Assignment I

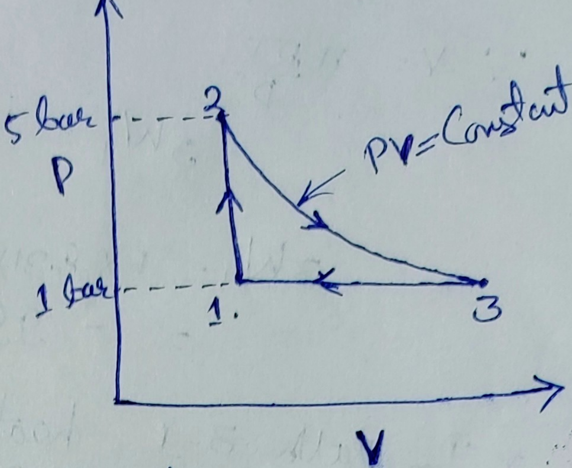
Q1)

Solⁿ:- Basis : 1 mol Ideal gas.

Given: $\gamma = 1.4$

$P_1 = 1 \text{ bar}$, $T_1 = 300K$

$P_2 = 5 \text{ bar}$ $T_2 = ?$



For Isochoric Process (1-2)

$$\boxed{\frac{P_1}{T_1} = \frac{P_2}{T_2}} \qquad T_2 = \frac{5 \text{ bar}}{1 \text{ bar}} \times 300 K$$

$$T_2 = 1500K$$

For Path 1-2, V = Constant so, $\boxed{{}_1W_2 = 0}$

hence ${}_1Q_2 = \Delta U = nC_v \Delta T$

$$\left\{ \because C_v = \frac{R}{\gamma - 1} \quad \therefore C_v = \frac{8.314}{1.4 - 1} = 20.785 \text{ J/mol K} \right.$$

$${}_1Q_2 = 1 \times 20.785 (1500 - 300)$$
$$= 24942 \text{ J}$$

For Path 2-3, T = Constant i.e $\Delta U = 0$

$$\& \ {}_2W_3 = PdV = nRT \ln\left[\frac{V_3}{V_2}\right]$$

$$\text{or } {}_2W_3 = nRT \ln\left[\frac{P_2}{P_3}\right]$$

$${}_2W_3 = 1 \times 8.314 \times 1500 \ln\left(\frac{5}{1}\right)$$
$$= 20,065.8 \text{ J}$$

$${}_2Q_3 = {}_2W_3 = 20065.8 \text{ J} \quad (\because \Delta U = 0)$$

For Path 3-1 (Isobaric)

$$_3W_1 = P\Delta V = 10^5\,Pa \times (V_1 - V_3)$$

$\because V = n\frac{RT}{P}$

$$_3W_1 = \cancel{10^5\,Pa}\left[\frac{nR}{\cancel{10^5\,Pa}}(T_1 - T_3)\right]$$

$$_3W_1 = 1 \times 8.314\,(300 - 1500)$$
$$= -9976.8\ J$$

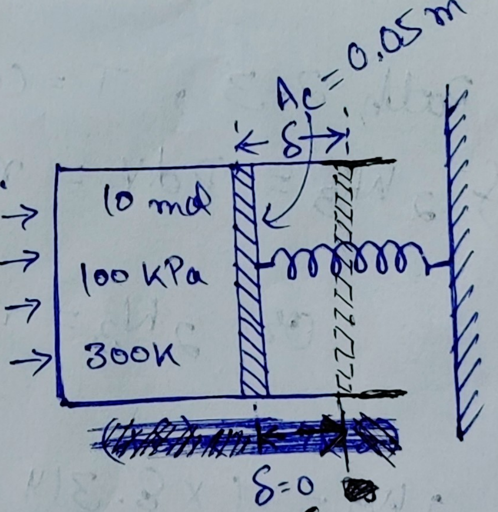
In Path 3-1 heat is released instead of absorbed

So. $$\eta = \frac{\cancelto{0}{_1W_2} + {_2W_3} + {_3W_1}}{_1Q_2 + {_2Q_3}}$$

$$= \frac{20{,}065.8 + (-9976.8)}{24942 + 20065.8}$$

$$\eta = 0.224$$

Q2)
Solution:



Given: No of moles of ideal gas = 10 mol.

$\gamma = 1.4$

Spring Constant $(K) = 50\left(\frac{KN}{m}\right)$

Piston area $A_c = 0.05\ m^2$.

Initial Pressure or Ambient Pressure $P_A = 100\ KPa.$, $T_I = 300K.$

Final Pressure $P_f = 500\ KPa.$

$$P_f = P_A + \frac{K\delta}{A_c}$$

$$\Rightarrow 500\ KPa = 100\ KPa + \frac{50\left(\frac{KN}{m}\right) \cdot \delta}{0.05\ m^2}$$

$$\Rightarrow \delta = 0.4\ m$$

$$V_I = \frac{10 \cdot 8.314 \times 300}{10^5} = 0.2494\ m^3$$

$\therefore$ Volume Increased $= A_c \times \delta$

$$= (0.05 \times 0.4)\ m^3$$

$$= 0.02\ m^3$$

$$V_f = (0.2494 + 0.02) = 0.2694\ m^3$$

(a) By $\boxed{PV = nRT}$

$$T_f = \frac{P_f \cdot V_F}{nR}$$

$$= \frac{5 \times 10^5\ Pa \times 0.2694}{10 \times 8.314} = \cancel{1617.7\ K}\ 1620.1\ K$$

$$\boxed{T_F \simeq 1620\ K}$$

$$\text{Workdone} = \frac{1}{2}(P_I + P_F)(V_F - V_I)$$

$$= \frac{1}{2}(500 + 100) \times 0.02 = 6\ KJ$$

(b) Energy transfered as heat 'Q'

$$Q = \Delta U + W \qquad \{\Delta U = nC_v \Delta T\}$$

$$= \{10 \times 20.875 \times (1618 - 300) + 6000\}\ J$$

$$= \cancel{275132}\ (6000 + 275583)\ J$$

$$= 281.13\ KJ$$

Q3)

Solu^n:

Given: No of moles of ideal gas = 10 moles.

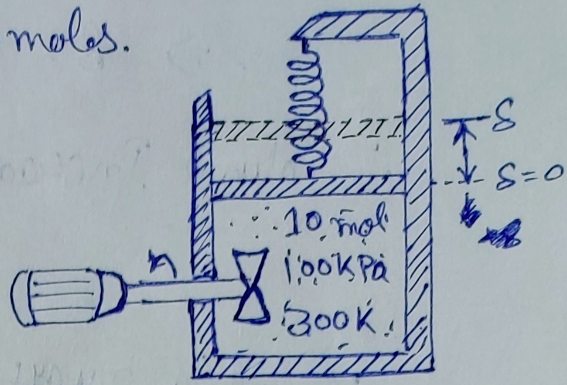
$\gamma = 1.4$

Spring Constant $(K) = 50 \left(\frac{kN}{m}\right)$

Piston area $A_c = 0.05\ m^2$

$P_A = 100\ KPa$, $T_I = 300\ k$.

$P_f = 500\ kPa$. $V_I = 0.2494\ m^3$ (from the previous problem)



$$P_f = P_A + \frac{K\delta}{A_c}$$

$$500 = 100 + \frac{50 \times \delta}{0.05}$$

$$\therefore \delta = 0.4m.$$

ⓐ Similar to the Problem (2)

$$V_f = (0.2494 + 0.02)$$
$$= 0.2694\ m^3.$$

ⓑ Work done by the gas $W_g = \frac{1}{2}(P_A + P_f)(V_f - V_I)$

$$= \frac{1}{2}(500+100) \times 0.02.$$

$$= 6\ KJ$$

ⓒ For the final temp^r of gas.

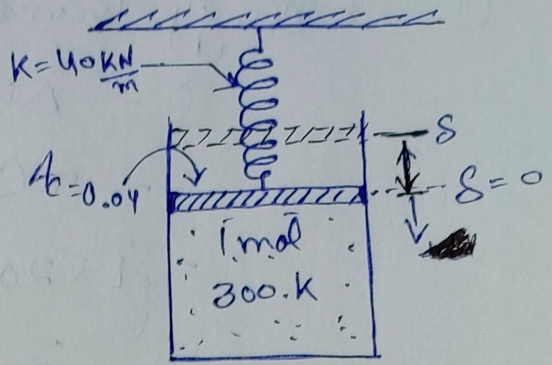
$$T_f = \frac{P_f V_f}{nR}$$

$$= \frac{5 \times 10^5\ Pa \times 0.2694}{10 \times 8.314}$$

$$\boxed{T_f = 1620\ K}$$

Q 4)

Solu^n :-



Given: No of moles of ideal Gas = 1 mol.

$\gamma = 1.4$

Spring Constant 'K' = $40 \left(\frac{KN}{m}\right)$

Piston area $A_C = 0.04\ m^2$.

Ambient Pressure $(P_A) = 100\ KPa$. , $T_I = 300K$, $V_I = \frac{nRT_I}{P_I} = \frac{8.314 \times 300}{10^5}$

P_f = ~~800~~ 400 KPa

$\boxed{V_I = 0.0249\ m^3}$

$\because\ P_f = P_A + \frac{K\delta}{A_c}$

$$400\ KPa = 100\ KPa + \frac{40 \frac{KN}{m} \times \delta}{0.04\ m^2}.$$

$\delta = 0.3\ m$

Volume Increase $= A_c \times \delta$

$= 0.04 \times 0.3 = 0.012\ m^3$

$V_f = (0.0249 + 0.012)\ m^3$

$= 0.0369\ m^3$

(a) Since the working fluid is an ideal gas.

it follows $PV = nRT$

$\therefore\ T_f = \frac{P_f V_f}{nR} = \frac{4 \times 10^5\ Pa \times 0.0369}{1 \times 8.314}$

$T_f = 1775.3\ K$

(b) Workdone by the gas $(W) = \frac{1}{2}(P_A + P_f)(V_f - V_I)$

$= \frac{1}{2}(400 + 100) \times 0.012$

$\boxed{W = 3\ KJ}$

(c) Energy Transferred as heat 'Q'

$$Q = \Delta U + W$$

$$= nC_v\Delta T + W$$

$$= \{1 \times 20.875(1775.3 - 300) + 3000\} J$$

$$= (30664.11 + 3000) J$$

$$\boxed{Q = 33664\ J \simeq 33.664\ KJ}$$