

Q. 1.

Assignment - 4.

$$\begin{cases} P_0 = 0.4 \text{ MPa}, & T_0 = 30^\circ\text{C} \\ v_0 = 0.00772 \text{ m}^3/\text{kg}, & h_0 = 205.58 \\ P_f = 0.3 \text{ MPa}, & T_f = ? \\ v_f = ? & h_f = ? \end{cases}$$

[as $\dot{Q} = 0$, & gas is going out from tank]

the only possibility is decrease in Temp;

from superheated steam table of Faren

$$T_f < T_0 = 30^\circ$$

from table for any $T < 30^\circ\text{C}$

$$h_f < h_0$$

$$\text{&} \quad v_f > v_0 \Rightarrow \frac{1}{v_f} < \frac{1}{v_0}$$

eqn. of discharging of a tank. (refer solution of Q.2)

$$\begin{aligned} O \rightarrow \oint = (P_0 - P_f) A + V \int \frac{dh}{v_e} & \text{ -ive} \\ \Rightarrow P_0 - P_f = - \int \frac{1}{v_e} dh & \end{aligned}$$

$$\text{L.H.S. } 0.1 \text{ MPa}$$

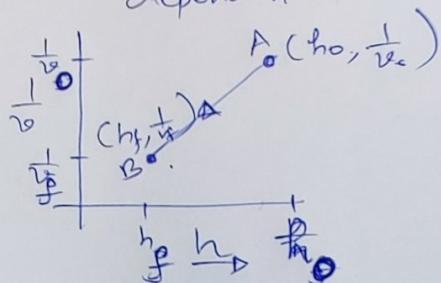
$$\begin{aligned} \text{R.H.S.} &= (-) (-) \text{ area under curve AB} \\ &= (-) (-) (h_0 - h_f) \left(\frac{\frac{1}{v_f} + \frac{1}{v_0}}{2} \right) \end{aligned}$$

Let us guess, $T_f = 20^\circ\text{C}$ at $P = 0.3 \text{ MPa}$

for which $v_f = 0.06273 \text{ m}^3/\text{kg}$ $h_f = 200.64 \text{ kJ/kg}$

$$\text{hence R.H.S.} = 90.86 \text{ kPa} = 0.0908 \text{ MPa}$$

Assumption
we are always in superheated regime
 $\frac{1}{v}$ & h are linearly dependent



LHS $\underline{\underline{\text{vs}}} \text{ RHS}$

one can be more precise

there mass entered in tank

$$m_f - m_o = \frac{(.05/.04797) - (.05/.06273)}{0.25229} = 0.245 \text{ kg}$$

Q2

from energy eqn

$$\frac{dE}{dt} = -\dot{m}_e h_e + \dot{Q}$$

$$\Rightarrow \int d(mu) = \int -\dot{m}_e h_e dt + \int \dot{Q} dt$$

enthalpy of
gas at exit
at time t

h_e = enthalpy of gas inside tank
at instant t

mass balance

$$\left\{ \begin{array}{l} \frac{dM}{dt} = -\dot{m}_e \\ dm = -\dot{m}_e dt \end{array} \right.$$

$$\Rightarrow m_f u_f - m_o u_o = \int h_e (-\dot{m}_e) dt + \dot{Q}$$

$$\Rightarrow m_f u_f - m_o u_o = \int h_e dm + \dot{Q}$$

$$\Rightarrow \dot{Q} = m_f u_f - m_o u_o - \int h_e dm$$

$$\Rightarrow \dot{Q} = m_f u_f - m_o u_o - \int \{ d(h_e m) - m dh_e \}$$

$$\begin{aligned} d(h_e m) &= h_e dm + m dh_e \\ \Rightarrow h_e dm &= d(h_e m) - m dh_e \end{aligned}$$

$$\Rightarrow \dot{Q} = m_f u_f - m_o u_o - \int d(h_e m) + \int m dh_e$$

$$\Rightarrow \dot{Q} = m_f u_f - m_o u_o - (m_f t_f - m_o t_o) + \int m dh_e$$

$$\begin{aligned}
 Q &= m_o(h_o - u_o) + m_f(h_f - u_f) + \int m d\Delta h_e \\
 Q &= m_o(P_o V_o) - m_f P_f V_f + \int m d\Delta h_e \\
 Q &= P_o(m_o V_o) - P_f(m_f V_f) + \int m d\Delta h_e \\
 Q &= P_o V - P_f V + \int \frac{V}{V} d\Delta h_e \\
 Q &= (P_o - P_f)V + V \left(\int \frac{1}{V} d\Delta h_e \right)
 \end{aligned}$$

at $T = 400^\circ C$
 P | \downarrow | $\frac{1}{V}$ | \downarrow | Δh_e
 50 bar
 :
 45 bar
 :
 40 bar
 :
 38 bar
 :
 36 bar
 :
 34 bar
 :
 32 bar
 :
 30 bar
 :
 28 bar
 :
 26 bar
 :
 24 bar
 :
 22 bar
 :
 20 bar

(from superheated steam table)
 +
 numerical integration
 +
 see next page

$Q = (50 - 20)(0.1)(1) + 0.607 \text{ MJ}$
 $Q = 3.607 \text{ MJ}$ Ans

P	V	$\frac{1}{V}$	h	Area under curve	
50 bar	0.057		3198.3	$\int_1^V \left(\frac{1}{V}\right) dh$	
45	0.064		3207.1	$\int_V^{45} \left(\frac{1}{V}\right) dh$	
40	0.073		3215.7	$\int_{40}^4 \left(\frac{1}{V}\right) dh$	
38	0.077		3219.1	$\int_{38}^3 \left(\frac{1}{V}\right) dh$	
36	0.082		3222.5	$\int_{36}^2 \left(\frac{1}{V}\right) dh$	
34	0.087		3225.9	$\int_{34}^1 \left(\frac{1}{V}\right) dh$	
32	0.092		3229.2	$\int_{32}^0 \left(\frac{1}{V}\right) dh$	
30	0.099		3232.5	$\int_0^1 \left(\frac{1}{V}\right) dh$	
28	0.106		3235.8	$\int_1^2 \left(\frac{1}{V}\right) dh$	
26	0.115		3239	$\int_2^3 \left(\frac{1}{V}\right) dh$	
24	0.125		3242.3	$\int_3^4 \left(\frac{1}{V}\right) dh$	
22	0.139		3245.5	$\int_4^5 \left(\frac{1}{V}\right) dh$	
20	0.151		3248.7	$\int_5^6 \left(\frac{1}{V}\right) dh$	

$$\begin{aligned}
 &= \sum_j \left[\left(\frac{1}{V_j} \right)_j + \left(\frac{1}{V} \right)_{j+1} \right] \frac{(h_j - h_{j+1})}{2} \\
 &= \sum_j \left[\left(\frac{1}{V_j} \right)_j + \left(\frac{1}{V} \right)_{j+1} \right] \frac{(h_j - h_{j+1})}{2} \\
 &= 607.525 \left(\frac{kg}{m^3} \frac{kg}{J} \right) \\
 &= 0.607 \text{ MJ}
 \end{aligned}$$

$$\text{Q.3} \quad m_0 = \frac{V}{v} = \frac{1}{v} = \frac{1}{1.694} \text{ kg} = 0.594 \text{ kg}$$

$$h_0 = h_0 - p_0 v_0 = 2506 \text{ kJ/kg} \quad (\text{at } 1 \text{ Ma}, 20^\circ\text{C})$$

~~= 2776.78 kJ/kg~~

tank charging eqn. $h_{\text{supply}} = 2826.8 \text{ kJ/kg}$

$$(m_f - m_0) h_{\text{supply}} = m_f u_f + m_0 u_0 \quad \text{(insulated)} \quad @$$

in the above eqn. we have two unknowns

$\boxed{m_f \quad u_f}$ \rightarrow they are correlated with eqn of state, which is not known to us.

Hence we have to adopt trial method.

we have to start guessing temp & have to check that for which temp. (at 1 Ma) we get L.H.S = R.H.S for

Anyway it is given in the problem that we have to prove that $T_f = 290^\circ\text{C}$

so at 1 Ma, 290°C we have super heated steam.

via interpolation

$$\left\{ \begin{array}{l} v_f = 0.25279 \text{ m}^3/\text{kg} \\ u_f = 3029.57 \text{ kJ/kg} \end{array} \right.$$

hence $u_f = h_f - p_f v_f = 2776.78 \text{ kJ/kg}$

$$\text{LHS} = \left(\frac{1}{0.25279} - 0.59 \right) 2826.8 \text{ kJ} = 9514.8 \text{ kJ}$$

$$\text{RHS} = \left(\frac{1 \times 2776.78}{0.25279} - (0.59) 2506 \right) \text{ kJ} = 9508 \text{ kJ/kg}$$

LHS \approx RHS

one can be more precise. $E = 0$
there mass entered in tank.

$$m_f - m_0 = \frac{1}{0.2522g} - 0.55 = 3.36 \text{ kg}$$

Q4.

we are assuming it constant

~~Assume~~
$$h_e (m_0 - m_f) = Q - (m_f u_f - m_0 u_0)$$

$$T = 120^\circ\text{C}, \quad P_{\text{sat}} = 1.9854 \text{ bar}$$

$$v_f = 0.001060 \text{ m}^3/\text{kg} \quad v_g = 0.8915 \text{ m}^3/\text{kg}$$

$$h_f = 503.72 \text{ kJ/kg} \quad h_g = 2706 \text{ kJ/kg}$$

$$m_{\text{eq}} = \frac{0.05}{0.001060} = 47.14 \text{ kg} \quad m_g = \frac{0.15}{0.8915}$$

$$x_0 (\text{initial quality}) = \frac{0.15}{47.308} = 0.168 \text{ kg}$$

$$v_0 = \frac{m}{m_0} = \frac{0.2}{47.308} = 4.2276 \times 10^{-3} \text{ m}^3 \text{ /kg}$$

$$h_0 = h_g x + h_f (1-x)$$

$$= 511.54 \text{ kJ/kg}$$

$$u_0 = 511.54 \times 10^3 - 1.9854 \times 10^5 \times 4.2276 \times 10^{-3}$$

$$= 510.7 \text{ kJ/kg.}$$

final state 1 bar 100°C

$$v_f = 1.696 \text{ m}^3/\text{kg} \quad h_f = 2676.2 \text{ kJ/kg}$$

$$\text{Assumption } u_f = 2676.2 \times 10^3 - 1 \times 10^5 \times 1.696$$

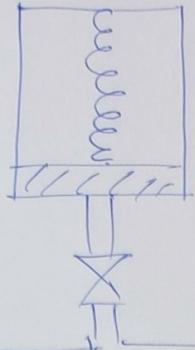
$$= 2506.6 \text{ kJ/kg}$$

$$h_e = \frac{h_0 + h_f}{2} = 2691.1 \text{ kJ/kg}$$

$$m_0 - m_f = 47.19 \text{ kg}$$

$$Q = 150.86 \text{ MJ}$$

Q. 5.



empty tank, $\underline{\underline{m_0 = 0}}$

$$\frac{dm}{dt} = \dot{m}_i$$

$$\int \dot{m} dt = \Delta m$$

$$h_i^o = h_{\text{supply}} - h_{\text{supr}} + P_s$$

we consider → lower part of tank (below piston)
as our $\underline{\underline{CV}}$ (note down CV boundary is moving)

Let's don't care whether we can say it CV in strict sense or not)

first law.

Insulated.

$$\cancel{\delta m_e(h_e) - m_i h_i^o} = \cancel{w_s} - \dot{w}_s - \frac{dE}{dt}$$

Shaft work.

\Rightarrow

$$-\dot{m}_i h_i^o + \dot{w}_s = -\frac{dE}{dt}$$

$$\Rightarrow -\dot{w}_s + \dot{m}_i h_i^o = \frac{dE}{dt}$$

\Rightarrow ~~Energy conservation~~

$$\int \dot{w}_s dt + h_i^o \int \dot{m}_i dt = E_f - E_i$$

$$-\frac{1}{2} k x_f^2 + h_i^o (\Delta m) = m_f u_f - m_0 u_0$$

$$\Rightarrow \boxed{m_f u_f + \frac{1}{2} k x_f^2 = h_i^o m_f}$$

$$\Rightarrow m_f u_f + \frac{P_f + x_f}{2} = h_i^o m_f$$

$$\int \dot{w}_s dt = \frac{1}{2} k x_f^2$$

(x is compression as air & gas is working on spring.)

(x_f is final compression in spring.)

from force balance

$$F = k x_f = P_f A$$

$$\Rightarrow k x_f^2 = P_f A x_f$$

$$\boxed{k x_f^2 = P_f x_f}$$

$$h_i^{\circ} m_f = m_f u_f + P_f \frac{v_f}{2}$$

$$\Rightarrow h_i^{\circ} = u_f + \frac{P_f}{2} \left(\frac{v_f}{m_f} \right)$$

$$\boxed{h_{\text{supply}} = u_f + \frac{P_f v_f}{2}}$$

Now u_f , P_f , v_f are correlated by eqn. of state, which is not known to us.

Hence we have to ~~not~~ guess final T , after such few ^{wise/logical} ~~guesses~~ we find that

$$\text{at } T = 362^{\circ}\text{C}$$

$$h_f = 3163.94 \text{ kJ/kg} \quad v_f = 0.14137 \text{ m}^3/\text{kg}$$

$$u_f = h_f - P_f v_f = 2881.2 \text{ kJ/kg}$$

$$\text{hence } R.H.S = u_f + \frac{P_f v_f}{2} = 3022.57 \text{ kJ/kg}$$

$$\text{Hence } T_f \approx 362^{\circ}\text{C}$$

$$\boxed{\text{LHS} = (3025 \text{ kJ/kg})}$$