ECE5554 – Computer Vision
Lecture 10c – Stereo Epipolar Geometry

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Today’s Objectives

Epipolar geometry

Stereo Constraints

- Uniqueness
- Ordering
- Disparity gradient

Challenges for Stereo Vision

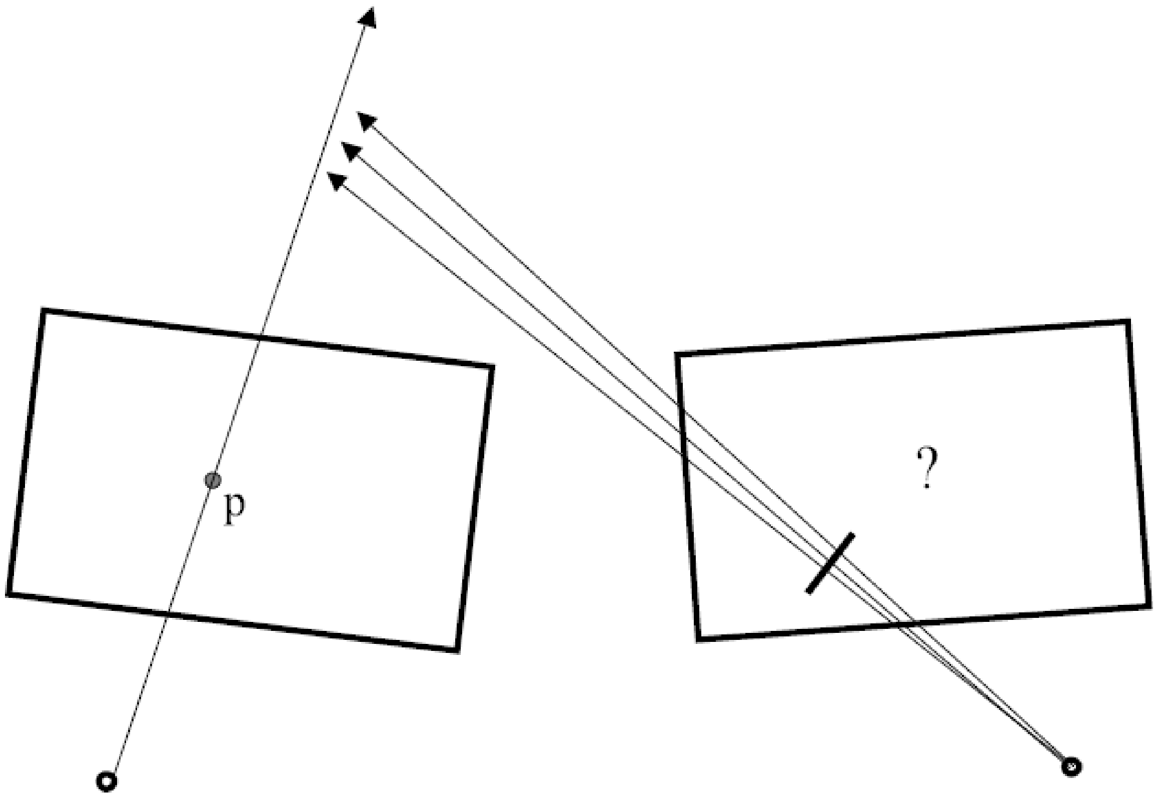
- Occlusion
- Repeating patterns

Stereo Rectification

- Thanks to Dr. A. L. Abbott for many of the following slides

EPIPOLAR GEOMETRY

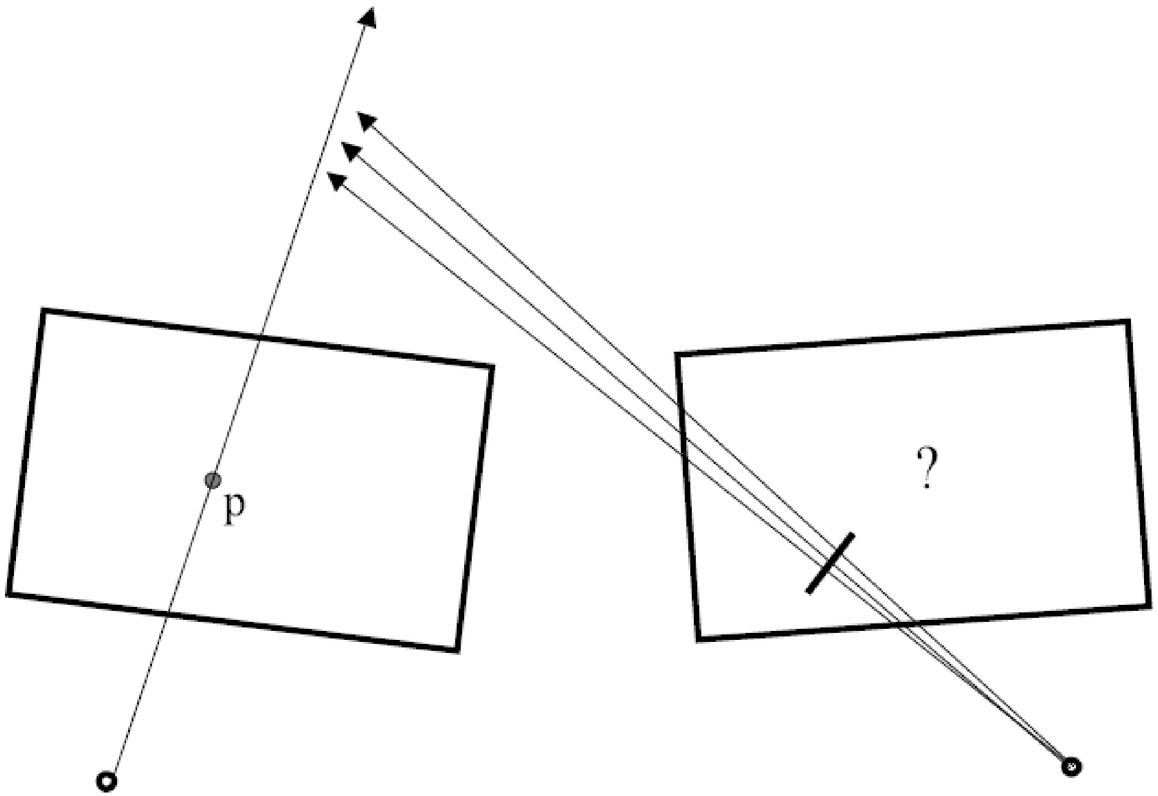
For a given point *p* in the left image,
what can be said about potential matches in the right image?



Source: Darrell

For a given point *p* in the left image,
what can be said about potential matches in the right image?

They are collinear!



Source: Darrell

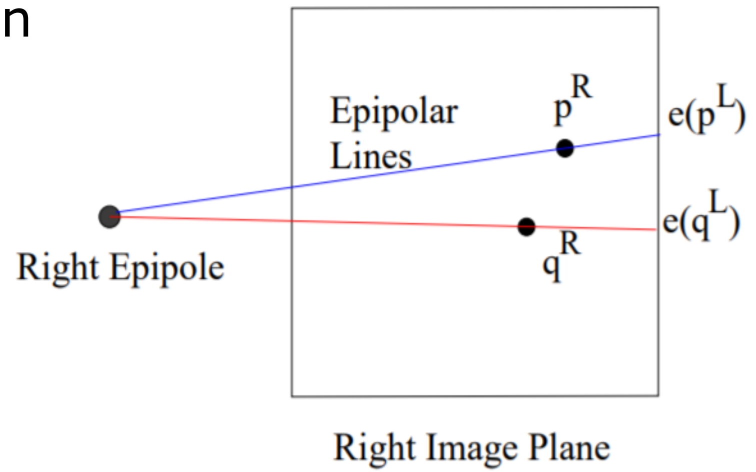
The epipolar constraint limits the relative pose of the two cameras so that correspondence between image points can be determined

Epipolar Constraint:

- Suppose p^L is the left image position for some scene point X^p. Then the corresponding point p^R in the right image must lie on the epipolar line $e(p^L)$.
- Notice the epipolar line $e(p^L)$ depends on the position of the point in the left image. For example, another image point q^L generally gives rise to a different epipolar line $e(q^L)$.

Epipole:

- All the epipolar lines in the right image pass through a single point (possibly at infinity) called the right epipole. This point is given by the intersection of the line containing the two nodal points d^L and d^R with the right image plane.



http://www.cs.toronto.edu/~jepson/csc420/notes/

Epipolar geometry: terms

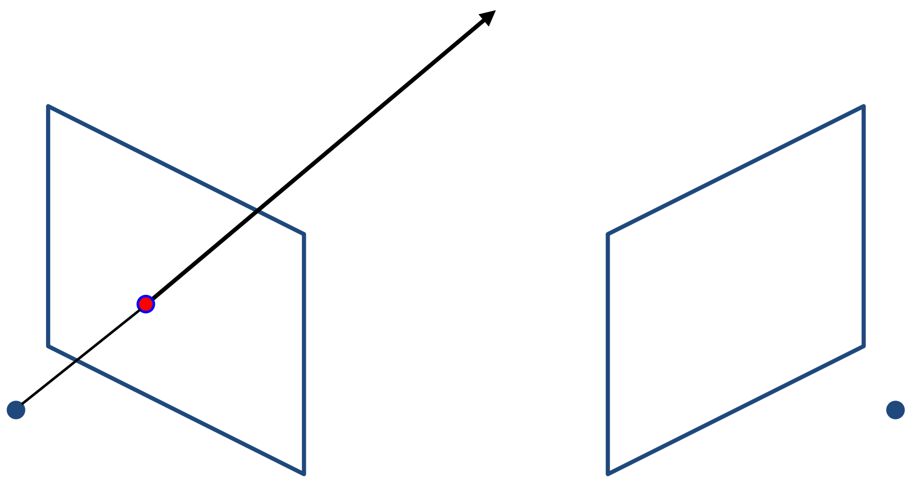
- **Baseline**: line joining the camera centers
- **Epipole**: point of intersection of the baseline with an image plane; projection of one camera center projection onto another camera's image plane
- **Epipolar plane**: a plane containing baseline and any third point (such as a 3D point of interest)
- **Epipolar line**: intersection of an epipolar plane with an image plane

- Note: For a given image and camera geometry, all epipolar lines intersect at the epipole

Credit: Grauman

Epipolar geometry

The geometry of two camera views causes corresponding image points to be <u>coplanar</u> with the original 3D point, and with the 2 points of projection



Source: S. Seitz

Epipolar geometry

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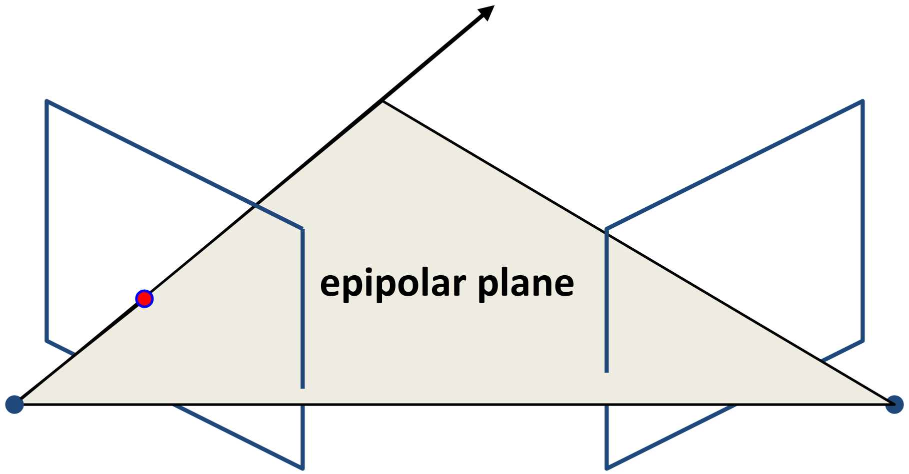

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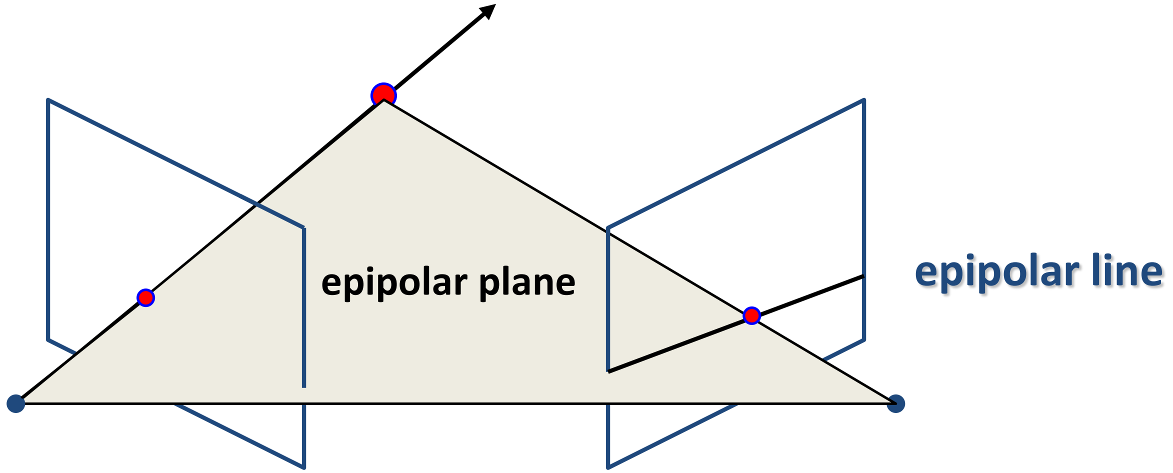
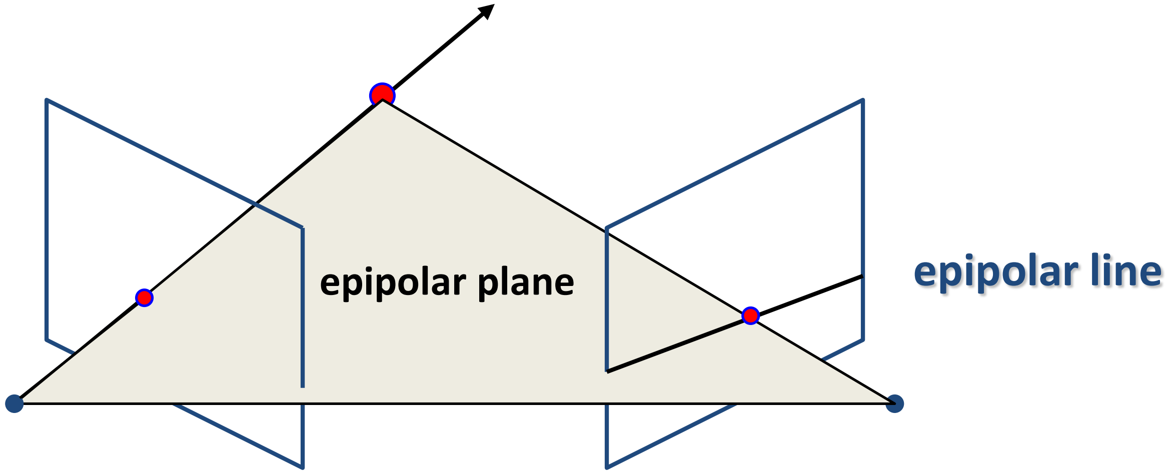


Source: S. Seitz

Epipolar geometry

Epipolar constraint: Why is this useful?

- Point A in one image determines an epipolar line in the other image
- The point corresponding to point A must lie on that epipolar line
- 1-dimensional search!



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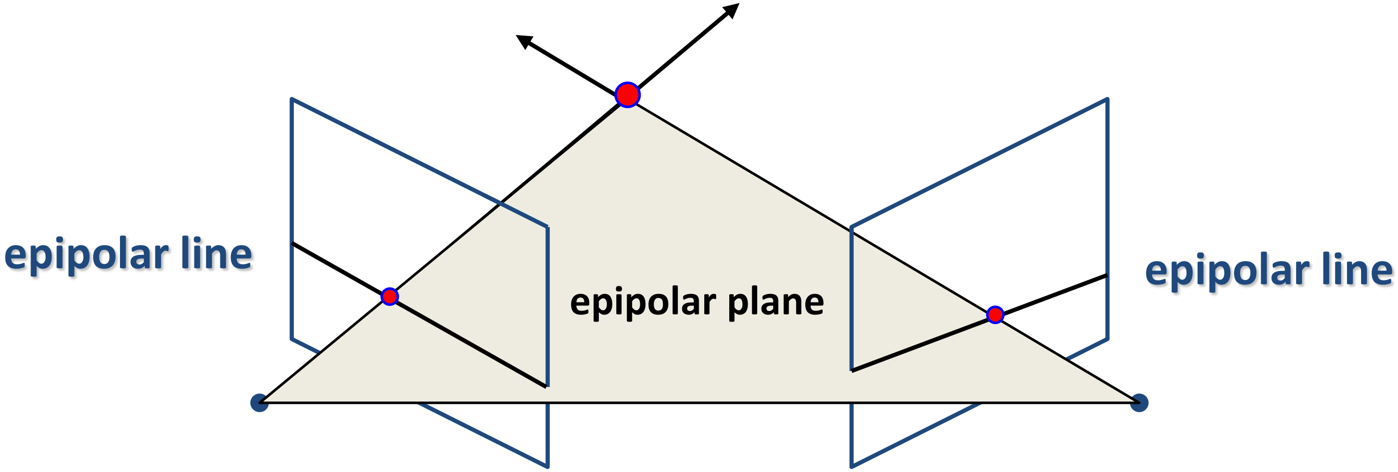
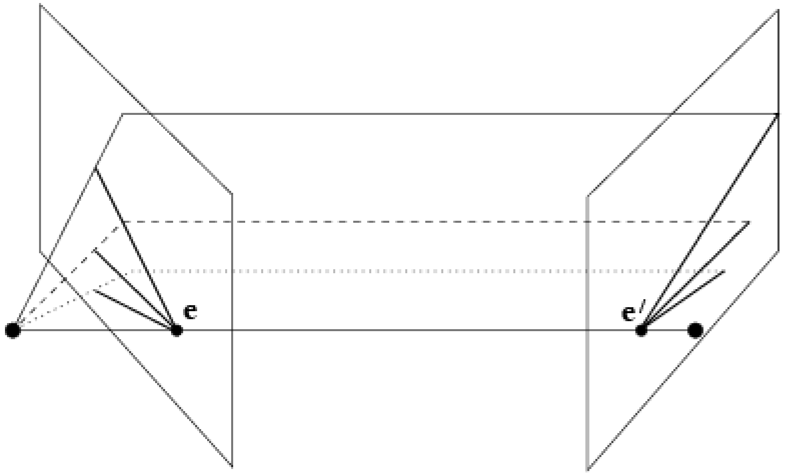
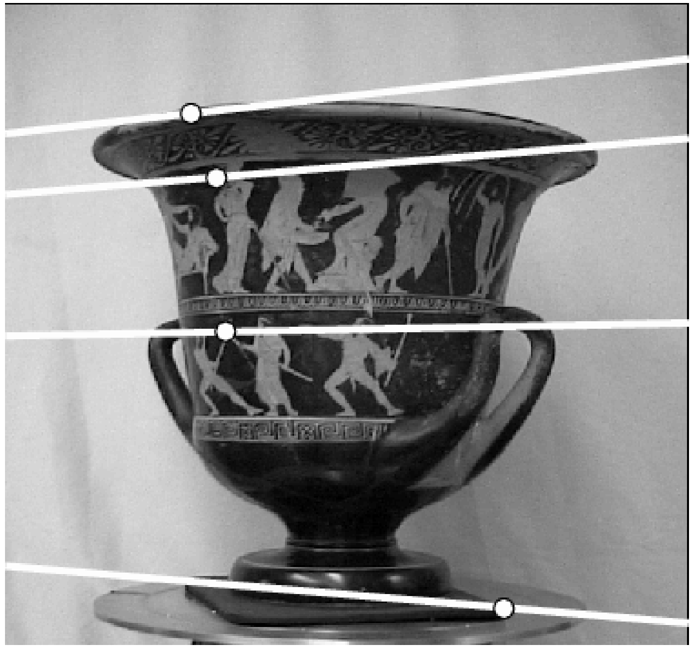


Source: S. Seitz

Example: converging cameras





Credit: Hartley & Zisserman

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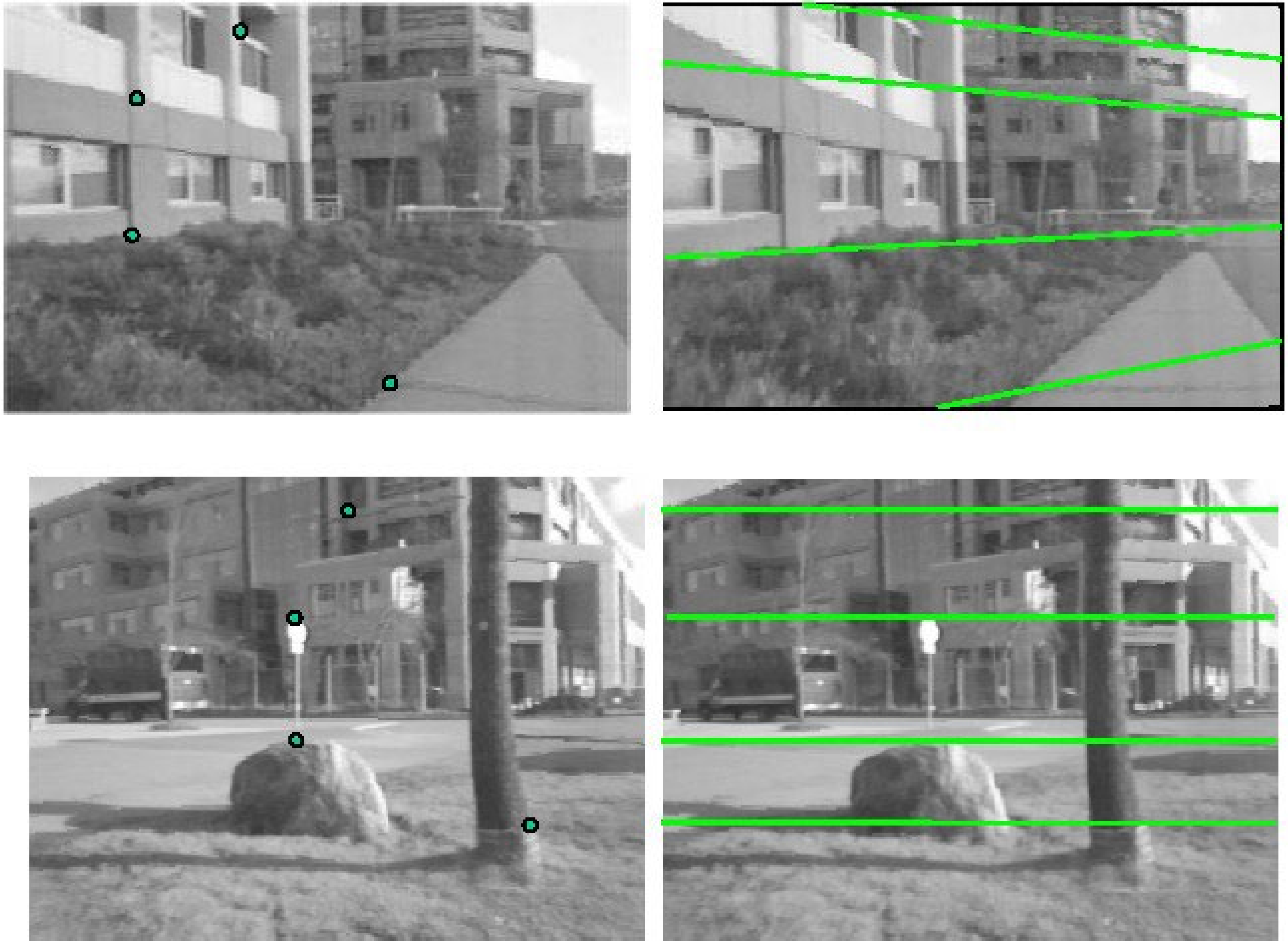


Credit: Darrell

Stereo constraints

- Most surfaces of interest are opaque
- Most surfaces are smooth, except for infrequent discontinuities at occlusion boundaries

- Therefore,
  - We expect correspondences to be unique
  - Correspondences along epipolar lines tend to appear in order, left to right

Uniqueness - For opaque objects, there is at most one match in the right image for every point in the left image

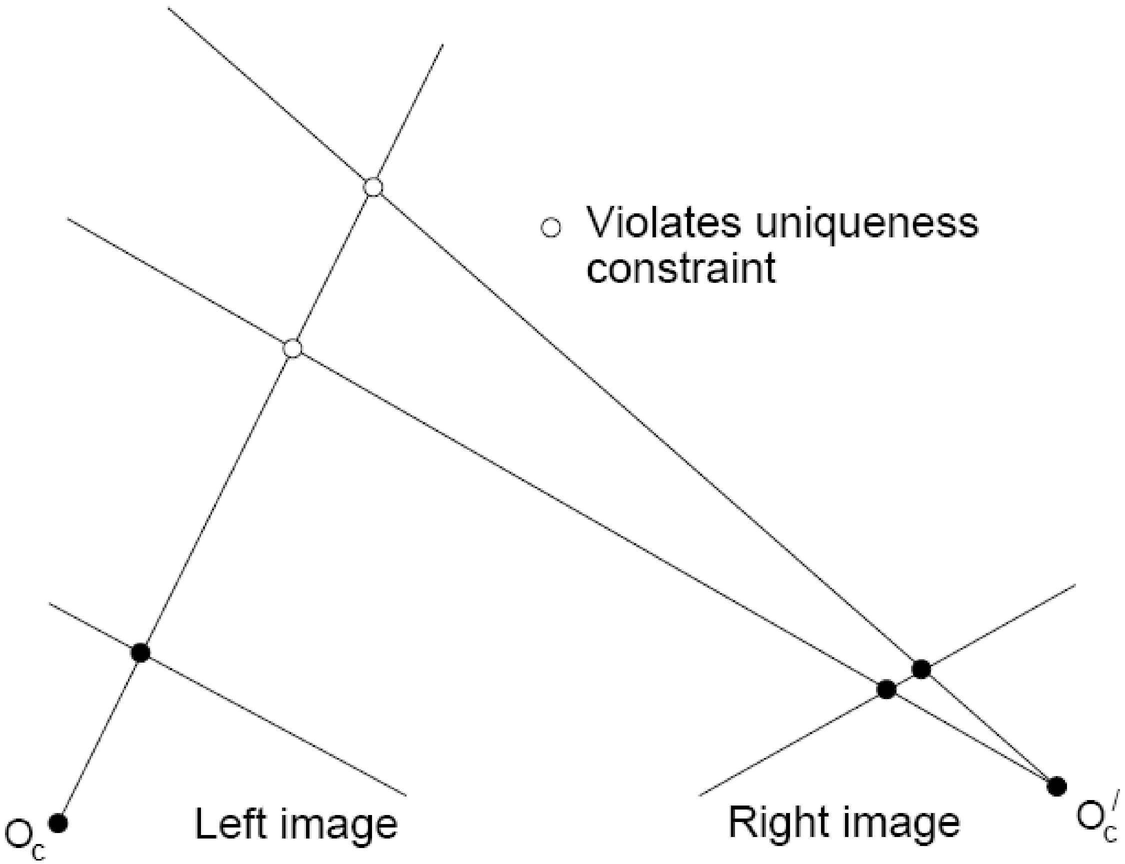


Figure from Gee & Cipolla

Ordering constraint - points on **same surface** (opaque object) will be in same order in both views

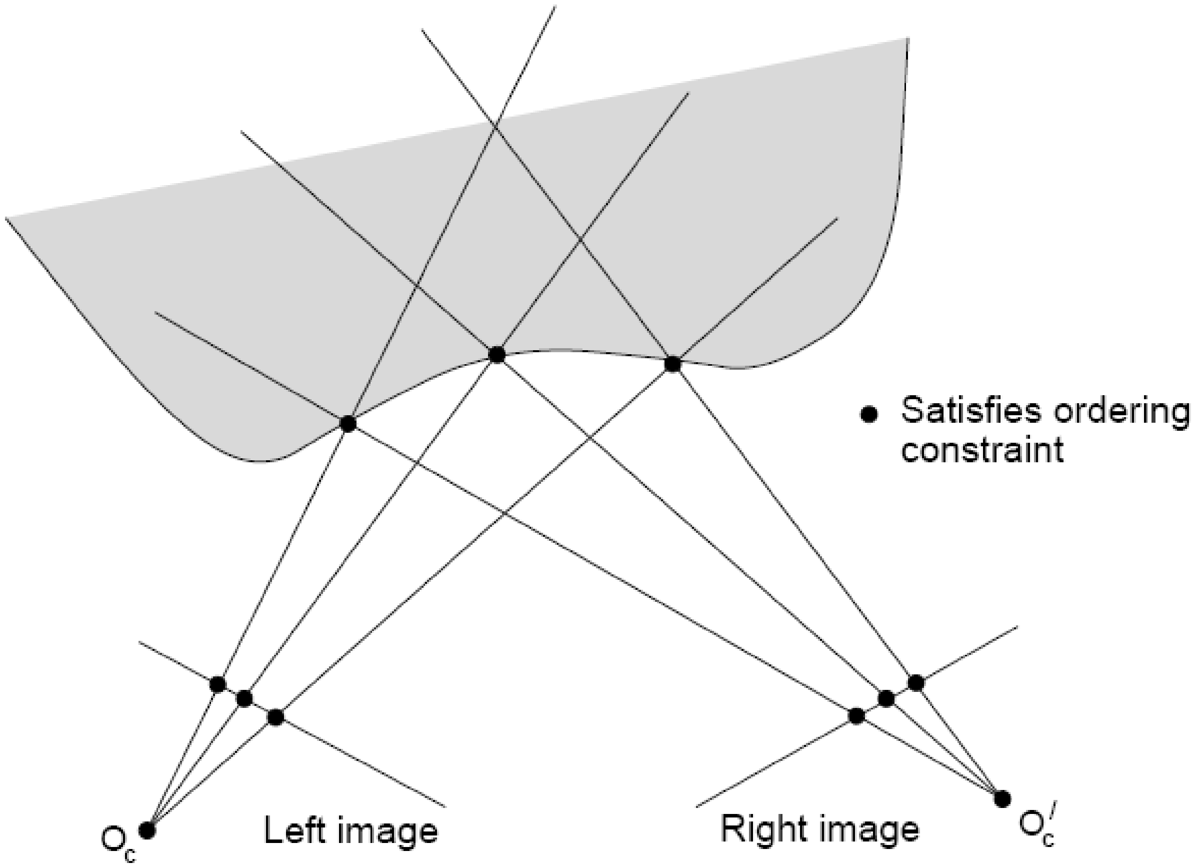
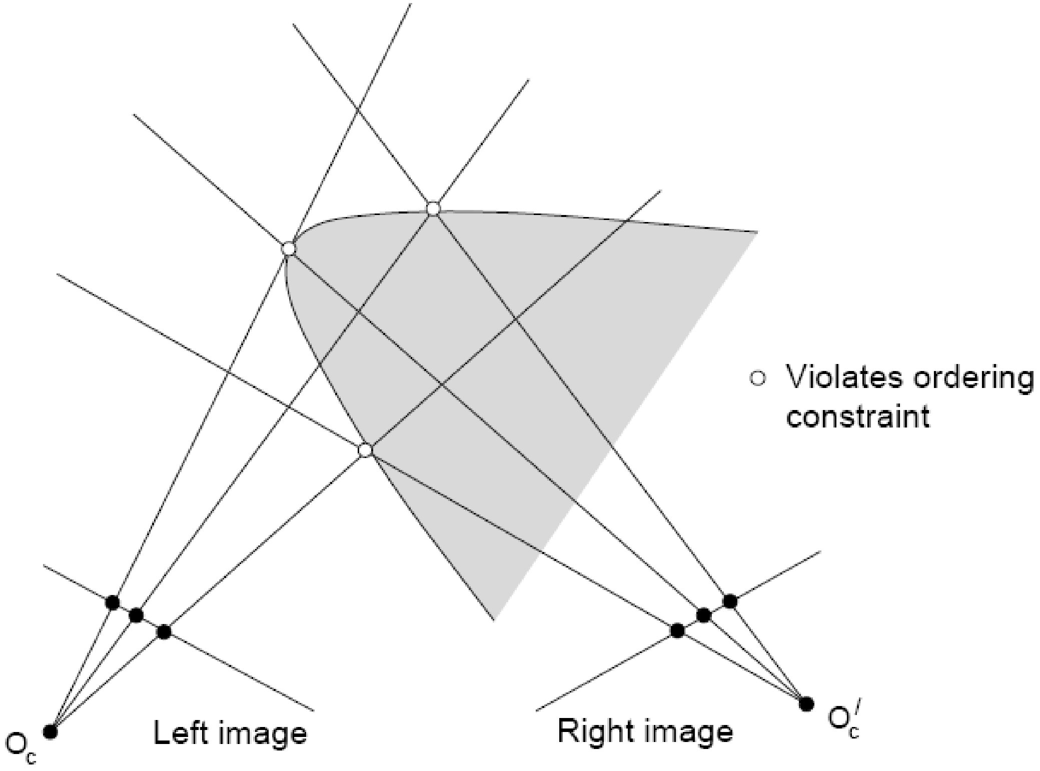
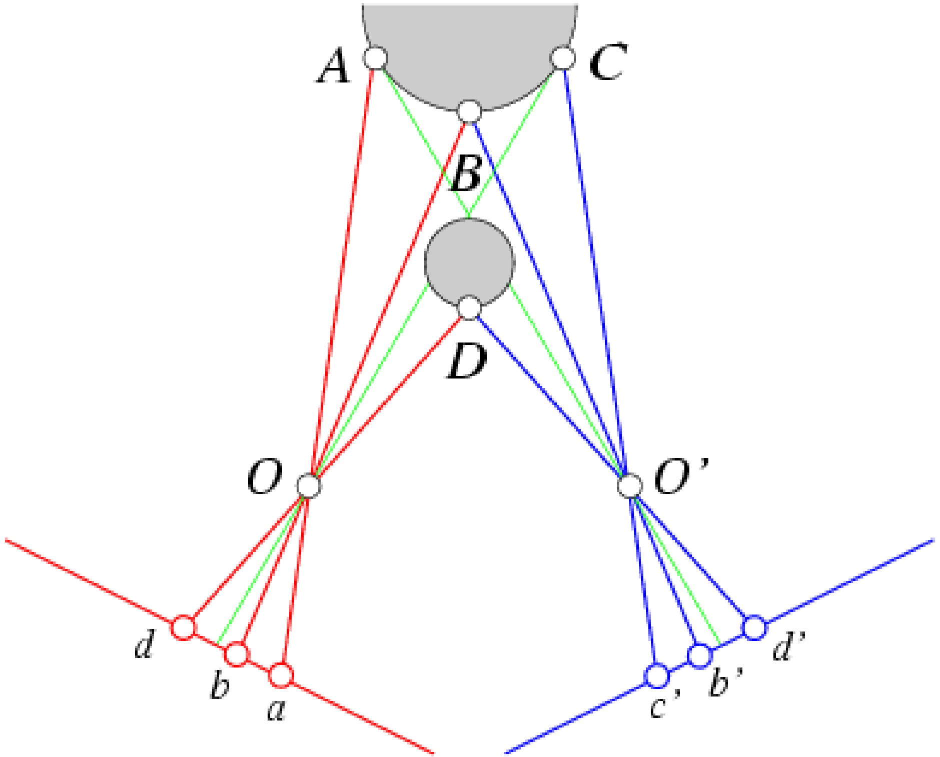


Figure from Gee & Cipolla

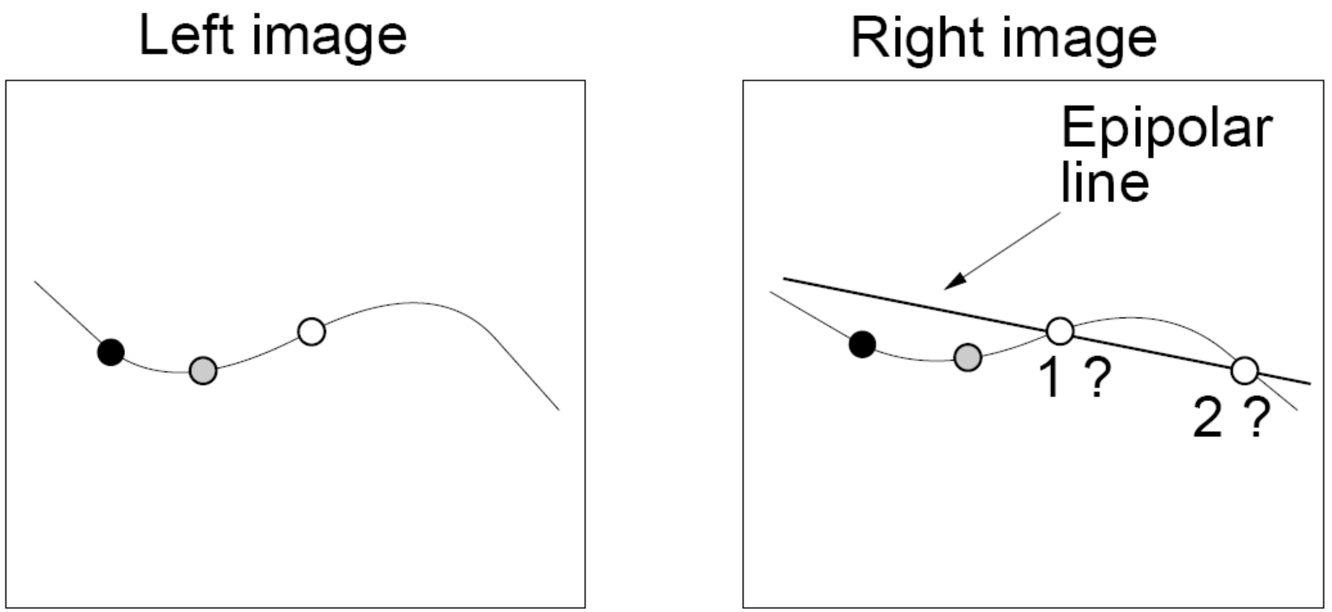
The ordering constraint won't always hold - consider transparent object, or an occluding surface





Figures from Forsyth & Ponce

Disparity gradient - Assume piecewise continuous surface, so want disparity estimates to be locally smooth



Given matches ● and ◉, point ○ in the left image must match point 1 in the right image. Point 2 would exceed the disparity gradient limit.

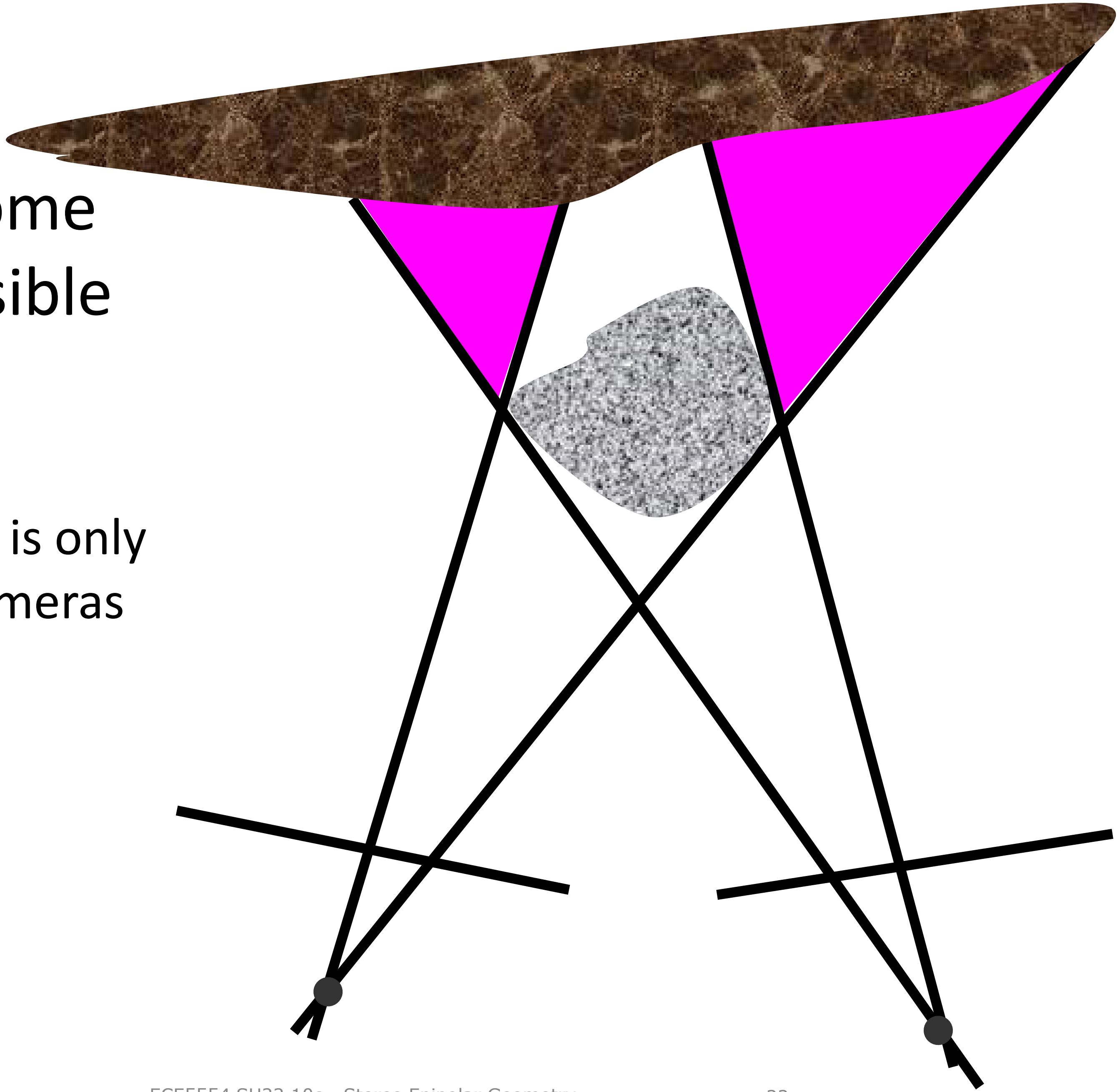
Figure from Gee & Cipolla

Stereo: more problems to consider

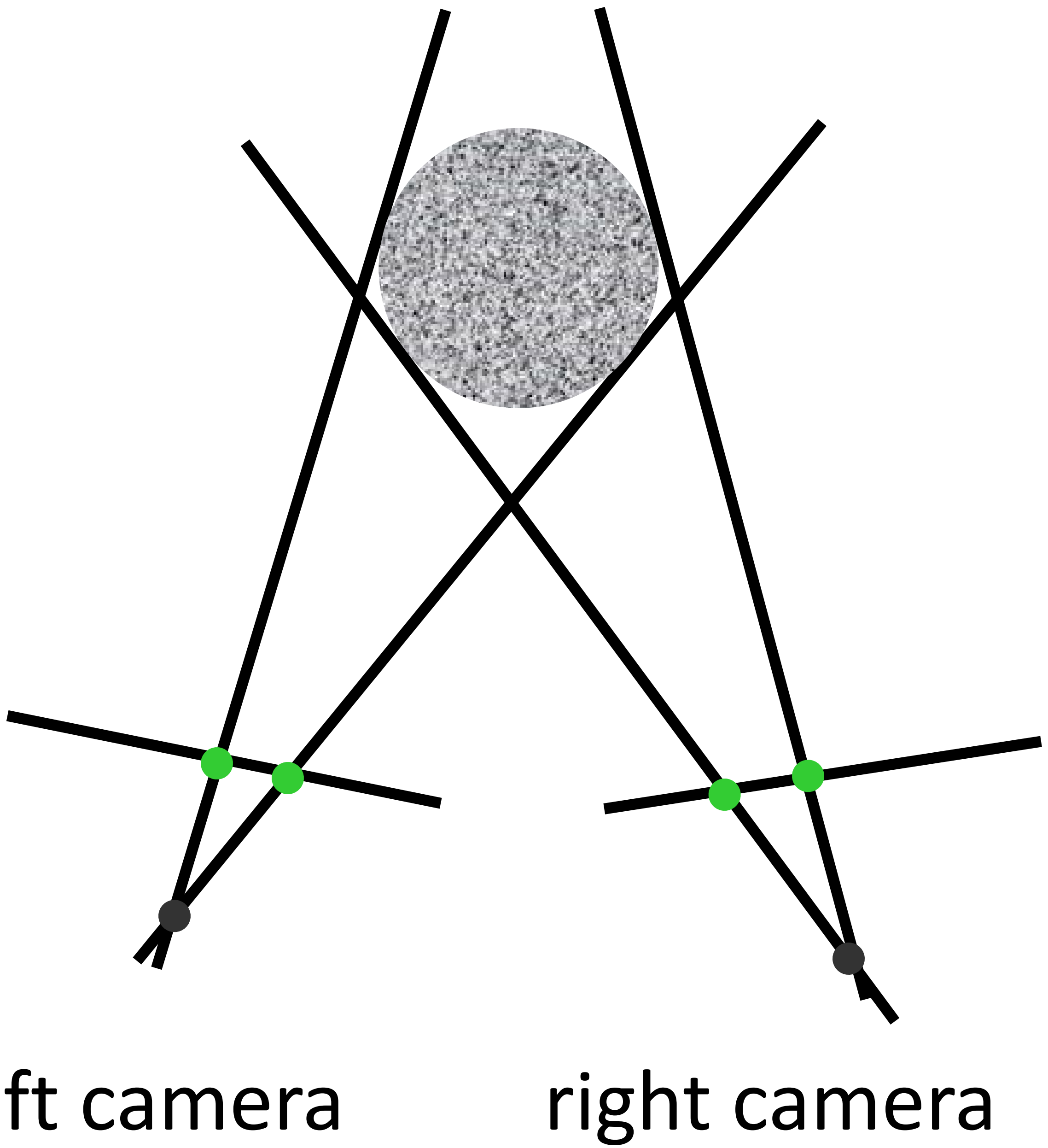
- Occlusion
- Lack of visual detail
- Partially transparent surfaces
- Specularities (mirror-like reflections)
- Repeating patterns
- Quantization error
- Limited field of view

Due to occlusion, some 3D points will be visible to one camera only

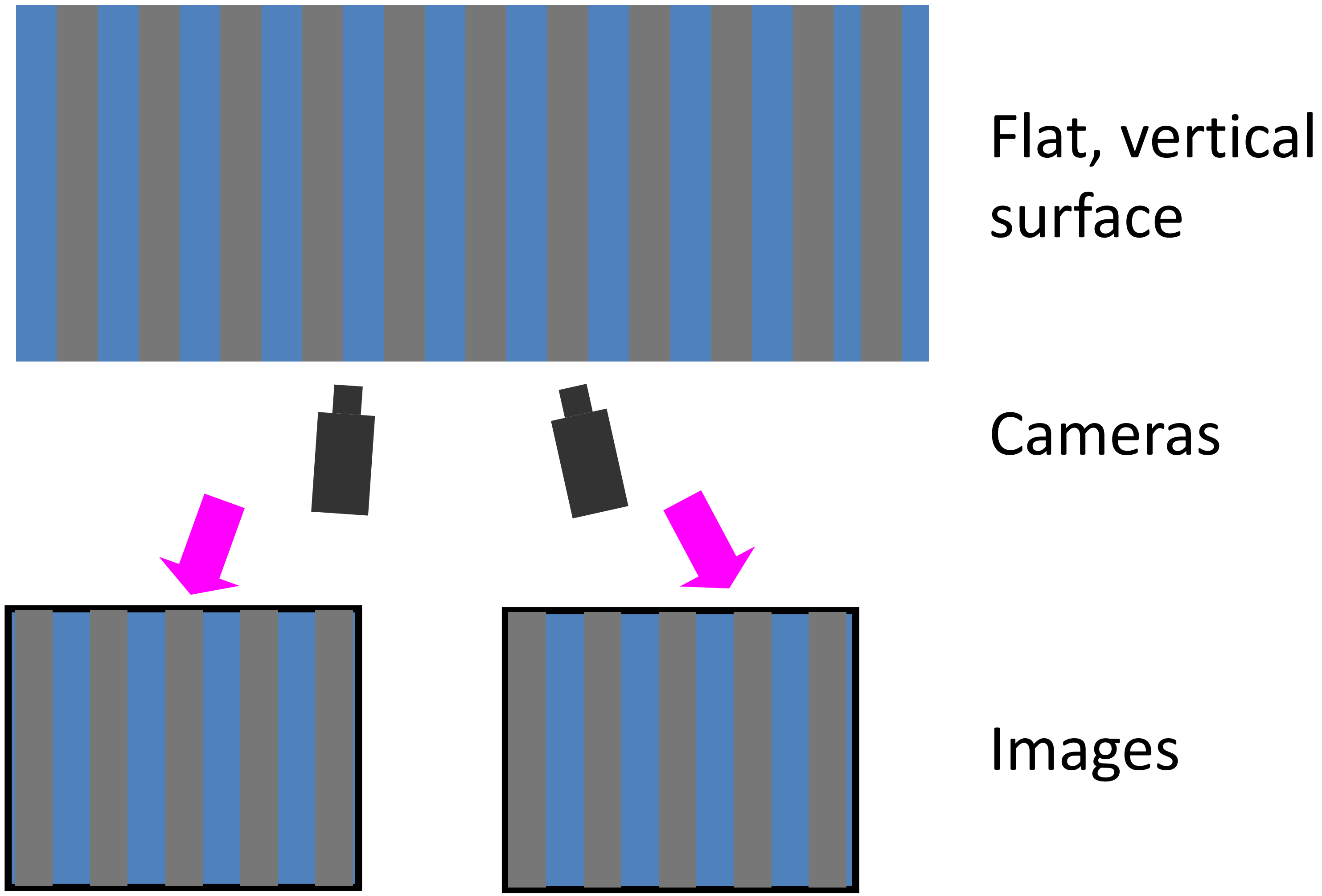
Anything in the pink areas is only seen by one of the two cameras



Self-occlusion:
For curved objects, occluding contours may not match



Periodic patterns in the image are difficult to handle; extreme cases can cause the "wallpaper illusion"



Stereo image rectification

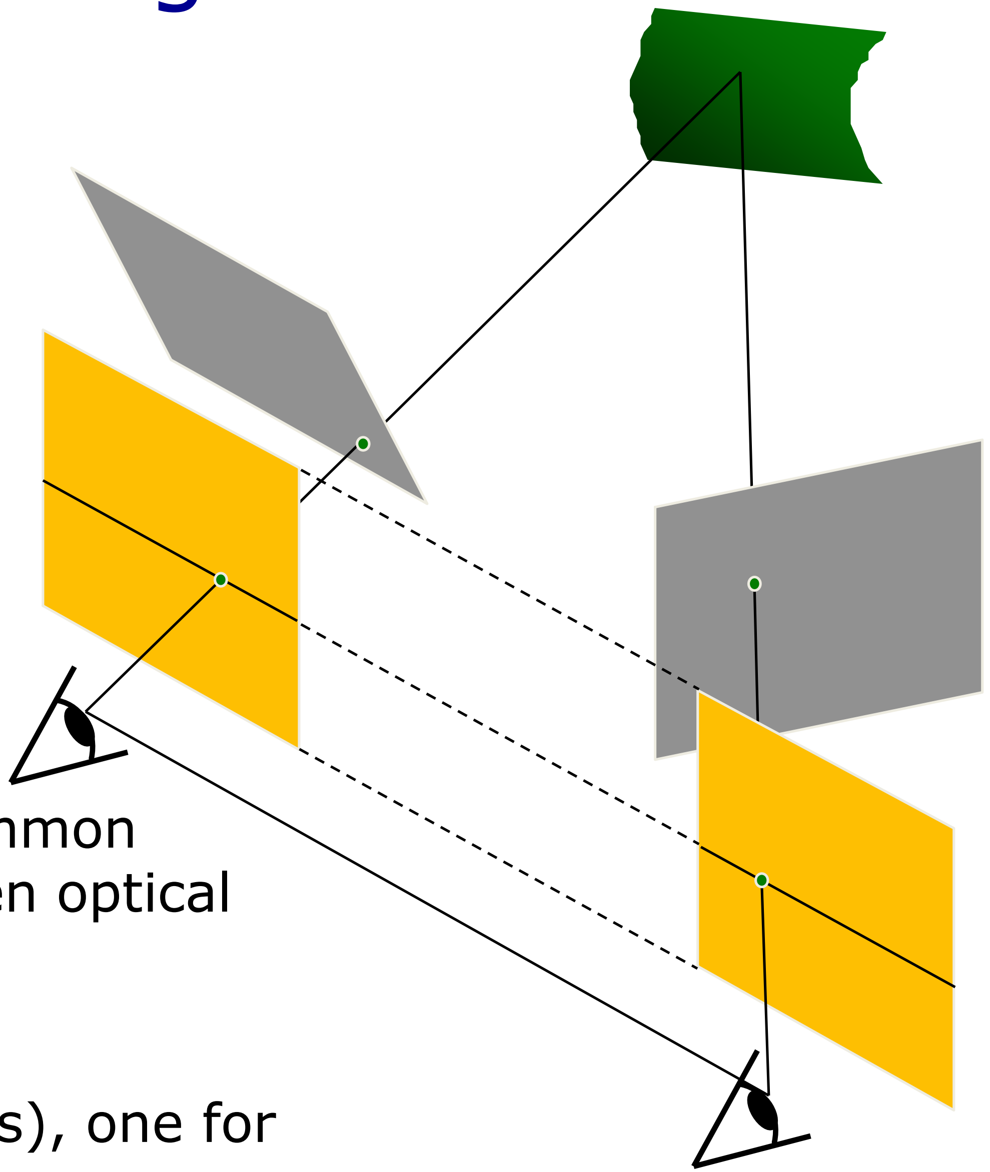
- *Given*: a stereo image pair, obtained using cameras in arbitrary orientations
- *Goal*: create a new stereo image pair such that the epipolar lines are horizontal and identical for the 2 new images

Stereo image rectification

- After rectification: For a given pixel in one image, the corresponding point, if present, must lie in the **same row** of the other image
- The new images will resemble those that would be obtained using the simple stereo geometry that we first discussed!

Stereo image rectification

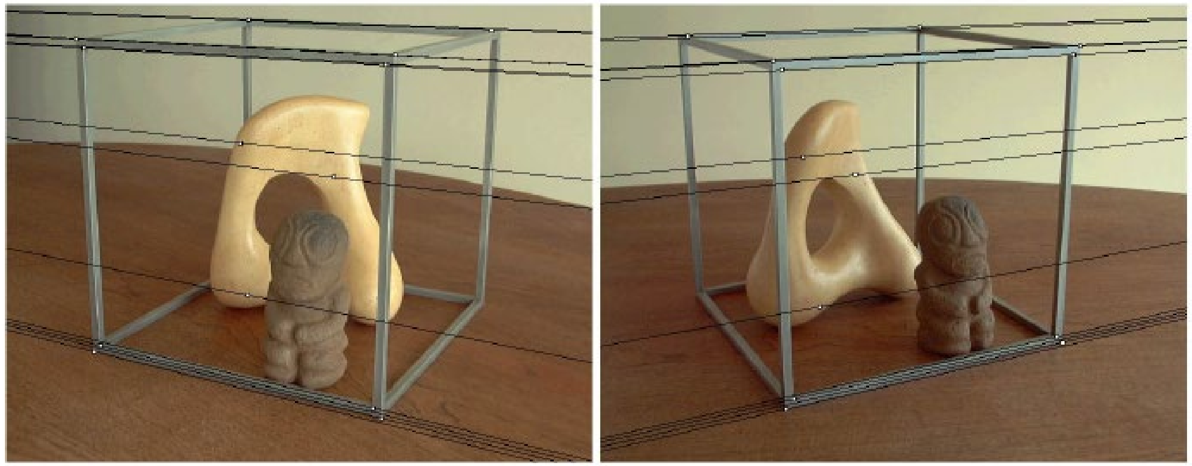
In practice, it is convenient if epipolar lines lie along image rows



- Reproject image planes onto a common plane parallel to the line between optical centers
- Disparities are horizontal after this transformation
- Two homographies (3x3 transforms), one for each input image reprojection

Adapted from Li Zhang

Stereo image rectification: example



Source: Alyosha Efros

Image rectification algorithm

- For each camera,
  - Select 4 original points (x_i, y_i) that will be well distributed over the new image
  - Project the 4 points onto the original image to obtain (u, v_i)
  - Solve for $\begin{bmatrix} sx \\ sy \\ s \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$ and create the new image

Main tasks in stereo ranging

- **Camera calibration**
- **Image rectification**
- **Feature extraction**
- **Matching** (= disparity estimation)
- **3D surface estimation**

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