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ASSIGNMENT-01

Ques.1 Suppose we have a fair coin and we toss it 10 times?

(a) what is the probability of getting exactly 7 heads and 3 tails?

(a) In a coin toss, there are two outcomes : heads and tails.

Since, the coin is fair, the probability of getting a head

$P(\text{Head}) = \frac{1}{2}$ or 0.5

and probability of getting a tail, $P(\text{Tail}) = \frac{1}{2}$ or 0.5.

using binomial distribution formula :

$$P_x = \binom{n}{x} p^x q^{n-x}$$

where

P = Binomial Probability

x = total number of times for a specific outcome within n trials.

$\binom{n}{x} = \frac{n!}{x!\,(n-x)!}$ is the number of ways to obtain x successes in n trials.

p = probability of success on a single trial

q = probability of failure on a single trial

n = number of trials.

P(Getting head) = P(Getting Tail) = $\frac{1}{2}$ or 0.5

Substituting values in binomial distribution formula,

$$P(\text{Getting tail}) = \frac{1}{2}$$

$$q = 1-p = 1-\frac{1}{2} = \frac{1}{2}$$

$$n = 10$$

$$x = 3$$

$$n-x = 7$$

$$P(3, 10, \tfrac{1}{2}) = \binom{10}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^7$$

$$= \frac{10!}{3!\,7!} \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^7 \qquad (n! = n\times(n-1)\times\ldots 1)$$

$$= \frac{10\times \cancel{9}^{3}\times \cancel{8}^{4}\times\cancel{7}\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}}{(\cancel{3}\times\cancel{2}\times 1)\times(\cancel{7}\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1})} \times \left(\frac{1}{2}\right)^{10}$$

(Cancelling common terms)

$$= (10\times 3\times 4) \times \frac{1}{2^{10}}$$

$$= \frac{\cancel{120}\ \cancel{60}\ \cancel{30}\ 15}{\cancel{1024}\ \cancel{512}\ \cancel{256}\ 128}$$

(Continuos division until no common factor is left)

$$\boxed{P(\text{Getting exactly 7 heads and 3 tails}) = \frac{15}{128}}$$

OR

$$\boxed{P(\text{Getting exactly 7 heads and 3 tails}) = 0.1172}$$

(b) What is the Probability of getting at least 4 head?

(b) In a coin toss, there are two outcomes: heads and tails.

Since the coin is fair,

The Probability of getting a head, $P(Head) = \frac{1}{2}$ or 0.5

The probability of getting a tail, $P(Tail) = \frac{1}{2}$ or 0.5

$P(Head) = P(Tail) = \frac{1}{2}$ or 0.5

using binomial distribution formula:

$$P_x = \binom{n}{x} p^x q^{n-x}$$

where,

P = Binomial probability

x = total number of times for a specific outcome within n trials.

$\binom{n}{x} = \frac{n!}{x!(n-x)!}$ is the number of ways to obtain x successes in n trials.

p = Probability of success on a single trial

q = Probability of failure on a single trial

n = number of trials.

$P(\text{Getting at least 4 head}) = P(x \geq 4)$

$P(x \geq 4) = P(\text{Getting 4 head}) + P(\text{Getting 5 head}) + P(\text{Getting 6 head}) + P(\text{Getting 7 head}) + P(\text{Getting 8 head}) + P(\text{Getting 9 head}) + P(\text{Getting 10 head})$

Substituting values in binomial distribution formula,

P(Getting 4 head):

$P(\text{Getting head}) = \frac{1}{2}$

$q = 1 - \frac{1}{2} = \frac{1}{2}$

$n = 10$

$x = 4$

$n - x = 10 - 4 = 6$

$$P(\text{Getting 4 Head}) = P(4, 10, \tfrac{1}{2}) = P(4H) = \binom{10}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^6 \quad \text{—} (1)$$

P(Getting 5 head):

$P(\text{Getting head}) = \frac{1}{2}$

$q = 1 - \frac{1}{2} = \frac{1}{2}$

$n = 10$

$x = 5$

$n - x = 10 - 5 = 5$

$$P(\text{Getting 5 head}) = P(5H) = P(5, 10, \tfrac{1}{2}) = \binom{10}{5}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^5 \quad \text{—} (2)$$

P(Getting 6 Head):

P(Getting Head) $= \frac{1}{2}$

$q = 1 - \frac{1}{2} = \frac{1}{2}$

$n = 10$

$x = 6$

$n - x = 10 - 6 = 4$

P(Getting 6 Head) $= P(6H) = P(6, 10, \frac{1}{2}) = \binom{10}{6} \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^4$ — (3)

P(Getting 7 Head):

P(Getting Head) $= \frac{1}{2}$

$q = 1 - \frac{1}{2} = \frac{1}{2}$

$n = 10$

$x = 7$

$n - x = 10 - 7 = 3$

P(Getting 7 Head) $= P(7H) = P(7, 10, \frac{1}{2}) = \binom{10}{7} \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^3$ — (4)

P(Getting 8 Head):

P(Getting Head) $= \frac{1}{2}$

$q = 1 - \frac{1}{2} = \frac{1}{2}$

$n = 10$

$x = 8$

$n - x = 10 - 8 = 2$

P(Getting 8 Head) $= P(8H) = P(8, 10, \frac{1}{2}) = \binom{10}{8} \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2$ — (5)

P(Getting 9 Head):

P(Getting Head) $= \frac{1}{2}$

$q = 1 - \frac{1}{2} = \frac{1}{2}$

$n = 10$

$x = 9$

$n - x = 10 - 9 = 1$

P(Getting 9 Head) $= P(9H) = P(9, 10, \frac{1}{2}) = \binom{10}{9}\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^1$ — 6

P(Getting 10 Head):

P(Getting Head) $= \frac{1}{2}$

$q = 1 - \frac{1}{2} = \frac{1}{2}$

$n = 10$

$x = 10$

$n - x = 10 - 10 = 0$

P(Getting 10 Head) $= P(10H) = P(10, 10, \frac{1}{2}) = \binom{10}{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^0$ — 7

Adding terms 1, 2, 3, 4, 5, 6, 7

$P(x \geq 4) = P(4H) + P(5H) + P(6H) + P(7H) + P(8H) + P(9H) + P(10H)$

$P(x \geq 4) = \binom{10}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^6 + \binom{10}{5}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^5 + \binom{10}{6}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^4 + \binom{10}{7}\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^3 + \binom{10}{8}\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 + \binom{10}{9}\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^1 + \binom{10}{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^0$

$$= \frac{10!}{4!\,6!}\left(\frac{1}{2}\right)^{10} + \frac{10!}{5!\,5!}\left(\frac{1}{2}\right)^{10} + \frac{10!}{6!\,4!}\left(\frac{1}{2}\right)^{10} + \frac{10!}{7!\,3!}\left(\frac{1}{2}\right)^{10} +$$

$$\frac{10!}{8!\,2!}\left(\frac{1}{2}\right)^{10} + \frac{10!}{9!\,1!}\left(\frac{1}{2}\right)^{10} + \frac{10!}{10!\,0!}\left(\frac{1}{2}\right)^{10}$$

$$= \Bigg[\frac{(10\times \cancel{9}^{3}\times \cancel{8}^{2}\times 7\times \cancel{6}\times \cancel{5}\times \cancel{4}\times \cancel{3}\times \cancel{2}\times 1)}{(\cancel{4}\times \cancel{3}\times \cancel{2}\times 1)(\cancel{6}\times \cancel{5}\times \cancel{4}\times \cancel{3}\times \cancel{2}\times 1)} + \frac{(\cancel{10}^{2}\times \cancel{9}^{3}\times \cancel{8}^{4}\times 7\times 6\times \cancel{5}\times \cancel{4}\times \cancel{3}\times \cancel{2}\times 1)}{(\cancel{5}\times \cancel{4}\times \cancel{3}\times \cancel{2}\times 1)(\cancel{5}\times \cancel{4}\times \cancel{3}\times \cancel{2}\times 1)} +$$

$$\frac{(10\times \cancel{9}^{3}\times \cancel{8}^{2}\times 7\times \cancel{6}\times \cancel{5}\times \cancel{4}\times \cancel{3}\times \cancel{2}\times 1)}{(\cancel{6}\times \cancel{5}\times \cancel{4}\times \cancel{3}\times \cancel{2}\times 1)(\cancel{4}\times \cancel{3}\times \cancel{2}\times 1)} + \frac{(10\times \cancel{9}^{3}\times \cancel{8}^{4}\times \cancel{7}\times \cancel{6}\times \cancel{5}\times \cancel{4}\times \cancel{3}\times \cancel{2}\times 1)}{(\cancel{7}\times \cancel{6}\times \cancel{5}\times \cancel{4}\times \cancel{3}\times \cancel{2}\times 1)(\cancel{3}\times \cancel{2}\times 1)} +$$

$$\frac{(\cancel{10}^{5}\times 9\times \cancel{8}\times \cancel{7}\times \cancel{6}\times \cancel{5}\times \cancel{4}\times \cancel{3}\times \cancel{2}\times 1)}{(\cancel{8}\times \cancel{7}\times \cancel{6}\times \cancel{5}\times \cancel{4}\times \cancel{3}\times \cancel{2}\times 1)(\cancel{2}\times 1)} + \frac{(10\times \cancel{9}\times \cancel{8}\times \cancel{7}\times \cancel{6}\times \cancel{5}\times \cancel{4}\times \cancel{3}\times \cancel{2}\times 1)}{(\cancel{9}\times \cancel{8}\times \cancel{7}\times \cancel{6}\times \cancel{5}\times \cancel{4}\times \cancel{3}\times \cancel{2}\times 1)(1)} +$$

$$\frac{(\cancel{10}\times \cancel{9}\times \cancel{8}\times \cancel{7}\times \cancel{6}\times \cancel{5}\times \cancel{4}\times \cancel{3}\times \cancel{2}\times 1)}{(\cancel{10}\times \cancel{9}\times \cancel{8}\times \cancel{7}\times \cancel{6}\times \cancel{5}\times \cancel{4}\times \cancel{3}\times \cancel{2}\times 1)(1)}\Bigg]\left(\frac{1}{2}\right)^{10}$$

→ (Expanding terms and taking common term $(\frac{1}{2})^{10}$ out)

→ (Cancelling common terms in denominator & numerator)

→ (Performing continuous division until no common factor is left)

$$= [(10\times 3\times 7) + (2\times 3\times 7\times 6) + (10\times 3\times 7) + (10\times 3\times 4) +$$

$$(5\times 9) + (10) + 1]\left(\frac{1}{2}\right)^{10}$$

→ (After Reduction)

$$= [210 + 252 + 210 + 120 + 45 + 10 + 1]\left(\frac{1}{2}\right)^{10}$$

$= 848 \times \left(\frac{1}{2}\right)^{10}$

$= \frac{848}{1024}$

$\boxed{P(\text{Getting at least 4 head}) = 0.828}$

Ques.2 What is the sample space for each one of the following statements?

(a) We have a bag containing white and black balls (number of balls are infinite). We keep drawing the balls until we see two white balls (or black balls) in consecutive draw.

(a) Let us assume,

W= White

B= Black

Bag contains infinite number of white and black balls. We keep drawing the balls until we see two consecutive white balls (WW) or two consecutive black balls (BB). The sample space is →

$S_1 = \{BB\}$ (when first two consecutive draws are Black)

$S_2 = \{WW)$ (when first two consecutive draws are white)

$S_3 = \{B, W, \ldots, BB\}$ (when first draw is black ball and we keep on drawing ball until two Black balls)

$S_4 = \{B, W, \ldots, WW\}$ (when first draw is black ball and draw ends in consecutive two white balls)

$S_5 = \{W, B, \ldots, BB\}$ (when first drawn ball is white and draw ends in two consecutive Black balls)

$S_6 = \{W, B, \ldots, WW\}$ (when first drawn ball is white and draw ends in two consecutive white balls)

Sample Space $(S) = S_1 + S_2 + S_3 + S_4 + S_5 + S_6$

$$\boxed{\begin{aligned} S = \{&(BB), (WW), (B, W, \ldots, BB), (B, W, \ldots, WW), \\ &(W, B, \ldots, BB), (W, B, \ldots, WW)\} \end{aligned}}$$

$\boxed{\text{Cardinality of Sample space } (S) = 6}$

(b) we have a bag containing 10 balls (numbered from 1 to 10) we choose 4 balls and save the order (of the numbers).

(b) Let us assume,

B = Ball

i = Ball number where i can be from 1 to 10

B_i = i^{th} number Ball

Bag contains 10 balls (numbered from 1 to 10). we choose 4 balls and save the order (of the number).

The sample space is →

$S_1 = \{B_1, B_2, B_3, B_4\}$

$S_2 = \{B_1, B_3, B_4, B_5\}$

$S_3 = \{B_1, B_4, B_5, B_6\}$

$S_4 = \{B_1, B_5, B_6, B_7\}$

$S_5 = \{B_1, B_6, B_7, B_8\}$

$S_6 = \{B_1, B_7, B_8, B_9\}$

$S_7 = \{B_1, B_8, B_9, B_{10}\}$

$\vdots$

$S_n = \{B_{10}, B_3, B_2, B_1\}$

where n is the total possible way of selecting 4 balls.

Total possible ways to choose 4 balls from 10 balls Can be Calculated using $^nP_r = \frac{n!}{(n-r)!}$

where ^nP_r = Permutation

n = total number of objects

r = number of objects selected

Hence, $n = 10$

$r = 4$

So, $^{10}P_4 = \frac{10!}{(10-4)!}$

$= \frac{10!}{6!}$

$$= \frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{6\times5\times4\times3\times2\times1}$$ (Expending factorial)

$$= 10\times9\times8\times7$$ (After cancelling common terms)

$$\boxed{{}^{10}P_4 = 5040}$$

⇒ There are 5040 possible ways to select 4 balls out of 10 balls.

⇒ This means $\boxed{\text{Cardinality of Sample space } (S) = 5040}$

Sample Space $(S) = S_1 + S_2 + S_3 + S_4 + S_5 + \dots S_{5040}$

$$\boxed{\begin{aligned} S = \{&(B_1, B_2, B_3, B_4), (B_1, B_3, B_4, B_5), (B_1, B_4, B_5, B_6),\\ &(B_1, B_5, B_6, B_7), (B_2, B_6, B_7, B_8), (B_1, B_7, B_8, B_9),\\ &(B_1, B_8, B_9, B_{10}), \dots\dots, (B_{10}, B_3, B_2, B_1)\} \end{aligned}}$$

Ques. 3 There are three rooms in a house : the kitchen, the bedroom and the living room. There is a radio transmitter outside the home that can be received in each room. A user is wearing a device that can measure the strength of the signal ; however, due to noise in the sensor and in the environment, each time the device reads the signal strength, the result may contain an error. we know that the bedroom should have a reading of 50 units ; the kitchen a reading of 51 units ; and the living room a reading of 52 units in 50% of the cases and 53 in the other 50% of the cases. Someone has made a lot of measurements and created the following table representing the probability distribution for a given reading is in a particular room (P (Reading | Room)):

Room\Reading	<=49	50	51	52	53	54<=
Bedroom	0.33	0.41	0.13	0.08	0.05	0
Kitchen	0.07	0.13	0.53	0.14	0.08	0.05
Living Room	0.06	0.09	0.13	0.28	0.33	0.11

(a) what are the probabilities (that we are inside) for each room given that we obtain a reading of 52 (and assuming that we spend about equal amounts of time in each room - i.e., that the sample comes equally likely from any of the three rooms) ?

(a) Let us assume,

l = living Room

K = Kitchen

b = bedroom

r = reading

To calculate Probability, $P(A) = \dfrac{\text{Total Outcomes in A}}{\text{Total Outcomes in Sample Space}}$

Total Outcomes in Sample Space = 3

Sample Space = $\{l, K, b\}$

Probability of being in living room = $P(l) = \frac{1}{3}$

Probability of being in kitchen = $P(K) = \frac{1}{3}$

Probability of being in bedroom = $P(b) = \frac{1}{3}$

Since equal time has been spent in each of three rooms, $P(l) = P(K) = P(b) = \frac{1}{3}$

To calculate probability of a being in a room when reading is 52 can be calculated using Bayes' theoram,

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

where,

A, B = events

$P(A|B)$ = Probability of A given B is true

$P(B|A)$ = Probability of B given A is true

$P(A), P(B)$ = the independent probabilities of A and B

Now we want,

$$P(b \mid r=52)$$

$$P(k \mid r=52)$$

$$P(l \mid r=52)$$

$$P(r=52) = P(r=52 \mid b)\,P(b) + P(r=52 \mid k)\,P(k) + P(r=52 \mid l)\,P(l)$$

$$= (0.08) * \left(\frac{1}{3}\right) + (0.14) * \left(\frac{1}{3}\right) + (0.28) * \left(\frac{1}{3}\right)$$

(Substituting values) ~~of [illegible]~~

$$= \frac{0.08 + 0.14 + 0.28}{3}$$

$$P(r=52) = 0.1667 \quad — (1)$$

Now,

Probability of Being in bedroom when reading is 52 →

$$P(b \mid r=52) = \frac{P(r=52 \mid b)\,P(b)}{P(r=52)} \quad \text{(using Bayes' theorem)}$$

$$= \frac{(0.08) * (1/3)}{0.1667} \quad \text{(Substituting values)}$$

$$\boxed{P(b \mid r=52) = 0.15996801}$$

$$\boxed{P(b \mid r=52) = 0.15997} \quad \text{(after rounding off)}$$

Probability of being in kitchen when reading is 52 →

$$P(k|r=52) = \frac{P(r=52|k)\,P(k)}{P(r=52)}$$ (using Bayes' theorem)

$$= \frac{(0.14) * (1/3)}{0.1667}$$ (substituting values)

$$\boxed{P(k|r=52) = 0.27994401}$$

$$\boxed{P(k|r=52) = 0.27994}$$ (after rounding off)

Probability of being in living room when reading is 52 →

$$P(l|r=52) = \frac{P(r=52|l)\,P(l)}{P(r=52)}$$ (using Bayes' theorem)

$$= \frac{(0.28) * (1/3)}{0.1667}$$ (substituting values)

$$\boxed{P(l|r=52) = 0.5598802}$$

$$\boxed{P(l|r=52) = 0.55988}$$ (rounding off)

So,

$$\boxed{\begin{aligned} P(l|r=52) &= 0.55988 \\ P(k|r=52) &= 0.27994 \\ P(b|r=52) &= 0.15997 \end{aligned}}$$

(b) For each reading we have our best guess (highest probability) determining which room we are in. What is our chance for being wrong when the reading is 52?

(b) As stated, the living room a reading of 52 units in 50% of the cases and 53 in the other 50% of the cases.

Also, <u>Probability of being in Bedroom when reading is 52</u> =

~~$P(b|r=52)\,P(r=52|b) = 0.08$~~ $P(b|r=52) = 0.15997$ (Calculated in Ques.3 Part a)

<u>Probability of being in living room when reading is 52</u> =

$P(l|r=52) = 0.55988$ (Calculated in Ques.3 Part a)

<u>Probability of being in kitchen when reading is 52</u> =

$P(K|r=52) = 0.27994$ (Calculated in Ques.3 Part a)

It can be noted that probability of being in living room when reading is 52 is highest, i.e. $P(l|r=52) = 0.5598$

And

$$P(l|r=52) > P(K|r=52) > P(b|r=52)$$

So, our best guess will be living room with 50% probability when reading is 52. Therefore, chance of making error is 50% as our best guess, i.e living room will only be correct 50% of the time when reading is 52 is for living room. For rest 50% (error chances) it can be either kitchen or bedroom.

(C) How would the result in B change if we knew that we are spending twice as much time in the bedroom than in the kitchen or living room (but the same in these two)?

(C) Let us assume,

b = bedroom

l = living room

k = kitchen

r = reading

As per the question,

$$P(b) = 2P(l) = 2P(k) \quad — (1)$$

$$P(l) = P(k) \quad — (2)$$

We know, sum of all probabilities of all outcomes is 1.

Therefore,

$$P(l) + P(k) + P(b) = 1$$

$$P(l) + P(l) + 2P(l) = 1 \quad (\text{from } (1) \,\&\, (2))$$

$$4P(l) = 1$$

$$\boxed{P(l) = 1/4}$$

$$P(l) = P(k) = 1/4$$

$$\boxed{P(k) = 1/4}$$

Substituting in (1)

$$P(b) = 2P(L)$$

$$P(b) = 2 \times 1/4$$

$$\boxed{P(b) = 1/2}$$

To calculate probability of being in a room when reading is 52 can be calculated using Bayes' theorem,

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

where,

A, B = events

$P(A|B)$ = Probability of A given B is true

$P(B|A)$ = Probability of B given A is true

$P(A), P(B)$ = The independent probabilities of A and B

We want to calculate,

$$P(b|r=52)$$

$$P(k|r=52)$$

$$P(l|r=52)$$

To calculate these, we need to calculate $P(r=52)$

$$P(r=52) = P(r=52|b)\,P(b) + P(r=52|l) + P(r=52|k)$$

$$= (0.08) * \left(\frac{1}{2}\right) + (0.28) * \left(\frac{1}{4}\right) + (0.14) * \left(\frac{1}{4}\right)$$

$$= 0.04 + 0.07 + 0.035$$ (Substituting values from table)

$$\boxed{P(r=52) = 0.145}$$

Now,

Probability of being in bedroom when reading is 52 →

$$P(b|r=52) = \frac{P(r=52|b)\,P(b)}{P(r=52)}$$ (using Bayes' theorem)

$$= \frac{(0.08) * (1/2)}{0.145}$$ (Substituting values)

$$\boxed{P(b|r=52) = 0.27586}$$

Probability of being in kitchen when reading is 52 →

$$P(K|r=52) = \frac{P(r=52|K)\,P(K)}{P(r=52)} \quad \text{(using Bayes' theorem)}$$

$$= \frac{(0.14) * (1/4)}{0.145} \quad \text{(substituting values)}$$

$$\boxed{P(K|r=52) = 0.24138}$$

Probability of being in living room when reading is 52 →

$$P(l|r=52) = \frac{P(r=52|l)\,P(l)}{P(l=52)} \quad \text{(using Bayes' theorem)}$$

$$= \frac{(0.28) * (1/4)}{0.145} \quad \text{(substituting values)}$$

$$\boxed{P(l|r=52) = 0.48276}$$

It can be noted that probability of being in living room when reading is 52 remains highest, $P(l|r) = 0.48276$.

$$P(l|r=52) > P(K|r=52) > P(b|r=52)$$

Therefore, our best guess will still remain living room with 50% probability when reading is 52. Therefore, chance of being wrong is 50% as our best guess, i.e. living room will only be correct 50% of the time when reading is 52 is for living room.