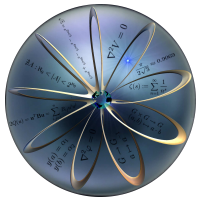


planetmath.org

Math for the people, by the people.

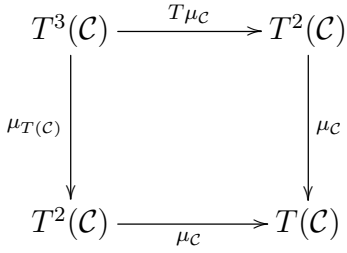
monad

Canonical name	Monad
Date of creation	2013-03-22 12:27:43
Last modified on	2013-03-22 12:27:43
Owner	mathcam (2727)
Last modified by	mathcam (2727)
Numerical id	11
Author	mathcam (2727)
Entry type	Definition
Classification	msc 68Q70
Classification	msc 18C15
Synonym	associative law of a monad
Synonym	left and right identity laws of a monad
Related topic	NaturalTransformation
Related topic	Functor
Related topic	Category
Related topic	FundamentalGroupoid2

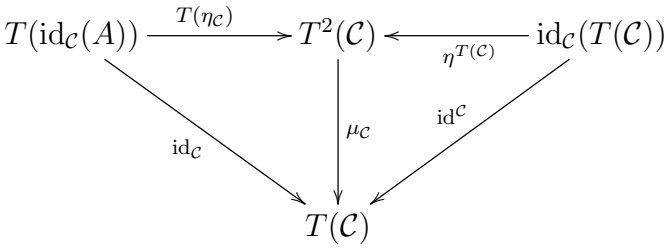
A *monad* over a category $\mathcal{C}$ is a triple (T, η, μ), where T is an endofunctor of $\mathcal{C}$, η is a natural transformation from the identity functor on $\mathcal{C}$, and μ is a natural transformations from $T \circ T$ to T, such that the following two properties hold:

- $\mu \circ (\mu \circ T) \equiv \mu \circ (T \circ \mu)$
- $\mu \circ (T \circ \eta) \equiv \mathrm{id}_{\mathcal{C}} \equiv \mu \circ (\eta \circ T)$

These laws are illustrated in the following diagrams.



$\mu \circ (\mu \circ T) \equiv \mu \circ (T \circ \mu)$



$\mu \circ (T \circ \eta) \equiv \mathrm{id}_{\mathcal{C}} \equiv \mu \circ (\eta \circ T)$

As an application, monads have been successfully applied in the field of functional programming. A pure functional program can have no side effects, but some computations are frequently much simpler with such behavior. Thus a mathematical model of computation such as a monad is needed. In this case, monads serve to represent state transformations, mutable variables, and interactions between a program and its environment. For further information in this regard, see http://www.nomaware.com/monads/html/.