Arithematic and Geometric Sequences

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1 Arithematic Progression

Defination

An arithematic progression (A.P.) is a sequence of numbers such that the difference between the consecutive terms is constant. For instance, the sequence $5, 7, 9, 11, 13, 15\ldots$ is an arithematic progression with common difference of 2.

Calculation

If the initial term of an arithematic progression is a_1 and the common difference of successive members is d , then the nth term of the sequence (a_n) is given by:

$$a_n = a_1 + (n-1)d,$$

or, in general

$$a_n = a_m + (n-m)d,$$

To derive the formula for sum of n terms(S_n) of an A.P.
Let

$$\begin{aligned} S_n &= a + (a+d) + (a+2d) + ...(a+(n-1)d) \\ S_n &= (a_n - (n-1)d) + (a_n - (n-2)d) + ... + a_n - d + a_n \end{aligned}$$

Adding Both sides of the equation, we get

$$\begin{aligned} 2S_n &= n(a + a_n) \\ 2S_n &= n(a + a + (n-1)d) \\ 2S_n &= n(2a + (n-1)d) \\ S_n &= \frac{n}{2}[2a + (n-1)d] \end{aligned}$$

2 Geometric Progression

Defination

A Geometric progression (GP) is a sequence of numbers such that the ratio between the consecutive terms is constant. For instance, the sequence $2, 4, 8, 16, 32, 64\ldots$ is an G.P. with common ratio of 2.
A generalized G.P. with first term a and common ratio r can be shown as,

$$a, ar, ar^2, ar^3, ar^4, ..., ar^{n-1}$$

with n_{th} term(a^n) being,

$$a_n = ar^{n-1}$$

Calculation

To derive the formula for sum of n terms(S_n) of an G.P..
Let

$$S_n = a + ar + ar^2 + ...ar^{n-1} \tag{1}$$

$$rS_n = ar + ar^2 + ar^3 + ... + ar^n \tag{2}$$

$(1) - (2)$, we get

$$S_n(1-r) = a - ar^n$$

$$S_n = a\frac{1-r^n}{1-r}$$