

Limits

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1 Introduction

This file contains a set of useful properties and formulas to solve limit problems. Any questions or concerns should be sent to the author at agflores1979@gmail.com.

2 Properties

An extensive list of the properties of limits.

Constant Function Rule:

$$(1) \lim_{x \rightarrow \alpha} C = C \text{ where } C \text{ is a constant}$$

Identity Function Rule:

$$(2) \lim_{x \rightarrow \alpha} x = \alpha$$

Constant Multiple Rule:

$$(3) \lim_{x \rightarrow \alpha} C f(x) = C \lim_{x \rightarrow \alpha} f(x) \text{ where } C \text{ is a constant}$$

Sum Rules:

$$(4) \lim_{x \rightarrow \alpha} [f(x) + g(x)] = \lim_{x \rightarrow \alpha} f(x) + \lim_{x \rightarrow \alpha} g(x)$$

$$(5) \lim_{x \rightarrow \alpha} [f(x) - g(x)] = \lim_{x \rightarrow \alpha} f(x) - \lim_{x \rightarrow \alpha} g(x)$$

Product Rule:

$$(6) \lim_{x \rightarrow \alpha} [f(x)g(x)] = \lim_{x \rightarrow \alpha} f(x) \lim_{x \rightarrow \alpha} g(x)$$

Quotient Rule:

$$(7) \lim_{x \rightarrow \alpha} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow \alpha} f(x)}{\lim_{x \rightarrow \alpha} g(x)}$$

provided that the numerator and denominator are not both equal to zero or infinity

Power Rules:

$$(8) \lim_{x \rightarrow \alpha} [f(x)]^n = \left[\lim_{x \rightarrow \alpha} f(x) \right]^n \text{ where } n \in \mathbb{R}$$

$$(9) \lim_{x \rightarrow \alpha} x^n = \alpha^n \text{ where } n \in \mathbb{R} \text{ and } \alpha \neq 0 \text{ if } n \text{ is not positive}$$

Replacement Rule:

If the functions f and g have the same values for all x near α , but not necessarily including $x = \alpha$, then

$$(10) \lim_{x \rightarrow \alpha} f(x) = \lim_{x \rightarrow \alpha} g(x)$$

Extended Sum Rule:

$$(11) \lim_{x \rightarrow \alpha} [f_1(x) + f_2(x) + \dots + f_n(x)] = \lim_{x \rightarrow \alpha} f_1(x) + \lim_{x \rightarrow \alpha} f_2(x) + \dots + \lim_{x \rightarrow \alpha} f_n(x)$$

Extended Product Rule:

$$(12) \lim_{x \rightarrow \alpha} [f_1(x) \cdot f_2(x) \cdot \dots \cdot f_n(x)] = \lim_{x \rightarrow \alpha} f_1(x) \lim_{x \rightarrow \alpha} f_2(x) \dots \lim_{x \rightarrow \alpha} f_n(x)$$

3 Formulas

A list of commonly seen limit problems.

$$(1) \lim_{x \rightarrow \alpha} \frac{x^n - \alpha^n}{x - \alpha} = n\alpha^{n-1} \text{ where } n \in \mathbb{R}$$

$$(2) \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

$$(3) \lim_{x \rightarrow 0} \frac{\alpha^x - 1}{x} = \ln \alpha$$

$$(4) \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$(5) \lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$$

$$(6) \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$(7) \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

$$(8) \lim_{x \rightarrow 0} [1 + x]^{\frac{1}{x}} = e$$

$$(9) \lim_{x \rightarrow \infty} \left[1 + \frac{1}{x}\right]^x = e$$