Strategy, An Introduction to Game Theory

Week 5: Bayesian Games

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Bertrand Duopoly

There are two firms. Firm 1's marginal cost of production is zero. Firm 2's marginal cost of production is 1. If the lowest price charged is p, the market demand is **Q = 8–p**. Each firm can choose only one of the three prices, 1, 4 or 6. Determine the Nash Equilibrium.

Jehle & Reny Ch 7, Ex 7.3

Bertrand Bayesian Duopoly

There are two firms. Firm 1's marginal cost of production is zero. Firm 1 believes that firm 2's marginal cost of production is either 1 or 4, and that each of these 'types' of firm 2 occur with probability ½. If the lowest price charged is p, the market demand is **Q = 8–p**. Each firm can choose only one of the three prices, 1, 4 or 6.

The payoffs of firms are given in the diagram. Firm is row player and firm 2 is column player. The first matrix corresponds to firm 2 having low marginal costs, and second matrix corresponds to firm 2 having high marginal costs. Determine the Bayesian Nash Equilibrium.

Jehle & Reny Ch 7, Ex 7.3

Firm 2 is of type “low” marginal cost

F1 \ F2	6	4	1
6	6,5	0,12	0,0
4	16,0	8,6	0,0
1	7,0	7,0	7,0

Firm 2 is of type “high” marginal cost

F1 \ F2	6	4	1
6	6,2	0,0	0, -21
4	16,0	16,0	0, -21
1	7,0	7,0	7,0

Spy Game

Two countries must simultaneously decide upon a course of action. Country 1 must decide to keep its weapons or destroy them. Country 2 must decide whether to spy on country 1 or not. Country 1 can be of two types: aggressive or non-aggressive. Country 1 knows its type but country 2 does not know country 1's type. Country 1 is of type "aggressive" with probability ε. The payoffs are given in the diagram. If $\varepsilon < 1/5$, find the BNE and determine if it is unique.

Jehle & Reny Ch 7, Ex 7.18

Country 1 is aggressive with probability ε

C1 \ C2	Spy	Don't Spy
Keep	10, -9	5, -1
Destroy	0, 2	0, 2

Country 1 is non-aggressive with probability 1-ε

C1 \ C2	Spy	Don't Spy
Keep	-1, 1	1, -1
Destroy	0, 2	0, 2

Cournot Duopoly

Consider a Cournot duopoly operating in a market with inverse demand $P(Q) = a - Q$ where $Q = q_1 + q_2$ is the aggregate quantity on the market. Both firms have total costs $c(q_i) = cq_i$ but demand is uncertain; it is high ($a = a_H$) with probability θ or low ($a = a_L$) with probability (1 − θ). Furthermore, information is asymmetric: **firm 1 knows whether demand is high or low, but firm 2 does not.** All of this is common knowledge. The two firms simultaneously choose quantities. What are the strategy spaces for the two firms? Make assumptions regarding a_H, a_L, θ such that q_i are greater than zero.

Gibbons Ch 3, Ex 3.2

Reference Reading

1. ***An Introduction to Game Theory*** by Martin Osborne
2. **Strategy, *An Introduction to Game Theory*** by Joel Watson
3. ***Advanced Microeconomic Theory (3e)*** by Jehle & Reny
4. ***A Primer in Game Theory*** by Robert Gibbons