Post correspondence problem

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Let Σ be an alphabet. As usual, Σ^+ denotes the set of all non-empty words over Σ. Let $P \subset \Sigma^+ \times \Sigma^+$ be finite. Call a finite sequence

$$(u_1, v_1), \ldots, (u_n, v_n)$$

of pairs in P a *correspondence* in P if

$$u_1 \cdots u_n = v_1 \cdots v_n.$$

The word $u_1 \cdots u_n$ is called a *match* in P.

For example, if $\Sigma = \{a, b\}$, and $P = \{(b, b^2), (a^2, a), (b^2a, b^3), (ab^2, a^2b)\}$. Then

$$(a^2, a), (ab^2, a^2b), (b^2a, b^3), (ab^2, a^2b), (b, b^2)$$

is a correspondence in P.

On the other hand, there are no correspondences in $\{(ab, a), (a, ba^2)\}$.

The Post correspondence problem asks the following:

> Is there an algorithm (Turing machine or any other equivalent computing models) such that when an arbitrary P is given as an input, returns 1 if there exists a correspondence in P and 0 otherwise.

The problem is named after E. Post because he proved

Theorem 1. *The Post correspondence problem is unsolvable (no such algorithms exist).*