1.6.7

EE24BTECH11002 - Agamjot Singh

Question:
The value of m which makes the points $(0,0)$, $(2m,-4)$, and $(3,6)$ collinear, is
Solution:
The points $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are collinear if

$$(\mathbf{AB}) = k(\mathbf{AC}) \tag{1}$$

$$(\mathbf{B}-\mathbf{A}) = k(\mathbf{C}-\mathbf{A}) \tag{2}$$

$$(\mathbf{B}-\mathbf{A}) - k(\mathbf{C}-\mathbf{A}) = 0 \tag{3}$$

Collinearity matrix is given by

$$\begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A}\end{pmatrix}^\top \tag{4}$$

For the vectors $\mathbf{AB}$ and $\mathbf{AC}$ to be linearly related, i.e. the points to be collinear, there exists a k which satisfies equation (4).
Row reduction on collinearity matrix should result in nullity of the matrix to be one, such that some k exists.
Let the collinearity matrix be $\mathbf{X}_{m\times n}$. By rank nullity theorem,

$$\text{rank}(\mathbf{X}) + \text{nullity}(\mathbf{X}) = n \tag{5}$$

$$\text{rank}(\mathbf{X}) = n - \text{nullity}(\mathbf{X}) \tag{6}$$

$$\text{rank}(\mathbf{X}) = n - 1 \tag{7}$$

Let the points be $\mathbf{A}(0,0)$,$\mathbf{B}(3,6)$ and $\mathbf{C}(2m,-4)$. The collinearity matrix $\mathbf{X}_{2\times 2}$ is given by

$$\begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A}\end{pmatrix}^\top = \begin{pmatrix}3 & 6\\ 2m & -4\end{pmatrix} \tag{8}$$

$$\xrightarrow{R_1=\frac{R_1}{2}} \begin{pmatrix}1 & 2\\ 2m & -4\end{pmatrix} \quad \xrightarrow{R_2=R_2-(2m)R_1} \begin{pmatrix}1 & 2\\ 0 & -4-4m\end{pmatrix} \tag{9}$$

For the points to be collinear, the rank of this matrix has to be one.

$$-4-4m = 0 \tag{10}$$

$$m = -1 \tag{11}$$

So, the point $\mathbf{C}$ is given by

$$\mathbf{C} = \begin{pmatrix}-2\\ -4\end{pmatrix} \tag{12}$$

The line joining **A**, **B** and **C** is given by

$$y = 2x \tag{13}$$

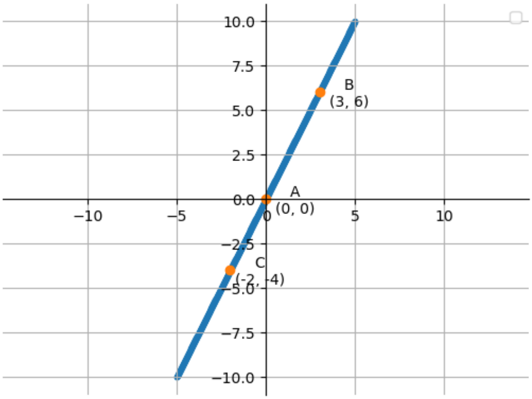

Fig. 0: Line containing points **A**, **B** and **C**