

Week-11
Tutorial-2

1.) Let $f, g : R \rightarrow R$ be defined by $f(x) = 4x - 1$ and $g(x) = x^2 + 2$.
 find.

vi.) $(g \circ g)(2)$

$$\begin{aligned} &= g(x^2 + 1) \\ &= (x^2 + 1)^2 + 1 \\ &= x^4 + 2x^2 + 1 + 1 \\ &= x^4 + 2x^2 + 2 \\ &= (2)^4 + 2(2)^2 + 2 \\ &= 16 + 2(4) + 2 \\ &= 16 + 8 + 2 \\ &= 26 \end{aligned}$$

vii.) $(f \circ g \circ f)(3)$

$$\begin{aligned} &= f \circ g(4x - 1) \\ &= f((4x - 1)^2 + 1) \\ &= 4((4x - 1)^2 + 1) - 1 \\ &= 4(16x^2 - 8x + 1 + 1) - 1 \\ &= 4(16x^2 - 8x + 2) - 1 \\ &= 4(144 - 24 + 2) - 1 \\ &= 4(122) - 1 \\ &= 488 - 1 \\ &= 487. \end{aligned}$$

viii.) (g

i)

$$\begin{aligned}
 &= (16x^4 + 8x^2) + 1 \\
 &= (16x(3)^4 + 8(3)^2) + 1 \\
 &= (16x81 + 8 \times 9) + 1 \\
 &= 1296 + 72 + 1 \\
 &= 1369.
 \end{aligned}$$

i.) $(g \circ f)(x)$
 $\underline{\underline{so}}$

$$\begin{aligned}
 &= g(4x - 1) \\
 &= (4x - 1)^2 + 1 \\
 &= 16x^2 - 8x + 1 + 1 \\
 &= 16x^2 - 8x + 2
 \end{aligned}$$

x.) $(f \circ g)(x)$
 $\underline{\underline{so}}$

$$\begin{aligned}
 &= f(x^2 + 1) \\
 &= 4(x^2 + 1) - 1 \\
 &= 4x^2 + 4 - 1 \\
 &= 4x^2 + 3.
 \end{aligned}$$

i. viii) $(g \circ f \circ g)(3)$

$$\begin{aligned} &= g \circ f(x^2 + 1) \\ &= g \circ g \circ (4(x^2 + 1) - 1) \\ &= g \circ (4x^2 + 4) - 1 \\ &= (4x^2 + 4)^2 - 1 \\ &= 16x^4 + 8x^2 + 16 \\ &= 16x^4 + 4 \cdot 2 \cdot 4x^2 + 4^2 - 1 \\ &= 1522 \end{aligned}$$

2.) let f, g and h be function $\mathbb{R} \rightarrow \mathbb{R}$ defined respectively.

$$f(x) = 2x+1, g(x) = \frac{1}{x^2+1} \quad \& \quad h(x) = \sqrt{x^2+1}.$$

find \exp^n for each of the following;

$$\text{vi.) } (gof)(x)$$

$$= g(2x+1)$$

$$= \frac{1}{(2x+1)^2+1}$$

$$= \frac{1}{4x^2+4x+1+1}$$

$$= \frac{1}{4x^2+4x+2}$$

$$\text{vii.) } (goh)(x)$$

$$= g(\sqrt{x^2+1})$$

$$= \frac{1}{(\sqrt{x^2+1})^2+1}$$

$$= \frac{1}{x^2+1+1}$$

$$= \frac{1}{x^2+2}$$

vii.) $(f \circ f)(x)$

$$\begin{aligned}f \circ f(x) &= f(2x+1) \\&= 2(2x+1)+1 \\&= 4x+2+1 \\&= 4x+3\end{aligned}$$

ix.) $((f \circ g) \circ h)(x)$

$$\begin{aligned}((f \circ g) \circ h)(x) &= f(g(h(x))) \\&= f(\quad)\end{aligned}$$

$$w i x.) ((f \circ g) \circ h)(x)$$

$$\stackrel{soj}{=} \left(f\left(\frac{1}{x^2+1}\right) \right) oh(x)$$

$$= \left(2 \left(\frac{1}{x^2+1} \right) \right) oh(x)$$

$$= \left(\frac{2}{x^2+1} + 1 \right) oh(x)$$

$$= \frac{2}{(\sqrt{x^2+1})^2+1} + 1$$

$$= \frac{2}{x^2+1+1} + 1$$

$$= \frac{2}{x^2+2} + 1$$

$$= \frac{2+1(x^2+2)}{x^2+2}$$

$$= \frac{2+x^2+4}{x^2+2}$$

$$x \cdot (f \circ (g \circ h))(x)$$

so,

$$= f \circ g (\sqrt{x^2 + 1})(x)$$

$$= f \circ \left(\frac{1}{(\sqrt{x^2 + 1})^2 + 1} \right)$$

$$= f \circ \left(\frac{1}{x^2 + 1 + 1} \right)$$

$$= 2 \left(\frac{1}{x^2 + 1 + 1} \right) + 1$$

$$= \frac{2}{x^2 + 2} + 1$$

$$= \frac{2 + x^2 + 2}{x^2 + 2}$$

$$= \frac{x^2 + 4}{x^2 + 2}$$

3.) Let $f(x) = \frac{1}{\sqrt{x^2+1}}$ & $g(x) = \frac{2}{x-3}$. Determine $(f \circ g)(x)$.

S01)

$$= f\left(\frac{2}{x-3}\right)$$

$$= \frac{1}{\sqrt{\left(\frac{2}{x-3}\right)^2 + 1}}$$

$$= \frac{1}{\sqrt{\frac{4}{x^2-6x+9} + 1}}$$

$$= \frac{1}{\sqrt{\frac{4 + (x^2 - 6x + 9)}{x^2 - 6x + 9}}}$$

$$= \frac{1}{\sqrt{\frac{x^2 - 6x + 13}{x^2 - 6x + 9}}}$$