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linear erasing

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It is well-known that, among all of the language families in the Chomsky hierarchy, the family $\mathscr{S}$ of context-sensitive languages is the only one that is not closed under arbitrary homomorphisms. Nevertheless, $\mathscr{S}$ is shown to be closed under a more restricted class of homomorphisms, namely the λ-free homomorphisms. Question: can we enlarge this class of homomorphisms so that $\mathscr{S}$ is still closed under the larger class? The answer is yes.

Definition. Let L be a language over an alphabet Σ, h a homomorphism over Σ, and k a non-negative integer. h is said to be a *k-linear erasing* on L if for any word $u \in L$, we have

$$|u| \leq k|h(u)|,$$

where $|u|$ stands for the length of u.

It is clear that if h is a k-linear erasing on L, then it is a m-linear erasing for any $m \geq k$. Also, if h is a 0-linear erasing on L, then L is either $\{\lambda\}$, or the empty set $\varnothing$. In addition, if h is a k-linear erasing on L, and $L' \subseteq L$, then it is a k-linear erasing on L'. Consequently, any λ-free homomorphism is a k-linear erasing on any L over Σ, for any $k \geq 1$.

However, the notion of linear erasing is language dependent. For example, let $\Sigma = \{a, b, c\}$. Let $L_1 = \{a^n b^n \mid n \geq 0\}$ and $L_2 = \{a^n c^n \mid n \geq 0\}$. Suppose h is the homomorphism on Σ^* with $h(a) = \lambda$, $h(b) = b^2$ and $h(c) = c$. Then h is a 1-linear erasing on L_1, and a 2-linear erasing on L_2.

Definition Let $\mathscr{L}$ be a family of languages over Σ. Then $\mathscr{L}$ is said to be *closed under linear erasing* if for any $L \in \mathscr{L}$, and any homomorphism h which is a k-linear erasing on L for some $k \geq 0$, then $h(L) \in \mathscr{L}$.

Clearly, if $\mathscr{L}$ is closed under homomorphism, it is closed under linear erasing, and thus the families of `http://planetmath.org/RegularLanguage`regular, context-free, and type-0 languages are all closed under linear erasing. We also have the following:

Theorem 1. *The family $\mathscr{S}$ of context-sensitive languages is closed under linear erasing.*

Remark. The theorem above can be generalized. Call a substitution s over Σ a k-linear erasing on a language L if $|u| \leq k|v|$ for any $v \in s(u)$. If L is context-sensitive such that $s(u)$ is context-sensitive for each $u \in L$, then $s(L)$ is context-sensitive provided that s is a k-linear erasing on L.

References

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[2] J.E. Hopcroft, J.D. Ullman, *Formal Languages and Their Relation to Automata*, Addison-Wesley, (1969).