Week 6

Tuesday, 5 March 2024 17:38

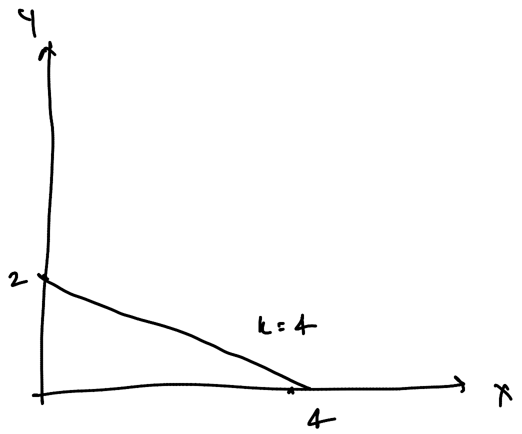
Q8 $u(x,y) = (x+2y)^2$

$MRS(x,y) = -\frac{u_x}{u_y}$

$u_x = \frac{\partial u}{\partial x} = 2(x+2y)$

$u_y = \frac{\partial u}{\partial y} = 4(x+2y)$

$MRS = -\frac{u_x}{u_y} = -\frac{2}{4} = -\frac{1}{2}$



$(x+2y)^2 = k^2$

$x+2y = k$

eg $x+2y = 4$

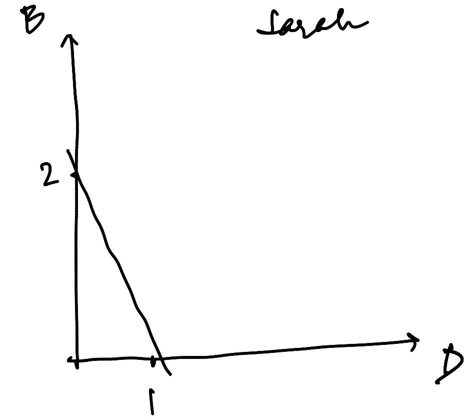
$MRS = -\frac{\Delta y}{\Delta x} = -\frac{1}{2}$

If $V(x,y) = x+2y$

$u(x,y) = (V(x,y))^2$ Positive monotonic transformation

∴ x & y are substitutes.

Q9



Sarah

$\Delta y = 2$

$\Delta x = 1$

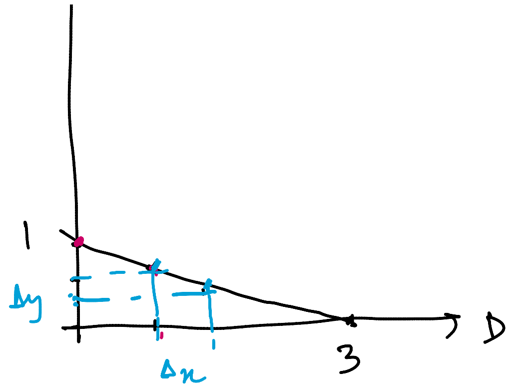
$MRS = -\frac{\Delta y}{\Delta x}$

$= -2$

Simon

$\Delta y = 1$

$\Delta x = 3$

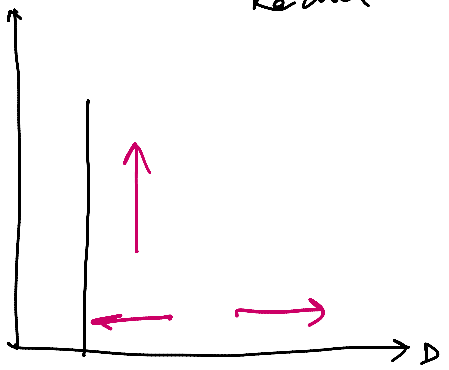


$\Delta x = 3$

$MRS = -\frac{\Delta y}{\Delta x} = -\frac{1}{3}$

B

Rachel.



$MRS = -\frac{u_x}{u_y}$

$MRS = \frac{-0}{k} = 0$

Q1

$u = 10x^2 z$ —(1)

$P_x = 1$

$P_z = 20$

$Y = 300$

$P_x X + P_z Z = Y$

$300 = X + 20Z$ —(2)

At the optimal bundle, both eqns (1) & (2) are simultaneously satisfied.

$Z = \frac{300 - X}{20}$ from eqn (2)

$u = 10X^2\left(\frac{300 - X}{20}\right)$ from (1) & (2)

$u = \frac{300\cancel{0}\, X^2}{2\cancel{0}} - \frac{X^3}{2}$

$\frac{du}{dX} = \frac{\cancel{2} \cdot 300}{\cancel{2}} X - \frac{3X^2}{2} = 0$

$$\frac{du}{dx} = 600x - 3x^2 = 0$$

$$\Rightarrow \quad 3x(200 - x) = 0$$

$$\Rightarrow \quad x = 0 \quad \text{or} \quad x = 200$$

$$300 = x + 20z$$

$$100 = 20z$$

$$z = 5$$

optimal bundle $(x, z) = (200, 5)$

Q2 $\quad u(B, Z) = AB^{\alpha} Z^{\beta}$

$P_B = 12$

$P_Z = 10$

$Y = 600$

$$MRS = -\frac{u_x}{u_y} = -\frac{u_B}{u_Z}$$

$$\frac{\partial u}{\partial B} = AZ^{\beta} . \alpha B^{\alpha-1}$$

$$\frac{\partial u}{\partial Z} = AB^{\alpha} \beta Z^{\beta-1}$$

$$MRS = \frac{\cancel{A}\alpha B^{\alpha-1} Z^{\beta}}{\cancel{A}\beta B^{\alpha} Z^{\beta-1}} = \frac{\alpha Z}{\beta B}$$

$$MRS = MRT = \frac{P_x}{P_y}$$

At the point of optimization & tangency,

$$\frac{\alpha Z}{\beta B} = \frac{P_B}{P_Z} = \frac{12}{10}$$

$$\frac{\alpha Z}{\beta B} = \frac{P_B}{P_Z} = \frac{12}{10}$$

$12B + 10Z = 600$ Budget constraint

$$Z = \frac{12\beta B}{10\alpha}$$

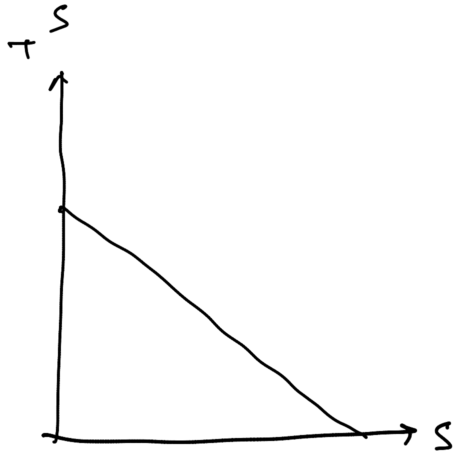
Solve for B & Z from eqn (1) or (2)

Q5 $u(S,T) = 2ST$

$P_S = P_T = 80$

$Y = 1280.$

$$MRS = -\frac{u_S}{u_T} = \frac{2T}{2S} = \frac{T}{S}$$



$P_S S + P_T T = Y$

$80S + 80T = 1280$

$MRS = MRT$

$$\frac{T}{S} = \frac{80}{80} = 1$$

$T = S$

$160S = 1280$

$$S = \frac{1280}{160} = 8$$

$T = 8$

Optimal bundle is $(S,T) = (8,8)$

Instead of money, time can also be a constraint.

$Y = 6$ hrs

$P_S = 1$ hr

$P_T = 1\,hr$

$u(S,T) = 2ST$

$MRS = \frac{T}{S}$

$P_S S + P_T T = Y$

$S + 2T = 6$

$\frac{T}{S} = \frac{P_S}{P_T} = \frac{1}{2} \Rightarrow 2T = S$

$2T + T = 6$

$T = 2$

$S = 4$

Q4 MU for coffee $= 5 = MU_c$

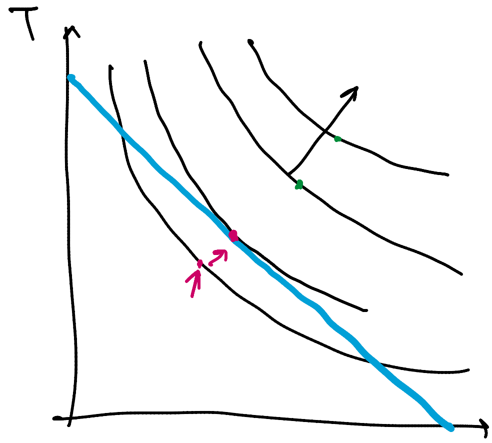
Price of coffee $= 20 = P_c$

MU for tea $= 5 = MU_t$

Price of tea $= 10 = P_t$

$\frac{MU_c}{P_c} = \frac{5}{20} = \frac{1}{4} = 0.25$

$\frac{MU_t}{P_t} = \frac{5}{10} = \frac{1}{2} = 0.5$



$\frac{MU_t}{P_t} > \frac{MU_c}{P_c}$

$\frac{MU_t}{MU_c} > \frac{P_t}{P_c}$

MU_c / pc

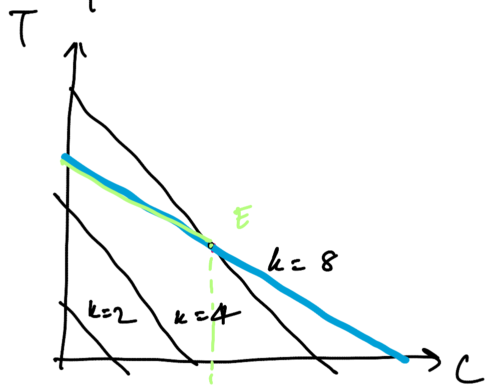
MRS MRT

At optimal bundle, MRS = MRT

slope of indiff curve

slope of budget constraint

If the consumer were indifferent to coffee,



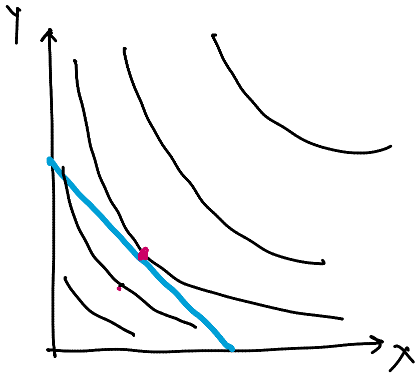
$MU_c = 0$

MRS is const for the 2 goods if perfect subst.

$x + y = k$

$$\frac{MU_c}{pc} > \frac{MU_t}{pt}$$

my: $u = k$



$$\frac{MU_c}{pc} = \frac{MU_t}{pt}$$

condition for tangency