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The probability distribution of the sum/average of three random variables is the linear convolution of the individual probability distribution of these three random variables.

Regardless of the shape of the individual distributions, please perform simply a "chain" of linear convolutions to illustrate the Gaussianity claim made by the central limit theorem.

Random variables : $(Y = X_1 + X_2 + \ldots + X_n)$

using Y_1, Y_2, and Y_3 as our RV's

$$Y_1 + Y_2 + Y_3 = Z f_{Y_1}(y) * Z f_{Y_2}(y) * Z f_{Y_3}(y)$$

$$= Z\left(f_{Y_1}(y) * f_{Y_2}(y) * f_{Y_3}(y)\right)$$

$$= Z\left[\left(\underbrace{\int_{-\infty}^{\infty} f_{Y_1}(y) f_{Y_2}(x-y)\,dy}_{\cancel{f_A(y)}}\right) * f_{Y_3}(y)\right]$$

$$\cancel{= Z\left[\int_{-\infty}^{\infty} f_A(y) f_{Y_3}(x-y)\,dy\right]}$$

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(z-y) f_Y(y)\,dy$$

$$= z\exp(-z)$$

$$= Z\left[x\exp(-x) * f_{Y_3}(y)\right]$$

$$= Z\left[\int_{-\infty}^{\infty} x\exp(-x) f_{Y_3}(x-y)\,dy\right]$$

$$= Z\left[x^2\exp(-x)\right]$$

$$= Z\left[x^2 e^{-x}\right]$$

Central Limit Thm

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$z = \frac{\bar{x} - \mu_x}{\sigma_{\bar{x}}}$$

$$= \frac{\bar{x} - \mu}{\frac{S_n}{\sqrt{n}}}$$