9.2.35

EE24BTECH11002 - Agamjot Singh

Question:
Sketch the region $(x, 0) : y = \sqrt{4-x^2}$ and x-axis. Find the area of the region using integration.
Solution:

Variable	**Description**
$\mathbf{O}$	Center of the circle
r	Radius of the circle
$\mathbf{u}$	$-\mathbf{O}$
f	$\|\|\mathbf{u}\|\|^2 - r^2$
D	Region for which the area has to be found
A	Area of the region

TABLE 0: Variables Used

The general equation of a circle is given by

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

The given curve $y = \sqrt{4-x^2}$ is that of a semicircle, since $y \geq 0$.
The equation of the curve can be written as

$$x^2 + y^2 - 4 = 0, y \geq 0 \tag{2}$$

The parameters of the circle are

$$\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -4 \implies r = 2, \mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3}$$

Boundary of D is the semicircle of radius r, which we can parameterize (in counter clock-wise orientation) using

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} r\cos t \\ r\sin t \end{pmatrix}, 0 \geq t \leq \pi \tag{4}$$

By Green's Theorem,

$$\text{area of D} = A = \int\int dA \tag{5}$$
$$= \frac{1}{2}\int_C x\,dy - y\,dx \tag{6}$$
$$= \frac{1}{2}\int_0^{\pi} r^2\left(\cos^2 t + \sin^2 t\right)\,dt \tag{7}$$
$$= \frac{r^2}{2}\int_0^{\pi} dt \tag{8}$$
$$= \frac{\pi r^2}{2} \tag{9}$$
$$= 2\pi \tag{10}$$

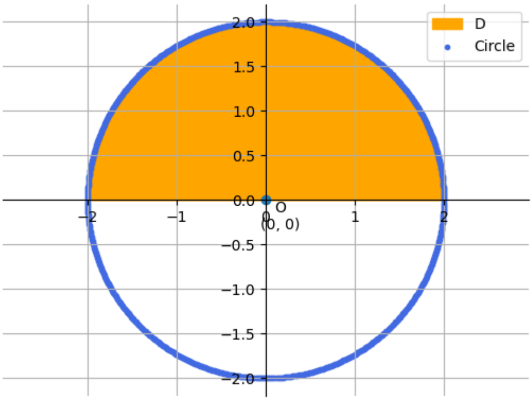

Fig. 0: Shaded area representing area of region given