pumping lemma (regular languages)

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Lemma 1. *Let L be a regular language (a.k.a. type 3 language). Then there exist an integer n such that, if the length of a word W is greater than n, then $W = ABC$ where A, B, C are subwords such that*

1. *The length of the subword B is less than n.*
2. *The subword B cannot be empty (although one of A or C might happen to be empty).*
3. *For all integers $k > 0$, it is the case that AB^kC belongs to L, where exponentiation denotes repetition of a subword k times.*

An important use of this lemma is to show that a language is not regular. (Remember, just because a language happens to be described in terms of an irregular grammar does not automatically preclude the possibility of describing the same language also by a regular grammar.) The idea is to assume that the language is regular, then arrive at a contradiction via this lemma.

An example of such a use of this lemma is afforded by the language

$$L = \{0^p 1^q 0^p \mid p, q > 0\}.$$

Let n be the number whose existence is guaranteed by the lemma. Now, consider the word $W = 0^{n+1}1^{n+1}0^{n+1}$. There must exist subwords A, B, C such that $W = ABC$ and B must be of length less than n. The only possibilities are the following

1. $A = 0^u, B = 0^v, C = 0^{n+1-u-v}1^{n+1}0^{n+1}$
2. $A = 0^{n+1-u}, B = 0^u 1^v, C = 1^{n+1-v}0^{n+1}$
3. $A = 0^{n+1}1^v, B = 1^u, C = 1^{n+1-u-v}0^{n+1}$
4. $A = 0^{n+1}1^{n+1-v}, B = 1^v 0^u, C = 0^{n+1-u}$
5. $A = 0^{n+1}1^{n+1}0^u, B = 0^v, C = 0^{n+1-u-v}$

In these cases, AB^2C would have the following form:

1. $AB^2C = 0^{n+1+v}1^{n+1}0^{n+1}$
2. $AB^2C = 0^{n+1}1^v 0^u 1^{n+1}0^{n+1}$
3. $AB^2C = 0^{n+1}1^{n+1+u}0^{n+1}$

4. $AB^2C = 0^{n+1}1^{n+1}0^u1^v0^{n+1}$

5. $AB^2C = 0^{n+1}1^{n+1}0^{n+1+u}$

It is easy to see that, in each of these five cases, $AB^2C \notin L$. Hence L cannot be a regular language.