Week 3

Tuesday, 13 August 2024 17:50

N players ; x_2 is strategy of other players

$$u_1(x_1, x_2) = x_1 \left(1 - (N-1)x_2 - x_1\right) \qquad x_2 \text{ as given}$$

$$\frac{\partial u_1}{\partial x_1} = 1 - (N-1)x_2 - 2x_1 = 0$$

$$x_1 = \frac{1 - (N-1)x_2}{2}$$

$$BR_1(x_2) = \frac{1 - (N-1)x_2}{2}$$

∵ Each player does the same calculation,
∴ we can take $x_1 = x_2$.

$$x_1 = \frac{1 - (N-1)x_1}{2}$$

$$\Rightarrow 2x_1 = 1 - Nx_1 + x_1$$

$$\Rightarrow x_1(N+1) = 1$$

$$\Rightarrow x_1 = \frac{1}{N+1}$$

Remaining resource $= 1 - Nx_1$

$$y_1 = 1 - \frac{N}{N+1}$$

$$\lim_{N\to\infty} y_1 = \lim_{N\to\infty} 1 - \frac{1}{1 + 1/N} = 0$$

Second Price Sealed Bid Auction

P_1 \ P_2	0	1	2	3	4
0	0,0	0,2	0,2	0,2	0,2
1	3,0	2,0	0,1	0,1	0,1
2	3,0	2,0	1,0	0,0	0,0
3	3,0	2,0	1,0	0,0	0,-1
4	3,0	2,0	1,0	0,0	-1,0
5					-1,0

Payoffs.

$V_1 = 3$, $V_2 = 2$

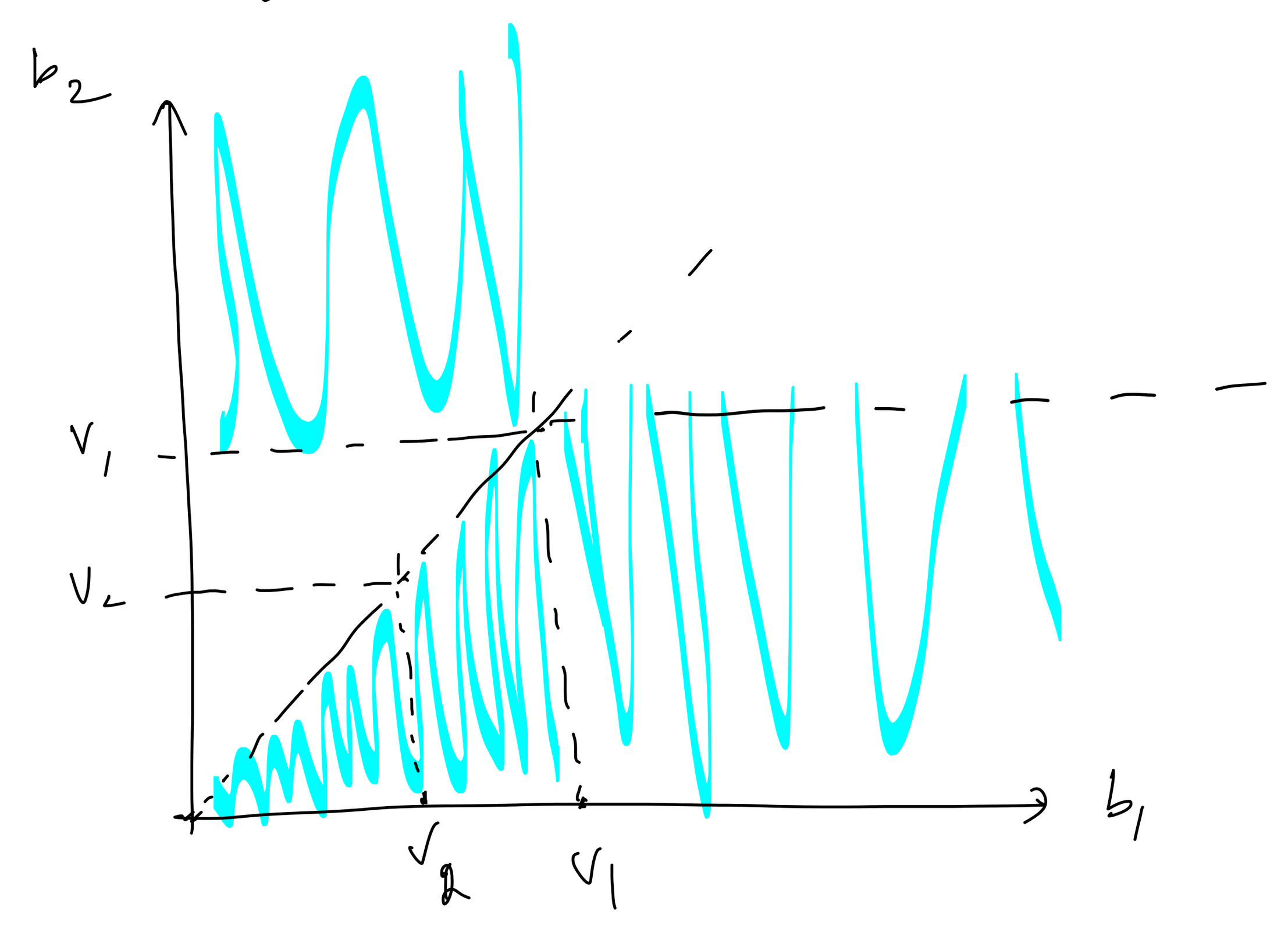
$$u_1(b_1, b_2) = \begin{cases} V_1 - b_2 & b_1 > b_2 \\ 0 & b_1 < b_2 \end{cases}$$

$$u_2(b_2, b_1) = \begin{cases} 0 & b_1 > b_2 \\ V_2 - b_1 & b_1 < b_2 \end{cases}$$

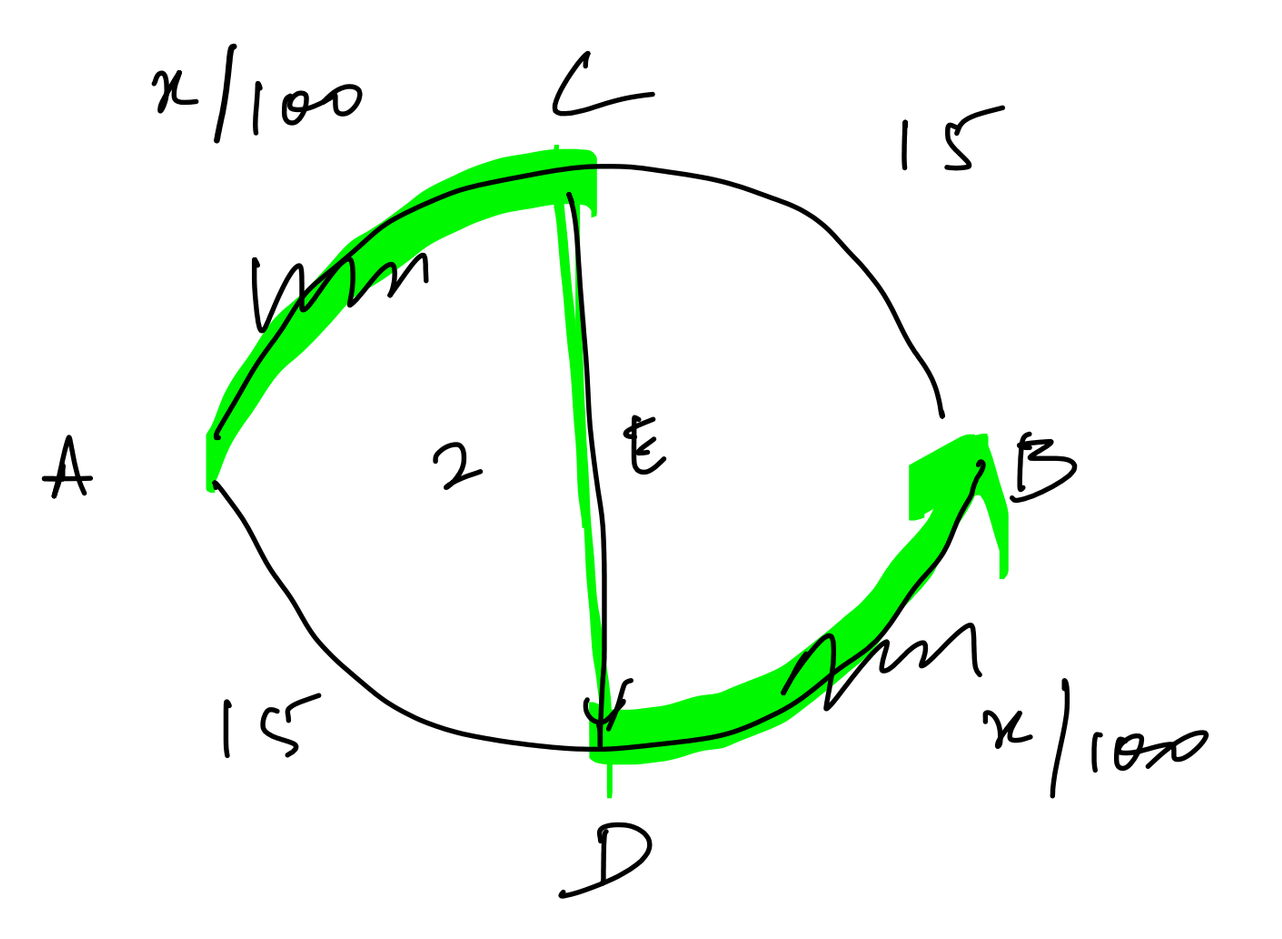
Cont Time

$$BR_1(b_2) = \begin{cases} b_1 < b_2 & b_2 > V_1 \\ [0, \infty) & b_2 = V_1 \\ b_1 > b_2 & b_2 < V_1 \end{cases}$$

$$BR_2(b_1) = \begin{cases} b_2 < b_1 & b_1 > V_2 \\ [0, \infty) & b_1 = V_2 \\ b_2 > b_1 & b_1 < V_2 \end{cases}$$



Braess' Paradox



Cars = 1000
x can go via route ACB
& 1000−x cars go via route ADB.

$$\frac{x}{100} + 15 = \frac{1000 - x}{100} + 15$$

$$x = 1000 - x$$

$$\Rightarrow x = 1000/2 = 500$$

∴ Travel time by either route $= \frac{500}{100} + 15$
$= 20$ mins

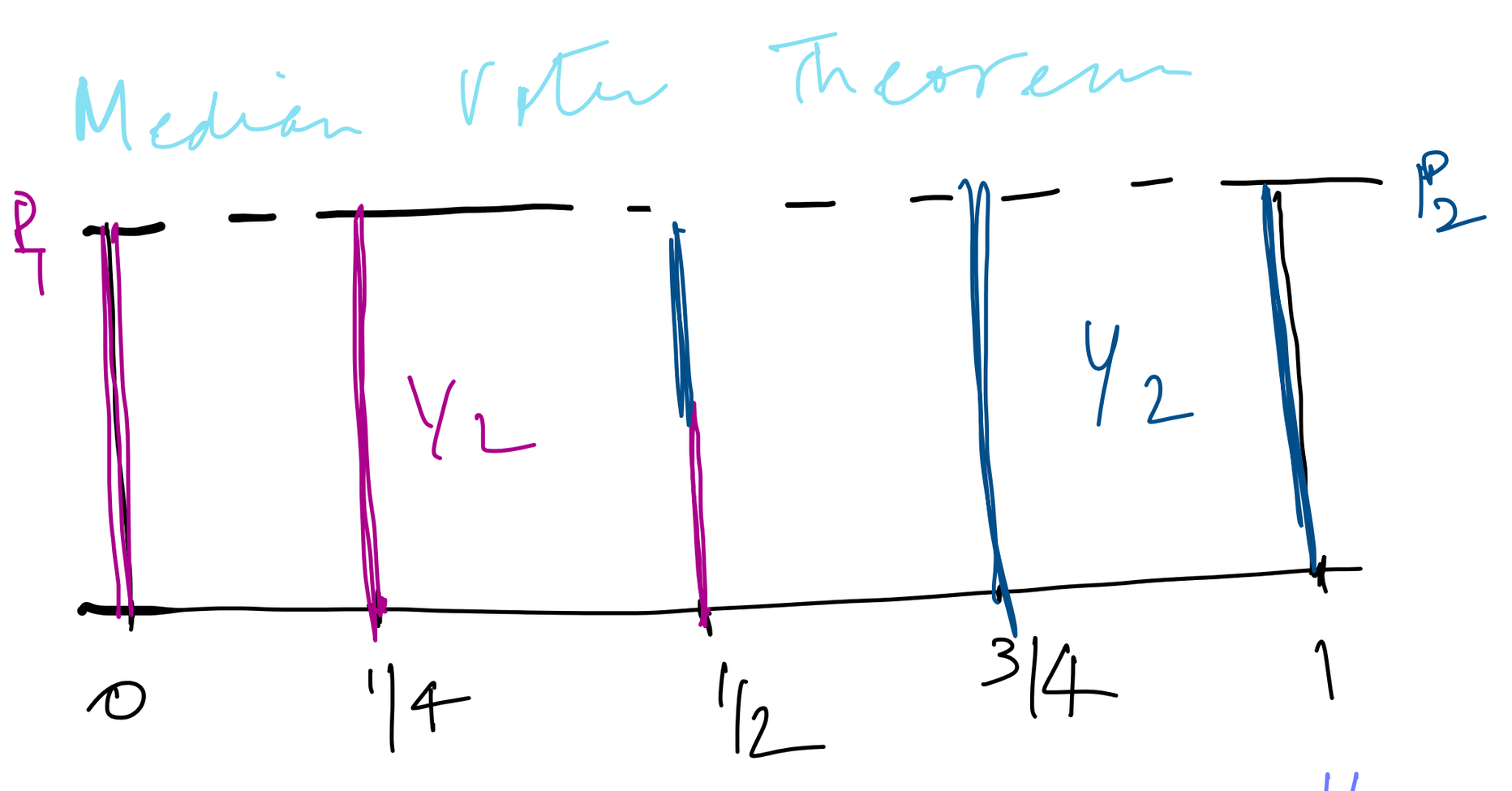
If they take the route ACEDB,
travel time $= \frac{x}{100} + 2 + \frac{x}{100}$
$= 10 + 2 + 10 = 22$ mins

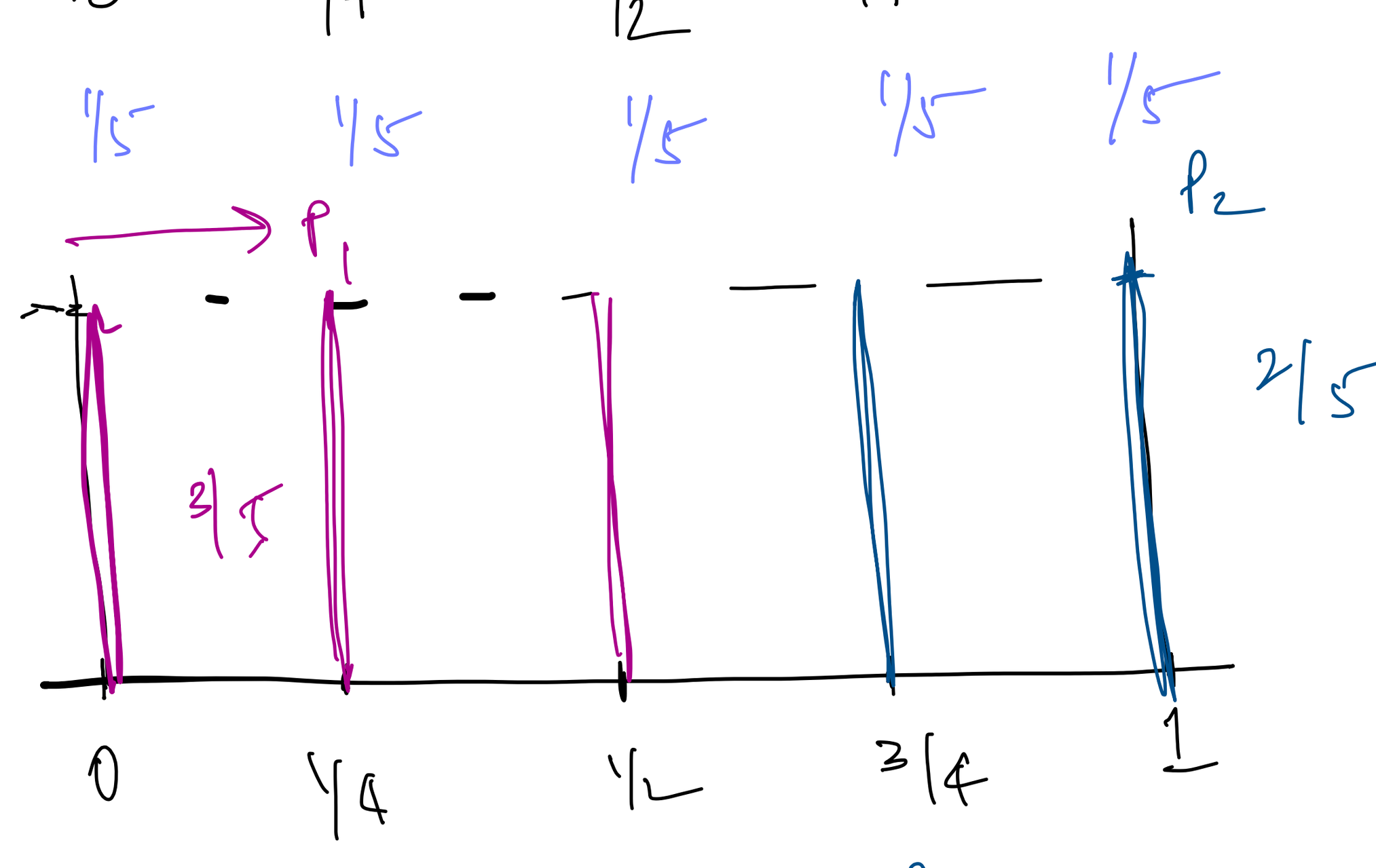
Travel time via ACB in new scenario
or ~~ADB~~

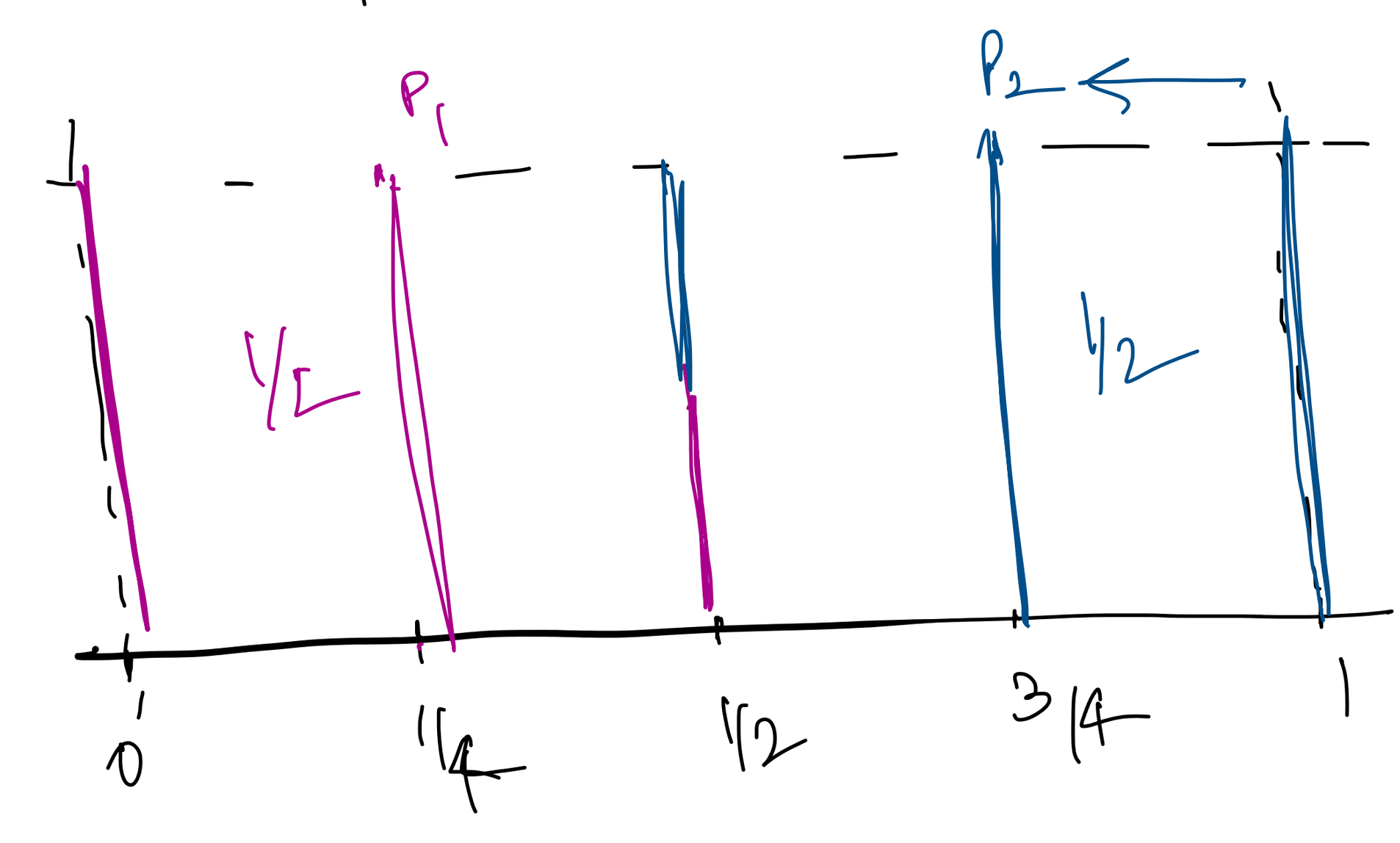
$$= \frac{1000}{100} + 15 = 10 + 15 = 25$$

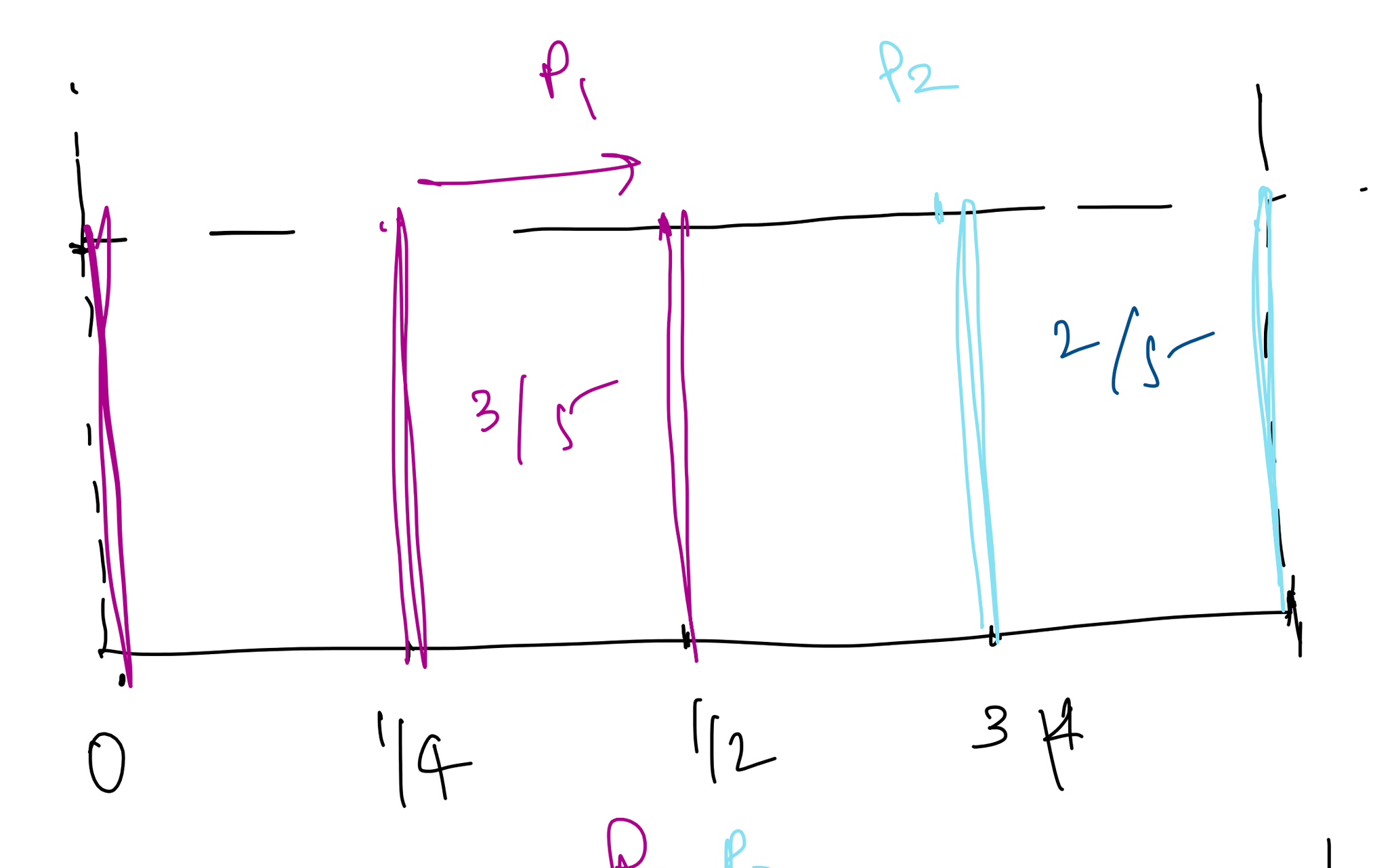
Median Voter Theorem

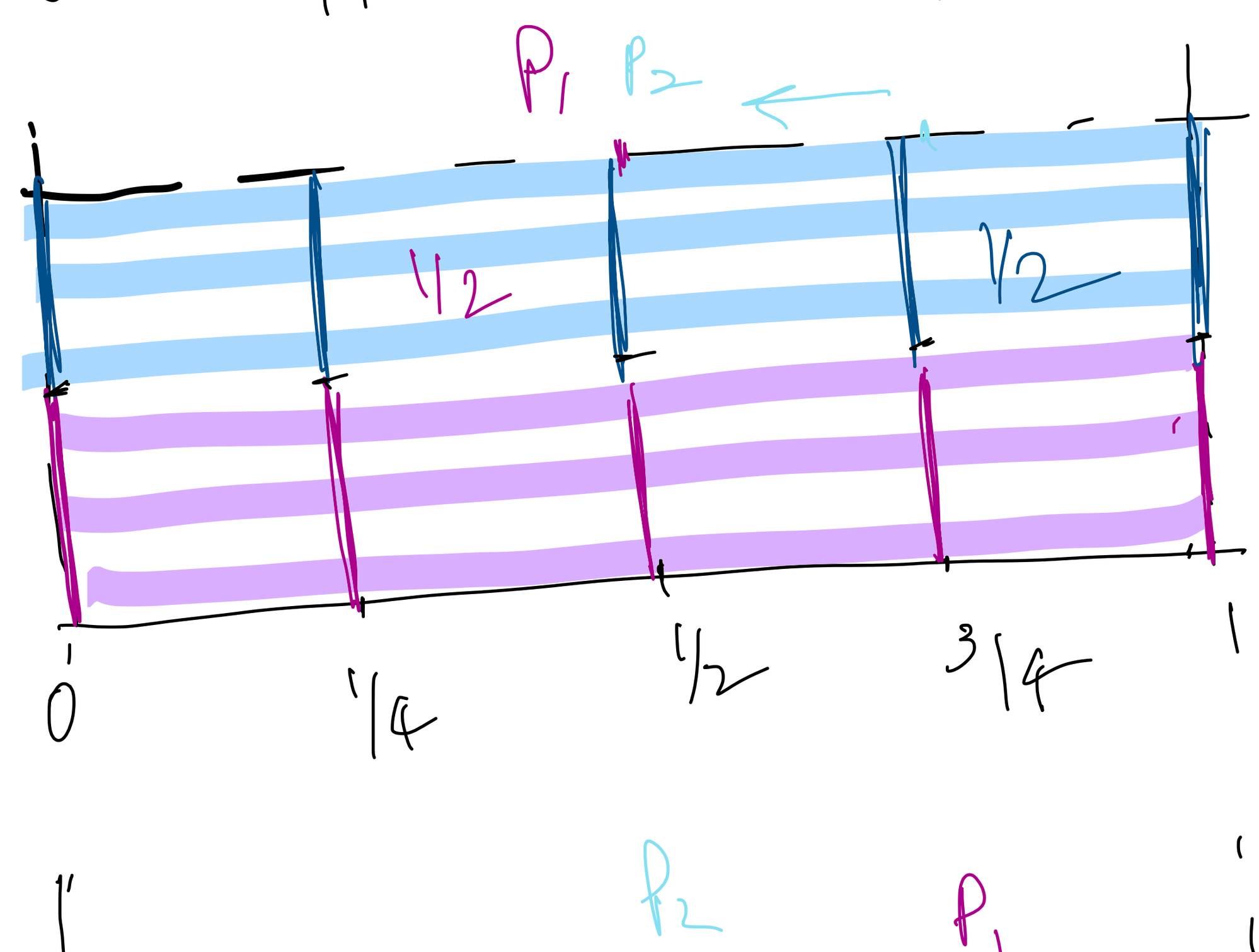
P_1 , P_2



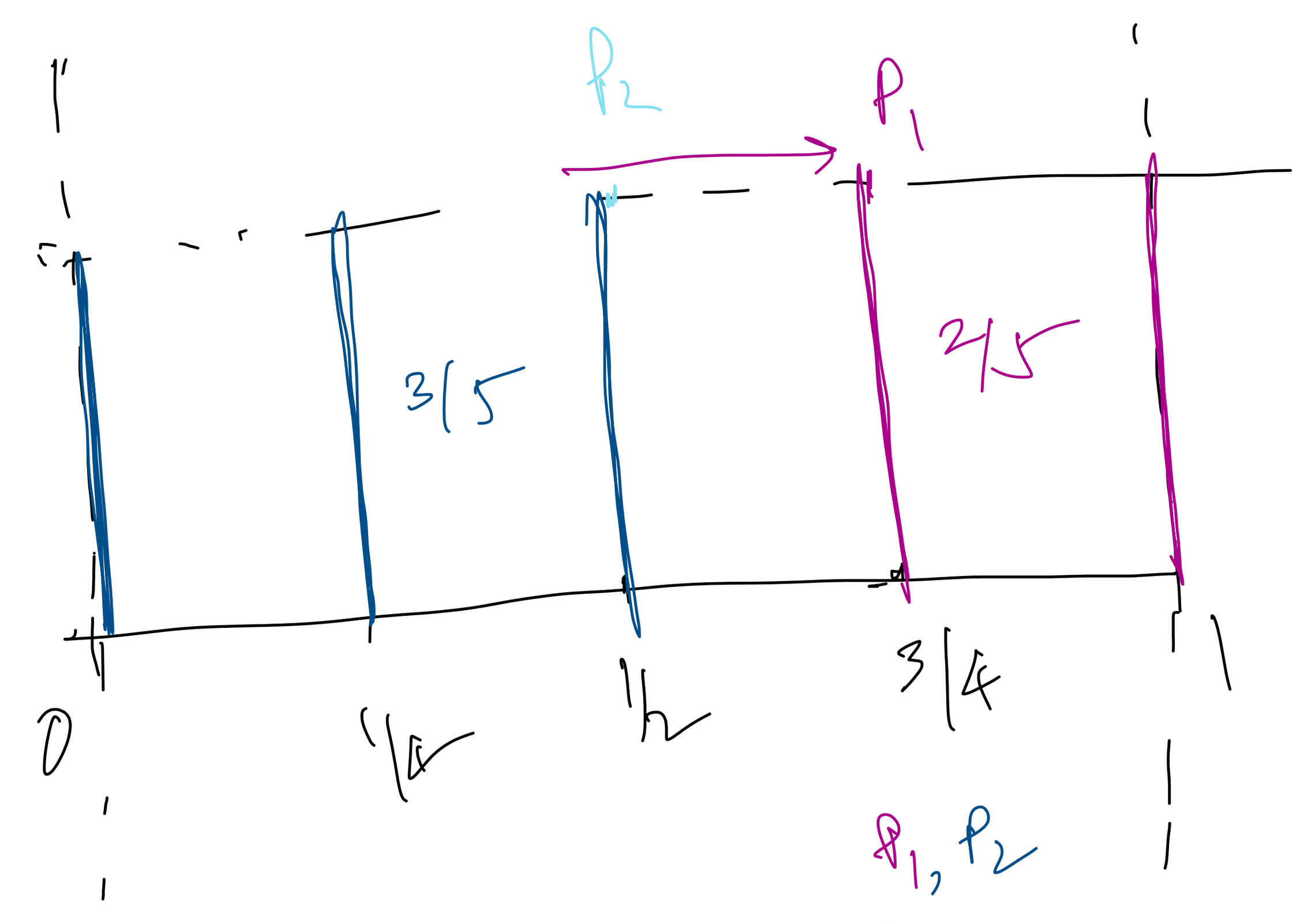


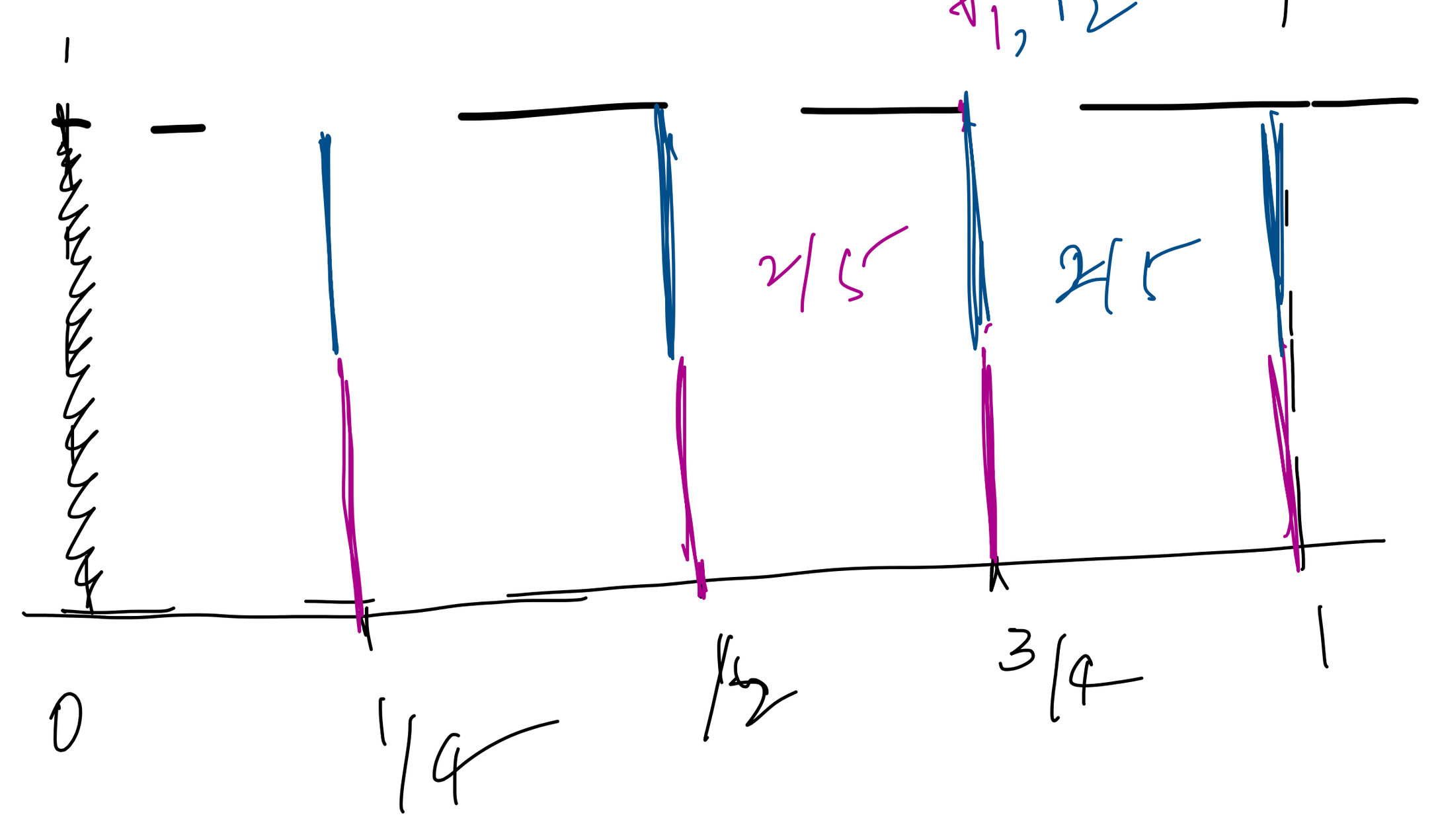






N.E ∵ no incentive to deviate.





Have incentive to deviate to increase vote share at policy ~~position~~ 0.
By symmetry, if both P_1 & P_2 are at 1/4, they lose votes of population at p = 1.
Again they have the same incentive to deviate.