

1.6.7

EE24BTECH11002 - Agamjot Singh

Question:

The value of m which makes the points $(0, 0)$, $(2m, -4)$, and $(3, 6)$ collinear, is

Solution:

The points \mathbf{A} , \mathbf{B} and \mathbf{C} are collinear if

$$(\mathbf{AB}) = k(\mathbf{AC}) \quad (1)$$

$$(\mathbf{B} - \mathbf{A}) = k(\mathbf{C} - \mathbf{A}) \quad (2)$$

$$(\mathbf{B} - \mathbf{A}) - k(\mathbf{C} - \mathbf{A}) = 0 \quad (3)$$

Collinearity matrix $\mathbf{X}_{m \times n}$ is given by

$$\mathbf{X} = (\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A})^T \quad (4)$$

Row reduction on collinearity matrix should result in nullity of the matrix to be one, which implies that there is some k which satisfies equation (4).

By rank nullity theorem,

$$\text{rank}(\mathbf{X}) + \text{nullity}(\mathbf{X}) = n \quad (5)$$

$$\text{rank}(\mathbf{X}) = n - \text{nullity}(\mathbf{X}) \quad (6)$$

$$\text{rank}(\mathbf{X}) = n - 1 \quad (7)$$

Let the points be $\mathbf{A}(0, 0)$, $\mathbf{B}(3, 6)$ and $\mathbf{C}(2m, -4)$. The collinearity matrix $\mathbf{X}_{2 \times 2}$ is given by

$$(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A})^T = \begin{pmatrix} 3 & 6 \\ 2m & -4 \end{pmatrix} \quad (8)$$

$$\xrightarrow{R_1 = \frac{R_1}{2}} \begin{pmatrix} 1 & 2 \\ 2m & -4 \end{pmatrix} \xrightarrow{R_2 = R_2 - (2m)R_1} \begin{pmatrix} 1 & 2 \\ 0 & -4 - 4m \end{pmatrix} \quad (9)$$

For the points to be collinear, the rank of this matrix has to be one.

$$-4 - 4m = 0 \quad (10)$$

$$m = -1 \quad (11)$$

So, the point \mathbf{C} is given by

$$\mathbf{C} = \begin{pmatrix} -2 \\ -4 \end{pmatrix} \quad (12)$$

The line joining **A**, **B** and **C** is given by

$$y = 2x \quad (13)$$

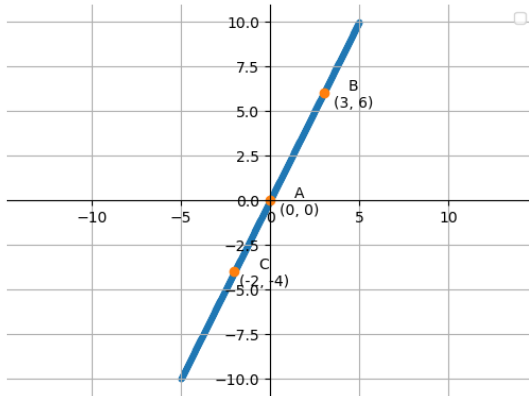


Fig. 0: Line containing points **A**, **B** and **C**