

# Strategy: An Introduction to Game Theory

## **Week 6: Bayesian Auctions**

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# Recap

- ❖ Dominant Strategies
- ❖ Nash Equilibrium
- ❖ Mixed Strategies
- ❖ Extensive Form Games
- ❖ Bayesian Games



# Random Variables

A uniform random variable  $X$  on the interval  $[a, b]$  has the property that the probability that  $X$  lies in any subinterval of length  $x$ , is equal to  $x/(b-a)$ . The probability density function  $f(x)$  on an interval  $[a, b]$  is given as:

$$f(x) = \frac{1}{b-a}, \quad a < x < b$$

$$f(x) = 0, \quad x < a \text{ or } x > b$$

# Random Variables

The cumulative density function (CDF)  $F(x)$  is defined as:

$$F(x) = P(X \leq x)$$

So the CDF of a uniform random variable over  $[0, 1]$  is given as:

$$\begin{aligned} F(x) &= 0 & x &\leq 0 \\ &= x & 0 &\leq x \leq 1 \\ &= 1 & x &\geq 1 \end{aligned}$$



# First Price Sealed Bid Auction

There are two players who want to own a prized object for auction. The valuation of each player  $i$  is  $v_i$  and  $v_1$  and  $v_2$  are independently and uniformly distributed on  $[0, 1]$ . The value  $v_i$  is known to player  $i$ . Each player bids  $b_i$ , a real number. The bidder with highest bid wins the object and pays the bid. The loser gets 0 payoff. Find a symmetric, linear Bayesian Nash Equilibrium where  $b_i = a + c v_i$  for some constants  $a$  and  $c$ . Determine  $a$  and  $c$ .

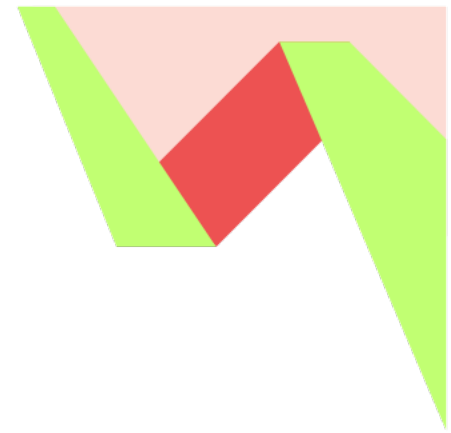


# First Price Sealed Bid Auction

Consider a first-price, sealed-bid auction in which the bidders' valuations are independently and uniformly distributed on  $[0,1]$ . Show that if there are  $n$  bidders, then the strategy of bidding  $(n-1)/n$  times one's valuation is a symmetric Bayesian Nash equilibrium of this auction.

Gibbons Ch 3, Ex 3.6

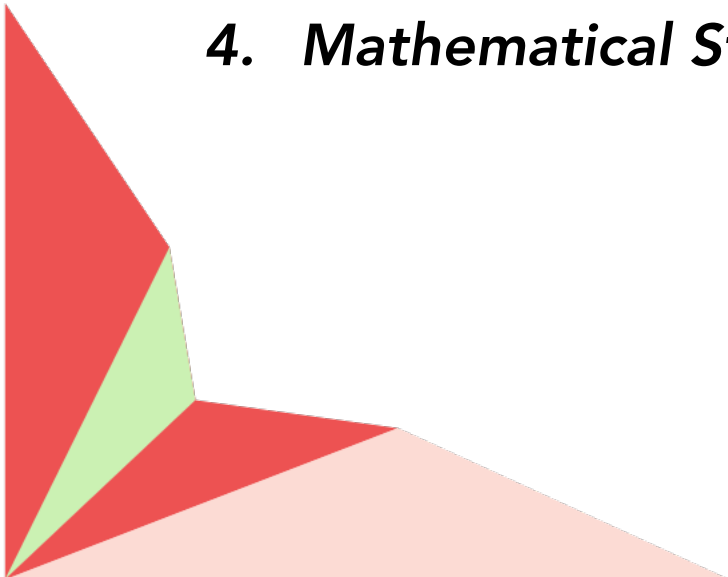
# Second Price Sealed Bid Auction



A house is to be sold in a second price sealed-bid auction. There are  $N$  bidders. The valuation of each bidder follows a probability distribution with a CDF of  $F_i$  for the  $i$ th bidder. Prove that in a symmetric Nash Equilibrium, bidding one's own valuation is a weakly dominant strategy in the auction, and is independent of the probability distribution of the valuation of other bidders.

# Reference Reading

1. *An Introduction to Game Theory* by Martin Osborne
2. *Strategy, An Introduction to Game Theory* by Joel Watson
3. *A Primer in Game Theory* by Robert Gibbons
4. *Mathematical Statistics and Data Analysis (3e)* by John A. Rice





If you have questions,  
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