SAS OPTIMIZATION CHALLENGE

Team

Pavan Kumar Ghantasala

Sri Nikhil Bolneyti

Anil Cavale

Rajib Mahato

Shubham Agarwal

Contents

1. Introduction

Building T is being supplied with water by two sources. One source is through contracts from Water Corporation (Main Water) and the other source is through internal precipitation mechanism (Cooling Water). We have water consumption data for 92 weeks of Building T from the source which it came from in all the corresponding weeks. Water corporation sells water to building T through two types of contracts, one with 15 cents per gallon with a minimum purchase of 25000 gallons per week and the other with 12 cents per gallon with a minimum purchase of 35000 gallons per week.

For precipitation cooling, it costs the company 18 cents per gallon in the first two weeks and 10 cents per gallon through process improvements in the second two weeks. The precipitated water is stored in the water tank whose level must be maintained at least 30000 gallons. Currently the tank is at 62500 gallons level. Because of environmental reasons, Building T should be using 25% water through precipitation.

The following are the predicted precipitation amounts in the coming four weeks:

Week	Precipitation
1	12000
2	18000
3	20000
4	22000

From the given data, we must forecast reliable estimates for the water demand for the next four weeks and use these estimates to find the optimal contract from the water corporation and how much precipitation water should be manufactured so that it reduces the total water cost for the company.

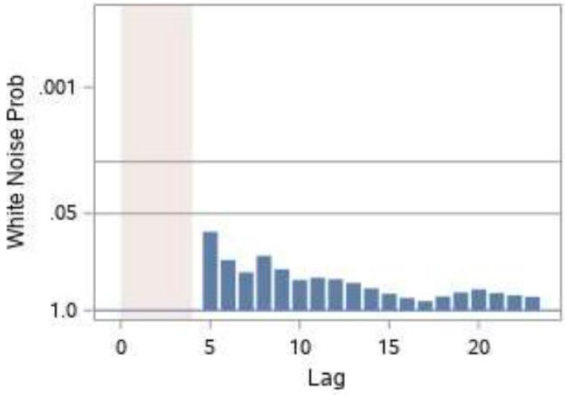
2. Forecasting

We have used the ARIMA model, Auto Regressive and Integrated Moving Average model, for forecasting. As can be seen from the below table, the probability value is

greater than the significance level of 0.05, so the model we chose, the ARIMA model, is very suitable for prediction.

Autocorrelation Check for White Noise									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	11.02	6	0.0878	-0.304	0.092	0.010	0.032	0.115	-0.005
12	16.64	12	0.1637	0.023	0.131	0.000	-0.051	0.141	-0.113
18	18.23	18	0.4407	-0.020	-0.022	-0.001	-0.007	0.035	-0.107

The white noise probability vs. lag plot below conveys the same message, the probability is greater than the significance level.



We have chosen the forecasting models which are reliable according to low white noise and consistent Auto Regression and Seasonal ARIMA models. We took the average ensemble of forecasts for three models addressing seasonality and trend components.

Forecasts for variable gallons				
Obs	Forecast	Std Error	95% Confidence Limits	
94	20739.9712	5342.4333	10268.9944	31210.9480
95	20266.9277	6472.4061	7581.2449	32952.6105
96	20206.0555	7721.3932	5072.4029	35339.7081
97	20009.5364	8708.5518	2941.0885	37077.9844

Forecasts for the Cooling Water

Forecasts for variable gallons				
Obs	Forecast	Std Error	95% Confidence Limits	
94	34392.0110	8040.7690	18632.3932	50151.6287
95	33943.3751	11217.786	11956.9176	55929.8326
96	33645.3572	12194.963	9743.6685	57547.0458
97	33610.8358	13160.770	7816.2011	59405.4706

Forecasts for the Main Water from Water Corp

The total water demand forecasts combining both cooling and main water are as follows:

Week	Forecasts
1	54172.24
2	54223.42
3	53514.76
4	53675.54

3. Optimization

From the forecast estimates from the model, we have the demand for water consumption in the next four weeks. We can use optimization and operations research techniques to find out the best contract alternative and respective amounts of water to be ordered from the water corporation and storage tank.

We can formulate the problem as an optimization problem trying to reduce the total cost and use the corresponding LP/MILP/NLP problems to solve it.

Our variables in the model would be the amount of water ordered from each source in each week.

W_{ij} – amount of water ordered from ith source in jth week

i -> {Water Corp., Storage Tank}

j -> { 1,2,3,4} (Next 4 weeks)

Let R_{ij} be the rate of water per gallon sourced from ith source and jth week same as above. For the first two weeks R_{precip} is 0.18 dollars per gallon and next two weeks it is 0.10 dollars as mentioned in the case. $R_{Water\ Corp,j}$ would be constant either 0.15 dollars if first contract is selected or 0.12 dollars if second contrat is selected.

We also need two more binary variables one for each contract (x_a,x_b), with their values suggesting whether that particular contract is selected or not. Only one of these variables should be having 1 and the other one should be 0 which can be implemented using a constraint. ($x_a + x_b = 1$) makes only one of the variable 1 and the other 0.

Given these values we have one more condition that water in the storage tank should always be greater than 30000. We were given the values of water precipitated each month, let that be WP_j for the month j. For this we can define, the storage at the beginning of the month as

$$S_j = S_{j-1} + WP_{j-1} - W_{precip,j-1}$$

Objective Function:

Minimize Total Cost = $\sum\{(W_{Water\ Corp,j} * (X_a * R_a + X_b * R_b)) + (W_{precip,j} * R_{precip,j})\}$

Subject to the following constraints

$W_{i,j}$ >= 0 for all i,j (non-negativity constraint)

S_j >= 30000 gallons for all j (Storage tank constraint)

$W_{precip,j} >= 0.25 * (W_{precip,j} + W_{Water\ Corp,\ j})$

$W_{precip,j} + W_{Water\ Corp,\ j}$ >= Forecasts

$X_a + X_b = 1$

X_a is binary

X_b is binary

$W_{Water\ Corp,\ j}$ >= (25000* X_a + 30000* X_b) (Water Corp base Constraint)

From the formulation of the problem above we can see that it is a nonlinear optimization problem as the objective function we are trying to solve has a multiplicative component between two variables. Using SAS optimization techniques, we have solved the problem and following are the results.

4. Results

The total number of gallons that Building T requires for the next 4 weeks are as follows:

Week	Forecasts
1	54172.24
2	54223.42
3	53514.76
4	53675.54

From solving the above optimization problem, we find that going with the second contract is more beneficial for the XYZ Corporation as the total cost is minimized when the company chooses the contract with 0.12 dollars per gallon and 35000 minimum purchase per week.

Choosing the first option the cost is $ 30,291 and second option costed $ 26,752.

Water ordered from the Water company each week is the following:

Week	Water Ordered
1	40629
2	40668
3	35000
4	35000

Water ordered from the storage tank each week are the following:

Week	Water Ordered
1	13543
2	13556
3	18515
4	18676

XYZ's projected total cost at the end of four weeks would be $ 26,752.

Week	Water in Tank
1	60956.93916
2	65401.08443
3	66886.36231
4	70210.86117

From choosing the second alternative instead of first, XYZ saved 30291 – 26752 = $ 3539.

If we had chosen the first contract, at the end of 4 weeks the storage tank inventory is 50210 gallons which is 20000 gallons less than choosing the second alternative (70210 gallons).

Week	Water in Tank (Option1)
1	60957
2	65401
3	56886
4	50210