Overview: The data contains the hourly and daily count of rental bikes between years 2011 and 2012 in Capital bikeshare system with the corresponding weather and seasonal information.

The data set has 3 files inside, day.csv, hour.csv and Readme.txt.

Aim: Based on the file day.csv propose a linear regression model for the response variable (count of total rental bike daily).

## **EXPLORATORY DATA ANALYSIS:**

- 1) Response variable: Count of total daily rental bikes
  - a) The response variable cnt is quantitative
  - b) Summary of cnt:

```
Min = 22
1st Quartile = 3152
2<sup>nd</sup> Quartile (median) = 4548
Mean = 4504
3rd Quartile = 5956
Max = 8714
> summary(cnt)
```

```
Min. 1st Qu. Median Mean 3rd Qu.
                                Max.
     3152 4548
                   4504 5956
                                 8714
```

- c) Nomarlity:
  - From the histogram of cnt in Fig 1.1, we can conclude that the distribution is bell-shaped and symmetric.
  - From the boxplot of cnt in Fig 1.2, we can conclude that the distribution has no outliers and is not skewed.

Thus, from the observations we can conclude that the distribution is normal.

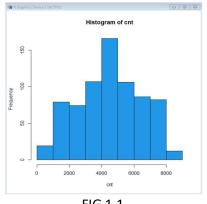


FIG 1.1

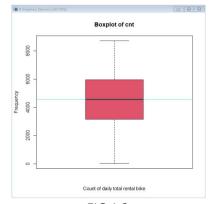


FIG 1.2

Hence, since the response is quantitative and its distribution is symmetric, the response is suitable to fit a linear regression model.

2) - Regressor: temp

Correlation between cnt and temp = 0.627494

Association between cnt and temp is positive and weak

From scatter plot: temp might be linear and have a constant variance

From histogram: multimodal

- Regressor: hum

Correlation between cnt and hum = -0.1006586

Association between cnt and hum is negative and weak

From scatter plot: hum might be linear but its variance might not be constant

From histogram: left skewed

## - Regressor: windspeed

Correlation between cnt and windspeed = -0.234545

Association between cnt and windspeed is negative and weak

From scatter plot: windspeed might be linear and have a constant variance

From histogram: right skewed

## - Regressor: season

From the boxplot in Fig 2.1 we can observe that the IQR for summer and winter season is almost same thus, the spread of data for both categories is almost same but, the median of winter vs cnt is higher than the median of summer vs cnt.

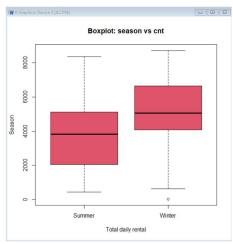


FIG 2.1

# - Regressor: workingday

From the boxplot in Fig 2.2 we can observe that the IQR for a non-working day is greater than the IQR for a working day thus, the spread of data for both categories is different. Moreover, the median of a working day is slightly more, almost same, than the median of a non-working day.

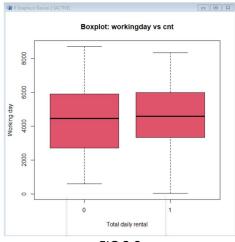


FIG 2.2

# - Regressor: weathersit

From the boxplot in Fig 2.3 we can observe that the IQR for good and bad weather is almost same

thus, the spread of data is almost same but, the median of good weather is higher than the median of bad weather.

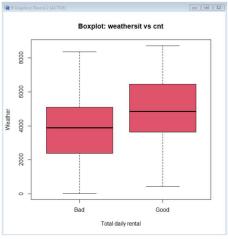


FIG 2.3

#### MODEL:

**3)** Proposed regressors for model M1 are: season, workingday, weathersit, temp, hum and windspeed

In R created a model M1 with the above mentioned regressors and got the following model summary:

## For the fitted model:

Let Y^ denoted the predicted value of cnt

B0 denote the intercept

B1 denote the coefficient and X1 denote the value for the regressor seasons

B2 denote the coefficient and X2 denote the value for the regressor workday

B3 denote the coefficient and X3 denote the value for the regressor weather

B4 denote the coefficient and X4 denote the value for the regressor temp

B5 denote the coefficient and X5 denote the value for the regressor hum

B6 denote the coefficient and X6 denote the value for the regressor windspeed

I denote an indicator variable

The fitted model equation:

 $Y^{A} = B0 + B1.I(X1=Winter) + B2.I(X2=Yes) + B3.I(X3=Good) + B4.X4 + B5.X5 + B6.X6$  $Y^{A} = 3409.6 + 466.7(1) + 150.1(1) + 296.6(1) + 6004.0(X4) - 2594.9(X5) - 4065.3(X6)$ 

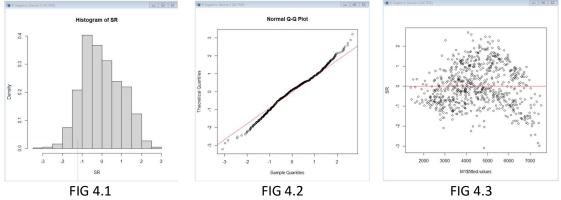
# 4) Checking if M1 is adequate:

- a) Calculated the standard residuals of the model and stored it in SR
- b) Using R we can infer that:

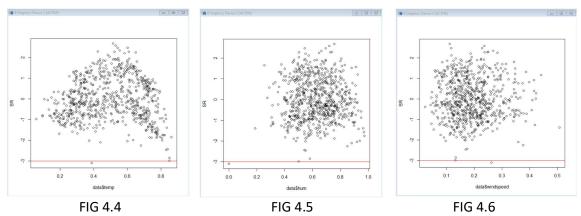
Outliers: at 69

Influential points: none

c) Checking whether the assumptions (normality, constant variance and linearity) hold:



From histogram of SR (Fig 4.1): Slightly left skewed, normality violated From QQ plot of SR (Fig 4.2): Both tails are slightly shorter than normal, normality violated From scatterplot of predicted Y^ vs SR: Slightly funnel shaped, constant variance violated



From scatterplots of regressors against SR (FIG 4.4, 4.5, 4.6): For temp linearity might be violated (FIG 4.4)

hum might be linear (FIG 4.5)

windspeed might be linear (FIG 4.6)

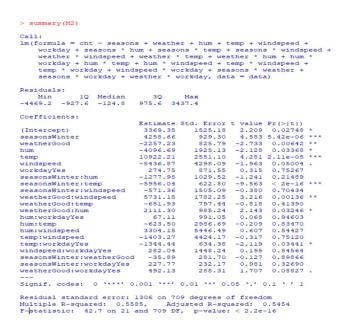
- d) Adjusted  $R^2 = 0.4732$
- e) F-statistic = 110.3 on a null distribution F(6,724)
- f) p-value < 0.05; f-test is significant
- g) The model **M1** is not adequate because the assumptions are violated and the adjusted R^2 suggests that the model is not good enough.
- 5) From model M1 we can see that: seasons- significant as p-value < 0.05 workday- insignificant as p-value> 0.05 weather- significant as p-value<0.05 temp- significant as p-value<0.05</p>

hum- significant as p-value<0.05 windspeed- significant as p-value<0.05

Proposal: Although the regressor workday is insignificant I did not choose to remove it from the model as in the following model M2 I decided to include the interaction regressors in the model and, since there exists significant interaction variables involving workday, I cannot remove workday from the model.

- 6) Series of linear models developed to reach the final linear model is as follows:
  - a) Model: M2

Difference from M1: Added interaction terms



#### Remarks:

- The F-test is significant and Adjusted R^2 increased.
- Interation terms that are insignificant: seasonsWinter\*windspeed, weatherGood\*temp, seasonsWinter\*hum, hum\*workdayYes,hum\*temp,hum\*windspeed, temp\*windspeed, workdayYes\*windspeed, seasonsWinter\*weatherGood and seasonsWinter\*workdayYes
- b) Model: M3

Difference from M2: Removed insignificant interaction terms

# 

## Remarks:

> summary(M3)

- F-test is significant and Adjusted R^2 increased
- hum and windspeed are insignificant but since they are involved in interaction terms, did not remove them
- temp\*workingday is insignificant
- Modifying M3 itself and removing the insignificant term
- Modified M3 contains another insignificant term seasonsWinter\*hum
- Thus, again modified M3 and removed the insignificant term

## Checking whether M3 is adequate:

- Calculated standard residuals and stored in SR3
- Outliers: at 69
- Influential point: none
- From

hist(SR3, prob=TRUE): the plot is slighlt left skewed

qqnorm(SR3, datax = TRUE): normality violated, tails are slightly shorter than normal qqline(SR3, datax = TRUE, col="red")

plot(M3\$fitted.values, SR3): slightly funnel shaped, equal variance might be violated abline(h=0, col="red")

#### Remarks:

- F-test is significant and Adjusted R^2 = 0.5469
- No insignificant terms except workday which is involved in significant interaction terms
- One outlier at 69 which is non-influential
- Model M3 is still not adequate as the assumptions of normality and constant variance are slightly violated plus the adjusted R^2 is also not good enough

#### c) Model: M4

Changes: Transformations made to the regressors and response to rectify the violations Transformations:

n\_hum = hum^3 #for higher powers the scatterplot might be funnel shaped
n\_wind = windspeed^2
n\_cnt = log(cnt)

## n temp = log(temp)

# Checking whether the model is adequate:

- Calculated standard residuals and stored in SR4

Outliers: at 2, 27, 69, 668Influential point: none

- From

hist(SR4, prob=TRUE): highly left skewed, might be due to outliers

qqnorm(SR4, datax = TRUE): left tail is longer & right tail is almost normal, normality violated, qqline(SR4, datax = TRUE, col="red")

```
plot(M4$fitted.values, SR4): normality violated abline(h=0, col="red")
```

```
plot(n_temp, SR4): linear, constant variance, normality violated
abline(h=3, col="red")
abline(h=(-3), col="red")
```

```
plot(n_hum, SR4): linear, constant variance, normality violated abline(h=3, col="red") abline(h=(-3), col="red")
```

```
plot(n_wind, SR4): linear, constant variance, normality violated
abline(h=3, col="red")
abline(h=(-3), col="red")
```

## Remarks:

- F-test is significant and Adjusted R^2 increased to 0.5976
- The transformations elimininated the moost of the violations
- Still the model is not adequate
- Suspected reason is due to outliers
- d) Model: M5

Changed: Removed Outliers

```
> summary (M5)
Inm(formula = n_cnt ~ n_season + n_weathersit + n_workingday +
n_wind + n_hum + n_temp + n_weathersit * n_wind + n_season *
n_temp + n_season * n_hum + n_weathersit * n_hum + n_weathersit *
n_workingday, data = new_data)
Min 1Q Median 3Q Max
-1.09090 -0.21015 0.01707 0.24189 0.79580
Coefficients:
                                                       (Intercept)
n_seasonWinter
n_weathersitGood
n_workingdayYes
n_wind
n_temp
n_weathersitGood:n_wind
n_seasonWinter:n_temp
                                                        -0.675396
                                                                           0.062916 -10.735
                                                                                            10.735 < 2e-16 ***
0.467 0.640545
n seasonWinter:n hum
                                                         0.063278
                                                                           0.135459
n_weathersitGood:n_hum 0.601752
n_weathersitGood:n_workingdayYes 0.091722
                                                                                            3.321 0.000943 ***
                                                                           0.181207
                                                                          0.053722
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.3175 on 715 degrees of freedom
Multiple R-squared: 0.6575, Adjusted R-squared: 0.65
F-statistic: 124.8 on 11 and 715 DF, p-value: < 2.2e-16
```

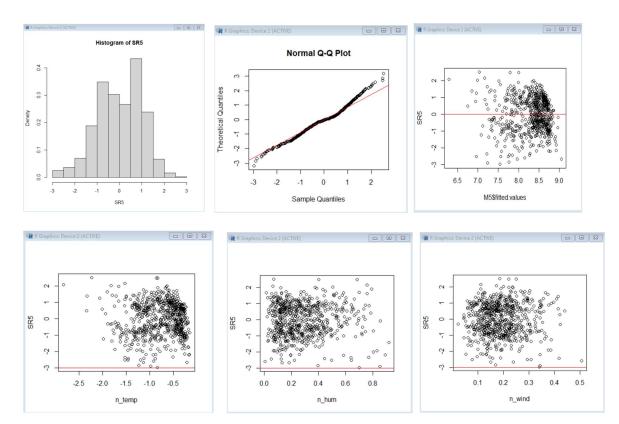
#### Remarks:

- While checking for the adequacy of M5, got more outliers that might be affecting the model and could eliminate the violations if these outliers were removed.
- Thus, on modifying M5 repeatedly removed the outliers at the following positions: 1,239,2,27,69,668, 302,328, 341, 358, 359, 669, 726, 299, 325, 338, 355, 356, 665, 722

Checking the adequacy of the final model:

- No outliers
- No influential points

\_



From these graphs we can conclude that:

Almost all violations have been removed and the regressors are linear with constant variance and are normally distributed whereas the response is also linear, has constant variance and is almost normal. QQ plot suggests that since the right tail is slightly shorter than normal thus, the response is slightly not normal.

#### Remarks:

- At last, the model has no outliers and almost all violations are eliminated from the model
- -So, M5 is the final model that has:
- -F-statistic = 127 and a null distribution of F(11,699)
- -Residual standard error = 0.2981
- -P-value of F-test < 0.05
- -Adjusted R^2 = 0.6613, suggests that the model is good
- -Thus, the model is almost adequate and significant
- 7) Had re-categorized season, weathersit and workingday and stored it in new variables seasons, weather and workday as follows:

```
weather = ifelse(weathersit == 1, "Good", "Bad")
seasons = ifelse(season == 1 | season == 2, "Summer", "Winter")
workday = ifelse(workingday == 1, "Yes", "No")
```

8) Final model: M5

Assumptions: - The data is random

- The response and regressors have a linear relationship
- The residuals of the build model are normal
- The regressors and response have constant variance

## R output:

## Final fitted equation:

X1: season (Winter=1, Summer=0) X2: weathersit (Good=1, Bad=0) X3: workingdays (Yes=1, No=0)

X4: windspeed

X5: hum X6: temp

Y^ = 9.75557 - 0.41681\*I(X1=Winter) - 0.43631\*I(X2=Good) - 0.01878\*(X3=Yes) - 1.74862\*X4

- 1.06483\*X5 + 1.03336\*X6 + 1.33020\*X4\*I(X2=Good) - 0.70099\*X5\*I(X1=Winter) + 0.16018\*X5\*I(X1=Winter) + 0.47214\*X5\*I(X2=Good) + 0.09092\*I(X3=Yes)\*I(X2=Good) +

## **Overall Significance of regression:**

- The first thing we should test for is whether overall, any terms in the model are significant or not.
- This is analogous F-test.
- The null hypothesis is

```
H0:_1 = _2 = _3 = 0
```

- The alternative is

#Q2

cor(cnt,temp) #positive and weak association

H1: at least one of \_1; \_2 or \_3 is not zero

- This F-test has test statistic F = 127 on a null distribution F(11,699) and has p-value < 0:00001.
- Thus, at \_ = 0:05, we would reject H0.

## **Conclusion: model adequacy:**

- As explained in the point number 6 d.
- The model M5 is adequate after removing the outliers.

## R CODE:

```
setwd("C:\Users\\Anjali\\Desktop\\NUS\\YEAR 1\\SEM 1\\ST1131 Introduction to Statistics and Statistical Computing\\Bike-Sharing-Dataset")

data = read.csv("day.csv")

attach(data)

#PART 1:

# RESPONSE VARIABLE: cnt

# REGRESSORS: season, workingday, weathersit, temp, hum and windspeed

weather = ifelse(weathersit == 1, "Good", "Bad")

seasons = ifelse(season == 1 | season == 2, "Summer", "Winter")

workday = ifelse(workingday == 1, "Yes", "No")

#Q1

summary(cnt)

hist(cnt, col=4) #symmetric distribution

boxplot(cnt, ylab= "Frequency", xlab= "Count of daily total rental bike", col= 2, main = "Boxplot of cnt")

abline(h=median(cnt), col= 5)
```

```
hist(temp) #multimodal
cor(cnt,hum) #negative and weak association
plot(cnt~hum) #linear, constant variance violated
hist(hum) #left skewed
cor(cnt, windspeed) #negative and weak association
plot(cnt~windspeed) #linear, variance might be constant
hist(windspeed) #right skewed
#IQR is almost same, median of winter> summer season
boxplot(cnt~seasons,ylab="Season", xlab="Total daily rental",, main="Boxplot: season vs cnt", col=2)
#IQR range of 0>1, spread of data 0>1, median alomost same, 1 had median slightly higher than 0
boxplot(cnt~workday, col=2, ylab="Working day", xlab="Total daily rental",, main="Boxplot: workingday vs
cnt")
#IQR is almost same, median of good weather>bad weather
boxplot(cnt~weather, ylab="Weather", xlab="Total daily rental",, main="Boxplot: weathersit vs cnt", col=2)
#PART 2:
#Q3,4,5,6,7
#MODEL 1
M1= lm(cnt~seasons+workday+weather+temp+hum+windspeed, data=data)
summary(M1) # adj r sqr = 0.4732, workday is insignificant
raw.res = M1$res
SR = rstandard(M1)
which(SR>3 | SR<(-3)) #one outlier at 69
c = cooks.distance(M1)
which(c>1) #no influential points
```

plot(cnt~temp) #linear, constant variance

```
hist(SR, prob=TRUE)#slightly left skewed, normality violated
```

```
qqnorm(SR, datax = TRUE) #normality violated, tails are slightly shorter than normal qqline(SR, datax = TRUE, col="red")
```

#PLOTTING PREDICTED Y^ VS SR TO CHECK FOR CONSTANT VARIANCE AND NORMALITY plot(M1\$fitted.values, SR) #slightly funnel shaped, equal variance violated abline(h=0, col="red")

#PLOTTING EACH X AGAINST SR TO CHECK FOR X'S LINEARITY ASSUMPTION plot(data\$temp, SR) #linearity might be violated abline(h=3, col="red") abline(h=(-3), col="red")

plot(data\$hum, SR) #linearity check of hum abline(h=3, col="red")#might be linear abline(h=(-3), col="red")

plot(data\$windspeed, SR) #linearity check of windspeed abline(h=3, col="red") #might be linear abline(h=(-3), col="red")

#Workingday is insignificant

#There is one outlier which is not influential

#The model is not adequate due to the violations of the assumptions

#There might interaction terms among the regressors

## #MODEL 2

#Since we added interaction terms of workday with other regressors, workday is still a regressor #Added interaction terms

M2 = Im(cnt~seasons+weather+hum+temp+windspeed+workday+ seasons\*hum+seasons\*temp+seasons\*windspeed+ weather\*windspeed+weather\*temp+weather\*hum+

```
hum*workday+hum*temp+hum*windspeed+
     temp*windspeed+temp*workday+
     windspeed*workday+
     seasons*weather+seasons*workday+weather*workday, data=data)
summary(M2)
# Adjusted R^2 = 0.5454
# Interation terms that are insignificant:
# seasonsWinter*windspeed, weatherGood*temp, seasonsWinter*hum,
hum*workdayYes,hum*temp,hum*windspeed,
# temp*windspeed, workdayYes*windspeed, seasonsWinter*weatherGood and
seasonsWinter*workdayYes
#MODEL 3
#Removed all insignificant regressors
M3 = Im(cnt~seasons+weather+hum+temp+windspeed+workday+
     seasons*temp+
     weather*windspeed+weather*hum+
     temp*workday+
     weather*workday, data=data)
summary(M3)
\#Adjusted R^2 = 0.5485
#hum and windspeed are insignificant but since they are involved in interaction terms, did not remove
them
#temp*workingday is insignificant
#Modifying model 3 itself and removing the insignificant term
M3 = Im(cnt~seasons+weather+hum+temp+windspeed+workday+
     seasons*temp+
     weather*windspeed+weather*hum+
     weather*workday, data=data)
summary(M3)
raw.res = M3$res
SR3 = rstandard(M3)
```

```
which(SR3>3 | SR3<(-3)) #no outliers
c3 = cooks.distance(M3)
which(c3>1) #no influential points
hist(SR3, prob=TRUE) #slighlt left skewed
qqnorm(SR3, datax = TRUE) #normality violated, tails are slightly shorter than normal
qqline(SR3, datax = TRUE, col="red")
plot(M3$fitted.values, SR3) #slightly funnel shaped, equal variance might be violated
abline(h=0, col="red")
#PLOTTING EACH X AGAINST SR
plot(data$temp, SR3) #constant variance might be violated
abline(h=3, col="red")
abline(h=(-3), col="red")
plot(data$hum, SR3)
abline(h=3, col="red")
abline(h=(-3), col="red")
plot(data$windspeed, SR3)
abline(h=3, col="red")
abline(h=(-3), col="red")
\#Adjusted R^2 = 0.5524
#Model is not adequate due to the violations
#Doing transformations to the response and regressors might help in eliminating the violations
#MODEL 4
n_hum = hum^3 #for higher powers the scatterplot might be funnel shaped
n_wind = windspeed^2
n_cnt = log(cnt)
```

```
n_temp = log(temp)
M4 =
lm(n_cnt~seasons+weather+workday+n_wind+n_hum+n_temp+weather*n_wind+seasons*n_temp+weat
her*n_hum+weather*workday, data=data)
summary(M4)
raw.res = M4$res
SR4 = rstandard(M4)
which(SR4>3 | SR4<(-3)) #outliers at 2, 27, 69, 668
c4 = cooks.distance(M4)
which(c4>1) #no influential point
hist(SR4, prob=TRUE) #highly left skewed, might be due to outliers
qqnorm(SR4, datax = TRUE) #left tail is longer & right tail is almost normal, normality violated,
qqline(SR4, datax = TRUE, col="red")
#PLOTTING PREDICTED Y^ VS SR TO CHECK FOR CONSTANT VARIANCE AND NORMALITY
plot(M4$fitted.values, SR4) #normality violated
abline(h=0, col="red")
#PLOTTING EACH X AGAINST SR TO CHECK FOR X'S LINEARITY ASSUMPTION
plot(n_temp, SR4) #linear
abline(h=3, col="red")
abline(h=(-3), col="red")
plot(n_hum, SR4) #linearity check of hum
abline(h=3, col="red")#linear
abline(h=(-3), col="red")
plot(n_wind, SR4) #linearity check of windspeed
abline(h=3, col="red") #linear
abline(h=(-3), col="red")
```

```
\#Adjusted R^2 = 0.5976
```

#After the transformations the adjusted R^2 increased and it also elimininated the linearity violations of the regressors

#Still the model is not adequate

#Suspected reason is due to outliers

```
#MODEL 5
new data = data[-c(1,239,2,27,69,668, 302,328, 341, 358, 359, 669, 726, 299, 325, 338, 355, 356, 665, 722
),]
n hum = (new data$hum)^3 #for higher powers the scatterplot might be funnel shaped
n wind = new data$windspeed
n_cnt = log(new_data$cnt)
n temp = log(new data$temp)
n_season = seasons[-c(1,239,2,27,69,668, 302,328, 341, 358, 359, 669, 726, 299, 325, 338, 355, 356, 665,
722)]
n weathersit = weather[-c(1,239,2,27,69,668, 302,328, 341, 358, 359, 669, 726, 299, 325, 338, 355, 356,
665, 722)]
n_workingday = workday[-c(1,239,2,27,69,668, 302,328, 341, 358, 359, 669, 726, 299, 325, 338, 355, 356,
665, 722)]
M5 =
lm(n cnt~n season+n weathersit+n workingday+n wind+n hum+n temp+n weathersit*n wind+n seas
on*n_temp+n_season*n_hum+n_weathersit*n_hum+n_weathersit*n_workingday, data=new_data)
summary(M5)
raw.res = M5$res
SR5 = rstandard(M5)
which(SR5>3 | SR5<(-3)) #outliers at 2, 239, 358, 669, 668, 667, 662, 353 (removed this from new data and
modified M5)
c5 = cooks.distance(M5)
which(c5>1) #no influential point
```

qqnorm(SR5, datax = TRUE) #both tails are slighlty shorter than normal, normality slighlty violated qqline(SR5, datax = TRUE, col="red")

hist(SR5, prob=TRUE) #slighlty left skewed, might be due to outliers

```
#PLOTTING PREDICTED Y^ VS SR TO CHECK FOR CONSTANT VARIANCE AND NORMALITY plot(M5$fitted.values, SR5) #constant variance but normality is minimally violated abline(h=0, col="red")
```

```
#PLOTTING EACH X AGAINST SR TO CHECK FOR X'S LINEARITY ASSUMPTION plot(n_temp, SR5) #linear, constant variance abline(h=3, col="red") abline(h=(-3), col="red")
```

plot(n\_hum, SR5) #linearity check of hum abline(h=3, col="red")#linear, constant variance abline(h=(-3), col="red")

plot(n\_wind, SR5) #linearity check of windspeed abline(h=3, col="red") #linear, constant variance abline(h=(-3), col="red")

#After removing outliers calculated from M4

#This model had an Adjusted R^2 = 0.6613 and almost all major violations eliminated

#Thus, removed more outliers, found in M5, from new\_data and modified M5 itself

#There are some insignificant regressors in the model but did not remove them for better estimation of Y^

#At last, the model has no outliers and almost all violations are eliminated from the model #So, M5 is the final model that has:
#F-statistic = 127 and a null distribution of F(11,699)
#Residual standard error = 0.2981
#P-value of F-test < 0.05
#Adjusted R^2 = 0.6613, suggests that the model is good

#Thus, the model is almost adequate and significant