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Church-Rosser property

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Let \rightarrow be a reduction (a binary relation) on a set S , and let \leftrightarrow^* be the reflexive transitive symmetric closure of \rightarrow . The reduction \rightarrow is said to have the *Church-Rosser property* provided that $a \leftrightarrow^* b$ implies that a and b are joinable, for any $a, b \in S$.

In terms of diagrams, the Church-Rosser property means the following, for any $a, b \in S$, if

$$a \leftrightarrow x_1 \leftrightarrow x_2 \leftrightarrow \cdots \leftrightarrow x_n \leftrightarrow b$$

where $u \leftrightarrow v$ means $u \rightarrow v$ or $u \leftarrow v$ ($:= v \rightarrow u$), then there is some $x \in S$ such that

$$a \rightarrow a_1 \cdots \rightarrow a_p \rightarrow x \leftarrow b_q \leftarrow \cdots \leftarrow b_1 \leftarrow b.$$

Remark. It can be shown that \rightarrow has the Church-Rosser property iff it is confluent.

References

- [1] F. Baader, T. Nipkow, *Term Rewriting and All That*, Cambridge University Press (1998).