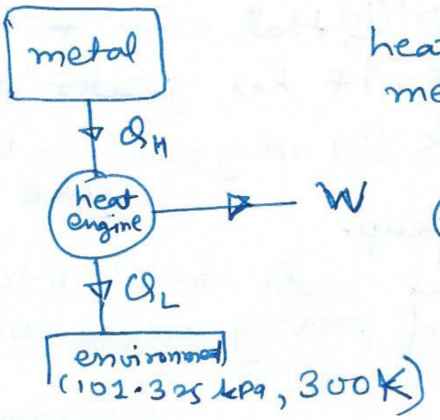
6.5



heat is getting removed from metal differentially.

$$\delta Q_H = c\,dT \;(kJ/kg)$$

ⓐ $Q_H = \int \delta Q_H = c(T_2 - T_1)$

$= 0.4(100-200)$

$= -40\,kJ/kg$

(−) sign means heating is getting rejected from metal.

ⓑ change in availability = max useful work which can be extracted while changing state

in this problem we want to calculate change in availability of metal $= \Phi_2 - \Phi_1$

$$= (u_2 + P_0 v_2 - T_0 s_2) - (u_1 + P_0 v_1 - T_0 s_1)$$

$$= (u_2 - u_1) + \cancel{P_0(v_2 - v_1)}^{0} - T_0(s_2 - s_1)$$

assuming no significant change in volume of metal, $\Delta v \approx 0$,

first law, $Q_H = \Delta u + \cancel{W}^{0}$ ← first law for metal

$\Rightarrow Q_H = u_2 - u_1$

$$\boxed{u_2 - u_1 = -40\,kJ/kg}$$

$$(s_2 - s_1) = \int ds = \int_{T_1}^{T_2} \frac{dQ}{T} = \int_{T_1}^{T_2} \frac{c\,dT}{T}$$

$$= c\ln\left(\frac{T_2}{T_1}\right)$$

Hence $\Phi_2 - \Phi_1 = (u_2 - u_1) - T_0(s_2 - s_1)$

$= \cancel{T_0(s_2 - s_1)}$

$$= -40 - (300)c\ln\left(\frac{T_2}{T_1}\right).$$

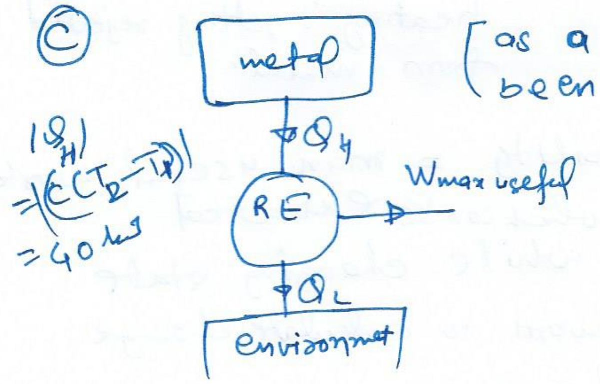
$$= -40 - (300)(0.4)\ln\left(\frac{373.15}{473.15}\right) kJ/kg$$

$$= -11.51\,kJ/kg.$$

what we learn from part (b), that when ~~w~~ the metal is at higher temperature, it has greater potential to provide ~~perform~~ useful work (by using some work device like engine) as compared to low temp.

It means when metal cools from high to low temp., the potential of proving useful work (availability) also decreases.

(c)



[as a reversible heat engine has ← mentioned in part (c)
been used, we will obtain max. possible useful work by heat engine]

$$\underbrace{\Delta S_{total}}_{0} = \underline{\Delta S_{metal} + \Delta S_{HR} = 0}$$

$$\Rightarrow \Delta S_{HR} = -\Delta S_{metal}$$

Why → because Heat engine is reversible & metal is cooling differentially.

$$\Delta S_{meate} = \int_{T_1}^{T_2} \frac{\delta Q}{T}$$

$$= c\int_{T_1}^{T_2} \frac{dT}{T} = c \ln \frac{T_2}{T_1}$$

$$\Delta S_{HR} = -\Delta S_{metal} = -c \ln \frac{T_2}{T_1} = c \ln \frac{T_1}{T_2}$$

hence $Q_L \rightarrow Q_{HR}/T_0$ $\Rightarrow$ $\boxed{Q_{HR} = T_0 \Delta S_{HR} = T_0\, c \ln \frac{T_1}{T_2}}$

$$Q_{HR} = (300)(0.4) \ln \frac{473.15}{373.15}$$

$$\boxed{Q_{HR} = 28.49 \text{ kJ/kg}}$$

using first law across heat engine:-

$$W_{max\ useful} = Q_H - Q_L = 40 - 28.49$$

$$= 11.51 \text{ kJ/kg}$$

which means that the maximum possible work which can be converted ~~heat~~ into useful work ~~using heat engine~~ is 11.51 kJ/kg by cooling down metal form 200° to 100°C.

It should be also noted down that maximum useful work (availability) also depends on the temp of environment (T_0), lower T_0 means higher conversion of heat (rejected from metal) to work using heat engine. And this work will be maximum when we use reversible heat engine.

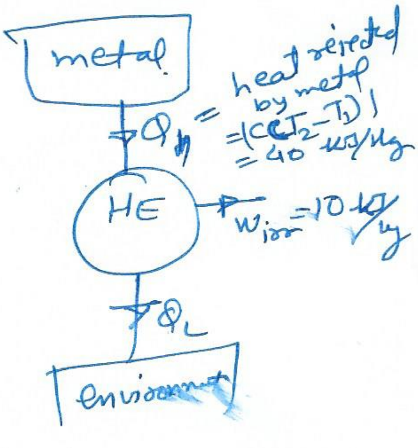
(d) in this part we are using an engine, whose output = 10 kJ/kg < 11.51 kJ/kg (max useful work for which we have to use rev. engine)

which implies there is irreversibility in engine.

hence $\Delta S_{total} > 0$

$$\boxed{\begin{aligned}\Delta S_{total} &= \Delta S_{metal} + \Delta S_{HR} \\ &= c \ln \frac{T_2}{T_1} + \frac{|Q_L|}{T_0}\end{aligned}}$$

applying first law across engine



$Q_L = Q_H - W_{irr}$

$Q_L = 40 - 10$

$Q_L = 30$ kJ/kg

hence $\Delta S_{total} = \left(0.4 \ln \frac{373.15}{473.15} + \frac{30}{300}\right)$ kJ/kg K.

hence Lost work = $T_0 \Delta S_{total} = 300 (\Delta S_{total})$

$= 1.51$ kJ/kg

which one can also obtain using answer obtained in part (c)

Lost work = max useful work (when we use rev. engine) − useful work delivered by irreversible heat engine

$= (11.51 - 10) = 1.51$ kJ/kg.