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Cook reduction

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Defines	Karp reduction

Given two (search or decision) problems π_1 and π_2 and a complexity class $\mathcal{C}$, a $\mathcal{C}$ *Cook reduction* of π_1 to π_2 is a Turing machine appropriate for $\mathcal{C}$ which solves π_1 using π_2 as an oracle (the Cook reduction itself is not in $\mathcal{C}$, since it is a Turing machine, not a problem, but it should be the class of bounded Turing machines corresponding to $\mathcal{C}$). The most common type are $\mathcal{P}$ Cook reductions, which are often just called Cook reductions.

If a Cook reduction exists then π_2 is in some sense "at least as hard" as π_1, since a machine which solves π_2 could be used to construct one which solves π_1. When $\mathcal{C}$ is closed under appropriate operations, if $\pi_2 \in \mathcal{C}$ and π_1 is $\mathcal{C}$-Cook reducible to π_2 then $\pi_1 \in \mathcal{C}$.

A $\mathcal{C}$ *Karp reduction* is a special kind of $\mathcal{C}$ Cook reduction for decision problems L_1 and L_2. It is a function $g \in \mathcal{C}$ such that:

$$x \in L_1 \leftrightarrow g(x) \in L_2$$

Again, $\mathcal{P}$ Karp reductions are just called Karp reductions.

A Karp reduction provides a Cook reduction, since a Turing machine could decide L_1 by calculating $g(x)$ on any input and determining whether $g(x) \in L_2$. Note that it is a stronger condition than a Cook reduction. For instance, this machine requires only one use of the oracle.