Review

Thursday, 12 September 2024 17:44

Bayesian Game

Player 1 —— only one type

Player 2 —— 2A, 2B } diff strategies, diff payoffs

BOS

2A \ 1	X	Y
X	3,1	0,0
Y	0,0	1,3

2B \ 1	X	Y
X	3,-1	1,1
Y	1,1	1,-3

Ex1

$p = 12 - q_1 - q_2$

$c_1 = 2q_1$

$c_2 = 4q_2$

} Common knowledge

Firms choose q_1 & q_2 simultaneously & independently.

$u_1 = (12 - q_1 - q_2)q_1 - 2q_1 = 12q_1 - q_1^2 - q_2q_1 - 2q_1$

$u_2 = (12 - q_1 - q_2)q_2 - 4q_2$

Each firm maximizes payoff wrt their strategy q_i. $(i = 1, 2)$

$$\frac{\partial u_1}{\partial q_1} = 12 - 2q_1 - q_2 - 2 = 0$$

$$\Rightarrow 5 - \frac{q_2}{2} = q_1 \Rightarrow q_1^* = 5 - \frac{q_2}{2}$$

$$BR_1(q_2) = 5 - \frac{q_2}{2} \quad \text{——(1)}$$

$$\frac{\partial u_2}{\partial q_2} = 12 - q_1 - 2q_2 - 4 = 0$$

$$\Rightarrow q_2^* = 4 - \frac{q_1}{2} \quad \text{——(2)}$$

$$BR_2(q_1) = 4 - \frac{q_1}{2}$$

At N.E. both firms are playing their best responses. ∴ Solve (1) & (2) simultaneously.

$$q_1^* = 5 - \frac{1}{2}\left[4 - \frac{q_1^*}{2}\right]$$

$$q_1^* = 3 + \frac{q_1^*}{4}$$

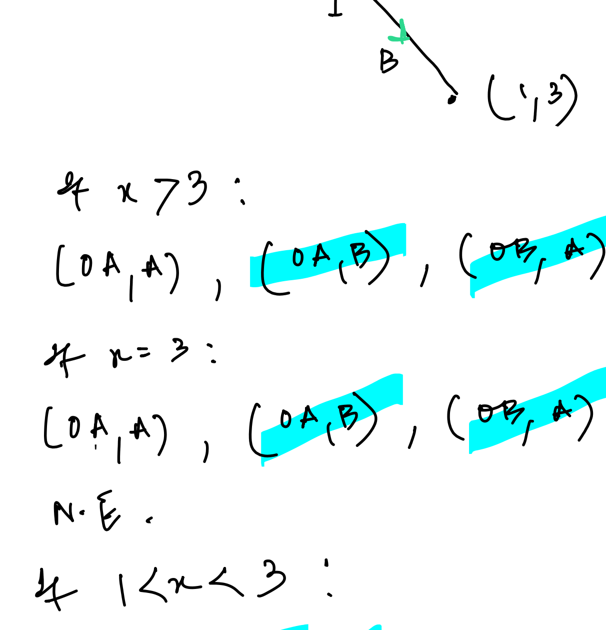
$$\frac{3q_1^*}{4} = 3$$

$$\Rightarrow q_1^* = 4$$

$$q_2^* = 4 - \frac{q_1^*}{2} = 4 - \frac{4}{2} = 2$$

∴ N.E is $(4, 2)$

Ex2



1 \ 2	A	B
OA	x, 1	x, 1
OB	x, 1	x, 1
IA	3, 1	0, 0
IB	0, 0	1, 3

x > 3

If $x > 3$:
(OA, A), (OA, B), (OB, A), (OB, B) are N.E.

If $x = 3$:
(OA, A), (OA, B), (OB, A), (OB, B), (IA, A) are N.E.

If $1 < x < 3$:
(IA, A), (OA, B), (OB, B) are N.E.

If $x = 1$:
+ (IB, B)

If $x < 1$:
(IA, A), (IB, B) are N.E.

Highlighted N.E. are not subgame perfect.

Ex3 ~~Bayesian~~ Auction

First Price, Sealed Bid, Private-Value

Utility of player 1:

$$u = \begin{cases} v - b & \text{if } b \geq b' \\ 0 & \text{otherwise} \end{cases}$$

Given: $v' \sim U[0, 1]$

$b'(v') = v'^2$

$v = 3/5 \qquad b = ?$

Expected payoff of player 1:

$EU_1 = (v - b) \cdot Pr(\text{win}) + 0 \times Pr(\text{not win})$

$EU_1 = (v - b) \cdot Pr(b \geq b')$

Now, $b' = v'^2$

∴ $Pr(b \geq b') = Pr(b \geq v'^2)$

∵ $v' > 0$, $b > 0$

∴ $Pr(v' \leq \sqrt{b})$

v' is the cont uniform R.V on $[0, 1]$

For any number $y \in [0, 1]$, CDF of v' is given as:

$Pr(v' \leq y) = y$

similarly, $Pr(v' \leq \sqrt{b}) = \sqrt{b}$

$$\therefore EU_1 = (v - b) \cdot \sqrt{b} = v\sqrt{b} - b^{3/2}$$

$$\therefore \frac{\partial EU_1}{\partial b} = \frac{v}{2\sqrt{b}} - \frac{3\sqrt{b}}{2} = 0$$

$\sqrt{b} > 0$

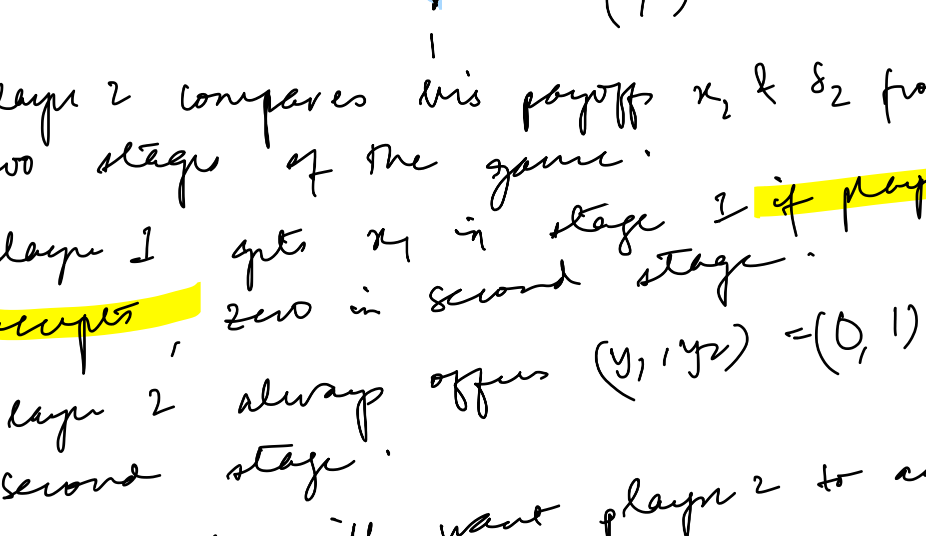
$$\Rightarrow \frac{v - 3b}{2\sqrt{b}} = 0$$

$$\therefore v - 3b = 0$$

$$\therefore b^* = \frac{v}{3}$$

$$\therefore b^* = \frac{3}{5} \cdot \frac{1}{3} = \frac{1}{5}$$

Ex4



Player 2 compares his payoffs x_2 & δ_2 from two stages of the game.

Player 1 gets x_1 in stage 1 if player 2 accepts, zero in second stage.

Player 2 always offers $(y_1, y_2) = (0, 1)$ in second stage.

∴ Player 1 will want player 2 to accept in first stage.

∴ Player 1 must offer $x_2 > \delta_2$ in first stage.

∴ Player 1 offers $(1 - \delta_2, \delta_2)$ & player 2 accepts. This is SPE.

Week 6, EX2

$u_i = v_i - b_i$ if $b_i \geq b_j \quad \forall j$

$j = 1, 2, \ldots i-1, i+1, \ldots N$

$$\therefore EU_i = (v_i - b_i) \times Pr(\text{win})$$

$$= (v_i - b_i) \times \left(Pr(b_i \geq v_j^2)\right)^{N-1}$$

$$= (v_i - b_i) \times (b_i)^{\frac{N-1}{2}}$$

Max EU_i wrt b_i