Stability Analysis of Switched DC-DC Boost Converters for Integrated Circuits

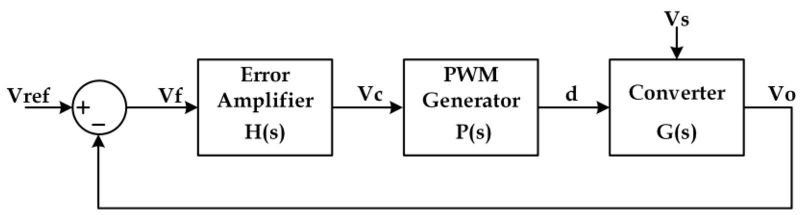
Kevin Fronczak

Advisor: Dr. Robert Bowman

Overview

Switched DC-DC Converters

- Why does stability matter?
- How does the architecture affect instability?
- How does component variability affect stability?
- What methods are used to minimize instability?
- How can stability be measured?

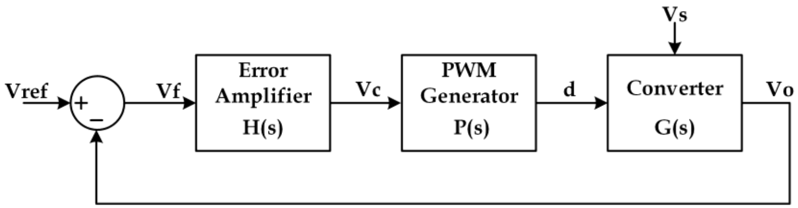


Switched Converter Block Diagram

Importance of Stability

Switched DC-DC Converters

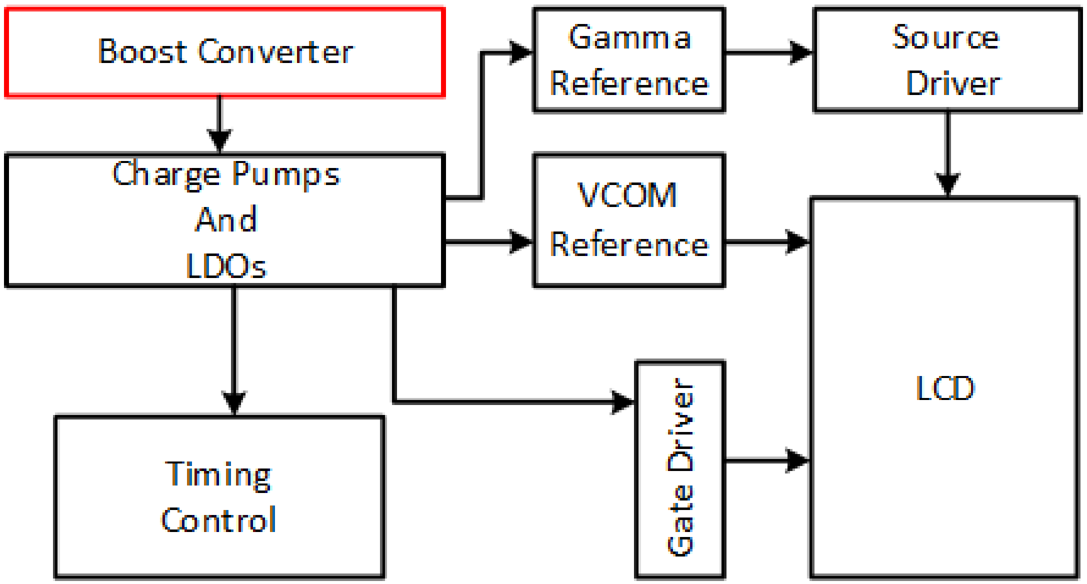
- Supply oscillations can couple into signal paths
  - Sensitive circuits suffer
- Unstable supply = inaccurate supply
- Inaccurate supply = performance degradation



Switched Converter Block Diagram

Importance of Stability

Display Driver Block Diagram

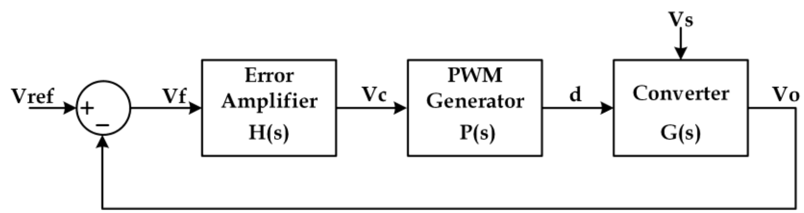


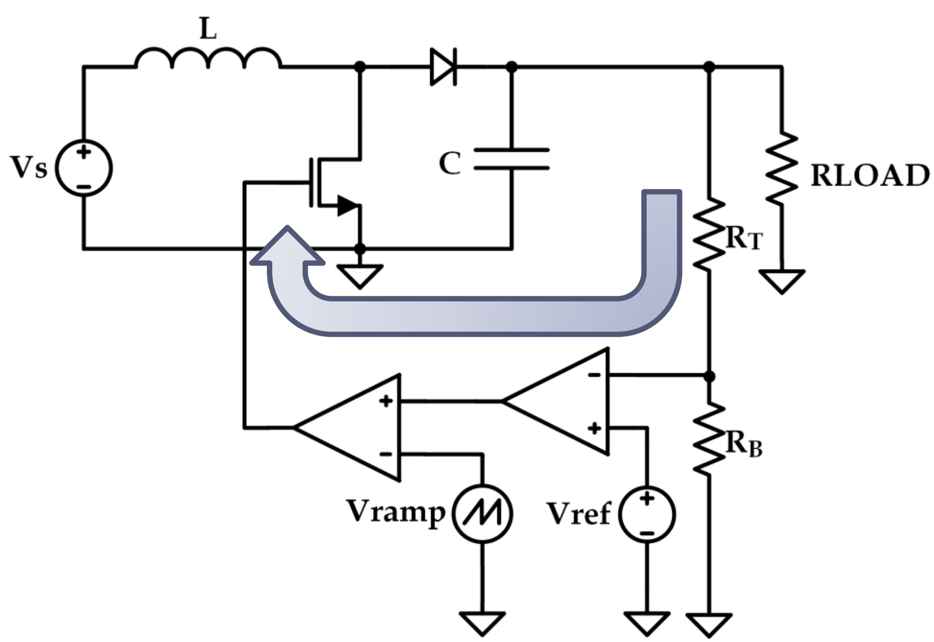
Adapted from Yang-Ching Lin *et. al.* (2012)

Importance of Stability

Boost Converter Architecture

- Feedback loop
  - Chance of oscillation at output

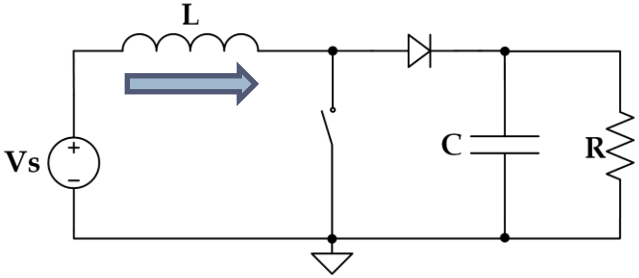


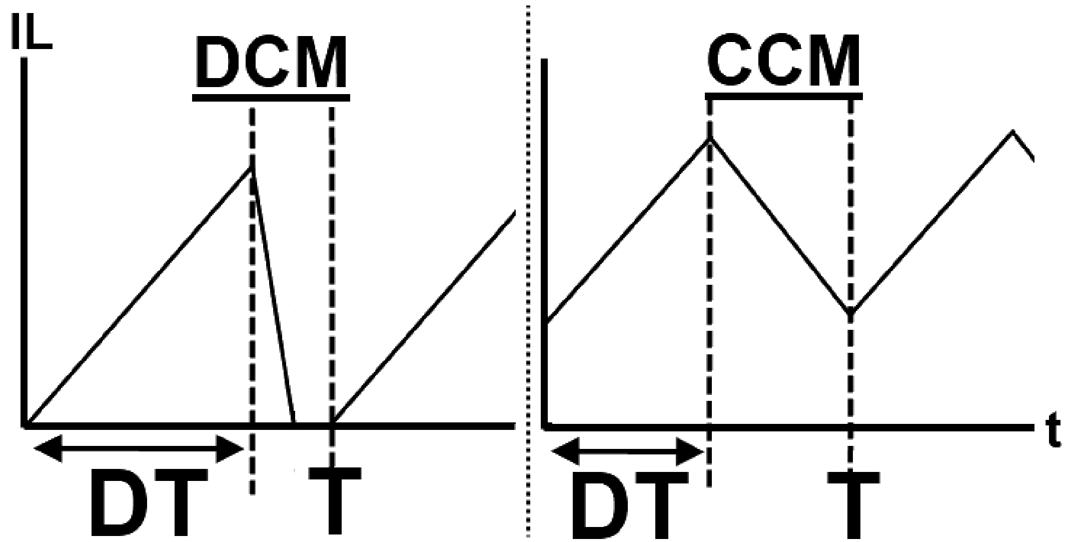


Causes of Instability

Operating Modes

- Operating Mode
  - CCM (Continuous Conduction Mode)
  - DCM (Discontinuous Conduction Mode)



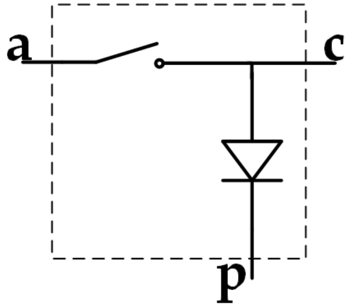


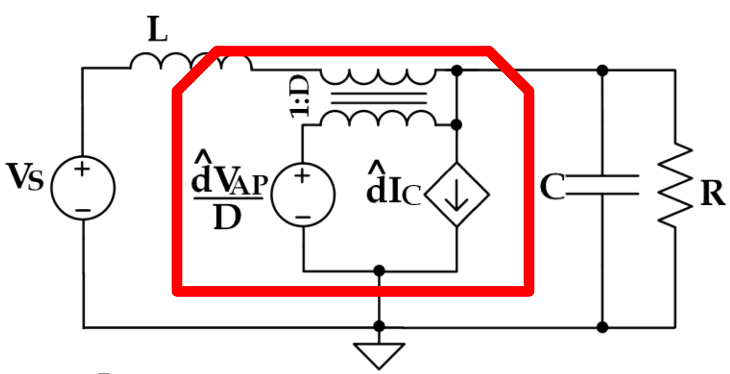
Causes of Instability

Small-Signal Modeling

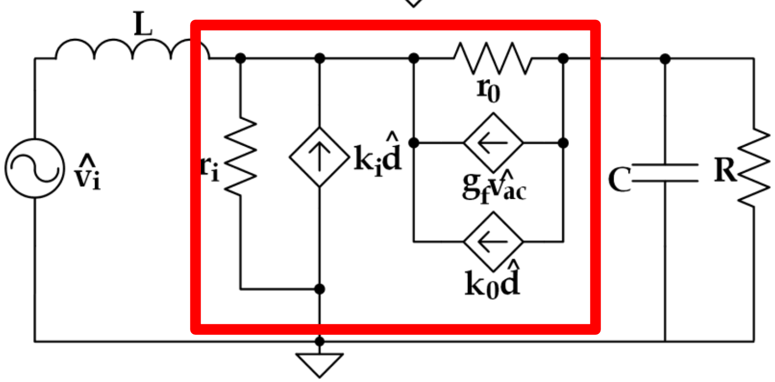
Average PWM Switch

Vorperian (1990)





Continuous Mode

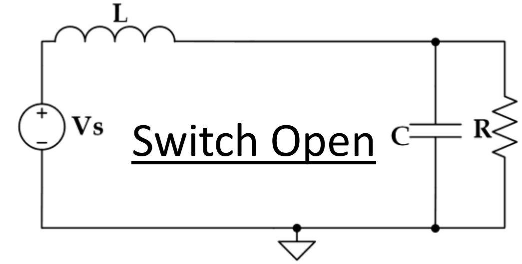


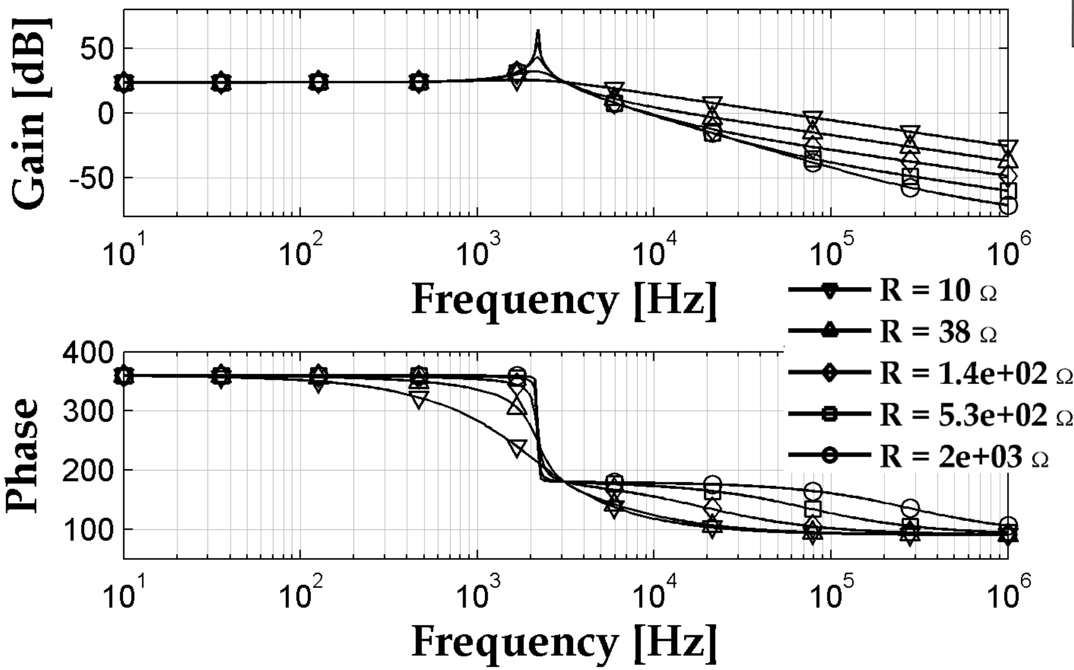
Discontinuous Mode

Causes of Instability (CCM)

Bode Plot for Continuous Mode

- CCM has conjugate pole
  - LC Resonance



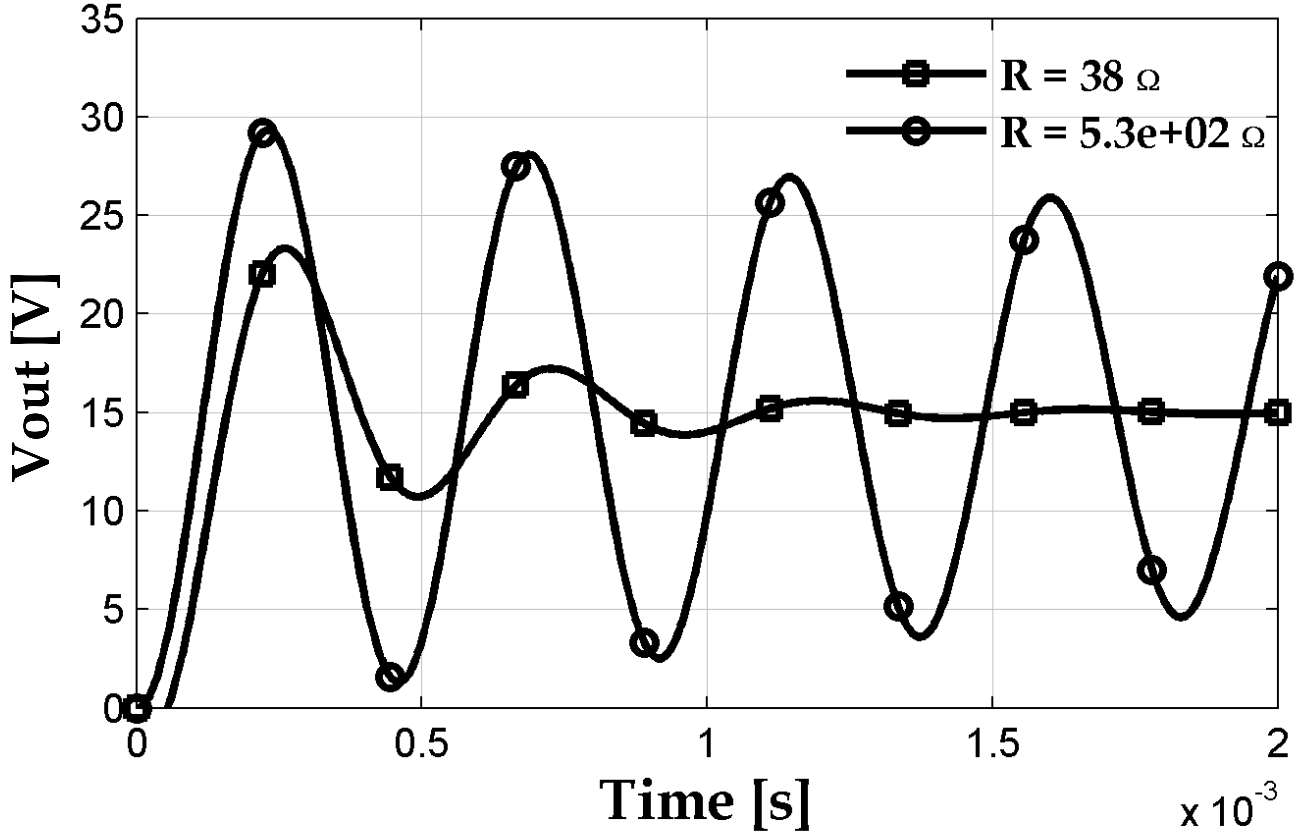


$$\omega_0 = \frac{1-D}{\sqrt{LC}}$$

$$Q = R(1-D)\sqrt{\frac{C}{L}}$$

Causes of Instability (CCM)

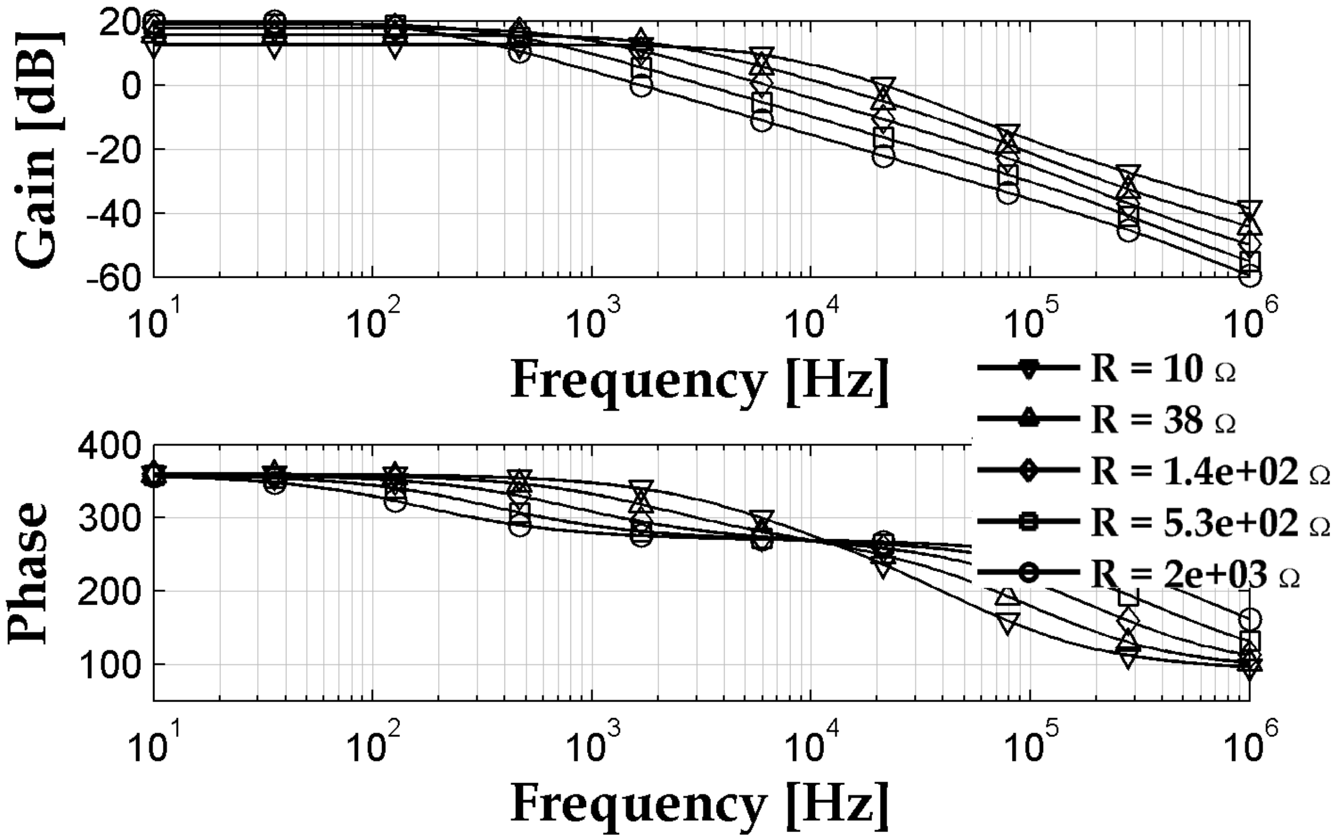
Input Step Response for Continuous Mode



Causes of Instability (DCM)

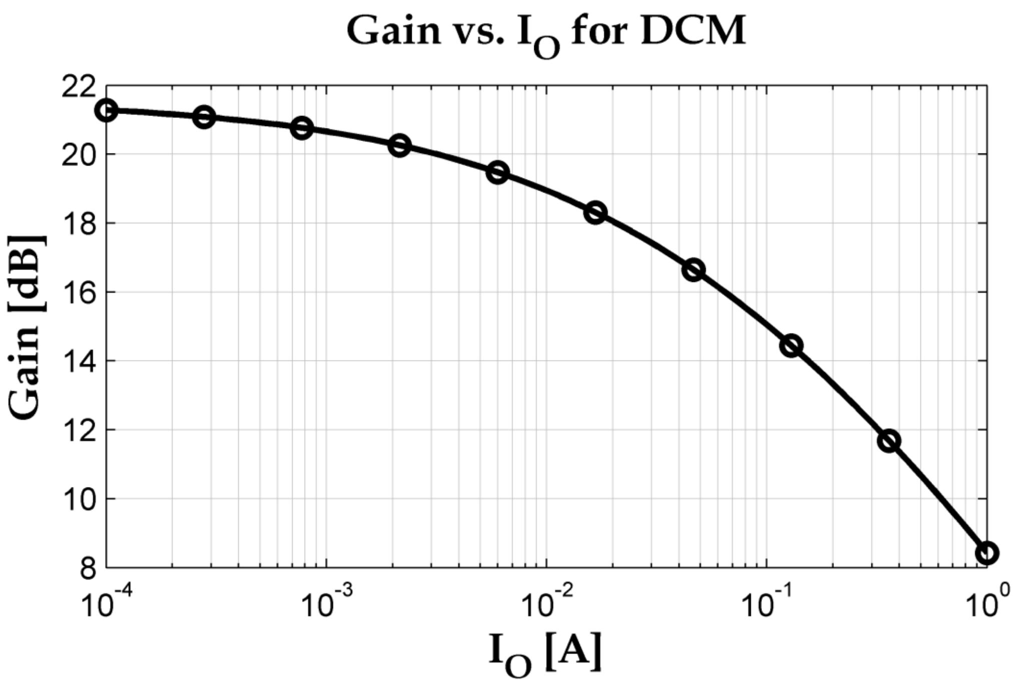
Bode Plot for Discontinuous Mode

- DCM Appears 1st order at low frequencies



Causes of Instability (DCM)

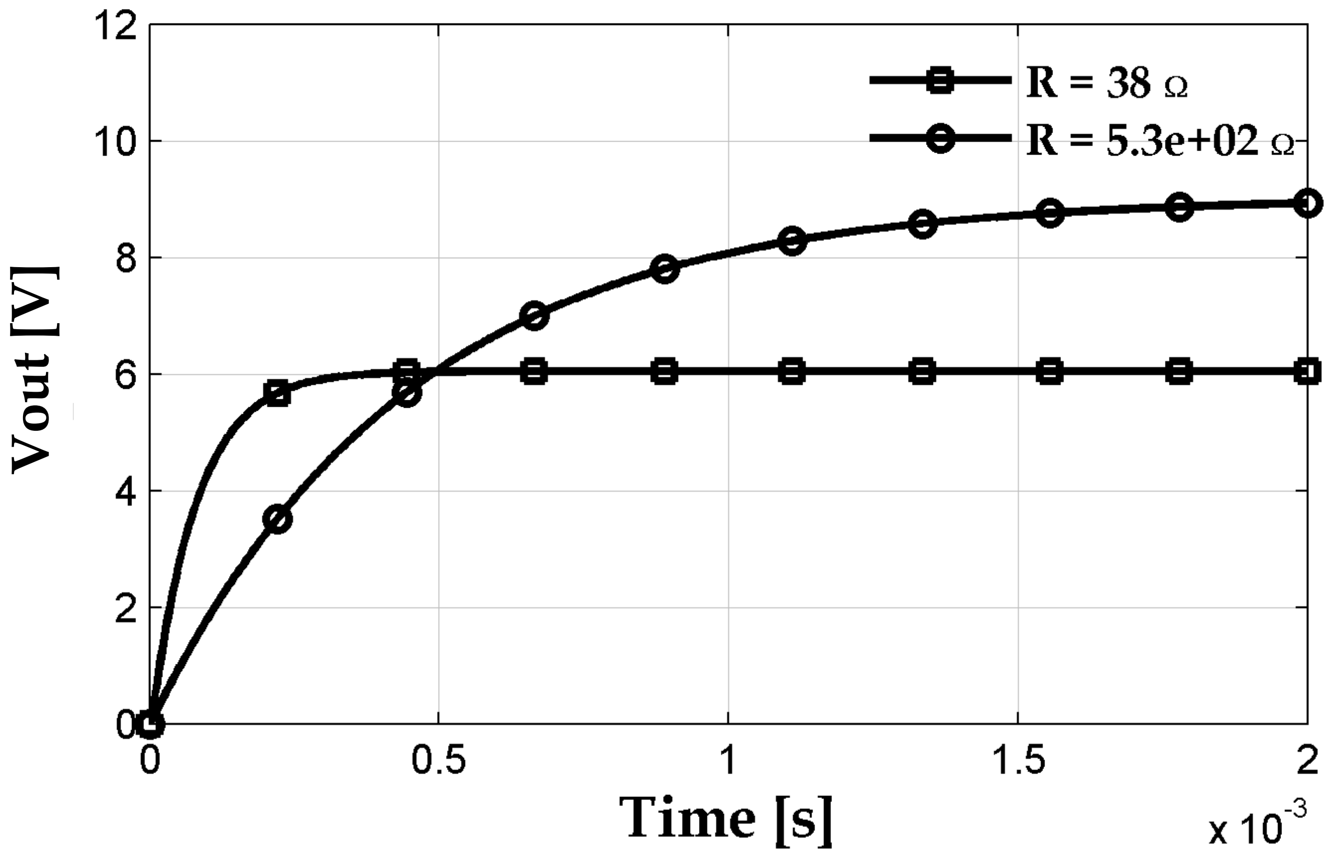
Change in Gain for Discontinuous Mode



$$G_{d0} = \frac{2V_O}{\sqrt{\frac{LF_S I_O}{2V_O}\left[(2\frac{V_O}{V_S} - 1)^2 - 1\right] + 1}}$$

Causes of Instability (DCM)

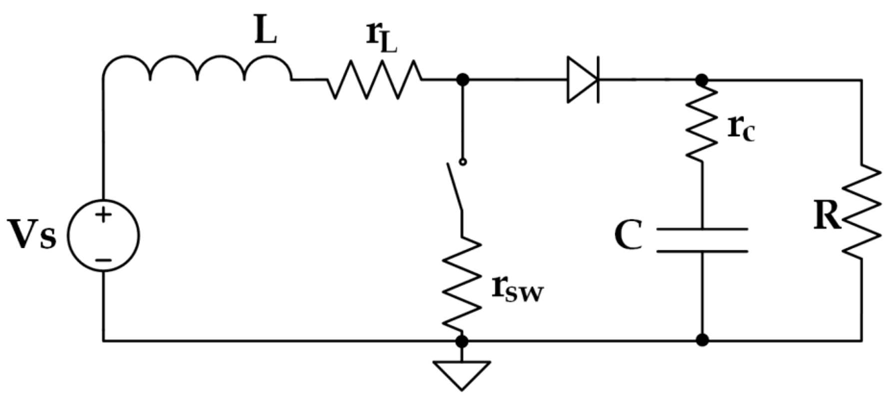
Input Step Response for Discontinuous Mode



Parasitic Component Effects

Non-ideal Converter Schematic

- Inductor has series resistance
  - Lowers height of resonant peak (CCM)
- Capacitor has series resistance
  - Adds high-frequency zero
  - Less attenuation at frequencies > Fs

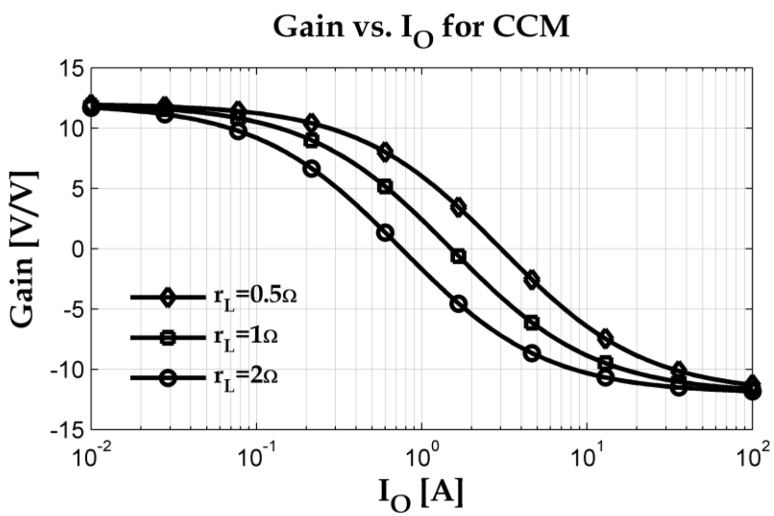


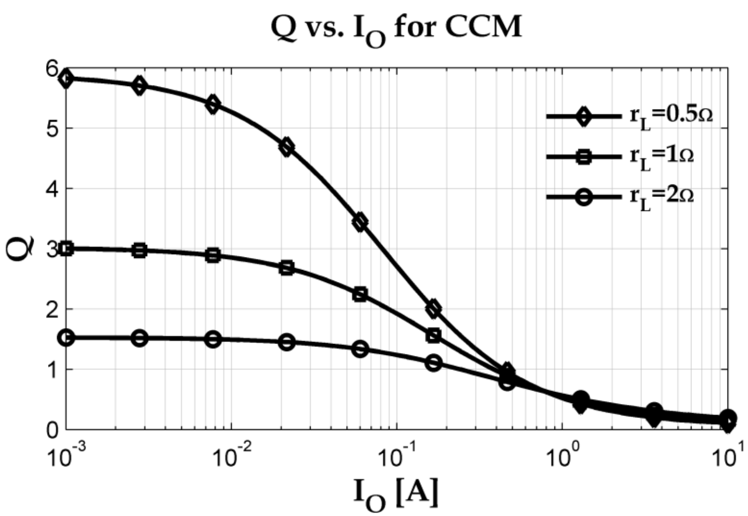
Parasitic Component Effects

Continuous Mode: Expected Behavior

$$G_{d0} = \frac{V_O}{1-D}\frac{R(1-D)^2 - r_L}{R(1-D)^2 + r_L}$$

$$Q = \sqrt{LC}\frac{\sqrt{(R+r_C)(R(1-D)^2 + r_L)}}{Rr_C C(1-D)^2 + Rr_L C + r_C r_L C + L}$$

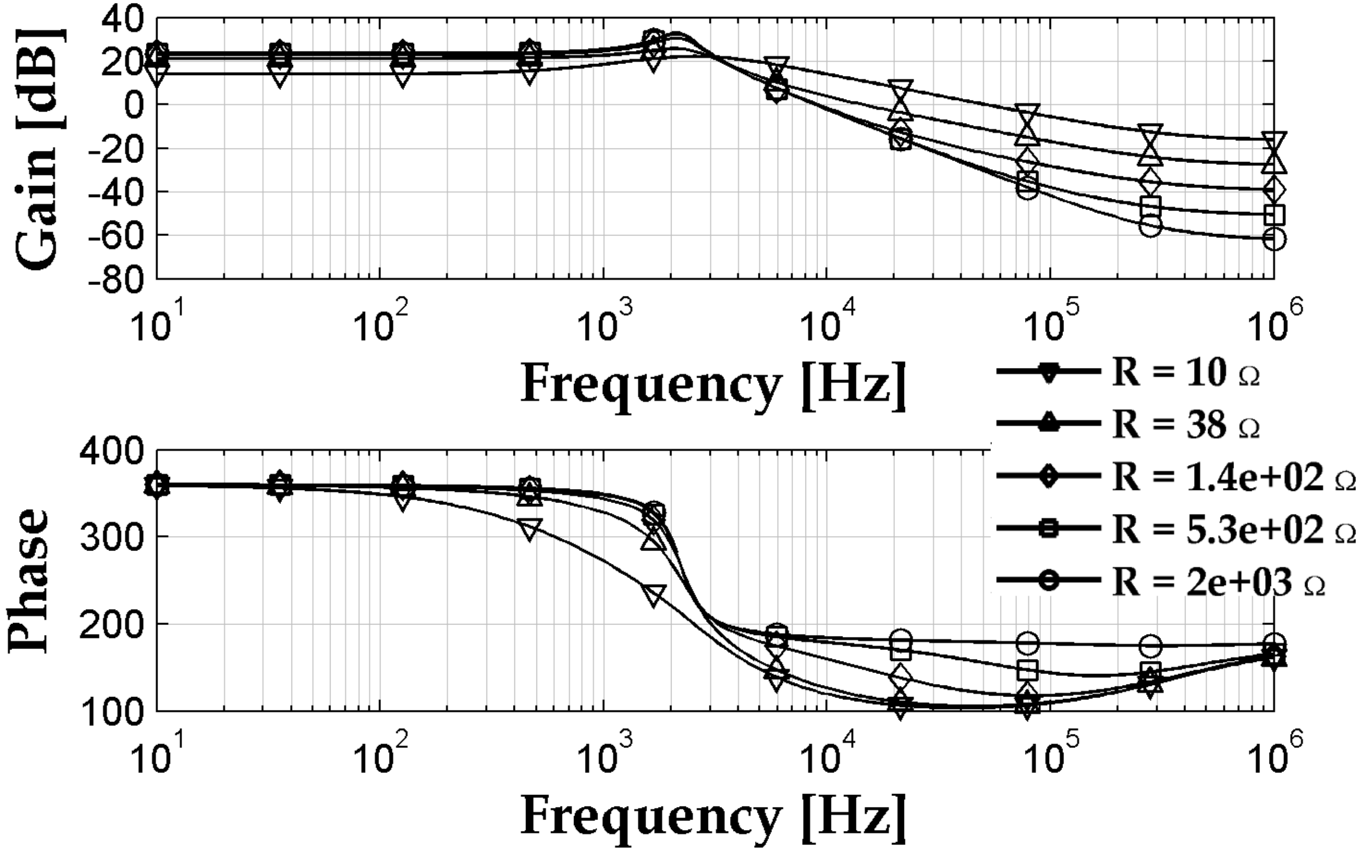




Parasitic Effects (CCM)

Bode Plot for Continuous Mode

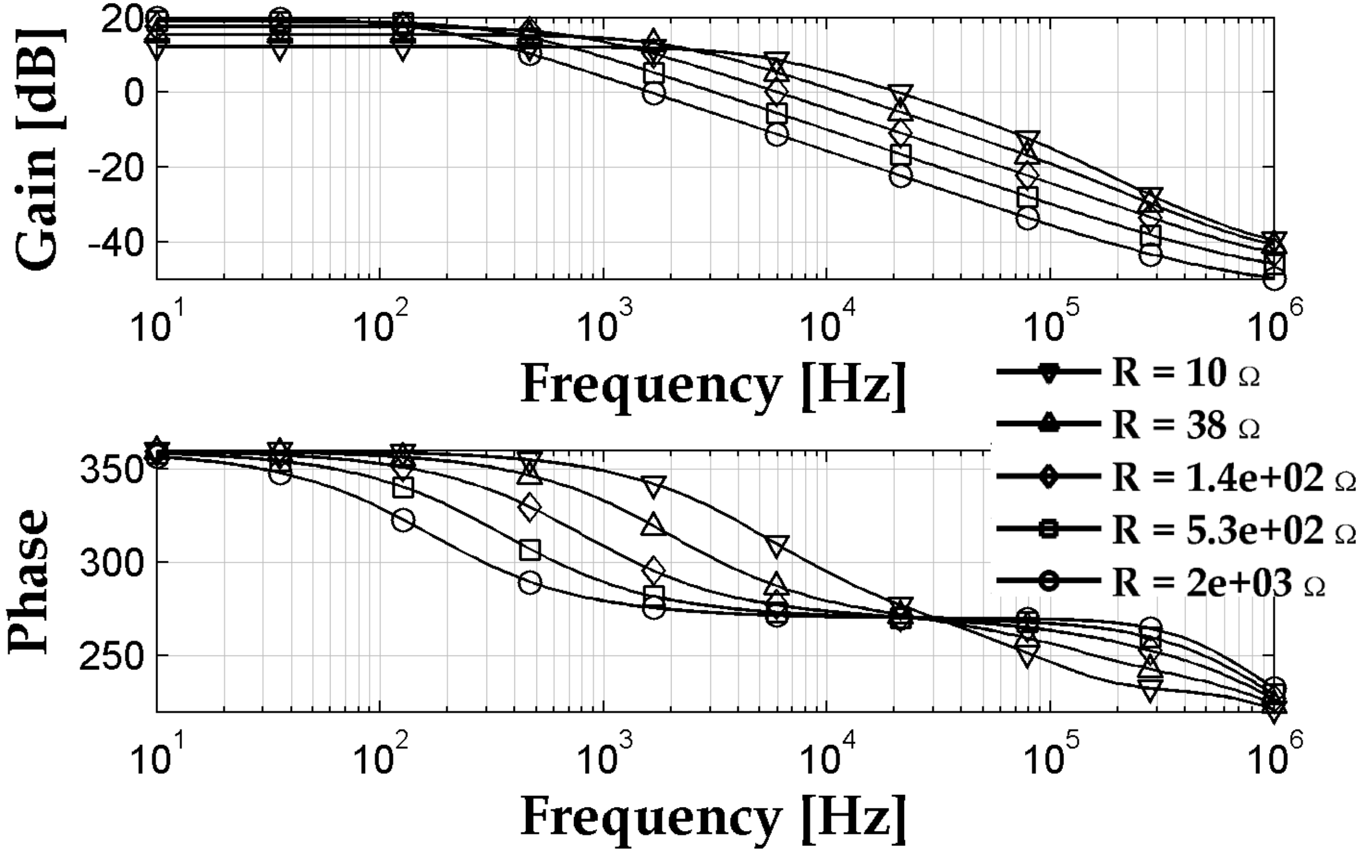
- Bode Plot



Parasitic Effects (DCM)

Bode Plot for Discontinuous Mode

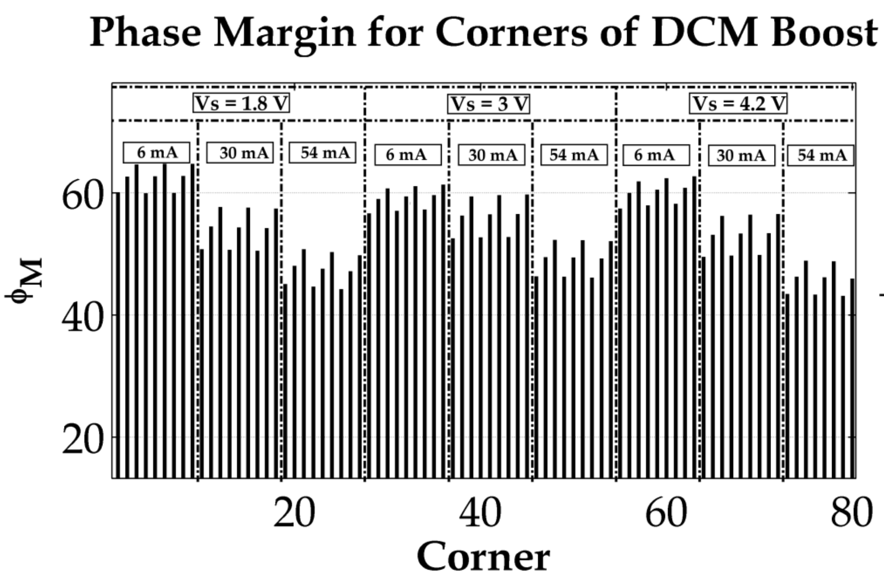
- Bode Plot

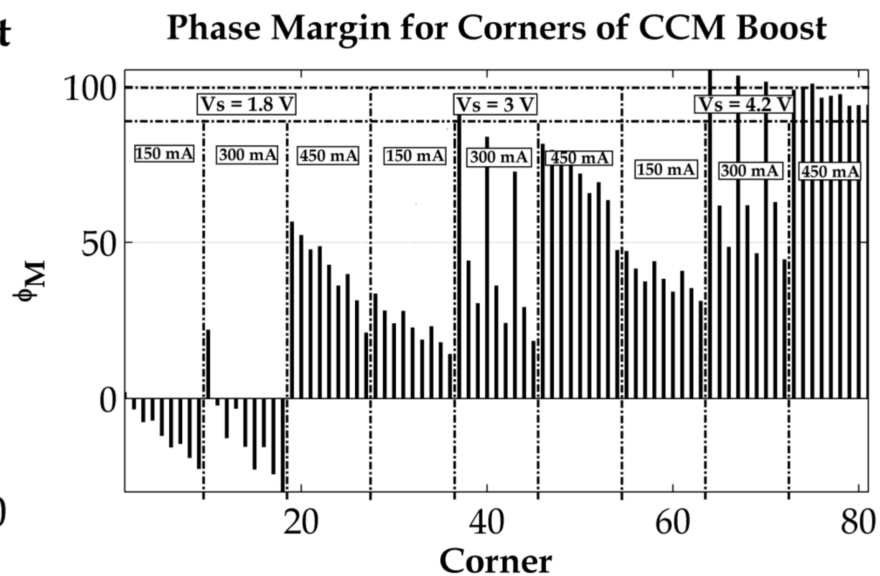


Component Variation

Effect on Phase Margin

- Input Voltage, Load Current, Inductance
  - Vo/Vs = [3.3, 2, 1.4]
  - Io = [6 mA, 30 mA, 54 mA] (DCM); [150 mA, 300 mA, 450 mA] (CCM)
  - L = [15 uH, 20 uH, 25 uH] (DCM); [170 uH, 200 uH, 230 uH] (CCM)



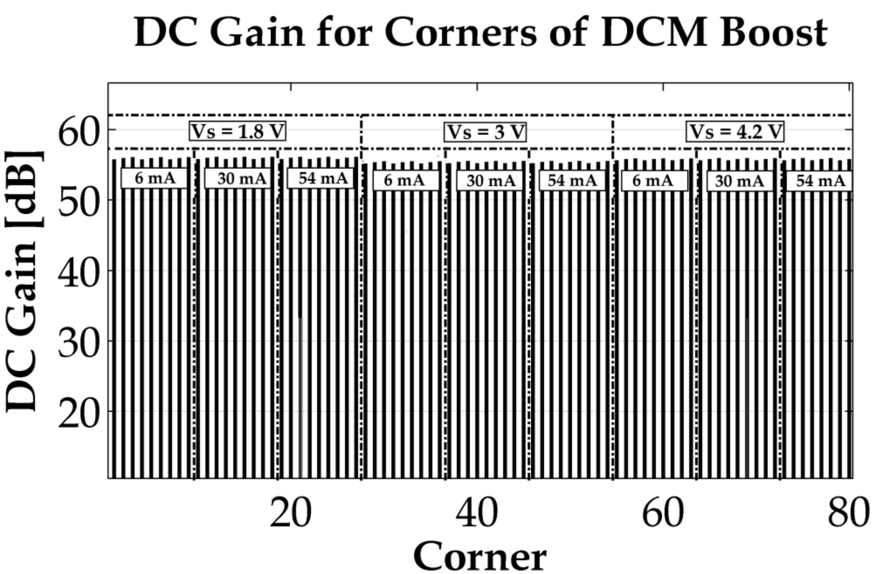


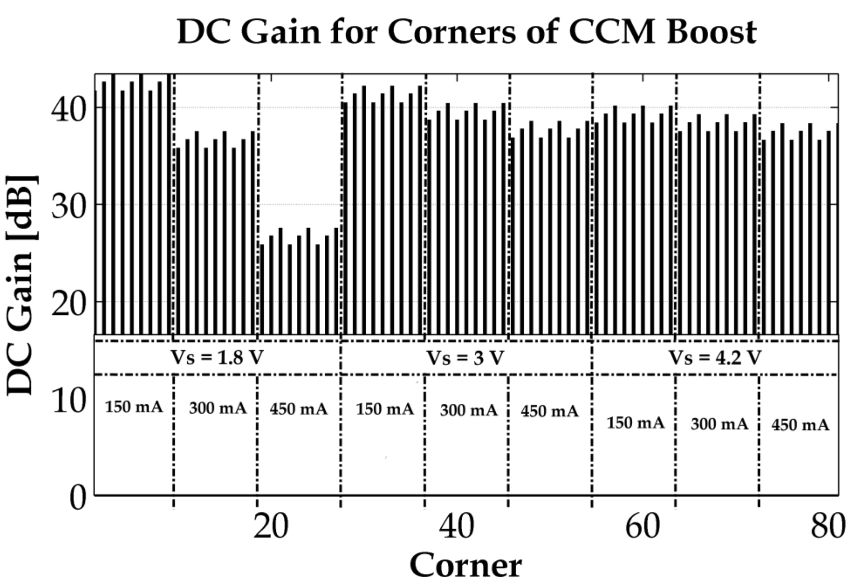
*Note – all values calculated for converter WITH control

Component Variation

Effect on DC-Gain

- Input Voltage, Load Current, Inductance
  - Vo/Vs = [3.3, 2, 1.4]
  - Io = [6 mA, 30 mA, 54 mA] (DCM); [150 mA, 300 mA, 450 mA] (CCM)
  - L = [15 uH, 20 uH, 25 uH] (DCM); [170 uH, 200 uH, 230 uH] (CCM)





*Note – all values calculated for converter WITH control

Control of Converters

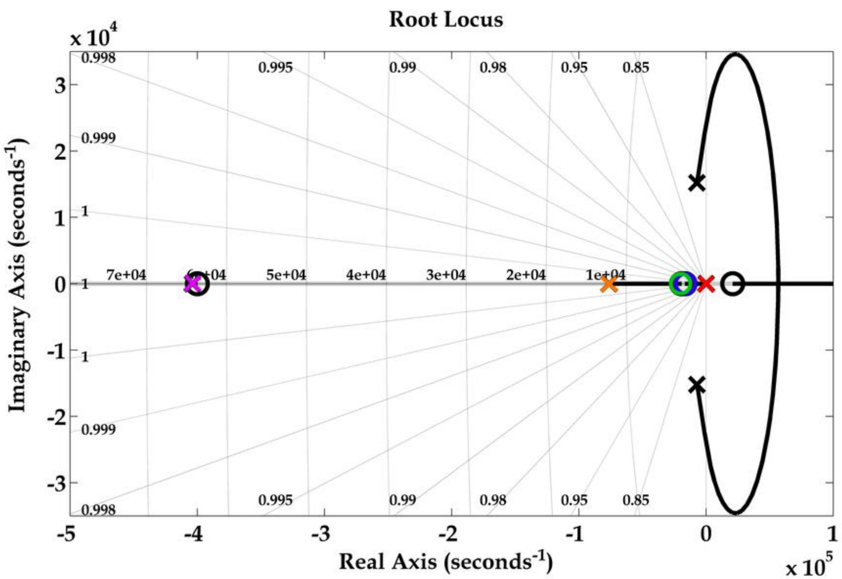
Error Amplifier Requirements

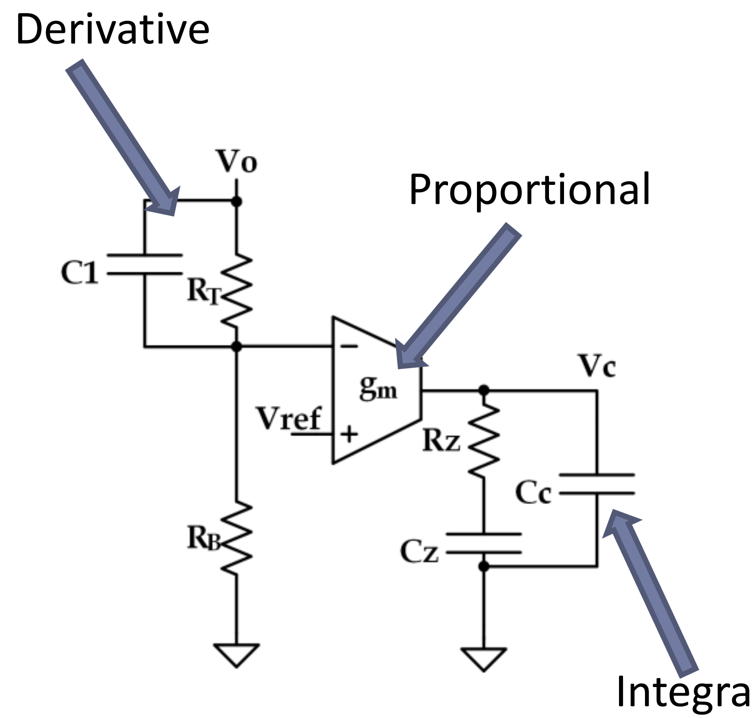
- Control needs to help provide stable output
  - Requires feedback loop → source of instability
  - Needs to minimize output error (large gain)
  - Needs to minimize instability (large phase margin)
  - Needs to maximize speed (large bandwidth)
- Op Amps/OTAs
  - Op Amps
    - Voltage buffer →Slows down performance
    - Can drive low impedances
  - OTAs
    - Can't drive resistive loads
    - Fast → does not have voltage buffer (response limited by load capacitance)

Control of Converters (CCM)

Proportial-Integral-Derivative Architecture

- PID Required for CCM
  - Two poles, two zeros
    - Minimizes conjugate pair effect



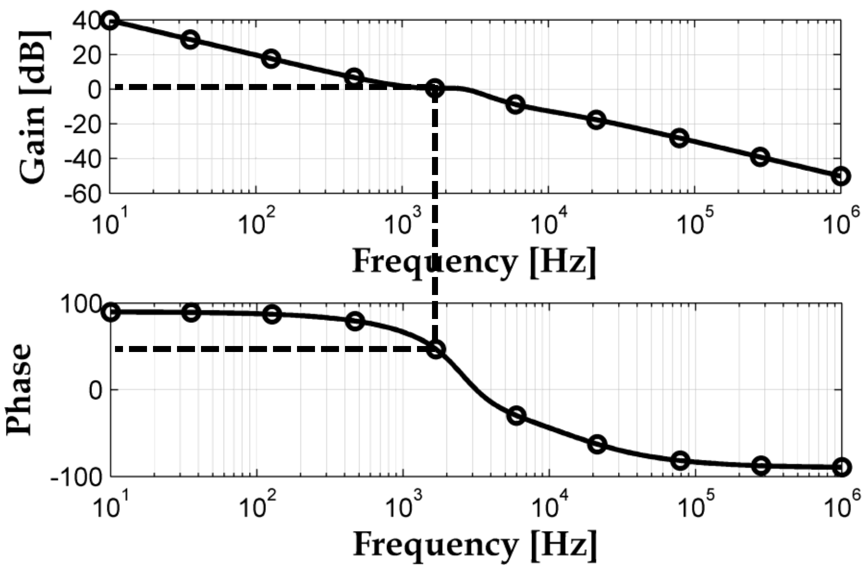


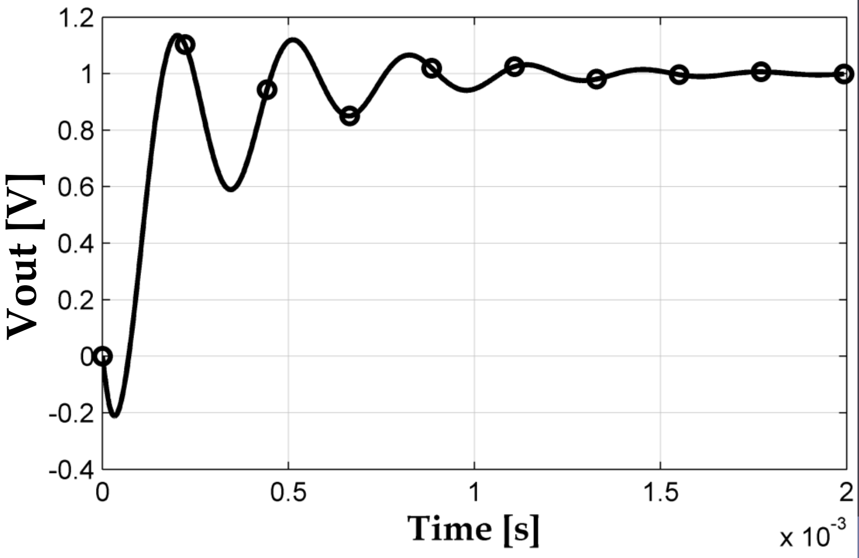
$$\frac{V_c}{V_o} = \frac{g_m R_B}{\boxed{s}(R_B + R_T)(C_Z + C_C)} \frac{\boxed{(sR_T C_1 + 1)}\boxed{(sR_Z C_Z + 1)}}{\boxed{(s\frac{R_Z C_Z C_C}{C_Z + C_C} + 1)}\boxed{(s(R_T \| R_B)C_1 + 1)}}$$

Control of Converters (CCM)

Bode Plot and Step Response

- Difficult to achieve all three requirements
  - Gain, Phase Margin, Bandwidth

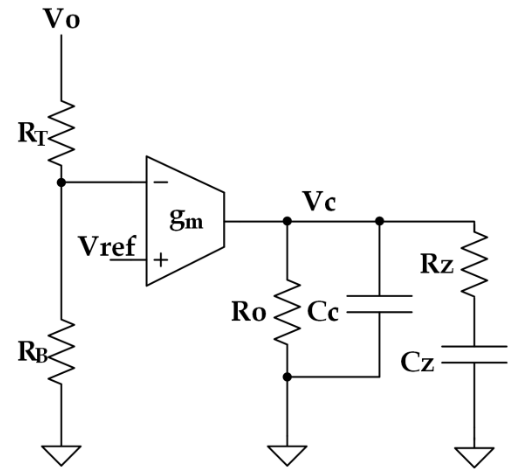


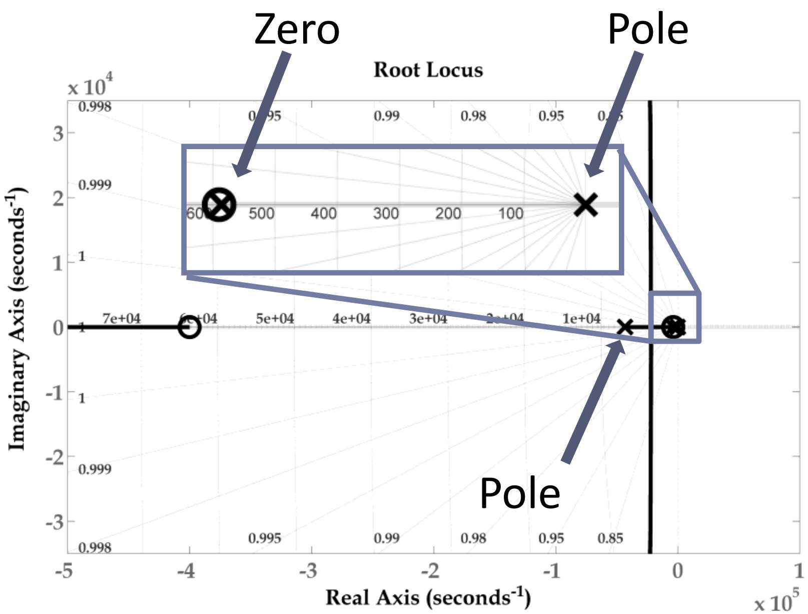


Control of Converters (DCM)

Lag Controller Architecture

- Lag Controller suitable for DCM
  - Zero to cancel converter Pole
  - Pole to attenuate switching noise





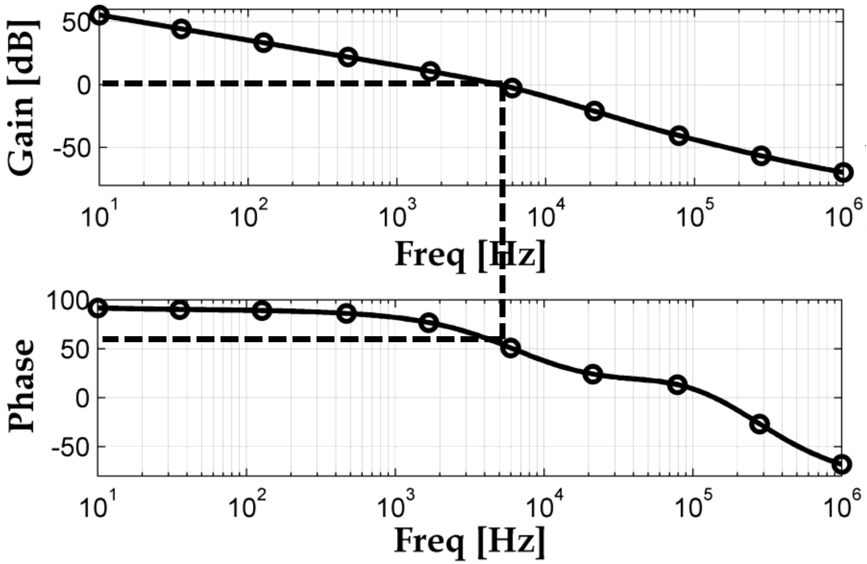
$$\frac{V_c}{V_o} = K \frac{sR_ZC_Z + 1}{s^2R_OR_ZC_CC_Z + sR_OC_Z + 1}$$

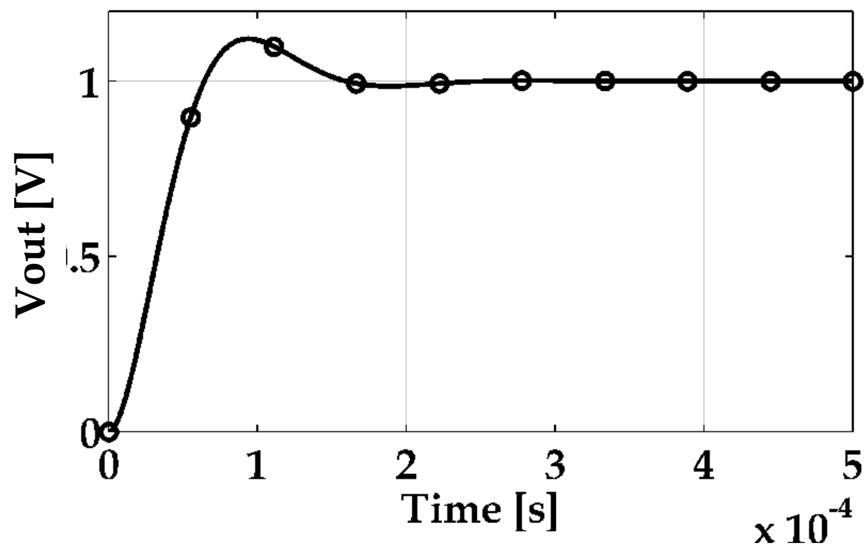
$$K = g_mR_O\frac{R_B}{R_T + R_B}$$

Control of Converters (DCM)

Bode Plot and Step Response

- Easier control compared to CCM
- More stable with fewer components





Measuring Stability

Stability Measurement Requirements and Possibilities

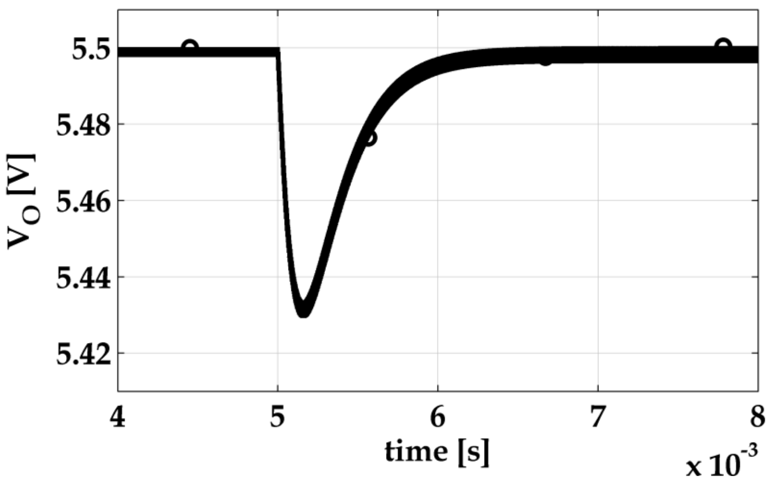
- Cannot “break the loop”
  - High loop gain
- Observe step response
- Superimpose voltage (Middlebrook’s Method)
- Cross-correlation

Measuring Stability

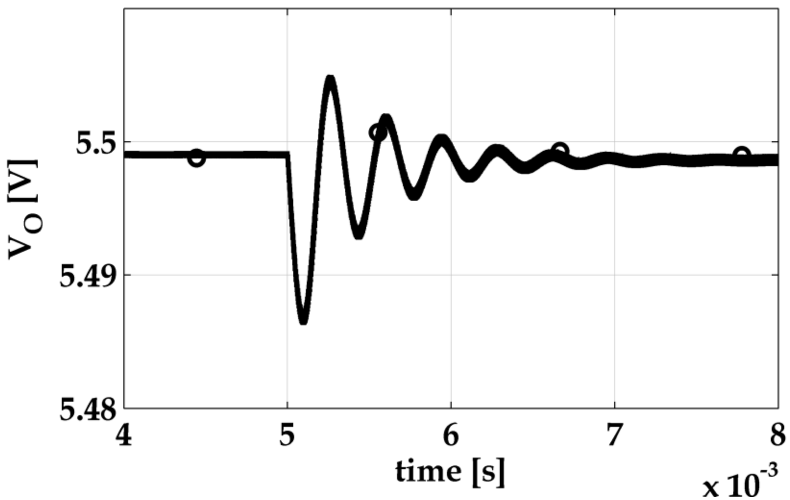
Load Step Response

- Converter reaches steady state
  - Step the load current
- Composite Response of System
  - No pole/zero information

Overdamped Response

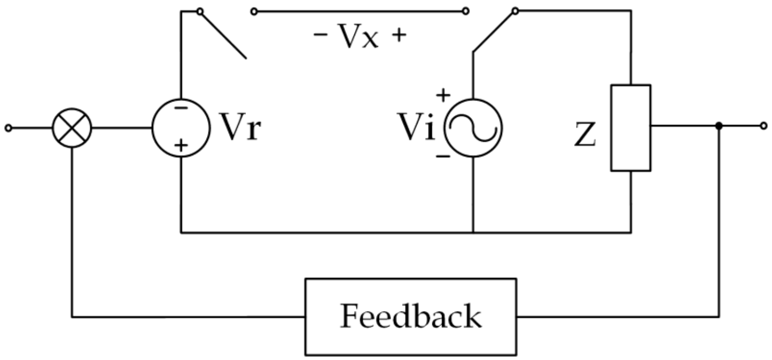


Underdamped Response

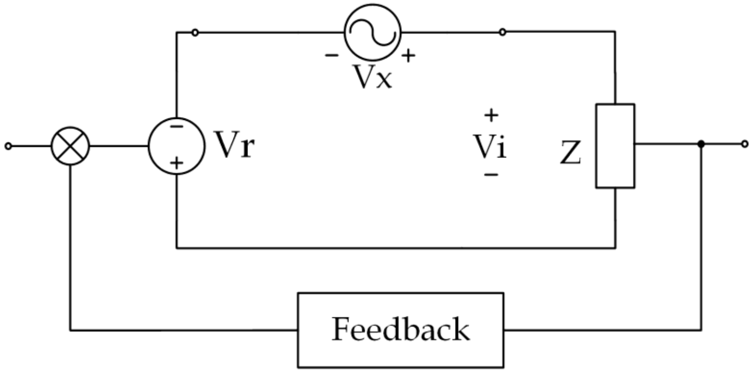


Measuring Stability

Middlebrook's Method (1975)



Conventional Approach
- Voltage Injection

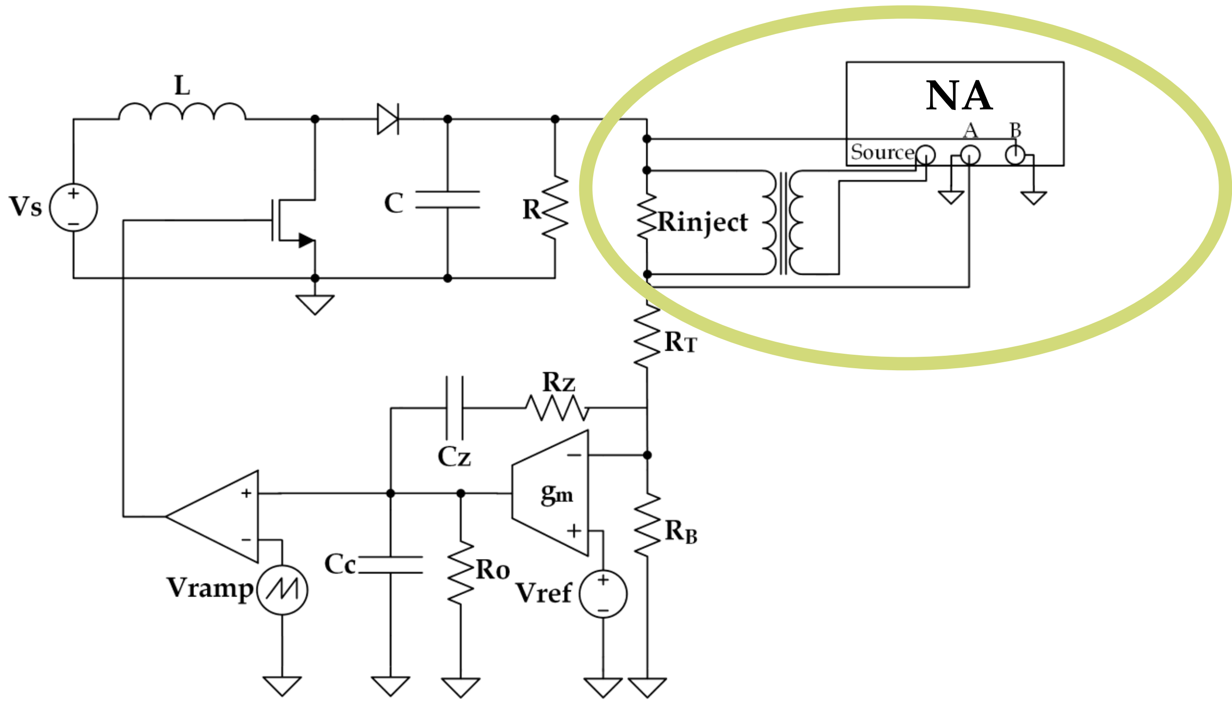


Middlebrook's Method
- Superposition

Measuring Stability

Implementation of Middlebrook's Method

- Can measure with Network Analyzer
- Bode plot can be compared to simulations
- Gives information on overall stability
  - Pole/Zero migration can be directly observed



Measuring Stability

Cross Correlation with White Noise

- Inject white noise in control signal path
  - White noise has autocorrelation of delta function
    - Yields impulse response
    - FFT yield frequency response

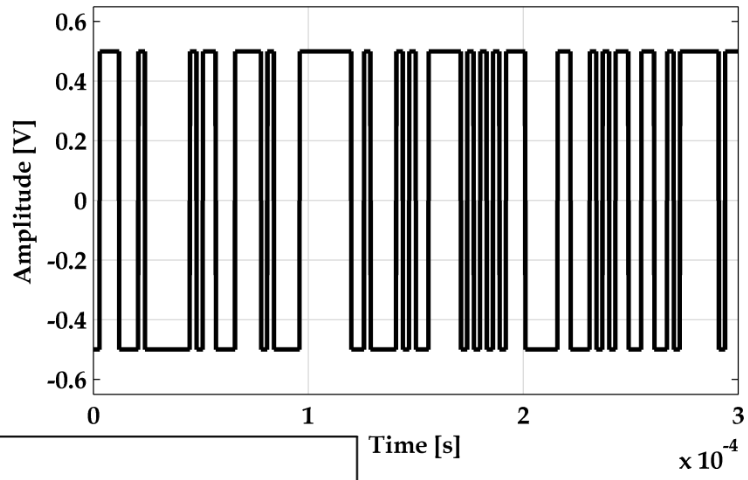
$$(1)\ R_{xy}[m] = \sum_{n=1}^{\infty} h[n] R_{xx}[m-n] + R_{xv}[m]$$

$$(2)\ R_{xx}[m] = \delta[m] \longrightarrow (3)\ R_{xy}[m] = h[m]$$

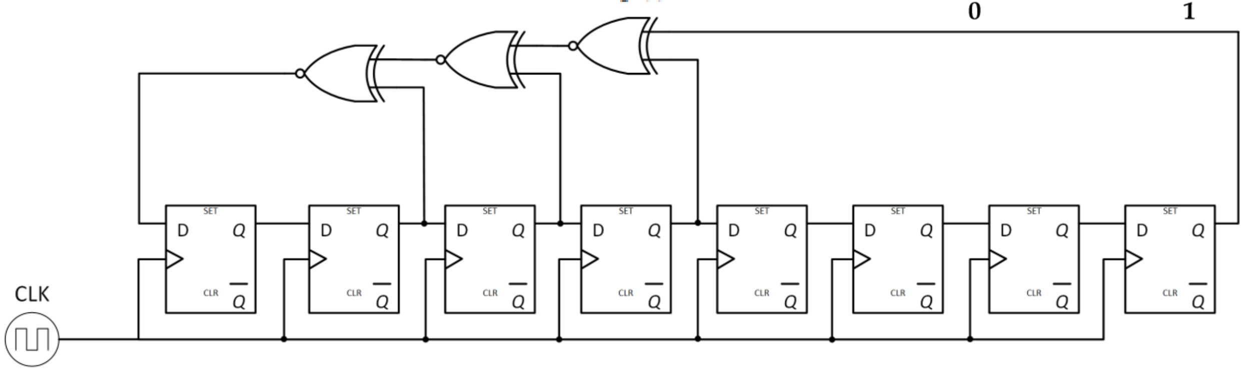
Measuring Stability

"White Noise" Circuit Implementation

- PRBS (Pseudo-Random Binary Sequence)
  - Periodic noise
    - Cross-correlation can only happen within one period
    - Period can be made larger by adding more bits to sequence
    - PRBS amplitude must be small percentage of control signal
    - Can be superimposed over Vref

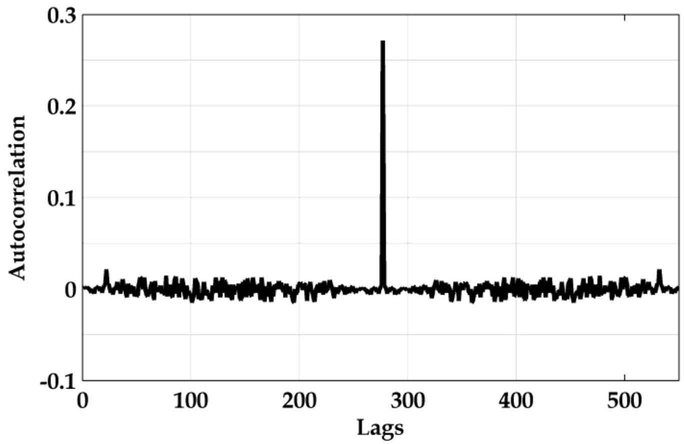


$$T_{PRBS} = \frac{2^n - 1}{f_k}$$

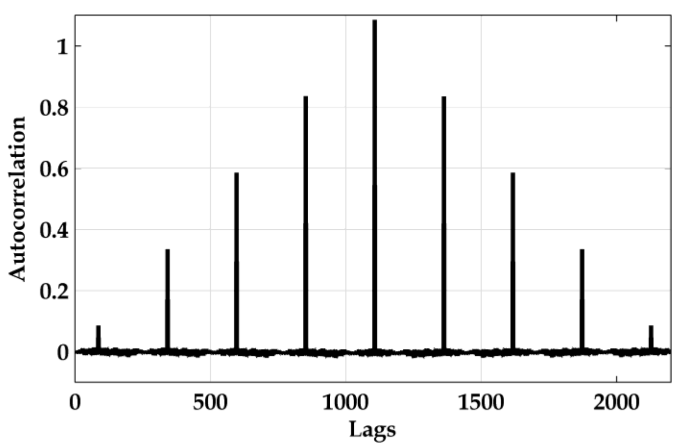


Measuring Stability
PRBS Autocorrelation

- Single period PRBS (a) has small autocorrelation value
  - Ideally infinite
- Multi-period PRBS (b) has larger value
  - Can sample the converter response multiple times
    - Average of results gives more accurate frequency response



(a)



(b)

Measuring Stability

Frequency Response Method Comparison

- Frequency Response gives most information on circuit
  - Can directly compare bode plot to simulations
  - Can see effect of resonance (for CCM)
  - Can see effect of filter capacitor ESR zero
- Methods
  - Middlebrook's Method
    - + Simple
    - - Requires Manual Capture
  - Cross-correlation (PRBS)
    - + Allows for Built-In Self-Test (BIST)
    - - Adds complexity

Summary

- DCM = Easier to Control
- Controller Design = Simple for DCM, Complex for CCM
- Frequency Response = Vital in determining stability
  - Two techniques: Middlebrook, Cross Correlation

Future Work

- Controller design via optimization algorithms
  - Genetic Algorithms, Particle Swarm Optimization
- More in-depth exploration of PRBS
  - On-chip Supply Testing

Thank You

- Comments/Questions?