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ASSIGNMENT-02

Ques.1 Given the following data sets. Compute their mean, variance and median.

(a) $\{5, 7, 2, 3, 1, 9, 5\}$

(a) $\text{Mean} = \dfrac{\text{Sum of the terms}}{\text{number of terms}}$

Sum of terms $= 5+7+2+3+1+9+5 = 32$

Number of terms $= 7$

$\text{Mean} = \dfrac{32}{7}$

$\boxed{\text{Mean} = 4.57}$

Variance

$$\sigma^2 = S^2 = \frac{\Sigma(X-\bar{X})^2}{n}$$

X = terms of dataset

$\bar{X}$ = mean

n = number of terms

classmate

$$\text{Mean} = \frac{\sum_{i=1}^{n} x_i}{n} = 4.57$$

$$\text{Sum} = \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$= (x_1-\bar{x})^2 + (x_2-\bar{x})^2 + (x_3-\bar{x})^2 + (x_4-\bar{x})^2 +$$
$$(x_5-\bar{x})^2 + (x_6-\bar{x})^2 + (x_7-\bar{x})^2$$

$$= (5-4.57)^2 + (7-4.57)^2 + (2-4.57)^2 +$$
$$(3-4.57)^2 + (1-4.57)^2 + (9-4.57)^2 +$$
$$(5-4.57)^2$$

$$= (0.43)^2 + (2.43)^2 + (-2.57)^2 + (-1.57)^2 +$$
$$(-3.57)^2 + (4.43)^2 + (0.43)^2$$

$$= 0.1849 + 5.9049 + 6.6049 + 2.4649 + 12.7449 +$$
$$19.6249 + 0.1849$$

$$\text{Sum} = 47.7143$$

classmate

$$\text{variance} = \sigma^2 = \frac{\Sigma(x_i - \bar{x})}{n}$$

$$\sigma^2 = \frac{47.7143}{7}$$

$$\boxed{\text{variance} = \sigma^2 = 6.8163}$$

<u>Median</u>

$\{5, 7, 2, 3, 1, 9, 5\}$

Arrange in ascending order

$\{1, 2, 3, 5, 5, 7, 9\}$

$$\text{Median} = \frac{\text{number of terms}(n) + 1}{2}$$

$$= \frac{7+1}{2}$$

$$= \frac{8}{2}$$

$$\boxed{\begin{array}{l}\text{median} = 4^{th} \text{ term} \\ \text{median} = 5\end{array}}$$

classmate

(b) $\{4, 2, 2, 1, 2, 1, 4, 1, 4, 5, 8\}$

(b) $\text{Mean} = \dfrac{\text{Sum of terms}}{\text{Number of terms}}$

Sum of terms $= 4+2+2+1+2+1+4+1+4+5+8$

Sum of terms $= 34$

Number of terms $= 11$

$\text{Mean} = \dfrac{34}{11}$

$\boxed{\text{Mean} = 3.09}$

Variance

$$\sigma^2 = s^2 = \frac{\Sigma(x-\bar{x})^2}{n}$$

x = terms of data set
$\bar{x}$ = mean
n = number of terms

classmate

$$\text{Mean} = \frac{\sum_{i=1}^{n} x_i}{n} = 3.09$$

$$\text{Sum} = \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$= (x_1-\bar{x})^2 + (x_2-\bar{x})^2 + (x_3-\bar{x})^2 +$$

$$(x_4-\bar{x})^2 + (x_5-\bar{x})^2 + (x_6-\bar{x})^2 +$$

$$(x_7-\bar{x})^2 + (x_8-\bar{x})^2 + (x_9-\bar{x})^2 +$$

$$(x_{10}-\bar{x})^2 + (x_{11}-\bar{x})^2$$

$$= (4-3.09)^2 + (2-3.09)^2 + (2-3.09)^2 +$$

$$(1-3.09)^2 + (2-3.09)^2 + (1-3.09)^2 + (4-3.09)^2$$

$$+ (1-3.09)^2 + (4-3.09)^2 + (5-3.09)^2 +$$

$$(8-3.09)^2$$

$$= (0.91)^2 + (-1.09)^2 + (-1.09)^2 + (-2.09)^2 +$$

$$(-1.09)^2 + (-2.09)^2 + (0.91)^2 + (-2.09)^2 +$$

$(0.91)^2 + (1.91)^2 + (4.91)^2$

$= 0.8281 + 1.1881 + 1.1881 + 4.3681 + 1.1881 + 4.3681 +$
$0.8281 + 4.3681 + 0.81991 + 3.6481 + 24.1081$

Sum $= 46.90091$

variance $= \sigma^2 = \dfrac{46.90091}{11}$

$\boxed{\text{variance} = 4.2637}$

Median

$\{4, 2, 2, 1, 2, 1, 4, 1, 4, 5, 8\}$

Arrange in asunding order

$\{1, 1, 1, 2, 2, 2, 4, 4, 4, 5, 8\}$

median $= \dfrac{\text{number of terms}(n) + 1}{2}$

$= \dfrac{11+1}{2}$

classmate

Median = 6th term

$\boxed{\text{Median} = 2}$

(c) $\{3, 5, -8, 0, 2, 3, -1\}$

$$\text{Mean} = \frac{\text{Sum of the terms}}{\text{number of terms}}$$

Sum of terms $= 3+5-8+0+2+3-1$

Sum of terms $= 4$

Number of terms $= 7$

$$\text{Mean} = \frac{4}{7}$$

$\boxed{\text{Mean} = 0.57}$

Variance

$$\sigma^2 = S^2 = \frac{\Sigma(x-\bar{x})^2}{n}$$

x = terms of data set

$\bar{x}$ = mean

n = number of terms

$$\text{Mean} = \frac{\sum_{i=1}^{n} x_i}{n} = 0.57$$

$$\text{Sum} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{}$$

$$= (x_1-\bar{x})^2 + (x_2-\bar{x})^2 + (x_3-\bar{x})^2 +$$
$$(x_4-\bar{x})^2 + (x_5-\bar{x})^2 + (x_6-\bar{x})^2 + (x_7-\bar{x})^2$$

$$= (3-0.57)^2 + (5-0.57)^2 + (-8-0.57)^2 +$$
$$(0-0.57)^2 + (2-0.57)^2 + (3-0.57)^2 +$$
$$(-1-0.57)^2$$

$$= (2.43)^2 + (4.43)^2 + (-8.57)^2 + (-0.57)^2 +$$
$$(1.43)^2 + (2.43)^2 + (-1.57)^2$$

$= 5.9049 + 19.6249 + 73.4449 + 0.3249 +$
$2.0449 + 5.9049 + 2.4649$

Sum $= 109.7143$

Variance $= \sigma^2 = \frac{109.7143}{7}$

$\boxed{\text{Variance} = \sigma^2 = 15.67347}$

Median

$\{3, 5, -8, 0, 2, 3, -1\}$

Arrange in ascending order

$\{-8, -1, 0, 2, 3, 3, 5\}$

Median $= \frac{\text{number of terms}(n) + 1}{2}$

$= \frac{7+1}{2}$

$= \frac{8}{2}$

Median $=$ 4th term $\boxed{\text{Median} = 2}$

classmate

Ques. 2 Suppose that we investigate the effects of some drug on Alzheimer disease and the probability of each drug being effective is 10%. Calculate the following.

1. The probability that we find the first effective drug in the first 10 experiments (each experiment is with a new drug).

1. To calculate the probability of the first effective drug in the first 10 experiments (each experiment is with a new drug) can be calculated by formula →

$$P(X=k) = p(1-p)^{k-1}$$ (Geometric Distribution)

where

p = Probability of success on each trial

$K = 1, 2, 3, \ldots$

Here,

$K = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$

p = Probability of each drug being effective

$= 10\%$

$p = 0.1$

classmate

Now, Substituting values in formula

$$P(X=1) = (0.1)(1-0.1)^{1-1}$$

$$= (0.1)(0.9)^0$$

$$P(X=1) = 0.1 \quad -(1)$$

$$P(X=2) = (0.1)(1-0.1)^{2-1}$$

$$= (0.1)(0.9)^1$$

$$P(X=2) = 0.09 \quad -(2)$$

$$P(X=3) = (0.1)(1-0.1)^{3-1}$$

$$= (0.1)(0.9)^2$$

$$P(X=3) = 0.081 \quad -(3)$$

$$P(X=4) = (0.1)(1-0.1)^{4-1}$$

$$= (0.1)(0.9)^3$$

$$P(X=4) = 0.0729 \quad -(4)$$

$$P(x=5) = (0.1)(1-0.1)^{5-1}$$

$$= (0.1)(0.9)^{4}$$

$$P(x=5) = 0.06561 \quad — (5)$$

$$P(x=6) = (0.1)(1-0.1)^{6-1}$$

$$= (0.1)(0.9)^{5}$$

$$P(x=6) = 0.059049 \quad — (6)$$

$$P(x=7) = (0.1)(1-0.1)^{7-1}$$

$$= (0.1)(0.9)^{6}$$

$$P(x=7) = 0.0531441 \quad — (7)$$

$$P(x=8) = (0.1)(1-0.1)^{8-1}$$

$$= (0.1)(0.9)^{7}$$

$$P(x=8) = 0.04782969 \quad — (8)$$

$$P(X=9) = (0.1)(1-0.1)^{9-1}$$

$$= (0.1)(0.9)^8$$

$$P(X=9) = 0.043046721 \quad \text{— (9)}$$

$$P(X=10) = (0.1)(1-0.1)^{10-1}$$

$$= (0.1)(0.9)^9$$

$$P(X=10) = 0.0387420489 \quad \text{— (10)}$$

P(first effective drug in first 10 experiments) =

$P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6) + P(X=7) + P(X=8) + P(X=9) + P(X=10)$

$= 0.1 + 0.09 + 0.081 + 0.0729 + 0.06561 + 0.059049 + 0.0531441 + 0.04782969 + 0.043046721 + 0.0387420489$

$$\boxed{P(\text{first effective drug in first 10 experiments}) = 0.65132155}$$

classmate

2. The probability that the first effective drug is the 5th one that we examine.

2. To calculate the probability that the first effective drug is the 5th one that we examine can be calculated using formula →

$$P(X=K) = P(1-P)^{K-1}$$ (Geometric Distribution)

where P = probability of success on each trial

$K = 1, 2, 3, 4, \ldots$

Here,

$K = 5$

P = probability of each drug being effective $= 10\%$

$P = 0.1$

Substituting values in formula,

$$P(X=5) = (0.1)(1-0.1)^{5-1}$$

$$= (0.1)(0.9)^4$$

classmate

$P(X=5) = 0.06561$

$\boxed{P(\text{first effective drug being } 5\text{th}) = 0.06561}$

3. Name the distribution

3. Geometric distribution

→ Drug is being tested in a sequence of independent trials.

→ drug will either be success or will fail

→ The probability of success, $p = 0.1$ same for every trial.

classmate

Ques. 3 Suppose that we investigate an apple store shop and saw on average that out of 20 customers, 15 of them want to buy the newest iphone. We select 50 customer randomly, find the probability that :

1. more than 40 customers want to buy the newest iphone.

1. To calculate probability that ~~the~~ more than 40 customers want to buy the newest iphone can be calculated using formula

$$P(K) = \frac{n!}{K!(n-K)!} p^K q^{n-K}$$ (Binomial Distribution)

where P = Binomial distribution probability
n = number of trials
k = number of times for a specific outcome within n trials
p = probability of success on a single trial
q = probability of failure on a single trial.

The probability of randomly selecting a person who wants to buy iphone $= \frac{15}{20}$

$$P(A) = \frac{15}{20}$$

$p = \boxed{P(A) = 0.75}$ $\boxed{q = 0.25}$ $(q = 1-p)$

$n = 50$

$K \sim \text{Binomial}(n=50, p=0.75)$

P(more than 40 people wanting to ~~by~~ buy iphone)

$= P(K > 40) = P(K=41) + P(K=42) +$

$P(K=43) + P(K=44) + P(K=45) + P(K=46)$

$+ P(K=47) + P(K=48) + P(K=49) +$

$P(K=50)$

$$= \frac{50!}{41!\,9!}(0.75)^{41}(0.25)^{9} + \frac{50!}{42!\,8!}(0.75)^{42}(0.25)^{8}$$

$$+ \frac{50!}{43!\,7!}(0.75)^{43}(0.25)^{7} + \frac{50!}{44!\,6!}(0.75)^{44}(0.25)^{6}$$

$$+ \frac{50!}{45!\,5!}(0.75)^{45}(0.25)^{5} + \frac{50!}{46!\,4!}(0.75)^{46}(0.25)^{4}$$

$$+ \frac{50!}{47!\,3!}(0.75)^{47}(0.25)^{3} + \frac{50!}{48!\,2!}(0.75)^{48}(0.25)^{2}$$

$$+ \frac{50!}{49!\,1!}(0.75)^{49}(0.25) + \frac{50!}{50!\,0!}(0.75)^{50}(0.25)^{0}$$

$$= 0.0721 + 0.0463 + 0.0258 + 0.0123 +$$

$$0.0049 + 0.0016 + 0.0004 + 0.000077 +$$

$$0 + 0$$

$$= \boxed{P(x > 40) = 0.1636}$$

classmate

2. Exactly 30 customers want to buy the newest iphone.

2. To calculate the probability that exactly 30 customers want to buy the newest iphone can be calculated using the formula →

$$P(K) = \frac{n!}{K!(n-K)!} p^K q^{n-K}$$ (Binomial distribution)

where
- P = Binomial probability
- n = number of trials
- K = number of times for a specific outcome within n trials
- p = probability of success on a single trial
- q = probability of failure on a single trial

The probability of randomly selecting a person who wants to buy iphone $= \frac{15}{20}$

$\boxed{p = P(A) = 0.75}$ $\boxed{q = 0.25}$

$(q = 1 - p)$

classmate

$n = 50$, $k = 30$, $n-k = 20$

Substituting values,

$$P(k=30) = \frac{50!}{20!\,30!} (0.75)^{30} (0.25)^{20}$$

$$\boxed{P(k=30) = 0.00765}$$

3. What is the variance on the number of purchased iphone.

3. For binomial distribution,

variance $= np(1-p)$

where,

n = number of trials

p = probability of success on a single trial

Here,

$n = 50$

$p = 0.75$

classmate

variance = $50 \times 0.75 \times 0.25$

$\boxed{\text{variance} = 9.375}$

4. Name the Distribution

4. Binomial distribution as customer will either buy new iphone or will not buy new iphone. There are only two possible outcomes.

Ques.4 Suppose that we saw thunderstorm on average rate of 4 per month during Fall. Calculate the following

1. The probability that during 2 month we see at most 6 thunderstorm.

1. To calculate probability during 2 months we see at most 6 thunderstorm can be calculated by formula

$$P(k \text{ events in interval}) = \frac{e^{-\lambda} \lambda^{k}}{k!} \quad \text{(Poisson Distribution)}$$

where λ is the average number of events per interval

e is the number 2.71828 (Euler's number).

k takes values 0, 1, 2, ...

Here,

$k = 0, 1, 2, 3, 4, 5, 6$ (As mentioned at most 6 thunderstorm)

$\lambda = 4$ (for a month)

$\Rightarrow \lambda = 8$ (for 2 months)

classmate

Now, substituting values in the formula

$$P(K=0) = \frac{e^{-8}(8)^0}{0!} = (2.71828)^{-8}$$

$$P(K=0) = \frac{1}{2980.941946}$$

$$\boxed{P(K=0) = 0.00033546}$$

$$P(K=1) = \frac{e^{-8}(8)^1}{1!} = (2.71828)^{-8}(8)$$

$$= \frac{8}{2980.941946}$$

$$\boxed{P(K=1) = 0.0026837}$$

$$P(K=2) = \frac{e^{-8}(8)^2}{2!} = \frac{(2.71828)^{-8}(64)}{2}$$

$$= \frac{32}{2980.941946}$$

$$\boxed{P(K=2) = 0.010734861}$$

classmate

$$P(K=3) = \frac{e^{-8}(8)^3}{3!} = \frac{(2.71828)^{-8}(512)}{6}$$

$$= \frac{512}{(2980.941946)(6)}$$

$$= \frac{512}{17885.65168}$$

$$\boxed{P(K=3) = 0.028626298}$$

$$P(K=4) = \frac{e^{-8}(8)^4}{4!} = \frac{(2.71828)^{-8}(4096)}{24}$$

$$= \frac{4096}{(2980.941946)(24)}$$

$$= \frac{4096}{71542.6067}$$

$$\boxed{P(K=4) = 0.057252596}$$

classmate

$$P(K=5) = \frac{e^{-8}(8)^5}{5!} = \frac{(2.71828)^{-8}(32768)}{120}$$

$$= \frac{32768}{(2980.941946)(120)}$$

$$= \frac{32768}{357713.0335}$$

$$\boxed{P(K=5) = 0.091604134}$$

$$P(K=6) = \frac{e^{-8}(8)^6}{6!} = \frac{(2.71828)^{-8}(262144)}{720}$$

$$= \frac{262144}{(2980.941946)(720)}$$

$$= \frac{262144}{2146278.201}$$

$$\boxed{P(K=6) = 0.122138872}$$

$P(K \le 6) = P(K=0) + P(K=1) + P(K=2) + P(K=3) + P(K=4) + P(K=5) + P(K=6)$

$= 0.00033546 + 0.0026837 + 0.010734861 + 0.02862 6298 + 0.057252596 + 0.091604154 + 0.122138872$

(Substituting values)

$= 0.313375941$

$\boxed{P(K \le 6) = 0.313375941}$

$\boxed{P(K \le 6) = 0.3134}$

2. The probability that during 3 months we see exactly 10 thunderstorms.

2. To calculate the probability that during 3 months we see exactly 10 thunderstorms can be calculated by formula

$$P(K \text{ events in interval}) = e^{-\lambda} \frac{\lambda^K}{K!}$$

(Poisson distribution)

classmate

where λ is the average number of events per interval

e is the number 2.71828 (Euler's Number)

K takes values 0, 1, 2, 3, ...

Here,

$K = 10$ (exactly 10 thunderstorms)

$\lambda = 4$ (per month)

$\Rightarrow \lambda = 12$ (for 3 months)

Now, substituting values in the formula

$$P(K=10) = \frac{e^{-12}(12)^{10}}{10!} = \frac{(2.71828)^{-12}(12)^{10}}{10!}$$

$$= \frac{1}{(2.71828)^{12}} \times \frac{61917364224}{3628800}$$

$$\boxed{P(K=10) = 0.104838}$$

classmate

3. What is the expected value on the number of the thunderstorms?

3. The expected value is arithmetic mean of a random variable.
For Poisson distribution we know,

$$Mean = \mu = \lambda = \text{average number of events per interval}$$

$$\lambda = 4$$

$\boxed{\text{Expected value} = 4}$ (for one month)

Therefore, expected value on the number of the thunderstorms $= 4$.
For Fall Expected value $= 12$ $(4 * 3)$ (3 months in Fall)

4. Name the distribution.

4. Poisson Distribution

→ In given question thunderstorm (event) can occur any number of times during a time period (4 per month)

→ Rate of occurrence is constant (4 per month)

→ The probability of an event occurring is proportional

classmate

to the length of the time period.

$\lambda = 4$ (per month)

$\lambda = 8$ (2 months)

$\lambda = 12$ (3 months)

→ Thunderstorm (event) occur independently.

classmate

Ques.5 we have a four sided die and die is not fair meaning that the probability of getting "1" is 3 times (*3) the probability of getting other values (2,3,4) but the probability of the other possibilities are equal. we define the random variable Z which shows the number when we roll the die. Calculate the following:

1. What is the expected value of Z ?

1. As per the question, it is given that a four sided die and die is not fair.

(Z) Sample space = {1, 2, 3, 4}

Probability of getting "1" = P(1)

Probability of getting "2" = P(2)

Probability of getting "3" = P(3)

Probability of getting "4" = P(4)

Given,

$$P(2) = P(3) = P(4)$$

And $$P(1) = 3P(2) = 3P(3) = 3P(4)$$

As we know,

$$P(1)+P(2)+P(3)+P(4)=1$$

Now,

$$3P(2)+P(2)+P(2)+P(2)=1$$

(from given)

$$6P(2)=1$$

$$\boxed{P(2)=1/6}$$

$$\boxed{P(2)=P(3)=P(4)=1/6}$$

$$P(1)=3P(2)$$

$$P(1)=3\times\frac{1}{6}$$

$$\boxed{P(1)=\frac{1}{2}}$$

$$\Rightarrow P(X)=\left\{\frac{1}{2},\frac{1}{6},\frac{1}{6},\frac{1}{6}\right\}$$

classmate

The expected value of z can be calculated as →

$$z = \sum_{n=1}^{4} z \times P(z)$$

$$= \left(1 \times \frac{1}{2}\right) + 2\left(\frac{1}{6}\right) + \left(3 \times \frac{1}{6}\right) + \left(4 \times \frac{1}{6}\right)$$

$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{2} + \frac{2}{3}$$

$$= \frac{2}{2} + \frac{3}{3}$$

$$= 1 + 1$$

$$\boxed{z = 2}$$

Therefore, expected value of z is 2.

Q. 2. What is the probability mass function for random variable z?

classmate

2. sample space $= \{1, 2, 3, 4\}$

since this is a finite (and thus a countable) set, the random variable z is a discrete random variable. The probability mass function is defined as

$$PZ(K) = P(Z=K) \text{ for } K = 1, 2, 3, 4$$

we have PMF,

$$PZ(1) = P(Z=1) = 1/2$$

$$PZ(2) = P(Z=2) = 1/6$$

$$PZ(3) = P(Z=3) = 1/6$$

$$P\cancel{\xi}Z(4) = P(Z=4) = 1/6$$

Probability mass function

$$P(Z) = \begin{cases} 1/6 & \text{when } Z = 2, 3, 4 \\ 1/2 & \text{when } Z = 1 \end{cases}$$

classmate

3. Is it uniform distribution? Explain your answer.

3. Uniform distribution refers to probability distribution in which all outcomes are equally likely.
In the given experiment we have,

$$P(Z=1) = 1/2$$

$$P(Z=2) = 1/6$$

$$P(Z=3) = 1/6$$

$$P(Z=4) = 1/6$$

Since here probability of getting "1" is three times the probability of getting "2", "3", "4", this is not uniform distribution.

$$P(Z=1) \neq P(Z=2)$$
$$P(Z=1) \neq P(Z=3)$$
$$P(Z=1) \neq P(Z=4)$$

Hence, this is not uniform distribution.

Ques. 6 We have a bag containing 5 red marbles and 8 blue marbles. We draw 6 marbles without replacement. Calculate the following:

1. The probability that we draw 4 blue and 2 red marbles.

1. Let us assume,

$N \rightarrow$ Total number of marbles in the bag

$K \rightarrow$ Total number of red marbles in bag

$N-K \rightarrow$ Total number of blue marbles in bag

$k \rightarrow$ Actual number of red marbles drawn in the experiment

$n \rightarrow$ Actual number of marbles drawn in the experiment

$n-k \rightarrow$ Actual number of blue marbles drawn in the experiment.

Contingency table:

	drawn	not drawn	Total
Red marbles	$k = 2$	$K-k = 3$	$K = 5$
Blue marbles	$n-k = 4$	$N+k-n-K = 4$	$N-K = 8$
Total	$n = 6$	$N-n = 7$	$N = 13$

The probability of drawing exactly 4 blue and 2 red marbles can be calculated by the formula

$$P(X=k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$$ (Hypergeometric distribution)

$$P(\text{Drawing 2 red and 4 blue marbles}) = \frac{\binom{5}{2}\binom{8}{4}}{\binom{13}{6}}$$

(Substituting values)

$$= \frac{\frac{5!}{2!\,3!} \times \frac{8!}{4!\,4!}}{\frac{13!}{6!\,7!}}$$

$$\left(\binom{n}{k} = \frac{n!}{k!\,(n-k)!} \right)$$

$$= \frac{\frac{(5\times 4\times 3\times 2\times 1)}{(2\times 1)(3\times 2\times 1)} \times \frac{(8\times 7\times 6\times 5\times 4\times 3\times 2\times 1)}{(4\times 3\times 2\times 1)(4\times 3\times 2\times 1)}}{\frac{(13\times 12\times 11\times 10\times 9\times 8\times 7\times 6\times 5\times 4\times 3\times 2\times 1)}{(6\times 5\times 4\times 3\times 2\times 1)(7\times 6\times 5\times 4\times 3\times 2\times 1)}}$$

classmate

$$= \frac{(5 \times 2) \times (2 \times 7 \times 5 \times 1)}{(13 \times 2 \times 11 \times 2 \times 3)}$$ (After cancelling common terms)

$$= \frac{10 \times 70}{1716}$$

$$= \frac{700}{1716}$$

$$= 0.4079$$

$$\boxed{P(\text{drawing 2 red and 4 blue marbles}) = 0.4079}$$

2. The probability that we draw at most 5 red marbles.

2. Probability that we draw at most 5 red marbles can be calculated by the formula

$$P(X=k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$$ (Hypergeometric distribution)

Different ways of drawing at most 5 red marbles are –

Red marbles	Blue marbles
0	6
1	5
2	4
3	3
4	2
5	1

Let us assume,

$N \rightarrow$ Total number of marbles in a bag

$K \rightarrow$ Total number of red marbles in a bag

$N-K \rightarrow$ Total number of blue marbles in a bag

$k \rightarrow$ Actual number of red marbles drawn in the experiment.

$n \rightarrow$ Actual number of marbles drawn in the experiment

$n-k \rightarrow$ Actual number of blue marbles drawn in the experiment.

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Contingency table:

When Red marbles drawn $(k) = 0$ and Blue marbles drawn $(n-k) = 6$ →

	Drawn	Not drawn	Total
Red marbles	$k = 0$	$K-k = 5$	$K = 5$
Blue marbles	$n-k = 6$	$N+k-n-K = 2$	$N-K = 8$
Total	$n = 6$	$N-n = 7$	$N = 13$

When Red marbles drawn $(k) = 1$ and blue marbles drawn $(n-k) = 5$ →

	Drawn	Not drawn	Total
Red marbles	$k = 1$	$K-k = 4$	$K = 5$
Blue marbles	$n-k = 5$	$N+k-n-K = 3$	$N-K = 8$
Total	$n = 6$	$N-n = 7$	$N = 13$

When Red marbles drawn $(k) = 2$ and blue marbles drawn $(n-k) = 4 \rightarrow$

	Drawn	Not Drawn	Total
Red marbles	$k = 2$	$K-k = 3$	$K=5$
Blue marbles	$n-k = 4$	$N+k-n-K = 4$	$N-K = 8$
Total	$n=6$	$N-n=7$	$N=13$

When Red marbles drawn $(k) = 3$ and blue marbles drawn $(n-k) = 3 \rightarrow$

	Drawn	Not Drawn	Total
Red marbles	$k = 3$	$K-k = 2$	$K=5$
Blue marbles	$n-k = 3$	$N+k-n-K = 5$	$N-K = 8$
Total	$n = 6$	$N-n = 7$	$N=13$

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When Red marbles drawn $(k) = 4$ and blue marbles drawn $(n-k) = 2 \rightarrow$

	Drawn	Not drawn	Total
Red marbles	$k = 4$	$K-k = 1$	$K=5$
Blue marbles	$n-k = 2$	$N+k-n-K = 6$	$N-K = 8$
Total	$n = 6$	$N-n = 7$	$N = 13$

When Red marbles drawn $(k) = 5$ and blue marbles drawn $(n-k) = 1 \rightarrow$

	Drawn	Not drawn	Total
Red marbles	$k = 5$	$K-k = 0$	$K=5$
Blue marbles	$n-k = 1$	$N-k-n-K = 7$	$N-K = 8$
Total	$n = 6$	$N-n = 7$	$N = 13$

Now, $P(\text{at most } 5 \text{ Red}) = P(X \leq 5 \text{ Red}) =$

$P(0 \text{ Red}, 6 \text{ Blue}) + P(1 \text{ Red}, 5 \text{ Blue}) + P(2 \text{ Red}, 4 \text{ Blue}) + P(3 \text{ Red}, 3 \text{ Blue}) + P(4 \text{ Red}, 2 \text{ Blue}) + P(5 \text{ Red}, 1 \text{ Blue})$

Substituting values from contigency table into formula for calculating probability for hypergeometric distribution →

$$P(\text{at most 5 red}) = \frac{\binom{5}{0}\binom{8}{6}}{\binom{13}{6}} +$$

$$\frac{\binom{5}{1}\binom{8}{5}}{\binom{13}{6}} + \frac{\binom{5}{2}\binom{8}{4}}{\binom{13}{6}} + \frac{\binom{5}{3}\binom{8}{3}}{\binom{13}{6}}$$

$$+ \frac{\binom{5}{4}\binom{8}{2}}{\binom{13}{6}} + \frac{\binom{5}{5}\binom{8}{1}}{\binom{13}{6}}$$

$$= \frac{\frac{5!}{0!\,5!} \times \frac{8!}{2!\,6!}}{\frac{13!}{6!\,7!}} + \frac{\frac{5!}{1!\,4!} \times \frac{8!}{5!\,3!}}{\frac{13!}{6!\,7!}} + \frac{\frac{5!}{2!\,3!} \times \frac{8!}{4!\,4!}}{\frac{13!}{6!\,7!}} +$$

$$\frac{\frac{5!}{3!\,2!} \times \frac{8!}{5!\,3!} + \frac{5!}{4!\,1!} \times \frac{8!}{2!\,6!} + \frac{5!}{5!\,0!} \times \frac{8!}{1!\,7!}}{\frac{13!}{6!\,7!} \qquad\qquad \frac{13!}{6!\,7!} \qquad\qquad \frac{13!}{6!\,7!}}$$

$$\left(\binom{n}{k} = \frac{n!}{k!\,(n-k)!} \right)$$

$$= \frac{\frac{(\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1)}{(1)(\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1)} \times \frac{(8\times7\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1)}{(2\times1)(\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1)}}{\frac{(13\times12\times11\times10\times9\times8\times\cancel{7}\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1)}{(6\times5\times4\times3\times2\times1)(\cancel{7}\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1)}} +$$

$$\frac{\frac{(5\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1)}{(1)(\cancel{4}\times\cancel{3}\times\cancel{2}\times1)} \times \frac{(8\times7\times6\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1)}{(\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1)(3\times2\times1)}}{\frac{(13\times12\times11\times10\times9\times8\times\cancel{7}\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1)}{(6\times5\times4\times3\times2\times1)(\cancel{7}\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1)}} +$$

$$\frac{\frac{(5\times4\times\cancel{3}\times\cancel{2}\times1)}{(2\times1)(\cancel{3}\times\cancel{2}\times1)} \times \frac{(8\times7\times6\times5\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1)}{(4\times3\times2\times1)(\cancel{4}\times\cancel{3}\times\cancel{2}\times1)}}{\frac{(13\times12\times11\times10\times9\times8\times\cancel{7}\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1)}{(6\times5\times4\times3\times2\times1)(\cancel{7}\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1)}} +$$

$$\frac{\dfrac{(5\times4\times3\times2\times1)}{(3\times2\times1)\,(2\times1)} \times \dfrac{(8\times7\times6\times5\times4\times3\times2\times1)}{(5\times4\times3\times2\times1)\,(3\times2\times1)}}{\dfrac{(13\times12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1)}{(6\times5\times4\times3\times2\times1)\,(7\times6\times5\times4\times3\times2\times1)}} +$$

$$\frac{\dfrac{(5\times4\times3\times2\times1)}{(4\times3\times2\times1)\,(1)} \times \dfrac{(8\times7\times6\times5\times4\times3\times2\times1)}{(2\times1)\,(6\times5\times4\times3\times2\times1)}}{\dfrac{(13\times12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1)}{(6\times5\times4\times3\times2\times1)\,(7\times6\times5\times4\times3\times2\times1)}} +$$

$$\frac{\dfrac{(5\times4\times3\times2\times1)}{(5\times4\times3\times2\times1)\,(1)} \times \dfrac{(8\times7\times6\times5\times4\times3\times2\times1)}{(1)\,(7\times6\times5\times4\times3\times2\times1)}}{\dfrac{(13\times12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1)}{(6\times5\times4\times3\times2\times1)\,(7\times6\times5\times4\times3\times2\times1)}}$$

(Expanding terms)

$$= \left[(1)\left(\frac{8\times7}{2}\right) + (5)\left(\frac{8\times7\times6}{3\times2}\right) + \left(\frac{5\times4}{2}\right)\left(\frac{8\times7\times6\times5}{4\times3\times2}\right) + \left(\frac{5\times4}{2}\right)\left(\frac{8\times7\times6}{3\times2}\right) + (5)\left(\frac{8\times7}{2}\right) + (1)\times(8) \right]$$

$$\times \frac{1}{\dfrac{13\times12\times11\times10\times9\times8}{(6\times5\times4\times3\times2\times1)}}$$

(After cancelling and taking common term out)

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$$= \frac{(28 + 280 + 700 + 560 + 140 + 8)}{1716}$$

$$= \frac{1716}{1716}$$

$$\boxed{P(\text{at most 5 red}) = 1}$$

3. Name the distribution

3. Hypergeometric distribution

The following conditions are being satisfied for being hypergeometric distribution →

(a) The result of given experiment can be classified into one of two mutually exclusive categories i.e. either ~~ball~~ marbles will be drawn or not drawn.

(b) In The given experiment sampling is being done without replacement from a finite population.

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