bisimilar automata

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Let $M = (S_M, \Sigma, \delta_M, I_M, F_M)$ and $N = (S_N, \Sigma, \delta_N, I_N, F_N)$ be two NDFA's (non-deterministic finite automata). Let $\approx \subseteq S_M \times S_N$ be a binary relation between the states of the automata M and N. We may extend $\approx$ to a binary relation between subsets of the states of the automata, as follows: for any $P \subseteq S_M$ and $Q \subseteq S_N$, set

$$C(P) := \{q \in S_N \mid p \approx q \text{ for some } p \in P\} \text{ and } C(Q) := \{p \in S_M \mid p \approx q \text{ for some } q \in Q\}.$$

Then, using the same notation, define $\approx \subseteq P(S_M) \times P(S_N)$ by

$$P \approx Q \text{ iff } P \subseteq C(Q) \text{ and } Q \subseteq C(P).$$

Definition. We say that M is *bisimilar* to N if there is a binary relation $\approx \subseteq S_M \times S_N$ such that

1. $I_M \approx I_N$,
2. if $p \approx q$, then $\delta_M(p, a) \approx \delta_N(q, a)$ for any $a \in \Sigma$,
3. if $p \approx q$, then $p \in F_M$ iff $q \in F_N$.

In other words, M is bisimilar to N as automata precisely when M is bisimilar to N as LTS, and satisfy conditions 1 and 3 above.

Any NDFA $M = (S, \Sigma, \delta, I, F)$ is bisimilar to itself, for the identity relation is clearly a bisimulation. Next, if M is bisimilar to N with bisimulation $\approx$, N is bisimilar to M with the converse relation $\approx^{-1}$. Finally, if M is bisimilar to N with bisimulation $\approx_1$ and N is bisimilar to P with bisimulation $\approx_2$, M is bisimilar to N with bisimulation $\approx_1 \circ \approx_2$. Therefore, bisimilarity is an equivalence relation on the class of NDFA's.

Another property of bisimulations on NDFA's is that the they are preserved by taking unions: an arbitrary non-empty union of bisimulations is again a bisimulation. From this property, it is not hard to show that if $A \approx B$, then $\delta(A, x) \approx \delta(B, x)$ for any word over Σ. As a result, bismilar NDFA's accept the same set of words.

By taking the union of all bisimulations on a given NDFA $M = (S, \Sigma, \delta, I, F)$, we get a bisimulation that is also an equivalence relation on the set of states of M. For each $p \in S$, let $[p]$ be the equivalence class containing p, and for any subset $A \subseteq S$, let $[A] := \{[p] \mid p \in A\}$. Then we get an NDFA $[M] := ([S], \Sigma, [\Delta], [I], [F])$, with

$$[\Delta]([p], a) := [\delta(p, a)]$$

for any $a \in \Sigma$. It can be shown that $[M]$ is minimal in the sense that $[[M]]$ is isomorphic to $[M]$, and that M is bisimilar to $[M]$. In addition, if M has no inaccessible states, then M is bisimilar to a unique minimal automaton, in the sense that, if N is any minimal automaton bisimlar to M, then N is isomorphic to $[M]$.