**Technical University of Denmark**

*Written examination*: 28. May 2016

*Course name and number*: **Introduction to Statistics (02323, 02402 and 02593)**

*Aids and facilities allowed:* All

The questions were answered by

________________ (student number) ________________ (signature) ________ (table number)

There are 30 questions of the "multiple choice" type included in this exam divided on 12 exercises. To answer the questions you need to fill in the prepared 30-question multiple choice form (on three seperate pages) in CampusNet

5 points are given for a correct answer and −1 point is given for a wrong answer. ONLY the following 5 answer options are valid: 1, 2, 3, 4 or 5. If a question is left blank or another answer is given, then it does not count (i.e. "0 points"). Also, if more answers are given to a single question, which in fact is technically possible in the online system, it will not count (i.e. "0 points"). The number of points corresponding to specific marks or needed to pass the examination is ultimately determined during censoring.

**The final answer of the exercises should be given by filling in and submitting via the exam module in CampusNet. The table sheet here is ONLY to be used as an "emergency" alternative.**

| **Exercise** | I.1 | I.2 | II.1 | III.1 | IV.1 | V.1 | V.2 | V.3 | VI.1 | VI.2 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Question** | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| **Answer** | 3 | 5 | 5 | 5 | 2 | | 4 | 3 | 5 | 3 |

| **Exercise** | VI.3 | VII.1 | VII.2 | VII.3 | VII.4 | VII.5 | VIII.1 | VIII.2 | IX.1 | IX.2 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Question** | (11) | (12) | (13) | (14) | (15) | (16) | (17) | (18) | (19) | (20) |
| **Answer** | 1 | 3 | 1 | 3 | 5 | 1 | 1 | 3 | 5 | 3 |

| **Exercise** | IX.3 | X.1 | X.2 | X.3 | XI.1 | XI.2 | XII.1 | XII.2 | XII.3 | XII.4 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Question** | (21) | (22) | (23) | (24) | (25) | (26) | (27) | (28) | (29) | (30) |
| **Answer** | 2 | 3 | 1 | 3 | 5 | 2 | 1 | 3 | 4 | 5 |

Remember to provide your **study number**. The questionnaire contains 45 pages. Please check that your questionnaire contains them all.

Continues on page 2

**Multiple choice questions:** *Note that not all the suggested answers are necessarily meaningful. In fact, some of them are very wrong but under all circumstances there is one and only one correct answer to each question.*

**Exercise I**

In an airport the security check screens exactly 10000 passengers each day. Based on data from a long period it was found that 8 out of 10000 passengers bring sharp objects in their carry on luggage. Let $X$ be a random variable denoting the number of passengers with sharp objects on a day (based on exactly 10000 checks). $X$ is assumed to follow a Binomial distribution.

**Question I.1 (1)**

What is the expected number of passengers with sharp objects on a day, and what is the variance of $X$?

1 ☐ $E[X] = 0.0008$ and $V[X] = 10000 \cdot 0.8 \cdot 0.2 = 1600$

2 ☐ $E[X] = 0.0008 \cdot 10000 = 8$ and $V[X] = 10000 \cdot 0.0008 \cdot 0.0002 = 0.0016$

3* ☐ $E[X] = 0.0008 \cdot 10000 = 8$ and $V[X] = 10000 \cdot 0.0008 \cdot 0.9992 = 7.994$

4 ☐ $E[X] = 0.0008 \cdot 10000 = 8$ and $V[X] = 10000 \cdot 0.0008 = 8$

5 ☐ $E[X] = 0.8 \cdot 10 = 8$ and $V[X] = 10 \cdot 0.8 \cdot 0.2 = 1.6$

——— FACIT-BEGIN ———

$E[X] = n * p = 10000 * 0.0008 = 8$ and $V[X] = n * p * (1 - p) = 10000 * 0.0008 * 0.9992 = 7.994$

——— FACIT-END ———

**Question I.2 (2)**

What is the probability of finding more than 10 passengers with sharp objects on a given day?

1 ☐ `qbinom(0.9, 10000, 0.0008)`

2 ☐ `1-dbinom(9990, 10000, 0.0008)`

3 ☐ `dbinom(10, 10000, 0.0008)`

4 ☐ `1-pbinom(9990, 10000, 0.0008)`

5* ☐ `1-pbinom(10, 10000, 0.0008)`

—————— FACIT-BEGIN ——————

The result is found by calculating 1 minus the probability of finding 10 or less sharp objects.

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Continues on page 4

**Exercise II**

A pharmaceutical company made a study in which 300 persons were randomly divided into 3 treatment groups of 100 patients each. One group was assigned to a placebo treatment, one group received the company's own product, and the last group got a competitor's product. For each patient the weight change over a period of time was measured and the final data set consists of 300 observations of weight changes. The focus is on comparing the average weight change in each group.

**Question II.1 (3)**

What kind of statistical analysis is most suitable for this?

1 ☐ Multiple linear regression analysis

2 ☐ Test for independence in a $r \times c$ frequency table (Contingency table)

3 ☐ Paired t-test

4 ☐ Toway analysis of variance

5* ☐ Oneway analysis of variance

———— FACIT-BEGIN ————

With the description this is clearly 3 independent samples of quantitative data, so the oneway anova is the right choice, so answer 5).

———— FACIT-END ————

Continues on page 5

**Exercise III**

A random variable $X$ follows a uniform distribution on the interval $[0; 1]$.

**Question III.1 (4)**

The expected value and the variance of $(X + 2) \cdot 4$ is

1 ☐ $\mu = \frac{5}{2}$ and $\sigma^2 = 4^2$

2 ☐ $\mu = 10$ and $\sigma^2 = 4^2$

3 ☐ $\mu = 8$ and $\sigma^2 = 4^2$

4 ☐ $\mu = 8$ and $\sigma^2 = \frac{1}{3}$

5* ☐ $\mu = 10$ and $\sigma^2 = \frac{4}{3}$

———— FACIT-BEGIN ————

The transformed variable is uniformly distributed on $[8, 12]$ so by Eq. 2-52 and 2-53 the answer is: $\mu = \frac{1}{2}(12 - 8) = 10$ and $\sigma^2 = \frac{1}{12}(12 - 8)^2 = \frac{16}{12} = \frac{4}{3}$.

———— FACIT-END ————

Continues on page 6

**Exercise IV**

A drone manufacturer is focusing on the feasible flight time between recharges. The flight time depends among other things on the weight of the drone. The drone basically consists of a battery($B$), a skeleton ($S$) and four engines with propellers ($M_1, \ldots, M_4$). It is assumed that the weights of the individual parts are independent and in the following all weights are in grams. The weights of the three types of components are given by the following Normal distributions: Battery: $B \sim N(100, 10^2)$, skeleton: $S \sim N(40, 5^2)$ and engines with propellers: $M_i \sim N(15, 2^2),\ i = 1, \ldots, 4$. (Each distribution is given on the usual form: $N(\mu, \sigma^2)$)

**Question IV.1 (5)**

The expected value and variance for the weight of the assembled drones are found to be

1 ☐ $\mu = 200$ and $\sigma^2 = 189$

2* ☐ $\mu = 200$ and $\sigma^2 = 141$

3 ☐ $\mu = 155$ and $\sigma^2 = 189$

4 ☐ $\mu = 155$ and $\sigma^2 = 129$

5 ☐ $\mu = 170$ and $\sigma^2 = 141$

———— FACIT-BEGIN ————

$\mu = 100 + 40 + 4 * 15 = 200$ and $\sigma^2 = 10^2 + 5^2 + 4 * 2^2 = 141$

———— FACIT-END ————

Continues on page 7

**Exercise V**

There is a recommendation to eat 600 grams of fruit and vegetables each day. Regularly surveys of Danish dietary habits are made to see if the recommendation is met.

The results of the daily intake of fruits and vegetables (in grams) for the last four of this kind of dietary studies (conducted in the years 1995, 2000-2002, 2003-2004 and 2005-2008) can be summarized by the following output from R.

| Survey | n | median | mean | var | std |
|---|---|---|---|---|---|
| 1995 | 1564 | 259.82 | 290.887 | 28861.55 | 169.887 |
| 2000-2002 | 3043 | 386.057 | 433.817 | 62029.21 | 169.887 |
| 2003-2004 | 1310 | 404.936 | 453.279 | 74159.29 | 272.322 |
| 2005-2008 | 1983 | 429.132 | 479.285 | 77166.51 | 277.789 |

| Survey | 2.5% | 5.0% | Q1 | Q3 | 95.0% | 97.5% |
|---|---|---|---|---|---|---|
| 1995 | 66.102 | 87.062 | 171.209 | 374.303 | 606.609 | 686.361 |
| 2000-2002 | 98.613 | 129.574 | 257.224 | 555.168 | 928.673 | 1055.419 |
| 2003-2004 | 83.48 | 127.528 | 256.286 | 583.723 | 974.246 | 1180.891 |
| 2005-2008 | 105.348 | 141.81 | 279.359 | 617.371 | 991.09 | 1189.367 |

In all the questions in this exercise one can assume that the data from each of the four studies are normally distributed.

**Question V.1 (6)**

The question is no longer part of the curriculum.

Continues on page 8

## Question V.2 (7)

A new dietary study is planned on the basis of the observed variation in dietary survey 2005-2008. What should the sample size be if the 90% confidence interval for the mean intake of fruits and vegetables is aimed to have a width of 20 grams?

1 ☐ $n \approx 77166.51/\left(\frac{20}{1.96}\right)^2 = 741.1$

2 ☐ $n \approx \left(\frac{479.285 \cdot 1.6449}{10}\right)^2 = 6215.4$

3 ☐ $n \approx \frac{77166.51}{1.96 \cdot 20} = 1968.5$

4* ☐ $n \approx \left(\frac{1.6449 \cdot 277.789}{10}\right)^2 = 2087.9$

5 ☐ $n \approx \left(\frac{1.6449 \cdot \sqrt{1983}}{1.6456}\right)^2 = 1981.3$

———— FACIT-BEGIN ————

Cf. Model in question V.1 (6), we are still in the same normal distribution model.

To determine the sample size, with 90% confidence interval for the mean intake of fruits and vegetables ($\mu$), so it not exceed a width of 20 grams based on the dietary survey from 2005 to 2008, used Method 3.45 in eNote 3 page 44:

$$n = \left(\frac{z_{1-\alpha/2} \cdot \sigma}{ME}\right)^2 = \left(\frac{1.6449 \cdot 277.789}{10}\right)^2 = 2087.9$$

Since $ME = 0.5 \cdot 20$ ($ME$ is half the width of the confidence interval), as variance we use the estimate from 2005-2008 dietary survey, ie $\sigma = 277.789$ and since it is 90% confidence interval, which is under consideration, we have that $1 - \alpha/2 = 0.95$, $z_{0.95} = 1.6449$. Thus we see that the correct answer is 4.

```
qnorm(0.95)

## [1] 1.644854
```

———— FACIT-END ————

## Question V.3 (8)

Determine the 95% confidence interval for the mean intake of fruit and vegetables in the 2003-2004 survey.

1 ☐ $404.936 \pm 1.9618 \cdot \sqrt{\frac{74159.29}{1310}} = [390.176; 419.697]$

2 ☐ $[83.48; 1180.891]$

3* ☐ $453.279 \pm 1.9618 \cdot 7.524 = [438.518; 468.040]$

4 ☐ $453.279 \pm 1.6460 \cdot \frac{272.322}{\sqrt{1310}} = [440.894; 465.664]$

5 ☐ $453.279 \pm 1.96 \cdot 272.322 = [-80.472; 987.030]$

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Consider 2003-2004 dietary survey.

Let $X_i$ be a random variable denoting the ith respondent's intake of fruits and vegetables per. day in 2003-2004 dietary survey. Assume $X_i$ is normally distributed N( $\mu$ ,$\sigma^2$), where the model parameters are estimated at: $\hat{\mu} = 453.279$ and $\hat{\sigma}^2 = 74159.29 = (272.322)^2$

To determine the $1 - \alpha$ confidence interval for the mean intake of fruits and vegetables in 2003-2004 dietary survey ($\mu$) used Method 3.8 eNote 3 page 12

$\bar{x} \pm t_{1-\alpha/2} \cdot s/\sqrt{n} = 453.279 \pm 1.9618 \cdot 7.524 = [438.518; 468.040]$

Since it is 95% confidence interval, we must determine, we have $\alpha = 0.05$, $t_{0.975} = 1.9618$, as it is 97.5% percentile of the t-distribution with 1309 degrees of freedom we should use. In addition, $s/\sqrt{n} = 272.322/\sqrt{1310} = 7.524$

```
qt(0.975,1309)

## [1] 1.961778
```

Thus we see that the correct answer is 3

———— FACIT-END ————

Continues on page 10

**Exercise VI**

A major company took a random sample of 20 employees and determined their daily intake of fruits and vegetables, and registered the following observations of the daily intake (in grams):

| | |
|---|---|
| 740.59 | 262.28 |
| 667.96 | 730.55 |
| 809.33 | 324.19 |
| 1138.12 | 421.93 |
| 489.42 | 561.23 |
| 352.78 | 552.96 |
| 1309.66 | 130.96 |
| 259.86 | 440.82 |
| 896.01 | 955.03 |
| 481.00 | 257.80 |

In all the questions in this exercise one can assume that the data is normally distributed.

Summary from R gives the following results for the intake of fruits and vegetables:

```
 n     median      mean        variance        Std. dev.
20    521.1898    589.1245    98996.08        314.6364


   2.5%        5.0%       Q1        Q3        95.0%     97.5%
 191.2095    251.4552   345.635  757.777   1146.697  1228.178
```

**Question VI.1 (9)**

Determine the 90% confidence interval for the variance $\sigma^2$ for the daily intake of fruits and vegetables of employees in the company

1 ☐ $[\frac{20\cdot 314.636}{32.852};\frac{20\cdot 314.636}{8.907}] = [191.548; 706.492]$

2 ☐ $98996.08 \pm 30.144 \cdot \frac{314.636}{\sqrt{20}} = [96875.31; 101116.90] = [311.248^2; 317.989^2]$

3 ☐ $98996.08 \pm 1.7959 \cdot \frac{314.636^2}{\sqrt{20}} = [59241.79; 138750.40] = [243.396^2; 372.492^2]$

4 ☐ $[314.636^2 - 10.117 \cdot 314.636; 314.636^2 + 30.144 \cdot 314.636] = [309.537^2; 329.364^2]$

5* ☐ $[\frac{19\cdot 314.636^2}{30.144};\frac{19\cdot 314.636^2}{10.117}] = [249.796^2; 431.181^2]$

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Consider the sample comprising 20 employees.

Let $X_i$ be a random variable, denoting the ith employee's daily intake of fruits and vegetables in this random sample. Assume $X_i$ is normally distributed N( $\mu$ ,$\sigma^2$), where the model parameters are estimated at: $\hat{\mu} = 589.1245$ and $\hat{\sigma}^2 = 98996.08 = 314.636^2$

To determine the $1-\alpha$ confidence interval for the variance ($\sigma^2$) for the daily intake of fruits and vegetables in the random sample used Method 3.18 eNote 3 page 24

$$\left[\frac{(n-1)\cdot s^2}{\chi^2_{1-\alpha/2}}; \frac{(n-1)\cdot s^2}{\chi^2_{\alpha/2}}\right] = \left[\frac{19\cdot 314.636^2}{\chi^2_{0.95}}; \frac{19\cdot 314.636^2}{\chi^2_{0.05}}\right] = \left[\frac{19\cdot 314.636^2}{30.144}; \frac{19\cdot 314.636^2}{10.117}\right] = [249.796^2; 431.181^2]$$

Since we should determine the 90% confidence interval, it is clear that $\alpha = 0.10$. $\chi^2_{\alpha/2}$ , $\chi^2_{1-\alpha/2}$ are percentiles of the chi-square / $\chi^2$-distribution with $\nu = n-1 = 19$ degrees of freedom. It follows that: $\chi^2_{0.05} = 10.117$ , $\chi^2_{0.95} = 30.144$ from

```
qchisq(0.05,19)

## [1] 10.11701

qchisq(0.95,19)

## [1] 30.14353
```

Thus we see that the correct answer is 5

———————— FACIT-END ————————

Continues on page 12

**Question VI.2 (10)**

Actually, the above data consist of 2 random samples, where the left column indicate the intakes of 10 men and the right column intakes for 10 women. One wants to investigate whether there are differences in men's and women's mean intake of fruits and vegetables.

The following R code is executed (not all necessarily sensible):

```
m <- c(740.59, 667.96, 809.33, 1138.12, 489.42, 352.78,
       1309.66, 259.86, 896.01, 481.00)
f <- c(262.28, 730.55, 324.19, 421.93, 561.23, 552.96,
       130.96, 440.82, 955.03, 257.80)
t.test(m, f, paired = TRUE)

##
##  Paired t-test
##
## data:  m and f
## t = 1.7378, df = 9, p-value = 0.1163
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -75.65101 577.04701
## sample estimates:
## mean of the differences
##                 250.698

mean(f) - mean(m)

## [1] -250.698

t.test(m, f)

##
##  Welch Two Sample t-test
##
## data:  m and f
## t = 1.9001, df = 16.481, p-value = 0.07506
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -28.33599 529.73199
## sample estimates:
## mean of x mean of y
##   714.473   463.775
```

Continues on page 13

```r
t.test(m, mu = median(f))

##
##  One Sample t-test
##
## data:  m
## t = 2.6577, df = 9, p-value = 0.02614
## alternative hypothesis: true mean is not equal to 431.375
## 95 percent confidence interval:
##  473.5091 955.4369
## sample estimates:
## mean of x
##   714.473

t.test(f)

##
##  One Sample t-test
##
## data:  f
## t = 5.957, df = 9, p-value = 0.0002135
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  287.6581 639.8919
## sample estimates:
## mean of x
##   463.775
```

Continues on page 14

What is the conclusion of the test of the hypothesis (at the level $\alpha = 0.05$)

$$H_0 : \mu_f = \mu_m$$
$$H_1 : \mu_f \neq \mu_m$$

corresponding to the examination of whether there are differences in men's and women's mean intake of fruits and vegetables.

1 ☐ Yes, there is a significant difference between men's and women's intake of fruits and vegetables per day, given that the relevant $p$-value is 0.1163

2 ☐ It is apparent that there is a significant difference between men's and women's intake of fruits and vegetables per day, with $\hat{\mu}_f - \hat{\mu}_m = 463.775 - 714.473 = -250.698$. It appears that men eat more fruits and vegetables per day than women

3* ☐ There is no significant difference between men's and women's intake of fruits and vegetables per day, as the relevant $p$-value is 0.07506

4 ☐ Yes, there is a significant difference between men's and women's intake of fruits and vegetables per day given that the relevant $p$-value is 0.02614

5 ☐ No, there is no significant difference in the intake of fruit and vegetables per day for men and women since the relevant $p$-value is 0.0002135

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Consider again the random sample comprising 20 employees. In fact, the 20 observations 2 random samples where the 10 observations in the first column is the data for men's daily intake of fruits and vegetables, while the 10 observations in the second column is for women's intake.

Let $M_i$ and $F_i$ be independent random variables, where $M_i$ indicates the ith man's daily intake of fruits and vegetables in this random sample and correspondingly $F_i$ ith woman's intake of fruits and vegetables in this random sample. Assume $M_i$ is normally distributed N( $\mu_m$ , $\sigma_m^2$) and correspondingly that $F_i$ are normally distributed N( $\mu_f$ , $\sigma_f^2$).The model parameters are estimated by: $\hat{\mu}_m = 714.473$ , $\hat{\sigma}_m^2 = 113464.1 = 336.84^2$ and $\hat{\mu}_f = 463.775$ , $\hat{\sigma}_f^2 = 60611.72 = 246.19^2$

We want to examine whether there are differences in men's and women's mean intake of fruits and vegetables, corresponding to the following hypothesis:

$$H_0 : \mu_f = \mu_m$$
$$H_1 : \mu_f \neq \mu_m$$

The hypothesis is tested on the level $\alpha = 0.05$

There must be made a Welch two-sample t-test Cf. Method 3.60 eNote 3 page 64. The test statistics is determined by:

$t_obs = \frac{(\bar{m}-\bar{f})}{\sqrt{s_m^2/n_m + s_f^2/n_f}} = \frac{714.473-463.775}{\sqrt{336.84^2/10+246.19^2/10}} = 1.9001$

The degrees of freedom is determined by:

$$\nu = \frac{\left(\frac{s_m^2}{n_m}+\frac{s_f^2}{n_f}\right)^2}{\frac{(s_m^2/n_m)^2}{n_m-1}+\frac{(s_f^2/n_f)^2}{n_f-1}} = \frac{\left(\frac{336.84^2}{10}+\frac{246.19^2}{10}\right)^2}{\frac{(336.84^2/10)^2}{9}+\frac{(246.19^2/10)^2}{9}} = 16.481$$

Since $T$ is t-distributed with $\nu = 16.481$ degrees of freedom, the p-value determined by

$p = 2 \cdot P(T > |t_{obs}|) = 2 \cdot P(T > 1.9001) = 0.07506$

```
2*(1-pt(abs(1.9001),16.481))

## [1] 0.07506054
```

As $p > 0.05$ we accept $H_0$, i.e. there is no significant difference between men and women mean intake of fruits and vegetables. It appears that the answer 3, is the right solution.

——————— FACIT-END ———————

## Question VI.3 (11)

What is the upper quartile (75% percentile) for the 10 intake for women, based on the textbook definition of this?

1* ☐ 561.23

2 ☐ 262.28

3 ☐ 431.375

4 ☐ 709.97

5 ☐ 246.19

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The ordered sample sample for the random sample of the 10 women daily intake of fruits and vegetables is determined by:

130.96, 257.80, 262.28, 324.19, 421.93, 440.82, 552.96, 561.23, 730.55, 955.03

According to Definition 1.4 Median eNote 1 page 10, respectively Definition 1.6 Quantiles and Percentiles eNote 1 page 12 the upper quartile is determined based on the ordered sample.

As n = 10, $np = 7.5$, the upper quartile is the 8th observation, that is, 561.23, so the correct answer is 1.

```
f <- c(262.28, 730.55, 324.19, 421.93, 561.23, 552.96, 130.96, 440.82, 955.03, 257.80)
quantile(f, type=2)

##      0%     25%     50%     75%    100%
## 130.960 262.280 431.375 561.230 955.030
```

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**Exercise VII**

A study investigated dioxin emissions from a Danish incineration plant. Parts of the measured variables are shown in the table below. The 3 variables are: Dioxin measured in "parts per millon", load of the plant measured as relative deviation from a reference, and the content of water in the emitted gas (measured in %). As seen in the table, there are in total 23 measurements. Average and empirical standard deviation ("sample standard deviation") are listed at the bottom of the table.

| | Dioxin (*ppm*) | Load | $H_20$ (%) |
|---|---|---|---|
| | DIOX | NEFF | H2O |
| 1 | 984.10 | 0.2560 | 13.78 |
| 2 | 662.00 | 0.3520 | 14.59 |
| 3 | 270.90 | -0.0200 | 12.55 |
| ⋮ | ⋮ | ⋮ | ⋮ |
| 21 | 112.70 | 0.0490 | 13.84 |
| 22 | 94.20 | 0.1350 | 14.18 |
| 23 | 323.20 | 0.2820 | 12.56 |
| $\bar{x}$ | 329.16 | -0.0266 | 12.589 |
| $s$ | 254.95 | 0.2105 | 1.980 |

The primary interest in the study is related to the question: can dioxin emissions be influenced by adjusting the load. For this purpose the following R code is executed (the data input is, however, omitted)

```
fit1 <- lm(DIOX ~ NEFF)
summary(fit1)

##
## Call:
## lm(formula = DIOX ~ NEFF)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -348.41 -116.61  -22.98  101.19  496.16
##
## Coefficients:
##             Estimate Std. Error t value    Pr(>|t|)
## (Intercept)    347.8       44.7   7.781 0.000000128 ***
## NEFF           702.2      215.3   3.262     0.00373 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 212.6 on 21 degrees of freedom
## Multiple R-squared:  0.3362,Adjusted R-squared:  0.3046
## F-statistic: 10.64 on 1 and 21 DF,  p-value: 0.00373
```

Continues on page 18

Hence, the following model is examined

$$\texttt{DIOX}_i = \beta_0 + \beta_1 \texttt{NEFF}_i + \epsilon_i; \quad \epsilon_i \sim N(0, \sigma^2)$$

**Question VII.1 (12)**

At significance level $\alpha = 0.05$, what is the conclusion about the effect of the load on dioxin emissions (both conclusions and argument must be correct)?

1 ☐ There is an effect since $1.3 \cdot 10^{-7} < 0.05$, and $\beta_1 > 0$ because $347.8 > 0$

2 ☐ There is an effect since $702.2 > 347.2$, and $\beta_1 > 0$ because $3.26 > 0$

3* ☐ There is an effect since $0.0037 < 0.05$, and $\beta_1 > 0$ because $702.2 > 0$

4 ☐ There is no evidence of an effect as $3.26 < 7.78$.

5 ☐ There is no evidence of an effect as $0.0037 > \frac{0.05}{100}$.

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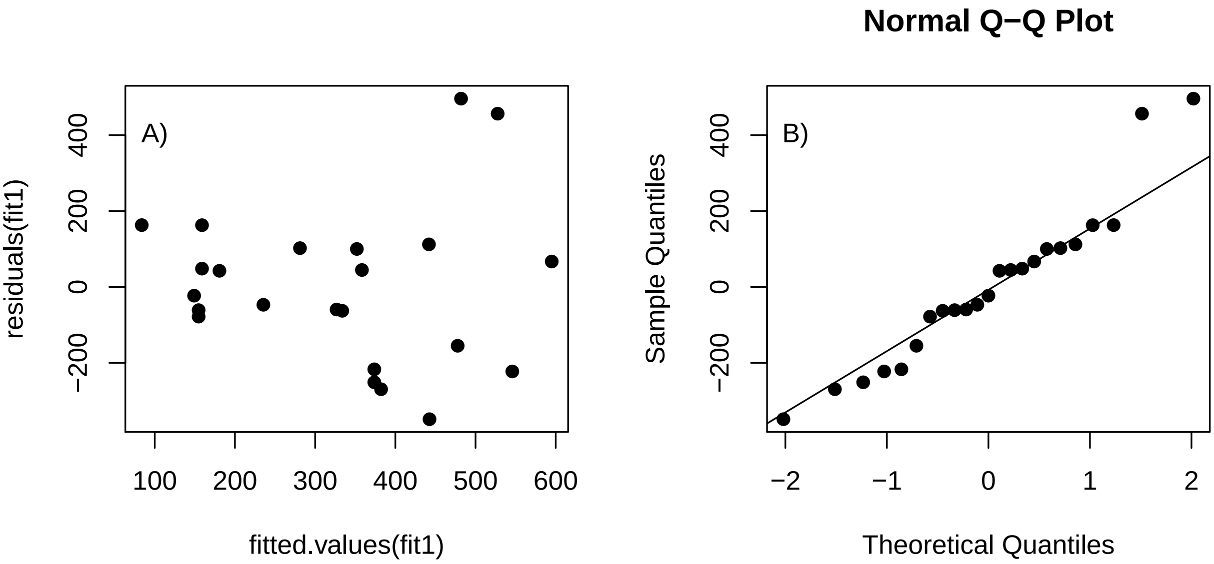
In this case we test the hypothesis

$$H_0: \quad \beta_1 = 0 \tag{1}$$

against the two-sided alternative. The $p$-value for this hypothesis is given directly in the table as 0.00373, since $0.00373 < 0.05$ there is an effect on the specified level. Since also $\hat{\beta}_1 = 702.2 > 0$ we have that $\beta_1 > 0$ (on the specified level). Hence the correct answer is no. 3.

———— FACIT-END ————

In order to investigate whether the conditions for using the model are satisfied, 2 residual plots are shown in the figure below.




Continues on page 20

**Question VII.2 (13)**

Which assumptions are primarily examined in each of the 2 plots (both assumptions and figure reference must be correct)?

1* ☐ Variance homogeneity (A) and the normal distribution assumption (B)

2 ☐ $E(\epsilon) = 0$ (A) and $V(\epsilon) = \sigma^2$ (B)

3 ☐ Variance-homogeneity (A) and assumption of linearity (B)

4 ☐ $E(\epsilon) = 0$ (A) and independence (B)

5 ☐ Independence (A) and variance homogeneity (B)

———— FACIT-BEGIN ————

Figure (A) is used to check variance homogeneity, independece or missing structures, while B is used for shecking the normal assumption, hence the correce answer is no. 1.

Lets just have a loook at the other answers for no. 2 the first part $E[\epsilon] = 0$ does not really make sense to test since $\sum e_i$ is always (by construction) equal 0. The second part is actually variance homogeneity (which is not tested in figure B)).

———— FACIT-END ————

Regardless of the outcome of the previous question it is decided to make the analysis on log-tranformed dioxin data. The result of the analysis conducted in R is shown below (some of the numbers are, however, replaced by letters)

```
> fit2 <- lm(log(DIOX) ~ NEFF)
> summary(fit2)

Call:
lm(formula = log(DIOX) ~ NEFF)

Residuals:
     Min       1Q   Median       3Q      Max
-1.29588 -0.44048  0.05093  0.49403  0.94119

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   5.5927          A      B  < 2e-16 ***
NEFF          1.8416          C      D     E
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.6266 on 21 degrees of freedom
Multiple R-squared:  0.2862,Adjusted R-squared:  0.2522
F-statistic:  8.42 on 1 and 21 DF,  p-value: 0.00853
```

Continues on page 22

**Question VII.3 (14)**

What is `D`?

1 ☐ `D`= $\frac{0.623^2}{21} = 0.019$

2 ☐ `D`= $0.623 \cdot \sqrt{\frac{1}{22 \cdot 0.211^2}} = 0.63$

3* ☐ `D`= $\frac{1.84}{\texttt{C}}$

4 ☐ `D`= $\frac{\texttt{C}}{\texttt{B}}$

5 ☐ `D`= $\frac{0.623}{\sqrt{22}} = 0.13$

———— FACIT-BEGIN ————

The model is in this case

$$\log(\texttt{DIOX}_i) = \beta_0 + \beta_1 \texttt{NEFF}_i + \epsilon_i; \quad \epsilon_i \sim N(0, \sigma^2)$$

`D` is the test statistic for the hypothesis

$$H_0: \quad \beta_1 = 0$$

against the twosided alternative, the teststatistic is in this case given by

$$t = \frac{\hat{\beta}_1}{\hat{\sigma}_{\beta_1}} = \frac{1.84}{\text{C}}$$

where 1.84 is the estimate for $\beta_1$ and `C` is the standard error for $\hat{\beta}_1$ ($\hat{\sigma}_{\beta_1}$). For completeness the full R-output is given below.

```
fit2 <- lm(log(DIOX) ~ NEFF)
summary(fit2)

##
## Call:
## lm(formula = log(DIOX) ~ NEFF)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.29588 -0.44048  0.05093  0.49403  0.94119
##
## Coefficients:
##             Estimate Std. Error t value             Pr(>|t|)
## (Intercept)   5.5927     0.1317  42.450 < 0.0000000000000002 ***
```

```
## NEFF          1.8416     0.6346   2.902              0.00853 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6266 on 21 degrees of freedom
## Multiple R-squared:  0.2862,Adjusted R-squared:  0.2522
## F-statistic:  8.42 on 1 and 21 DF,  p-value: 0.00853
```

———— FACIT-END ————

**Question VII.4 (15)**

What is the usual 95% confidence interval for the slope in the model for log(DIOX)?

1 ☐ $1.84 \pm 1.72 \cdot \texttt{B}$

2 ☐ $1.84 \pm 2.08 \cdot 0.2862$

3 ☐ $1.84 \pm 1.72 \cdot \texttt{D}$

4 ☐ $1.84 \pm 2.08 \cdot 0.623$

5* ☐ $1.84 \pm 2.08 \cdot \texttt{C}$

———— FACIT-BEGIN ————

The slope is 1.84 (directly from the R-output), and the standard error for the slope is C, to to get a 95% confidence interval we need to multiply the C by the 0.975 quantile in the t-distribution with 21 degrees of freedom,

$$1.85 \pm C \cdot t_{0.975} \tag{2}$$

the quantile in the t-distribution is calculated by

```
qt(0.975,df=21)

## [1] 2.079614
```

This is answer no. 5.

———— FACIT-END ————

Continues on page 24

It is now decided to investigate whether water vapor should be included in the model. For this purpose, a multiple regression model is formulated

$$\log(\text{DIOX}_i) = \beta_0 + \beta_1 \text{NEFF}_i + \beta_2 \text{H2O}_i + \epsilon_i; \quad \epsilon_i \sim N(0, \sigma^2)$$

In order to investigate the model, the following R-code has been executed (including the result)

```
fit3 <- lm(log(DIOX) ~ NEFF + H2O)
summary(fit3)

##
## Call:
## lm(formula = log(DIOX) ~ NEFF + H2O)
##
## Residuals:
##      Min        1Q    Median        3Q       Max
## -1.11709 -0.36741  0.05337  0.36192  0.90410
##
## Coefficients:
##             Estimate Std. Error t value     Pr(>|t|)
## (Intercept)   7.4704     0.8098   9.225 0.0000000121 ***
## NEFF          2.1963     0.5955   3.688      0.00146 **
## H2O          -0.1484     0.0633  -2.345      0.02948 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5687 on 20 degrees of freedom
## Multiple R-squared:  0.4401,Adjusted R-squared:  0.3841
## F-statistic:  7.86 on 2 and 20 DF,  p-value: 0.003028
```

## Question VII.5 (16)

What are the parameter estimates for the model?

1* ☐ $\hat{\beta}_0 = 7.47,\ \hat{\beta}_1 = 2.20,\ \hat{\beta}_2 = -0.148$ og $\hat{\sigma} = 0.569$

2 ☐ $\hat{\beta}_0 = 9.22,\ \hat{\beta}_1 = 3.69,\ \hat{\beta}_2 = -2.35$ og $\hat{\sigma} = 0.4401$

3 ☐ $\hat{\beta}_0 = 7.47,\ \hat{\beta}_1 = 2.20,\ \hat{\beta}_2 = -0.148$ og $\hat{\sigma} = 0.384$

4 ☐ $\hat{\beta}_0 = 9.22,\ \hat{\beta}_1 = 3.69,\ \hat{\beta}_2 = -2.35$ og $\hat{\sigma} = 0.569$

5 ☐ $\hat{\beta}_0 = 9.22,\ \hat{\beta}_1 = 3.69,\ \hat{\beta}_2 = -2.35$ og $\hat{\sigma} = 7.86$

———— FACIT-BEGIN ————

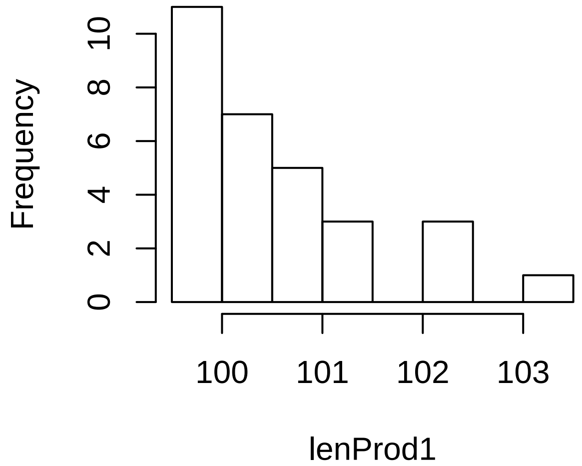
The estimates of $\beta_0$-$\beta_1$ can be read in the first column of the result (the one denoted "`Estimate`"), this is answer no. 1. The estimate of the residual standard deviation ($\hat{\sigma}$) is 0.569 (the one denoted "`Residual standard deviation`"). This is also in no. 1, hence the correct answer is no. 1.

———— FACIT-END ————

Continues on page 26

## Exercise VIII

In a production, it is anticipated that part of the production must be discarded due to a minimum length requirement. It is found that it is economically feasible if not more than 25% of the produced elements are discarded. An experiment is carried out with a particular production method and the length of 50 produced items are observed. The observations are loaded and stored in the vector `lenProd1`. A histogram of the observations is




A confidence interval for the lower quartile (i.e. the 25% quantile) must be calculated without any assumptions of the distribution. The following R code is run:

```
## Simulate 10000 samples
k = 10000
simSamples = replicate(k, sample(lenProd1, replace = TRUE))

simStat = apply(simSamples, 2, quantile, probs=0.25)
quantile(simStat, c(0.005,0.025,0.05,0.95,0.975,0.995))

##     0.5%     2.5%       5%      95%    97.5%    99.5%
##  99.5825  99.7225  99.7600 100.0575 100.1025 100.2300

simStat = apply(simSamples, 2, quantile, probs=0.5)
quantile(simStat, c(0.005,0.025,0.05,0.95,0.975,0.995))

##     0.5%     2.5%       5%      95%    97.5%    99.5%
##  99.9450  99.9850 100.0000 100.5900 100.6550 100.8801

simStat = apply(simSamples, 2, quantile, probs=0.75)
quantile(simStat, c(0.005,0.025,0.05,0.95,0.975,0.995))

##     0.5%     2.5%       5%      95%    97.5%    99.5%
## 100.3000 100.4625 100.5125 101.4300 101.9550 102.1450
```

Continues on page 27

Note that the option `probs` is "passed on" to the `quantile` function, such that for each of the three calls to `apply` a different quantile is calculated by the `quantile` function.

**Question VIII.1 (17)**

What is the 95% confidence interval for the lower quartile (i.e. the 25% quantile) for the length?

1* ☐ $[99.72, 100.10]$

2 ☐ $[100.00, 100.59]$

3 ☐ $[99.59, 100.23]$

4 ☐ $[100.46, 101.96]$

5 ☐ $[100.49, 101.43]$

———— FACIT-BEGIN ————

To find the correct estimate of the 95% confidence interval for the lower quantile, we need to first find the one of the three calculation of `simStat` which is of the lower quartile (i.e. the 25% quantile). The argument `probs` indicate the quantile to be calculated, hence the first which has `probs=0.25` is the right one. Next we need to find the 2.5% and 97.5% quantile of the simulated statistic, hence the estimated interval is

$$[99.72, 100.10]$$

———— FACIT-END ————

**Question VIII.2 (18)**

In the following $Q$ denotes a quartile, such that $Q_1$ is the lower quartile, $Q_2$ is the median and $Q_3$ is the upper quartile. In which of the following two-sided tests would the null hypothesis have been rejected on significance level $\alpha = 0.01$ under the assumptions and simulation results presented above?

1 ☐ $H_0 : Q_1 = 100$ vs. $H_1 : Q_1 \neq 100$

2 ☐ $H_0 : Q_2 = 100$ vs. $H_1 : Q_2 \neq 100$

3* ☐ $H_0 : Q_2 = 101$ vs. $H_1 : Q_2 \neq 101$

4 ☐ $H_0 : Q_3 = 101$ vs. $H_1 : Q_3 \neq 101$

5 ☐ $H_0 : Q_3 = 102$ vs. $H_1 : Q_3 \neq 102$

——————— FACIT-BEGIN ———————

The null hypothesis will be rejected if the value tested for falls out of the confidence interval calculated with the same significance level, as used for the test. Hence, the null hypothesis $H_0 : Q_2 = 101$ is the only one falling outside the respective 0.5% and 99.5% CI.

——————— FACIT-END ———————

Continues on page 29

**Exercise IX**

A new wind turbine is to be build on a site and some investigations of the wind conditions on the site have been carried out. The outcome is that the average hourly wind speed on the site can be represented with the probability density function plotted below in Figure 1:

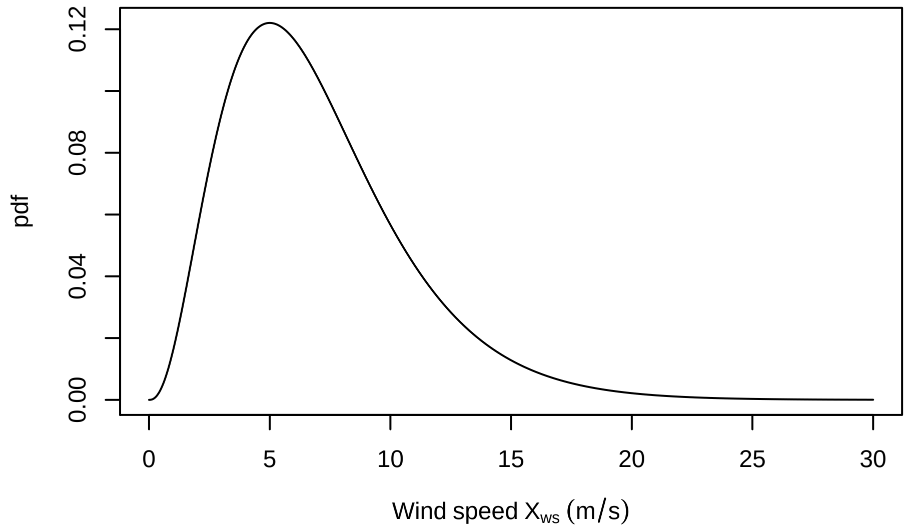



Figure 1: *Probability density function (pdf) for the wind speed* $X_{\mathrm{ws}}$.

In order to investigate the power production of a wind turbine build on the site a function called the 'power curve' for the wind turbine is used (it is the power output as a function of the wind speed, it has nothing to do with the power of a statistical test). The power curve used is plotted below in Figure 2:

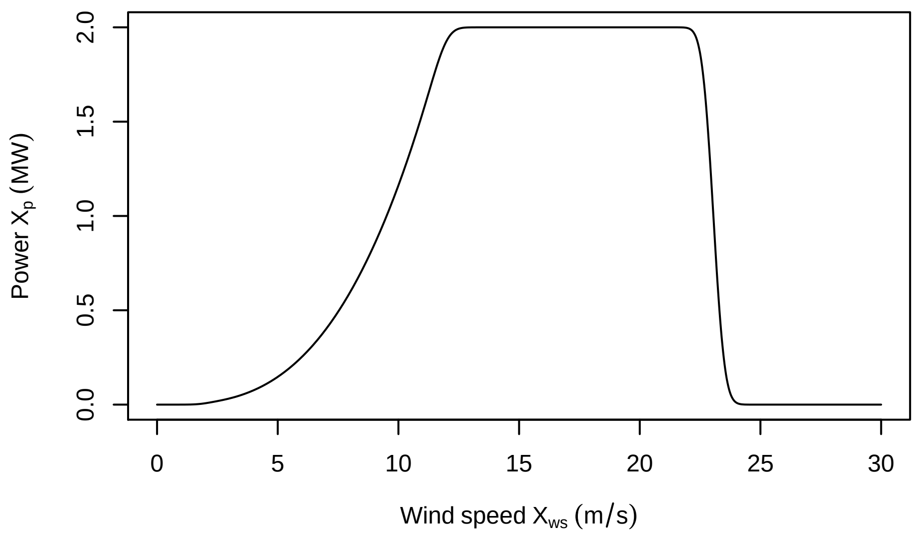



Figure 2: *Power curve, i.e. the function between the wind speed* $X_{\mathrm{ws}}$ *and the output power* $X_{\mathrm{p}}$.

It can be seen that if the wind speed is 5 m/s the output power will be around 0.15 MW and at 15 m/s the output power will be 2 MW. This function can be applied directly on average hourly values of the wind speed and gives then hourly average values of output power.

Continues on page 30

Let $X_{\text{ws}}$ be the average hourly wind speed in m/s and the power output in MW

$$X_{\text{p}} = f_{\text{powercurve}}(X_{\text{ws}})$$

where $f_{\text{powercurve}}()$ is the power curve function.

## Question IX.1 (19)

From the plot of the pdf in Figure 1 conclude which of the following statements is not correct (Note: you must mark the FALSE statement - four of the statements are correct!):

1 ☐ $P(X_{\text{ws}} > 12) \approx 0.10$

2 ☐ $P(X_{\text{ws}} < 5) \approx 0.34$

3 ☐ $P(X_{\text{ws}} > 10) \approx 0.19$

4 ☐ $P(X_{\text{ws}} > 0) \approx 1$

5* ☐ $P(X_{\text{ws}} < 15) \approx 0.04$

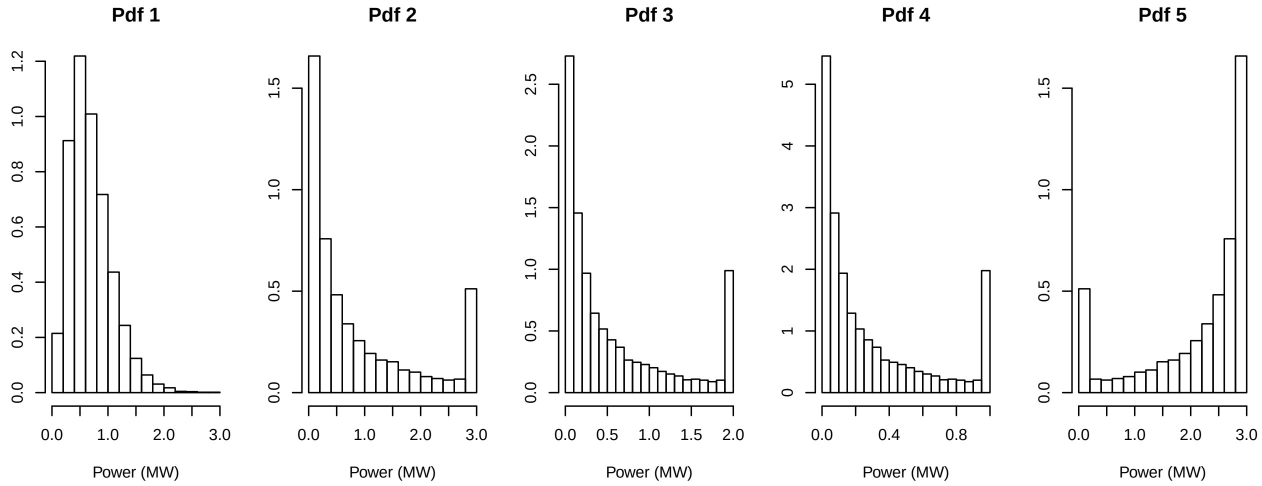
————— FACIT-BEGIN —————

Reading from the plot of the wind speed pdf is clear that the statement $P(X_{\text{ws}} < 15) \approx 0.04$ is not correct, since most of the probability mass (the area below the pdf) is below 15 m/s.

————— FACIT-END —————

## Question IX.2 (20)

The probability density function of the hourly output power $X_{\text{p}}$ is found by simulation. Which of the following pdfs can be the probability density function of the hourly output power at the site, i.e. the pdf of $X_{\text{p}}$?



Continues on page 31

1 ☐ Pdf 1

2 ☐ Pdf 2

3* ☐ Pdf 3

4 ☐ Pdf 4

5 ☐ Pdf 5

————— FACIT-BEGIN —————

Since the power cannot be higher that 2 MW, which can be seen from the power curve which saturates at 2 MW, Pdf 1, 2 and 5 can be excluded. Further, Pdf 4 has zero probability of being higher than 1 MW, which cannot be the case either. The correct pdf is Pdf 3: it has most probablity mass below 0.5 MW, which because most probability mass of the wind speed pdf is below 7 m/s, which is the point where output power is approximately 0.5 MW. Further, the saturation of the power curve at 2 MW gathers all probability mass in the range approx. 12 to 23 m/s around 2 MW, which creates the peak in the output power pdf around 2 MW.

————— FACIT-END —————

## Question IX.3 (21)

In wind power forecasting it is very important to include the forecast uncertainty. It can be described by the variance of the power output forecast $\sigma_{\mathrm{p}}^2$. In the range from 5 to 10 m/s the power curve is constructed by the following relation

$$f_{\mathrm{powercurve}}(X_{\mathrm{ws}}) = aX_{\mathrm{ws}}^3 \quad \textit{for } 5 < X_{\mathrm{ws}} < 10 \tag{3}$$

Further, it is known that the variance of the wind speed forecast in the same range is $\sigma_{\mathrm{ws}}^2$.

Which of the following expressions calculates an approximation to the variance of the output power forecast $\sigma_{\mathrm{p}}^2$ for a wind speed $X_{\mathrm{ws}}$ in the range from 5 to 10 m/s?

1 ☐ $\sigma_{\mathrm{p}}^2 = X_{\mathrm{ws}}^3 \sigma_{\mathrm{ws}}^2$

2* ☐ $\sigma_{\mathrm{p}}^2 = 9a^2 X_{\mathrm{ws}}^4 \sigma_{\mathrm{ws}}^2$

3 ☐ $\sigma_{\mathrm{p}}^2 = \int_5^{10} \sigma_{\mathrm{ws}}^2 3ax^2 dx$

4 ☐ $\sigma_{\mathrm{p}}^2 = \int_5^{10} \sigma_{\mathrm{ws}}^2 x^3 dx$

5 ☐ $\sigma_{\mathrm{p}}^2 = a^2 \sigma_{\mathrm{ws}}^2$

—————— FACIT-BEGIN ——————

We need to consider error propagation through a non-linear function and the power curve $aX_{\text{ws}}^3$ is a non-linear function. Hence we can use the error propagation rule in Method 4.6. We need the derived function with respect to $X_{\text{ws}}$

$$\frac{\partial f_{\text{powercurve}}}{\partial x_{\text{ws}}} = 3aX_{\text{ws}}^2$$

We know the variance of the wind speed forecast $\sigma_{\text{ws}}^2$, hence the correct expression is found by inserting (and squaring the partial derivative) in the Method 4.6 formula

$$\sigma_{\text{p}}^2 = 9a^2X_{\text{ws}}^4\sigma_{\text{ws}}^2$$

—————— FACIT-END ——————

Continues on page 34

**Exercise X**

A supermarket chain would like to track the trend in sales of organic meat. Therefore, they have for four years conducted a survey among their customers, asking whether the customers bought organic meat. The distribution of the answers is seen in the table below.

| | 2011 | 2012 | 2013 | 2014 |
|---|---|---|---|---|
| Bought organic meat | 68 | 72 | 81 | 90 |
| Bought non-organic meat | 432 | 428 | 419 | 410 |

**Question X.1 (22)**

The supermarket chain wants to test the hypothesis that the proportion buying organic meat is the same each year.

$$H_0 : p_1 = p_2 = p_3 = p_4$$

Here $p_1$ is the proportion that buys organic meat in 2011, $p_2$ is the proportion that buys organic meat in 2012 etc.

What is the expected number of organic meat purchases in 2014, under the hypothesis of equal proportions each year?

1 ☐ 144.69

2 ☐ 250.00

3* ☐ 77.75

4 ☐ 43.48

5 ☐ 422.25

———— FACIT-BEGIN ————

We are looking for the number

$$\begin{aligned} e_{1,4} &= \frac{\text{Row 1 total} \cdot \text{Column 4 total}}{\text{Grand total}} \\ &= \frac{(68+72+81+90)\cdot(90+410)}{68+72+81+90+432+428+419+410} \\ &= \frac{311 \cdot 500}{2000} = \frac{155500}{2000} = 77.75 \end{aligned}$$

So the correct answer is

3 ☐ 77.75

——— FACIT-END ———

Continues on page 36

**Question X.2 (23)**

A $\chi^2$ distributed test statistic is used in order to test the hypothesis

$$H_0 : p_1 = p_2 = p_3 = p_4$$

What is the contribution $q_{No,2011}$ to the test statistic $\chi^2_{obs}$ from the respondents, who answer that they bought non-organic meat in 2011?

1* ☐ $q_{No,2011} = 0.2251$

2 ☐ $q_{No,2011} = 1.2227$

3 ☐ $q_{No,2011} = 9.75$

4 ☐ $q_{No,2011} = 0.0231$

5 ☐ $q_{No,2011} = 0.2201$

———————— FACIT-BEGIN ————————

From Method 7.21 we know that the $\chi^2$-test statistic $\chi^2_{obs}$ is calculated as a sum

$$\chi^2_{obs} = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(o_{ij} - e_{ij})^2}{e_{ij}}$$

The contribution from the respondents in 2011 who answered no:

$$q_{No,2011} = \frac{(432 - 422.25)^2}{422.25} = 0.2251$$

So the correct answer is

1 ☐ $q_{No,2011} = 0.2251$

———————— FACIT-END ————————

**Question X.3 (24)**

The supermarket chain has now conducted the test of the hypothesis

$$H_0 : p_1 = p_2 = p_3 = p_4$$

to track the trend in sales of organic meat.

In this case the relevant test statistic becomes 4.3977.

Which of the following R commands calculates the p-value for the hypothesis test?

1 ☐ `pchisq(4.3977, 3)`

2 ☐ `2*(1-pnorm(4.3977))`

3* ☐ `1-pchisq(4.3977, 3)`

4 ☐ `2*(1-pchisq(4.3977, 4))`

5 ☐ `1-pchisq(4.3977, 6)`

———————— FACIT-BEGIN ————————

From Method 7.21 we see that the test statistic should be compared with a $\chi^2$-distribution with (2-1)(4-1)=3 degrees of freedom. The test probability is now:

```
  1-pchisq(4.3977, 3)

## [1] 0.2215987
```

So the correct answer is

3 ☐ `1-pchisq(4.3977, 3)`

Doing the analysis in R

```
 study <- matrix(c(  68 ,72, 81 ,90,432, 428, 419 , 410 ), nrow=2, byrow=TRUE)
  colnames(study) <- c("2011", "2012", "2013", "2014")
  rownames(study) <- c("Organic", "Non-Organic")
chi <- chisq.test(study); chi

##
##  Pearson's Chi-squared test
##
## data:  study
## X-squared = 4.3977, df = 3, p-value = 0.2216
```

———————— FACIT-END ————————

Continues on page 38

**Exercise XI**

Studies have shown that teenage girls have a lower life satisfaction than boys. Therefore, a team of first-year students decided to study life satisfaction among their peers. The results of their study were as follows.

| | High life satisfaction | Lower life satisfaction |
|---|---|---|
| Men | 68 | 208 |
| Women | 18 | 74 |

**Question XI.1 (25)**

What is the correct 95% confidence interval for the estimate of the difference between the proportion of high life satisfaction for men and women?

1 ☐ $(0.2464 - 0.1957) \pm 1.64 \cdot \sqrt{\frac{0.2464(1-0.2464)}{276} + \frac{0.1957(1-0.1957)}{92}} = (-0.029; 0.131)$

2 ☐ $(0.2464 - 0.1957) \pm 1.96 \cdot \sqrt{(\frac{0.2464(1-0.2464)}{276})^2 + (\frac{0.1957(1-0.1957)}{92})^2} = (0.047; 0.054)$

3 ☐ $\frac{0.2464}{0.1957} \pm 1.96 \cdot \sqrt{\frac{0.2464(1-0.2464)}{276} + \frac{0.1957(1-0.1957)}{92}} = (1.16; 1.35)$

4 ☐ $(0.2464 - 0.1957) \pm 3.84 \cdot \sqrt{\frac{0.2464(1-0.2464)}{276} + \frac{0.1957(1-0.1957)}{92}} = (-0.137; 0.238)$

5* ☐ $(0.2464 - 0.1957) \pm 1.96 \cdot \sqrt{\frac{0.2464(1-0.2464)}{276} + \frac{0.1957(1-0.1957)}{92}} = (-0.045; 0.146)$

——————— FACIT-BEGIN ———————

Let $p_1$ be the proportion of high life satisfaction in men and $p_2$ the proportion of high life satisfaction in women. According to Method 7.14 the 95% confidence interval for the estimated difference $\hat{p}_1 - \hat{p}_2$ is given as

$$(\hat{p}_1 - \hat{p}_2) \pm z_{1-\alpha/2}\hat{\sigma}_{\hat{p}_1 - \hat{p}_2}$$

The estimates $\hat{p}_1$ and $\hat{p}_2$ are the observed proportions with high life satisfaction.

$$\begin{aligned} \hat{p}_1 &= \frac{68}{68+208} = 0.2464 \\ \hat{p}_2 &= \frac{18}{18+74} = 0.1957 \end{aligned}$$

The estimated standard error is

$$\hat{\sigma}_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

If we insert the number of men $n_1 = 276$ and the number of women $n_2 = 92$ then we get

$$\hat{\sigma}_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{0.2464(1-0.2464)}{276} + \frac{0.1957(1-0.1957)}{92}}$$

Finally, notice $z_{1-\alpha/2} = 1.96$ is the 97.5% percentile in a standard normal distribution.

So the correct answer is

5 ☐ $(0.2464 - 0.1957) \pm 1.96 \cdot \sqrt{\frac{0.2464(1-0.2464)}{276} + \frac{0.1957(1-0.1957)}{92}} = (-0.045; 0.146)$

———— FACIT-END ————

Continues on page 40

**Question XI.2 (26)**

Now we want to test the hypothesis that the proportion of high life satisfaction is the samen for men and women. I.e. we testing the hypothesis (at significance level $\alpha = 0.05$).

$$H_0 : p_1 = p_2$$
$$H_A : p_1 \neq p_2$$

Here $p_1$ is the proportion of high life satisfaction amongst men and $p_2$ is the proportion of high life satisfaction amongst women.

What is the conclusion to this test? (Both the conclusion and the argumentation must be correct).

1 ☐ $H_0$ is rejected, since the test statistic $z_{obs} = \frac{(0.2464-0.1957)}{\sqrt{0.2337(1-0.2337)(\frac{1}{276+92})}} = 2.298$ leads to a p-value of 0.02

2* ☐ $H_0$ is accepted, since the test statistic $z_{obs} = \frac{(0.2464-0.1957)}{\sqrt{0.2337(1-0.2337)(\frac{1}{276}+\frac{1}{92})}} = 0.995$ leads to a p-value of 0.32

3 ☐ $H_0$ is accepted, since the test statistic $z_{obs} = \frac{(0.2464-0.1957)}{\sqrt{\frac{0.2464(1-0.2464)}{276}+\frac{0.1957(1-0.1957)}{92}}} = 1.038$ leads to a p-value of 0.15

4 ☐ $H_0$ is accepted, since the test statistic $z_{obs} = \frac{(0.2464-0.1957)}{\sqrt{0.2337(1-0.2337)(\frac{1}{276+92})}} = 2.298$ leads to a p-value of 0.02

5 ☐ $H_0$ is rejected, since the test statistic $z_{obs} = \frac{(0.2464-0.1957)}{\frac{0.2464(1-0.2464)}{276}+\frac{0.1957(1-0.1957)}{92}} = 21.3$ leads to a p-value $< 0.0001$

———— FACIT-BEGIN ————

According to Method 7.17 the hypothesis is tested using the test statistic

$$z_{obs} = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1}+\frac{1}{n_2})}}$$

Here we have

$$n_1 = 276$$
$$n_2 = 92$$
$$\hat{p}_1 = \frac{68}{68+208} = 0.2464$$
$$\hat{p}_2 = \frac{18}{18+74} = 0.1957$$
$$\hat{p} = \frac{68+18}{276+92} = 0.2337$$

So

$$z_{obs} = \frac{0.2464 - 0.1957}{\sqrt{0.2337(1 - 0.2337)(\frac{1}{276} + \frac{1}{92})}} = 0.995$$

We want a two-sided test and calculate $2P(Z > z_{obs})$

```
2*(1-pnorm(0.995))

## [1] 0.3197363
```

So the correct answer is

2 ☐ $H_0$ is accepted, since the test statistic $z_{obs} = \frac{(0.2464-0.1957)}{\sqrt{0.2337(1-0.2337)(\frac{1}{276}+\frac{1}{92})}} = 0.995$ leads to a p-value of 0.32

——————— FACIT-END ———————

Continues on page 42

**Exercise XII**

18 test persons evaluated the bass quality of 3 different headphones, so that all 18 persons evaluated all 3 headphones such that the data consist of 54 observations of bass quality on a scale between 0 and 150. The average of the three headphone bass qualities were:

| Headphone | Average |
|---|---|
| 1 | 53.5 |
| 2 | 55.5 |
| 3 | 97.1 |

**Question XII.1 (27)**

How is the $SS(Tr)$ calculated in the 2-way analysis of variance, which compares the mean bass quality for the three headphones? (Where "Tr" now refers to the 3 headphones)

1* ☐ $18 \cdot (53.5 - 68.7)^2 + 18 \cdot (55.5 - 68.7)^2 + 18 \cdot (97.1 - 68.7)^2$

2 ☐ $(53.5 - 68.7)^2 + (55.5 - 68.7)^2 + (97.1 - 68.7)^2$

3 ☐ $\frac{(53.5-68.7)^2}{53.5} + \frac{(55.5-68.7)^2}{55.5} + \frac{(97.1-68.7)^2}{55.5}$

4 ☐ $\frac{(53.5-68.7)}{53.5} + \frac{(55.5-68.7)}{55.5} + \frac{(97.1-68.7)}{55.5}$

5 ☐ $3 \cdot (53.5 - 68.7)^2 + 3 \cdot (55.5 - 68.7)^2 + 3 \cdot (97.1 - 68.7)^2$

——————— FACIT-BEGIN ———————

With 18 observations ($b = 18$) for each treatment in a 2-way ANOVA the defining formula for $SS(Tr)$ gives:

$$18 \cdot (53.5 - 68.7)^2 + 18 \cdot (55.5 - 68.7)^2 + 18 \cdot (97.1 - 68.7)^2$$

, since the mean of the three means become 68.7. So the correct answer is 1).

——————— FACIT-END ———————

**Question XII.2 (28)**

If, in line with the above, we let "persons" constitute "blocks", we are given that $SS(Bl) = 6003.5$ and that $SSE = 7160.3$ in the 2-way analysis of variance. What will the F-test statistic for the hypothesis that the 18 persons have the same mean value be?

1 ☐ $F_{obs} = \frac{18 \cdot 6003.5}{210.6/3}$

2 ☐ $F_{obs} = \frac{3 \cdot 6003.5}{7160.3/17}$

3* ☐ $F_{obs} = \frac{6003.5/17}{7160.3/34}$

4 ☐ $F_{obs} = \frac{(6003.5 - 210.6)^2}{7160.3}$

5 ☐ $F_{obs} = \frac{(6003.5/18 - 210.6)}{\sqrt(210.6)}$

——— FACIT-BEGIN ———

The $F$-statistic is

$$F_{obs,Bl} = \frac{MS(Bl)}{MSE} = \frac{6003.5/17}{210.6},$$

as the $MSE = 210.6$ (PBB: I will have to change this!!!!)

——— FACIT-END ———

Continues on page 44

**Question XII.3 (29)**

The hypothesis of no difference in mean bass quality of the three headphones is by the usual test evaluated by which sampling distribution?

1 ☐ $z$-distribution (= standard normal distribution)

2 ☐ $t$-distribution with 53 degrees of freedom

3 ☐ $\chi^2$-distribution with 53 degrees of freedom

4* ☐ $F$-distribution with 2 and 34 degrees of freedom

5 ☐ $F$-distribution with 3 and 51 degrees of freedom

——————— FACIT-BEGIN ———————

According to Theorem 8.22 the right sampling distrubution is the F-distribution with $l-1=17$ and $(k-1)(l-1) = 2 \cdot 17 = 34$, so the correct answer is 4).

——————— FACIT-END ———————

**Question XII.4 (30)**

What will the 95% confidence interval be for the mean difference between headphone 2 and 1? (It can be assumed that this is a "pre-planned" comparison)

1 ☐ $2 \pm 2 \cdot 210.6$

2 ☐ $2 \pm 2.03 \cdot \sqrt{210.6}$

3 ☐ $2 \pm 1.96 \cdot \frac{210.6}{54}$

4 ☐ $2 \pm 1.96$

5* ☐ $2 \pm 2.03 \cdot \sqrt{2 \cdot 210.6 \frac{1}{18}}$

——————— FACIT-BEGIN ———————

We use the post hoc method box for oneway anova combined with the 2-way adaption:

1. Use the MSE and/or SSE from the two-way analysis
2. Use $(l-1)(k-1)$ as denominator DF

So:

$$2 \pm 2.03 \cdot \sqrt{2 \cdot 210.6 \frac{1}{18}}$$

So the correct answer is 5).

———————— FACIT-END ————————

THE EXAM IS FINISHED. ENJOY THE SUMMER!