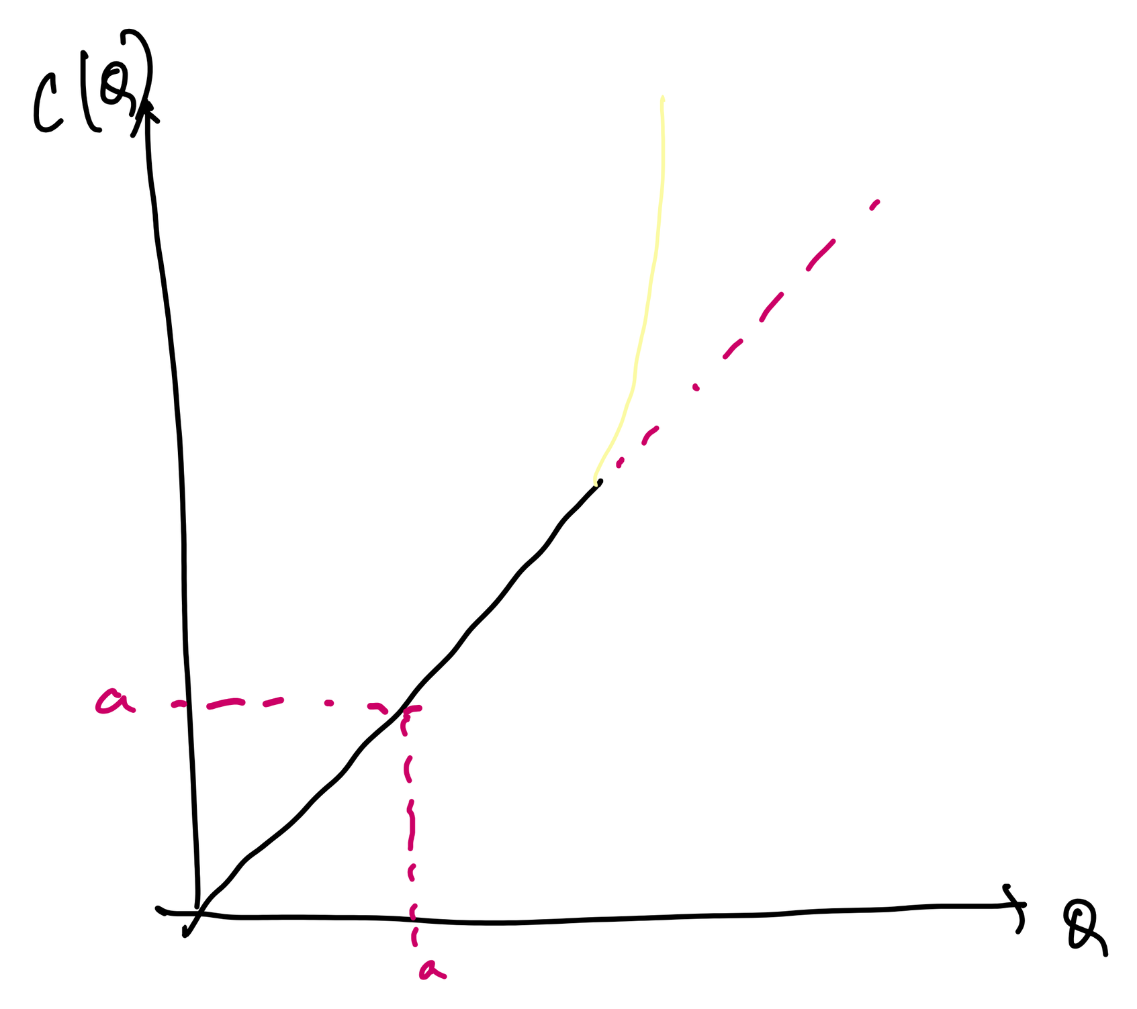
Week 10

Tuesday, 2 April 2024 18:33

$\pi = TR - TC$ → only opportunity costs.

If $TR = TC \Rightarrow \pi = 0$



IRS:

$Q = F(K, L)$

$\theta F(K, L) < F(\theta K, \theta L)$

$F(\theta K, \theta L)$ in CRS

DRS:

$\theta F(K, L) > F(\theta K, \theta L)$

$Q = F(K, L)$

$Q = 2K + 5L \Rightarrow \theta Q = \theta[2K + 5L] = F(\theta K, \theta L)$

$C = rK + wL = F(Q)$

$Q = f(K, L)$

Q2) 1) $Q = KL$

$r = 100, \quad w = 15$

$Q_0 = 500$

$\min_{K,L} \; rK + wL = C$ ——(1)

s.t. $Q = KL$ ——(2)

For optimization:

$$\frac{MP_K}{r} = \frac{MP_L}{w}$$

$MP_K = L$

$MP_L = K$

$$\frac{L}{100} = \frac{K}{15}$$

$$K = \frac{15L}{100}$$ ——(3)

Substituting (3) in eqn (2),

$$Q_0 = \frac{15L^2}{100} = 500$$

$$\Rightarrow L^2 = \frac{500 \times 100}{15} \Rightarrow L^* = 57.73 \approx 58$$

$$\therefore K^* = \frac{15}{100} \times 57.73 = 8.66 \approx 9$$

2) $Q_0 = 500 = \bar{K}L$ $\qquad \bar{K} = 5$

$\therefore L = 100$

$C_{SR} = r\bar{K} + wL$

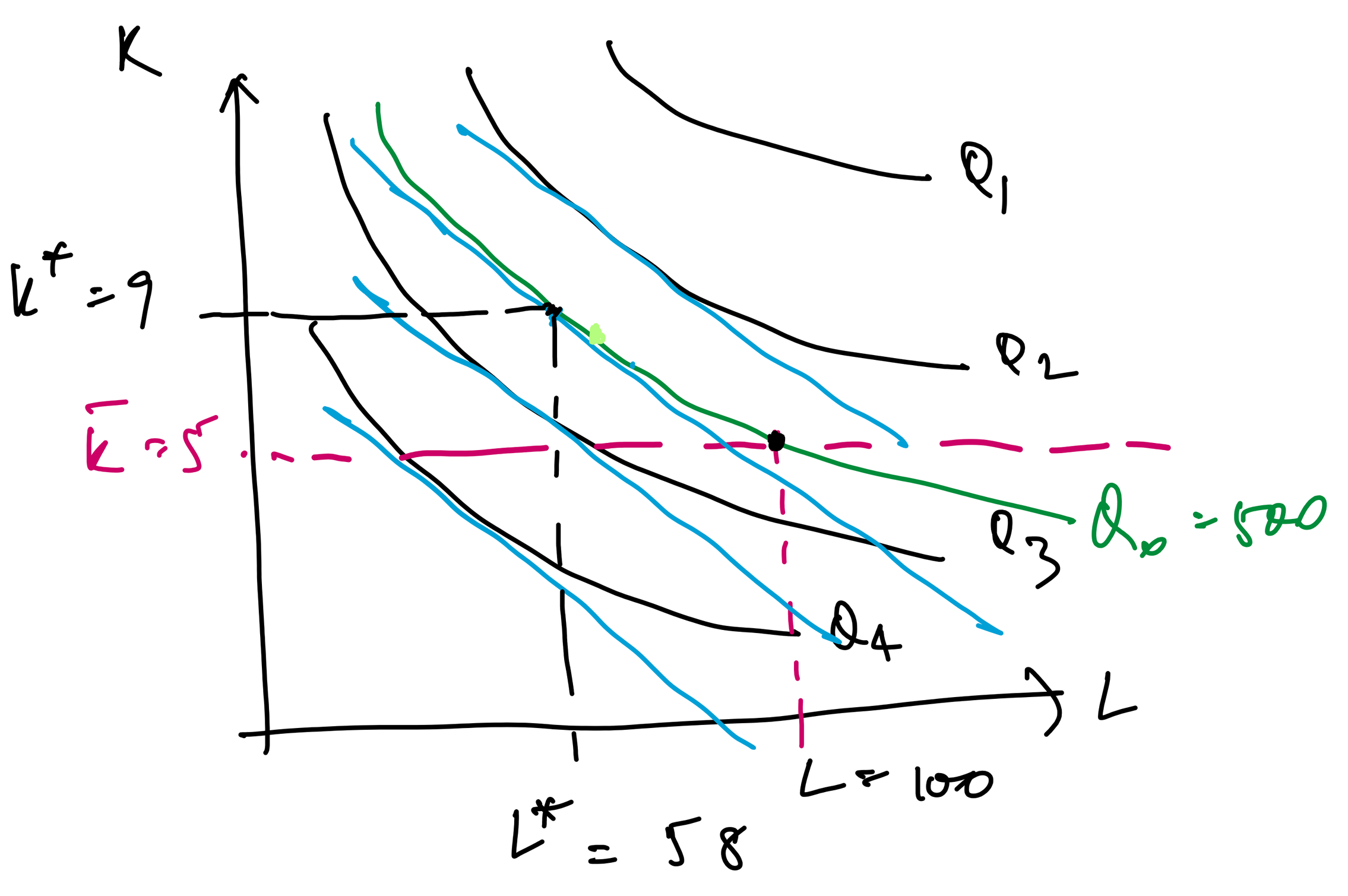
$C_{SR} = (100 \times 5) + (15 \times 100)$

$C_{SR} = 2000$

$C_{LR} = rK^* + wL^*$

$C_{LR} = (100 \times 8.66) + (15 \times 57.73)$

$C_{LR} = 1731.95$



3) $\bar{K} = 9$

$Q_0 = \bar{K}L$

$500 = 9 \times L$

$$L = \frac{500}{9} = 55.55$$

Q3)

$\min_{K,L} \; C = rK + wL$

s.t. $Q = 4\sqrt{KL}$

$$\frac{MP_K}{r} = \frac{MP_L}{w}$$

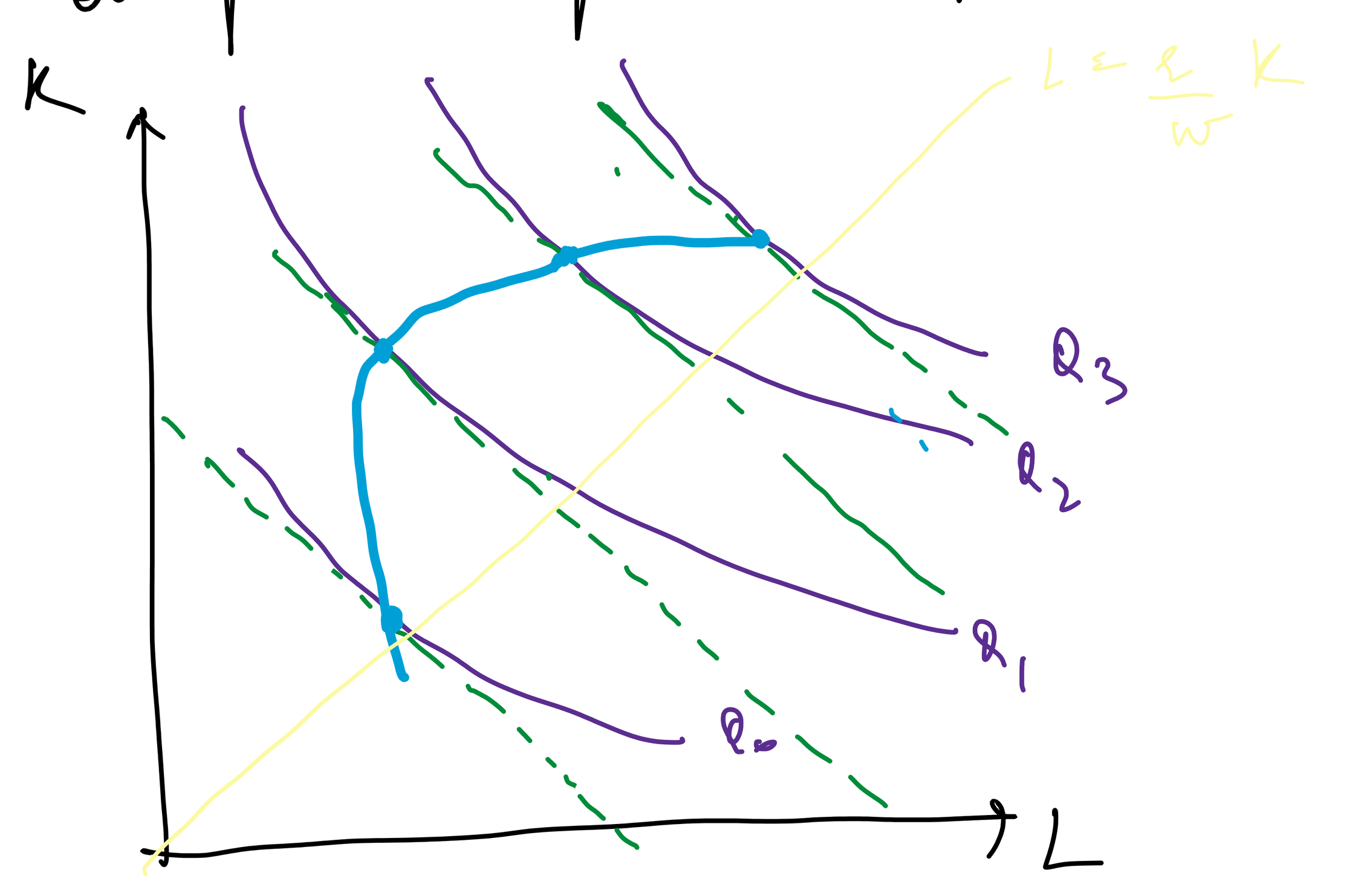
$$MP_K = \frac{2}{r}\sqrt{\frac{L}{K}}$$

$$MP_L = \frac{2}{w}\sqrt{\frac{K}{L}}$$

$$\frac{2}{r}\sqrt{\frac{L}{K}} = \frac{2}{w}\sqrt{\frac{K}{L}}$$

$$L = \frac{r}{w}K \qquad \alpha = \beta$$

Output expansion path



2) $Q_0 = 4\sqrt{KL}$

$$Q_0 = 4\sqrt{K \cdot \frac{r}{w}K}$$

$$Q_0 = 4K\sqrt{\frac{r}{w}}$$

$$K = \frac{Q_0}{4}\sqrt{\frac{w}{r}} \quad ✓$$

$$L = \frac{r}{w}K = \frac{Q_0}{4}\sqrt{\frac{w}{r}} \cdot \frac{r}{w}$$

$$L = \frac{Q_0}{4}\sqrt{\frac{r}{w}} \quad ✓$$

3) $C = rK + wL$

$$C = r \cdot \frac{Q_0}{4}\sqrt{\frac{w}{r}} + w \cdot \frac{Q_0}{4}\sqrt{\frac{r}{w}}$$

$$C = \frac{Q_0}{4}\left[\sqrt{rw} + \sqrt{rw}\right]$$

$$C = \frac{Q_0\sqrt{rw}}{2}$$