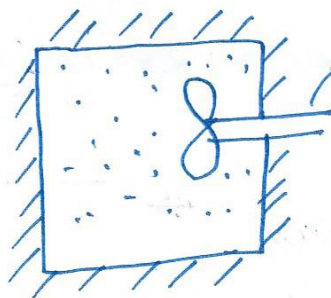


Consider today's class-illustration & try to see it from another perspective, as explained below.

Our main goal is to increase the temp. of ideal gas from T_1 to T_2 ($> T_1$). In other words, the gas will go to more disordered state (by increasing kinetic energy of molecules). The above goal can be achieved in multiple ways, we are going to adopt two such ways, one using paddle wheel (an irreversible method) & another using ^{rev} heat pump (a reversible way).

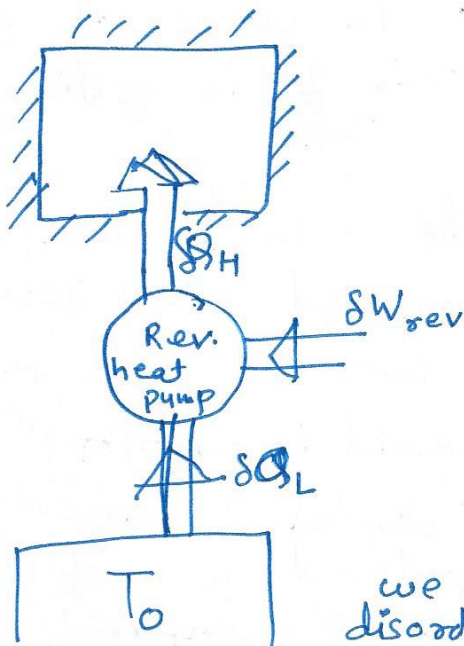
Further the ^{extent of} disorder of gas can be increased by two mode of energy transfer:

(a) either by pure work transfer (paddle wheel)



the blades of wheel (paddles) hit the gas molecules & increase their K.E., leading to higher disorderliness.

or (b) pure heat transfer (with the help of a reversible heat pump,



It is a device which can extract heat (a lower grade/more disorganized form of energy) from low temp. reservoir to high temp object, however this device also needs some work (δW_{rev}). The advantage of this method is that we are taking the gas to more disordered state by providing lower grade energy (heat), Q_H whose some part comes

from low temp. reservoir & remaining part ($\delta Q_H - \delta Q_L = \delta W_{rev}$) comes from external work input to pump.

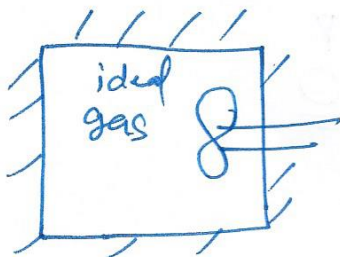
Thus in case of paddle wheel the extent of disorderiness has been increased by using pure work (which is an organized form / high grade energy) whereas in case of heat pump same extent of disorderiness (in gas) has been achieved by transferring heat, which is disorganized form / lower grade energy (Q_H)

In conclusion in case of rev. heat pump, disorderiness in gas is created by two contributions Q_L & δW_{rev} , whereas in case of paddle wheel this whole disorderiness is getting created by only work.

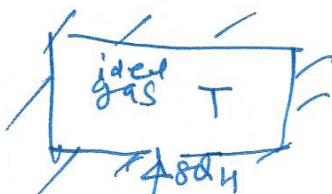
Hence while using paddle wheel more work input is required as compared to reversible heat pump in order to increase gas temp $T_1 \rightarrow T_2$
 → Same statement can be said in another way
 "in case of paddle wheel, larger amount of work is going to disordered form of energy (K.E. of gas molecules). as compared to rev. heat pump. Hence entropy generation (degradation of energy) is more for paddle wheel."

Up to this point we understood that while using paddle-wheel we need to invest larger amount of work which will lead to degradation of high quality energy (work) to ^{increase} disorganized form of ~~energy~~ energy, hence more entropy generation.

Since we are saying entropy generation is manifestation of degradation of quality of energy, Let us define a term "Lost work", & let's try to achieve its mathematical formulation.



$$\begin{aligned} \text{First Law} \quad \delta W_{\text{paddle}} &= dU_{\text{gas}} = C_v dT \\ W_{\text{paddle}} &= \int_{T_1}^{T_2} C_v dT = C_v (T_2 - T_1) \end{aligned}$$



$$|\delta Q_H| = dU_{\text{gas}} \quad \text{--- (9)}$$

$$\text{COP}_{\text{rev heat pump}} = \frac{|\delta Q_H|}{\delta W_{\text{rev}}} = \frac{1}{1 - \frac{T_0}{T}}$$

$$\begin{aligned} \Rightarrow \delta W_{\text{rev}} &= \left(\frac{1}{1 - \frac{T_0}{T}} \right) |\delta Q_H| \xrightarrow{\text{from (9)}} dU_{\text{gas}} \\ &= \left(\frac{1}{1 - \frac{T_0}{T}} \right) dU_{\text{gas}} = \left(1 - \frac{T_0}{T} \right) dU_{\text{gas}} \end{aligned}$$

$$\Rightarrow \delta W_{\text{rev}} = \left(1 - \frac{T_0}{T} \right) C_v dT$$

$$\Rightarrow W_{\text{rev}} = \int_{T_1}^{T_2} \left(1 - \frac{T_0}{T} \right) C_v dT = C_v (T_2 - T_1) - C_v T_0 \ln \frac{T_2}{T_1}$$

So we have

$$\Rightarrow W_{\text{paddle}} = W_{\text{rev heat pump}} + \left(C_v \ln \frac{T_2}{T_1} \right) T_0$$

Now Let us calculate ΔS_{total} for both cases:-

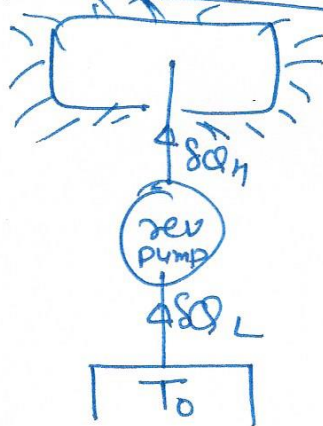
paddle wheel case (Case 1)

$$\Delta S_{\text{total}} = \Delta S_{\text{gas}} + \cancel{\Delta S_{\text{paddle wheel}}}$$

0 (work reservoir)

$$\Rightarrow \boxed{\Delta S_{\text{total}} = \Delta S_{\text{gas}} = C_V \ln \frac{T_2}{T_1}}$$

reversible heat pump case (Case 2)



$$\Delta S_{\text{total}} = \Delta S_{\text{gas}} + \cancel{\Delta S_{\text{rev pump}}} + \Delta S_{\text{cool reservoir}}$$

(via heat transfer) (cyclic device)

$$\Rightarrow \Delta S_{\text{total}} = \Delta S_{\text{gas}} + \Delta S_{\text{cool reservoir}}$$

(via heat transfer)

Since we have reversible heat pump.

we know that for any

$$T > T_0$$

$$\frac{|Q_H|}{T_H} = \frac{|Q_L|}{T_L} \quad \text{--- (a)}$$

$$= \int_{T_1}^{T_2} \frac{\delta Q_H}{T} + \int \left(-\frac{\delta Q_L}{T_0} \right)$$

from eqn (a)

$$= \int_{T_1}^{T_2} \frac{\delta Q_H}{T} + \int (-) \frac{\delta Q_H}{T}$$

$$= \int_{T_1}^{T_2} \left(\frac{\delta Q_H}{T} - \frac{\delta Q_H}{T} \right) = 0$$

which implies $\Delta S_{\text{total}} = 0$

Therefore ΔS_{total} in case 2 = 0,

hence whole process is reversible (both externally & internally)

★ $\Delta S_{\text{total}} = 0$ is the only criterion for a fully reversible process.

$\Delta S_{\text{total}} > 0$ is the only criteria for process to be irreversible.

Now what we notice is that

we already have
$$W_{\text{paddle wheel}} = W_{\text{rev (heat pump)}} + T_0 \left(C_v \ln \frac{T_2}{T_1} \right)$$

$$W_{\text{paddle wheel}} = W_{\text{rev (heat pump)}} + T_0 \left(\Delta S_{\text{total (paddle wheel)}} \right)$$

it means ^{net} due to entropy generation in case of paddle-wheel we need to put more work. Or otherwise, in case of paddle wheel, there is net entropy generation, which leads to degradation of quality of energy

by amount $T_0 (\Delta S_{\text{total}})$

↳ this term is known as Lost Work

The concept of Lost Work will be more clear when we will solve the examples where useful work will be an output by the system or device rather than a input

Nevertheless \star Lost work is always $T_0 \Delta S_{\text{total (univ)}}$

\star (assignment 6 will make Lost work more clear)

formula sheet for solving assign=6

closed system

$$dS_{\text{system}} = \frac{\delta Q_{\text{rev}}}{T}, \quad T dS = dU + \delta W_{\text{rev}} + \delta Q_{\text{rev}}$$

$$dS_{\text{sys}} > \frac{\delta Q_{\text{irr}}}{T}$$

or simply $dS_{\text{sys}} > \frac{\delta Q}{T}$ if process is irreversible involving δQ heat transfer

hence $dS_{\text{sys}} = \frac{\delta Q}{T} + S_G$ (entropy generation)

flow process:

$$\frac{dS_{\text{cv}}}{dt} = \frac{\delta \dot{Q}}{T} + \dot{S}_G + \dot{m}_i s_i - \dot{m}_e s_e$$

for steady state: $\frac{dS_{\text{cv}}}{dt} = 0,$

$$\Rightarrow \dot{S}_G = -\frac{\delta \dot{Q}}{T} - \dot{m}_i s_i + \dot{m}_e s_e$$

A device is feasible if it satisfies first law of thermodynamics as well as second law

$S_G > 0$ for closed system

$\dot{S}_G > 0$ for flow processes

environment temp or cold reservoir temp

$$\text{Lost Work} = (T_0 \Delta S_{\text{total}})_{\text{total (sys + surroundings)}}$$

$\Delta S_{\text{total}} = 0$ process reversible

$\Delta S_{\text{total}} > 0$ " irreversible.