1.8.19

EE24BTECH11002 - Agamjot Singh

Question:

Find the points on X axis which are at a distance of $2\sqrt{5}$ from the point $(7,-4)$. How many such points are there?

Solution:

Let $\mathbf{A}\begin{pmatrix}7\\-4\end{pmatrix}$ and $\mathbf{B}\begin{pmatrix}x\\y\end{pmatrix}$ be the desired point on the X axis.

If $\mathbf{B}$ is at a distance of $2\sqrt{5}$ from point $\mathbf{A}$.

$$\|\mathbf{B}-\mathbf{A}\| = 2\sqrt{5} \tag{1}$$

$$\implies \|\mathbf{B}-\mathbf{A}\|^2 = \left(2\sqrt{5}\right)^2 \tag{2}$$

$$\implies \|\mathbf{B}\|^2 - 2\mathbf{B}^\top\mathbf{A} + \|\mathbf{A}\|^2 = 20 \tag{3}$$

$$x^2 + y^2 - 2\begin{pmatrix}x & y\end{pmatrix}\begin{pmatrix}7\\-4\end{pmatrix} + 65 = 20 \tag{4}$$

$$x^2 + y^2 - 2(7x - 4y) + 45 = 0 \tag{5}$$

$$x^2 + y^2 - 14x + 8y + 45 = 0 \tag{6}$$

$$(x-7)^2 + (y+4)^2 = 20 \tag{7}$$

By equation (7), we can say that locus of $\mathbf{B}$ is a circle with centre at $\mathbf{A}\begin{pmatrix}7\\-4\end{pmatrix}$ and radius $2\sqrt{5}$ units.

As $\mathbf{B}$ lies on the X axis, $y = 0$. By solving equation (7) with $y = 0$, we get,

$$\mathbf{B} = \begin{pmatrix}9\\0\end{pmatrix} or \begin{pmatrix}5\\0\end{pmatrix} \tag{8}$$

There are two such points.

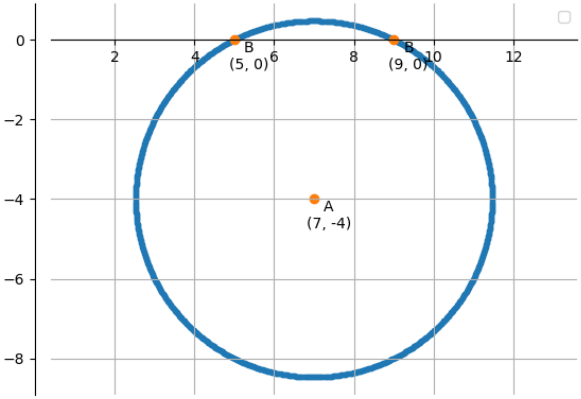

Fig. 0: Circle representing the locus of **B**