

4.3.2

EE24BTECH11002 - Agamjot Singh

Question:

The line $x + 3y = 0$ is a diameter of the circle $x^2 + y^2 + 6x + 2y = 0$.

Solution:

Variable	Description
\mathbf{O}	Center of the circle
r	Radius of the circle
\mathbf{u}	$-\mathbf{O}$
f	$\ \mathbf{u}\ ^2 - r^2$

TABLE 0: Variables Used

The general equation of a circle is given by

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^\top \mathbf{x} + f = 0 \quad (1)$$

The given equation can be represented as

$$\|\mathbf{x}\|^2 + 2 \begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = 0 \quad (2)$$

By comparing equations (1) and (2),

$$\mathbf{u} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} = -\mathbf{O} \quad (3)$$

$$f = 0 \implies r = \|\mathbf{u}\| = \sqrt{10} \quad (4)$$

$$\implies \mathbf{O} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (5)$$

$$\implies x + 3y \neq 0 \quad (6)$$

The line $x + 3y = 0$ does not go through the center of the circle, hence it is not the diameter of the circle.

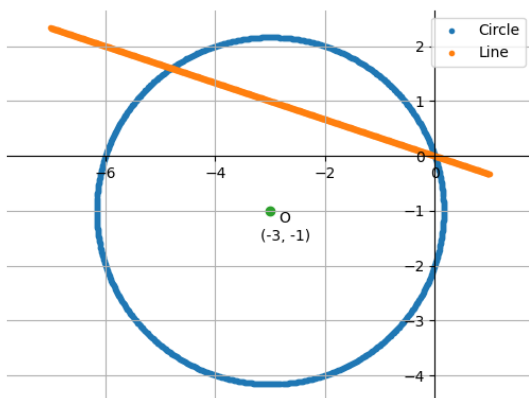


Fig. 0: Circle with the given line