sin 2x$
b) $3e^{-4t}\sin 2x$
c) $3e^{-9t}\sin 2x$
d) $3e^{-2t}\sin 2x$