alternative treatment of concatenation

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It is possible to define words and concatenation in terms of ordered sets. Let A be a set, which we shall call our alphabet. Define a *word* on A to be a map from a totally ordered set into A. (In order to have words in the usual sense, the ordered set should be finite but, as the definition presented here does not require this condition, we do not impose it.)

Suppose that we have totally ordered sets $(u, <)$ and $(v, \prec)$ and words $f \colon u \to A$ and $g \colon v \to A$. Let $u \coprod v$ denote the disjoint union of u and v and let $p \colon u \to u \coprod v$ and $q \colon u \to u \coprod v$ be the canonical maps. Then we may define an order $\ll$ on $u \coprod v$ as follows:

- If $x \in u$ and $y \in u$, then $p(x) \ll p(y)$ if and only if $x < y$.
- If $x \in u$ and $y \in v$, then $p(x) \ll q(y)$.
- If $x \in v$ and $y \in v$, then $q(x) \ll q(y)$ if and only if $x \prec y$.

We define the *concatenation* of f and g, which will be denoted $f \circ g$, to be map from $u \coprod v$ to A defined by the following conditions:

- If $x \in u$, then $(f \circ g)(p(x)) = f(x)$.
- If $y \in u$, then $(f \circ g)(q(x)) = g(x)$.