



Strategy, An Introduction to Game Theory

Week 5: Bayesian Games

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Bertrand Duopoly

There are two firms. Firm 1's marginal cost of production is zero. Firm 2's marginal cost of production is 1. If the lowest price charged is p , the market demand is $Q = 8 - p$. Each firm can choose only one of the three prices, 1, 4 or 6. Determine the Nash Equilibrium.

Bertrand Bayesian Duopoly

There are two firms. Firm 1's marginal cost of production is zero. Firm 1 believes that firm 2's marginal cost of production is either 1 or 4, and that each of these 'types' of firm 2 occur with probability $\frac{1}{2}$. If the lowest price charged is p , the market demand is $Q = 8 - p$. Each firm can choose only one of the three prices, 1, 4 or 6.

The payoffs of firms are given in the diagram. Firm 1 is row player and firm 2 is column player. The first matrix corresponds to firm 2 having low marginal costs, and second matrix corresponds to firm 2 having high marginal costs. Determine the Bayesian Nash Equilibrium.

Firm 2 is of type “low”
marginal cost

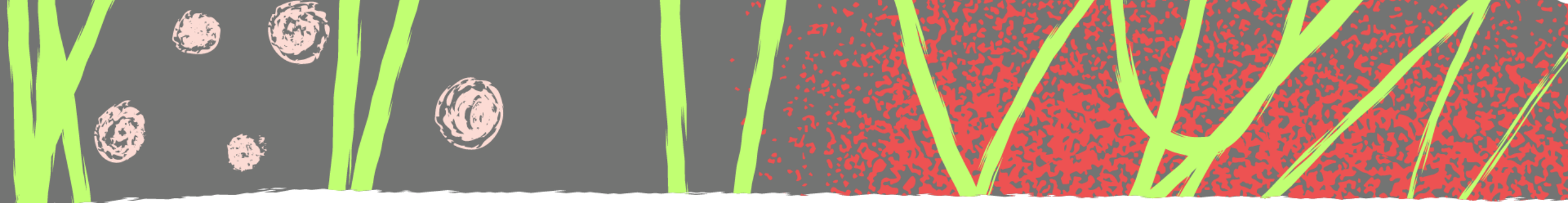
F1 \ F2			
	6	4	1
6	6,5	0,12	0,0
4	16,0	8,6	0,0
1	7,0	7,0	7,0

Firm 2 is of type “high”
marginal cost

F1 \ F2			
	6	4	1
6	6,2	0,0	0, -21
4	16,0	16,0	0, -21
1	7,0	7,0	7,0

Spy Game

Two countries must simultaneously decide upon a course of action. Country 1 must decide to keep its weapons or destroy them. Country 2 must decide whether to spy on country 1 or not. Country 1 can be of two types: aggressive or non-aggressive. Country 1 knows its type but country 2 does not know country 1's type. Country 1 is of type “aggressive” with probability ε . The payoffs are given in the diagram. If $\varepsilon < 1/5$, find the BNE and determine if it is unique.



Country 1 is aggressive with
probability ε

		C2	
		Spy	Don't Spy
C1	Keep	10, -9	5, -1
	Destroy	0, 2	0, 2

Country 1 is non-aggressive with
probability $1-\varepsilon$

		C2	
		Spy	Don't Spy
C1	Keep	-1, 1	1, -1
	Destroy	0, 2	0, 2

Cournot Duopoly

Consider a Cournot duopoly operating in a market with inverse demand $P(Q) = a - Q$ where $Q = q_1 + q_2$ is the aggregate quantity on the market. Both firms have total costs $c(q_i) = cq_i$ but demand is uncertain; it is high ($a = a_H$) with probability θ or low ($a = a_L$) with probability $(1 - \theta)$. Furthermore, information is asymmetric: **firm 1 knows whether demand is high or low, but firm 2 does not**. All of this is common knowledge. The two firms simultaneously choose quantities. What are the strategy spaces for the two firms? Make assumptions regarding a_H , a_L , θ such that q_i are greater than zero.

Reference Reading

1. ***An Introduction to Game Theory*** by Martin Osborne
2. ***Strategy, An Introduction to Game Theory*** by Joel Watson
3. ***Advanced Microeconomic Theory (3e)*** by Jehle & Reny
4. ***A Primer in Game Theory*** by Robert Gibbons

