explicit form for currying

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In lambda calculus, we may express Currying functions and their inverses explicitly using lambda expressions. Suppose that f is a function of two arguments. Then, if c_2 is the currying function which maps of two arguments to higher order functions, we have, by definition,

$$f(x,y) = ((c_2(f))(x))(y).$$

We then have

$$c_2(f) = \lambda_v(\lambda_u f(u,v)),$$

hence

$$c_2 = \lambda_w(\lambda_v(\lambda_u w(u,v))).$$

Likewise, from the original equation, we see that

$$c_2^{-1} = \lambda_w(\lambda_{ab}(w(x))(y)).$$

We can write similar expressions for any number of arguments:

$$\begin{aligned}
c_3 &= \lambda_w(\lambda_c(\lambda_b(\lambda_a w(a,b,c)))) \\
c_4 &= \lambda_w(\lambda_d(\lambda_c(\lambda_b(\lambda_a w(a,b,c,d))))) \\
c_5 &= \lambda_w(\lambda_e(\lambda_d(\lambda_c(\lambda_b(\lambda_a w(a,b,c,d)))))),
\end{aligned}$$

etc.

Their inverses look as follows:

$$\begin{aligned}
c_3^{-1} &= \lambda_w(\lambda_{abc}((w(a))(b))(c)) \\
c_4^{-1} &= \lambda_w(\lambda_{abcd}(((w(a))(b))(c))(d)) \\
c_4^{-1} &= \lambda_w(\lambda_{abcde}((((w(a))(b))(c))(d))(e))
\end{aligned}$$