CI-1

Given:

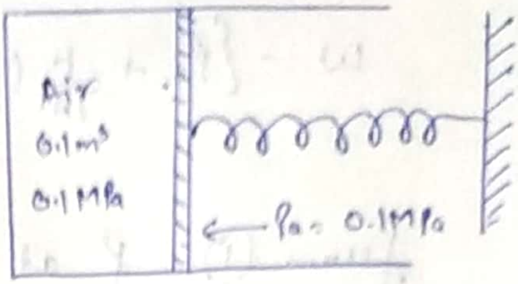
(1) $V_0 = 0.1 m^3 = V_1$

$P_a = 0.1 MPa$

$P_1 = 0.1 MPa$

$V_2 = 0.3 m^3$

$P_2 = 0.6 MPa$



So, force balance on piston-

$$PA = P_a A + kx \qquad \{A \rightarrow \text{cross sectional area}\}$$

$$P = P_a + \frac{kx}{A}$$

x can be taken as change in volume at any instant

$$\therefore \quad P = P_a + \frac{k(V-V_0)}{A} \longrightarrow (1)$$

and, work done by the gas-

$$W = \int_1^2 P\,dv$$

$$= \int_1^2 \left[P_a + \frac{K}{A^2}(V-V_0)\right]dv$$

$$= \left[P_a V + \frac{K}{2A^2}(V-V_0)^2\right]_1^2$$

$$= P_a(V_2-V_1) + \frac{K}{2A^2}\{(V_2-V_0)^2 - (V_1-V_0)^2\}$$

$$= P_a(V_2-V_1) + \frac{K}{2A^2}\{(V_2-V_0+V_1-V_0)(V_2-\cancel{V_0}-V_1+\cancel{V_0})\}$$

$$= P_a(V_2-V_1) + \frac{K}{2A^2}\{(V_2-V_1)(V_2+V_1-2V_0)\}$$

$$W = \left\{P_a + \frac{K}{2A^2}(V_2 + V_1 - 2V_0)\right\}(V_2 - V_1) \quad -(2)$$

from (1) P at position 1 & 2.

$$P_1 = P_a + \frac{K}{A^2}(V_1 - V_0)$$

$$P_2 = P_a + \frac{K}{A^2}(V_2 - V_0)$$

By (1) + (2)

$$P_1 + P_2 = 2P_a + \frac{K}{A^2}(V_2 + V_1 - 2V_0)$$

$$\frac{P_1 + P_2}{2} = P_a + \frac{K}{2A^2}(V_2 + V_1 - 2V_0) \quad -(3)$$

On substituting (3) in (2)

$$W = \frac{P_1 + P_2}{2}(V_2 - V_1)$$

$$W = \frac{(0.1 + 0.6) \times 10^6}{2}(0.3 - 0.1)$$

$$\boxed{W = 70\,kJ}$$

CI-2

(2) Given:-

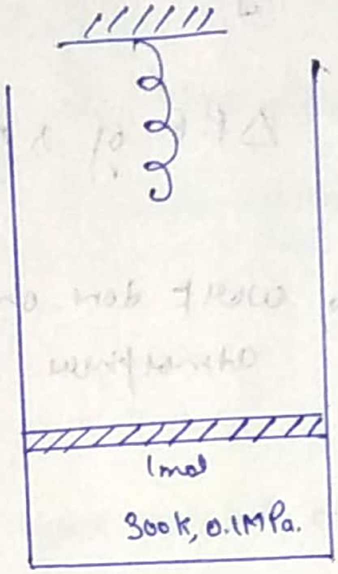
$V_2 = 2V_1$ {V_1 is the initial volume}

$V_3 = 3V_1$

& $T_1 = 300K$

$P_1 = 0.1$ MPa

$P_2 = 0.3$ MPa

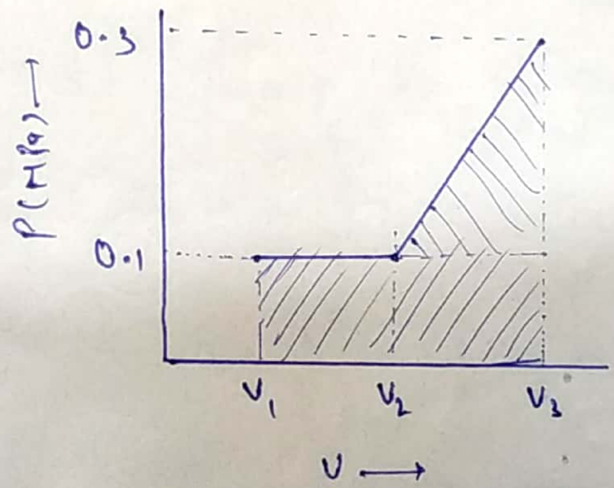


So,

$$V_1 = \frac{RT_1}{P_1} \quad \{PV = RT\}$$

$$V_1 = \frac{8.314 \times 300}{0.1 \times 10^6} = 0.024942\ m^3$$

Now, let's draw the PV-graph.



work done by gas = Area under PV curve.

$$= P_1 \times (V_2 - V_1) + P_1 \times (V_3 - V_2) + \tfrac{1}{2} \times (P_2 - P_1) \times (V_3 - V_2)$$

$$= \frac{V_1}{2}(P_2 + 3P_1) \quad \left\{ \begin{array}{l} V_2 = 2V_1 \\ V_3 = 3V_1 \end{array} \right\}$$

$$= \frac{0.02494}{2}(0.3 + 3 \times 0.1) \times 10^6$$

$$= 0.0074826 \times 10^6 = 7.4826\ KJ$$

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Now,

Change in P.E of spring:-

$\therefore$ ΔP.E of spring = work done by gas - work done atmosphere

So, work done on atmosphere = $P_1(V_3 - V_1)$

$= 0.1 \times 10^6 (2 \times 0.02494)$

$= 4.9884$ KJ

$\therefore$ Δ P.E of spring = $(7.4826 - 4.9884)$ KJ

$= 2.4942$ KJ