Horner's rule

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Horner's rule is a technique to reduce the work required for the computation of a polynomial at a particular value. Its simplest form makes use of the repeated factorizations

$$\begin{aligned} y &= a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n \\ &= a_0 + x(a_1 + x(a_2 + x(a_3 + \cdots + x a_n)) \cdots) \end{aligned}$$

of the terms of the nth degree polynomial in x in order to reduce the computation of the polynomial $y(a)$ (at some value $x = a$) to n multiplications and n additions.

The rule can be generalized to a finite series

$$y = a_0 p_0 + a_1 p_1 + \cdots + a_n p_n$$

of orthogonal polynomials $p_k = p_k(x)$. Using the recurrence relation

$$p_k = (A_k + B_k x) p_{k-1} + C_k p_{k-2}$$

for orthogonal polynomials, one obtains

$$y(a) = (a_0 + C_2 b_2) p_0(a) + b_1 p_1(a)$$

with

$$\begin{aligned} b_{n+1} &= b_{n+2} = 0, \\ b_{k-1} &= (A_k + B_k \cdot a) b_k + C_{k+1} + b_{k+1} + a_{k-1} \end{aligned}$$

for the evaluation of y at some particular a. This is a simpler calculation than the straightforward approach, since a_0 and C_2 are known, $p_0(a)$ and $p_1(a)$ are easy to compute (possibly themselves by Horner's rule), and b_1 and b_2 are given by a backwards-recurrence which is linear in n.

References

- Originally from The Data Analysis Briefbook (`http://rkb.home.cern.ch/rkb/titleA.html`