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polynomial hierarchy

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The *polynomial hierarchy* is a hierarchy of complexity classes generalizing the relationship between $\mathcal{P}$ and $\mathcal{NP}$.

We let $\Sigma_0^p = \Pi_0^p = \Delta_0^p = \mathcal{P}$, then $\Delta_{i+1}^p = \mathcal{P}^{\Sigma_i^p}$ and $\Sigma_{i+1}^p = \mathcal{NP}^{\Sigma_i^p}$. Define Π_i^p to be $co\Sigma_i^p$.

For instance $\Sigma_1^p = \mathcal{NP}^{\mathcal{P}} = \mathcal{NP}$.

The complexity class $\mathcal{PH} = \bigcup_{i \in \mathbb{N}} \Sigma_i^p$.

The polynomial hierarchy is closely related to the arithmetical hierarchy; indeed, an alternate definition is almost identical to the definition of the arithmetical hierarchy but stricter rules on what quantifiers can be used.

When there is no risk of confusion with the arithmetical hierarchy, the superscript p can be dropped.