9.2.35

EE24BTECH11002 - Agamjot Singh

Question:
Sketch the region $(x, 0) : y = \sqrt{4 - x^2}$ and x-axis. Find the area of the region using integration.
Solution:

Variable	Description
$\mathbf{O}$	Center of the circle
r	Radius of the circle
$\mathbf{u}$	$-\mathbf{O}$
f	$\|\mathbf{u}\|^2 - r^2$
A	Area of the region

TABLE 0: Variables Used

The general equation of a circle is given by

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

The given curve $y = \sqrt{4 - x^2}$ is that of a semicircle, since $y \geq 0$.
The equation of the curve can be written as

$$x^2 + y^2 - 4 = 0, y \geq 0 \tag{2}$$

The parameters of the circle are

$$\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -4 \implies r = 2, \mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3}$$

The area between the curve and x-axis can be given by

$$A = \int_{-2}^{2} \sqrt{4 - x^2}\, dx \tag{4}$$

Substituting $x = 2\sin\theta$, (5)

$$A = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} 4\cos^2\theta\, d\theta \tag{6}$$

$$A = 8\int_{0}^{\frac{\pi}{2}} \cos^2\theta\, d\theta \tag{7}$$

$$A = 8\int_{0}^{\frac{\pi}{2}} \sin^2\theta\, d\theta \tag{8}$$

$$\implies 2A = 8\left(\frac{\pi}{2}\right) \tag{9}$$

$$\implies A = 2\pi \tag{10}$$

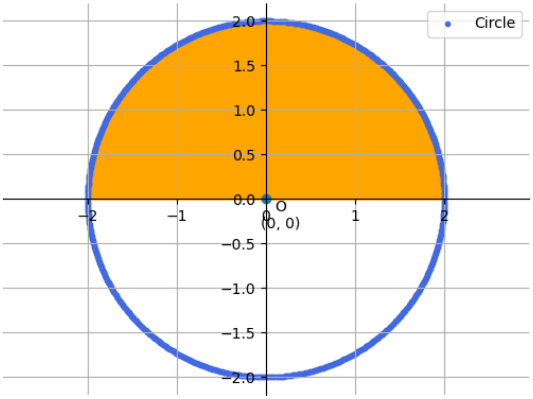

Fig. 0: Shaded area representing area of region given