Week 1

Saturday, 3 August 2024 01:15

The Grade Game

P1 \ P2	α	β
α	B⁻, B⁻	A, C
β	C, A	B+, B+

The pink & blue circles denote the best responses of players 2 and 1 respectively.

The Nash Eqbm (N.E.) is when both players are playing their best-responses at the same time.

∴ N.E. is (α, α).

Pareto Optimality

The following scenario shows payoff (2,2) for a strategy in a 2-player game when it is Pareto optimal (P.O.).

P1 \ P2	A	B
A	1, 1	2, 2
B	1, 3	0, 4

It is also the N.E. You cannot improve payoff of both players simultaneously by choosing a strategy other than (A, B).

The following game shows a N.E that is not P.O. You can improve payoffs of both players simultaneously by choosing an alternate strategy (B, A).

P1 \ P2	A	B
A	1, 1	3, 2
B	3, 3	0, 4

Rock Paper Scissors

There's no pure strategy N.E.

P1 \ P2	R	P	S
R	0, 0	−1, 1	1, −1
P	1, −1	0, 0	−1, 1
S	−1, 1	1, −1	0, 0

Hawk Dove Game

P1 \ P2	H	D
H	0, 0	4, 2
D	2, 4	3, 3

(H, D) & (D, H) both are N.E.

Humanities vs. Sciences.

P.S.: There was an error in the game table which has been corrected. Please use the corrected payoffs.

Sc. \ Hum.	Lab	Theater
Lab	4, 2	0, 0
Theater	0, 0	1, 5

∴ N.E are (Lab, Lab) & (Theater, Theater).

Joint Project

Arthur \ Tom	Work	Goof off
Work	$\frac{R}{2}-c_1, \frac{R}{2}-c_2$	$\frac{R}{2}-c_1$, R/2
Goof off	$\frac{R}{2}, \frac{R}{2}-c_2$	1, 1

$x_i \in \{1, 2, 3\}$

If Arthur works,

$$u_1 = \frac{(x_1+x_2)^2}{2} - 2x_1$$

we want to maximize u_1 wrt x_1

$$\therefore \frac{du_1}{dx_1} = x_1 + x_2 - 2 = 0$$

$$\Rightarrow x_1 + x_2 = 2$$

$$\frac{d^2u_1}{dx_1^2} = 1$$

∴ it gives a minima ∴ not useful

by symmetry, $x_1 = x_2 = 1$ is a minima.

we write payoff table for $x_1 = x_2 = 1$

Arthur \ Tom	W	G
W	0, 0	0, 2
G	2, 0	1, 1

This yields, that for $x_1 = x_2 = 1$, (G, G) is the N.E.

If we want to find overall N.E, we can make a larger game table: we simply write x_1 & x_2 as strategies.

A \ T	$x_2=1$	$x_2=2$	$x_2=3$	$x_2=0$
$x_1=1$	0, 0	2.5, 0.5	6, 2	0, 2
$x_1=2$	½, 2.5	4, 4	8.5, 6.5	−2, 2
$x_1=3$	2, 6	6.5, 8.5	12, 12	−2.5, 4.5
$x_1=0$	2, 0	2, −2	4.5, −2.5	1, 1

$$\pi_1 = \frac{(x_1+x_2)^2}{2} - 2x_1$$

$$\pi_2 = \frac{(x_1+x_2)^2}{2} - 2x_2$$

The matrix shows the payoffs (π_1, π_2).

The overall N.E are (3, 3) and (0, 0).