deterministic finite automaton

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A *deterministic finite automaton* (or DFA) is a deterministic automaton with a finite input alphabet and a finite number of states. It can be formally defined as a 5-tuple $(S, \Sigma, \delta, q_0, F)$, where

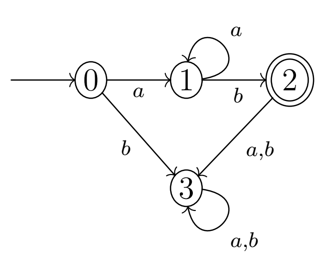
- S is a non-empty finite set of *states*,
- Σ is the alphabet (defining what set of input strings the automaton operates on),
- $\delta : S \times \Sigma \to S$ is the *transition function*,
- $q_0 \in S$ is the starting state, and
- $F \subseteq S$ is a set of final (or accepting states).

A DFA works exactly like a general automaton: operation begins at q_0, and movement from state to state is governed by the transition function δ. A word is accepted exactly when a final state is reached upon reading the last (rightmost) symbol of the word.

DFAs represent regular languages, and can be used to test whether any string in Σ^* is in the language it represents. Consider the following regular language over the alphabet $\Sigma := \{\mathtt{a}, \mathtt{b}\}$ (represented by the regular expression `aa*b`):

$$
\begin{aligned}
<S> \quad &\texttt{::=} \quad \mathtt{a}\,A \\
<A> \quad &\texttt{::=} \quad b \mid \mathtt{a}\,A
\end{aligned}
$$

This language can be represented by the DFA with the following state diagram:



The vertex 0 is the initial state q_0, and the vertex 2 is the only state in F. Note that for every vertex there is an edge leading away from it with a label for each symbol in Σ. This is a requirement of DFAs, which guarantees that operation is well-defined for any finite string.

If given the string `aaab` as input, operation of the DFA above is as follows. The first `a` is removed from the input string, so the edge from 0 to 1 is followed. The resulting input string is `aab`. For each of the next two `a`s, the edge is followed from 1 to itself. Finally, `b` is read from the input string and the edge from 1 to 2 is followed. Since the input string is now λ, the operation of the DFA halts. Since it has halted in the accepting state 2, the string `aaab` is accepted as a sentence in the regular language implemented by this DFA.

Now let us trace operation on the string `aaaba`. Execution is as above, until state 2 is reached with `a` remaining in the input string. The edge from 2 to 3 is then followed and the operation of the DFA halts. Since 3 is not an accepting state for this DFA, `aaaba` is *not* accepted.

Remarks.

- A DFA can be modified to include *ϵ-transitions*EpsilonTransitions. But the resulting DFA can be simulated by another DFA (without any epsilon transitions).

- Although the operation of a DFA is much easier to compute than that of a non-deterministic automaton, it is non-trivial to directly generate a DFA from a regular grammar. It is much easier to generate a non-deterministic finite automaton from the regular grammar, and then *transform* the non-deterministic finite automaton into a DFA.