

Arithmetic and Geometric Sequences

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1 Arithmetic Progression

Defination

An arithmetic progression (A.P.) is a sequence of numbers such that the difference between the consecutive terms is constant. For instance, the sequence 5, 7, 9, 11, 13, 15... is an arithmetic progression with common difference of 2.

Calculation

If the initial term of an arithmetic progression is a_1 and the common difference of successive members is d , then the n th term of the sequence (a_n) is given by:

$$a_n = a_1 + (n - 1)d,$$

or, in general

$$a_n = a_m + (n - m)d,$$

To drive the formula for sum of n terms(S_n) of an A.P.
Let

$$\begin{aligned} S_n &= a + (a + d) + (a + 2d) +(a + (n - 1)d) \\ S_n &= (a_n - (n - 1)d) + (a_n - (n - 2)d) + + a_n - d + a_n \end{aligned}$$

Adding Both sides of the equation, we get

$$\begin{aligned} 2S_n &= n(a + a_n) \\ 2S_n &= n(a + a + (n - 1)d) \\ 2S_n &= n(2a + (n - 1)d) \\ S_n &= \frac{n}{2}[2a + (n - 1)d] \end{aligned}$$

2 Geometric Progression

Defination

A Geometric progression (GP) is a sequence of numbers such that the ratio between the consecutive terms is constant. For instance, the sequence 2, 4, 8, 16, 32, 64... is an G.P. with common ratio of 2. A generalized G.P. with first term a and common ratio r can be shown as,

$$a, ar, ar^2, ar^3, ar^4,, ar^{n-1}$$

with n_{th} term(a^n) being,

$$a_n = ar^{n-1}$$

Calculation

To drive the formula for sum of n terms(S_n) of an G.P..

Let

$$S_n = a + ar + ar^2 +ar^{n-1} \quad (1)$$

$$rS_n = ar + ar^2 + ar^3 + + ar^n \quad (2)$$

(1) – (2) , we get

$$S_n(1 - r) = a - ar^n$$

$$S_n = a \frac{1 - r^n}{1 - r}$$