Center for Quantum Networks

Bell States, GHZ States, and W-States: Exploring the Robustness of Entanglement for Noisy Quantum Systems

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Motivation

- The production and distribution of entanglement among several users is a prerequisite for building quantum networks

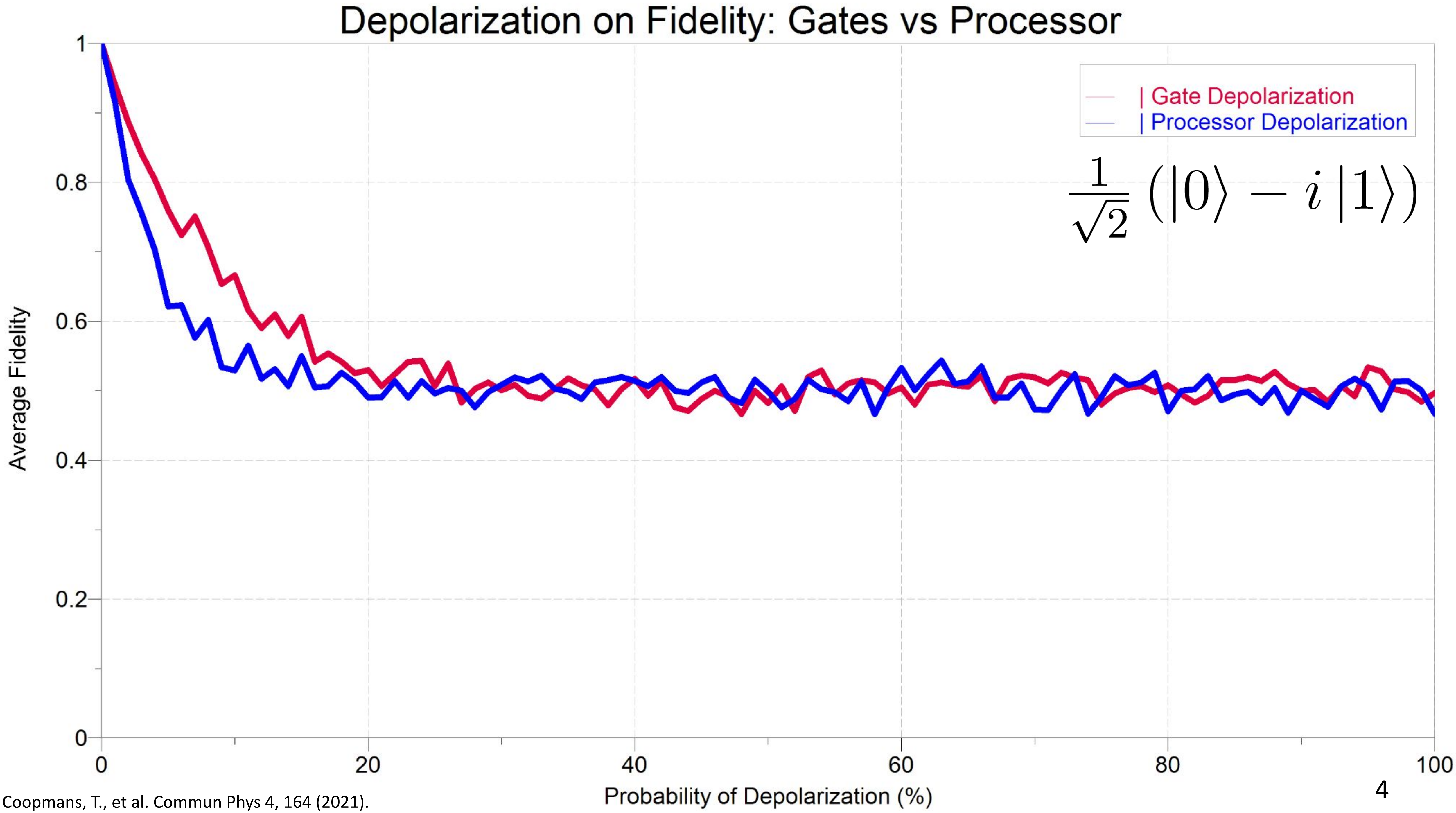
- **Net**work **S**imulator for **Qu**antum **I**nformation using **D**iscrete events
  - Models the impact of time on the performance of quantum networks

NetSquid

Coopmans, T., et al. Commun Phys 4, 164 (2021).

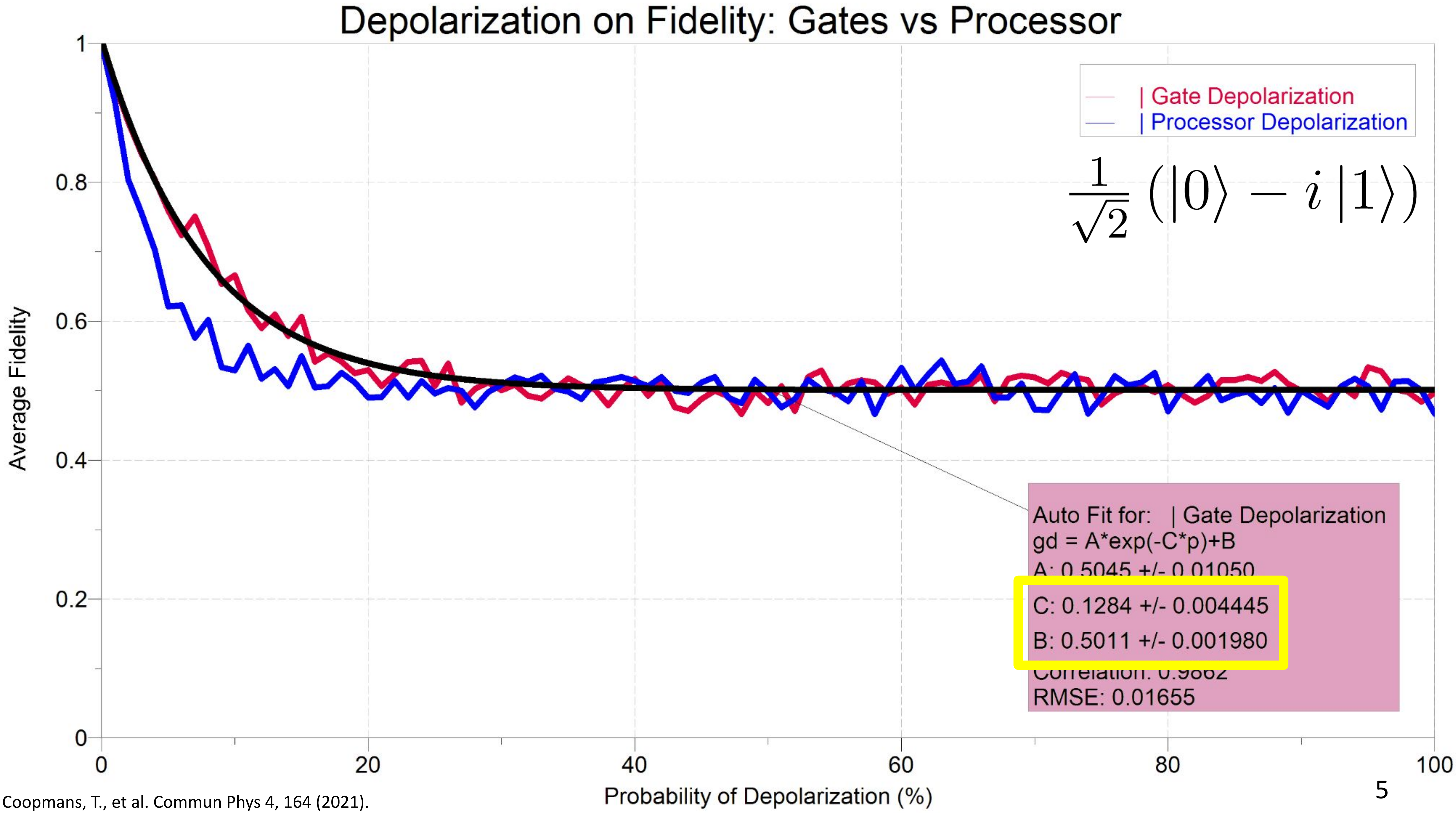
Research Project

- **How Bell, GHZ, and W-states react to system flaws, and how this affects their viability for QIS applications**
  - Utilizing quantum teleportation
  - Fidelity of entangled states can rapidly diminish due to noise
    - E.g. memory decoherence, noisy quantum gates, and noise (or lossy) quantum channels



$$\frac{1}{\sqrt{2}}\left(|0\rangle - i\,|1\rangle\right)$$

Coopmans, T., et al. Commun Phys 4, 164 (2021).



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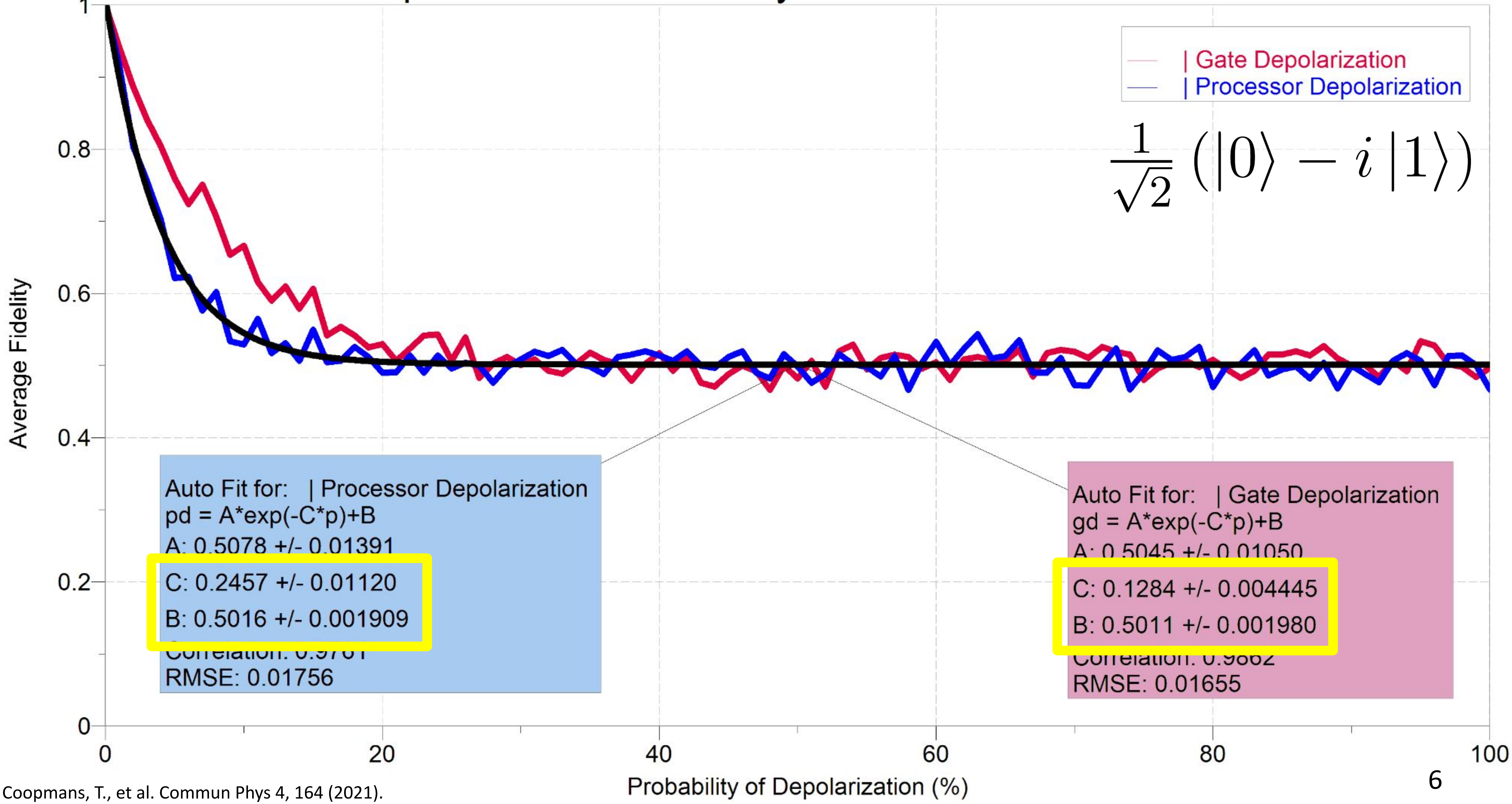
Depolarization on Fidelity: Gates vs Processor



Coopmans, T., et al. Commun Phys 4, 164 (2021).

Teleportation Using GHZ State

State to Teleport	Gate Depolarization: Decay Rate (C)	Gate Depolarization: Plateau (B)	Processor Depolarization: Decay Rate (C)	Processor Depolarization: Plateau (B)
$\|0\rangle$	0.0855	0.4988	0.1568	0.4999
$\frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)$	0.1147	0.4974	0.2049	0.4966
$\frac{1}{\sqrt{2}}(\|0\rangle - i\|1\rangle)$	0.1738	0.5007	0.3084	0.4978

Teleportation Using W-State

State to Teleport	Gate Depolarization: Decay Rate (C)	Gate Depolarization: Plateau (B)	Processor Depolarization: Decay Rate (C)	Processor Depolarization: Plateau (B)
$\|0\rangle$	0.1098	0.5004	0.2211	0.4986
$\frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)$	0.1433	0.6634	0.2652	0.6666
$\frac{1}{\sqrt{2}}(\|0\rangle - i\|1\rangle)$	0.2111	0.6627	0.397	0.6647

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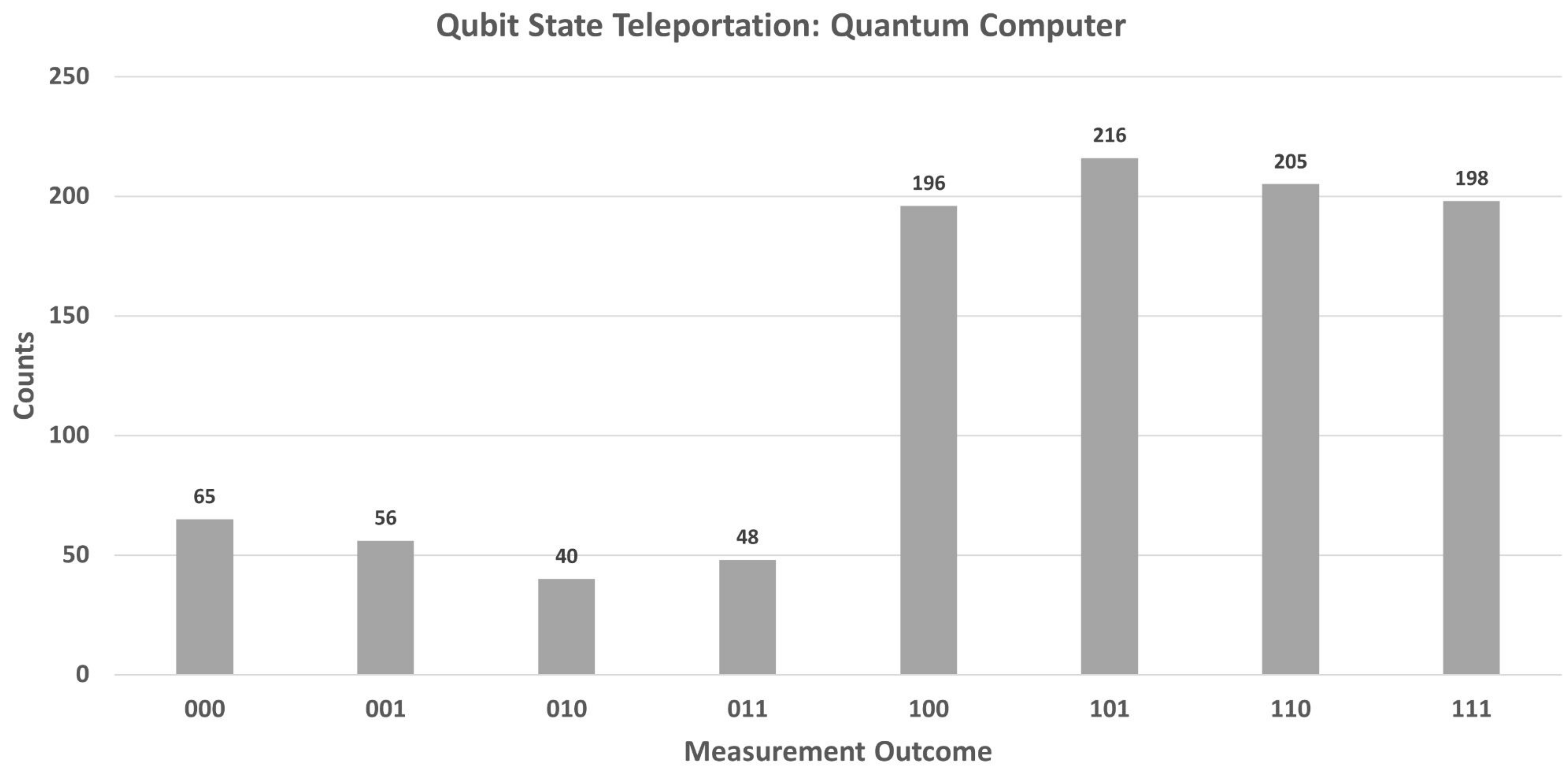
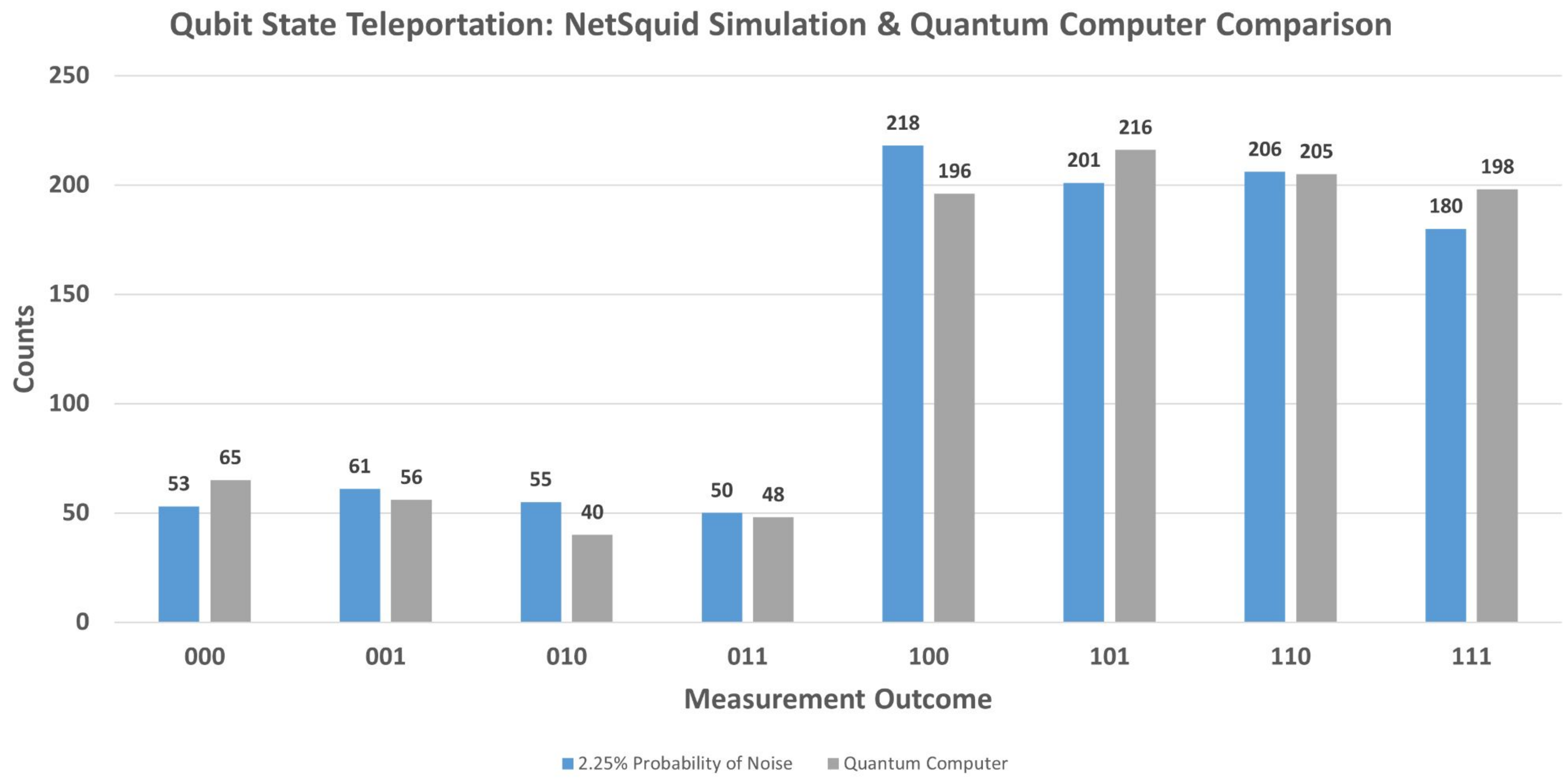
How can we use this information to simulate quantum computer noise environments?



IBM Quantum (2023). ibmq_lima (Version 1.0.49) [Falcon r4T].



Coopmans, T., et al. Commun Phys 4, 164 (2021).
IBM Quantum (2023). ibmq_lima (Version 1.0.49) [Falcon r4T].

Conclusions

Average Values of All Teleported States

Teleportation Circuit	Gate Depolarization: Fidelity Decay Rate	Gate Depolarization: Fidelity Plateau	Processor Depolarization: Fidelity Decay Rate	Processor Depolarization: Fidelity Plateau
Bell State	0.0921	0.4972	0.1704	0.4996
GHZ State	0.1055	0.4157	0.1899	0.4151
W-State	0.1585	0.6084	0.2967	0.6101

Thank you!

Dr. Inès Montaño

Jake Navas

Jaden Brewer

Alexander Hardin

Alan Valladares

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Background: NetSquid

- **Net**work **S**imulator for **Qu**antum **I**nformation using **D**iscrete events
- Nodes
  - Computers
  - Quantum sensors
- Models the impact of time on the performance of quantum networks and quantum computing systems

Fidelity of Quantum States

- A measure of the "closeness" of two quantum states

$$F(\rho, \sigma) \equiv \mathrm{tr}\sqrt{\rho^{1/2}\sigma\rho^{1/2}}$$

2.25% Noise = ~20% Error?

$$\text{If } P(A_1) = P(A_2) = \ldots = P(A_n):$$

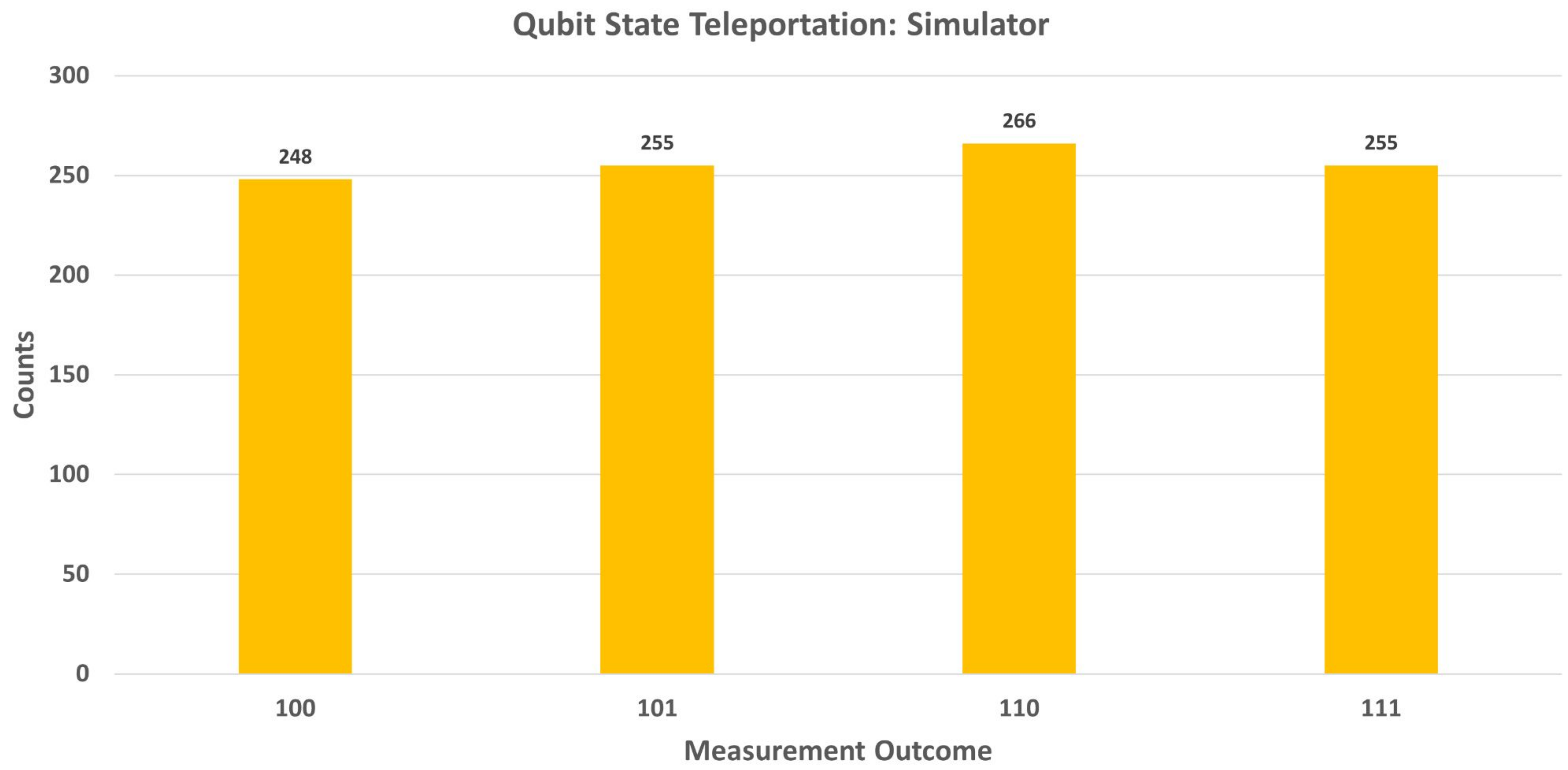
$$P(A_1 \cup A_2 \cup \ldots \cup A_n) = \sum_{i=1}^{n} (-1)^{i+1} \binom{n}{i} p^i$$

$$\text{where } p = P(A_1) = P(A_2) = \ldots = P(A_n).$$

2.25% Noise = ~20% Error?

$$\text{When } n = 10, p = 0.0225:$$

$$P(A_1 \cup A_2 \cup \ldots \cup A_{10}) = \sum_{i=1}^{10} (-1)^{i+1} \binom{10}{i} 0.0225^i = \boxed{0.2035}.$$



IBM Quantum (2023). ibmq_qasm_simulator (Version 0.1.547).

ibmq_lima: calibration data (Bell - 7/19/2023)

Qubit	T1 (us)	T2 (us)	Frequency (GHz)	Anharmonicity (GHz)	Readout assignment error
0	97.87596485	129.6657947	5.029748739	-0.335741553	0.022
1	142.567242	116.3382729	5.127740171	-0.318349123	0.0408
2	65.70349121	70.81956078	5.247360452	-0.333603661	0.0297
3	111.1314363	85.13604973	5.302580432	-0.331241932	0.0577
4	26.18833536	23.61742353	5.092087686	-0.334469657	0.0451
Mean	88.69329393	85.11542032	5.159903496	-0.330681185	0.03906
Median	97.87596485	85.13604973	5.127740171	-0.333603661	0.0408
Of only the first three:					
Mean	102.0488993	105.6078761	5.134949787	-0.329231445	0.030833333
Median	97.87596485	116.3382729	5.127740171	-0.333603661	0.0297

T1: Relaxation time

T2: Dephasing time

Frequency (GHz): The frequency(energy) associated with the transition between the qubit's ground state ($|0\rangle$) and first excited state ($|1\rangle$).

Readout error: The probability of preparing a $|0\rangle(|1\rangle)$ and measuring a $|1\rangle(|0\rangle)$, ie., of having an error in your readout

ibmq_lima: calibration data (Bell - 7/19/2023)

	Prob meas0 prep1	Prob meas1 prep0	Readout length (ns)	ID error	\sqrt{x} (sx) error	Pauli-X error
	0.0348	0.0092	5912.888889	0.000570268	0.000570268	0.000570268
	0.0676	0.014	5912.888889	0.00103715	0.00103715	0.00103715
	0.0528	0.0066	5912.888889	0.000825927	0.000825927	0.000825927
	0.084	0.0314	5912.888889	0.001394263	0.001394263	0.001394263
	0.0764	0.0138	5912.888889	0.000656327	0.000656327	0.000656327
Mean	0.06312	0.015	5912.888889	0.000896787	0.000896787	0.000896787
Median	0.0676	0.0138	5912.888889	0.000825927	0.000825927	0.000825927
Of only the first three:						
Mean	0.051733333	0.009933333	5912.888889	0.000811115	0.000811115	0.000811115
Median	0.0528	0.0092	5912.888889	0.000825927	0.000825927	0.000825927

ibmq_lima: calibration data (Bell - 7/19/2023)

CNOT error	Gate time (ns)
0_1:0.007159745408065854	0_1:305.77777777777777
1_0:0.007159745408065854; 1_3:0.01741388385831824; 1_2:0.007884061799106723	1_0:341.3333333333333; 1_3:497.77777777777777; 1_2:334.22222222222223
2_1:0.007884061799106723	2_1:298.66666666666663
3_4:0.030634203062174348; 3_1:0.01741388385831824	3_4:519.1111111111111; 3_1:462.2222222222222
4_3:0.030634203062174348	4_3:483.55555555555554

ibmq_lima: calibration data (GHZ - 8/2/2023)

Qubit	T1 (us)	T2 (us)	Frequency (GHz)	Anharmonicity (GHz)	Readout assignment error
0	127.2228592	192.2952686	5.029744087	-0.335741553	0.0279
1	122.3811919	126.3815463	5.127745261	-0.318349123	0.0169
2	68.41142026	142.5541098	5.247353298	-0.333603661	0.018
3	125.953597	110.8255667	5.302551534	-0.331241932	0.0333
4	17.21016343	18.72160498	5.092086016	-0.334469657	0.047
Mean	92.23584637	118.1556193	5.159896039	-0.330681185	0.02862
Median	122.3811919	126.3815463	5.127745261	-0.333603661	0.0279

Of only the first four:

Mean	110.9922671	143.0141228	5.176848545	-0.329734067	0.024025
Median	124.1673945	134.467828	5.18754928	-0.332422796	0.02295

T1: Relaxation time

T2: Dephasing time

Frequency (GHz): The frequency(energy) associated with the transition between the qubit's ground state ($|0\rangle$) and first excited state ($|1\rangle$).

Readout error: The probability of preparing a $|0\rangle(|1\rangle)$ and measuring a $|1\rangle(|0\rangle)$, ie., of having an error in your readout

ibmq_lima: calibration data (GHZ - 8/2/2023)

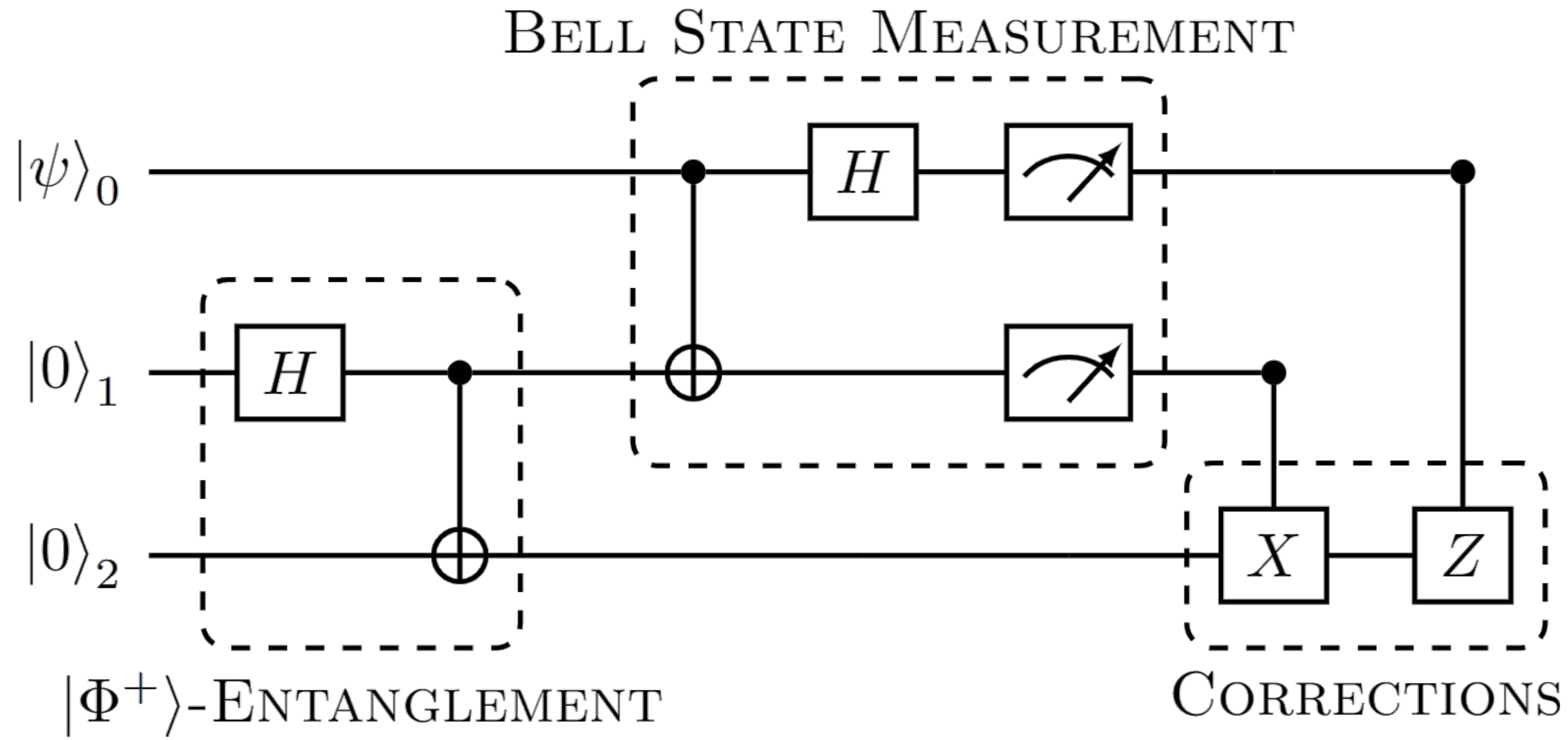
	Prob meas0 prep1	Prob meas1 prep0	Readout length (ns)	ID error	\sqrt{x} (sx) error	Pauli-X error
	0.0434	0.0124	5912.888889	0.000765315	0.000765315	0.000765315
	0.0262	0.0076	5912.888889	0.000246975	0.000246975	0.000246975
	0.0282	0.0078	5912.888889	0.000486996	0.000486996	0.000486996
	0.048	0.0186	5912.888889	0.000507311	0.000507311	0.000507311
	0.0758	0.0182	5912.888889	0.000722864	0.000722864	0.000722864
Mean	0.04432	0.01292	5912.888889	0.000545892	0.000545892	0.000545892
Median	0.0434	0.0124	5912.888889	0.000507311	0.000507311	0.000507311
Of only the first four:						
Mean	0.03645	0.0116	5912.888889	0.000501649	0.000501649	0.000499762
Median	0.0358	0.0101	5912.888889	0.000497154	0.000497154	0.000486996

ibmq_lima: calibration data (GHZ - 8/2/2023)

CNOT error	Gate time (ns)
0_1:0.008877829144317506	0_1:305.77777777777777
1_3:0.015287333729115837; 1_2:0.006275413908105404; 1_0:0.008877829144317506	1_3:497.77777777777777; 1_2:334.22222222222223; 1_0:341.3333333333333
2_1:0.006275413908105404	2_1:298.66666666666663
3_4:0.01879316874610462; 3_1:0.015287333729115837	3_4:519.1111111111111; 3_1:462.2222222222222
4_3:0.01879316874610462	4_3:483.55555555555554

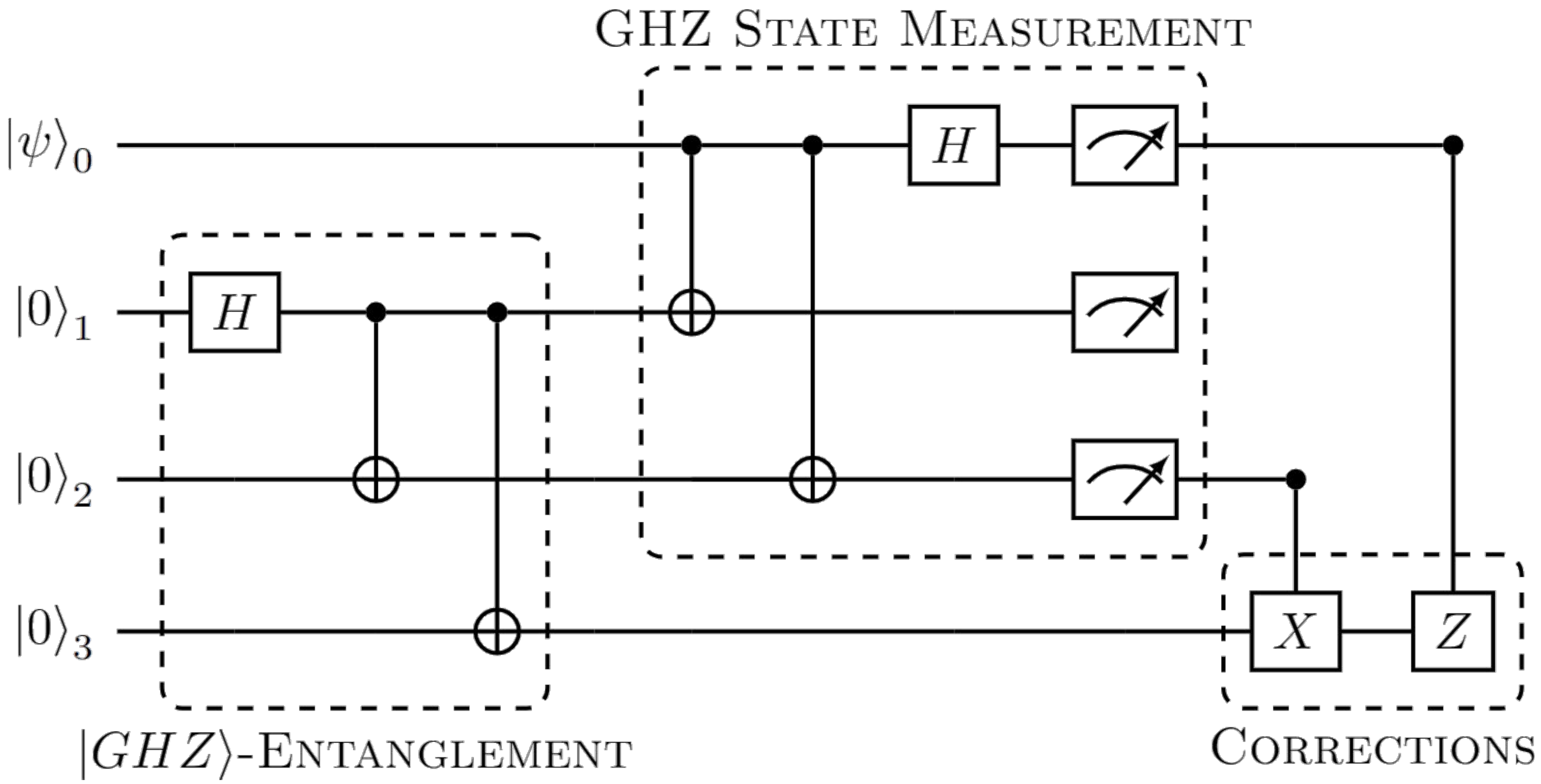
Teleportation Circuits

- Bell State:



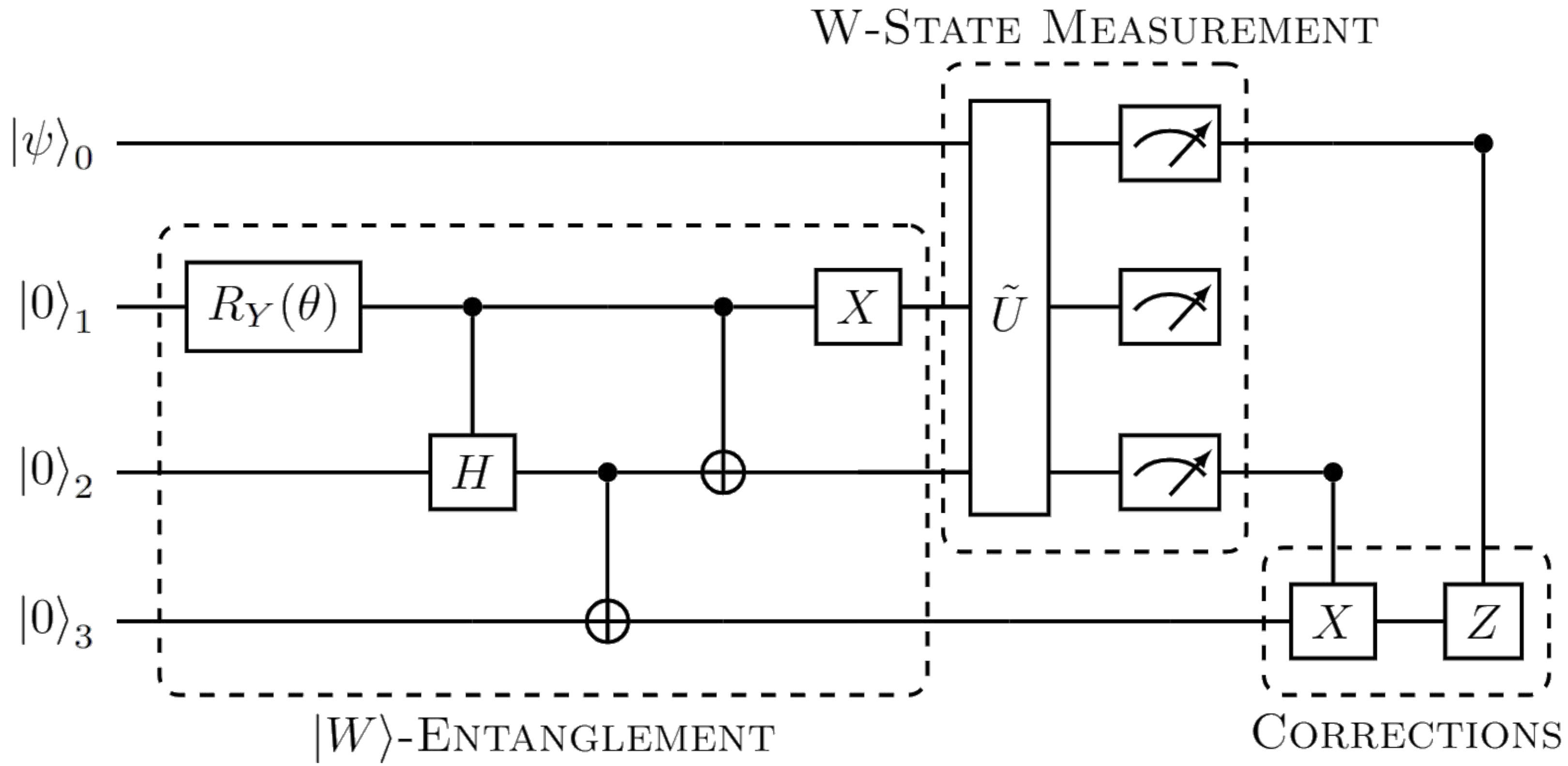
Teleportation Circuits

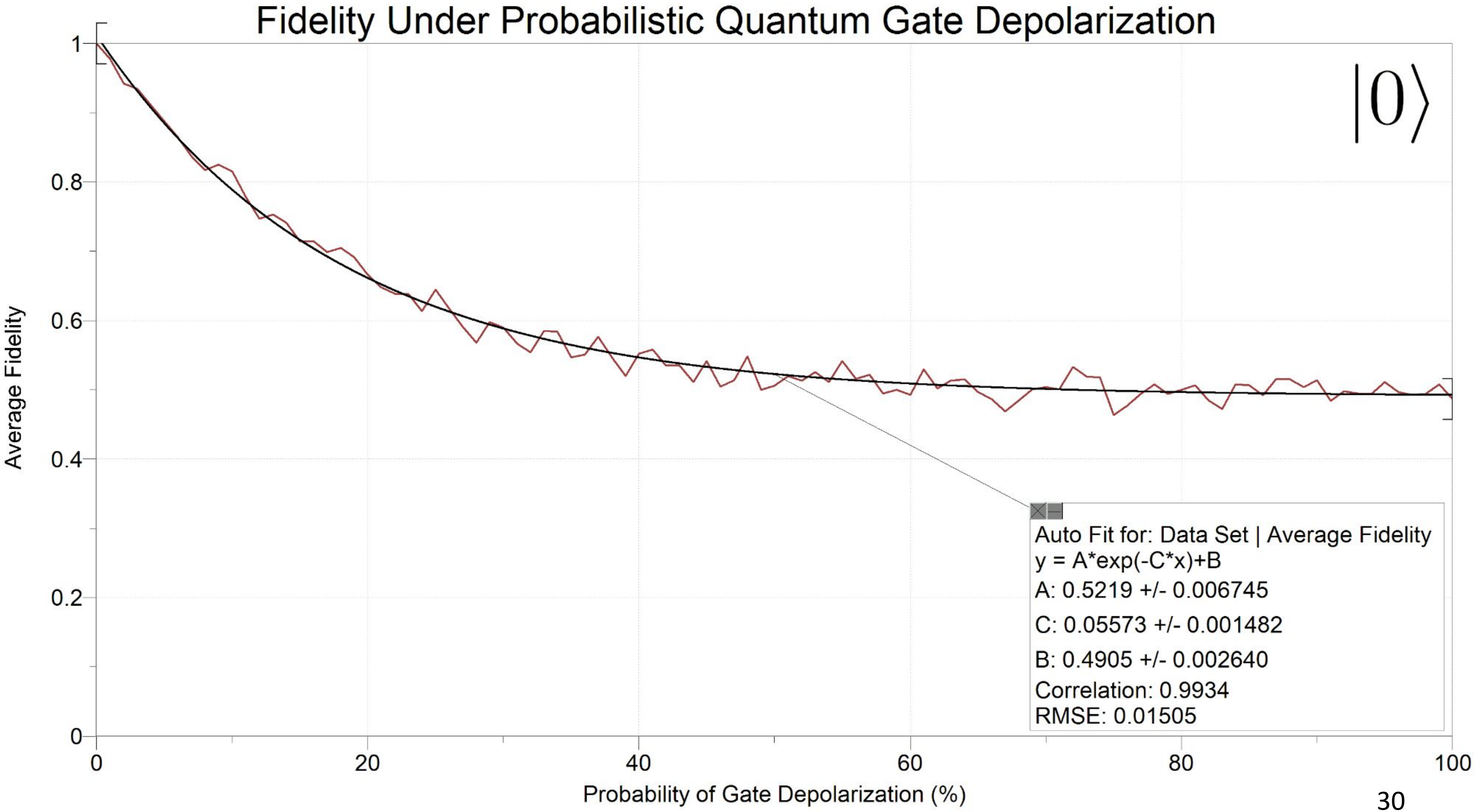
- GHZ State:

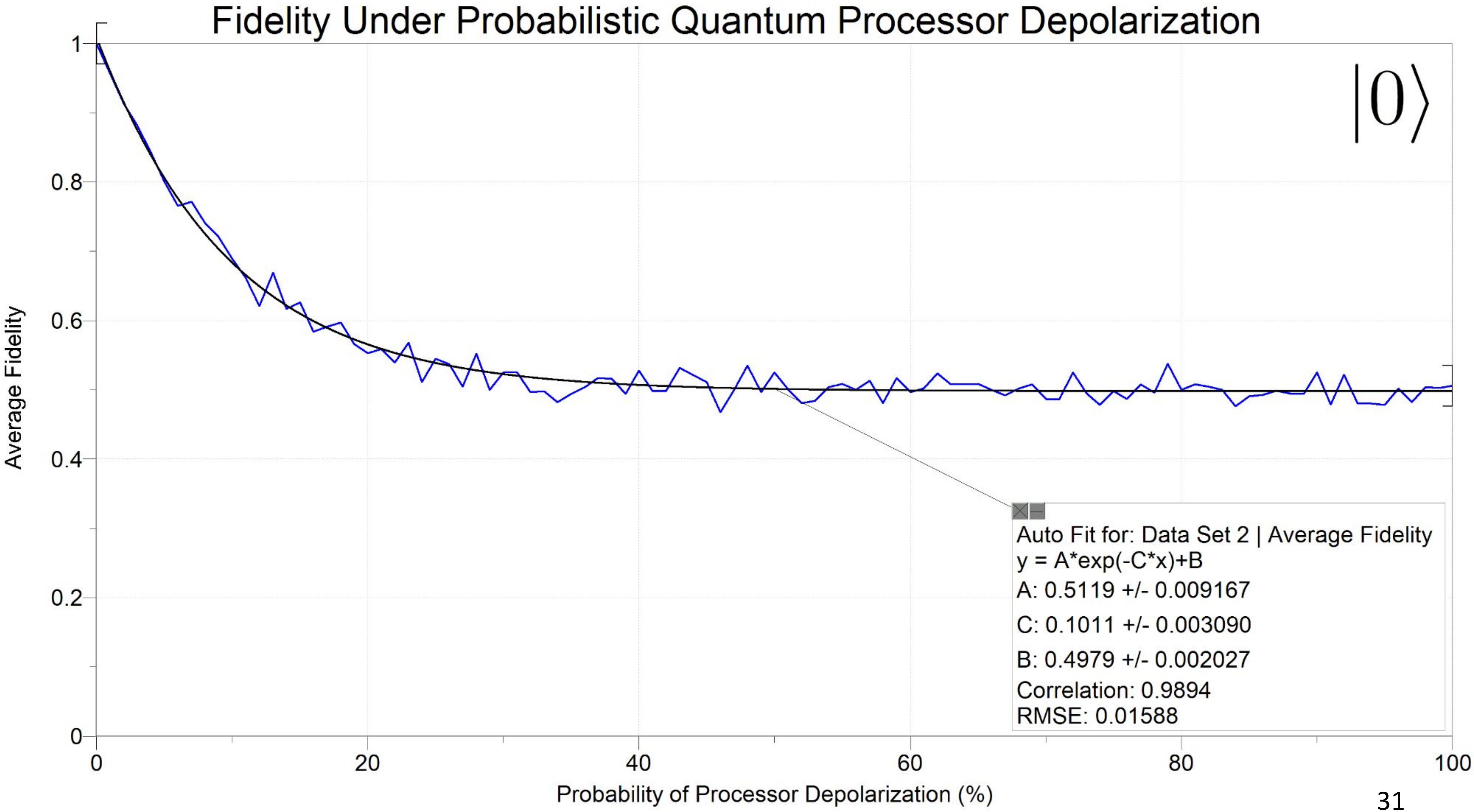


Teleportation Circuits

- W-State:




Fidelity Under Probabilistic Quantum Gate Depolarization
$|0\rangle$
Average Fidelity
Probability of Gate Depolarization (%)
0
0.2
0.4
0.6
0.8
1
0
20
40
60
80
100
Auto Fit for: Data Set | Average Fidelity
y = A*exp(-C*x)+B
A: 0.5219 +/- 0.006745
C: 0.05573 +/- 0.001482
B: 0.4905 +/- 0.002640
Correlation: 0.9934
RMSE: 0.01505


Fidelity Under Probabilistic Quantum Processor Depolarization
|0⟩
Average Fidelity
1
0.8
0.6
0.4
0.2
0
0
20
40
60
80
100
Probability of Processor Depolarization (%)
Auto Fit for: Data Set 2 | Average Fidelity
y = A*exp(-C*x)+B
A: 0.5119 +/- 0.009167
C: 0.1011 +/- 0.003090
B: 0.4979 +/- 0.002027
Correlation: 0.9894
RMSE: 0.01588