Name: PRAGYA AGARWAL

STUDENT ID: 1001861779

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ASSIGNMENT-04

Ques. 1 Suppose that we have a sample of 100 people and the random variable x_i is the height of the i^{th} person (in centimeters). Knowing that all x_i's are identically distributed and $Ex_i = \mu = 160$ and $\sigma x_i = \sigma = 25$, find the probability that the total height of these people is smaller than 17,000 cm.

Ans. 1 Given that,

$n = 100$

$\mu = Ex_i = 160$

$\sigma = \sigma_{x_i} = 25$

we know Central limit theorem (C.L.T.),

$$P(Y<y) = P\left(\frac{Y-n\mu}{\sqrt{n}\,\sigma} < \frac{y-n\mu}{\sqrt{n}\,\sigma}\right)$$

$$P(Y<17000) = P\left(\frac{Y-n\mu}{\sqrt{n}\,\sigma} < \frac{17000-(100\times160)}{\sqrt{100}\times 25}\right)$$

$$= P\left(\frac{Y-n\mu}{\sqrt{n}\,\sigma} < \frac{17000-16000}{10\times25}\right)$$

$$= P\left(\frac{Y-n\mu}{\sqrt{n}\,\sigma} < \frac{1000}{250}\right)$$

$$= P\left(\frac{Y-n\mu}{\sqrt{n}\,\sigma} < 4\right)$$

$$P(Y<17000) \approx \Phi(4)$$

$P(Y < 17000) = 0.99997$ (from Z table)

$$\boxed{P(Y<17000) = 1.0000}$$

Ques.2

Assuming that our random variable Y belongs to binomial distribution and $Y \sim$ Binomial $(n=36, p=1/3)$. find the probability that $Y>14$.

Ans.2

Given,

$n = 36$

$p = \frac{1}{3}$

We know that a Binomial $(n=36, p=\frac{1}{3})$ can be written as the sum of n events

Bernoulli (p) random variables:

$$Y = x_1 + x_2 + \dots + x_n$$

Since, $x_i \sim$ Bernoulli $(p=\frac{1}{3})$, we have

$$Ex_i = \mu = p = \frac{1}{3} \quad , \quad var(x_i) = \sigma^2 = p(1-p) = \frac{1}{3}\left(1-\frac{1}{3}\right)$$

$$= \frac{1}{3} \times \frac{2}{3}$$

$$var(x_i) = \sigma^2 = p(1-p) = \frac{2}{9}$$

Thus, Applying central limit theorem -

$$P(Y>y) = P\left(\frac{Y-n\mu}{\sqrt{n}\,\sigma} > \frac{y-n\mu}{\sqrt{n}\,\sigma}\right)$$

$$P(Y>14) = P\left(\frac{Y-n\mu}{\sqrt{n}\,\sigma} > \frac{14-(36\times\frac{1}{3})}{\sqrt{36}\times\sqrt{\frac{2}{9}}}\right)$$

$$= P\left(\frac{Y-n\mu}{\sqrt{n}\,\sigma} > \frac{14-12}{6\times\frac{\sqrt{2}}{3}}\right)$$

$$= P\left(\frac{Y-n\mu}{\sqrt{n}\,\sigma} > \frac{2}{2\times\sqrt{2}}\right)$$

$$P(Y>14) = P\left(\frac{Y-n\mu}{\sqrt{n}\,\sigma} > \frac{1}{\sqrt{2}}\right)$$

$$P(Y>14) \approx 1-\Phi\left(\frac{1}{\sqrt{2}}\right)$$

$$\approx 1-\Phi(0.71)$$

$$= 1-0.76115$$ (using z table)

$$P(Y>14) = 0.23885$$

$$\boxed{P(Y>14) = 0.2389}$$

Ques.3 The random variable x_i is the waiting time (in seconds) for the elevator to arrive once the i^{th} person pushes the button and $Ex_i = \mu = 20$ and $\sigma x_i = \sigma = 8$. Find the probability that the waiting time for 50 people is between 900 & 1100 seconds $(P(900 \le Y \le 1100))$?

Ans. 3

Given,

$n = 50$

$\mu = E x_i = 20$

$\sigma = \sigma_{x_i} = 8$

using central limit theoram,

$$P(y_1 \le Y \le y_2) = P\left(\frac{y_1 - n\mu}{\sqrt{n}\,\sigma} \le \frac{Y - n\mu}{\sqrt{n}\,\sigma} \le \frac{y_2 - n\mu}{\sqrt{n}\,\sigma}\right)$$

$$P(900 \le Y \le 1100) = P\left(\frac{900 - (50 \times 20)}{\sqrt{50} \times 8} \le \frac{Y - n\mu}{\sqrt{n}\,\sigma} \le \frac{1100 - (50 \times 20)}{\sqrt{50} \times 8}\right)$$

$$= P\left(\frac{900 - 1000}{56.5685} \le \frac{Y - n\mu}{\sqrt{n}\,\sigma} \le \frac{1100 - 1000}{56.5685}\right)$$

$$= P\left(\frac{-100}{56.5685} \le \frac{Y - n\mu}{\sqrt{n}\,\sigma} \le \frac{100}{56.5685}\right)$$

$$\approx \Phi(1.7678) - \Phi(-1.7678)$$

$$= \left[\frac{0.96080 + 0.96164}{2}\right] - \left[\frac{0.03920 + 0.03836}{2}\right]$$

$$= \frac{1.92244}{2} - \frac{0.07756}{2}$$

$$= 0.96122 - 0.03878$$

$$P(900 \le Y \le 1100) = 0.92244$$

$$\boxed{P(900 \le Y \le 1100) = 0.9224}$$

Ques.4 Suppose that the weight of people in a specific town are normally distributed with the mean of 140 pounds and standard deviation of 20 pounds. Answer the following questions:

A. Find the percentage of the people who weight less than 130 pounds.
B. Find the percentage of people who weight more than 160 pounds.

Ans.4 Given,

$\mu = 140$

$\sigma = 20$

we can write $X \sim N(140, 20)$

A) using central limit theoram (C.L.T.),

$$P(Y < y) = P\left(\frac{Y - n\mu}{\sqrt{n}\,\sigma} < \frac{y - n\mu}{\sqrt{n}\,\sigma}\right)$$

$$P(Y < 130) = P\left(\frac{Y - n\mu}{\sqrt{n}\,\sigma} < \frac{130 - 140}{20}\right)$$

$$= P\left(\frac{Y - \mu}{\sigma} \le \frac{-1}{2}\right)$$

$$= \Phi(-0.5)$$

$$P(Y < 130) = 0.30854$$

$$\boxed{P(Y < 130) = 30.8540\,\%}$$

(B) find the percentage of people who weight more than 160 pounds.

(B) Using central limit theorem (C.L.T.),

$$P(Y>160) = P\left(\frac{Y-n\mu}{\sqrt{n}\,\sigma} > \frac{y-n\mu}{\sqrt{n}\,\sigma}\right)$$

$$= P\left(\frac{Y-n\mu}{\sqrt{n}\,\sigma} > \frac{160-140}{20}\right)$$

$$= P\left(\frac{Y-\mu}{\sigma} > \frac{20}{20}\right)$$

$$\approx 1 - \Phi(1)$$

$$= 1 - 0.84134$$

$$P(Y>160) = 0.15866$$

$$\boxed{P(Y>160) = 15.8660\%}$$