Week 9

Tuesday, 26 March 2024 18:01

Week 8 Assignment

Q9) $f(L,T) = L^{1/2} T^{1/2} = q$

$q = 4$

$4 = \sqrt{LT}$

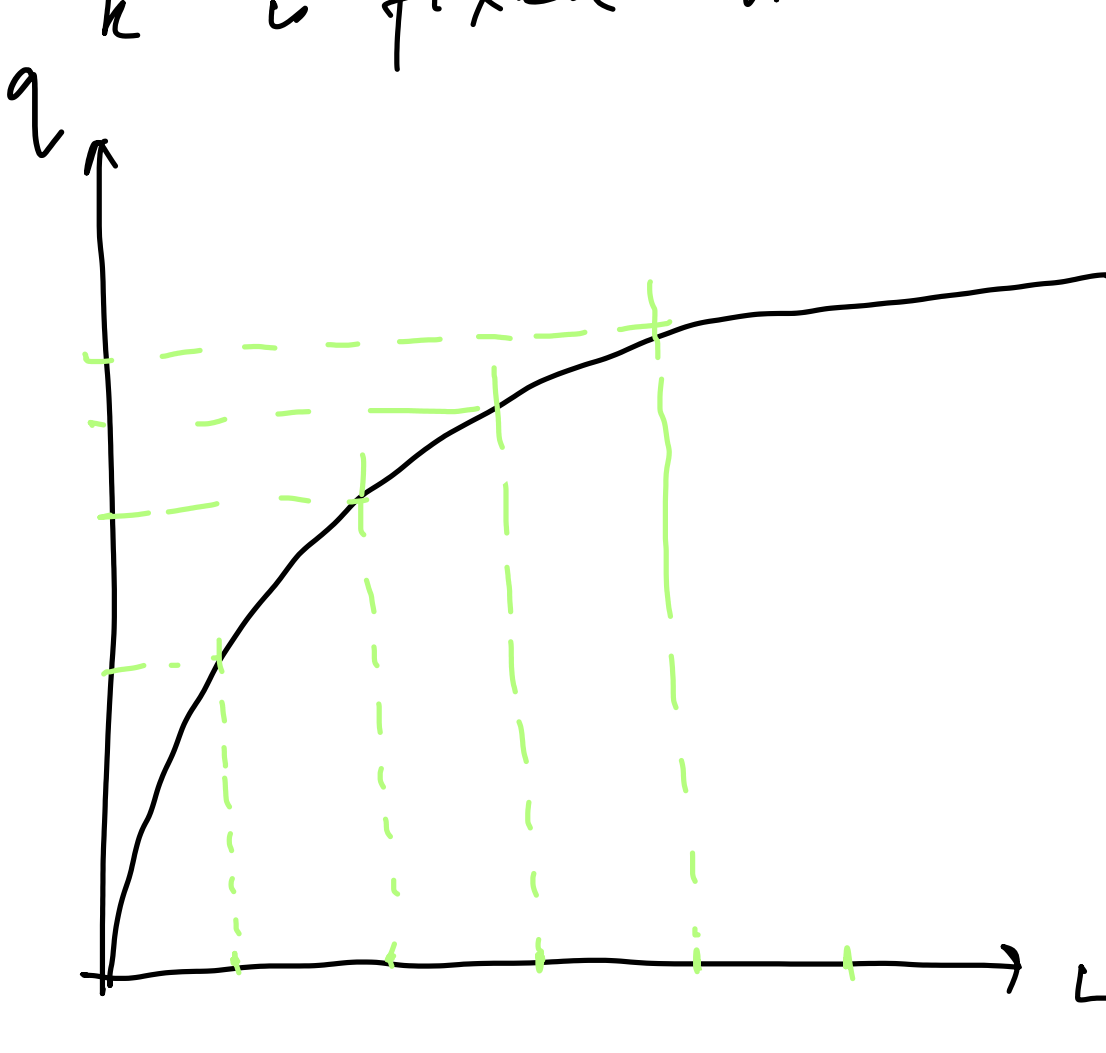
$16 = L.T$

$\Rightarrow T = 16/L$

Q10) $f(L,K) = q$

$\bar{k}$ is fixed in short run

$MP_L = \frac{dq}{dL} = \frac{\Delta q}{\Delta L}$



L	q
1	5
2	9
3	12
4	14

(differences: 4, 3, 2)

Q1) a) Avg Total Cost $= \frac{TC}{q}$

Total Cost (TC) = ATC × q

TC = VC + FC
(VC ↓ Variable; FC → Fixed.)

c) $\frac{TC}{q} = \frac{VC}{q} + \frac{FC}{q}$

Avg Total Cost = Avg Variable Cost + Avg Fixed Cost

D) TC = VC + FC
(VC ↪ Varying with q)

For eg $C(q) = \underbrace{(4q^2 + 2q)}_{VC} + \underbrace{10}_{FC}$ ← TC

D) Avg Total Cost ⟶

$\frac{C}{q} = \underbrace{(4q+2)}_{\text{Avg VC}} + \underbrace{\left(\frac{10}{q}\right)}_{\text{Avg FC}}$

AFC is declining with increase in q

E) $TC = C(q) = 4q^2 + 2q + 10$ $q > 0$

Q4) $MP_L = \frac{\Delta q}{\Delta L}$ or $\frac{dq}{dL}$

$MC = \left(w \frac{\Delta L}{\Delta q}\right)$

For ΔL units of labour, cost is $w\Delta L$

$MC = \frac{w\Delta L}{\Delta q} = \frac{w}{\Delta q/\Delta L} = \frac{w}{MP_L}$

$MC = \frac{w}{MP_L}$

Q5) $q = \sqrt{K} + \sqrt{L}$ —(1)

$r = 5$, $w = 50$, $q = 110$

$MRTS = \frac{MP_L}{MP_K}$

$MP_L = \frac{\partial q}{\partial L} = \frac{1}{2\sqrt{L}}$

$MP_K = \frac{\partial q}{\partial K} = \frac{1}{2\sqrt{K}}$

$MRTS = \frac{MP_L}{MP_K} = \frac{2\sqrt{K}}{2\sqrt{L}} = \sqrt{\frac{K}{L}}$

$MRTS = \frac{w}{r} = \frac{MP_L}{MP_K}$ ✓ Cost Minimization $C = wL + rK$

$\sqrt{\frac{K}{L}} = \frac{w}{r} = \frac{50}{5} = 10$

$K = 100L$ —(2)

From (1) & (2)

$q = \sqrt{100L} + \sqrt{L} = 10\sqrt{L} + \sqrt{L} = 11\sqrt{L}$

$110 = 11\sqrt{L} \Rightarrow L = 100$

$K = 100 \times 100 = 10{,}000$

$q_0 = \sqrt{K} + \sqrt{L}$

$(q_0 - \sqrt{L}) = \sqrt{K}$

$K = (q_0 - \sqrt{L})^2$

$C = wL + rK$

$C = wL + r(q_0 - \sqrt{L})^2$

$\min_L C =$

Q6) $C = aq + b$ (aq: ↑ Aq; b: long run = 0, rent per yr) $c < b$

Q7) $q = \min(aK, bL)$ $a, b > 0$

$C = wL + rK$

If $K = 100$, $L = 1$ (If $a = b = 1$)

then $q = 1$ (means only 1 unit of K is used, 99 goes waste)

If $a \neq b$

$q = \begin{cases} 100a & \text{if } 100a < b \\ b & \text{if } 100a > b \end{cases}$

For cost minimization $aK = bL$ —(1)

$q = aK = bL$

$K = \frac{q}{a}$

$L = \frac{q}{b}$

$C = \left(\frac{wq}{b} + \frac{rq}{a}\right)$

$C = \left(\frac{w}{b} + \frac{r}{a}\right)q$

$C(w, r, q) = \left(\frac{w}{b} + \frac{r}{a}\right)q$

Q8) $q = (L^\rho + K^\rho)^{1/\rho}$ | $\rho = \frac{1}{2}$, $w = r = 1$, $q = 4$

$MRTS = \frac{w}{r} = \frac{MP_L}{MP_K}$

$MP_L = \frac{\partial q}{\partial L} = \frac{L^{\rho-1}}{(L^\rho + K^\rho)^{\frac{1}{\rho}-1}}$

$MP_K = \frac{\partial q}{\partial K} = \frac{K^{\rho-1}}{(L^\rho + K^\rho)^{\frac{1}{\rho}-1}}$

$MRTS = \frac{MP_L}{MP_K} = \left(\frac{L}{K}\right)^{\rho-1}$

$MRTS = \frac{w}{r} = 1 = \left(\frac{L}{K}\right)^{\frac{1}{2}-1}$

$\left(\frac{K}{L}\right)^{1/2} = 1 \Rightarrow K = L$

$q = \left(K^{1/2} + K^{1/2}\right)^2 = \left(2\sqrt{K}\right)^2 = 4K$

$q = 4$

$\therefore K = L = 1$

Q7 of Assignment

$f(x_1, x_2) = C x_1^a x_2^b$ $C, a, b > 0$

$\frac{\partial f}{\partial x_1} = Ca x_1^{a-1} x_2^b = MP_1 = g(x_1, x_2)$

$g_1'(x_1, x_2) = \frac{\partial MP_1}{\partial x_1} = Ca(a-1) x_1^{a-2} x_2^b$

For MP_1 to be decreasing, $MP_1' < 0$

$\therefore Ca(a-1) x_1^{a-2} x_2^b < 0$

(C ↑ +, a ↑ +, (a−1) ↓ ?, x_1^{a-2} ↑ +, x_2^b ↑ +)

If $a - 1 < 0$, MP_1 is decreasing. $C > 0$, $b > 0$

$\therefore a < 1$