JEE Main 2022 June 27 Shift 1

EE24BTECH11002 - Agamjot Singh

1) The area of the polygon, whose vertices are the non-real roots of the equation $\bar{z} = iz^2$ is
   a) $\frac{3\sqrt{3}}{4}$
   b) $\frac{3\sqrt{3}}{2}$
   c) $\frac{3}{2}$
   d) $\frac{3}{4}$
2) Let the system of linear equations $x + 2y + z = 2$, $\alpha x + 3y - z = \alpha$ and $-\alpha x + y + 2z = -\alpha$ be inconsistent. Then α is equal to
   a) $\frac{5}{2}$
   b) $\frac{-5}{2}$
   c) $\frac{7}{2}$
   d) $\frac{-7}{2}$
3) If $x = \sum_{n=0}^{\infty} a^n, y = \sum_{n=0}^{\infty} b^n, z = \sum_{n=0}^{\infty} c^n$ where a, b, c are in A.P. and $|a| < 1, |b| < 1, |c| < 1, abc \neq 0$, then
   a) x, y, z are in A.P.
   b) x, y, z are in G.P.
   c) $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.
   d) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1 - (a + b + c)$
4) Let $\frac{dy}{dx} = \frac{ax-by+a}{bx+cy+a}$, where a, b, c are constants, represent a circle passing through the point $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$. Then the shortest distance of the point $\begin{pmatrix} 11 \\ 6 \end{pmatrix}$ from this circle is
   a) 10
   b) 8
   c) 7
   d) 5
5) Let a be an integer such thtat $\lim_{x\to 7} \frac{18-[1-x]}{[x-3a]}$ exists, where $[t]$ is greatest integer $\leq t$. Then a is equal to
   a) -6
   b) -2
   c) 2
   d) 6
6) The number of distinct real roots of $x^4 - 4x + 1 = 0$ is
   a) 4
   b) 2
   c) 1
   d) 0
7) The lengths of the sides of a triangle are $10 + x^2$, $10 + x^2$ and $20 - 2x^2$. If for $x = k$, the area of the triangle is maximum, then $3k^2$ is equal to
   a) 5
   b) 8
   c) 10
   d) 12
8) If $\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5, |y| < 2$, then
   a) $x^2y'' + xy' - 25y = 0$
   b) $x^2y'' - xy' - 25y = 0$
   c) $x^2y'' - xy' + 25y = 0$
   d) $x^2y'' + xy' + 25y = 0$
9) $\int \frac{(x^2+1)e^x}{(x+1)^2}\, dx = f(x)\, e^x + C$, where C is a constant, then $\frac{d^3 f}{dx^3}$ at $x = 1$ is equal to
   a) $\frac{-3}{4}$
   b) $\frac{3}{4}$

c) $\frac{-3}{2}$
d) $\frac{3}{2}$

10) The value of the integral $\int_{-2}^{2} \frac{|x^3+x|}{(e^{x|x|}+1)}\, dx$ is equal to
a) $5e^2$
b) $3e^{-2}$
c) 4
d) 6

11) If $\frac{dy}{dx} + \frac{2^{x-y}(2^y-1)}{2^x-1} = 0, x, y > 0, y(1) = 1$, then $y(2)$ is equal to
a) $2 + \log_2 3$
b) $2 + \log_2 2$
c) $2 - \log_2 3$
d) $2 - \log_2 3$

12) In an isosceles triangle ABC, the vertex A is $\begin{pmatrix} 6 \\ 1 \end{pmatrix}$ and the equation of the base BC is $2x + y = 4$. Let the point B lie on the line $x + 3y = 7$. If $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ is the centroid of $\triangle ABC$, then $15(\alpha + \beta)$ is equal to
a) 39
b) 41
c) 51
d) 63

13) Let the eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$, be $\frac{1}{4}$. If this ellipse passes through the point $\begin{pmatrix} -4\left(\sqrt{\frac{2}{5}}\right) \\ 3 \end{pmatrix}$, then $a^2 + b^2$ is equal to
a) 29
b) 31
c) 32
d) 34

14) If two straight lines whose direction cosines are given by the relations $1 + m - n = 0, 3l^2 + m^2 + cnl = 0$ are parallel, then the positive value of c is
a) 6
b) 4
c) 3
d) 2

15) Let $\mathbf{a} = \mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{c} = 2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$. Then the number of vectors $\mathbf{b}$ such that $\mathbf{b} \times \mathbf{c} = \mathbf{a}$ and $|\mathbf{b}| \in \{1, 2, \ldots, 10\}$ is
a) 0
b) 1
c) 2
d) 3