Strategy: An Introduction to Game Theory

Week 6: Bayesian Auctions

TA: Arti Agarwal

Recap

- Dominant Strategies
- Nash Equilibrium
- Mixed Strategies
- Extensive Form Games
- Bayesian Games

Random Variables

A uniform random variable X on the interval [a, b] has the property that the probability that X lies in any subinterval of length x, is equal to x/(a-b). The probability density function $f(x)$ on an interval [a, b] is given as:

$$f(x) = \frac{1}{b-a}, \qquad a < x < b$$

$$f(x) = 0, \qquad x < a \; or \; x > b$$

Random Variables

The cumulative density function (CDF) $F(x)$ is defined as:

$$F(x) = P(X \leq x)$$

So the CDF of a uniform random variable over [0, 1] is given as:

$$\begin{array}{lll} F(x) & = 0 & x \leq 0 \\ & = x & 0 \leq x \leq 1 \\ & = 1 & x \geq 1 \end{array}$$

First Price Sealed Bid Auction

There are two players who want to own a prized object for auction. The valuation of each player i is v_i and v_1 and v_2 are independently and uniformly distributed on [0, 1]. The value vi is known to player i. Each player bids b_i, a real number. The bidder with highest bid wins the object and pays the bid. The loser gets 0 payoff. Find a symmetric, linear Bayesian Nash Equilibrium where $b_i = a + c\ v_i$ for some constants a and c. Determine a and c.

First Price Sealed Bid Auction

Consider a first-price, sealed-bid auction in which the bidders' valuations are independently and uniformly distributed on [0,1]. Show that if there are n bidders, then the strategy of bidding (n-l)/n times one's valuation is a symmetric Bayesian Nash equilibrium of this auction.

Gibbons Ch 3, Ex 3.6

Second Price Sealed Bid Auction

A house is to be sold in a second price sealed-bid auction. There are N bidders. The valuation of each bidder follows a probability distribution with a CDF of F_i for the ith bidder. Prove that in a symmetric Nash Equilibrium, bidding one's own valuation is a weakly dominant strategy in the auction, and is independent of the probability distribution of the valuation of other bidders.

Reference Reading

1. ***An Introduction to Game Theory*** by Martin Osborne
2. ***Strategy, An Introduction to Game Theory*** by Joel Watson
3. ***A Primer in Game Theory*** by Robert Gibbons
4. ***Mathematical Statistics and Data Analysis (3e)*** by John A. Rice

If you have questions, please contact:

arti21@iitk.ac.in