

# 1.8.19

EE24BTECH11002 - Agamjot Singh

## Question:

Find the points on  $X$  axis which are at a distance of  $2\sqrt{5}$  from the point  $(7, -4)$ . How many such points are there?

## Solution:

Let  $\mathbf{A} \begin{pmatrix} 7 \\ -4 \end{pmatrix}$  and  $\mathbf{B} \begin{pmatrix} x \\ y \end{pmatrix}$  be the desired point on the  $X$  axis.

The  $L_p$  norm, written as  $\|x\|_p$ , is defined as

$$\|x\|_p = \sqrt[p]{\sum_{i=1}^n |x_i|^p}, p \geq 1 \quad (1)$$

If  $\mathbf{B}$  is at a distance of  $2\sqrt{5}$  from point  $\mathbf{A}$ .

$$\|\mathbf{B} - \mathbf{A}\|_p = 2\sqrt{5} \quad (2)$$

$$\Rightarrow \sqrt[p]{|x - 7|^p + |y + 4|^p} = 2\sqrt{5} \quad (3)$$

$$(4)$$

Taking  $y = 0$ , as  $\mathbf{B}$  lies on the  $X$  axis.

$$\sqrt[p]{|x - 7|^p + |4|^p} = 2\sqrt{5} \quad (5)$$

$$|x - 7|^p + (4)^p = (2\sqrt{5})^p \quad (6)$$

$$|x - 7| = \sqrt[p]{(2\sqrt{5})^p - (4)^p} \quad (7)$$

$$x = 7 \pm \sqrt[p]{(2\sqrt{5})^p - (4)^p} \quad (8)$$

By solving equation (7) with  $y = 0$ , we get,

$$\mathbf{B} = \begin{pmatrix} 7 \pm \sqrt[p]{(2\sqrt{5})^p - (4)^p} \\ 0 \end{pmatrix}, p \geq 1 \quad (9)$$

There are infinitely many such points as  $p$  varies from 1 to  $\infty$ .

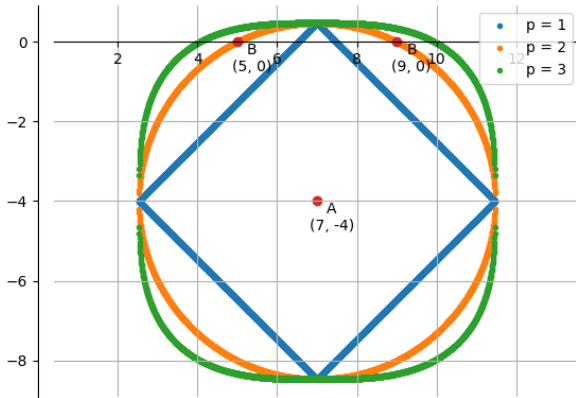


Fig. 0: Graph representing the locus of  $\mathbf{B}$ , taking  $p = 1, 2, 3$