Greibach normal form

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A formal grammar $G = (\Sigma, N, P, \sigma)$ is said to be in *Greibach normal form* if every production has the following form:

$$A \to aW$$

where $A \in N$ (a non-terminal symbol), $a \in \Sigma$ (a terminal symbol), and $W \in N^*$ (a word over N).

A formal grammar in Greibach normal form is a context-free grammar. Moreover, any context-free language not containing the empty word λ can be generated by a grammar in Greibach normal form. And if a context-free language L contains λ, then L can be generated by a grammar that is in Greibach normal form, with the addition of the production $\sigma \to \lambda$.

Let L be a context-free language not containing λ, and let $G = (\Sigma, N, P, \sigma)$ be a grammar in Greibach normal form generating L. We construct a PDA M from G based on the following specifications:

1. M has one state p,
2. the input alphabet of M is Σ,
3. the stack alphabet of M is N,
4. the initial stack symbol of M is the start symbol σ of G,
5. the start state of M is the only state of M, namely p
6. there are no final states,
7. the transition function T of M takes (p, a, A) to the singleton $\{(p, W)\}$, provided that $A \to aW$ is a production of G. Otherwise, $T(p, a, A) = \varnothing$.

It can be shown that $L = L(M)$, the language accepted on empty stack, by M. If we further define $T(p, \lambda, \sigma) := \{(p, \lambda)\}$, then M accepts $L \cup \{\lambda\}$. As a result, any context-free language is accepted by some PDA.

References

[1] J.E. Hopcroft, J.D. Ullman, *Formal Languages and Their Relation to Automata*, Addison-Wesley, (1969).