Week 7

Tuesday, 12 March 2024 18:05

Week 6

A & B are two goods

$$\frac{MU_A}{P_A} > \frac{MU_B}{P_B}$$

$a x_1 + b x_2 = u(x_1, x_2)$

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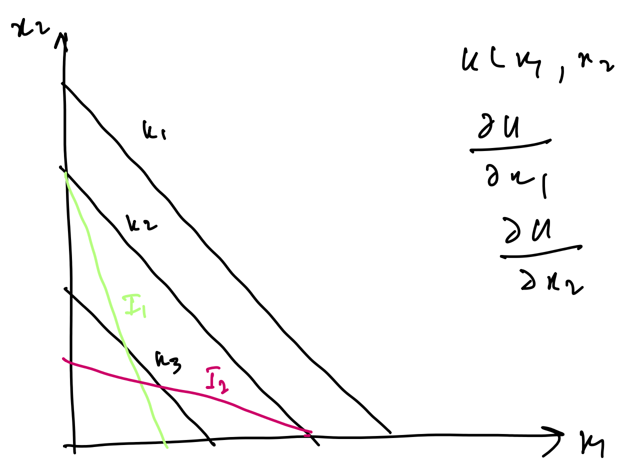
"Perfect" substitutes ← (in coursework)

Perfect substitutes



$u(x_1, x_2) = a x_1 + b x_2$

$\frac{\partial u}{\partial x_1} = a = MU_A$

$\frac{\partial u}{\partial x_2} = b = MU_B$

$\frac{MU_A}{P_A} = \text{Const} = \frac{a}{P_A}$

$\frac{MU_B}{P_B} = \frac{b}{P_B} = \text{const}$

$I_1 = P_A x_1 + P_B x_2$

u_2 is the highest level of utility possible $= u_2$

$I_2 = P_A x_1 + P_B x_2$

u_2 is the highest level of utility possible.

$u = x_1^3 x_2$ prices are P_A, P_B

$\frac{\partial u}{\partial x_1} = 3x_1^2 x_2$

$\frac{\partial u}{\partial x_2} = x_1^3$

$MU_1 / P_A = \frac{3x_1^2 x_2}{P_A}$

$MU_2 / P_B = \frac{x_1^3}{P_B}$

As long as $\frac{MU_1}{P_A} > \frac{MU_2}{P_B}$ ⇒ consume more of A

If $\frac{3x_1^2 x_2}{P_A} > \frac{x_1^3}{P_B}$

$\underbrace{\frac{3x_2}{P_A}}_{MU_1} \gtrless \underbrace{\frac{x_1}{P_B}}_{MU_2}$ ⇒ consume more of good 1

At point of optimization, (s.t. $x_1, x_2 > 0$) $\frac{MU_1}{P_A} = \frac{MU_2}{P_B}$

Q1 I = 30 for every 10 days.

$P_1 = 0.5$, $P_2 = 10$

$P_1' = 1.25$, $P_2 = 10$

A) Atleast 20 potatoes per 10 days ~ 10\$

2kgs of rice for 20\$

$(x_1, x_2) = (20, 2)$

B) If $P_1' = 1.25$

Then $(x_1', x_2') = (20, 0.5)$

C) Giffen good → Income & substitution effects cancel each other.

Q2 A) $u(x_A, x_B) = x_A \cdot x_B$ —(1)

$P_A = 1$, $P_B = 2$

$I = 40$

$x_A + 2x_B = 40$ —(2) ⇒ $x_A = (40 - 2x_B)$

$u = x_B(40 - 2x_B)$ from (1) & (2)

$\frac{du}{dx_B} = 40 - 4x_B = 0$

$\Rightarrow x_B = 10$

$\therefore x_A = 20$

$(x_A, x_B) = (20, 10)$

B) $x_A + x_B = 40$

$x_A = x_B = \frac{40}{2} = 20$

$(x_A', x_B') = (20, 20)$

c) Substitution effect is dominant

Solving B)

$x_A + x_B = 40$

$u = x_A \cdot x_B$

$u = x_A(40 - x_A) = 40x_A - x_A^2$

$\frac{du}{dx_A} = 40 - 2x_A = 0$

$x_A = \frac{40}{2} = 20$

$x_B = 40 - 20 = 20$

$(x_A', x_B') = (20, 20)$

Q3) $u = x + y$

$P_x = 2$, $P_y = 3$, $E = 10$

$E = P_x x + P_y y$

$10 = 2x + 3y$

Since $P_y > P_x$, $y = 0$ amount consumed.

$10 = 2x \Rightarrow x = 5$

$\therefore (x, y) = (5, 0)$

Now, $P_x = 4$; $P_y = 3$

Now consumer consumes $x = 0$

$10 = 3y \Rightarrow y = 10/3$

$\therefore (x_M, y_M) = (0, 10/3)$

Hicksian demand.

Utility remains constant at initial level $= \bar{u}$

$\bar{u} = x + y = 5 + 0$

$\therefore \bar{u} = 5$

$P_x > P_y \therefore x = 0$

$\bar{u} = x + y \Rightarrow 5 = 0 + y \Rightarrow y = 5$

$\therefore (x_h, y_h) = (0, 5)$

General Case

$u = Axy$ —(1) P_x & P_y are prices. $A > 0$

$E = xP_x + yP_y$ —(2)

$\frac{u_x}{u_y} = \frac{P_x}{P_y}$

$\frac{\cancel{A}y}{\cancel{A}x} = \frac{P_x}{P_y}$

$yP_y = xP_x$

$E = 2xP_x \Rightarrow x = \frac{E}{2P_x}$

Similarly,

$y = \frac{E}{2P_y}$.

$\bar{u} = A x . y$

$\bar{u} = A\left(\frac{E}{2P_x}\right)\left(\frac{E}{2P_y}\right)$

$\Rightarrow E^2 = \frac{4\bar{u} P_x P_y}{A}$

$\Rightarrow E = 2\sqrt{\frac{\bar{u} P_x P_y}{A}}$

Hicksian demand.

$x_h = \frac{\partial E(\bar{u}, P_x, P_y)}{\partial P_x}$

$x_h = \frac{\partial}{\partial P_x}\left[2\left(\sqrt{\frac{\bar{u} P_y}{A}}\right) \cdot \sqrt{P_x}\right]$

$x_h = \sqrt{\frac{\bar{u} P_y}{A P_x}}$

$y_h = \frac{\partial E(\bar{u}, P_x, P_y)}{\partial P_y}$

$y_h = \sqrt{\frac{\bar{u} P_x}{A P_y}}$