proof of Stirling's approximation

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Computing the Taylor expansion with remainder of the functions log and $x \mapsto x \log x - x$, we have

$$\begin{aligned}(n+1)\log(n+1) - \log(n+1) &= \qquad n\log n - n + \log n + \frac{1}{2n} + \frac{1}{6\xi_n^2} \\ \log(n+1) &= \qquad\qquad\qquad\qquad \log n + \frac{1}{n} - \frac{1}{2\eta_n^2}\end{aligned}$$

where $n \leq \xi_n \leq n+1$ and $n \leq \eta_n \leq n+1$. Summing the first equation from 1 to $n-1$, we have

$$n\log n - n = -1 + \log(n-1)! + \frac{1}{2}\sum_{m=1}^{n-1}\frac{1}{m} + \frac{1}{6}\sum_{m=1}^{n-1}\frac{1}{\xi_n^2}.$$