ECE5554 – Computer Vision
Lecture 8b – Active Contours

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Today’s Objectives

- Active Contours (gray-scale images)
  - Concept
  - Internal and external energy
  - Optimization
  - Uses

also called Deformable Models or “Snakes”

ACTIVE CONTOURS

[Kass, Witkin, & Terzopoulos, “Snakes: Active contour models,” ICCV 1987]

Active contours

- Goal: Fit a curve to an object’s boundary (of arbitrary shape) in an image
- The image need not be binary!
- Given: An initial curve (model) near the desired object
- Procedure: Iteratively modify the curve, based on
  1) pixel values in the image, and
  2) the curve’s shape

Active contours

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Slide credit: Boykov

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Main idea: An elastic band is iteratively adjusted so as to

- be near image locations of interest (usually with high gradient magnitudes), and
- satisfy shape "preferences", such as to prefer low curvature

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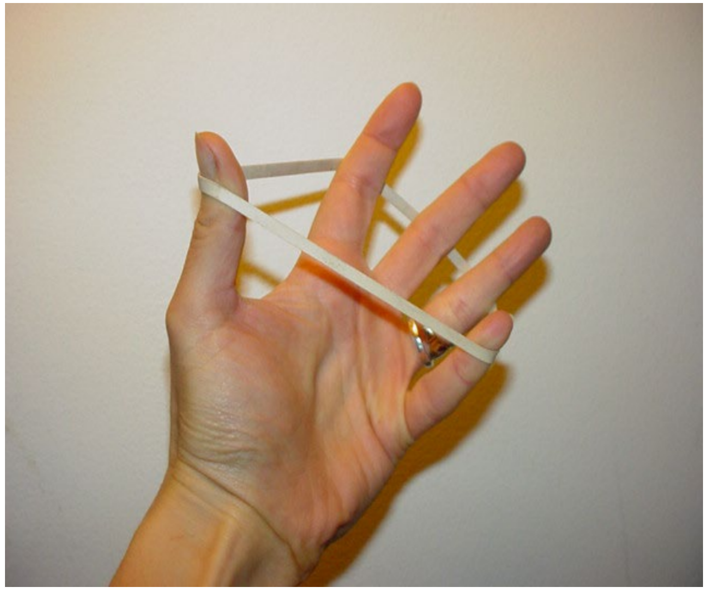
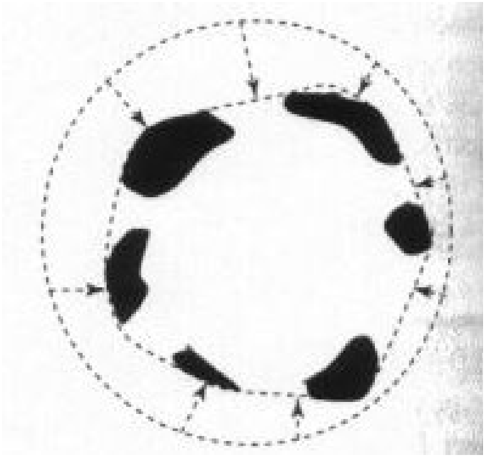
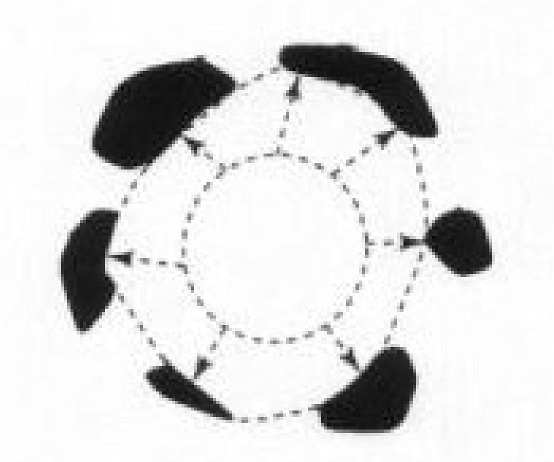
Active contours: intuition







Slide credit: Grauman

Another example

Image from: https://raw.github.com/pmneila/morphsnakes/master/examples/starfish.gif

Tracking heart ventricles
(multiple frames)

Why do we want to fit deformable shapes? Non-rigid, deformable objects can change their shapes over time

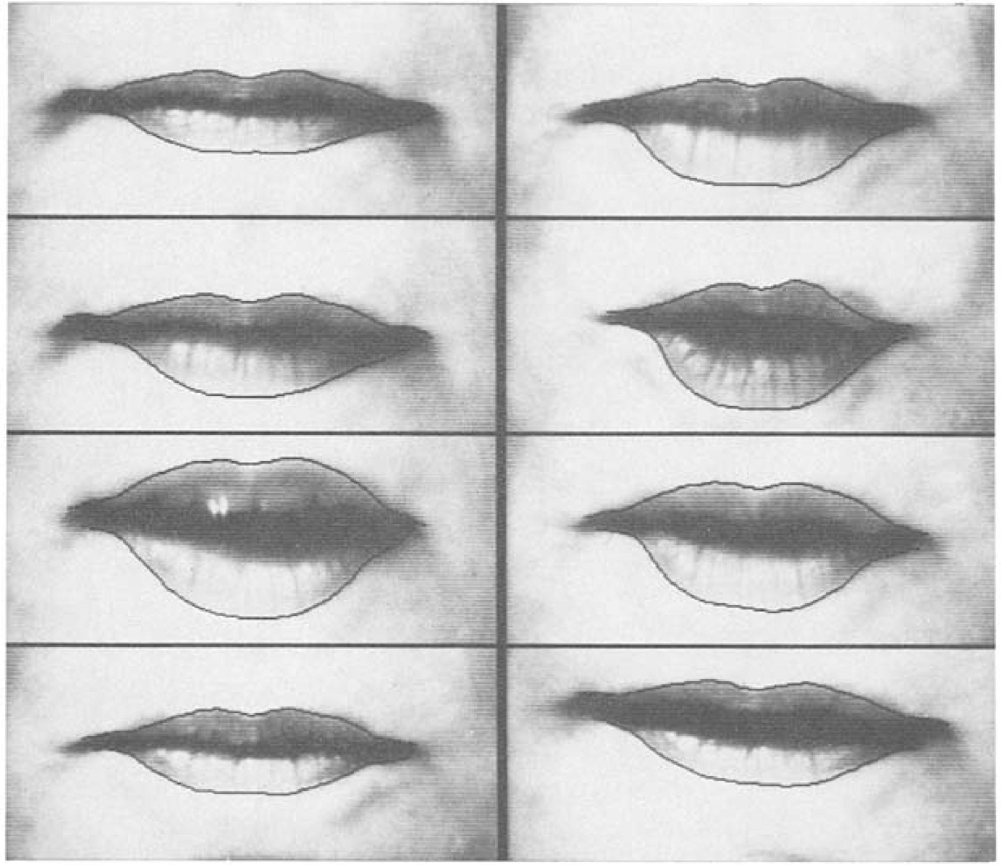
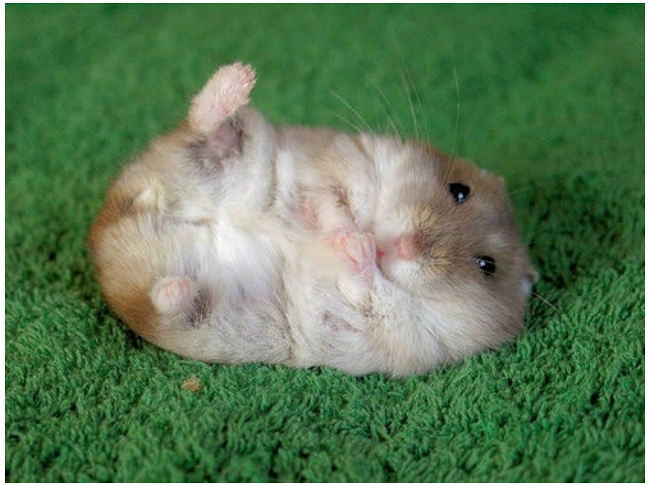


Figure from Kass et al. 1987



Slide credit: Grauman

Why do we want to fit deformable shapes?

- Non-rigid, deformable objects can change their shapes over time.

Figure credit: Jomier

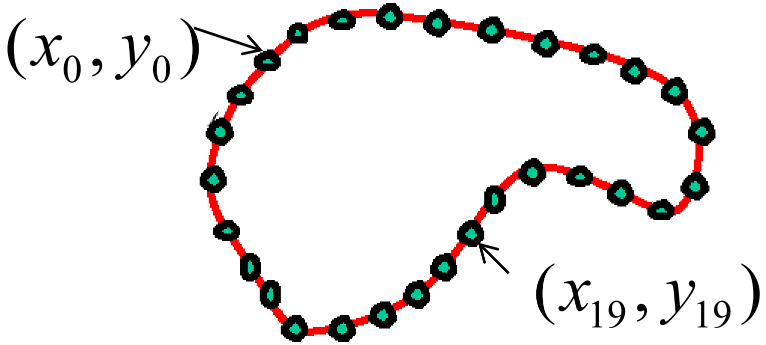
Aspects we need to consider

- Representation of the contours
- Formulation as a problem of energy minimization
- Defining the energy functions
  - External
  - Internal
- Minimizing the energy function
- Extensions:
  - Tracking
  - Interactive segmentation

Slide credit: Grauman

Representation

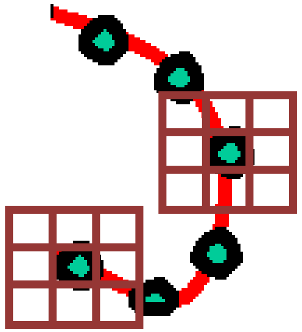
- Consider a discrete representation of the contour, consisting of a list of 2D point positions ("vertices")



$$v_i = (x_i, y_i),$$

for $\quad i = 0, 1, \ldots, n-1$

- At each iteration, we'll have the option to move each vertex to another nearby location ("state")

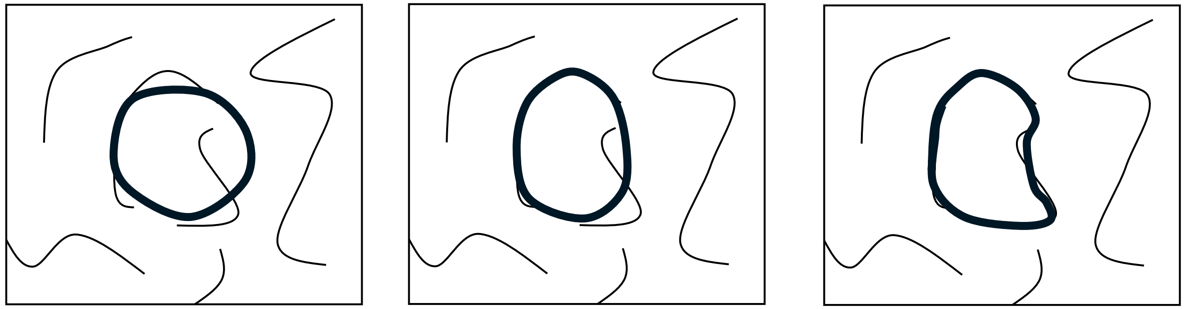


Slide credit: Grauman

Fitting deformable contours

How should we adjust the current contour to form the new contour at each iteration?

- Define a cost function (“energy” function) that says how good a candidate configuration is
- “Greedy” search: Select the next configuration that gives the best improvement in cost

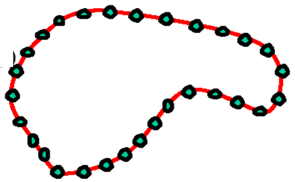


initial intermediate final

Slide credit: Grauman

Energy function

The total energy (cost) of the current snake is often defined as:



$$E_{total} = E_{internal} + E_{external}$$

Internal energy: encourage shape preferences:
e.g., smoothness, elasticity, similarity to a particular known shape

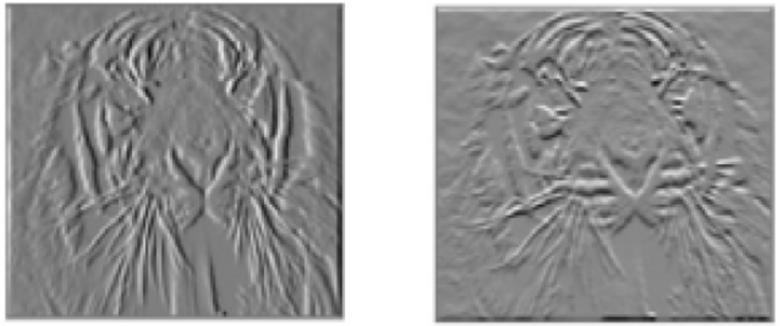
External energy ("image" energy): encourage contour to fit on places where image structures exist (e.g., edges)

The idea is to find a balance between
our prior preferences (such as low curvature) and
what is actually observed (such as image edges)

Slide credit: Grauman

External image energy

- Gradient images $G_x(x,y)$ and $G_y(x,y)$



- External energy at a point on the curve is:

$$E_{external}(v) = -(\,|G_x(v)|^2 + |G_y(v)|^2\,)$$

Slide credit: Grauman

External image energy

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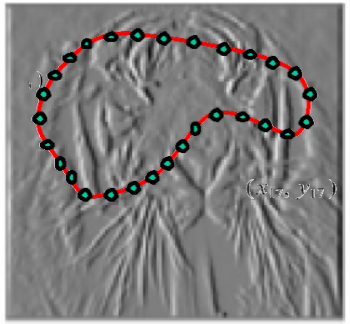
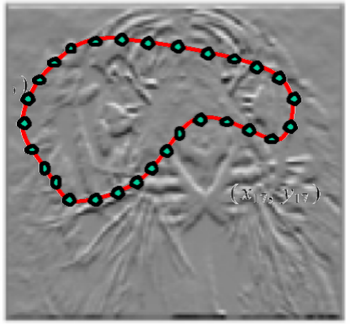




- External energy at a point on the curve is:

$$E_{external}(\nu) = -(\,|\,G_x(\nu)\,|^2 + |\,G_y(\nu)\,|^2\,)$$

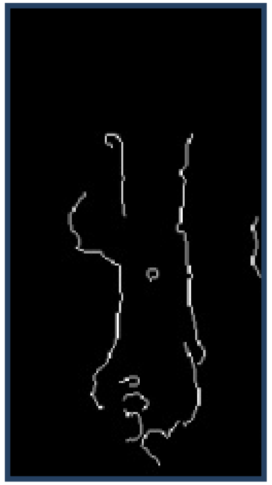
- External energy for the whole curve:

$$E_{external} = -\sum_{i=0}^{n-1} |\,G_x(x_i, y_i)\,|^2 + |\,G_y(x_i, y_i)\,|^2$$

Slide credit: Grauman

Internal energy: intuition

What underlying shape should be detected here?



Slide credit: Grauman

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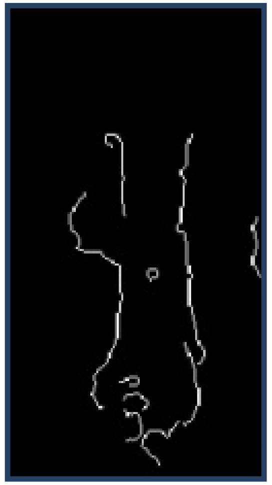

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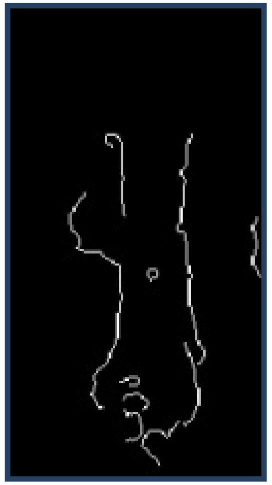


Slide credit: Grauman

Internal energy

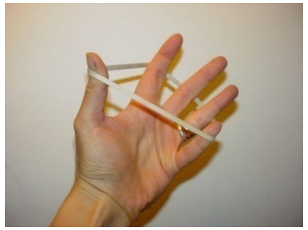
For a *continuous* curve, a common internal energy term is the "bending energy".

At some point $v(s)$ on the curve, this is:

$$E_{internal}(v(s)) = \boxed{\alpha \left| \frac{dv}{ds} \right|^2} + \boxed{\beta \left| \frac{d^2 v}{d^2 s} \right|^2}$$

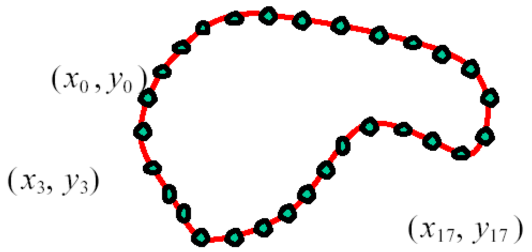
Tension, Elasticity

Stiffness, Curvature



Slide credit: Grauman

Internal energy



- For our discrete representation,

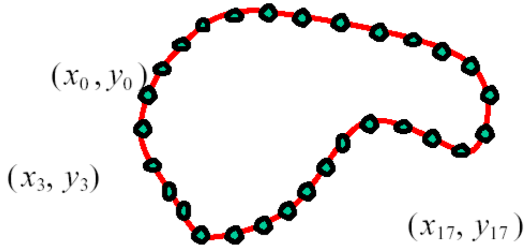
$$v_i = (x_i, y_i) \qquad i = 0 \ \dots, \ n-1$$

$$\frac{dv}{ds} \approx v_{i+1} - v_i \qquad \frac{d^2v}{ds^2} \approx (v_{i+1} - v_i) - (v_i - v_{i-1}) = v_{i+1} - 2v_i + v_{i-1}$$

Note that these are derivatives relative to position on the curve --- not gradients of image intensity

Slide credit: Grauman

Internal energy



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- Internal energy for the whole curve:

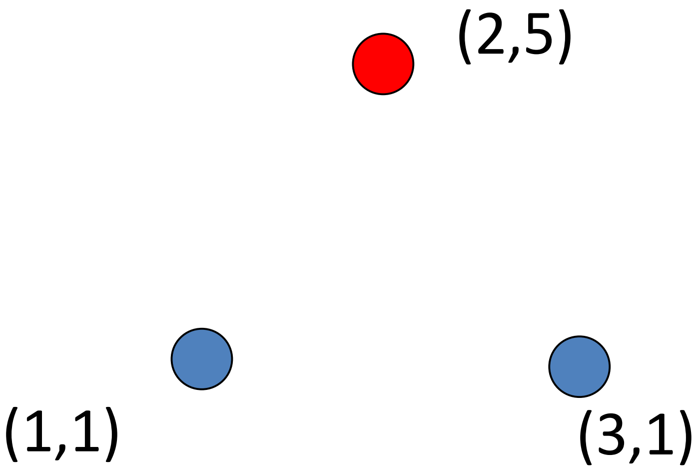
$$E_{internal} = \sum_{i=0}^{n-1} \alpha \left\| v_{i+1} - v_i \right\|^2 + \beta \left\| v_{i+1} - 2v_i + v_{i-1} \right\|^2$$

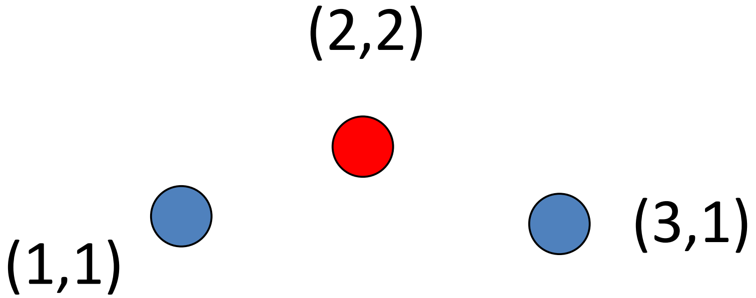
Why do these reflect tension and curvature?

Slide credit: Grauman

Example: compare curvature

$$E_{curvature}(v_i) = \left\| v_{i+1} - 2v_i + v_{i-1} \right\|^2$$
$$= (x_{i+1} - 2x_i + x_{i-1})^2 + (y_{i+1} - 2y_i + y_{i-1})^2$$





$$(3 - 2(2) + 1)^2 + (1 - 2(5) + 1)^2$$
$$= (-8)^2 = 64$$

$$(3 - 2(2) + 1)^2 + (1 - 2(2) + 1)^2$$
$$= (-2)^2 = 4$$

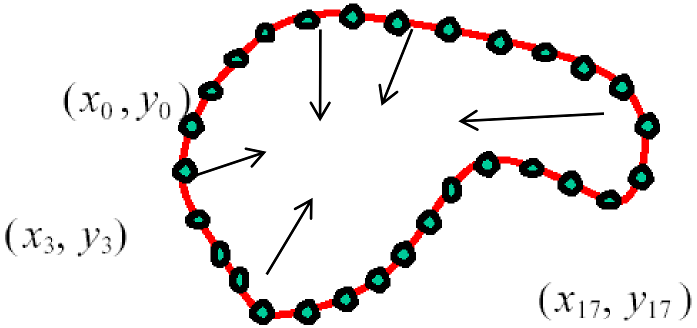
Slide credit: Grauman

Penalizing elasticity

- Current elastic energy definition uses a discrete estimate of the derivative:

$$E_{elastic} = \sum_{i=0}^{n-1} \alpha \left\| v_{i+1} - v_i \right\|^2$$

$$= \alpha \cdot \sum_{i=0}^{n-1} (x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2$$



What is the possible problem with this definition?

Slide credit: Grauman

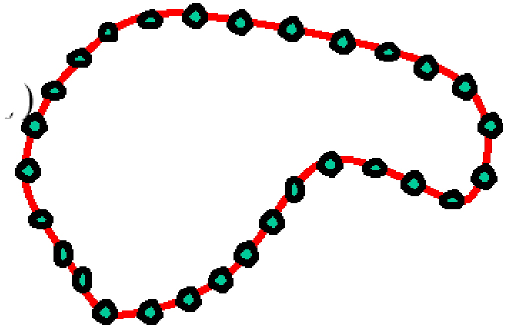
Penalizing elasticity

- Current elastic energy definition uses a discrete estimate of the derivative:

$$E_{elastic} = \cancel{\sum_{i=0}^{n-1} \alpha \left\| v_{i+1} - v_i \right\|^2}$$

Instead:

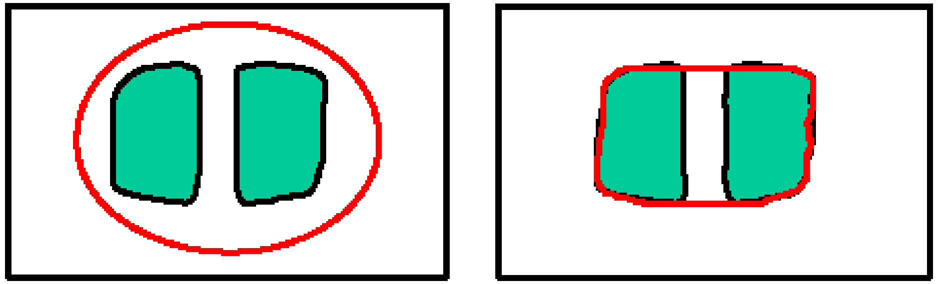
$$= \alpha \cdot \sum_{i=0}^{n-1} \left((x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 - \overline{d} \right)^2$$

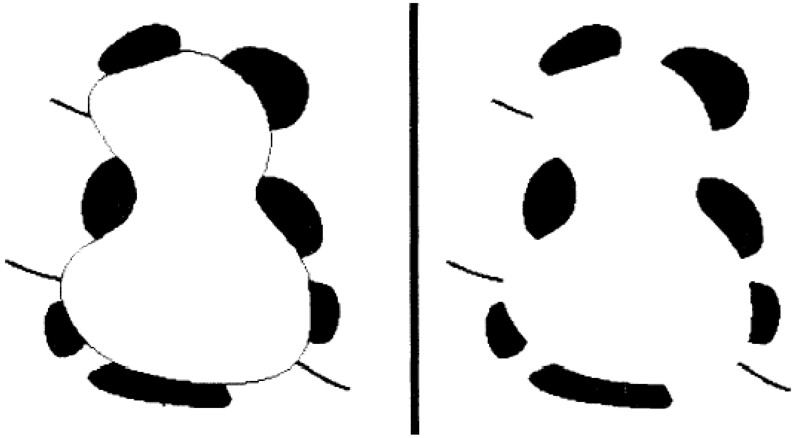


where *d* is the average distance between pairs of points – updated at each iteration.

Slide credit: Grauman

- The preferences for low-curvature, smoothness help deal with missing data:





Illusory contours found!

[Figure from Kass et al. 1987]

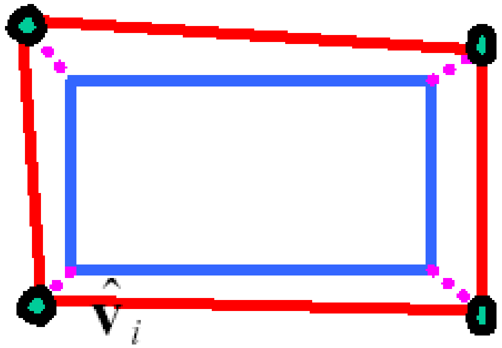
Slide credit: Grauman

Extending the internal energy: capture shape prior

- If object is some smooth variation on a known shape, we can use a term that will penalize deviation from that shape:

$$E_{internal} += \alpha \cdot \sum_{i=0}^{n-1} (v_i - \hat{v}_i)^2$$

where $\{\hat{v}_i\}$ are the points of the known shape



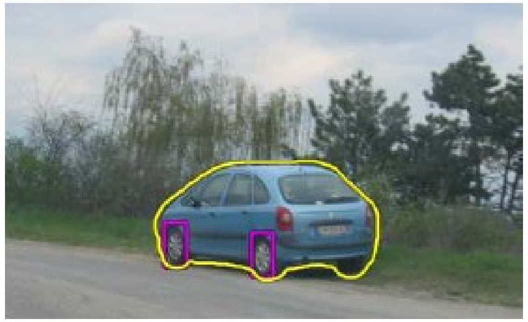


Fig from Y. Boykov

Slide credit: Grauman

Total energy depends on choice of weights

$$E_{total} = E_{internal} + \gamma E_{external}$$

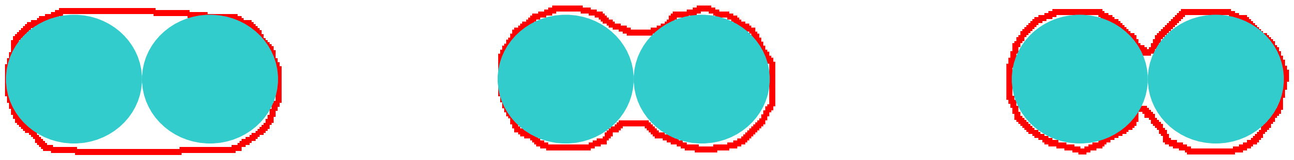
$$E_{external} = -\sum_{i=0}^{n-1} |G_x(x_i, y_i)|^2 + |G_y(x_i, y_i)|^2$$

$$E_{internal} = \sum_{i=0}^{n-1} \alpha\left(\bar{d} - \|v_{i+1} - v_i\|\right)^2 + \beta\|v_{i+1} - 2v_i + v_{i-1}\|^2$$

Slide credit: Grauman

Total energy depends on choice of weights

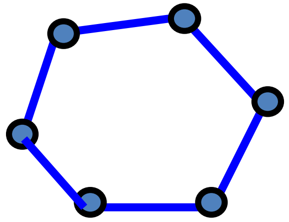
- e.g., α controls the penalty for internal elasticity



large α medium α small α

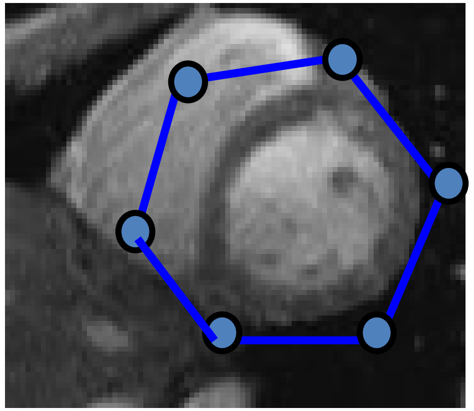
Credit: Boykov

Recap



- A simple elastic snake is defined by:
  - A set of n points
  - An internal energy term (tension, bending, plus optional shape prior)
  - An external energy term (gradient-based)

- To use to segment an object:
  - Initialize in the vicinity of the object
  - Modify the points to minimize the total energy



Slide credit: Grauman

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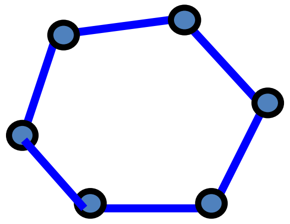

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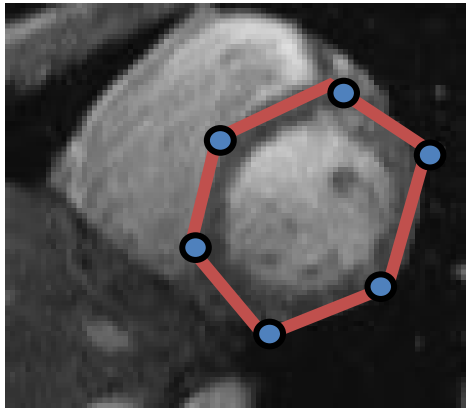

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Slide credit: Grauman

Active contours: pros and cons

Pros:

- Useful to track and fit non-rigid shapes
- Contour remains connected
- Possible to fill in “subjective” contours
- Flexibility in how energy function is defined, weighted

Cons:

- May get stuck in local minima!
- Must have decent initialization near true boundary
- Parameters of energy function must be set well based on prior information

Slide credit: Grauman

Summary

- Deformable shapes and active contours are useful for
  - Segmentation: fit or "snap" to boundary in image
  - Tracking: previous frame's estimate serves to initialize the next
- Fitting active contours:
  - Define terms to encourage certain shapes, smoothness, low curvature, push/pulls, …
  - Use weights to control relative influence of each component cost
  - Can optimize 2D snakes with Viterbi algorithm.
- Image structure (esp. gradients) can act as attraction force for interactive segmentation methods.

Slide credit: Grauman

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  - Optimization
  - Uses