Departure function :- measure of deviation from ideal gas behavior.

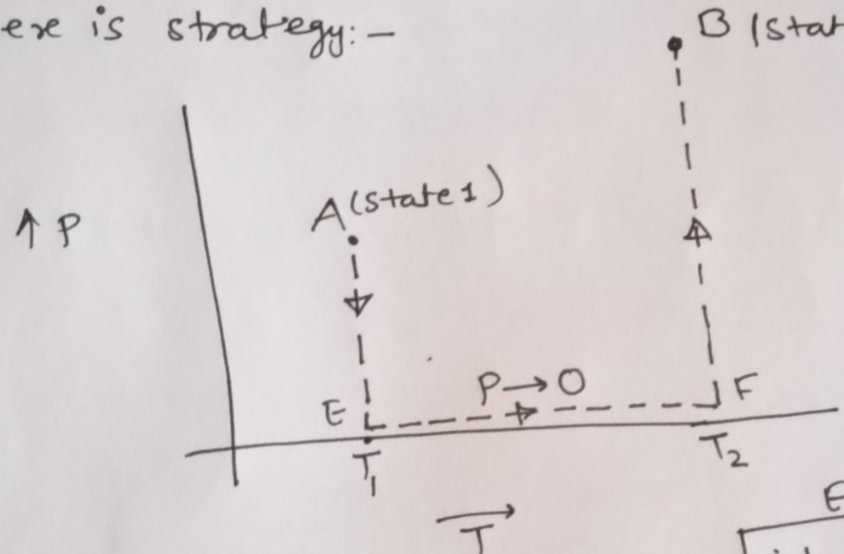
for example :- if a real gas is making a change from state 1 to 2, we are interested in $h_2 - h_{20}$ or $h_1 - h_{10}$ or $(h_2 - h_{20}) - (h_1 - h_{10})$

where 'o' signifies ideal gas behavior.

we will start with the following relation;

$$\left(\frac{\partial h}{\partial P}\right)_T = v - T\left(\frac{\partial v}{\partial T}\right)_P \quad \text{(derive it)}$$

Here is strategy :-



we are choosing an alternative path A–B

E → F (P → 0)

$\boxed{\text{ideal gas behavior}}$

$$h_2 - h_1 = \int_{P_1}^{P \to 0} dh + \underbrace{\int_{T_1\,(P\to 0)}^{T_2} dh}_{\text{ideal gas behavior}} + \int_{P \to 0}^{P_2} dh$$

$dh = C_{P0}\, dT \Rightarrow h_F - h_E = C_{P0}(T_2 - T_1)$

$\Rightarrow h_{20} - h_{10} = C_{P0}(T_2 - T_1)$

$$\Rightarrow h_2 - h_1 = \int_{P_1}^{P\to 0}\left[v - T\left(\frac{\partial v}{\partial T}\right)_P\right]_{T_1} dP + (h_{20} - h_{10}) + \int_{P\to 0}^{P_2}\left(v - T\left(\frac{\partial v}{\partial T}\right)_P\right)_{T_2} dP$$

$$\Rightarrow (h_2 - h_{20}) - (h_1 - h_{10}) = \int_{P\to 0}^{P_2}\left(v - T\left(\frac{\partial v}{\partial T}\right)_P\right)_{T_2} dP - \int_{P\to 0}^{P_1}\left[v - T\left(\frac{\partial v}{\partial T}\right)_P\right]_{T_2} dP$$

$$\Rightarrow \boxed{(h - h_0) = \int_{P\to 0}^{P}\left(v - T\left(\frac{\partial v}{\partial T}\right)_P\right)_T dP} \quad \text{— (a)}$$

which implies for a system of given state: (h, P, T, v, s)

the deviation from ideal gas behavior is given by

the above formulla can be used for property estimation of real fluids; provided eqn. of state is known.

more simplication:-

$$v\,dP = d(Pv) - P\,dv \quad \text{— (b)}$$

cyclic relation between $v, T, P \rightarrow \left(\frac{\partial v}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_v \left(\frac{\partial P}{\partial v}\right)_T = -1$

$$\Rightarrow \left(\frac{\partial v}{\partial T}\right)_P = -\left(\frac{\partial P}{\partial T}\right)_v \left(\frac{\partial v}{\partial P}\right)_T$$

$$\Rightarrow \left[\left(\frac{\partial v}{\partial T}\right)_P dP\right]_T = \left[\left(\frac{\partial P}{\partial T}\right)_v dv\right]_T \quad \text{— (c)}$$

from eqn (a), (b) & (c)

$$\left[(h - h_0) = Pv - RT + \int_{v\to\infty}^{v}\left[T\left(\frac{\partial P}{\partial T}\right)_v - P\right]_T dv\right]$$