

Strategy: An Introduction to Game Theory

Week 3: Rationality, Auctions,
Extensive Form Games

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Recap

- ❖ Dominant Strategies
- ❖ Nash Equilibrium
- ❖ Tragedy of Commons
- ❖ Mixed Strategies



Second Price Sealed Bid Auction

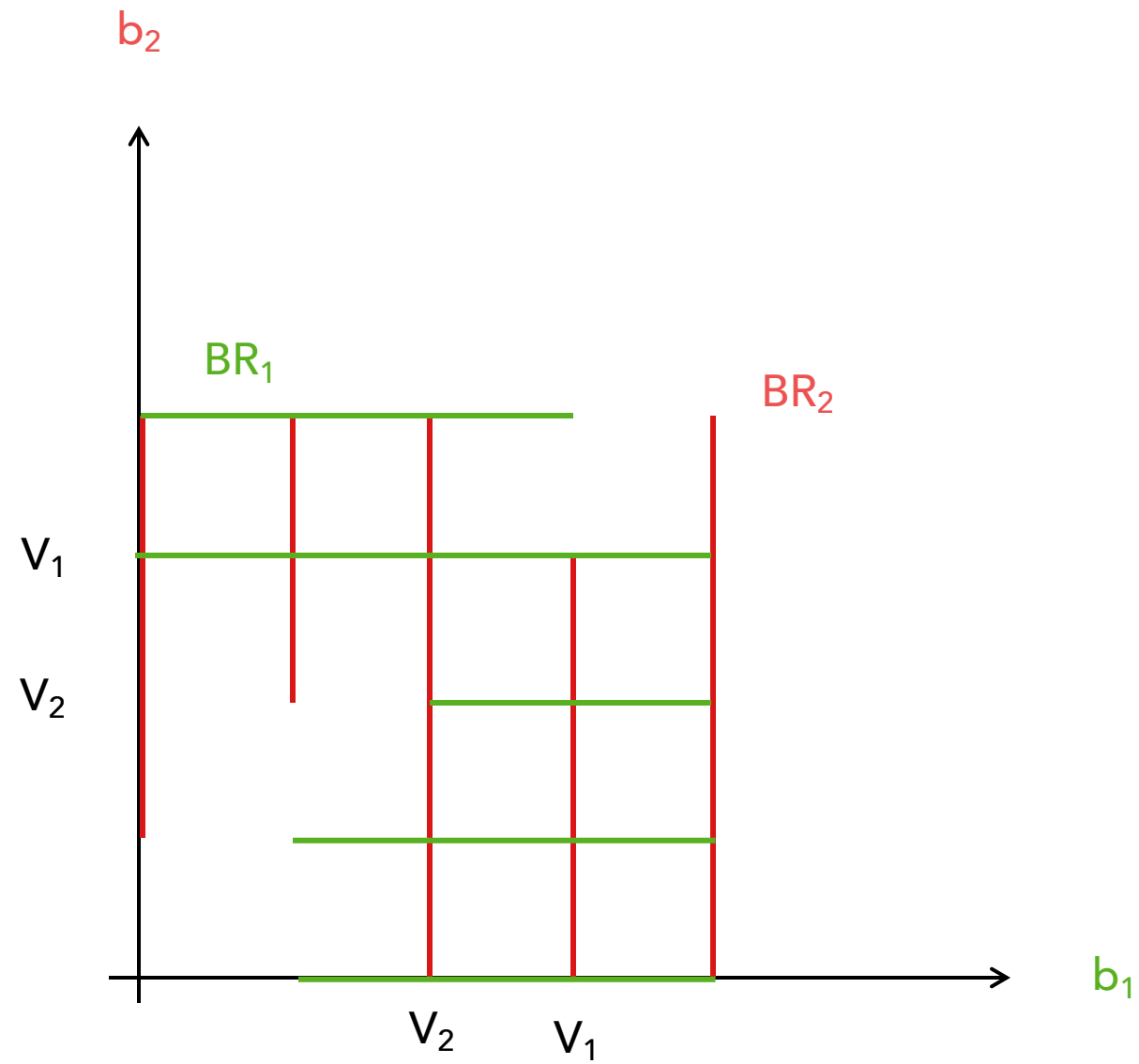
Two buyers wish to buy an antique piece of furniture in a second price, sealed bid auction. They are the only two potential buyers. Their individual valuations of the furniture are $V_1 = 3$ and $V_2 = 2$ (assume valuations and bids are in 1000s of \$). Their strategies are bids $b_i = \{0, 1, 2, 3, 4\}$. If they both bid the same amount, P_1 wins and pays the bid amount. Find the N.E.

Second Price Sealed Bid Auction

P1 \ P2	0	1	2	3	4
0	0,0	0,2	0,2	0,2	0,2
1	3,0	2,0	0,1	0,1	0,1
2	3,0	2,0	1,0	0,0	0,0
3	3,0	2,0	1,0	0,0	0,-1
4	3,0	2,0	1,0	0,0	-1,0

Second Price Sealed Bid Auction

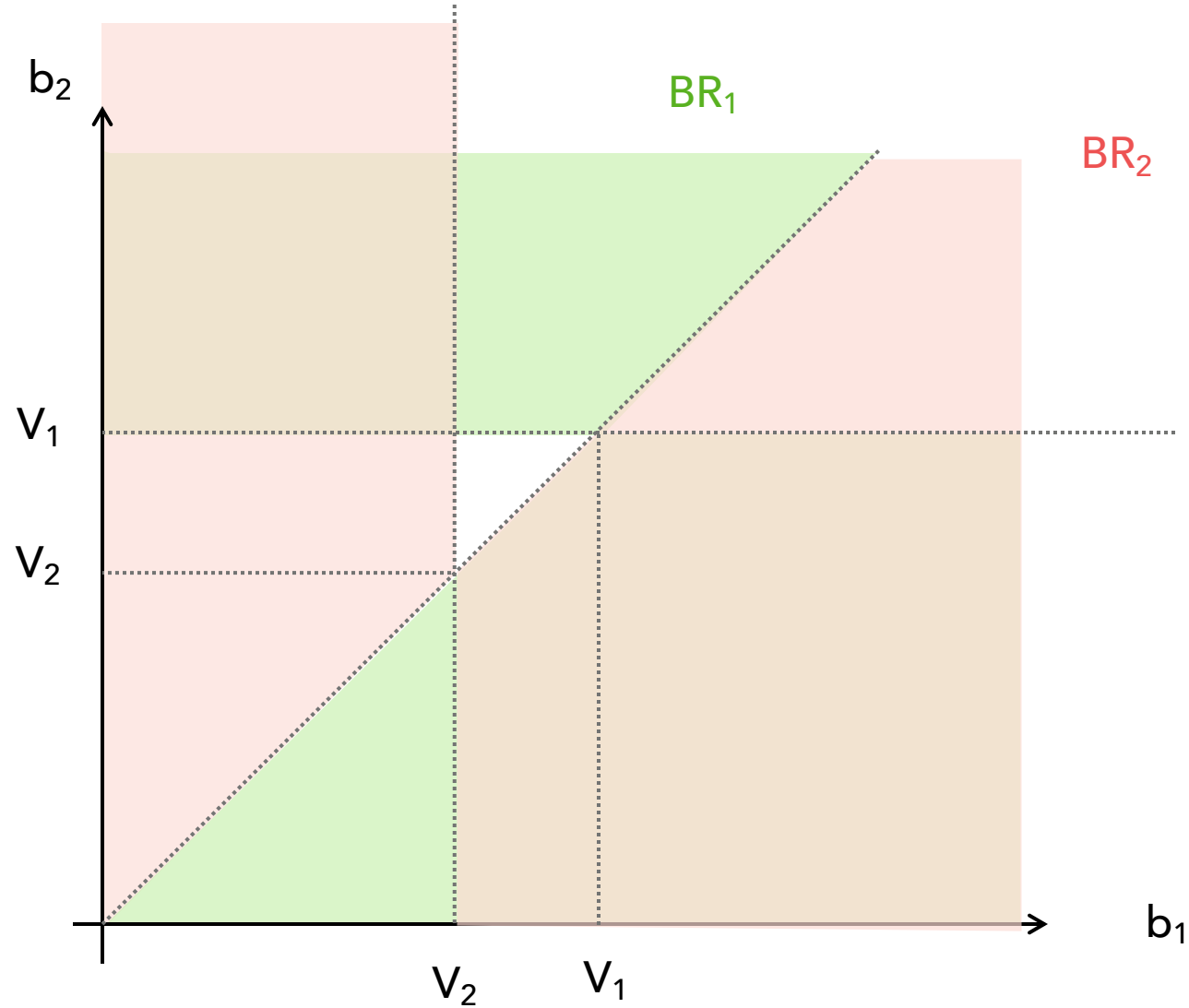
P1 \ P2	0	1	2	3	4
0	0,0	0,2	0,2	0,2	0,2
1	3,0	2,0	0,1	0,1	0,1
2	3,0	2,0	1,0	0,0	0,0
3	3,0	2,0	1,0	0,0	0,-1
4	3,0	2,0	1,0	0,0	-1,0



Second Price Sealed Bid Auction (Cont. Time)

Two buyers wish to buy an antique piece of furniture in a second price, sealed bid auction. They are the only two potential buyers. Their individual valuations of the furniture are V_1 and V_2 where $V_1 > V_2$. Their strategies are bids $b_i \in [0, \infty)$. They can pick any positive number as a bid. If they both bid the same amount, P_1 wins and pays the bid amount. Find the N.E.

Second Price Sealed Bid Auction (Cont. Time)



Two Firms

Consider two competing firms in a declining industry that cannot support both firms profitably. Each firm has three possible choices as it must decide whether or not to exit the industry immediately (E), at the end of this quarter (T), or at the end of the next quarter (N). If a firm chooses to exit then its payoff is 0 from that point onward. Every quarter that *both* firms operate yields each a loss equal to -1 , and each quarter that a firm operates alone yields a payoff of 2. For example, if firm 1 plans to exit at the end of this quarter while firm 2 plans to exit at the end of the next quarter then the payoffs are $(-1, 1)$ because both firms lose -1 in the first quarter and firm 2 gains 2 in the second. The payoff for each firm is the sum of its quarterly payoffs. Represent the game in matrix form. Find the pure strategy NE and msNE.

Two Firms

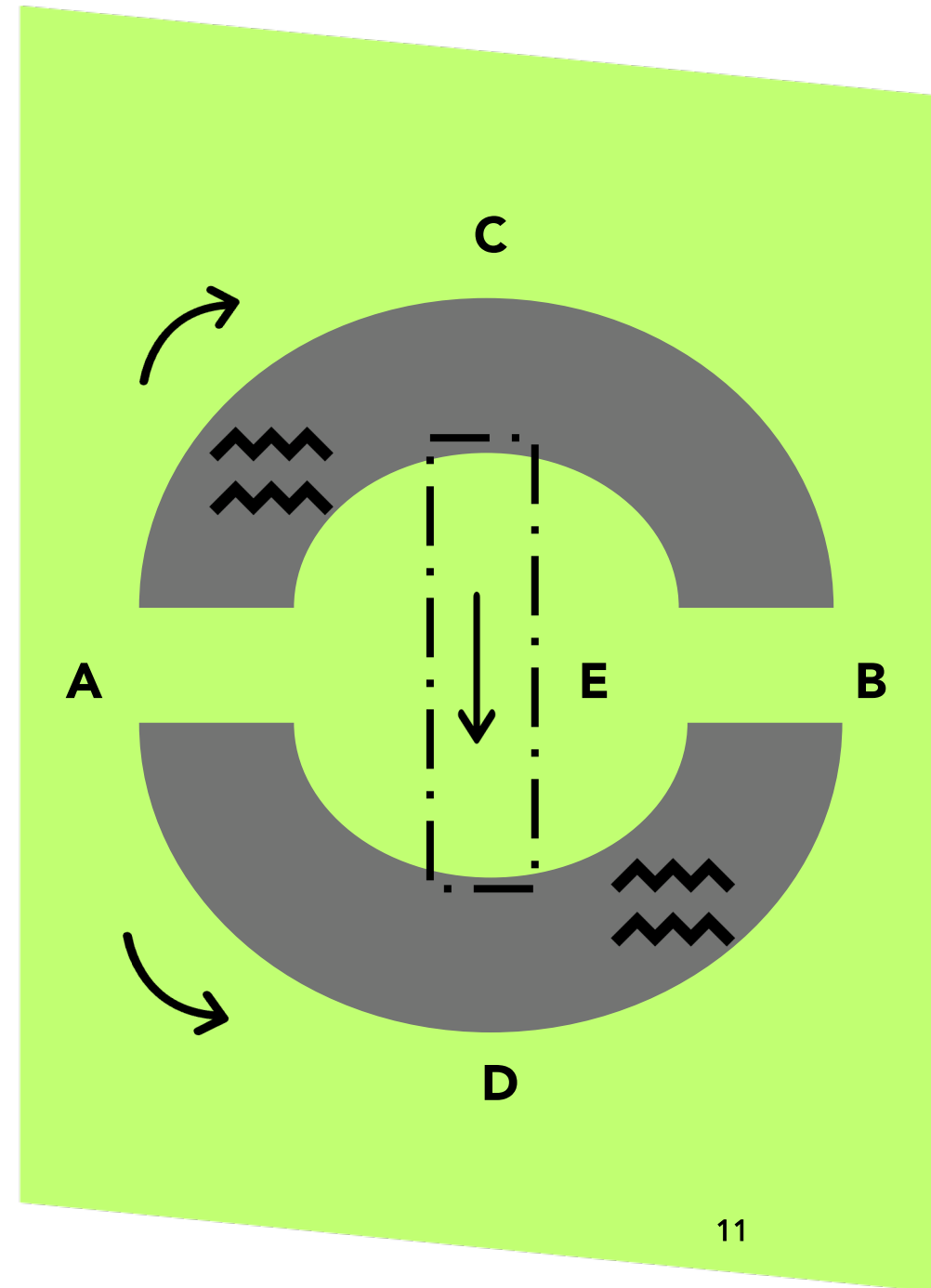
F1 \ F2	F2		
	E	T	N
E	0, 0	0, 2	0, 4
T	2, 0	-1, -1	-1, 1
N	4, 0	1, -1	-2, -2



F1 \ F2	F2	
	E	N
E	0, 0	0, 4
N	4, 0	-2, -2

Braess' Paradox

1000 cars/hr are traveling from A to B via two routes. Travel time from C to B and A to D is 15 mins. The time to cross parts of roads under construction (A to C and D to B) is $x/100$ mins where x is traffic flow per hr. What is the travel time from A to B? If a new expressway E is built as shown, with 2 mins commute time, what is the new travel time from A to B?



Median Voter Theorem

On the planet Jakku, there are two political candidates, P_1 and P_2 , with choice of policy positions $p_i \in [0, 0.25, 0.5, 0.75, 1]$, where zero is extreme left, and 1 is extreme right. Voters are uniformly distributed between $[0,1]$, such that, there are 1/5 voters at each policy position. Voters are committed to their ideologies. Each voter can vote for only one candidate, or for neither, and will vote for the candidate closest to their ideological position, as long as the candidate does not differ more than 0.5 from their ideological position. If both candidates are equally close, votes are equally divided between the two. The candidate with highest votes wins. What is the NE strategy of the two candidates?

Reference Reading

1. *Games of Strategy (3e to 5e)* by Avinash Dixit, Susan Skeath, David Reiley.

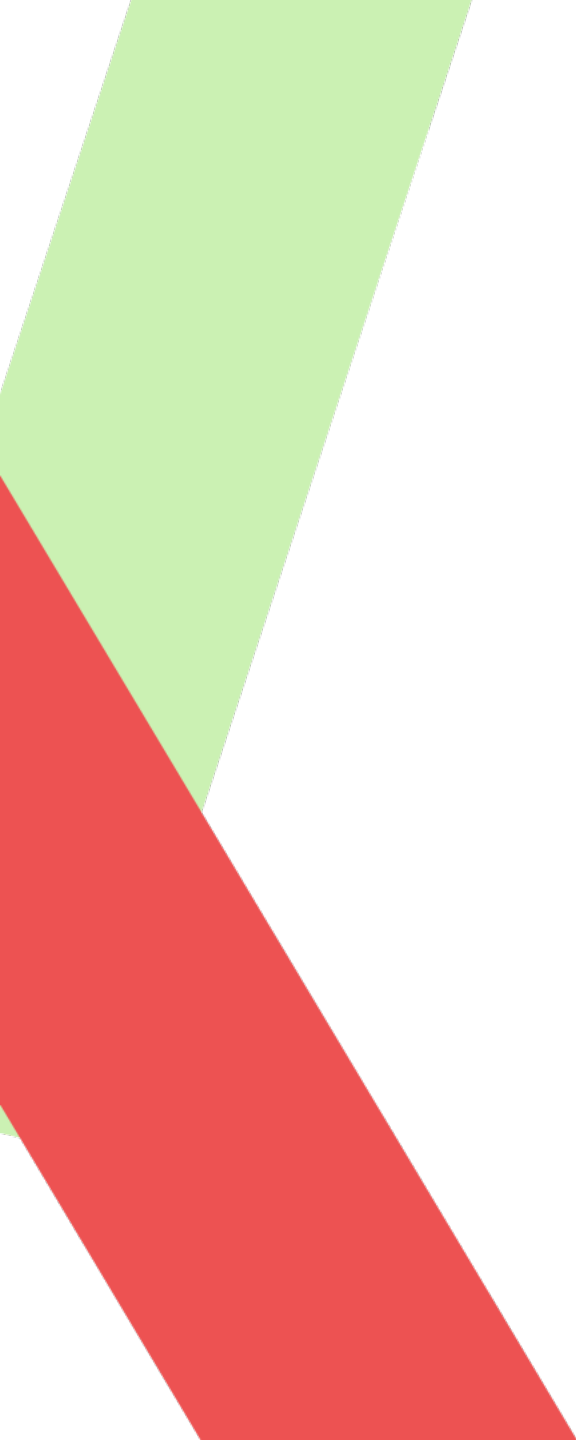
Ebook link (partial)

2. *An Introduction to Game Theory* by Martin Osborne

3. *Strategies and Games. Theory and Practice* by Prajit K. Dutta

4. *Strategy and Game Theory Practice Exercises with Answers* by Felix

Munoz-Garcia, Daniel Toro-Gonzalez



If you have questions,
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