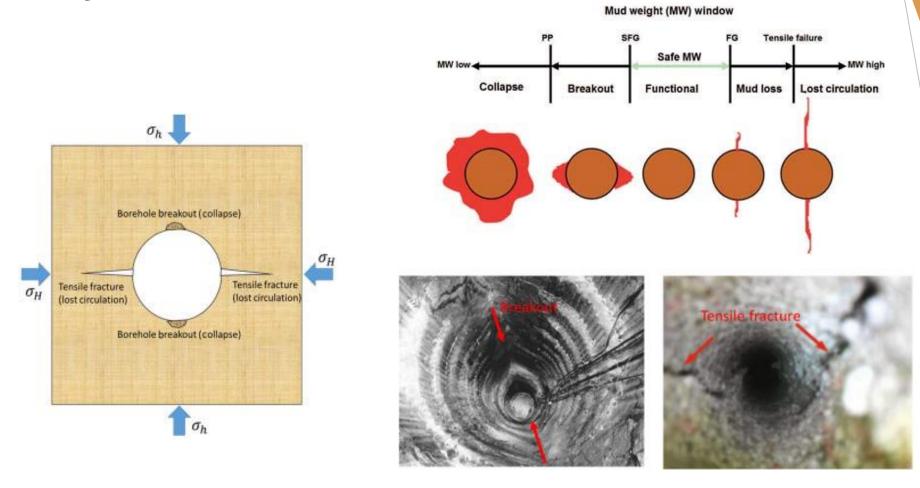
# Analysis of Wellbore Stability with consideration of Stress Anisotropy and Wellbore Deviation'

Python for Geoscience Research Final Project

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# Project Goals and Motivations



- Wellbore breakouts can lead to well collapse, while tensile fractures can bring important fluid losses
- Calculating the stress limits for both events becomes crucial for calculating the mud weight window during drilling operations
- Stress anisotropy along the wellbore must be accounted for to get accurate results

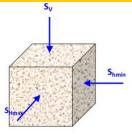
# ► Theory & Steps

### 1. Principal Stresses

$$S_v(z) = \int_0^z \rho_{bulk}(z)g \, dz$$

Assuming elasticity and isotropy of mechanical properties:

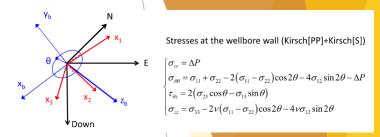
$$\begin{cases} \sigma_{Hmax} = \frac{\nu}{1 - \nu} \sigma_v + E' \varepsilon_{Hmax} + \nu E' \varepsilon_{hmin} \\ \sigma_{hmin} = \frac{\nu}{1 - \nu} \sigma_v + \nu E' \varepsilon_{Hmax} + E' \varepsilon_{hmin} \end{cases}$$



### Rock mechanical properties:

$$E_{dyn} = \rho V_s^2 \frac{3V_p^2 - 4V_s^2}{V_p^2 - V_s^2}$$
$$v_{dyn} = \frac{V_p^2 - 2V_s^2}{V_p^2 - 2V_s^2}$$

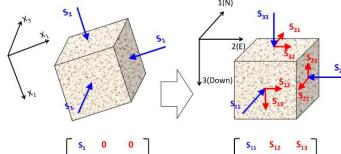
### 4. Kirsch Equations - Well Principal Stresses



### 2. Rotation of Principal Stresses into Geographical Coordinates

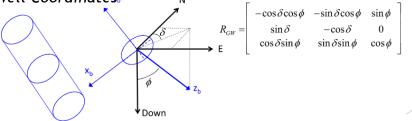
$$\mathbf{R}_{PG} = \begin{bmatrix} \cos\alpha\cos\beta & \sin\alpha\cos\beta & -\sin\beta \\ \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \cos\beta\sin\gamma \\ \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma & \cos\beta\cos\gamma \end{bmatrix}$$

$$\underline{\underline{S}}_{C} = R_{PG}^{T} \underline{\underline{S}}_{P} R_{PG}$$

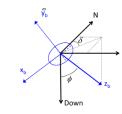


### 3. Rotation of Principal Stresses from Geo to Well Coordinates.

$$\underline{\underline{S}}_W = R_{GW}\underline{\underline{S}}_G R_{GW}^T$$



### 5. Principal Stresses around the deviated wellbore perimeter



Wellbore principal stresses calculated for all possible values of azimuth and inclination



 $UCS = \max(\max\_shear\ stress)$  $TS = \min(\min\_shear\ stress)$ 

Calculations - Input Datasets

The project was tested with the data from a deviated well (the information was provided by a professor from a different course):

- Well logs: includes density, sonic and interpreted Pore pressure
  - This curves come from a real well near the Gulf of Mexico (well name is confidential)
- Well Deviation survey: Table with azimuth, Easting, Northing and inclination for the well
  - This data was obtained from the same well, but was modified synthetically to increase the deviation in this well for the results to be more applicable to deviated wells

Well Deviation survey - .xlsx file

| Measured<br>Depth<br>(m)                       | Inclination<br>(°)                                  | Azimuth  | Vertical<br>Depth<br>(m)                       | +N/-S<br>(m)                              | +E/-W<br>(m)  |
|--|---|--|--|---|---|
| 0.00<br>66.73<br>95.62<br>108.44<br>136.81     | 0.000 0.360 0.330 0.310 0.670                       | 0.00 °<br>310.89 °<br>287.90 °<br>306.29 °<br>302.69 ° | 0.00 66.73 95.62 108.44 136.81                 | 0.00<br>0.14<br>0.22<br>0.25<br>0.39      | 0.00<br>-0.16<br>-0.31<br>-0.37<br>-0.57            |
| 154.70<br>172.91<br>201.62<br>220.14<br>238.17 | 0.800 F<br>1.060 F<br>1.670 F<br>1.810 F<br>1.950 F | 262.36   | 154.70<br>172.90<br>201.61<br>220.12<br>238.14 | 0.43<br>0.37<br>0.26<br>0.14<br>-0.03     | -0.78 -1.07 -1.75 -2.30 -2.86 -                     |
| 257.13<br>285.98<br>314.75<br>343.33<br>371.60 | 1.400 F<br>0.850 F<br>0.680 F<br>0.320 F<br>0.530 F | 254.74   | 257.09 285.93 314.70 343.28 371.55             | -0.18<br>-0.42<br>-0.58<br>-0.58<br>-0.45 | -3.39 -<br>-3.90 -<br>-4.23 -<br>-4.47 -<br>-4.47 - |

Well logs (Density, Sonic) - .LAS file

```
Neutron Porosity
                        Photo-Electric Factor
                        Shallow Resistivity
                                Resistivity
                        Spontaneous
        213.0: CSG
                                ELEVATION
                        SOURCE
                SERVICE
114.34880
                 -999.2500
                                                                 5.2109
                                  -999.250000
                                                                          -999,2500
                                                                                                           -999,2500
114.50126
                 -999.2500
                                 -999.250000
                                                                  5.2422
                                                                          -999.2500
                                                                                                           -999.2500
114,65366
                 -999,2500
                                 -999,250000
                                                 -999,2500
                                                                  5.8164 -999.2500
                                                                                                           -999.2500
                                                                  5.2539 -999.2500
```

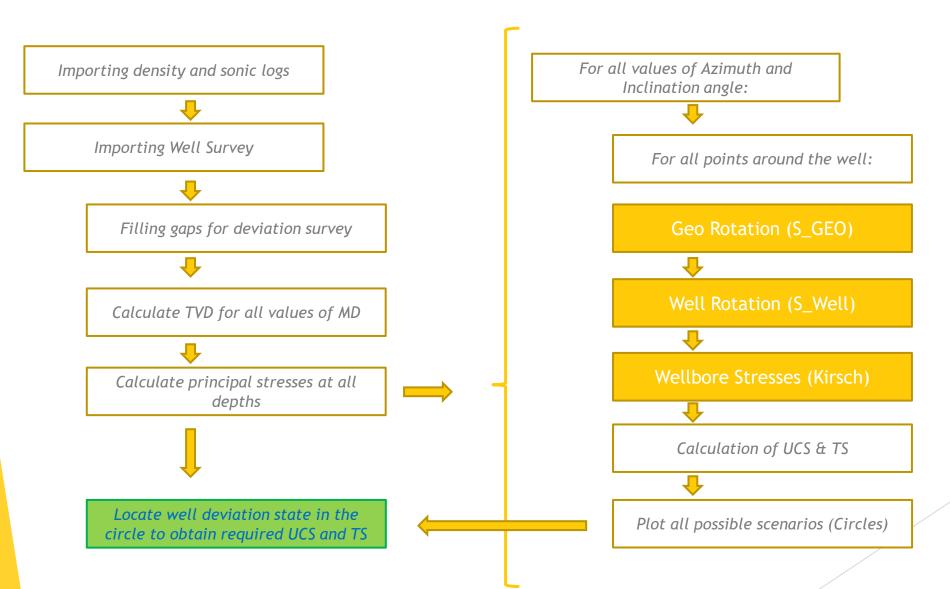
# Calculations - Input Datasets

### Some important definitions:

- **Azimuth**: the direction of an object/vector with respect to the North
- Inclination: the degree of slope with respect to the vertical
- **Density log**: a well logging tool that can provide a continuous record of a formation's bulk density along the length of a borehole
- **Sonic log**: an acoustic log that emits sound waves which start at the source, travel through the formation, and return back to the well
- Young Modulus: the slope of the linear part of the stress-strain curve for a material under tension or compression; it tells how easily a material can stretch and deform
- Poisson Ratio: the amount of transversal elongation divided by the amount of axial compression; it shows how the cross-section of a deformable body changes under lengthwise stretching (or compression)
- **Principal Stresses**: Set of 3 applied normal stresses in the principal planes, in a system defined where shear stresses are equal to zero

# Methods - Workflow

### Workflow



# **Python Libraries:**

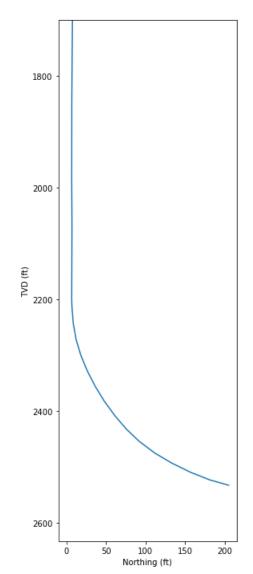
- Pandas
- Numpy
- Scipy
- Matplotlib
- Math
- Welly: Useful to read .las files and convert into Pandas Dataframe



# **INPUTS**

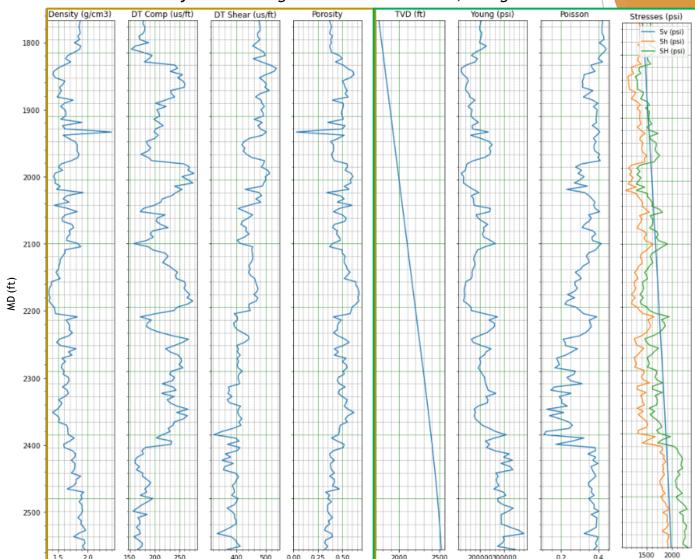
### **OUTPUTS**

### Well Deviation Survey



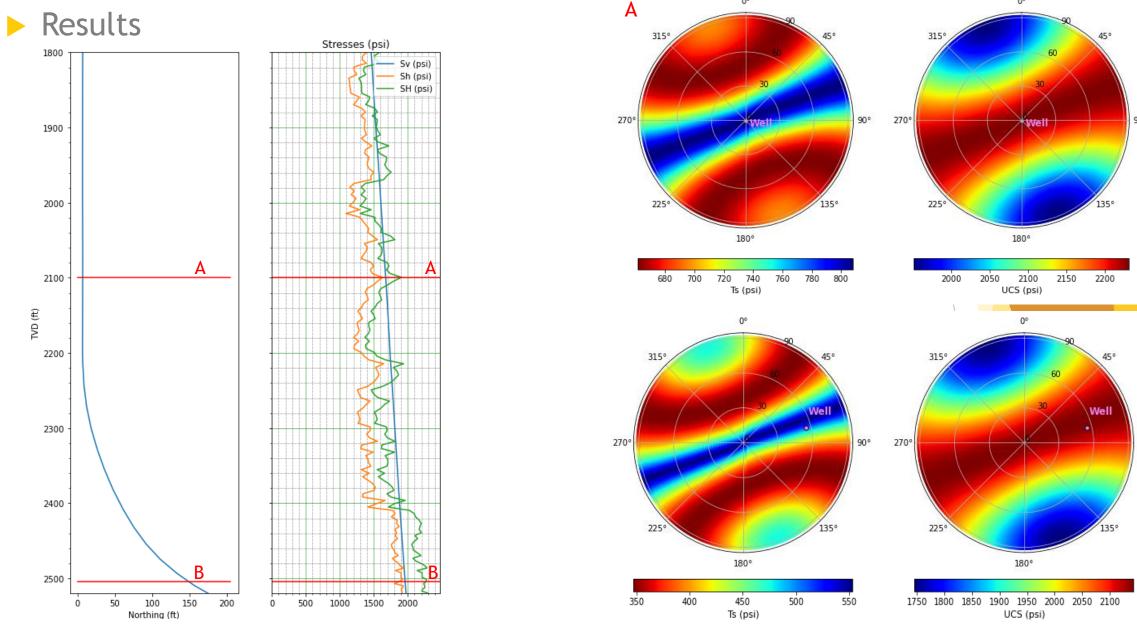
### Density and sonic logs

### Poisson Ratio, Young Modulus and Total Stresses



Stresses
are
calculated
at all
depths

- The figure on the left is a vertical cross section of the well trajectory (y axis is true vertical depth)
- The figures on the right are the well logs and the calculated properties and stresses at each depth (y axis is total well length, not vertical depth)



- In this case both well trajectory and logs are displayed in true vertical depth
- The point represents well current deviation compared to all possible states of deviation
- The colors are a reference for UCS and TS requirements at current deviation and depth

# **Conclusions**

- The deviation of the wellbore has a strong impact on the stresses around the well
- Depending on the depth, deviation of the wellbore and stress regime, different values for mud pressure are required to avoid tensile fractures and breakouts
- For different deviation angles, the necessary unconfined compressive strength and the tensile strength of the well can become sensitively higher
- It becomes critical to account for these differences in order to avoid tensile fractures and breakouts in the wellbore

# **Future Work**

- A possibility for future work is considering the effect of temperature on stresses, to assess how much wellbore stability calculations are affected by mud temperature
- Another interesting approach would be addressing how overpressure affects these results by comparing a well in an overpressured formation with a normal pressure reservoir

Appendix - Code

# **Python Libraries:**

- Pandas
- Numpy
- Scipy
- Welly
- Matplotlib
- Math

### Function for GEO rotation of S matrix

### Function for WELL rotation of S\_GEO matrix

```
#I define a function for rotation of the stress tensor in geographical coordinates to wellbore coordinates
def Well_rot(delta, phi, STens): #phi is the angle with respect to the vertical axis, delta is the azimuth
   delta = delta*2*math.pi/360
   phi = phi*2*math.pi/360
   RB = np.array([[-np.cos(delta)*np.cos(phi),
                                                          -np.sin(delta)*np.cos(phi).
                                                                                              np.sin(phi)],
                                                                  -np.cos(delta) ,
          [np.sin(delta),
           [np.cos(delta)*np.sin(phi) ,
                                                                  np.sin(delta)*np.sin(phi)
                                                                                                     np.cos(phi)]])
   # Transnose RR
    RB_T = RB.transpose()
   # Calculate Stress Tensor in Wellbore Coordinate System
   # SWell = RB * STens * RB_T
   SWell = np.round(np.dot(RB,np.dot(STens,RB_T)),2)
   return SWell
```

### Function for calculating Kirsch stresses around the wellbore

```
def kirsch(tens, theta, PW, PP, Poisson):
    theta = theta*2*math.pi/360
    sigma11 = tens(0)[0] - PP
    sigma12 = tens(0)[1]
    sigma13 = tens(0)[1]
    sigma13 = tens(0)[1]
    sigma23 = tens(0)[1]
    sigma33 = tens(0)[1]
    sigma31 = sigma31 = sigma20 = 2*(sigma11 = sigma22)*np.cos(2*theta) = 4*Poisson*sigma12*np.sin(2*theta)
    sigma_tt = sigma11 + sigma20 = 2*(sigma11 = sigma22)*np.cos(2*theta) = 4*Sigma12*np.sin(2*theta) = tau_to_0 = tau
```

# Function that calls previous functions and calculates UCS and TS for all possible scenarios of well deviation

```
# Tensile Fractures Likelihood
def Tensile(Alpha, Beta, Gamma, S1, S2, S3, PW, PP, Poisson, inc = 0, azi = 0):
    s_tensor = np.array([[S1, 0 , 0], [0, S2, 0], [0, 0, S3]] )
     geo_s = Geo_rot(Alpha, Beta, Gamma, s_tensor) #principal stresses to geographical coordinates
     phi - np.linspace(0, 90, 90) #well dip array
delta - np.linspace(0,180, 180) #well azimuth array
     angle - np.linspace(0,180, 180) #well border angle
    OCS = [] #OCS array

T0 = [] # Tensite strength array

Mohr_w = [] # width by Mohr criterium

DP_w = [] # width by Drucker Prager criterium

ML_w = [] #width by Modified Lade criterium
    x = [] #x coordinate for plotting
     y = [] #y coordinate for plotting
deltar = []
     phir = []
     for p in phi:
          for d in delta
               x.append(p*math.sin(d*2*math.pi/360)) #cos and sin are inverted because of the plot
                deltar.append(d*2*math.pi/360)
                phir.append(p)
               well_s - Well_rot(d, p, geo_s) #principal stress tensor to well coordinates
              max_s_list = []
min_s_list = []
               width MC - A
               width ML = 0
                for ang in angle:
                    max s, min s = kirsch(well_s, ang, PW, PP, Poisson) #calculate min and max stress for each point in the well max s, list.append(max s)
                UCS n = max(max s list)
                UCS.append(UCS_n)
T0.append(T0_n)
                Mohr_w.append(width_MC/2)
                DP_w.append(width_DP/2)
ML_w.append(width_ML/2)
      # Duplicating the different arrays to get the results for the full circle
    UCS += UCS.copy()
T0 += T0.copy()
Mohr_w += Mohr_w.copy()
DP_w += DP_w.copy()
ML_w += ML_w.copy()
     x= np.array(x)
x_m = x*(-1)
      x = np.append(x, x_m)
     y= np.array(y)
     y_m = y*(-1)
y = np.append(y, y_m)
      deltar = np.array(deltar)
      deltar_m = deltar+np.pi
      deltar = np.append(deltar, deltar_m
      phir = np.array(phir)
     phir_m = phir
phir = np.append(phir, phir_m )
```

# Code inside this function that plots UCS and TS for all possible scenarios - 'Polar' subplot was used

```
fig, ax - plt.subplots(ncols - 2, subplot_kw-{'projection': 'polar'}, figsize - (10, 8))
  fig.tight layout(pad-3.0)
  color map = plt.cm.get cmap('jet')
 reversed color map = color map.reversed()
plot_T0 = ax[0].scatter(deltar, phir, c-T0, cmap-reversed_color_map)
                                                          'deltar), np.min(deltar), np.min(phir), np.max(phir)), origin-'lower')
  cb_0 = plt.colorbar(plot_T0, ax-ax[0], orientation = 'horizontal')
  ax[0].set_rticks([0, 30, 60, 90]) # Less radial ticks
# ax.set rlabel position(-22.5) # Move radial labels away from plotted line
 ax[0].set_theta_zero_location('N')
ax[0].set_theta_direction(-1)
  cb_0.set_label('Ts (psi)', fontsize=11)
  #Tendency for breakouts considering a constant UCS criterium
  plot_UCS - ax[1].scatter(deltar, phir, c-UCS , cmap-'jet')
                                               ent=(np.min(deltar).np.min(deltar).np.min(phir).np.max(phir)), origin='lower')
  cb_1 = plt.colorbar(plot_UCS, ax = ax[1], orientation = 'horizontal')
ax[1].set_rticks([0, 30, 60, 90]) # Less radial ticks
# ax.set_rlabel_position(-22.5) # Move radial labels away from plotted line
 ax[1].set_theta_zero_location('N')
ax[1].set_theta_direction(-1)
  ax[1].grid(True)
  cb_1.set_label('UCS (psi)', fontsize=11)
      ax[0].scatter(azi, inc, c-'violet', s-20)
       ax[0].annotate('Well', (azi, inc+10), color = 'violet', size=12)
ax[1].scatter(azi, inc, c='violet', s=20)
ax[1].annotate('Well', (azi, inc+10), color='violet', size=12)
```



### .LAS transformation into Pandas Dataframe - Welly Library

#### Import las file

0 0.0 0.0000000000 0.0000000000

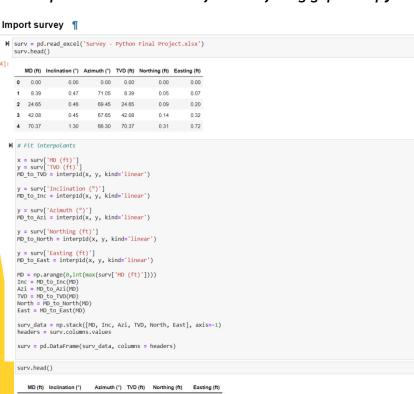
2 2.0 0.1120381406 16.9368295590

3 3.0 0.1680572110 25.4052443385

4 4.0 0.2240762813 33.8736591180



### .xlsx import into Pandas Dataframe & filling gaps - scipy Library



0.0 0.0000000000 0.00000000000

2.0 0.0119189511 0.0166865316

3.0 0.0178784267 0.0250297974

4.0 0.0238379023 0.0333730632

### Calculus of Mechanic Properties and Stresses from logs

#### Process logs

```
#I calculate TVD based on the survey

data['TVD (ft)'] = np.round(MD_to_TVD(data['DEPTH (ft)'].values),2)

data.head()

DEPTH (ft) Res Press (ps) Density (gicm3) DT Comp (us/ft) DT Shear (us/ft) Porosity TVD (ft)

0 1750 0 700 187 177.0 477.0 037 1749.38
```

| 1750.0 | 700.0 | 1.87 | 177.0 | 477.0 | 0.37 | 1749.38 |
|--------|-------|------|-------|-------|------|---------|
| 1755.0 | 702.0 | 1.86 | 176.0 | 479.0 | 0.38 | 1754.37 |
| 1760.0 | 704.0 | 1.85 | 180.0 | 481.0 | 0.39 | 1759.37 |
| 1765.0 | 706.0 | 1.86 | 180.0 | 482.0 | 0.38 | 1764.37 |
| 1770.0 | 708.0 | 1.87 | 177.0 | 473.0 | 0.37 | 1769.37 |
|        |       |      |       |       |      |         |

# Density is in g/cm3
# Depth is in ft
#Sv is in psi

```
### and the proof of the proof
```

#Assuming Total Isotropy

#I calculate dynamic Young modulus (in psi, for which I set dens in kg/cm3, which gives Pa,
# and then I convert Pa into Mpa, then into psi) and Poisson's rotio
Ed = np.round(dens\*(1800) \* V\*\*\*2 (\*2\*\*y\*\*2) (\*4\*\*\*2)/(Vp\*\*2-Vs\*\*2)/(166)\*145.038,0) #psi
P\_d = np.round((Vp\*\*2 - 2\*Vs\*\*2)/(2\*(Vp\*\*2-Vs\*\*2))/(3\*)

#I calculate Static Young Modulus and Poisson' rotio
E\_S = 0.63\*E\_d
P\_S = P\_d

data[ Young (psi)'] = E\_S

data[ Young (psi)'] = E\_S

PP = data['Res Press (psi)'].values
Blot = 0.9

#Compute total maximum and minimum horizontal stress assuming theory of elasticity

TectStrain\_H = 0

TectStrain\_H = 0.0015

$$\label{eq:control_energy} \begin{split} & \text{TectStrain}_H = 0.0015 \\ & \text{Sh} = P_S/(1.P_S)^n(\text{Sv} - \text{Biot}^n\text{PP}) + \text{E}_S/(1.P_S^{n*2})^n(\text{TectStrain}_H + P_S^{n*TectStrain}_H) + \text{Biot}^n\text{PP} \\ & \text{Sh} = P_S/(1.P_S)^n(\text{Sv} - \text{Biot}^n\text{PP}) + \text{E}_S/(1.P_S^{n*2})^n(\text{TectStrain}_H + P_S^{n*TectStrain}_H) + \text{Biot}^n\text{PP} \\ & \text{data}[Sv (psi)^n] = \text{Sw} \\ & \text{data}[Sv (psi)^n] = \text{data}[Sv (psi)^n]. \text{cumsum}() \\ & \text{data}[Sh (psi)^n] = \text{Sh} \\ & \text{data}[Sh (psi)^n] = \text{Sh} \end{split}$$

### Plotting the logs

```
Depth = 2100
# Initialise the subplot function using number of rows and columns
#figure, axis = plt.subplots(2, 2)
logs = data.columns.values
cols = data.shape[1]
fig,ax = plt.subplots(ncols=cols-3, figsize= (20,15), sharey=True)
for i in range(1, cols-3):
    ax[i-1].plot(data.iloc[:,i].values,data.iloc[:,0].values)
    ax[i-1].set ylim(max(data.iloc[:,0]),min(data.iloc[:,0]))
    ax[i-1].minorticks_on()
    ax[i-1].grid(which='major',linestyle='-',linewidth='0.5',color='green')
    ax[i-1].grid(which='minor',linestyle=':',linewidth='0.5',color='black')
    ax[i-1].set_title('%s' %logs[i])
    ax[i-1].plot([max(data.iloc[:,i]), min(data.iloc[:,i])], [Depth, Depth], color = 'red')
ax[cols-4].plot(data.iloc[:,cols-3].values,data.iloc[:,0].values, label = ('%s' %logs[cols-3]) )
ax[cols-4].plot(data.iloc[:,cols-2].values,data.iloc[:,0].values, label = ('%s' %logs[cols-2]))
ax[cols-4].plot(data.iloc[:,cols-1].values,data.iloc[:,0].values, label = ('%s' %logs[cols-1]))
ax[cols-4].set_ylim(max(data.iloc[:,0]),min(data.iloc[:,0]))
ax[cols-4].minorticks_on()
ax[cols-4].grid(which='major',linestyle='-',linewidth='0.5',color='green')
ax[cols-4].grid(which='minor',linestyle=':',linewidth='0.5',color='black')
ax[cols-4].set_title('Stresses (psi)')
ax[cols-4].legend(loc - True)
#ax[cols-4].plot([max(data.iloc[:,cols-3]),0],[Depth, Depth], color = 'red')
```

### Calling previous functions from a different phyton notebook

```
# Do a definitions-only import
import ipynb.fs # Boilerplate required
from .defs.Geomechanics_functions import Mohr
from .defs.Geomechanics_functions import Geo_rot
from .defs.Geomechanics_functions import Well rot
from .defs.Geomechanics_functions import Well rot
from .defs.Geomechanics_functions import Winsch
from .defs.Geomechanics_functions import Tensile
```

# Defining principal stresses directions and inputting into the circles function

```
#Normal Regime
Alpha =160
Beta = 90
Gamma = 0
#Strike-slip Regime
#Alpha =70
#Beta = 0
#Gamma = 96
#Reverse Reaime
 #Alpha =70
#Beta = 0
#Gamma = 0
#Stresses
Sh = round(float(data[data['DEPTH (ft)']==Depth]['Sh (psi)'].values),2) #MPa
SH = round(float(data[data['DEPTH (ft)']==Depth]['SH (psi)'].values),2) #MPa
Sv = round(float(data[data['DEPTH (ft)']==Depth]['Sv (psi)'].values),2) #MPa
Stresses = [Sh. SH. Sv]
S1 = max(Stresses)
S3 = min(Stresses)
S2 = Stresses[Stresses not in [S1, S3]]
PP = round(float(data[data['DEPTH (ft)']--Depth]['Res Press (psi)'].values),2) #psi
PW = PP #psi mud pressure
Poisson = 0.3
inc = surv['Inclination (°)'][Depth]
azi = surv['Azimuth (°)'][Depth]
s_tensor = np.array([[S1, 0 , 0], [0, S2, 0], [0, 0, S3]] )
Tensile(Alpha, Beta, Gamma, S1, S2, S3, PW, PP, Poisson, inc, azi)
```