Electrostatics Q1

Part a

Let $\vec{r} = z\hat{k}$ and $k = \frac{1}{4\pi\epsilon_0}$.

Electric Field due to q_1

$$\begin{aligned}
\vec{E}_1 &= k\frac{q_1}{|\vec{r} - \vec{r}_1|^3}(\vec{r} - \vec{r}_1) \\
&= \frac{kq_1}{\left|z\hat{k} - (-\frac{d}{2})\hat{k}\right|^3}\left(z\hat{k} - (-\frac{d}{2})\hat{k}\right) \\
&= \frac{kq_1}{\left|z + \frac{d}{2}\right|^3}\left(z + \frac{d}{2}\right)\hat{k} \\
&= \frac{kq_1}{\left(z + \frac{d}{2}\right)^2}\frac{\left(z + \frac{d}{2}\right)}{\left|z + \frac{d}{2}\right|}\hat{k}
\end{aligned}$$

Electric Field due to q_2

$$\begin{aligned}
\vec{E}_2 &= k\frac{q_2}{|\vec{r} - \vec{r}_2|^3}(\vec{r} - \vec{r}_2) \\
&= \frac{kq_2}{\left|z\hat{k} - (\frac{d}{2})\hat{k}\right|^3}\left(z\hat{k} - (\frac{d}{2})\hat{k}\right) \\
&= \frac{kq_2}{\left|z - \frac{d}{2}\right|^3}\left(z - \frac{d}{2}\right)\hat{k} \\
&= \frac{kq_2}{\left(z - \frac{d}{2}\right)^2}\frac{\left(z - \frac{d}{2}\right)}{\left|z - \frac{d}{2}\right|}\hat{k}
\end{aligned}$$

By superposition principle: $\vec{E} = \vec{E}_1 + \vec{E}_2$

Case 1: $z > \frac{d}{2}$

$$\begin{aligned}
\vec{E} &= \vec{E}_1 + \vec{E}_2 \\
&= \frac{kq_1}{\left(z + \frac{d}{2}\right)^2}\hat{k} + \frac{kq_2}{\left(z - \frac{d}{2}\right)^2}\hat{k} \\
&= \left(\frac{kq_1}{\left(z + \frac{d}{2}\right)^2} + \frac{kq_2}{\left(z - \frac{d}{2}\right)^2}\right)\hat{k}
\end{aligned}$$

Since $q_1 = q_2 = q$,

$$\vec{E} = kq\left[\frac{1}{\left(z+\frac{d}{2}\right)^2} + \frac{1}{\left(z-\frac{d}{2}\right)^2}\right]\hat{k}$$

Case 2: $-\frac{d}{2} < z < \frac{d}{2}$

$$\begin{aligned}\vec{E} &= \vec{E}_1 + \vec{E}_2 \\ &= \frac{kq_1}{\left(z+\frac{d}{2}\right)^2}\hat{k} + \frac{kq_2}{\left(z-\frac{d}{2}\right)^2}(-\hat{k}) \\ &= kq\left[\frac{1}{\left(z+\frac{d}{2}\right)^2} - \frac{1}{\left(z-\frac{d}{2}\right)^2}\right]\hat{k}\end{aligned}$$

Case 3: $z < -\frac{d}{2}$

$$\begin{aligned}\vec{E} &= \vec{E}_1 + \vec{E}_2 \\ &= \frac{kq_1}{\left(z+\frac{d}{2}\right)^2}(-\hat{k}) + \frac{kq_2}{\left(z-\frac{d}{2}\right)^2}(-\hat{k}) \\ &= -kq\left[\frac{1}{\left(z+\frac{d}{2}\right)^2} + \frac{1}{\left(z-\frac{d}{2}\right)^2}\right]\hat{k}\end{aligned}$$