reversal

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Let Σ be an alphabet and w a word over Σ. The *reversal* of w is the word obtained from w by "spelling" it backwards. Formally, the *reversal* is defined as a function $\mathrm{rev} : \Sigma^* \to \Sigma^*$ such that, for any word $w = a_1 \cdots a_n$, where $a_i \in \Sigma$, $\mathrm{rev}(w) := a_n \cdots a_1$. Furthermore, $\mathrm{rev}(\lambda) := \lambda$. Oftentimes w^R or $\mathrm{mi}(w)$ is used to denote the reversal of w.

For example, if $\Sigma = \{a, b\}$, and $w = aababb$, then $\mathrm{rev}(w) = bbabaa$.

Two properties of the reversal are:

- it fixes all $a \in \Sigma$: $\mathrm{rev}(a) = a$.
- it is idempotent: $\mathrm{rev} \circ \mathrm{rev} = 1$, and
- it reverses concatenation: $\mathrm{rev}(ab) = \mathrm{rev}(b)\,\mathrm{rev}(a)$.

In other words, the reversal is an antihomomorphism. In fact, it is the antihomomorphism that fixes every element of Σ. Furthermore, g is an antihomomorphism iff $g \circ \mathrm{rev}$ is a homomorphism. By the second property above, h is a homomorphism iff $h \circ \mathrm{rev}$ is an antihomomorphism.

A word that is fixed by the reversal is called a palindrome. The empty word λ as well as any symbol in the alphabet Σ are trivially palindromes. Also, for any word w, the words $wx\,\mathrm{rev}(w)$ and $\mathrm{rev}(w)xw$ are both palindromes, where x is either a symbol in Σ or the empty word. In fact, every palindrome can be written this way.

The language consisting of all palindromes over an alphabet is context-free, and not regular if Σ has more than one symbol. It is not hard to see that the productions are $\sigma \to \lambda$, $\sigma \to a$ and $\sigma \to a\sigma a$, where a ranges over Σ.

Reversal of words can be extended to languages: let L be a language over Σ, then

$$\mathrm{rev}(L) := \{\mathrm{rev}(w) \mid w \in L\}.$$

A family $\mathscr{F}$ of languages is said to be *closed under reversal* if for any $L \in \mathscr{F}$, $\mathrm{rev}(L) \in \mathscr{F}$. It can be shown that regular languages, context-free languages, context-sensitive languages, and type-0 languages are all closed under reversal.