Q1) $q_1 = 100 - p$
$q_2 = 60 - 4p$
$q_3 = 0$
$q_T = 0$
$100 - p > 0$
$\Rightarrow p < 100$ —(1)
$q_2 > 0 \Rightarrow$
$60 - 4p > 0$
$\Rightarrow p < 30$ —(2)

q_1 = Kumar
q_2 = Sharma
q_3 = Singh
q_T = Total demand

$\sum q_i = q_D$
s.t. $q_i \geq 0$

$$q_D = \begin{cases} q_1 + q_2 & 0 \leq p \leq 30 \\ q_1 & 30 < p < 100 \\ 0 & p > 100 \end{cases}$$

$$p = \begin{cases} 100 - 2q & 0 \leq q \leq 30 \\ 100 - q & 30 < q \leq 100 \\ 0 & q > 100 \end{cases}$$

Q2) a) $p = 100 - 2q_D$
$p = 100 + q_S$
$q_S = q_D$

$100 - 2q = 100 + q$
$\Rightarrow 3q = 3q$
$q = 300$
$p = 100 + q = 400$
$(q^*, p^*) = (300, 400)$

b) $p^* = 400$ (price floor)
$p = 100 - 2q_D$
$\Rightarrow q_D = \frac{100 - p}{2}$
$q_D = \frac{100 - 500}{2} = 250$
$q_S = p - 100$
Excess $= q_S - q_D = 400 - 300 = 150$
When $p^* = 400 < 500$
If there were price ceiling, no change in equilibrium.
If price ceiling $= 500 > p^*$
$q_D = \frac{100 - p}{2} = \frac{100 - 500}{2} = 250$
$q_S = p - 100 = 500 - 100 = 400$
Shortage $= 400 - 250 = 150$

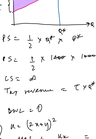
Q3) a) $q = \frac{100}{p} + 4p^2$
p, q are parameters
$\varepsilon = \frac{\partial q / \partial p}{q / p}$
$\varepsilon = \frac{-100/p^2}{q/p} = \frac{-100}{qp}$
$\varepsilon = \frac{-100}{100 + 4p^3} = -\frac{1}{4p^3}$

b) $q = \frac{100}{p} + 4p^2$
Cross price elasticity $= \varepsilon_{qx} = \frac{\partial q / \partial p_x}{q / p_x}$
$\varepsilon_{qx} = \frac{10 p_x}{q / p_x} = \frac{10 p_x^2}{q}$
$\varepsilon_{qx} = \frac{10 \times 100 \times 3}{1000} = 3$

c) $\varepsilon_d > 0 \longrightarrow \varepsilon > 1$ ∴ elastic index.
∴ ?

Q4) a) $q_D = 1000$
$p = 500 + q_S$
$p = 500 + 1000 = 1500$
$(q^*, p^*) = (1000, 1500)$

b) $q = 50$



Quantity demanded is beyond consumer surplus area
∴ $q^* = 1000$
$q_S = p^* + 50$
$p^* = 1000 + 50 = 1050$

c) $TS = \frac{1}{2} \times q \times p^* \times q^*$
$TS = \frac{1}{2} \times 1000 \times 1000$
$CS = 0$
Total revenue $= TR^* = 1000 \times 1050 = 5,250,000$
$DWL = 0$

Q5) a) $M = (2 \times 4 \times y)^2$
$MRS = \frac{M_x}{M_y} = \frac{2M/x}{2M/y}$
$MRS = \frac{2(2 \times 4 \times y)}{1 \cdot (2 \times 4 \times y)} = 2$

b) $p_x = 300$
$p_y = 150$
$y = 1500$
$p_x x = p_y y$ $\quad x = 0$
$1500 = y \times \frac{150}{300}$
$y = \frac{1500}{150} = 10$
$(x^*, y^*) = (0, 10)$

$M = (2 \times 4 \times y)^2$ —(1)
$Y = xp_x + yp_y$ —(2)

c) $M^* = 400$
$\bar{M} = (2 \times 4 \times y)^2 = 8^2 = 64$
$p_y > p_x$ $\quad \therefore y = 0$
Maximum demand:
$\bar{M} = (2 \times 4 \times y)^2$
$\bar{M} = 4x^2$ $\quad = 64$
$x^2 = \frac{64}{4} = 16$
$x_D = 4$
$\therefore (x_m, y_m) = (4, 0)$
Remainder income:
$p_y > p_x \quad \therefore y = 0$
$Y = xp_x + yp_y$
$1500 = x \times 300$
$x_M = \frac{1500}{300} = 4$
$(x_M, y_M) = (4, 0)$

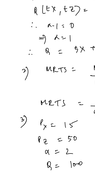
Q6) $u = 4\sqrt{x_1} + x_2$
$Y = p_1 x_1 + p_2 x_2$
$MRS = MRT$
$\frac{MU_1}{MU_2} = \frac{p_1}{p_2}$
$\frac{2/\sqrt{x_1}}{1} = \frac{p_1}{p_2}$
$x_1 = \frac{4p_1^2}{p_2^2}$

$f(x \times y)$
↓
Substitution
$x^2 = 4p_1^2$
↓
Find subs.

$p_1 = p_2 = 1$, $Y = 10$
$x_1 = 4 \times \frac{1}{1} = 4$
$Y = p_1 x_1 + p_2 x_2$
$10 = 1 \times 4 + 1 \times x_2$
$\Rightarrow x_2 = 6$
$\therefore (x_1, x_2) = (4, 6)$
$\bar{u} = 4\sqrt{x_1} + x_2$
$\bar{u} = 4 \times 2 + 6 = 14$
Then $p_1 = 2$ change.
$x_1' = \frac{4p_1^2}{p_2^2} = \frac{4 \times 1}{4} = 1$
$Y = p_1 x_1' + p_2 x_2'$
$10 = 2 \times 1 + 1 \times x_2'$
$x_2' = 8$
$\therefore (x_1', x_2') = (1, 8)$
Total effect on $x_1 = x_1' - x_1^*$
$= 1 - 4 = -3$

To find substitution effect, we find x_1 under constant utility.

$x_1 = \frac{4p_1^2}{p_2^2} = \frac{4 \times 1}{4} = 1$



SE $= x_1 - x_1^*$
$SE = 1 - 4 = -3$
$\because TE = SE + IE$
$-3 = -3 + IE$
$\therefore IE = 0$

Q7) a) $Q = 5K + L^2$
For LRS, $f(\lambda K, \lambda L) = \lambda f(K, L)$
$Q(\lambda K, \lambda L) = 5(\lambda K) + (\lambda L)^2$
$Q(\lambda K, \lambda L) = 5\lambda K + \lambda^2 L^2$
$Q(\lambda K, \lambda L) = \lambda \underbrace{[5K + \lambda^{n-1} L^2]}$

For LRS,
$Q(\lambda K, \lambda L) = \lambda Q(K, L)$
If $\lambda^{n-1} = 1 = \lambda^0$, then,
$Q(\lambda K, \lambda L) = \lambda Q(K, L)$
$\therefore n - 1 = 0$
$\Rightarrow n = 1$
$\therefore Q = 5K + L$

b) $MRTS = \frac{MP_L}{MP_K} = \frac{\partial Q / \partial L}{\partial Q / \partial K}$
$MRTS = \frac{5}{2L^{0.5}}$

c) $P_L = 15$
$P_K = 50$
$Q = 2$
$Q_0 = 100$
$MRTS = \frac{5}{2L} = \frac{P_L}{P_K} = \frac{15}{50}$
$\therefore L = \frac{5 \times 50}{2 \times 15} = \frac{25}{3} = 8.33$
$Q = 5K + L^2$
$100 = 5K + \left(\frac{25}{3}\right)^2$
$K = \frac{100 - 69.44}{5} = 6.112$

d) $C = 300 + 2q^2$
$MC = 4q$
$AVC = \frac{VC}{q}$
$MC = AVC \Rightarrow q = 0$
$\therefore MC > AVC$ $\forall P > 0$, firm operates

e) At $P = 100$
$P = MC$
$100 = 4q \Rightarrow q = 25$

f) $\pi = Pq - C$
$\pi = (100 \times 25) - (300 + 2(25)^2)$
$\pi = 2500 - 1300$
$\pi = 1200$

g) Firm operates at $P > 0$