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Week 10 (Tutorial 1)

1. Let $A = \{x : x \in N, x \le 9\}$ and let R be defined on A by aRb if $a \equiv b \pmod 7$. Write down the relation in set listing notation.

$A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
R is defined by $a \equiv b \pmod 7$ means 7 divides $a-b$ with no remainder. Then,
$R = \{(0,0), (1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8), (9,9), (0,7), (7,0), (1,8), (8,1), (2,9), (9,2)\}$

2. Let $A = \{x : x \in Z^+, x < 8\}$ and let R be the "has the same parity" as relation on A. Write down R in set listing notation.

$A = \{1, 2, 3, 4, 5, 6, 7\}$

R is defined by "has the same parity" which means if first element is an odd, second must be an odd and same as even. Then,
$R = \{(1,3), (1,5), (1,7), (2,4), (2,2), (2,6), (3,1), (3,3), (3,5), (3,7), (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (5,7), (6,2), (6,4), (6,6), (7,1), (7,3), (7,5), (7,7), (1,1)\}$

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iii) $A = \{1, 2, 3, 4, 5\}$
$R = \{(1,1), (1,2), (1,3), (2,1), (2,2), (3,1), (3,3), (\ldots$
$(5,5)\}$

→

a. Reflexive
R is reflexive because for all $a \in A$ then $(a,a) \in$ R

b. Symmetric:
R is symmetric because for $(1,2) \in R$ and $(2,1)$ also belongs to R. and for $(1,3) \in R$ and $(3,1)$ also $\in$ R.

c. Transitive:
R is transitive because when
$(1,2)$ and $(2,1) \in R$ then $(1,1)$ also $\in R$
$(1,3)$ and $(3,1) \in R$ then $(1,1)$ also $\in R$.

→ R is equivalence relation because it is reflexive, symmetric, transitive.

4. which of the following statements are true?

i) $12 \equiv 75 \pmod 7$
True, because $12 - 75 = -63$ is divisible by 7.

ii) $6 \equiv 42 \pmod 6$
True, because $6 - 42 = -36$ is divisible by 6.

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3. In each of the following determine whether the relation R on the given set A is an equivalence relation.

i) $A = \{a, b, c, d\}$
$R = \{(a,a), (a,b), (b,a), (b,b), (c,a), (c,c), (a,c), (d,a), (d,d)\}$

→ R is not equivalence because it is not symmetric and transitive.

ii) $A = \{1, 2, 3, 4, 5\}$
$R = \{(1,1), (2,2), (2,3), (3,2), (3,3), (4,4), (5,5), (4,5), (5,4)\}$

→

a. Reflexive:
R is reflexive because for all $a \in A$ then (a,a) also belongs to R.

b. Symmetric:
R is symmetric because for all $(a,b) \in R$ and also $(b,a) \in R$.

c. Transitive:
R is transitive because when
$(2,3)$ and $(3,2) \in R$ then $(2,2)$ also $\in R$.
$(4,5)$ and $(5,4) \in R$ then $(4,4)$ also $\in R$.

→ R is equivalence relation because it is reflexive, symmetric and transitive.

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iii) $43 \equiv 97 \pmod 3$

True, because $43 - 97 = -54$ is divisible by 3

iv) $19 \equiv 14 \pmod 5$

False, because $19 + 14 = 33$ is not divisible by 5.

v) $-14 \equiv 31 \pmod 9$

True, because $-14 - 31 = -45$ is divisible by 9.

5. Determine whether each of the following is an equivalence relation on the set $A = \{x : x \in \mathbb{Z}^+, x < 9\}$.

$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

i) $R = \{(x, y) : x, y \in A,\ 3 \mid (x+y)\}$

$R = \{(1,2), (1,5), (1,8), (2,1), (2,4), (2,7), (3,3), (3$
$(4,2), (4,5), (4,8), (5,1), (5,4), (5,7), (6,3), (6$
$(7,2), (7,5), (7,8), (8,1), (8,4), (8,7)\}$

a. Reflexive:
R is not reflexive because for all $a \in A$ but $(a,a) \notin R$

b. Symmetric:
R is symmetric because for all $(a,b) \in R$ and (b
also $\in R$.

c. Transitive:

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R is not transitive (1,2) and (2,1) $\in$ R but (1,1) $\notin$ R.

→ So, R is not an equivalence relation because it not reflexive and transitive.

ii) $R = \{(x,y) : x, y \in A, (x-y)^2 \leq 1\}$

$R = \{(1,1), (1,2), (2,1)\ (2,2), (3,3), (3,2), (2,3), (3,$
$(4,4), (4,3), (4,5), (5,5), (5,4), (5,6), (6,6), ($
$(6,7), (7,7), (7,6), (7,8), (8,8), (8,7)\}$

→ R is ~~not~~ an equivalence relation because it is transitive, reflexive, symmetric.

iii) $R = \{(x,y) : x, y \in A, 3 \mid (x+2y)\}$

$R = \{(1,1), (1,4), (1,7), (2,2), (2,5), (2,8), (3,3), ($
$(4,1), (4,4), (4,7), (5,2), (5,5), (5,8), (6,3),$
$(6,6), (7,1), (7,4), (7,7), (8,2), (8,5), (8,8)\}$

→ R is an equivalence relation because it is reflexive, symmetric and transitive.

iv) $R = \{(x,y) : x, y \in A, x \mid y\}$

$R = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8)$
$(2,2), (2,4), (2,6), (2,8), (3,3), (3,6), (4,4),$
$\cancel{(4,2)}, (4,8), (5,5), (6,6), (7,7), (8,8)\}$

→ R is ~~a~~not an equivalence ~~reaction~~ relation because it is not symmetric.

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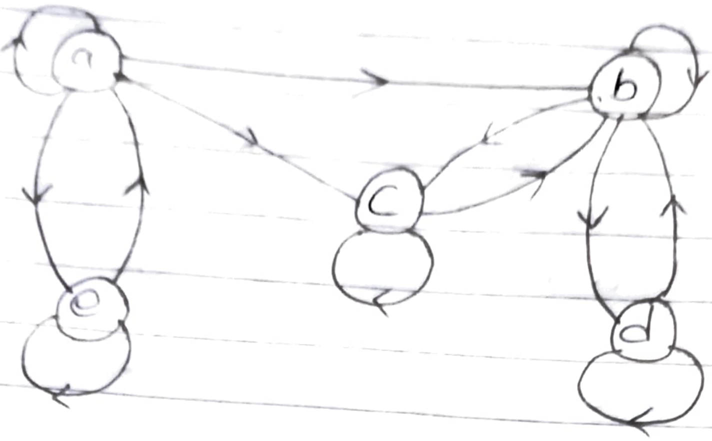
6. A relation R is defined on the set A = {a, b, c, d, e} by the digraph below:



Write down the matrix of the relation and say whether the relation is

~~(i) reflexive:~~

R = {(a,a), (a,b), (a,c), (a,e), (b,b), (b,c), (b,d), (c,c), ~~(c,a)~~, (c,b), (d,d), (d,b), (e,e), (e,a)}

$$M = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The relation is reflexive (each diagonal element is 1),
Not symmetric ((a,b) = 1 but (b,a) = 0),
not transitive ((a,b) = 1, (b,d) = 1 but (a,d) = 0)

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Not anti-symmetric ((b,c) = 1 and (c,b) = 1)).
R is not an equivalence relation because it is not symmetric and transitive.

7. A relation R is defined on the set A = {a,b,c,d,e} by R = {(a,a), (b,b), (c,c), (d,d), (e,e), (d,b), (e,d), (b,e), (d,e), (b,d), (e,b)}

Check that R is an equivalence relation and find

i) [b] = {b,d,e} (the set of second element of the relation whose first element is b)

ii) [a] = {a} and [e] = {e,d,b}
[a] ∪ [e] = {a,b,d,e}

R is equivalence because it is symmetric, reflexive, transitive.

8. Determine whether the relations on {a,b,c} defined by the following matrices are equivalence:

i) (100011011)

R = {(a,a), (b,b), (b,c), (c,b), (c,c)}

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

⇒ The relation defined by the matrix is reflexive (each diagonal element is 1), symmetric (each (x,y

$=(y,x)$ and transitive (for every $(x,y),(y,z)$ there is (x,z)). Hence the relation is an equivalence relation.

Equivalence class of $[a] = \{a\}$

Equivalence class of $[b] = \{b, c\}$

Equivalence class of $[c] = \{b, c\}$

iii) (1010 11101)

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$R = \{(a,a), (a,c), (b,b), (b,c), (c,a), (c,c)\}$

→ The relation defined by the matrix is reflexive (each diagonal element is 1), is not symmetr (each $(x,y) \neq (y,x)$) and not transitive (for every $(x,y), (y,z)$ there is not (x,z)). Hence the relation is not an equivalence relation.