Strategy: An Introduction to Game Theory

Week 1: Nash Equilibrium, Dominant Strategies, Multiple NE

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Reference Reading

1. *Games of Strategy (3e to 5e)* by Avinash Dixit, Susan Skeath, David Reiley. Ebook link (partial)
2. *An Introduction to Game Theory* by Martin Osborne
3. *Strategy. An Introduction to Game Theory* by Joel Watson
4. *Strategies and Games. Theory and Practice* by Prajit K. Dutta

What is a Strategic Game?

Assumptions

- Rational preferences.
  - Max payoff always
  - $A \succ B$ & $B \succ C$ then $A \succ C$
- Complete information

- Players/ Agents
- Competitive setting, actions/ strategies of one player affects the other and they are aware of it
- Maximize their utility/ profit/ payoff
- Follow a set of rules
- Choose actions/ strategies

Notation

N players

ith player playing then all other players are mentioned as $-i$.

Actions (strategies of players

$a_i \rightarrow$ action of player i

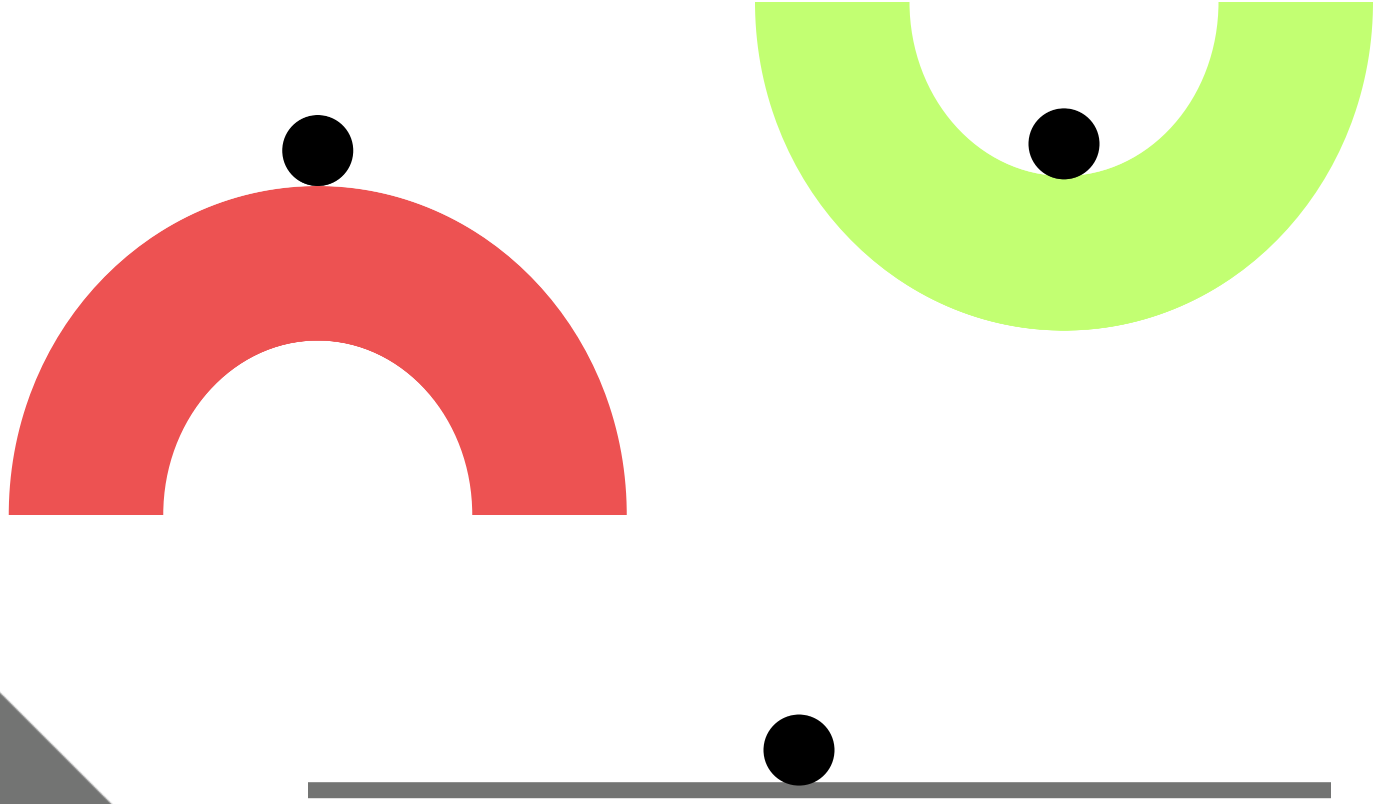
$a_{-i} \rightarrow$ " " " except i

$u_i(a_i, a_{-i})$

Outcome Matrix

P1 \ P2	a	b
x	2, 4	,
y	,	,

What is an Equilibrium?



The Grade Game

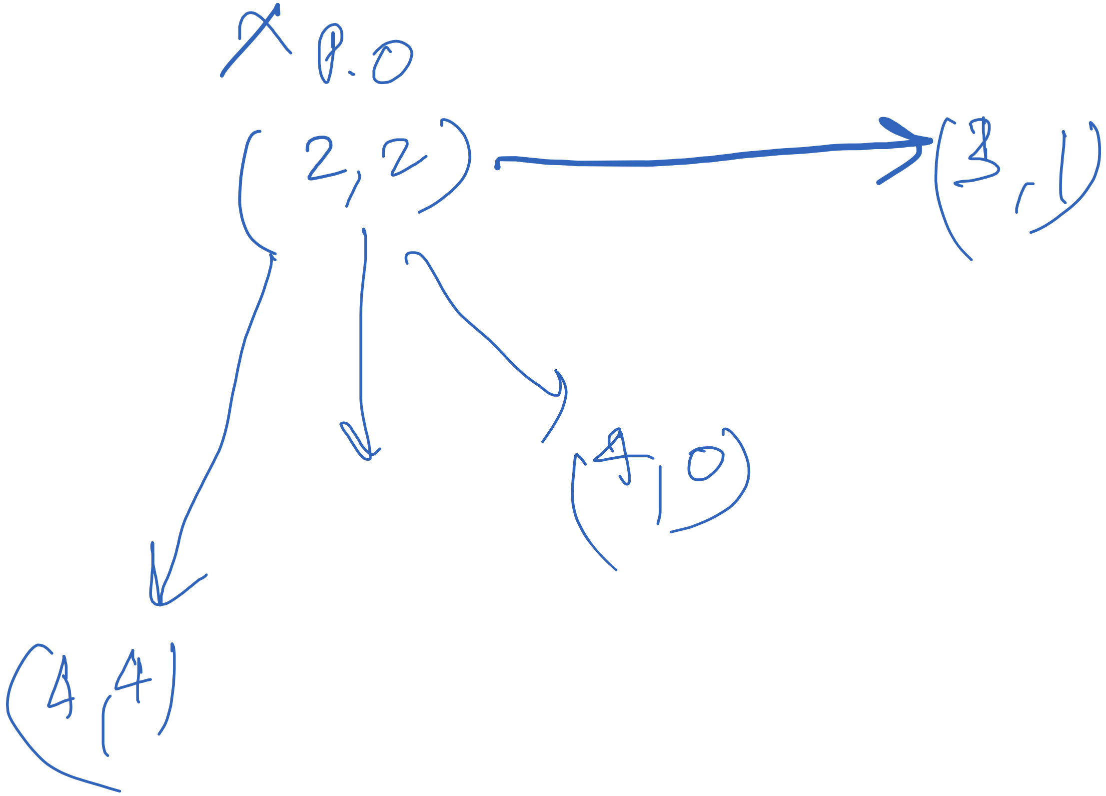
Two players are asked to choose to pick one out of {**α**, **β**}. They are awarded grades based on what they and their pair pick.

[Pollack Lecture 1]

P2 / P1	**α**	**β**
α	B-, B-	A, C
β	C, A	B+, B+

Nash Equilibrium

- When **all players are playing their best responses** to each other
- No player has any incentive to deviate
- Self-enforcing
- Not always Pareto optimal
- NE does not always exist



Prisoners' Dilemma

- Players/ Agents: prisoners
- Competitive setting/ actions/ strategies of one player affects the other and they are aware of it: they know their actions affect both of them
- Maximize Utility/ Profit/ Payoff: each wants to get lowest sentence
- Follow a set of rules: cannot communicate with each other
- Choose actions/ strategies: confess or deny

Prisoners' Dilemma

Payoff is the utility of prisoners

Prisoner 2 / Prisoner 1	C	D
C	-5, -5	0, -10
D	-10, 0	1, 1

-2, -2

Dominant Strategies

A strategy s_i^* of player i strictly dominates a strategy s_i' of player i if:

$$U_i(s_i^*, s_{-i}) > U_i(s_i', s_{-i})$$

s_i^* is dominating $\hat{s}_i$

$$U_i(\hat{s}_i, s_{-i}) > U_i(s_i^*, s_{-i})$$

$\hat{s}_i$ is dominating s_i^*

Dominant Strategies

A is dominant strategy for 1

& Z is dominant for 2

Player 1 \ Player 2	X	Y	Z
A	10, 2	8, 3	7, 5
B	5, 7	4,4	5, 8
C	7, 3	3, 2	2, 9
D	3, 2	5, 2	-1, 3

Identify the dominant strategies of the two players, if any.

(A, Z) is the NE

Rock Paper Scissors

P2 / P1	R	P	S
R	0,0	-1, 1	1, -1
P	1, -1	0, 0	-1, 1
S	-1, 1	1, -1	0,0

Hawk-Dove Game

Two animals are fighting over some prey. Each can be passive (dove) or aggressive (hawk). Each prefers to be aggressive if its opponent is passive, and passive if its opponent is aggressive; given its own stance, it prefers the outcome when its opponent is passive to that in which its opponent is aggressive.

[Osborne Ch 2, Ex 31.2]

Hawk-Dove Game

P2 / P1	Hawk	Dove
Hawk	0, 0	4, 2
Dove	2, 4	3, 3

Humanities vs Sciences

A university is contemplating whether to build a new lab or a theater on campus. The science faculty would rather see a new lab built, and the humanities faculty would rather see a new theater built. However, funding is contingent on unanimous support. If there is disagreement, neither project will be built. The meetings of two faculty groups occur simultaneously. Find the NE.

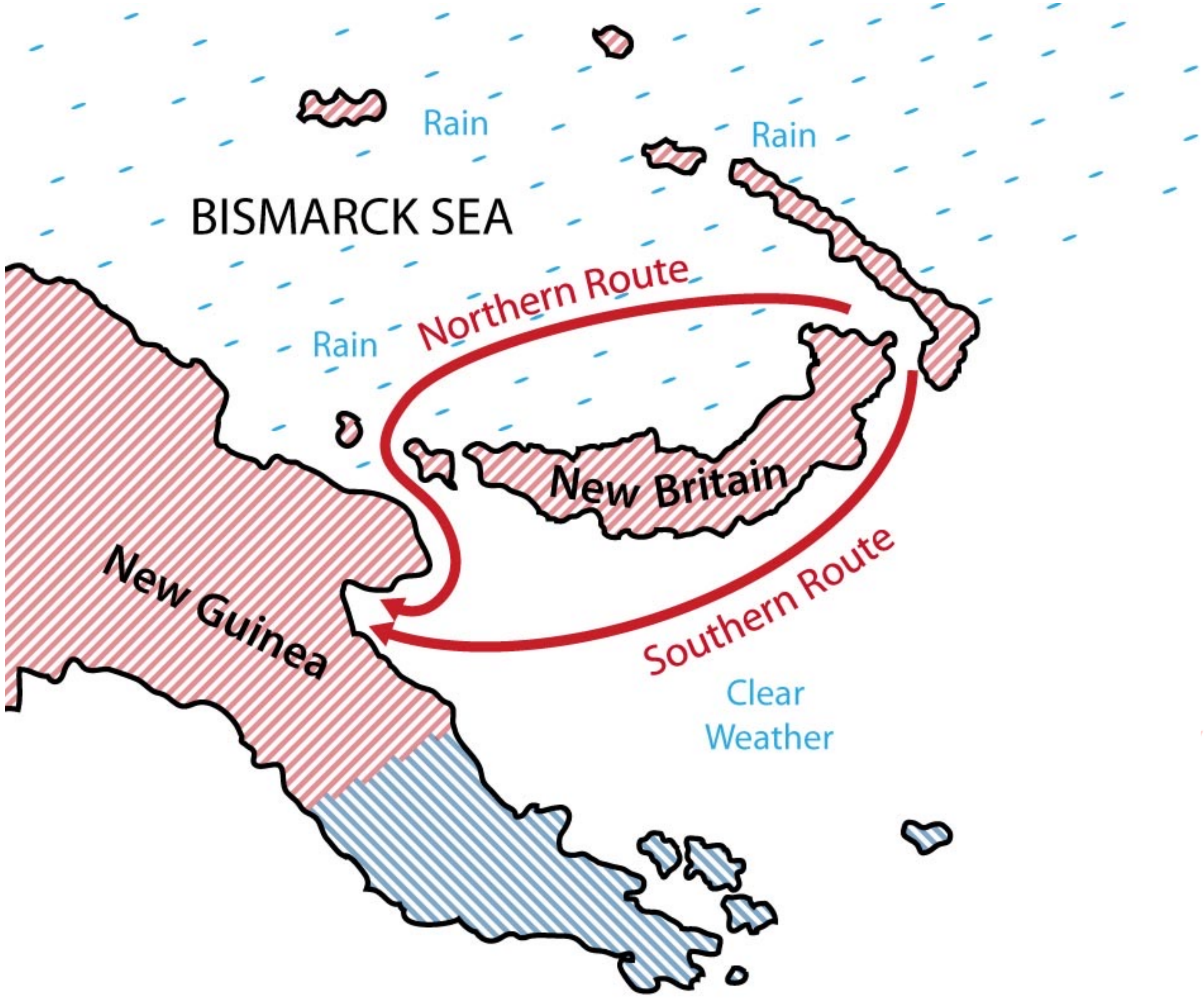
[Dixit et al, Ch 4, S8]

Humanities vs Sciences

Humanities / Science	Lab	Theater
Lab	4,2	0,0
Theater	0,0	1,5

Battle of Bismarck Sea

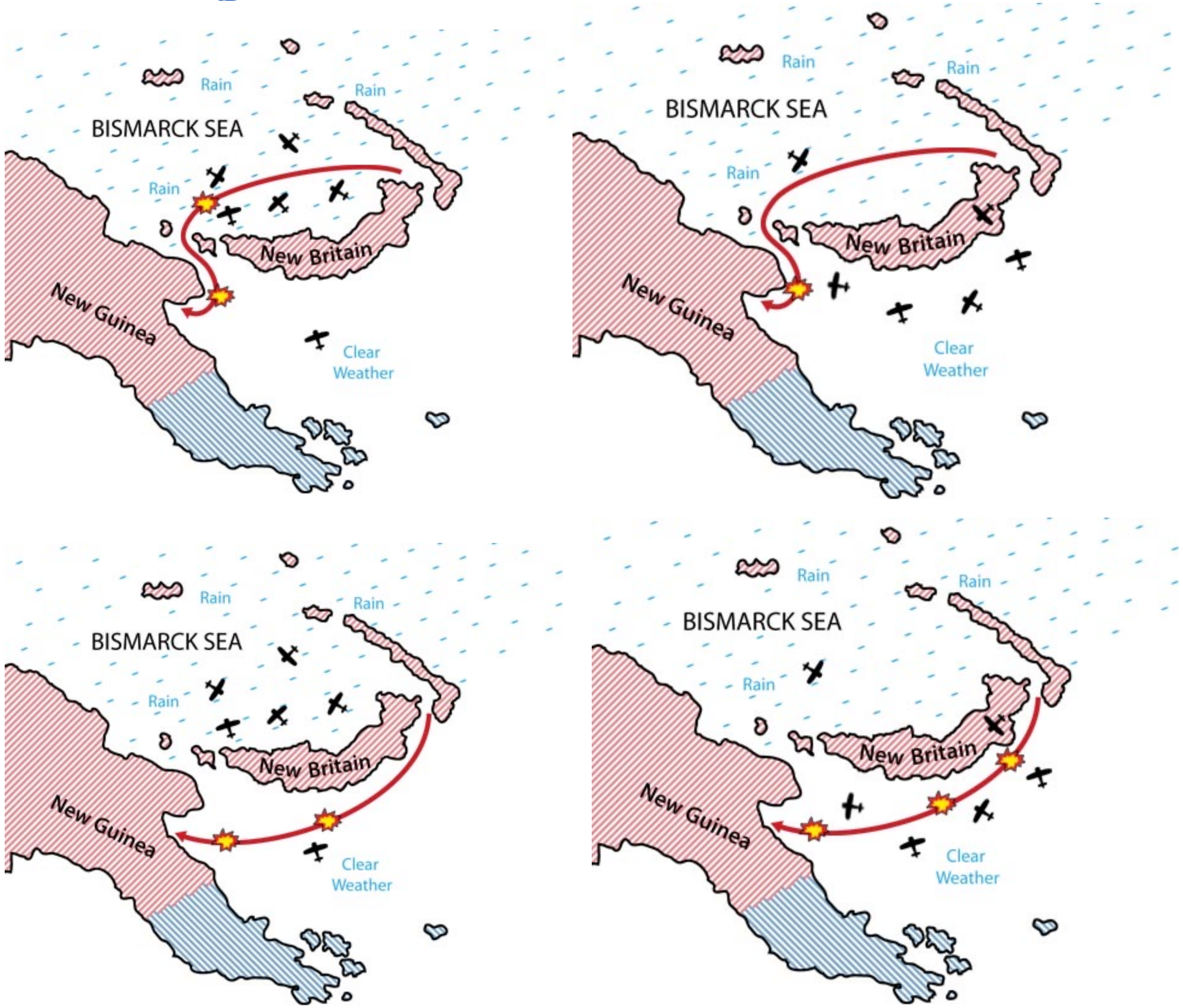
The Battle of the Bismarck Sea was a military game during WW2. The Japanese and the Allies forces had to simultaneously decide which route to take to reach New Guinea.



Source: https://policonomics.com/battle-of-the-bismarck-sea/

Battle of Bismarck Sea

Japanese / Allied	North	South
North	2, -2	2, -2
South	1, -1	3, -3



Joint Project

Tom and Arthur work on a joint project. For an effort x_1 by Tom, he pays a cost of $C_1 = 2x_1$ and for an effort of x_2 by Arthur, he bears a cost of $C_2 = 2x_2$. The reward of the project is $R = (x_1 + x_2)^2$, shared equally between both. If they don't work and goof off, they enjoy leisure with payoff = 1 and no reward.

$x_i \in \{1, 2, 3\}$. Find NE.

Tom/ Arthur	Work	Goof off
Work	$R/2-C_1$, $R/2-C_2$	$R/2-C_1$, $R/2$
Goof off	$R/2$, $R/2-C_2$	1, 1

If Tom works

$$u_1 = \frac{(x_1 + x_2)^2}{2} - 2x_1$$

$$u_2 = \frac{(x_1 + x_2)^2}{2} - 2x_2$$

Joint Project

$R_i = \frac{(2+2)^2}{2} = 8$ $\quad C_i = 2x_i = 4$

$x_1 = x_2 = 2$

A \ I	W	G
W	4, 4	4, 8
G	8, 4	?, 1

(W, G), (G, W)

If you have questions, please contact

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