7.2.30

EE24BTECH11002 - Agamjot Singh

Question:
The line $x + 3y = 0$ is a diameter of the circle $x^2 + y^2 + 6x + 2y = 0$.
Solution:

Variable	Description
$\mathbf{O}$	Center of the circle
r	Radius of the circle
$\mathbf{u}$	$-\mathbf{O}$
f	$\|\|\mathbf{u}\|\|^2 - r^2$

TABLE 0: Variables Used

The general equation of a circle is given by

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^\top\mathbf{x} + f = 0 \tag{1}$$

The given equation can be represented as

$$\|x\|^2 + 2\begin{pmatrix}3 & 1\end{pmatrix}\mathbf{x} = 0 \tag{2}$$

By comparing equations (1) and (2),

$$\mathbf{u} = \begin{pmatrix}3\\1\end{pmatrix} = -\mathbf{O} \tag{3}$$

$$f = 0 \implies r = \|u\| = \sqrt{10} \tag{4}$$

$$\implies \mathbf{O} = \begin{pmatrix}-3\\-1\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix} \tag{5}$$

$$\implies x + 3y \neq 0 \tag{6}$$

The line $x + 3y = 0$ does not go through the center of the circle, hence it is not the diameter of the circle.

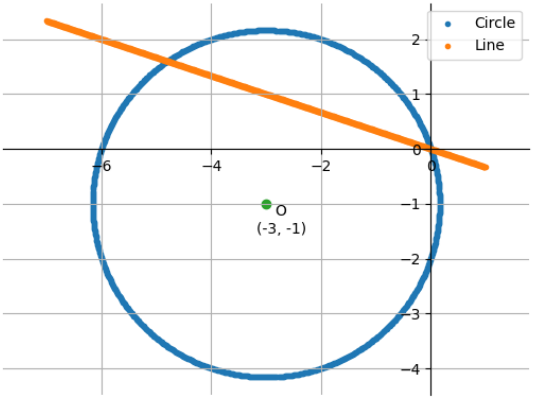

Fig. 0: Circle with the given line