Tutorial 2

Q1 A = {1, 2, 3, 4, 5}

a

R = {(1,1), (2,2) (3,3), (1,2), (2,1), (1,3), (3,1), (2,3), (3,2)}

b.

R = {(1,1), (2,2), (3,3), (4,4), (5,5), (1,2), (2,1), (2,3), (3,2),
(2,4), (4,2), (5,2), (2,5)}

c

R = {(1,1), (2,2), (3,3), (4,4), (5,5), (1,2), (2,3), (1,3), (1,4), (4,5)
(1,5)}

d.

R = {(1,1), (2,2), (3,3), (4,4), (5,5), (2,1), (1,2), (1,3), (3,4), (1,4),
(4,2), (2,5), (4,5)}

e

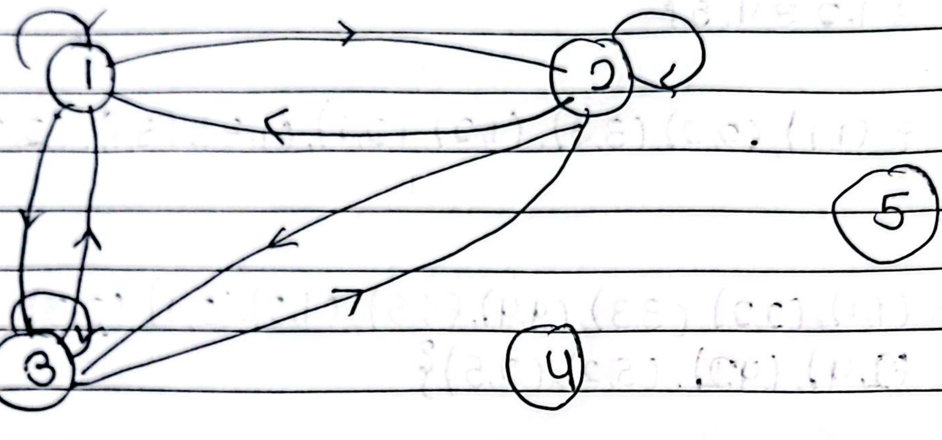
R = {(1,1), (2,2), (1,2), (2,3), (1,3), (3,4), (4,5), (3,5), (2,4)
(1,4)}

f.

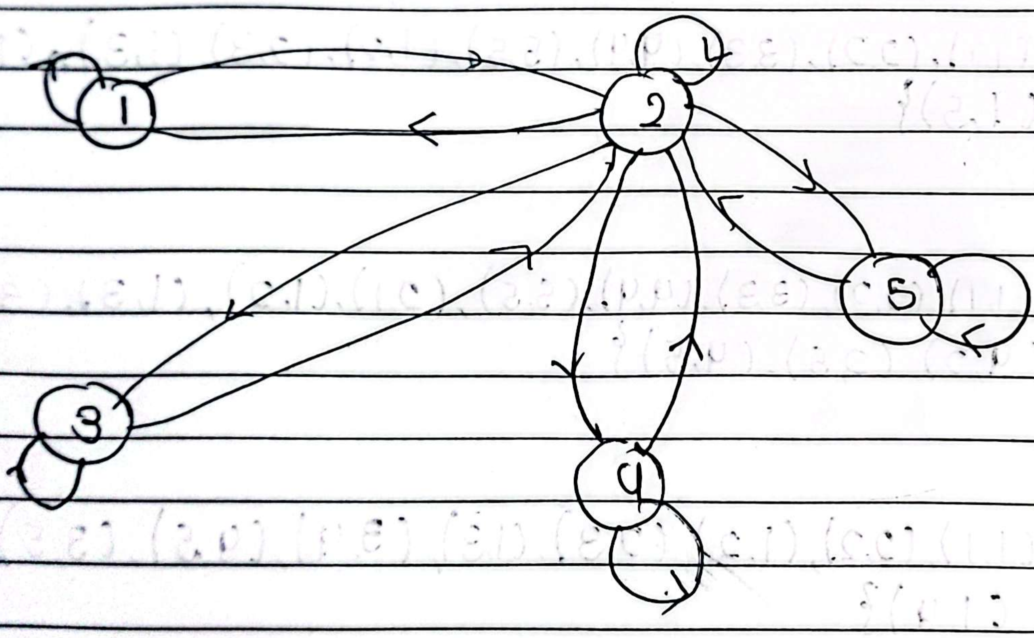
R = {(1,1), (2,2), (3,3), (4,4), (5,5), (2,1), (1,5), (1,3), (4,1),
(2,3), (3,4), (5,2), (2,4)}

Diagraphs

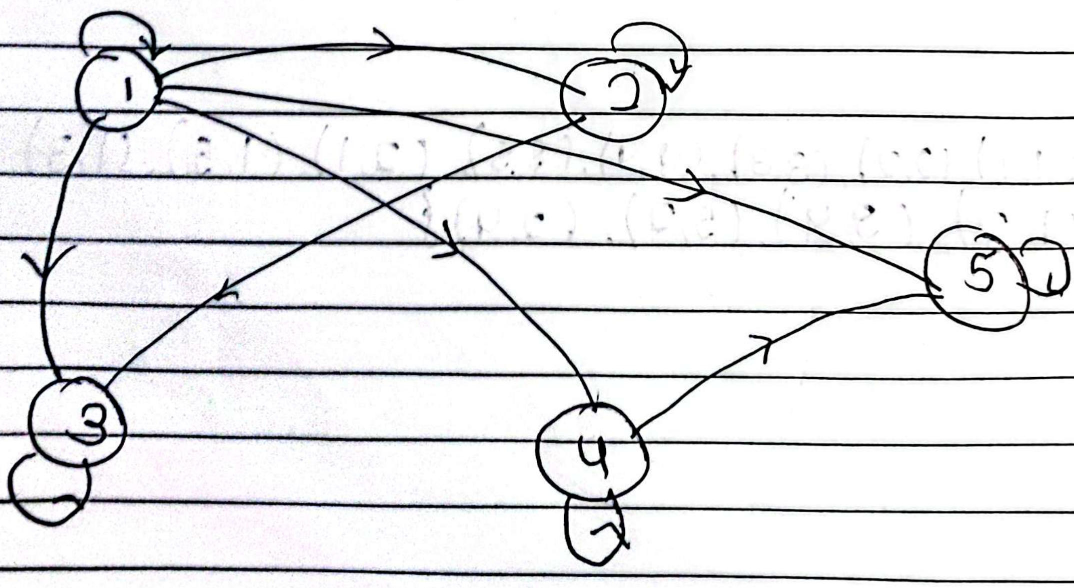
a.



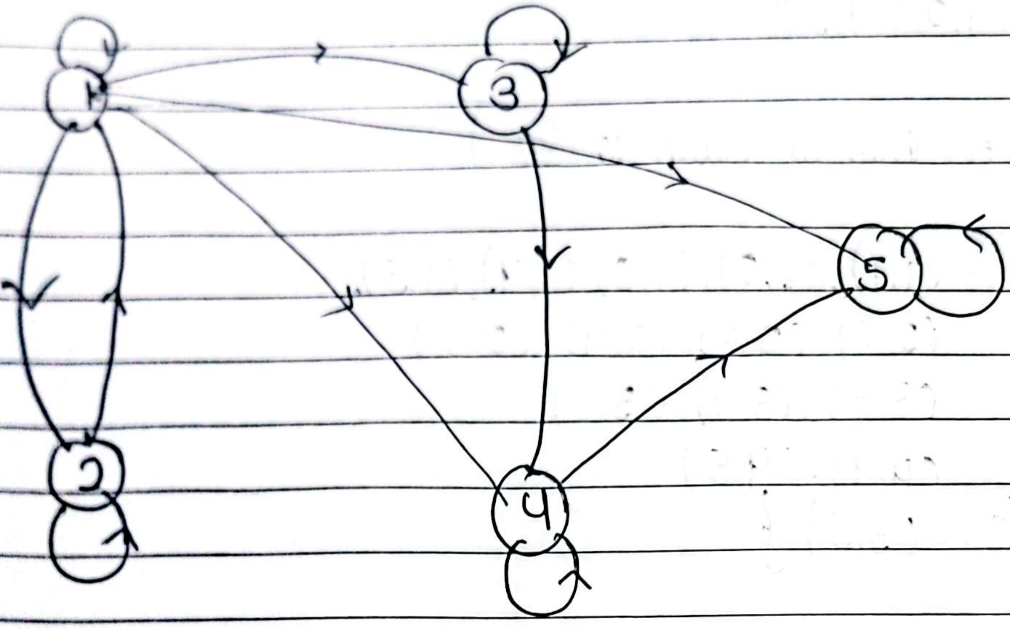

b.



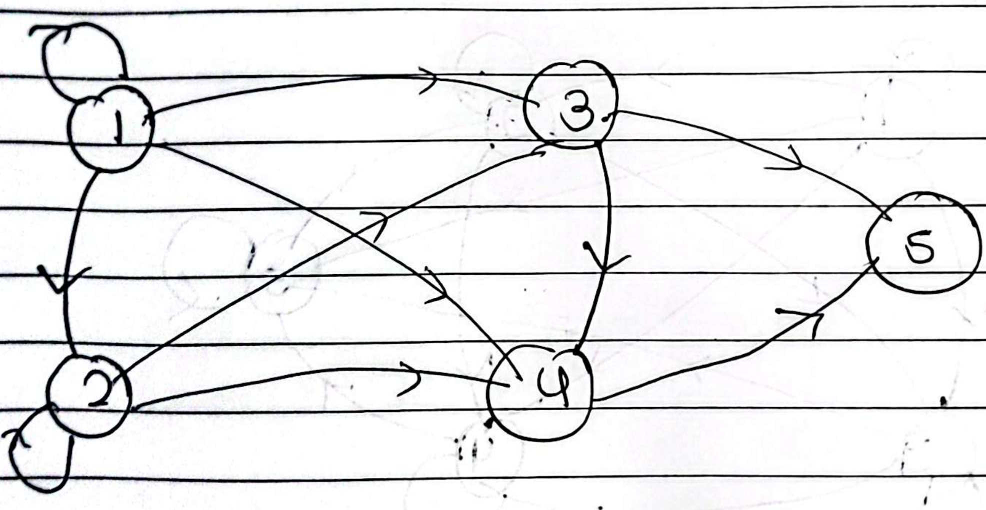

c.



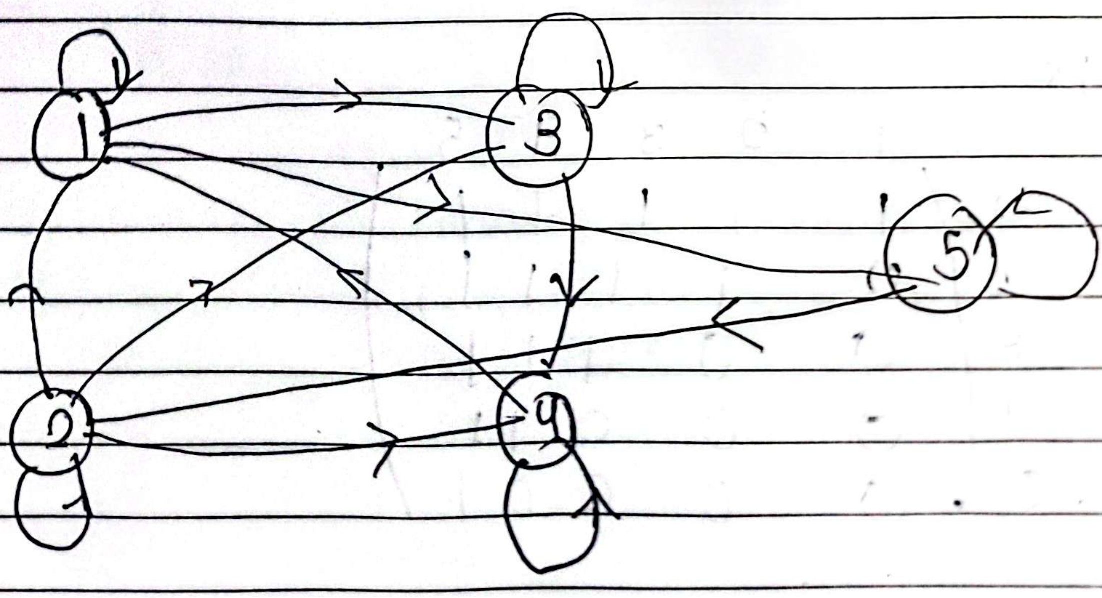
d




e




f

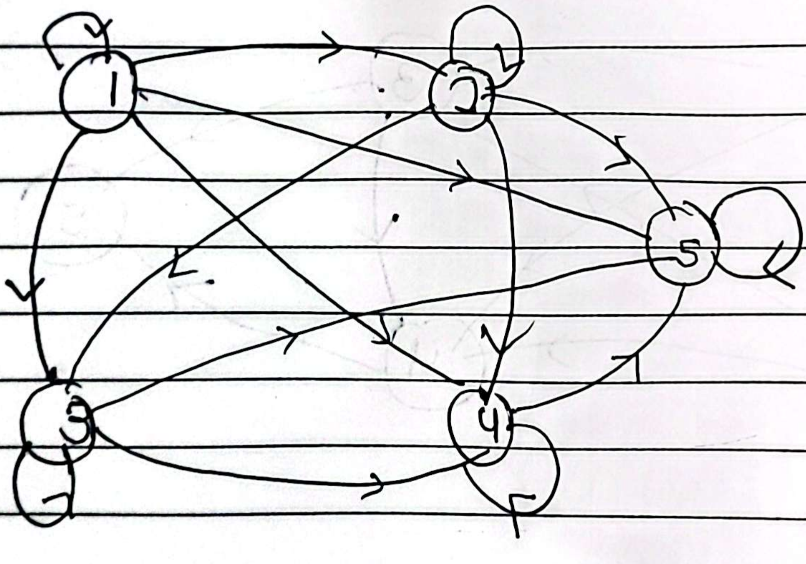


Q2 $A = \{1,2,3,4,5\}$

$R$ (less than or equal to $\leq$)

$R = \{(1,1),(1,2),(1,3),(1,4),(1,5),$
$(2,2),(2,3),(2,4),(2,5)$
$(3,3),(3,4),(3,5)$
$(4,4),(4,5)$
$(5,5)\}$

Diagraph




Matrix

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 1 & 1 & 1 & 1 & 1 \\
2 & 0 & 1 & 1 & 1 & 1 \\
3 & 0 & 0 & 1 & 1 & 1 \\
4 & 0 & 0 & 0 & 1 & 1 \\
5 & 0 & 0 & 0 & 0 & 1
\end{array}
$$

checking equivalence

It is reflexive since for all $a \in A$, $(a,a) \in R$
It is not symmetric since for all $(a,b)$, $(b,a) \in R$
For transitive,

$(1,2)(2,3) \rightarrow (1,3)$
$(1,3),(3,4) \rightarrow (1,4)$
$(1,4),(4,5) \rightarrow (1,5)$
~~(1,3)~~ $(2,3)(3,4) \rightarrow (2,4)$
$(2,4),(4,5) \rightarrow (2,5)$
$(3,4),(4,5) \rightarrow (3,5)$
~~(3,5)~~

since for all $(a,b),(b,c)$ there exist $(a,c)$
R is transitive
therefore,
R is not equivalence Relation since not symmetric.

Q3.
a " is the son of" in a set of real people
[~~suppose a father b son~~] ($x$ is the son of $y$] $(x,y)$
$x(x,x)$ → Not reflexive since can't be son and father at same time
✓ → Irreflexive since no $(a,a)$ exist
$(x,y)$ $(y,x)$ ✗ → Not symmetric since a son
→ Not antisymmetric since no Reflexive
$(x,y)$ $(y,z)$ $(x,z)$ ✗ → Not transitive since can't be son (only Grand son)

(1, 2) ( <

a, b (b, c)

b "greater than" set of real num

R : (a ≥ b)

eg R : { (1,1), ~~(1,2), (1,3)~~ ...

(2,2), (2,1)

(3,3), (3,2), (3,1) ... }

→ Reflexive since for all a exist, (a,a) exist

→ Not reflexive since (a,a) exist

→ Not symmetric since for (a,b), (b,a) dosen't exist

→ Anti symmetric since no (a,b) (b,a) exist

→ Not transitive since no (a,b) (b,c) → (a,c) exist

c. "is multiple of" positive integer

R : { (1,1),

(2,1), (2,2) (4,1), (4,2) (5,1) (12,4), (12,3)

(3,1), (3,3) (6,1), (6,2), (6,3), (12,1), (12,2) (12,6)

✓ → Reflexive since (a,a) exist [(1,1) (2,2)]

→ Not reflexive since R is reflexive

→ Not symmetric since (a,b) (b,a) dosen't exist [(4,2) then no (2,4)]

✓ → Antisymmetric since at least one loop and not symmetric

✓ → Transitive since (a,b), (b,c) → (a,c)

eg. (8,4), (4,2) → (8,2)

(possible)

[8 is multiple of 4 ✓

4 is multiple of 2 ✓

8 is multiple of 2 ✓]

$u = 6$

d. $R = \{(x,y) : x^2 = y^2\}$,

$R = \{(1,1), (2,2), (3,3), (4,4) \dots \}$

✓ Reflexive since loop exists $[1^2 = 1, (2,2) = 2^2 = 4]$

Not irreflexive since R is reflexive

✓ symmetric since (a,b) (b,a) exist since all reflexive

Not antisymmetric since it is reflexive

✓ Transitive since all reflexive

e. "Has the same parity" (both odd or even) set of integer.

R = "Has the same parity"

✓ Reflexive since (x,x) can both be odd or even

Not irreflexive since R is reflexive

✓ Symmetric since (x,y) (y,x) both exist. [(2,6) (6,2)

(1,3) (3,1)]

Not antisymmetric since R is symmetric

✓ Transitive since $(a,b)\ (b,c) \to (a,c)$,

$[(1,3), (3,5) \to (1,5)$

$(2,4), (4,6) \to (2,6)]$

Date
Page

4 $A = \{1, 2, 3, 4, 5\}$

By Q.

$R = \{B_1, B_2, B_3)$

where,

$B_1 = \{1, 3)$ means,

$(1,1), (1,3), (3,1), (3,3)$

$B_2 = \{5\}$ means

$(5,5)$

$B_3 = \{2, 4\}$ means

$= (2,2), (2,4), (4,2), (4,4)$

then,

$R = \{(1,1), (3,3), (1,3), (3,1), (5,5), (2,2), (2,4), (4,2), (4,4)\}$

5. $A = \{1, 2, 3, 4, 5, 6\}$

$\left[\begin{array}{l} R = x\ R\ y \\ \text{when, } x^2 - 4x = y^2 - 4y \end{array}\right]$

So, from first eq^n

$R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

Second equation $(x + y = 4)$

$R_2 = \{(1,3), (2,2), (3,1)\}$

combining $R_1 + R_2$

$R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (1,3), (3,1)\}$

solving

$x^2 - 4x = y^2 - 4y$

$x^2 - y^2 = -4y + 4x$

$(x+y)(x-y) = -4y + 4x$

$(x+y)(x-y) + 4y - 4x = 0$

$(x+y)(x-y) - 4(x-y) = 0$

$(x-y)(x+y-4) = 0$

either, OR,

$x = y$ 1st

$x + y - 4 = 0$

$x + y = 4$ 2nd

checking for equivalence Relation

R is Reflexive since (x,x) exist for all x ∈ R
R is symmetric since (x,y) (y,x) exist for all
R is transitive since (x,y) (y,z) → (x,z)

[ (3,1), (1,3) = (3,3)
(1,3) , (3,1) = (1,1) ]

Therefore,

R is equivalence relation
since it is reflexive, symmetric and transitive

Then,

equivalence classes

|1| = { 1,3 }
|2| = { 2 }
|3| = { 3,1 }
|4| = { 4 }
|5| = { 5 }
|6| = { 6 }
//