Week 5

Tuesday, 27 August 2024 17:50

Bertrand Duopoly

$c_1 = 0$

$c_2 = 1$

$Q = 8 - P$

$p_i \in \{1, 4, 6\}$

$$\pi_1 = \begin{cases} 0 & \text{if } p_1 > p_2 > 0 \\ \frac{p(8-p)}{2} & \text{if } p_1 = p_2 = p \\ (8-p_1).p_1 & \text{if } p_1 < p_2 \end{cases}$$

$$\pi_2 = \begin{cases} (p_2 - c)(8-p_2) & \text{if } p_1 > p_2 > c_2 > 0 \\ (p_2 - c_2)\left(\frac{8-p_2}{2}\right) & \text{if } p_1 = p_2 > c_2 \\ (p_2 - c_2)\left(\frac{8-p_2}{2}\right) & \text{if } p_1 = p_2 < c_2 \\ 0 & \text{if } p_1 < p_2 \end{cases}$$

F1 \ F2	1	4	6
1	7/2, 0	7/2, 0	7/2, 0
4	0, 0	8, 6	16, 0
6	0, 0	0, 12	6, 5

If $p_1 = p_2 = 1$; $Q = 8 - P = 7$; $q_1 = q_2 = 7/2$

$\pi_1 = 1 \times 7/2$

$\pi_2 = 0$

If $p_1 = p_2 = 4$ $Q = 8 - 4 = 4$; $q_1 = q_2 = 4/2$

$\pi_1 = 4 \times 2 = 8$

$\pi_2 = (4-1) \times 2 = 6$

If $p_1 = p_2 = 6$, $Q = 8 - 6 = 2$; $q_1 = q_2 = 2/2 = 1$

$\pi_1 = 6 \times 1 = 6$

$\pi_2 = (6-1) \times 1 = 5$

$\therefore$ From BR analysis, N.E = (4, 4)

Bayesian Bertrand Game

F1 \ F2	6	4	1
6	6, 5	0, 12	0, 0
4	16, 0	8, 6	0, 0
1	7, 0	7, 0	7, 0

Firm 2 of type L

F1 \ F2	6	4	1
6	6, 2	0, 0	0, -21
4	16, 0	16, 0	0, -21
1	7, 0	7, 0	7, 0

Firm 2 of type H

Strategy of firm 2 = (p_L, p_H)

For Firm 2, type L, $p_2 = 1$ is a weakly dominated strategy. Similarly, for firm 2 of type H also, $p_H = 1$ is a weakly dominated strategy $\therefore$ we eliminate it. In the reduced game, for firm 1, $p_1 = 4$ is a strictly dominant strategy. $\therefore$ We compute payoffs of firm 1 for the reduced game.

Payoff of firm 1:

$u_1(4, (6,6)) = \frac{1}{2} \times 16 + \frac{1}{2} \times 16 = 16$

$u_1(4, (6,4)) = \frac{1}{2} \times 16 + \frac{1}{2} \times 16 = 16$

$u_1(4, (4,6)) = \frac{1}{2} \times 8 + \frac{1}{2} \times 16 = 12$

$u_1(4, (4,4)) = \frac{1}{2} \times 8 + \frac{1}{2} \times 16 = 12$

F1 \ F2	6, 6	6, 4	4, 6	4, 4
4	16	16	12	12

Payoffs of Firm 1

$p_1 = 4$ is already the B.R of firm 1. For firm 2, from prev game tables, we can see that for $p_1 = 4$, only (4, 6) & (4, 4) are B.R. $\therefore$ BNE are (4, (4, 6)) & (4, (4, 4)).

Spy games

(K, D) → possible eqbm

C1 \ C2	Spy	Don't
Keep	10, -9	5, -1
~~Destroy~~	0, 2	0, 2

Type "aggressive" for 1 with prob = ε

C1 \ C2	Spy	Don't
Keep	-1, 1	1, -1
Destroy	0, 2	0, 2

Type "non-aggressive" for 1 with prob = 1 − ε

(D, S) → possible eqbm

Payoff of country 2:

$u_2(s_2, (s_{11}, s_{12}))$

type 1 prob ε; type 2 prob 1−ε

$u_2(S, (K, K)) = -9 \times \varepsilon + 1(1-\varepsilon) = 1 - 10\varepsilon$

$u_2(S, (K, D)) = -9 \times \varepsilon + 2(1-\varepsilon) = 2 - 11\varepsilon$

$u_2(D, (K, K)) = -1 \times \varepsilon + (-1(1-\varepsilon)) = -1$

$u_2(D, (K, D)) = -1 \times \varepsilon + 2(1-\varepsilon) = 2 - 3\varepsilon$

1 \ 2	Spy	Don't
K, K	1 − 10ε	−1 ✓
K, D	2 − 11ε	2 − 3ε ✓
D, K		
D, D		

given ε < 1/5

$\therefore$ Only one possible B.N.E.

→ ((K, D), D)

none chosen by country 1 of type "aggressive".

Cournot Duopoly

$P(Q) = a - Q = a - (q_1 + q_2)$

where $a = a_H$ with prob θ

$a = a_L$ with prob 1−θ

$c_i = cq_i$

$\pi_1 = (P - c)q_1$

$\pi_1 = [a_i - (q_1 + q_2) - c]\, q_1$

$\pi_1 = a_i q_1 - q_1^2 - q_2 q_1 - cq_1$

$$\frac{\partial \pi_1}{\partial q_1} = a_i - 2q_1 - q_2 - c = 0$$

$$q_1^* = \frac{a_i - c - q_2}{2}$$

$$q_1^* = \begin{cases} \left(\frac{a_H - c - q_2^*}{2}\right) \to q_{1H} & \text{if } a_i = a_H \\ \left(\frac{a_L - c - q_2^*}{2}\right) \to q_{1L} & \text{if } a_i = a_L \end{cases}$$

Expression for best response of firm 1 wrt to demand high or low

For firm 2:

$\pi_2 = (P - c)q_2 = (a_i - (q_1 + q_2) - c)\, q_2$

$$\frac{\partial \pi_2}{\partial q_2} = a_i - c - 2q_2 - q_1 = 0$$

$$\Rightarrow q_2^* = \frac{a_i - c - q_1}{2}$$

$$\Rightarrow q_2^* = \frac{a_H\theta + a_L(1-\theta) - c}{2} - \frac{1}{2}\left[\theta q_{1H} + (1-\theta) q_{1L}\right]$$

$$\Rightarrow q_2^* = \frac{a_H\theta + a_L(1-\theta) - c}{2} - \frac{1}{2}\left[\frac{\theta a_H + (1-\theta)a_L - c - q_2^*}{2}\right]$$

$$\Rightarrow q_2^* - \frac{q_2^*}{4} = \frac{a_H\theta}{4} + \frac{a_L(1-\theta)}{4} - \frac{c}{4}$$

$$\Rightarrow \frac{3q_2^*}{4} = \frac{a_H\theta + a_L(1-\theta) - c}{4}$$

$$\Rightarrow q_2^* = \frac{a_H\theta + a_L(1-\theta) - c}{3}$$

$q_2^* > 0$

$$\Rightarrow \frac{a_H\theta + a_L(1-\theta) - c}{3} > 0$$

This gives best response of firm 2.

For firm 1, when $a_i = a_H$

$$q_{1H}^* = \frac{a_H - c - q_2^*}{2}$$

$$q_{1H}^* = \frac{a_H - c}{2} - \frac{1}{2}\left[\frac{a_H\theta + a_L(1-\theta) - c}{3}\right]$$

$$q_{1H}^* = \frac{a_H(1-\theta)}{2} - \frac{c}{3} - \frac{a_L(1-\theta)}{6} > 0$$

Similarly compute for when market demand is low.