concept lattice

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Defines	object
Defines	attribute
Defines	context
Defines	concept
Defines	extent
Defines	intent

Let G and M be sets whose elements we call *objects* and *attributes* respectively. Let $I \subseteq G \times M$. We say that object $g \in G$ has attribute $m \in M$ iff $(g, m) \in I$. The triple (G, M, I) is called a *context*. For any set $X \subseteq G$ of objects, define

$$X' := \{m \in M \mid (x, m) \in I \text{ for all } x \in G\}.$$

In other words, X' is the set of all attributes that are common to all objects in X. Similarly, for any set $Y \subseteq M$ of attributes, set

$$Y' := \{g \in G \mid (g, y) \in I \text{ for all } y \in M\}.$$

In other words, Y' is the set of all objects having all the attributes in M. We call a pair $(X, Y) \subseteq G \times M$ a *concept* of the context (G, M, I) provided that

$$X' = Y \quad \text{and} \quad Y' = X.$$

If (X, Y) is a concept, then X is called the *extent* of the concept and Y the *intent* of the concept.

Given a context (G, M, I). Let $\mathbb{B}(G, M, I)$ be the set of all concepts of (G, M, I). Define a binary relation $\leq$ on $\mathbb{B}(G, M, I)$ by $(X_1, Y_1) \leq (X_2, Y_2)$ iff $X_1 \subseteq X_2$. Then $\leq$ makes $\mathbb{B}(G, M, I)$ a lattice, and in fact a complete lattice. $\mathbb{B}(G, M, I)$ together with $\leq$ is called the *concept latice* of the context (G, M, I).