Graph Theory

Trival Graph :

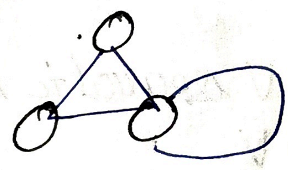
only one vertex & no edge.

Null Graphs :

number of vertices and no edges.

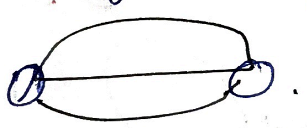
Self loop :

Edges having same vertex as both its end vertices.



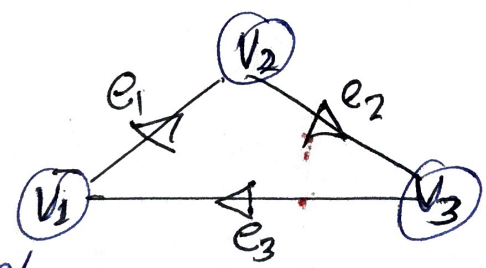
multiedge :

Two or more edges Having same end point.



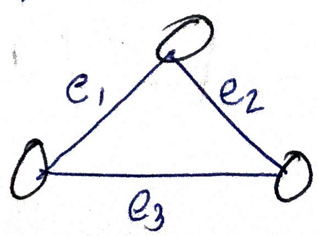
Directed Graph :

Direction of edges in the graphs.



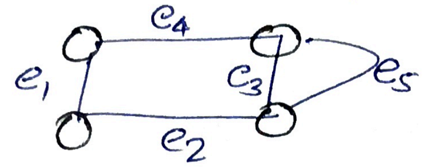
Simple graph

Graph does not have any self loop & multiedge



Multi Graph :

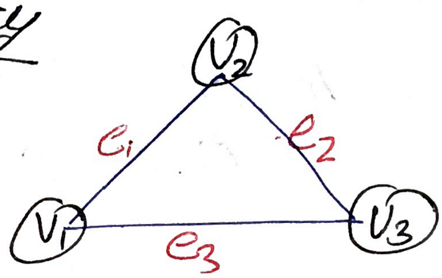
A Graph having multiple edge.



Incidence and Adjacency

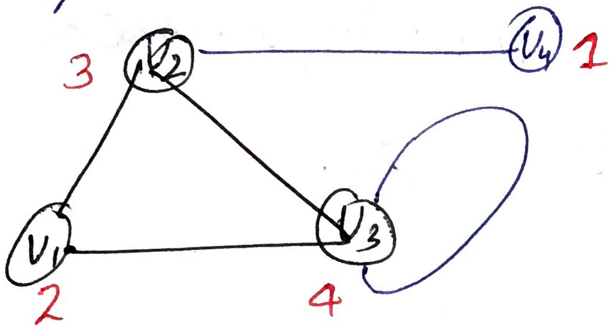
e_1 = Incidence.

V_1 & V_2 = Adjacent

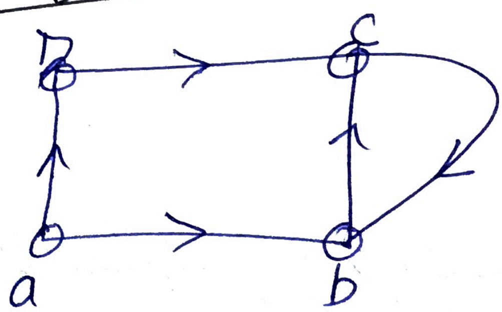


Degree of Vertex

- Degree of vertex V is equal to NO. of edges which are incident on V.
- ★ Self loop counted twice.



Indegree & outdegree



Vertices	in	out
a	0	2
b	2	1
c	2	1
d	1	1

Isolated Vertex :-

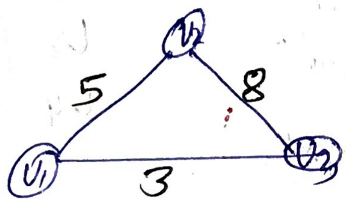
A Vertex = Degree 0

Pendax Vertex :-

A Vertex = Degree 1

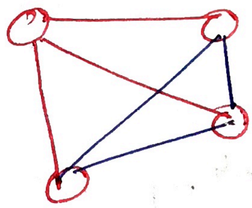
Weighted graph :-

A Graph G in which some weight is Assigned to every edge of G.



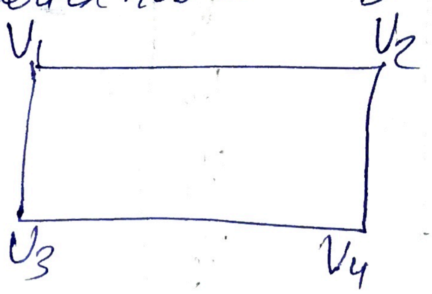
Complete Graph

A graph in which each vertex connected to every other vertex.



Regular Graph

every vertex has same degree.



Order & Size of Graph

No. of vertices in finite graph = order.
No. of edges = size

If Graph $G(P, q)$ $\quad P = V, \; q = E$

Adjacency Matrix

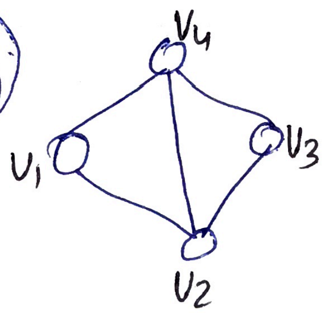
$$\begin{array}{c|cccc} & v_1 & v_2 & v_3 & v_4 \\ \hline v_1 & & & & \\ v_2 & & & & \\ v_3 & & & & \\ v_4 & & & & \end{array}$$

directed graph mein aane wale par dhyan dena hai vertex

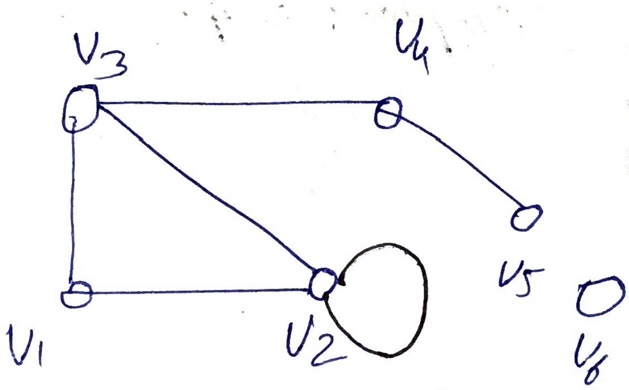
Incidence Matrix

$$\begin{array}{c|cccc} & e_1 & e_2 & e_3 & e_4 \\ \hline v_1 & & & & \\ v_2 & & & & \\ v_3 & & & & \\ v_4 & & & & \end{array}$$

eg (1)



$$\begin{array}{c|cccc} & v_1 & v_2 & v_3 & v_4 \\ \hline v_1 & 0 & 1 & 0 & 1 \\ v_2 & 1 & 0 & 1 & 1 \\ v_3 & 0 & 1 & 0 & 1 \\ v_4 & 1 & 1 & 1 & 0 \end{array}$$



$$\begin{array}{c|cccccc} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \hline v_1 & 0 & 1 & 1 & 0 & 0 & 0 \\ v_2 & 1 & 1 & 1 & 0 & 0 & 0 \\ v_3 & 1 & 1 & 0 & 1 & 0 & 0 \\ v_4 & 0 & 0 & 1 & 0 & 1 & 0 \\ v_5 & 0 & 0 & 0 & 1 & 0 & 0 \\ v_6 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

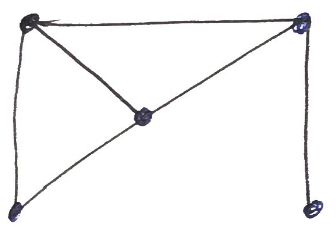
Isomorphism of Graph

Two graph G_1 & G_2 are isomorphic iff.

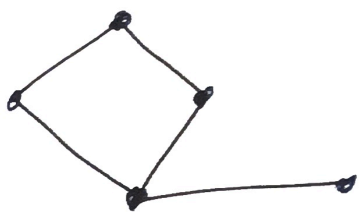
1. No. of vertices. (Same)
2. No. of edges (Same)
3. Degree seq^n
4. one to one
(vertex correspondance & edge correspondance)

eg.①

Not Isomorphic



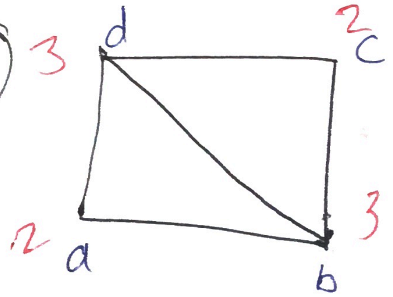
G_1

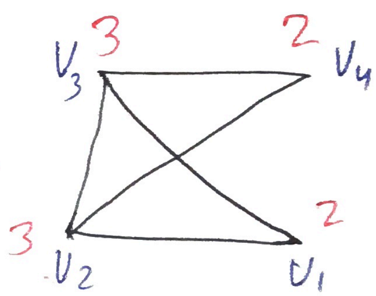


G_2

1	v	5	v 5
2	e	6	e 5

eg.②





1	4	4	✓
2	5	5	✓
3	3,3,2,2	3,3,2 2	✓

$a \rightarrow v_1$
$b \rightarrow v_2$
$c \rightarrow v_4$
$d \rightarrow v_3$

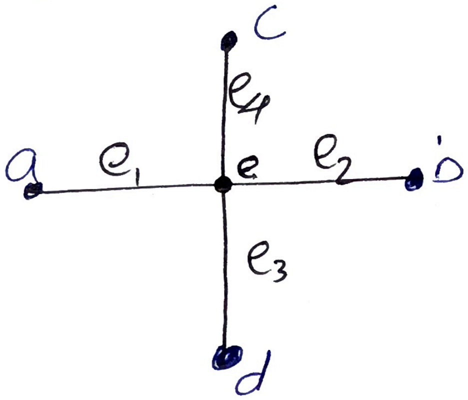
Eulerian Graph (only edge see)

Eulerian Path
- 0 or 2 vertices of odd degree
- every edges of graph appears exactly once in the path

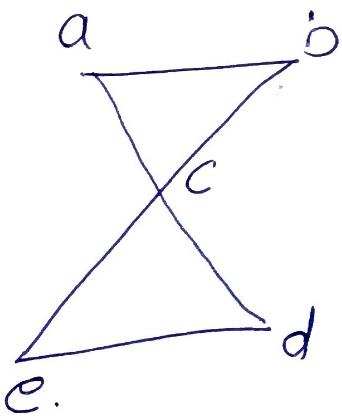
eulerian circuit
- each vertices must be even degree
- The circuit which contains every edge of the graph exactly once.

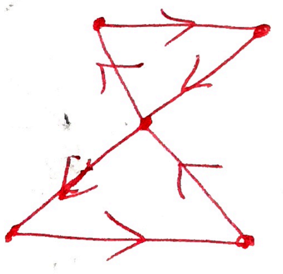
eulerian graph : A graph which has an Eulerian circuit

eg:



ae_1, be_2 X





Hamiltonian Graph (isme ~~edge~~ vertex dekhte hai)

Hamiltonian path

every vertex of Graph appears exactly once in the path.

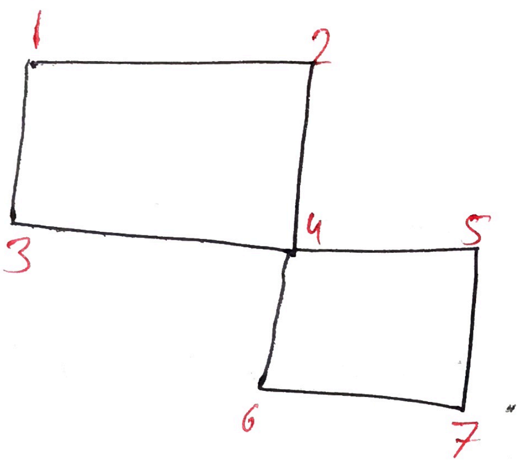
Hamiltonian circuit

The circuit which contains every vertex of the graph exactly once.

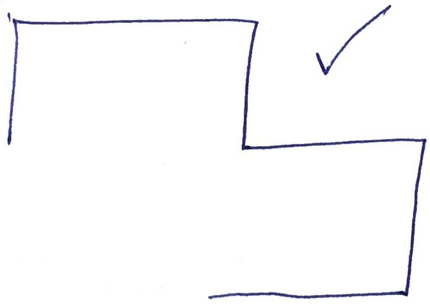
Hamiltonian Graph

A graph which has an Hamiltonian graph

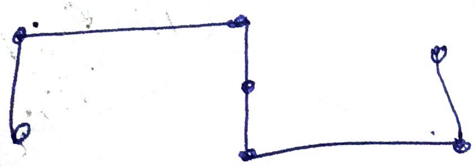
eg:



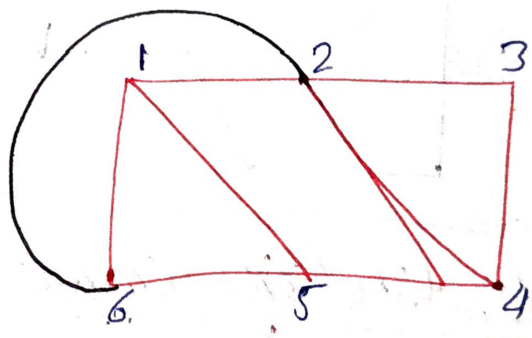
HP



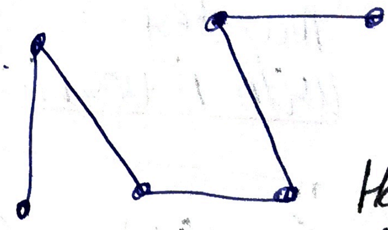
✓



HC ✗



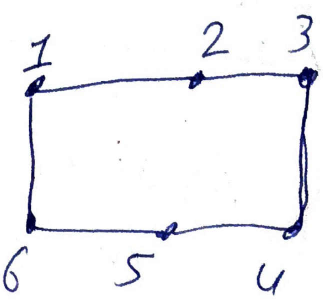
① HP.



✓

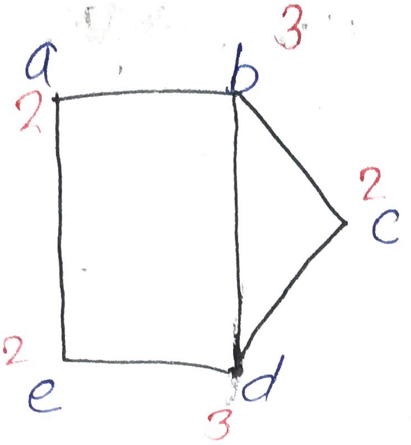
Ham. Graph ✓

② HC



✓

eg:

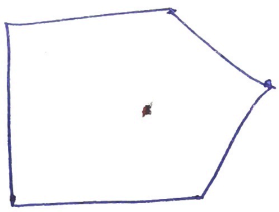


eulerian circuit
&
Hamiton circuit.

① eulerian circuit:

degree of all the edges have even degree but there is b & d edge have odd degree ∴ ✗ eluerian circuit

② Hamilton circuit



✓

⇒ Handshaking lemma

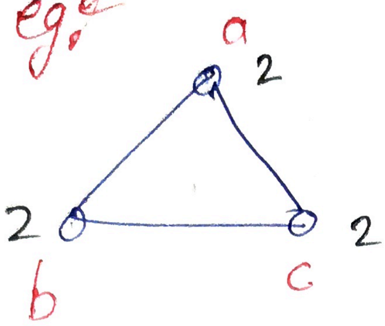
Max degree of any vertex in a simple graph with n vertices is $(n-1)$

$$\boxed{\sum_{i=1}^{n} d(V_i) = 2e}$$

Maximum no of edges in simple graph $= \dfrac{n(n-1)}{2}$

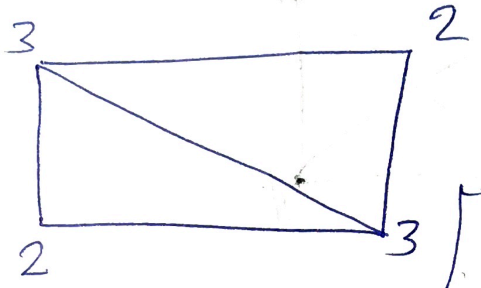
n = vertix no.

eg:



$= \quad 3 \times 2 = 6$

$\boxed{\sum \text{degree} = 2+2+2 = 6 = 3 \times 2 = 6}$

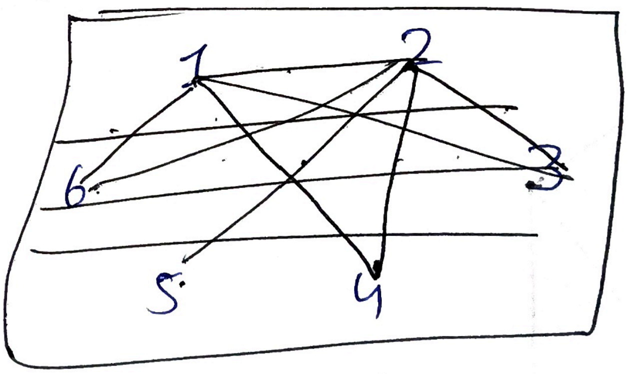


$2 \times 2 + 3 \times 2 = 4 + 6 = 10$

$\boxed{\sum \text{degree} = 5 \times 2}$

Que no. of edges = ? 6 nodes, 2 of degree 4 and 4 of degree 2, Draw.

$V_1 = 4$
$V_2 = 4$
$\left.\begin{matrix} V_3 \\ V_4 \\ V_5 \\ V_6 \end{matrix}\right\} = 2$



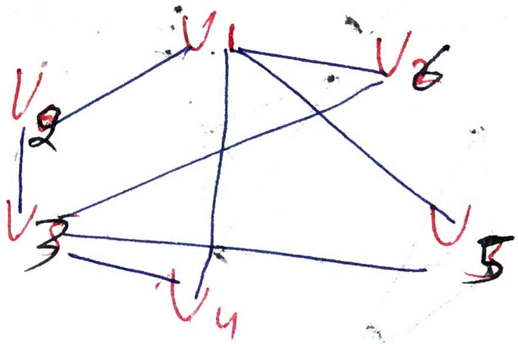
$$\sum_{i=1} d(V_i) = 2 \times e$$

$V_1 + V_2 + V_3 + V_4 + V_5 + V_6 = 2 \times e$

$4 + 4 + 2 + 2 + 2 + 2 = 2 \times e$

$16 = 2e$

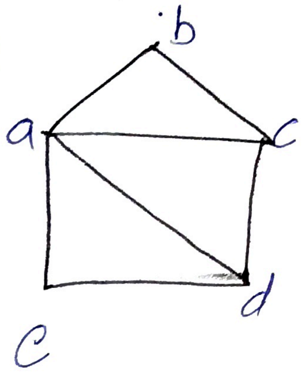
$\boxed{e = 8}$



Palanar Graph :-

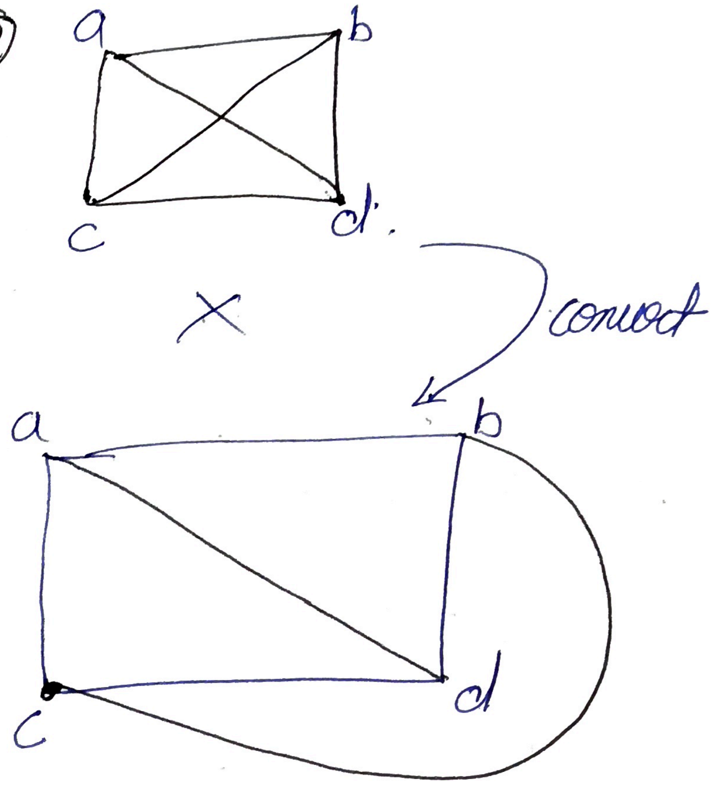
A graph is said to be planar if it can be drawn on a plane is such a way that no edges cross one another.

(1)

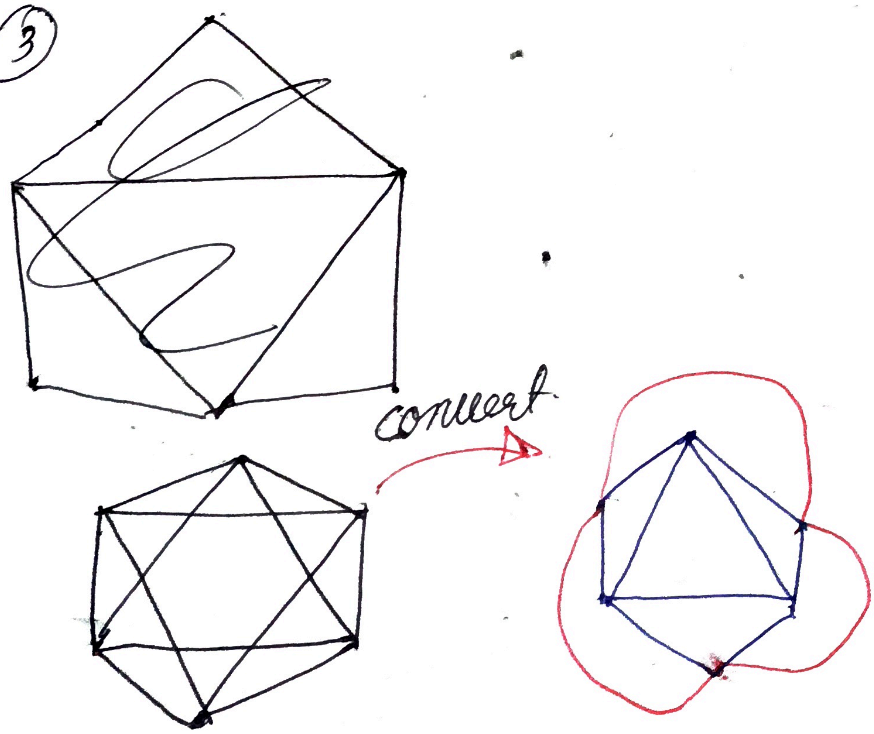


✓

(2)



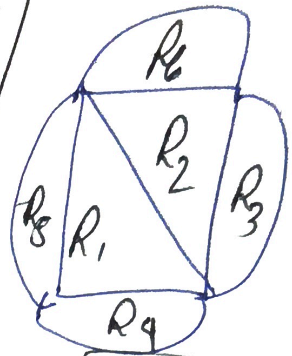
(3)



Euler's formula

$\boxed{V - E + R = 2}$

V = Vertix
E = edge.
R = Region



R=6.
V=4
E=9

eg:- 6=V , 12=e R=?

$6 - 12 + R = 2$

$-6 + R = 2$

$\boxed{R = 8}$

⇒ Dijkstra's algorithm

→ Shortest Path b/w 2 Point of vertix

⇒ Nearest Neighbor ~~math~~ method

→ Sabee choti value use karni hai har vertex par jakar. and har bar graph alag alag banana hai, tab tak Jab tak sabhi vertex cover Naa ho jaye