Strategy: An Introduction to Game Theory

Week 2: Commons, Mixed Strategies

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Recap

- Notation
- Best Response
- Nash Equilibrium
- Pareto Optimality
- Dominant Strategies

Tragedy of Commons

Two herds of livestock use the same pasture for grazing. The size of pasture is limited, normalized to 1. Let x_1 and x_2 denote the intensity of grazing by the two herds such that $0 < x_i < 1$ and $x_1 + x_2 < 1$. Assume that the cost of looking after each herd while grazing, to the owners, is $c*x_i$ where where c is a positive constant. Each herd gets the benefit of grazing in proportion of its intensity of grazing. Find the N.E. values of x_i. Is the N. E. Pareto Optimal?

Renewable Resource

Suppose now that there is a fishery of size $y = 1$. Each player extracts an amount x_i in the first period, $i = 1, 2$. Whatever is not extracted, that is, the amount $y - x_1 - x_2$, regenerates and becomes an amount equal to $\sqrt{(y - x_1 - x_2)}$ in period 2. Find the Nash equilibrium.

[Dutta Ch8, Ex 8.2]

N Players

Suppose now that there are N players in the grazing problem game, and costs are negligible. Intensity of grazing of a herd in question is x_1. All other herds graze with the same intensity x. Show that the "tragedy of commons" game leads to complete depletion of the pasture and compute the Nash Eqbm strategies.

[Dutta Ch8, 8.4]

Chicken Game

P1 \ P2	Straight	Swerve
Straight	0,0	3,1
Swerve	1,3	2,2

Two players driving their cars against each other must decide whether to swerve or not.

[Garcia et. al, Ch3, Ex 1]

Mixed Strategies

P1 \ P2	Straight	Swerve
Straight	0,0	3,1
Swerve	1,3	2,2

- A player will choose a mixed strategy only when it gives a higher expected payoff than some pure strategy.
- A player will choose a mixed strategy when the likelihood of their opponent choosing some strategy is not known.
- A mixed strategy is when a player randomly picks any of the available strategies.

Squash

P1 \ P2	F	B
f	0.2	0.8
b	0.7	0.3

[Dutta Ch9]

Imagine that you are playing the game of squash. In the middle of a rally, you have to decide whether to position your next shot in the front (f) of the court or to the back (b). Your opponent likewise has to move in anticipation of your shot; s/he could move forward (F) or backward (B). In the following matrix are displayed your chances of winning the rally in the four possible cases. Find the msNE.

Rock Paper Scissors

P2 / P1	R	P	S
R	0,0	-1, 1	1, -1
P	1, -1	0, 0	-1, 1
S	-1, 1	1, -1	0,0

Up, Middle, Down

P1 \ P2	L	C	R
Up	3,2	2,1	1,3
Middle	2,1	1,5	0,3
Down	1,3	4,2	2,2

Two players have payoffs given in the table. Find the Mixed Strategy NE (msNE).

Penalty Kick

The game table shows the chances of landing a goal for the soccer playing taking the penalty kick. Find the Mixed Strategy Nash Equilibrium.

Kicker \ Goalie	l	m	r
L	0.4	0.5	0.9
M	0.5	0.2	0.5
R	0.9	0.5	0.3

[Dutta Ch9]

Lobbying Game

Two firms simultaneously and independently decide whether to lobby Congress in favor a particular bill. When both firms lobby, Congress' decisions are unaffected. Each firm earns a profit of 10 if none of them lobbies and −5 each if both choose to lobby. If only one firm lobbies its payoff is 15 (since it is the only beneficiary of the policy), while that of the firm that did not lobby is zero. Find pure strategy NE and msNE.

F1 \ F2	Lobby	Not Lobby
Lobby	-5, -5	15, 0
Not Lobby	0, 15	10, 10

[Garcia et. al, Ch3, Ex 2]

Two Firms

Consider two competing firms in a declining industry that cannot support both firms profitably. Each firm has three possible choices as it must decide whether or not to exit the industry immediately (E), at the end of this quarter (T), or at the end of the next quarter (N). If a firm chooses to exit then its payoff is 0 from that point onward. Every quarter that *both* firms operate yields each a loss equal to −1, and each quarter that a firm operates alone yields a payoff of 2. For example, if firm 1 plans to exit at the end of this quarter while firm 2 plans to exit at the end of the next quarter then the payoffs are (−1, 1) because both firms lose −1 in the first quarter and firm 2 gains 2 in the second. The payoff for each firm is the sum of its quarterly payoffs. Represent the game in matrix form. Find the pure strategy NE and msNE.

Two Firms

F1 \ F2	E	T	N
E	0,0	0, 2	0, 4
T	2, 0	-1, -1	-1, 1
N	4, 0	1, -1	-2, -2

Reference Reading

1. *Games of Strategy (3e to 5e)* by Avinash Dixit, Susan Skeath, David Reiley.

   Ebook link (partial)

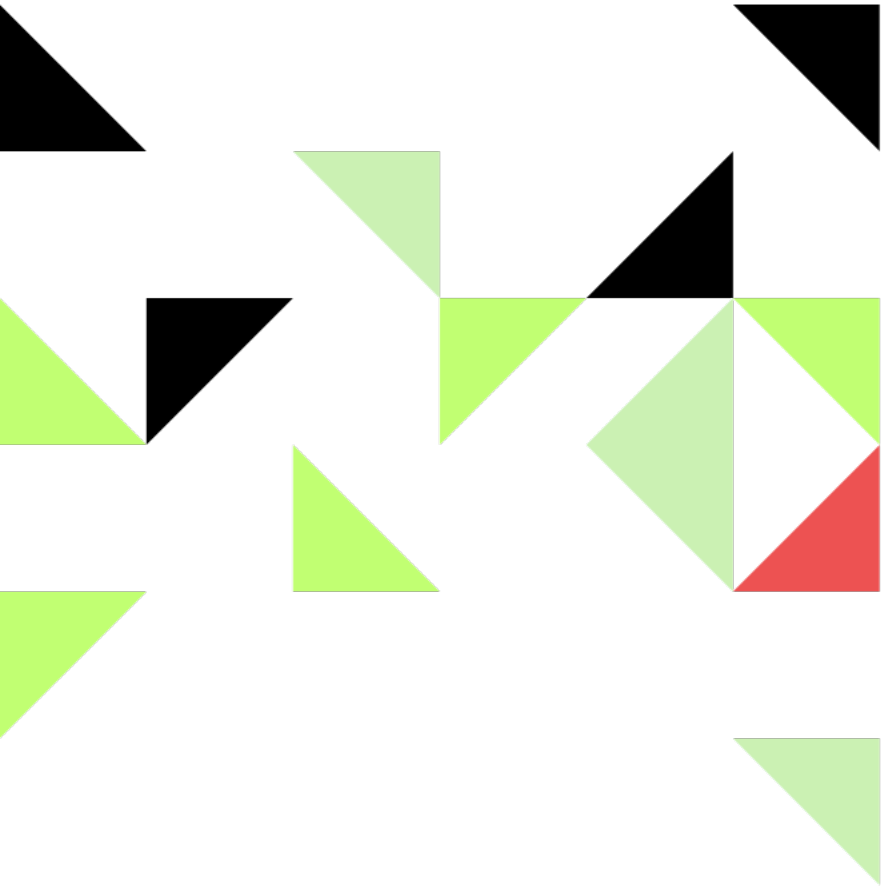
2. *An Introduction to Game Theory* by Martin Osborne

3. *Strategies and Games. Theory and Practice* by Prajit K. Dutta

4. Strat*egy and Game Theory Practice Exercises with Answers* by Felix

   Munoz-Garcia, Daniel Toro-Gonzalez

5. *Introduction to Economic Analysis v 1.0* by R. Preston McAfee.



If you have questions,

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