

1.3.4

EE24BTECH11002 - Agamjot Singh

Question:

If $\mathbf{A}(1, 3)$, $\mathbf{B}(-1, 2)$, $\mathbf{C}(2, 5)$ and $\mathbf{D}(x, 4)$ are the vertices of a parallelogram $ABCD$, then the value of x is

Solution: By definition of parallelogram, we say that the line \mathbf{AD} is parallel to the line \mathbf{BC} . Therefore, the slopes of these lines must be equal.

The direction vector of \mathbf{PQ} is defined as

$$\mathbf{m} = \mathbf{Q} - \mathbf{P} = k \begin{pmatrix} 1 \\ m \end{pmatrix}$$

where m is the slope of \mathbf{PQ} . We also say that

$$\mathbf{m} \equiv \begin{pmatrix} 1 \\ m \end{pmatrix}$$

Direction vector of $\mathbf{m} = \mathbf{BC}$ is given as

$$\mathbf{m} = \mathbf{C} - \mathbf{B} \quad (1)$$

$$\mathbf{m} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \quad (2)$$

$$\mathbf{m} \equiv \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (3)$$

$$m = 1 \quad (4)$$

So, the slope of \mathbf{BC} is 1, which is equal to the slope of \mathbf{AD} .

Direction vector of $\mathbf{m}_1 = \mathbf{AD}$ is given as

$$\mathbf{m}_1 = \mathbf{D} - \mathbf{A} \quad (5)$$

$$\mathbf{m}_1 = \begin{pmatrix} x \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} x-1 \\ 1 \end{pmatrix} \quad (6)$$

$$\mathbf{m}_1 \equiv \begin{pmatrix} 1 \\ \frac{1}{x-1} \end{pmatrix} \quad (7)$$

By equation (7), we get the slope of \mathbf{AD} is $\frac{1}{x-1}$.

By equation (4), we get

$$\frac{1}{x-1} = 1 \quad (8)$$

$$x = 2 \quad (9)$$

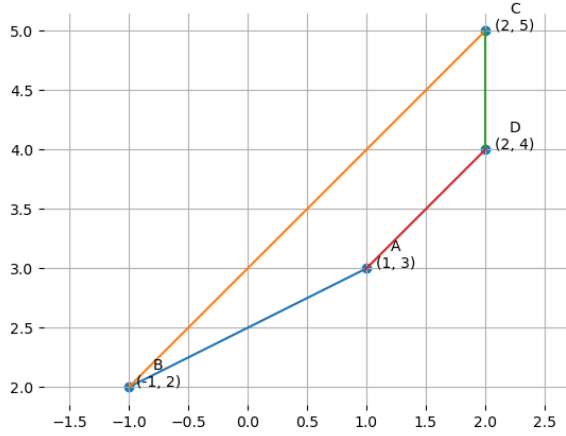


Fig. 0: Quadrilateral ABCD formed with given equations

As we can see in the graph, the given quadrilateral formed with x found in equation (9) is not a parallelogram. Therefore, we conclude that there is no x for which $ABCD$ is a parallelogram.