restricted homomorphism

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Let h be a homomorphism over an alphabet Σ. Let L be a language over Σ. We say that h is *k-restricted* on L if

1. there is a letter $b \in \mathrm{Alpha}(L)$ such that no word in L begins with b and contains more than $k-1$ consecutive occurrences of b in it,

2. for any $a \in \mathrm{Alpha}(L)$,

$$h(a) = \begin{cases} \lambda & \text{if } a = b \\ a & \text{otherwise.} \end{cases}$$

Here, $\mathrm{Alpha}(L)$ is the set of all letters in Σ that occur in words of L.

It is easy to see that any k-restricted homomorphism on L is a k-linear erasing on L, for if $u \in L$ is a non-empty word, then we may write $u = v_1 b^{m_1} v_2 b^{m_2} \cdots v_n b^{m_n}$, where each $0 < m_i \leq k-1$, and each v_i is a non-empty word not containing any occurrences of b. Then

$$|u| = |v_1 \cdots v_{n-1}| + \sum_{i=1}^{n} m_i \leq |h(u)| + n(k-1) \leq |h(u)| + (k-1)|h(u)| = k|h(u)|.$$

Note that $n \leq |h(u)|$ since $1 \leq |v_i|$ for each $i = 1, \ldots, n$. A k-linear erasing is in general not a k-restricted homomorphism, an example of which is the following: $L = \{a, ab\}^*$ and $h : \{a, b\} \to \{a, b\}$ given by $h(a) = a^2$ and $h(b) = \lambda$. Then h is a 1-linear erasing, but not a 1-restricted homomorphism, on L.

A family $\mathscr{F}$ of languages is said to be *closed under restricted homomorphism* if for every $L \in \mathscr{F}$, and every k-restricted homomorphism h on L, $h(L) \in \mathscr{F}$. By the previous paragraph, we see that if $\mathscr{F}$ is closed under linear erasing, it is closed under restricted homomorphism. The converse of this is not necessarily true.

References

[1] A. Salomaa, *Formal Languages*, Academic Press, New York (1973).