Week 7

Tuesday, 10 September 2024 15:32

Ex 1

B1 \ B2	S ($1-x$)	L (x)
S	5,5	1,8
L	8,1	3,3

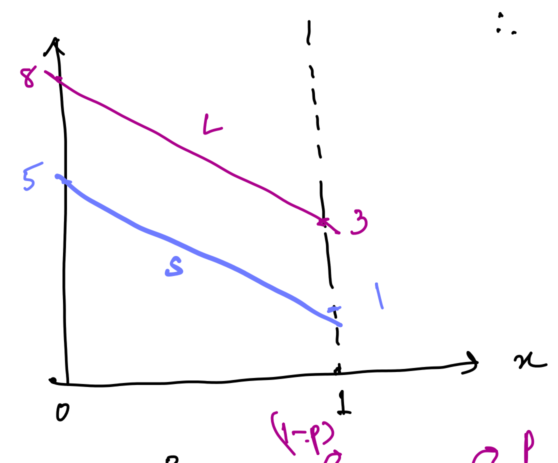
In a small beetle population, x fraction of large beetles are introduced.

$EU_1(S) = 5(1-x) + x$
$= 5 - 4x$

$EU_1(L) = 8(1-x) + 3x = 8 - 5x$

where $0 \le x \le 1$

$\therefore$ L is the ESS.



Ex 2

A \ B	X ($1-p$)	Y (p)
X	1,1	2,x
Y	x,2	3,3

$x \in \{0, 1, 2\}$

Fraction p of mutant Y is introduced.

Recall $\rightarrow$

1 \ 2	S	T (x)
S	a, a	b, c
T	c, b	d, d

N.E. $\Rightarrow$

$a \geq c$; If $x = 0$, (x, x) is N.E.

ESS $\Rightarrow$

$EU_1(S) = a(1-x) + bx$

$EU_1(T) = c(1-x) + dx$

For S to be ESS,

$EU_1(S) \geq EU_1(T)$

$a(1-x) + bx \geq c(1-x) + dx$

$\Rightarrow a \geq c$

$b \geq d$

In given question, $b < d$, we solve.

$EU_A(X) = 1.(1-p) + 2p = 1+p$

$EU_A(Y) = x(1-p) + 3p$

For $x = 2$,

$EU_A(Y) = 2+p > EU_A(X) \quad \forall p$

$\therefore$ For $x=2$, Y is ESS.

For $x = 1$,

$EU_A(Y) = 1 + 2p > EU_A(X) \quad \forall p$

$\therefore$ For $x = 1$, Y is ESS.

For $x = 0$,

$EU_A(Y) = 3p$

$\therefore EU_A(Y) > EU_A(X)$

$\Rightarrow 3p > 1+p$

$\Rightarrow p > 1/2$

For $x=0$, if $p > 1/2$, Y is ESS

else X is ESS.

Check result for when X is introduced into an existing population of Y.

Ex 3

1 \ 2	L	M	R
U	8,8	0,9	0,0
C	9,0	0,0	3,1
D	0,0	1,3	3,3

$(U, L) \longrightarrow (C, R) \longrightarrow$
$(11, 9)$

$(U, L) \rightarrow (D, M) \rightarrow (9, 11)$

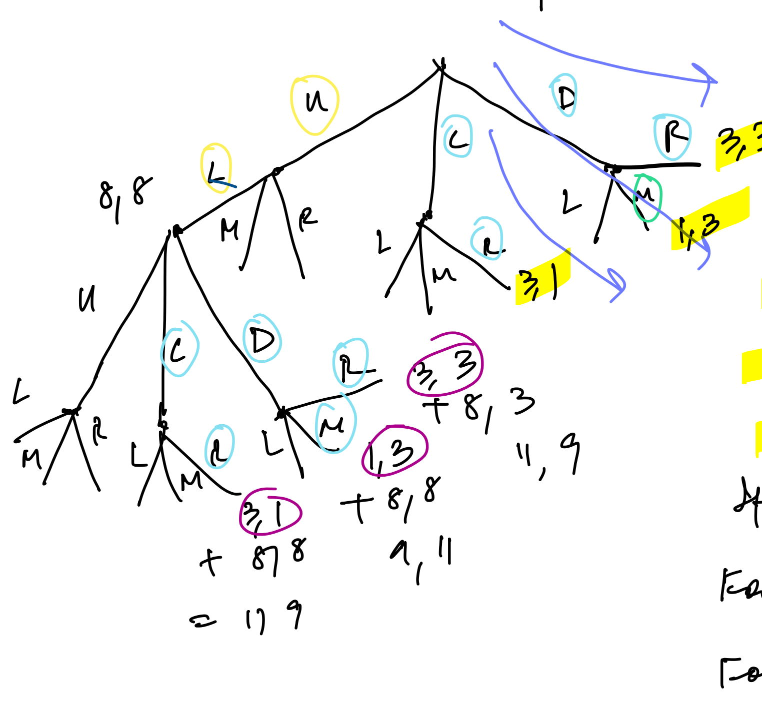
$(U, L) \rightarrow (D, R) \rightarrow (11, 11)$

1st stage

If player 1 plays U, & player 2 plays M (BR)

$\rightarrow (0, 9)$

Is it possible to cheat?



Next game:

If player 1 plays C, pl 2 has to play R

$\rightarrow (3,1) + (0,9) = (3, 10)$

If pl 1 plays D, pl 2 plays M or R

For $(D, M) \rightarrow (1,3) + (0,9) = (1, 12)$

For $(D, R) \rightarrow (3,3) + (0,9) = (3, 12)$

If players play (U, L) followed by (D, R), they can maximize payoff of both. & there is no incentive to deviate.

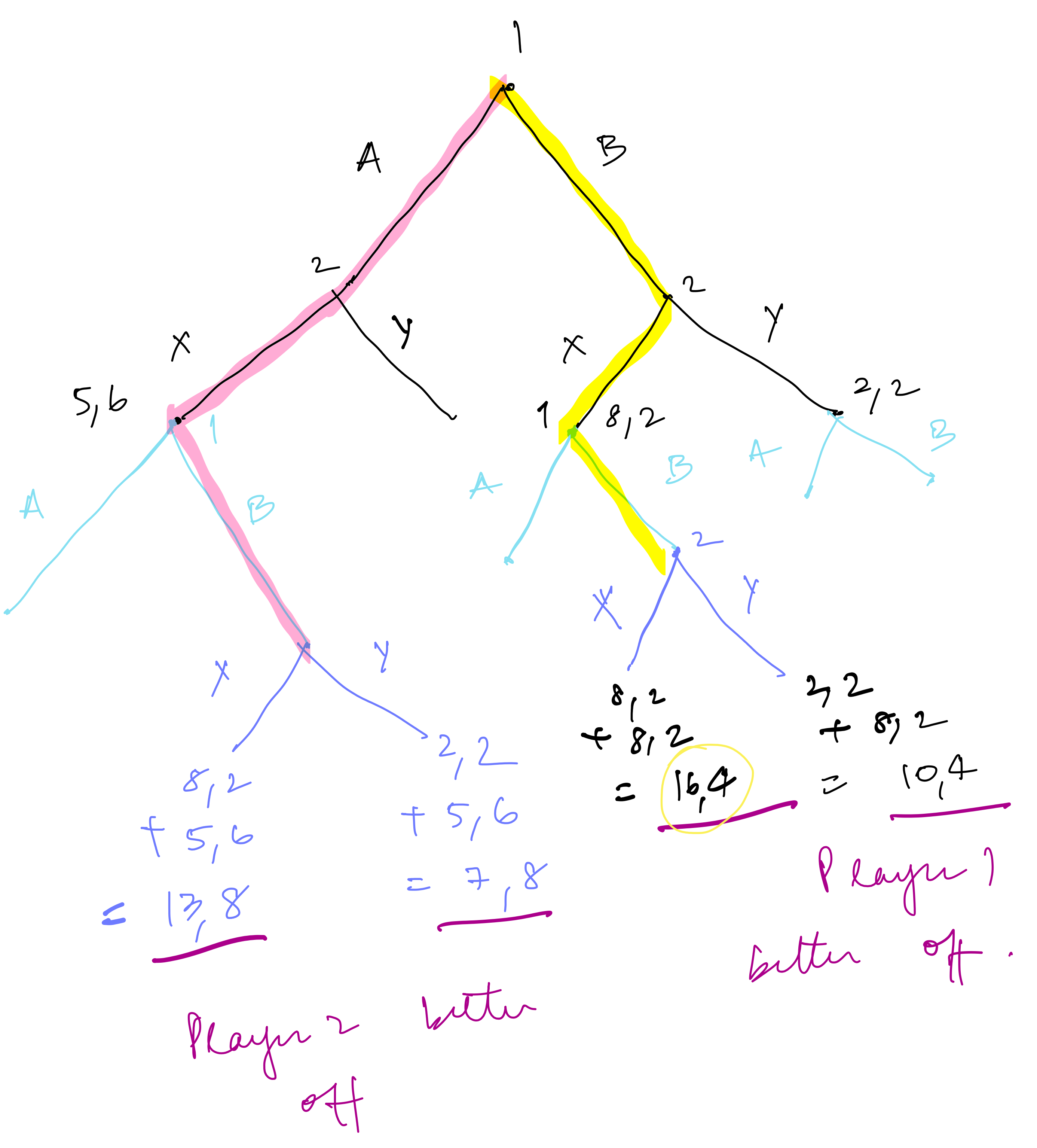
Ex 4

P1 \ P2	X	Y
A	5,6	0,0
B	8,2	2,2

N.E $\Rightarrow$ (B, X), (B, Y)

$EU_1(AB, XX/Y) = \frac{1}{2} \times 13 + \frac{1}{2} \times 7$
$= 11$

$EU_1(BB, XX/Y) = \frac{1}{2} \times 16 + \frac{1}{2} \times 10$
$= 13$



Ex

$u_1(x_1, x_2) = x_2^2 + x_2 - x_1 x_2 \qquad x_1, x_2 \geq 0$
$= x_2(x_2 + 1 - x_1)$

$0 \le \delta \le 1$

Utility is decreasing in effort.

Similarly for x_2.

$u_2(x_2, x_1) = x_1^2 + x_1 - x_2 x_1$

For any $x_1 > 0 \Rightarrow$ utility is decreasing

If $x_1 = k$, $x_2 = 0$,

$u_1 = 0$

No incentive to make an effort $x_i > 0$

$\therefore$ N.E $= (x_1, x_2) = (0, 0)$

For infinite game, u_i is discounted by δ.

If $x_1 = x_2 = k > 0$

$u_1 = k^2 + k - k^2 = k$

$u_1 = \frac{k}{1-\delta}$

If player 1 cheats & picks $x_1 = 0$ while $x_2 = k$,

$u_1 = k^2 + k - 0 = k^2 + k$

For player 1 not to cheat & sustain an effort of k, $\frac{k}{1-\delta} > k^2 + k$

$\therefore k > (k^2 + k)(1 - \delta)$

$k > k^2 + k - \delta(k^2 + k)$

$\delta(k^2 + k) > k^2$

$\delta > \frac{k^2}{k^2 + k}$

$\delta > \frac{k}{k+1} \qquad k \neq 0$