

CS345 Theoretical Assignment 4

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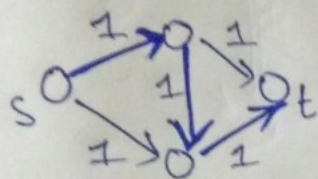
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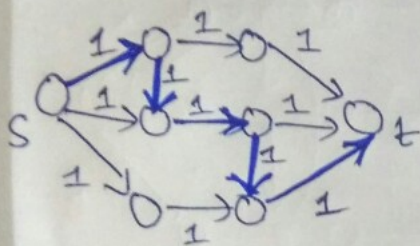
1 Any Guarantee of our First-Attempt Algorithm

1.1 Counter Example

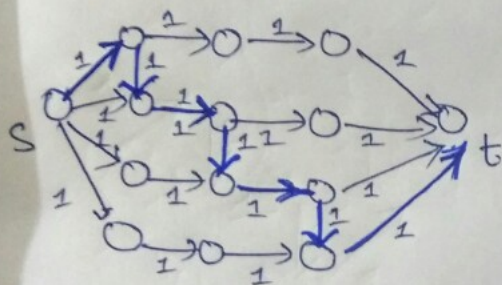
Counter Example



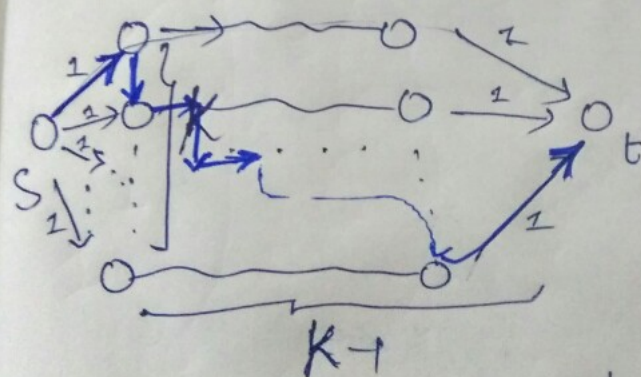
for the shown path chosen
first $\rightarrow \frac{\text{flow returned}}{\text{Actual max flow}} = \frac{1}{2}$



" = $\frac{1}{3}$



" = $\frac{1}{4}$



" = $\frac{1}{k}$

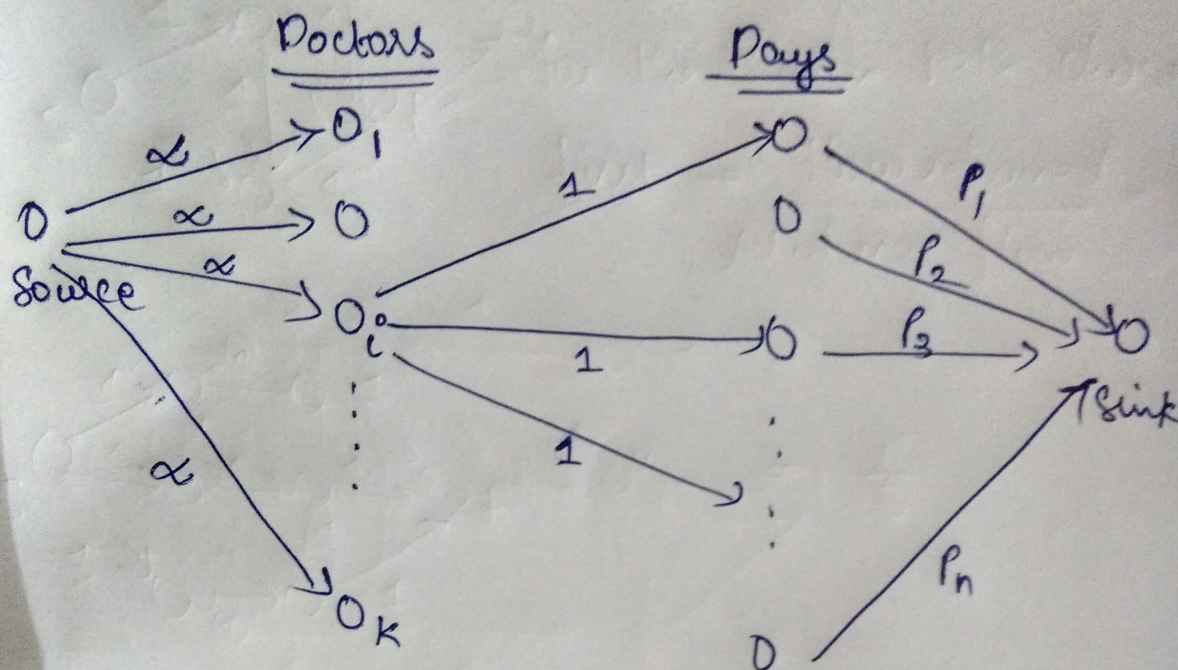
That is, we have to construct one path such that it blocks all other $k-1$ paths possible. This construction possible $\forall k > 1$
 \Rightarrow Approximation Not possible

2 A max flow application

2.1 Without Extra Constraint

2.1.1 Overview

A max flow Application



Doctors $[i]$ to Days $[j]$ there is an edge
iff j lies in the list L_i .

All these edges inserted between Doctors
& Days hold capacity 1.

L'_1, \dots, L'_k exists if Max flow of
above graph is $\sum_{i=1}^n P_i$.

2.1.2 Notations

p_i denotes the exact number of doctors required on day i

L_i denotes the list of days where doctor i is available

L'_i denotes the list of days that doctor i has to work to produce the required match. Note, $L'_i \subseteq L_i$

$D = \sum_{i=1}^n p_i$

n = Number of days in total

k = Number of Doctors in total

2.1.3 Claim

Construction of L'_i s is possible if and only if the max-flow in the source-sink graph constructed (in image) is D .

2.1.4 Proof - Part 1

Given L'_i s list for all doctors, show that the max flow of source-sink graph shown above is D

Construct a flow f of the source-sink graph as follows.

$f(\text{source}, \text{Doctor}[i])$ is $|L'_i|$ - satisfies capacity constraint as these edges had infinite capacity

$f(\text{Doctor}[i], \text{Day}[j])$ is 1 if j lies in L'_i , otherwise is 0 - satisfies capacity constraint

$f(\text{Day}[j], \text{sink})$ is p_j - satisfies capacity constraint

Flow Conservation is ensured as it is given to us that L'_i s of such definitions exist.

Hence, the above is a valid flow.

As the *cut - capacity* between *sink* and the rest of the graph is D , by *min - cut - max - flow* theorem, f should be a max flow of the source-sink graph. Hence, proved.

2.1.5 Proof - Part 2

Given the max flow of source-sink graph to be D , show that L'_i s list for all doctors exist

Let f be the integral max-flow of the source-sink graph. (Note: Integral flow exists was proved in class)

Construct L'_i as follows.

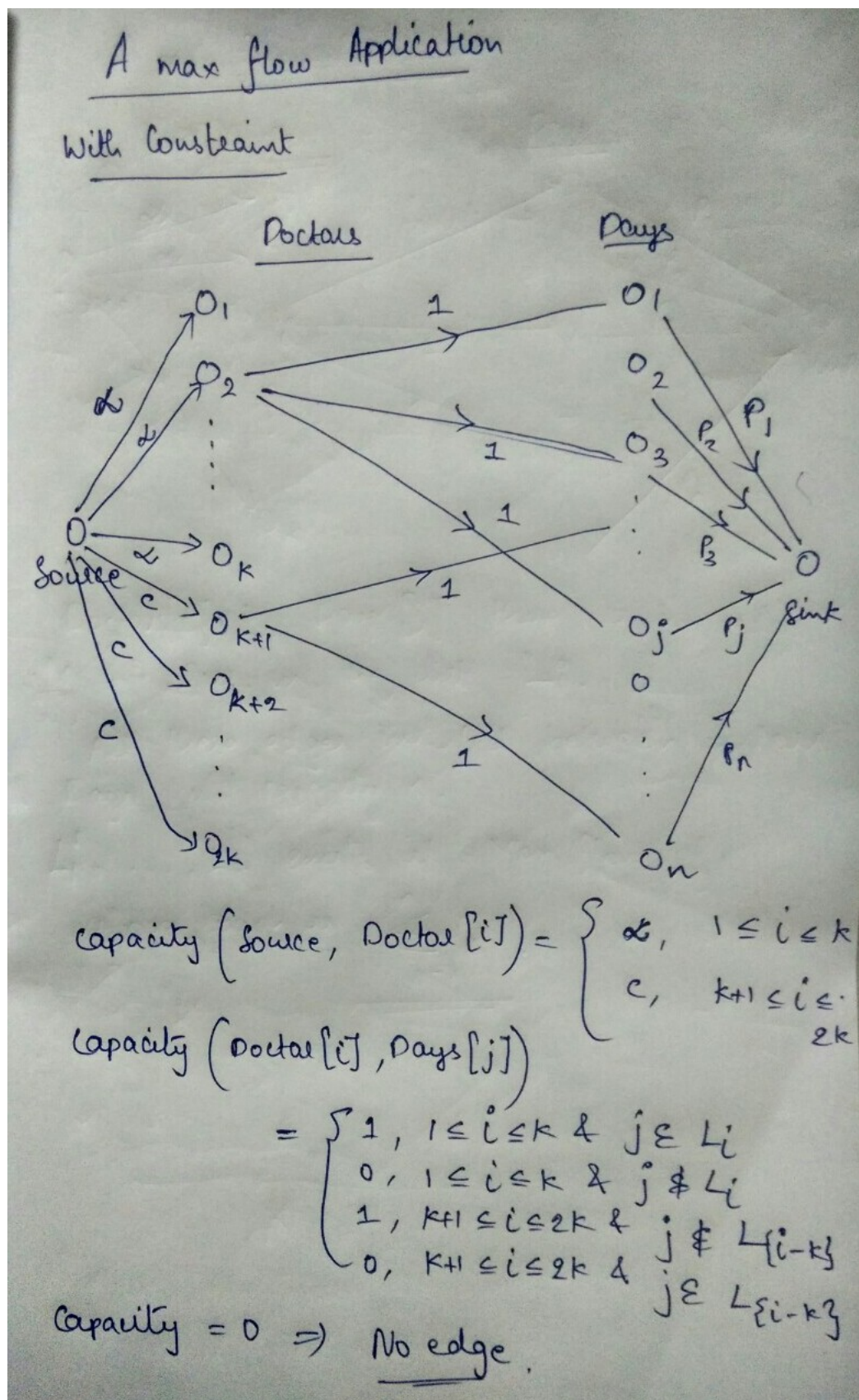
If $f(\text{Doctor}[i], \text{Day}[j])$ is 1 then add j to L'_i otherwise do nothing.

Note: $f(\text{Doctor}[i], \text{Day}[j])$ can be only 0 or 1 by integral flow property.

The L'_i s thus constructed are valid as $\text{value}(f) = D$ implying every day j has got the exact number of doctors wanted (p_j). Moreover, $L'_i \subseteq L_i$ is ensured from the construction of the source-sink graph itself. Hence, proved.

2.2 With Extra Constraint

2.2.1 Overview



2.2.2 Claim

2.2.3 Proof