Mathematical Optimization for Economic Applications

Lecture 1

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Topics

- Introduction
- Economic Applications: Examples
- Types of Optimization
- Method of Optimization

What is Optimization?

- The process of choosing the best option from a set of alternatives.
- The most *feasible* and *desirable* way of doing a task.
- For eg, you love eating chocolates. But you cannot eat chocolates for breakfast, lunch, and dinner. There is a budget constraint and it is not good for your health. So, you eat as much as your pocket and your stomach will allow to maximize your "utility."

Economic Applications

- We study the economics of ***scarcity***.
- If everything is available in abundance and no damage accrues from any production or consumption, this economics will not apply.
- But since that does not happen, we need to **optimize**, and make informed decisions to maximize or minimize certain outcomes.

Economic Applications

- Utility Maximization (Microeconomics)
- Profit Maximization (Microeconomics)
- Resource Allocation (Macroeconomics)
- Payoff Maximization in multi-player setting (Game Theory)
- Intertemporal Payoff Maximization (All)

Types of Optimization

- Single variable. Eg, profit maximization where profit π is $\pi(q)$ where q is quantity produced.
- Multi-variable. Eg, consumer utility maximization.
- Constrained. Eg, budget constraint in utility maximization.
- Unconstrained. Eg, profit maximization for production of single good.
- Single agent. Eg, a firm, a consumer.
- Multi-agent. Eg, firms competing in a market.

Types of Optimization

- Linear.
- Non-linear.
- Static or not varying with time.
- Dynamic or time-varying.

Method of Optimization

A few things form the basic construct of an optimization problem. These are:

- What type of optimization is it?
- What are you choosing—the choice variables?
- What is the domain for choice variables?
- What is the **objective function**?
- Are there any constraints?

Method of Optimization: Example 1

A simple example in profit maximization of a single firm with a single good with price P, quantity produced q, total cost cq and inverse demand function $P(q)$. Here $q \geq 0$.

Eg: Profit Maximization

$$\max_q \pi(q) = P \cdot q - c \cdot q = P(q) \cdot q - c \cdot q$$

This is a single variable, single agent, static, unconstrained maximization problem.

Method of Optimization: Example 2

We can also sometimes minimize costs $C(q)$, instead of maximizing profit.

Here $q \geq 0$.

Eg: Cost Minimization

$$\min_q C(q) = \alpha q^2 - \beta q + c_0$$

This is a single variable, single agent, static, unconstrained minimization problem.

Method of Optimization: Example 3

A consumer choosing between two goods x and y, priced at P_x and P_y subject to a budget constraint of $B = P_x \cdot x + P_y \cdot y$.

Here $x, y \geq 0$.

Utility is given by $U(x, y) = f(x, y)$

Eg: Utility Maximization

$$\max_{x,y} U(x, y) = f(x, y)$$

$$s.t. \quad B = P_x \cdot x + P_y \cdot y$$

This is a 2-variable, single agent, static, constrained maximization problem.

Reference Reading

1. *Essential Mathematics for Economic Analysis (5e)* by Sydsaeter, Hammond, Strom and Carvajal.
2. *Fundamental Methods of Mathematical Economics (4e)* by Chiang and Wainwright.
3. *Mathematics for Economists* by Simon and Blume.

Thank you!

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