MatGeo Assignment Presentation

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Outline

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Problem

Problem Statement

Sketch the region $(x, 0) : y = \sqrt{4 - x^2}$ and x-axis. Find the area of the region using integration.

Solution

General Equation of Circle and Variable Description

The general equation of circle is given by

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{3.1}$$

Variable	Description
$\mathbf{O}$	Center of the circle
r	Radius of the circle
$\mathbf{u}$	$-\mathbf{O}$
f	$\|\mathbf{u}\|^2 - r^2$
D	Region for which the area has to be found
A	Area of the region

Equation of given circle as a general equation in matrix form

The given curve $y = \sqrt{4 - x^2}$ is that of a semicircle, since $y \geq 0$. The equation of the curve can be written as

$$x^2 + y^2 - 4 = 0, y \geq 0 \tag{3.2}$$

By comparing with equation (3.1), the parameters of the circle are

$$\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -4 \implies r = 2, \mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3.3}$$

Expressing boundary of D in parametric form

Boundary of D is the semicircle of radius r, which we can parameterize (in counter clock-wise orientation) using

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} r\cos t \\ r\sin t \end{pmatrix}, 0 \leq t \leq \pi \tag{3.4}$$

Using Green's theorem to find area using line integral

Green's Theorem:

Let C be a curve in the plane, and D be the region bounded by it. If L and M are the functions of (x, y) defined on an open region containing D, then

$$\oint_C (L\,dx + M\,dy) = \int\int_D \left(\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}\right)\,dA \tag{3.5}$$

In this case, to find the area of the region D, we substitute $L = -\frac{y}{2}$ and $M = \frac{x}{2}$, then

$$\frac{1}{2}\oint_C (x\,dy - y\,dx) = \int\int_D dA = \text{area of region } D \tag{3.6}$$

Calculating the area using line integral

By Green's Theorem (3.6),

$$\text{area of D} = A = \int\int dA \tag{3.7}$$

$$= \frac{1}{2}\int_C x\,dy - y\,dx \tag{3.8}$$

$$= \frac{1}{2}\int_0^{\pi} r^2\left(\cos^2 t + \sin^2 t\right)\,dt \tag{3.9}$$

$$= \frac{r^2}{2}\int_0^{\pi} dt \tag{3.10}$$

$$= \frac{\pi r^2}{2} \tag{3.11}$$

$$= 2\pi \tag{3.12}$$

Graph

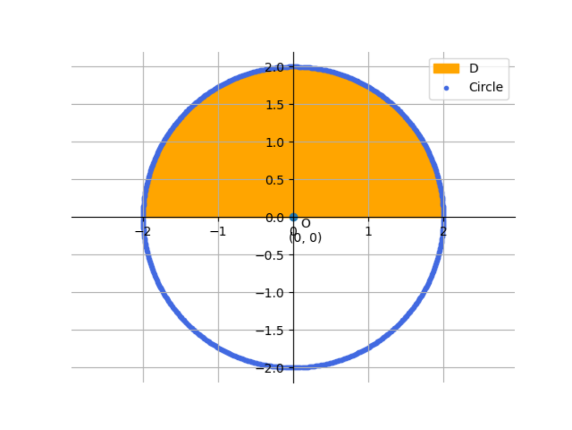

Figure 1: Shaded area representing area of region given

C Code

C function for getting 'n' number of points to graph a circle of radius r and center (x, y):

```
float **circleGet(int n, float x, float y, float r) {
    float **pts = (float **) malloc(sizeof(float *) * n);
    float theta = 0;
    for(int i = 0; i < n; i++){
        pts[i] = (float *) malloc(sizeof(float) * 2 * n);
        pts[i][0] = x + r*cos(theta);
        pts[i][1] = y + r*sin(theta);
        theta += 2*PI/n;
    }
    return pts;
}
```

Python Code

The python code for generating the graph can be found at:

https://github.com/agamjotsingh1/EE1030/blob/main/matgeo_questions/q9/codes/graph.py