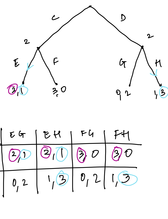
Week 4

Tuesday, 19 August 2024 18:08

Q1) Backward Induction

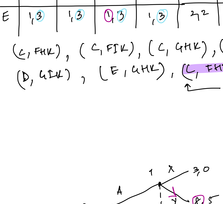
Game tree -?
NE -?
SPE -?



1 \ 2	EG	EH	FG	FH
C	2,1	2,1	3,0	3,0
D	0,2	1,3	0,2	1,3

N.E:
(C, EG)
(C, EH)
SPE: (C, EH)

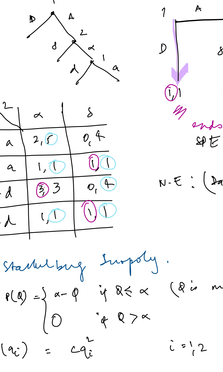
Ex



1 \ 2	FHK	FIK	GHK	GIK	FHJ	GHJ	FIJ	GIJ
C	3,0	3,0	1,0	1,0	3,0	1,0	3,0	1,0
D	1,1	2,1	1,1	2,1	1,1	1,1	2,1	2,1
E	1,3	1,3	1,3	1,3	2,2	2,2	2,2	2,2

N.E: (E, FHK), (C, FIK), (C, GHK), (C, GIK),
(D, GIK), (E, GHK), (C, FHJ), (C, FIJ), (D, GIJ)
not S.P.E.

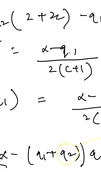
Ex

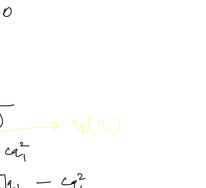


1 \ 2	AC	AD	BC	BD
W X	3,0	3,0	4,6	4,6
W Y	2,1	3,2	2,1	2,1
Z X	6,4	3,2	6,4	3,2
Z Y	6,4	3,2	5,4	3,2

SPE:
(WY, AC),
(ZX, BC)

Ex





cuts here
SPE

1 \ 2	α	δ
A a	2,5	0,4
D a	1,1	1,1
A d	3,3	0,4
D d	1,1	1,1

N.E: (Aa, δ), (Dd, δ)

Ex Stackelberg Duopoly.

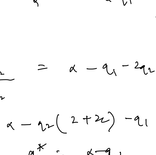
$$P(Q) = \begin{cases} \alpha - Q & \text{if } Q \le \alpha \\ 0 & \text{if } Q > \alpha \end{cases}$$ (Q is market demand)

$$C_i(q_i) = cq_i^2 \qquad i = 1,2$$

$Q = q_1 + q_2$

$q_1, q_2 \in \mathbb{R}^+$

1 is leader
2 is follower



$$P = \alpha - (q_1 + q_2)$$

$$\pi_1 = Pq_1 - Cq_1$$

$$\pi_1 = (\alpha - (q_1 + q_2))q_1 - cq_1^2$$

$$\pi_2 = Pq_2 - Cq_2$$

$$\pi_2 = (\alpha - (q_1 + q_2))q_2 - cq_2^2$$

$$\frac{d\pi_2}{dq_2} = \alpha - q_1 - 2q_2 - 2cq_2 = 0$$

$$\Rightarrow \alpha - q_2(2 + 2c) - q_1 = 0$$

$$\Rightarrow q_2^* = \frac{\alpha - q_1}{2(c+1)}$$

$$BR_2(q_1) = \frac{\alpha - q_1}{2(c+1)} \rightarrow q_2(q_1)$$

$$\pi_1 = (\alpha - (q_1 + q_2))q_1 - cq_1^2$$

$$\pi_1 = \left[\alpha - \left(q_1 + \frac{\alpha - q_1}{2(c+1)}\right)\right]q_1 - cq_1^2$$

$$\pi_1 = \alpha q_1 - q_1^2 - \frac{\alpha q_1}{2(c+1)} + \frac{q_1^2}{2(c+1)} - cq_1^2$$

$$\frac{d\pi_1}{dq_1} = \alpha - 2q_1 - \frac{\alpha}{2(c+1)} + \frac{2q_1}{2(c+1)} - 2cq_1 = 0$$

$$q_1\left[-2 + \frac{1}{c+1} - 2c\right] = \alpha\left[\frac{1}{2(c+1)} - 1\right]$$

Bertrand Duopoly.

Two firms. Each firm sets price p_i.
The firm with $p_i < p_j$ gets market share as per demand curve. The other firm cannot sell.

Demand for

$$Q_i(p_1, p_2) = \begin{cases} A - p_i & \text{if } p_i < p_j \\ \frac{A - p_i}{2} & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j \end{cases} \qquad A > 0$$

Costs are zero.

If $p_1 > p_2$

$\pi_1 = 0$

$\pi_2 = (A - p_2)p_2$

If $p_1 = p_2$

$\pi_1 = \left(\frac{A - p_1}{2}\right) \cdot p_1$

$\pi_2 = \left(\frac{A - p_2}{2}\right) \cdot p_2$