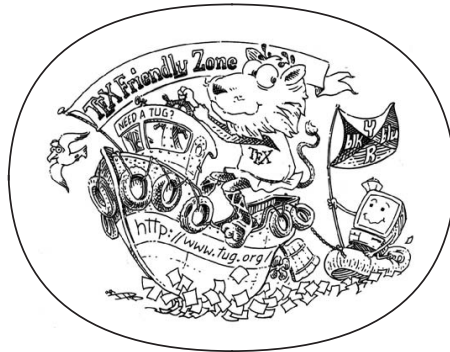


GENERAL RELATIVITY RAYTRACER

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A massively parallel free software alternative

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Omnia sunt communia

— Thomas Müntzer, 1524

ABSTRACT

Short summary of the contents in English...a great guide by Kent Beck how to write good abstracts can be found here:

<https://plg.uwaterloo.ca/~migod/research/beck00PSLA.html>

*We have seen that computer programming is an art,
because it applies accumulated knowledge to the world,
because it requires skill and ingenuity, and especially
because it produces objects of beauty.*

— knuth:1974 [knuth:1974]

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Put your acknowledgments here.

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¹ Members of GuIT (Gruppo Italiano Utilizzatori di T_EX e L^AT_EX)

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ACRONYMS

Part I

INTRODUCTION

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INTRODUCTION TO DIFFERENTIAL GEOMETRY

1.1 DIFFERENTIABLE MANIFOLDS

Roughly speaking, a manifold is a topological space that, locally, looks like the Euclidean space \mathbb{R}^n . This similitude is essential, and will let us control the manifold as if we were working in the Euclidean space; generally, its properties will be proved using the known properties of \mathbb{R}^n .

The following definition specifies the formal concept of a topological manifold:

Definition 1 (N-dimensional topological manifold) *Let M^n be an n -dimensional topological space. The space M^n is called a topological manifold if the following properties are satisfied:*

1. M^n is locally homeomorphic to \mathbb{R}^n .
2. M^n is a Hausdorff space.
3. M^n has a countable topological basis.

The first property states that, for every point $p \in M^n$, there exists an open neighbourhood $U \subset M^n$ of p and a homeomorphism

$$h: U \rightarrow V$$

with $V \subset \mathbb{R}^n$ an open set.

The local homeomorphism does not imply the manifold to be Hausdorff, and this will be an essential property throughout the study of these spaces. The usual counterexample is the line with two origins: let $M = \mathbb{R} \cup p$ be the union of the real line and a point $p \notin \mathbb{R}$. Define a topology in this space with $\mathbb{R} \subset M$ as an open set and the neighbourhoods of p being the sets $(U \setminus \{0\}) \cup \{p\}$, where U is a neighbourhood of $0 \in \mathbb{R}$. This space is locally Euclidean but not Hausdorff: the intersection of any two neighbourhoods of the points $0 \in \mathbb{R}$ and p is non-empty.

The last property will prove to be key in our study, as it will let us define metrics on the manifold.

1.1.1 Charts

We can now focus on the characteristic of the manifolds that will let us

Definition 2 (Coordinate chart) A coordinate chart —or coordinate system— in a topological manifold M^n is a homeomorphism $h: U \rightarrow V$ from an open subset of the manifold $U \subset M$ onto an open subset of the Euclidean space $V \subset \mathbb{R}^n$.

We call U a coordinate neighbourhood in M .

Definition 3 (Coordinate atlas) Let

$$A = \{h_\alpha: U_\alpha \rightarrow V_\alpha / \alpha \in I\}$$

be a set of coordinate charts in a topological manifold M^n , where I is a family of indices and the open subsets $U_\alpha \subset M$ are the corresponding coordinate neighbourhoods.

A is said to be an atlas of M if every point is covered with a coordinate neighbourhood; i.e., if $\bigcup_{\alpha \in I} U_\alpha = M$.

1.1.2 Differentiable structures

Definition 4 (Transition maps)

Definition 5 (Smooth coordinate atlas)

Definition 6 (Maximal atlas)

Proposition 7 (Maximal atlas uniqueness)

Definition 8 (Differentiable structure)

Definition 9 (Differentiable manifold)

Part II

APPENDIX

APPENDIX TEST

Lorem ipsum at nusquam appellantur his, ut eos erant homero concludaturque. Albucius appellantur deterruisset id eam, vivendum partiendo dissentiet ei ius. Vis melius facilisis ea, sea id convenire referrentur, takimata adolescens ex duo. Ei harum argumentum per. Eam vidit exerci appetere ad, ut vel zzril intellegam interpretaris.

More dummy text.

A.1 APPENDIX SECTION TEST

Test: [Table 1](#) (This reference should have a lowercase, small caps A if the option floatperchapter is activated, just as in the table itself → however, this does not work at the moment.)

LABITUR BONORUM PRI NO	QUE VISTA	HUMAN
fastidii ea ius	germano	demonstratea
suscipit instructor	titulo	personas
quaestio philosophia	facto	demonstrated

Table 1: Autem usu id.

A.2 ANOTHER APPENDIX SECTION TEST

Equidem detraxit cu nam, vix eu delenit periculis. Eos ut vero constituto, no vidit propriae complectitur sea. Diceret nonummy in has, no qui eligendi recteque consetetur. Mel eu dictas suscipiantur, et sed placerat oporteat. At ipsum electram mei, ad aequae atomorum mea. There is also a useless Pascal listing below: [Listing 1](#).

Listing 1: A floating example (listings manual)

```
for i:=maxint downto 0 do
begin
{ do nothing }
end;
```


BIBLIOGRAPHY

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DECLARATION

Put your declaration here.

Granada, September 2016

Alejandro García Montoro

COLOPHON

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