2. Suppose two equally probable one-dimensional densities are of the form $p(x|\omega_i) \propto e^{-|x-a_i|/b_i}$ for $i = 1, 2$ and $0 < b_i$.

   (a) Write an analytic expression for each density, that is, normalize each function for arbitrary a_i and positive b_i.

   (b) Calculate the likelihood ratio as a function of your four variables.

   (c) Sketch a graph of the likelihood ratio $p(x|\omega_1)/p(x|\omega_2)$ for the case $a_1 = 0$, $b_1 = 1$, $a_2 = 1$ and $b_2 = 2$.

6. Consider the Neyman-Pearson criterion for two univariate normal distributions: $p(x|\omega_i) \sim N(\mu_i, \sigma_i^2)$ and $P(\omega_i) = 1/2$ for $i = 1, 2$. Assume a zero-one error loss, and for convenience let $\mu_2 > \mu_1$.

   (a) Suppose the maximum acceptable error rate for classifying a pattern that is actually in ω_1 as if it were in ω_2 is E_1. Determine the single-point decision boundary in terms of the variables given.

   (b) For this boundary, what is the error rate for classifying ω_2 as ω_1?

   (c) What is the overall error rate under zero-one loss?

   (d) Apply your results to the specific case $p(x|\omega_1) \sim N(-1/2, 1)$ and $p(x|\omega_2) \sim$
(1/2,1) $N(1/2, 1)$ and $E_1 = 0.05$. (-1/2,1)

   (e) Compare your result to the Bayes error rate (i.e., without the Neyman-Pearson conditions).