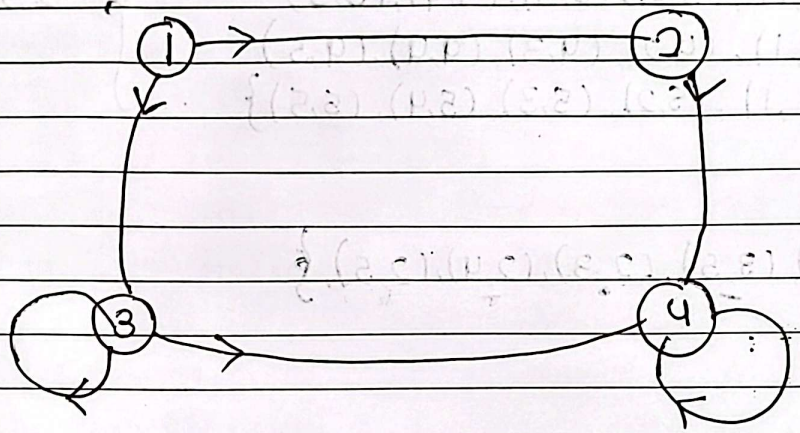
Tutorial 2

Q1. $A = \{1,2,3,4\}$

$R = \{(1,2),(1,3),(2,4),(3,3),(3,4),(4,4)\}$

a. diagraph




b. Matrix

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0 | 1 | 1 | 0 |
| 2 | 0 | 0 | 0 | 1 |
| 3 | 0 | 0 | 1 | 1 |
| 4 | 0 | 0 | 0 | 1 |

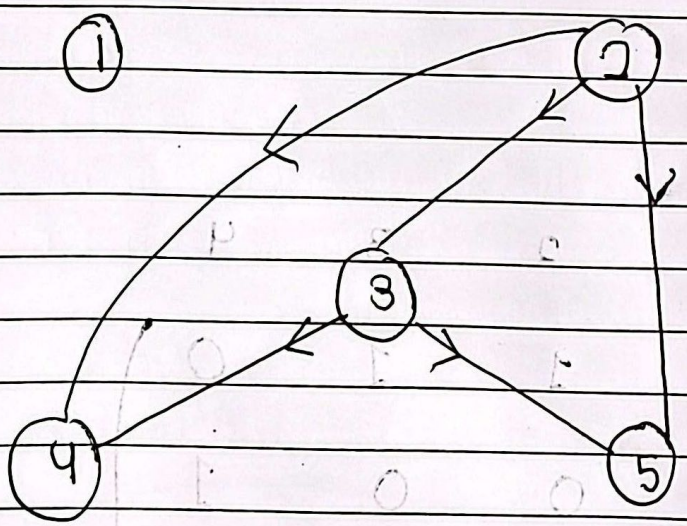
Q2 $A = \{1,2,3,4,5\}$

$R = \{(x,y) : x < y \text{ and } x \text{ is prime}\}$

$A \times A = \{(1,1),(1,2),(1,3),(1,4),(1,5)$
$(2,1),(2,2),(2,3),(2,4),(2,5)$
$(3,1),(3,2),(3,3),(3,4),(3,5)$
$(4,1),(4,2),(4,3),(4,4),(4,5)$
$(5,1),(5,2),(5,3),(5,4),(5,5)\}$ $\quad 25 = n$

a) $R = \{(3,4),(3,5),(2,3),(2,4),(2,5),\}$

b. Matrix




c) Not reflexive since for every $a \in A$, $(a,a)$ doesn't exist
Not symmetric since $a R b \not\rightarrow bRa$
Transitive since $aRb$ and $bRc \rightarrow aRc$
~~and~~ Not antisymmetric since not even one loop/reflexive
Asymmetric since $aRb \not\rightarrow bRa$ and no loop
Irreflexive since for $a \in A$, no $(a,a)$ exists

Q3 $A = \{1,2,3\}$

- $R = \{(1,1), (1,2), (1,3), (3,3)\}$

not Reflexive since missing (2,2)
not Symmetric since for (1,2), (2,1) doesn't exist
transitive since aRb, bRa → aRc relation
(1,3), (3,3) → (1,3)
(1,1), (1,2) → (1,2)
not equivalence since not reflexive and not symmetric
Anti Symmetric since contains loop,
aRb ↛ bRa.
Not asymmetric since contains loop (1,1), (3,3)
not irreflexive since (1,1) loop exists

- $S = \{(1,1), (1,2), (2,1), (2,2), (3,3)\}$

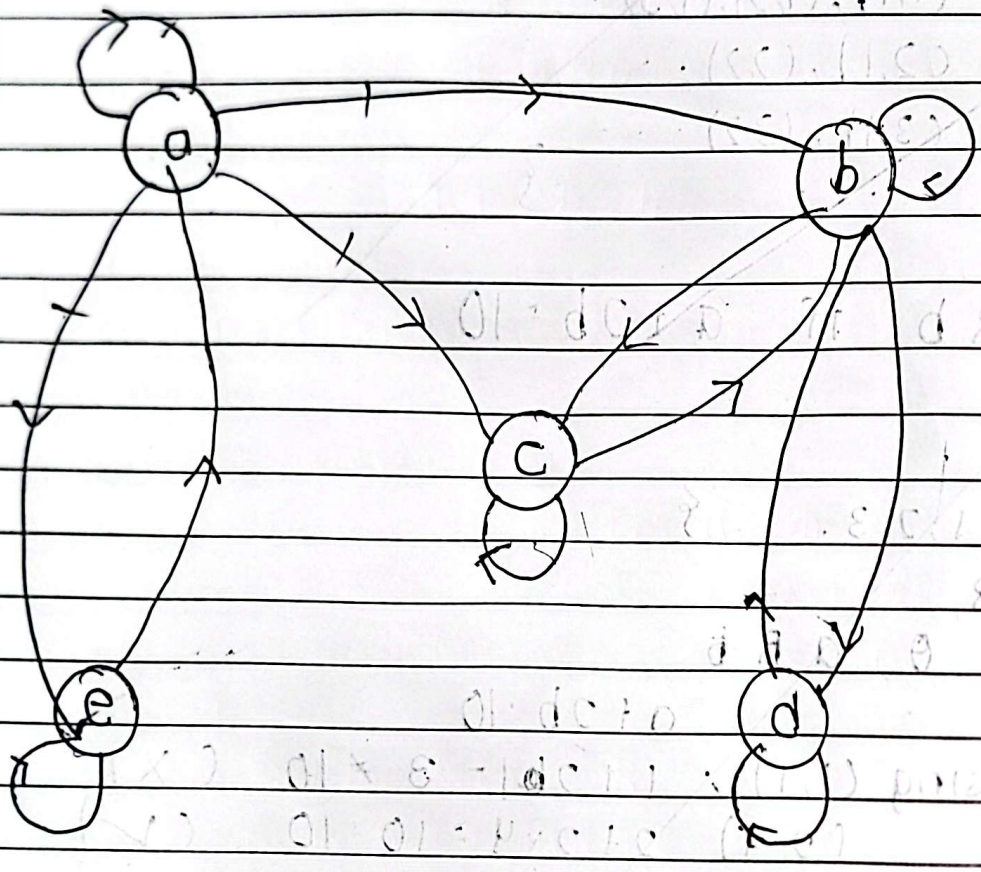
Reflexive sine contains loop for all element [a∈A, (a,a)]
Symmetric since aRb → bRa (1,2) → (2,1)
transitive since aRb ↛ bRc → aRc (1,2), (2,1) → (1,1)
(2,1) ↛ (1,2) → (2,2)
(2,2) ↛ (2,1) → (2,1)
Equivalence since it is Reflexive
Symmetric
transitive
Not Anti-symmetric since aRb ↛ bRa
Not Asymmetric since contains loop
not irreflexive since Relation R, a∈A, (a,a) exists.

$\{(4,5),(5,5)\}$
a b b r

• $I = \{(1,1),(1,2),(2,2),(2,3)\}$

Not Reflexive since R doesn't contain $(a,a)$ for $a \in A$
not symmetric since I doesn't contain $aRb \rightarrow bRa$
where $a \in A, b \in A$
Not transitive since I doesn't contain $aRb, bRC \rightarrow aRC$
where $a \neq b, b \neq C$
$(1,2)(2,3) \not\rightarrow (1,3)$
Not equivalence since Relation is not reflexive
not symmetric
not transitive
Anti-symmetric since Relation contains $(a,a)$ where
$a \in A$ and doesn't contain $aRb \rightarrow bRa$
$(1,2) \not\rightarrow (2,1)$
Not anti-symmetric since Relation contains loop $(a,a)$
where $a \in A$
Not Irreflexive since Relation contains loop $(a,a)$ where
$a \in A$.

Q5 $A = \{a, b, c, d, e\}$




$R = \{(a,a), (a,b), (a,c), (a,e), (b,b), (b,c), (b,d), (c,b), (c,c), (d,b), (d,d), (e,a), (e,e)\}$

i) Reflexive

Since Relation contains $(a,a)$ for every $a \in A$

ii) Symmetric

Not symmetric since Relation doesn't contain $aRb \rightarrow bRa$

$(a,b) \nrightarrow (b,a)$

Not

iii) antisymmetric

since Relation contains loop and

$aRb \Rightarrow bRa$ $(a,e) - (e,a)$

iv) Transitive

$(a,b), (b,c) \rightarrow (a,c)$
$(a,c), (c,b) \rightarrow (a,b)$
$(a,e), (e,a) \rightarrow (a,a)$
$(b,c), (c,b) \rightarrow (b,b)$
$(b,d), (d,b) \rightarrow (b,b)$
$(c,b), (b,c) \rightarrow (c,c)$
$(c,b), (b,d) \not\rightarrow (c,d)$

Since Relation R dosen't fulfill the condition,
when $(a,b)$ and $(b,c) \in R$
then, $(a,c) \in R$,,
It is not transitive

- Matrix

| | a | b | c | d | e |
|---|---|---|---|---|---|
| a | 1 | 1 | 1 | 0 | 1 |
| b | 0 | 1 | 1 | 1 | 0 |
| c | 0 | 1 | 1 | 0 | 0 |
| d | 0 | 1 | 0 | 1 | 0 |
| e | 1 | 0 | 0 | 1 | |

Q4

using condition

$a + 2b = 10$

where $aRb$

we get

$R = \{(8,1), (6,2), (4,3), (2,4)\}$

a) R is not reflexive

To be reflexive, R should contain $(a,a)$ for all $a \in A$

Since the current doesn't contain the loops.

It is not reflexive

b) R is not symmetric

To be symmetric, R should contain $(a,b)$ and $(b,a)$

$[(a,b) \in R$ and $(b,a) \in R]$

but the current R doesn't contain the required pair

so it is not symmetric. $((8,1) \not\to (1,8))$

c) R is not transitive

To be transitive, R should contain $(a,b)$ $(b,c)$ along with $(a,c)$

$[(a,b), (b,c) \in R$ and $(a,c) \in R]$

but the current R doesn't contain the required ordered pair

$((2,4), (4,3) \not\to (2,3))$