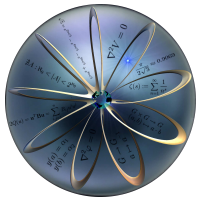

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natural numbers identified with binary strings

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It is convenient to identify a natural number n with the nth binary string in lexicographic order:

0	ϵ
1	0
2	1
3	00
4	01
5	10
6	11
7	000
...	...

The more common *binary notation* for numbers fails to be a bijection because of leading zeroes. Yet, there is a close relation: the nth binary string is the result of stripping the leading 1 from the binary notation of $n + 1$.

With this correspondence in place, we can talk about such things as the length $l(n)$ of a number n, which can be seen to equal $\lfloor \log(n + 1) \rfloor$.