weak bisimulation

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Defines	silent step
Defines	weak simulation

Let $M = (S, \Sigma, \rightarrow)$ be a labelled state transition system (LTS). Recall that for each label $\alpha \in \Sigma$, there is an associated binary relation $\stackrel{\alpha}{\rightarrow}$ on S. Single out a label $\tau \in \Sigma$, and call it the *silent step*. Define the following relations:

1. Let $\Rightarrow$ be the reflexive and transitive closures of $\stackrel{\tau}{\rightarrow}$. In other words, $p \Rightarrow q$ iff either $p = q$, or there is a positive integer $n > 1$ and states $r_1, \ldots, r_n$ such that $p = r_1$ and $q = r_n$ and $r_i \stackrel{\tau}{\rightarrow} r_{i+1}$, where $i = 1, \ldots, n-1$.

2. Next, for any label α that is not the silent step τ in Σ, define

$$\stackrel{\alpha}{\Rightarrow} \;:=\; \stackrel{\alpha}{\rightarrow} \circ \Rightarrow \circ \stackrel{\alpha}{\rightarrow},$$

   where $\circ$ denotes the relational composition operation. In other words, $p \stackrel{\alpha}{\Rightarrow} q$ iff there are states r and s such that $p \stackrel{\alpha}{\rightarrow} r$, $r \Rightarrow s$, and $s \stackrel{\alpha}{\rightarrow} q$.

3. Finally, for any label $\alpha \in \Sigma$, let

$$\stackrel{(\alpha)}{\Rightarrow} \;:=\; \begin{cases} \Rightarrow & \text{if } \alpha = \tau \\ \stackrel{\alpha}{\Rightarrow} & \text{otherwise.} \end{cases}$$

Definition. Let $M = (S_1, \Sigma, \rightarrow_1)$ and $N = (S_2, \Sigma, \rightarrow_2)$ be two labelled state transition systems, with $\tau \in \Sigma$ the silent step. A relation $\approx \subseteq S_1 \times S_2$ is called a *weak simulation* if whenever $p \approx q$ and any labelled transition $p \stackrel{\alpha}{\rightarrow}_1 p'$, there is a state $q' \in S_2$ such that $p' \approx q'$ and $p' \stackrel{(\alpha)}{\Rightarrow}_2 q$. $\approx$ is a *weak bisimulation* if both $\approx$ and its converse $\approx^{-1}$ are weak simulations.