

CS345 Theoretical Assignment 4

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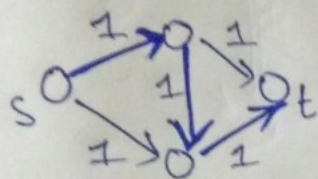
Contents

| | | |
|----------|---|----------|
| 1 | Any Guarantee of our First-Attempt Algorithm | 2 |
| 1.1 | Counter Example | 2 |
| 2 | A max flow application | 3 |
| 2.1 | Without Extra Constraint | 3 |
| 2.1.1 | Overview | 3 |
| 2.1.2 | Notations | 4 |
| 2.1.3 | Claim | 4 |
| 2.1.4 | Proof | 4 |
| 2.2 | With Extra Constraint | 5 |
| 2.2.1 | Overview | 5 |
| 2.2.2 | Claim | 6 |
| 2.2.3 | Proof | 6 |

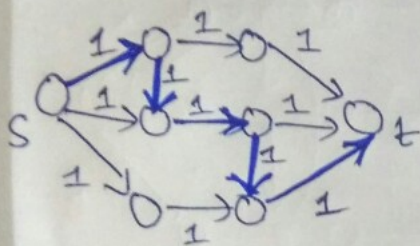
1 Any Guarantee of our First-Attempt Algorithm

1.1 Counter Example

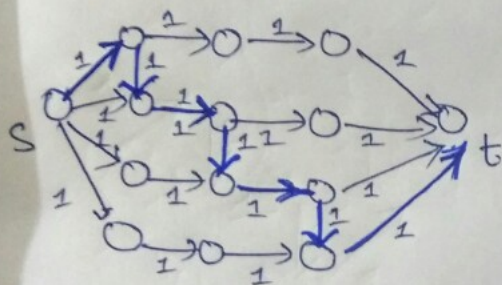
Counter Example



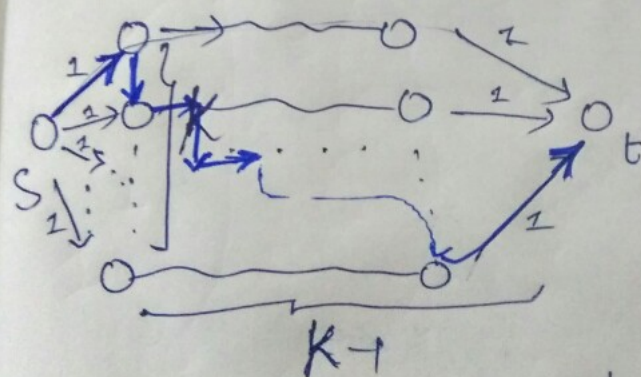
for the shown path chosen
first $\rightarrow \frac{\text{flow returned}}{\text{Actual max flow}} = \frac{1}{2}$



" = $\frac{1}{3}$



" = $\frac{1}{4}$



" = $\frac{1}{k}$

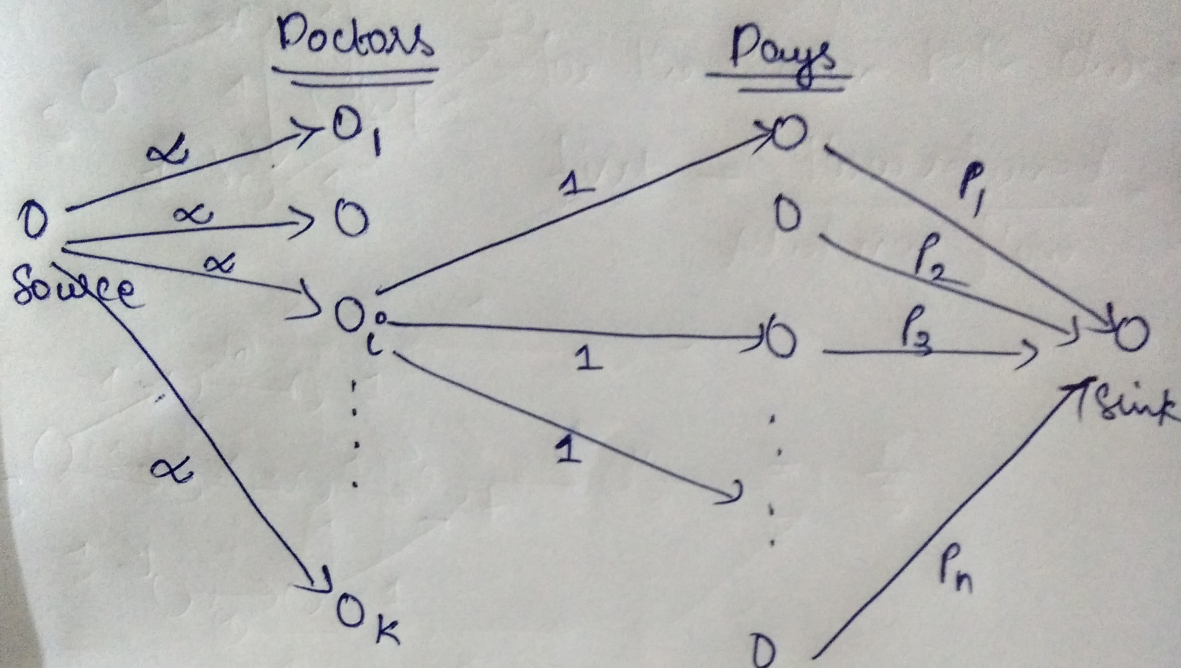
That is, we have to construct one path such that it blocks all other $k-1$ paths possible. This construction possible $\forall k > 1$
 \Rightarrow Approximation Not possible

2 A max flow application

2.1 Without Extra Constraint

2.1.1 Overview

A max flow Application



Doctors $[i]$ to Days $[j]$ there is an edge
iff j lies in the list L_i .

All these edges inserted between Doctors
& Days hold capacity 1.

L_1, \dots, L_k exists if Max flow of
above graph is $\sum_{i=1}^n P_i$.

2.1.2 Notations

p_i denotes the exact number of doctors required on day i

L_i denotes the list of days where doctor i is available

L'_i denotes the list of days that doctor i has to work to produce the required match. Note, $L'_i \subseteq L_i$

$D = \sum_{i=1}^n p_i$

n = Number of days in total

k = Number of Doctors in total

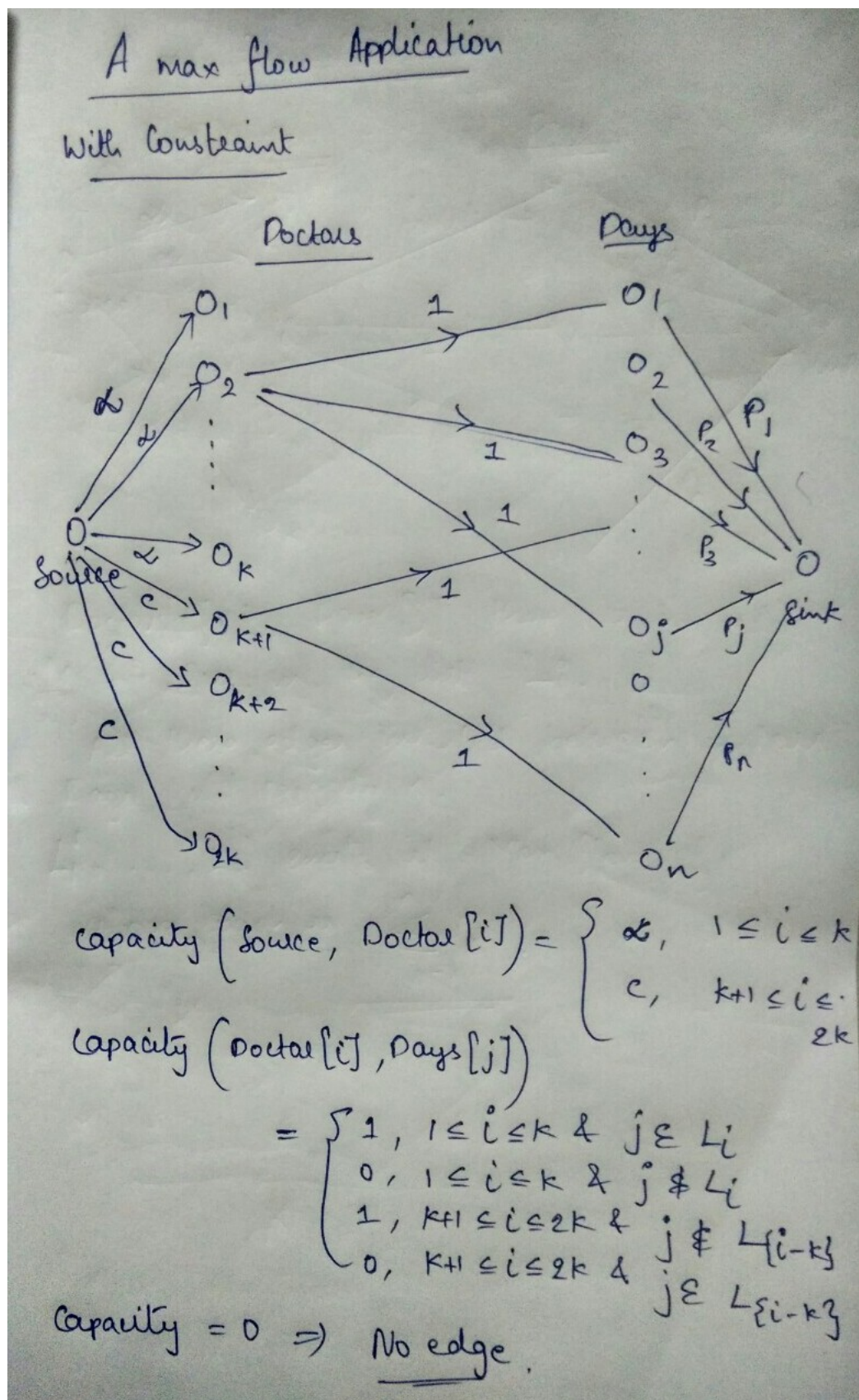
2.1.3 Claim

Construction of L'_i s is possible if and only if the max-flow in the s-t graph is D .

2.1.4 Proof

2.2 With Extra Constraint

2.2.1 Overview



2.2.2 Claim

2.2.3 Proof