1. (a) $A = \{a, b, c, d\}$

$P(A) =$
$\{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}$
$\{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}$
$\{b, c, d\}, \{a, b, c, d\}\}$

2. i) $2^4 = 16$

ii) $2^4 = 16$

$|P(B)| = 2^5 = 32$

3. $n = 1, \quad |P(A)| = 2^1 = 2$
$n = 2, \quad |P(A)| = 2^2 = 4$
$n = 3, \quad |P(A)| = 2^3 = 8$
$n = 4. \quad |P(A)| = 2^4 = 16$
$n = 5 \quad |P(A)| = 2^5 = 32$

When, $A = \{x : x \in Z^+, x < 9\}$:
$n = 8, \quad |P(A)| = 2^8 = 256$

4) $P(\emptyset) = \{\emptyset\}$

$|P(A)| = 2^n$

$n = |\emptyset| = 0$

$2^0 = 1$

5) $A = \{a, b\}$

$P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

$P(P(A)) = \{\emptyset, \{\emptyset\}, \{\{a\}\}, \{\{b\}\}, \{\{a, b\}\}, \{\emptyset, \{a\}\},$

~~$\{\emptyset, \{b\}\}$~~

$\{\emptyset, \{b\}\}, \ldots \ldots \{\emptyset, \{a\}, \{b\}, \{a, b\}\}\}$

6) $A = \{1, 2, 3\}, \quad B = \{a, b\}$

Cartesian Product $\Rightarrow$

$A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$

$|A \times B| = 6$

7) $A \times \emptyset$

If A is set with n elements $A \times \emptyset = \emptyset$

$|A \times B| = n \times m$

$m = |\emptyset| = 0$

$= n \times 0$

$= 0$

8) $A = \{x : x = 2k, k \in N\}$

$k = \{0, 1, 2, 3 \ldots\}$

$x = 2k = \{0, 2, 4, 6 \ldots\}$

$A = \{0, 2, 4, 6 \ldots\}$

$B = \{x : x = 2k+1, k \in N\}$

$k = \{0, 1, 2, 3 \ldots\}$

$x = \{1, 3, 5, 7, \ldots\}$

$B = \{1, 3, 5, 7 \ldots\}$

A and B are disjoint

$A \cap B = \emptyset$.

$A \cup B = N$

$A \cup B = Z^+$

But.

$A \cup B \neq Z$ (~~not~~ There are no negative integers)

10) $A = \{1, 2, 3, 4, 5, 6, 7\}$

$A_1 = \{1, 2, 3\}$ $A_4 = \{5, 1, 4\}$

$A_2 = \{3, 6, 7\}$ $A_5 = \{5, 6\}$

$A_3 = \{4, 7\}$. $A_6 = \{2\}$

11) Which of the following are partitions of A?

$\{A_1, A_2, A_3\}$

$\{\{1, 2, 3\}, \{3, 6, 7\}, \{4, 7\}\}$

$\Rightarrow$ No (overlap)

(ii) $\{A_1, A_3, A_5\}$

$= \{\{1, 2, 3\}, \{4, 7\}, \{5, 6\}\}$

→ Yes (donot overlap, covers all elements)

(11) list all partitions of $A = \{1, 2, 3\}$

$\{\{1\}, \{2\}, \{3\}\}$

$\{\{1, 2\}, \{3\}\}$
$\{\{1, 3\}, \{2\}\}$
$\{\{2, 3\}, \{1\}\}$
$\{\{1, 2, 3\}\}$.