



planetmath.org

Math for the people, by the people.

Chomsky-Schützenberger theorem

Canonical name	ChomskySchutzenbergerTheorem
Date of creation	2013-03-22 18:55:40
Last modified on	2013-03-22 18:55:40
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	5
Author	CWoo (3771)
Entry type	Theorem
Classification	msc 68Q42
Classification	msc 68Q45

An important characterization of context-free languages is captured in what is known as the Chomsky-Schützenberger theorem. It shows the intimate connection between context-free and parenthesis languages.

Theorem 1 (Chomsky-Schützenberger). *A language L over an alphabet Σ is context-free iff for some $n \geq 0$, there is a homomorphism $h : \Sigma_n^* \rightarrow \Sigma^*$ such that*

$$L = h(\mathbf{Paren}_n \cap R),$$

where \mathbf{Paren}_n is the parenthesis language over Σ_n , and R is a regular language (over Σ_n).

Note that the “if” part is the trivial consequence of the following facts: parenthesis languages are context-free; any homomorphic image of a context-free language is context-free; any intersection of a context-free language with a regular language is context-free.

References

- [1] N. Chomsky, M.P. Schützenberger, *The Algebraic Theory of Context-Free Languages, Computer Programming and Formal Systems*, North-Holland, Amsterdam (1963).
- [2] D. C. Kozen, *Automata and Computability*, Springer, New York (1997).