Bell States, GHZ States, and W-States: Exploring the Robustness of Entanglement for Noisy Quantum Systems

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Motivation

Due to the sensitivity of quantum information to external perturbations, we investigate how noise affects Bell, GHZ, and W-states, and which types of entangled states are most resilient to depolarization. Our findings can help us determine the best entangled state to employ for exchanging quantum information between quantum devices.

Background

Quantum entanglement is a nonlocal property of two or more qubits that permits a collection of qubits to express stronger correlation than is feasible in classical systems. Three examples of quantum entanglement states are shown below:

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\mathrm{GHZ}\rangle_M = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes M} + |1\rangle^{\otimes M})$$

$$|\mathrm{W}\rangle_3 = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$$

We can use quantum circuits to achieve **quantum teleportation**:

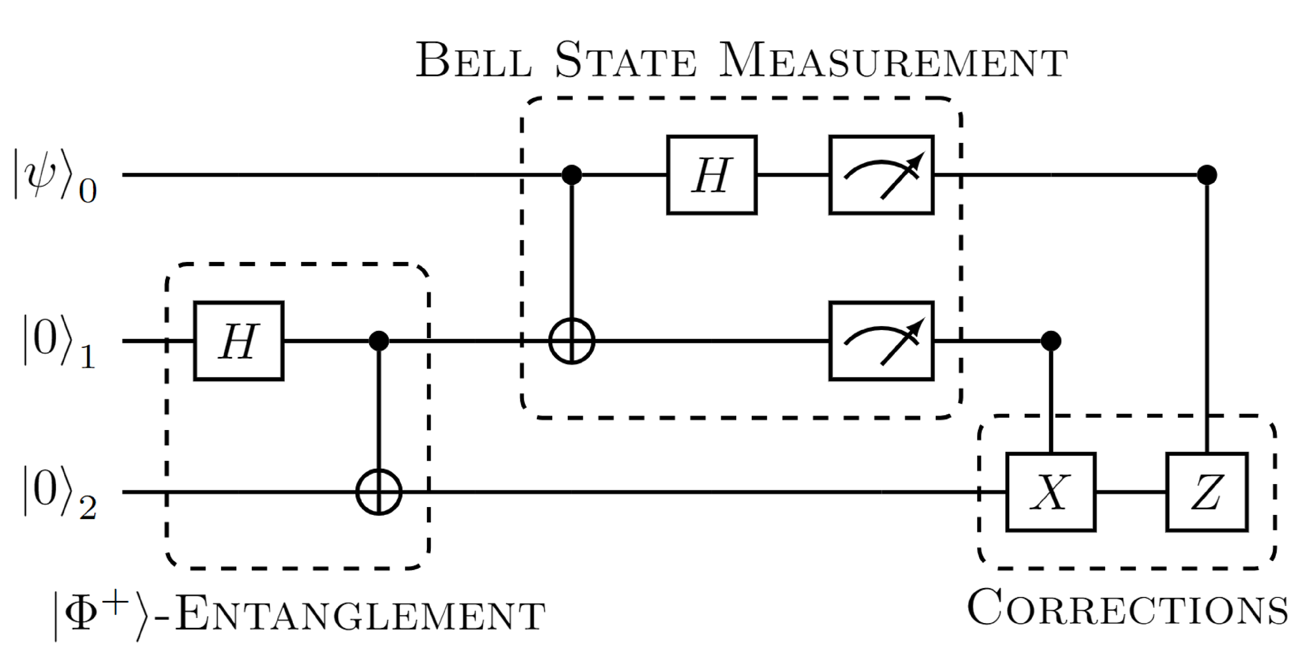

Figure 1: A quantum teleportation circuit that utilizes the $|\Phi^+\rangle$ bell state to teleport some arbitrary state, $|\psi\rangle$, from qubit 0 onto qubit 2.

Fidelity generates a helpful metric that reveals information about the closeness of two arbitrary qubit states:

$$F(\rho, \sigma) \equiv tr\sqrt{\rho^{1/2}\sigma\rho^{1/2}}$$

Source: [1]

The fidelity of the state we are teleporting and the teleported state yields information on the robustness of the teleportation circuit.

How are the different states of entanglement compared?

Methods

The Network Simulator for Quantum Information Using Discrete Events (NetSquid) is a software tool for modeling and simulation of scalable quantum networks.

NetSquid
Source: [2]

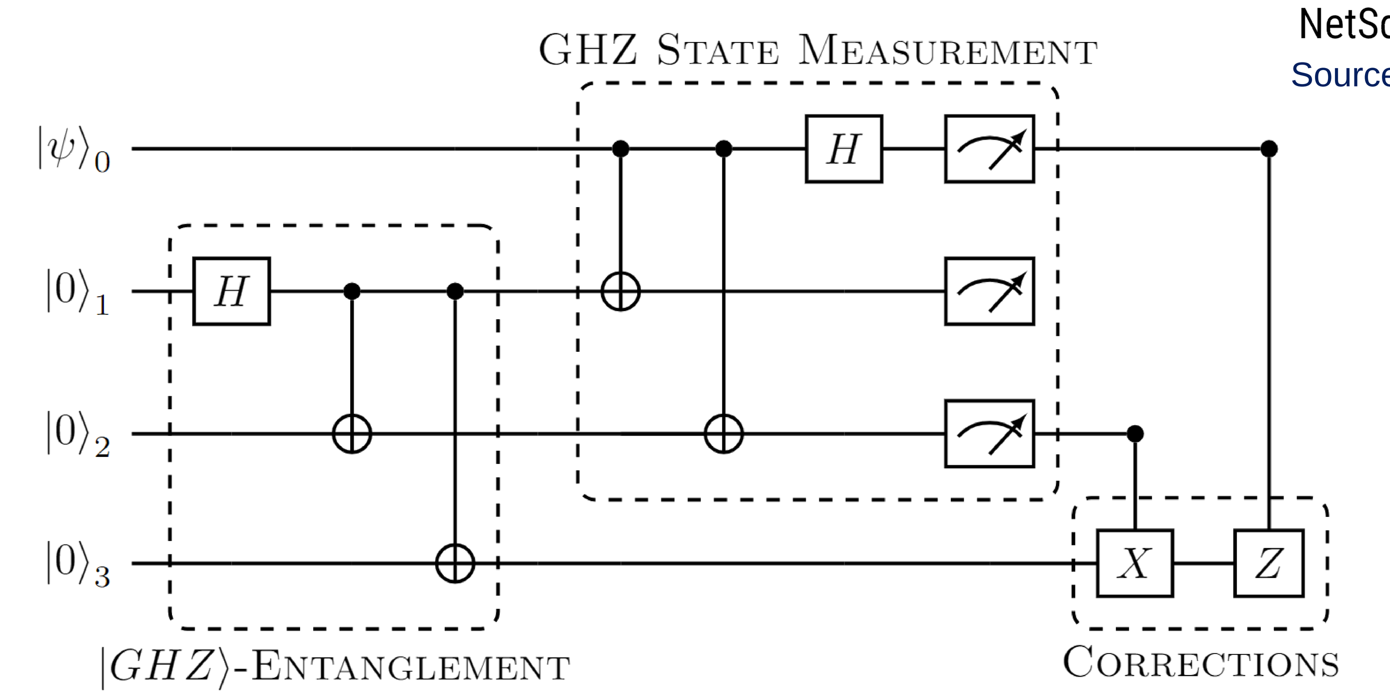

Figure 2: A quantum teleportation circuit that utilizes a three-qubit $|GHZ\rangle$ state to teleport some arbitrary state, $|\psi\rangle$, from qubit 0 onto qubit 3.[3]

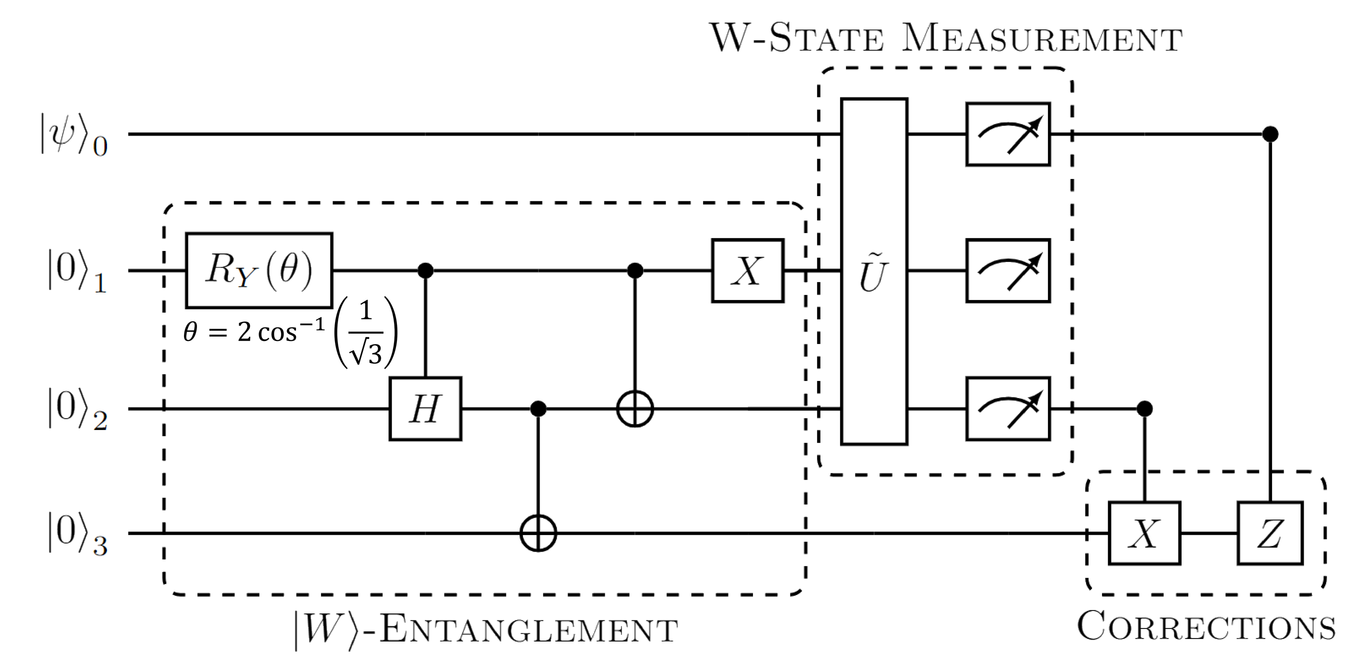

Figure 3: A quantum teleportation circuit that utilizes a $|W\rangle$ state to teleport some arbitrary state, $|\psi\rangle$, from qubit 0 onto qubit 3. Here, the $\tilde{U}$ gate is an $8x8$ unitary matrix.[3]

Depolarization randomly turns a quantum state into a totally mixed state with a specific probability.

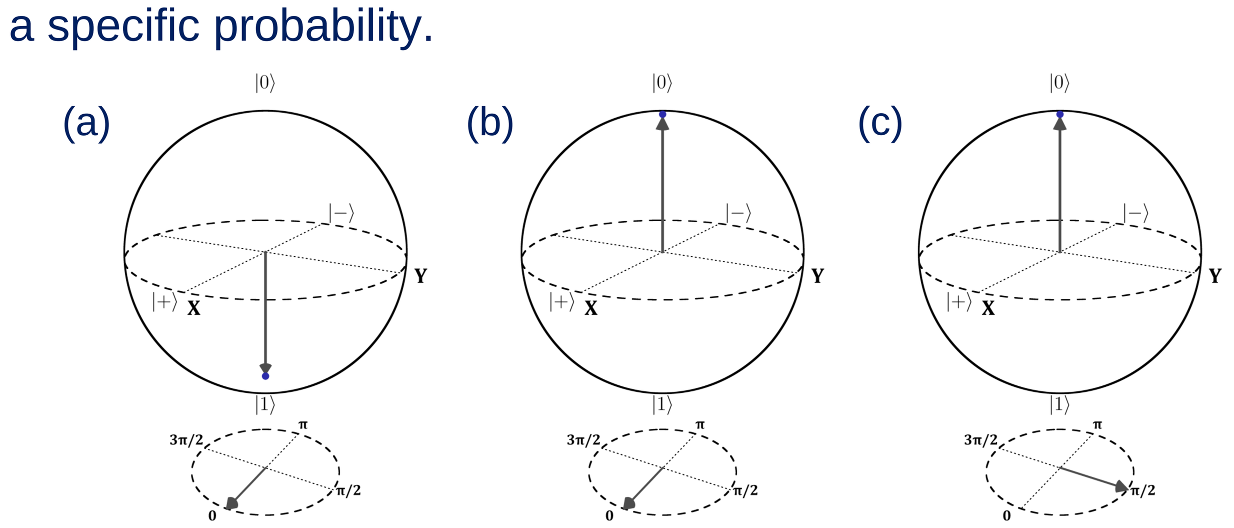

Figure 4: A Bloch sphere representation of a qubit after applying an X gate. (a) shows the qubit when no depolarization is applied to the operation. (b) and (c) show the qubit when a 100% probability of depolarization is applied to the X gate operation.

How does probabilistic noise affect fidelity?

Results

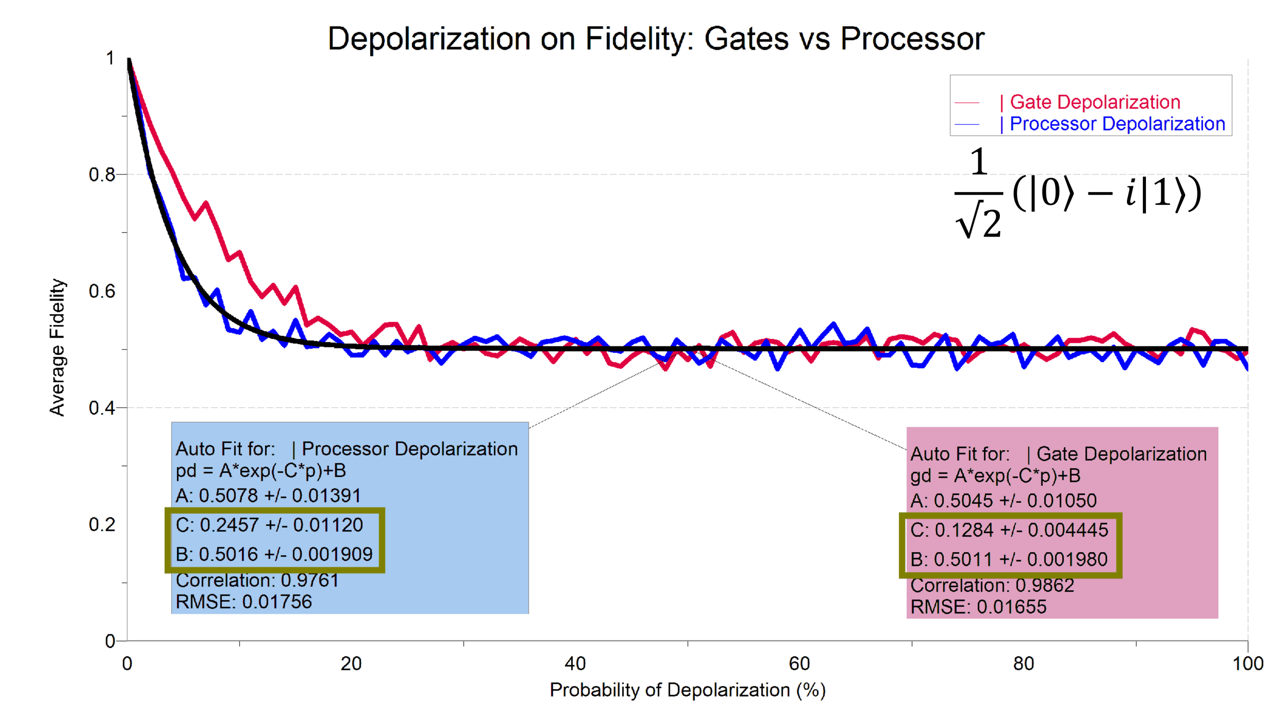

Figure 5: Illustrating the individual impact of gate and processor depolarization on fidelity. We emphasize the decay rate (C term) and the fidelity plateau value (B term).[2,4]

Average Values of All Teleported States

Teleportation Circuit	Gate Depolarization: Fidelity Decay Rate	Gate Depolarization: Fidelity Plateau	Processor Depolarization: Fidelity Decay Rate	Processor Depolarization: Fidelity Plateau
Bell State	0.0921	0.4972	0.1704	0.4996
GHZ State	0.1055	0.4157	0.1899	0.4151
W-State	0.1585	0.6084	0.2967	0.6101

Table 1: A comparison of the average values for all three variations of teleportation circuits. The boxes in blue indicate optimal performance.

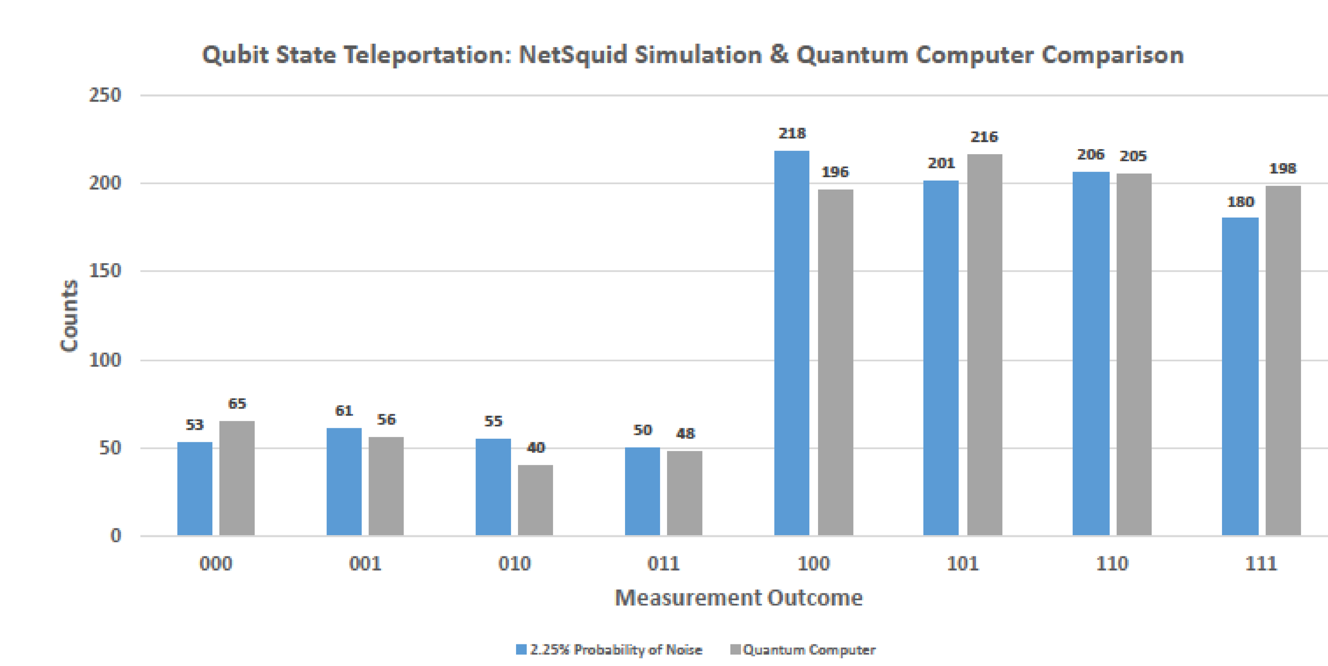

Figure 6: Comparing our NetSquid simulation data to that of an actual quantum computer.[2,5]

Conclusions

- Quantum computer noise environments can be replicated with simplified noise models.
- Depending on the probability of depolarization within a quantum system, either the Bell States or the W-state would be optimal for communicating information.

Acknowledgements

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