Week 2

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Q1) Herd 1 & 2 graze with intensity x_1 & x_2.

$u_1(x_1, x_2) = x_1(1 - x_1 - x_2) - cx_1$

$u_2(x_2, x_1) = x_2(1 - x_1 - x_2) - cx_2$

$\frac{\partial u_1}{\partial x_1} = 1 - 2x_1 - x_2 - c = 0$

$\Rightarrow x_1^* = \frac{1 - c - x_2}{2}$

By symmetry,

$x_2^* = \frac{1 - c - x_1}{2}$

For N.E, x_1^* & x_2^* will be the strategies.

$\therefore x_1^* = \frac{1-c}{2} - \frac{1}{2}\left[\frac{1 - c - x_1^*}{2}\right]$

$x_1^* = \frac{1-c}{3}$

$x_2^* = \frac{1-c}{3}$

$\therefore$ N.E. is $(x_1^*, x_2^*) = \left(\frac{1-c}{3}, \frac{1-c}{3}\right)$

Q2) Renewable Resource

Fishery of size $y=1$

$u_1(x_1, x_2) = x_1\sqrt{1 - x_1 - x_2}$

$u_2(x_2, x_1) = x_2\sqrt{1 - x_1 - x_2}$

$\frac{\partial u_1}{\partial x_1} = \sqrt{1 - x_1 - x_2} - \frac{x_1}{2\sqrt{1 - x_1 - x_2}} = 0$

$\frac{2 - 3x_1 - x_2}{2\sqrt{1 - x_1 - x_2}} = 0$

$x_1^* = \frac{2 - x_2}{3}$

By symmetry, $x_2^* = \frac{2 - x_1}{3}$

N.E is when both eqns solve each other.

$x_1^* = \frac{2}{3} - \frac{1}{3}\left[\frac{2 - x_1^*}{3}\right]$

$x_1^* = \frac{2}{5} > \frac{1}{3}$

N.E is $(x_1^*, x_2^*) = \left(\frac{2}{5}, \frac{2}{5}\right)$

Q3) N- Player Game

Intensity of grazing for a herd is x_i

$u_i(x_i, x_{-i}) = x_i(1 - (N-1)x - x_i)$

Chicken Game

P2 / P1	St	Sw
St	0,0	3,1
Sw	1,3	2,2

Belief of P1 about P2 swerving is p.

$EU_1(St) = 0 \times (1-p) + 3p$

$EU_1(Sw) = 1 \times (1-p) + 2p$

$EU_1(St) = EU_2(Sw)$

$3p = 1 + p$

$\Rightarrow p = 1/2$

For P2, if we go by symmetry, & if q is the probability of P1 swerving, we get

$q = 1/2$

ms NE $= \left[\left(\frac{St}{2}, \frac{Sw}{2}\right), \left(\frac{St}{2}, \frac{Sw}{2}\right)\right]$

Squash

P2 / P1	F	B
f	0.2, 0.8	0.8, 0.2
b	0.7, 0.3	0.3, 0.7

If p is prob of P2 going to back of court,

$EU_1(f) = 0.2(1-p) + 0.8p$

$EU_1(b) = 0.7(1-p) + 0.3p$

For a mixed strategy, $EU_1(f) = EU_1(b)$

$0.2 + 0.6p = 0.7 - 0.4p$

$p = 0.7 - 0.2 = 0.5 = 1/2$

Now assume belief of P_2 abt P1 going fwd is q.

$EU_2(F) = 0.8q + 0.3(1-q)$

$EU_2(B) = 0.2q + 0.7(1-q)$

$EU_2(F) = EU_2(B)$

$\therefore 0.8q + 0.3(1-q) = 0.2q + 0.7(1-q)$

$\Rightarrow 0.5q + 0.3 = 0.7 - 0.5q$

$q = 0.4 = 2/5$

ms NE $= \left[\left(\frac{2f}{5}, \frac{3b}{5}\right), \left(\frac{F}{2}, \frac{B}{2}\right)\right]$

Payoffs at ms NE $= [EU_1(f), EU_2(F)]$

$= \left[\left(\frac{0.2 \times 1}{2} + \frac{0.8 \times 1}{2}\right), \left(\frac{0.8 \times 2}{5} + \frac{0.3 \times 3}{5}\right)\right]$

Rock Paper Scissors

P2 / P1	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

$EU_1(P) = 0 \times (1-p-q) + (p \times -1) + 1 \times q$

$= q - p$ —— (1)

$EU_1(R) = 1 \times (1-p-q) + 0 \times p + (q \times -1)$

$= 1 - p - 2q$ —— (2)

$EU_1(S) = -1 \times (1-p-q) + 1 \times p + 0 \times q$

$= 2p + q - 1$ —— (3)

From equating (1) & (2)

$q - \cancel{p} = 1 - \cancel{p} - 2q$

$\Rightarrow 3q = 1 \Rightarrow q = 1/3$

Equating (1) & (3),

$\cancel{q} - p = 2p + \cancel{q} - 1$

$\Rightarrow 3p = 1 \Rightarrow p = 1/3$

From symmetry, same applies to P1 choosing between P, R, S & P2's belief about it.

ms NE $= \left[\left(\frac{P}{3}, \frac{R}{3}, \frac{S}{3}\right), \left(\frac{P}{3}, \frac{R}{3}, \frac{S}{3}\right)\right]$

Up, Middle Down

P2 / P1	L	C	R
U	3,2	2,1	1,3
~~M~~	~~2,1~~	~~1,5~~	~~0,3~~
D	1,3	4,2	2,2

Iterated Elimination of Dominated strategies (IEDS)

Game reduces to:

P2 / P1	L	R
U	3,2	1,3
D	1,3	2,2

$EU_1(U) = 3(1-p) + p$

$EU_1(D) = 1 \times (1-p) + 2p$

$= 1 + p$

$3 - 2p = 1 + p$

$2 = 3p$

$\Rightarrow p = 2/3$

$EU_2(L) = 2(1-q) + 3q = 2 + q$

$EU_2(R) = 3(1-q) + 2q = 3 - q$

(At msNE)

$\therefore 2 + q = 3 - q$

$2q = 1 \Rightarrow q = 1/2$

Lobbying Game

F2 / F1	Lobby	Not Lobby
Lobby	-5,-5	15, 0
Not Lobby	0,15	10, 10

Two Firms

F2 / F1	E	N
E	0,0	0,4
N	4,0	-2,-2