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decision problem

Canonical name	DecisionProblem
Date of creation	2013-03-22 13:01:33
Last modified on	2013-03-22 13:01:33
Owner	Henry (455)
Last modified by	Henry (455)
Numerical id	12
Author	Henry (455)
Entry type	Definition
Classification	msc 68Q25
Defines	enumerates
Defines	decide

Let  $T$  be a Turing machine and let  $L \subseteq \Gamma^+$  be a language. We say  $T$  *decides*  $L$  if for any  $x \in L$ ,  $T$  accepts  $x$ , and for any  $x \notin L$ ,  $T$  rejects  $x$ .

We say  $T$  *enumerates*  $L$  if:

$$x \in L \text{ iff } T \text{ accepts } x$$

For some Turing machines (for instance non-deterministic machines) these definitions are equivalent, but for others they are not. For example, in order for a deterministic Turing machine  $T$  to decide  $L$ , it must be that  $T$  halts on every input. On the other hand  $T$  could enumerate  $L$  if it does not halt on some strings which are not in  $L$ .

$L$  is sometimes said to be a *decision problem*, and a Turing machine which decides it is said to solve the decision problem.

The set of strings which  $T$  accepts is denoted  $L(T)$ .