The Spline-GARCH Model for Low Frequency Volatility and its Global Macroeconomic Causes

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ABSTRACT

25 years of volatility research has left the macroeconomic environment playing a minor role. This paper proposes modeling equity volatilities as a combination of macroeconomic effects and time series dynamics. High frequency return volatility is specified to be the product of a slow moving component, represented by an exponential spline, and a unit GARCH. This slow moving component is the low frequency volatility, which in this model coincides with the unconditional volatility. This component is estimated for nearly 50 countries over various sample periods of daily data.

Low frequency volatility is then modeled as a function of macroeconomic and financial variables in an unbalanced panel with a variety of dependence structures. It is found to vary over time and across countries. The low frequency component of volatility is greater when the macroeconomic factors GDP, inflation and short term interest rates are more volatile or when inflation is high and output growth is low. Volatility is higher for emerging markets and for markets with small numbers of listed companies and market capitalization, but also for large economies.

The model allows long horizon forecasts of volatility to depend on macroeconomic developments, and delivers estimates of the volatility to be anticipated in a newly opened market.

1. Introduction

After more than 25 years of research on volatility, the central unsolved problem is the relation between the state of the economy and aggregate financial volatility. The number of models that have been developed to predict volatility based on time series information is astronomical, but the models that incorporate economic variables are hard to find. Using various methodologies, links are found but they are generally much weaker than seems reasonable. For example, it is widely recognized that volatility is higher during recessions and following announcements but these effects turn out to be a small part of measured volatility.

Officer(1973) tried to explain the high volatility during the 30's based on leverage and the volatility of industrial production. Schwert(1989) sought linkages between financial volatility and macro volatility but concluded that "The puzzle highlighted by the results in this paper is that stock volatility is not more closely related to other measures of economic volatility."

An alternative approach examines the effects of news or announcements on returns. With simple or elaborate regression models, contemporaneous news events are included in return regressions. Roll(1988), and Cutler, Poterba and Summers(1990) for example developed such models which are found to explain only a fraction of volatility ex post, and more recent versions such as Andersen and Bollerslev(1998a), Fleming and

Remolona(1999), Balduzzi, Elton and Green(2001), or Andersen, Bollerslev, Diebold and Vega(2005) use intraday data but with more or less similar results.

This paper will introduce a simple model of the relation between macroeconomics and volatility and then apply this to the problem of explaining the financial volatility of nearly 50 markets over time. Along the way a new volatility model, the Spline-GARCH, will be introduced to allow the high frequency financial data to be linked with the low frequency macro data. As a result it will be possible to forecast the effect of potential macroeconomic events on equity volatility and to forecast the volatility that could be expected in a new market. Moreover, the assumption that volatility is mean reverting to a constant level, which underlies almost all GARCH and SV models estimated over the last 25 years, will be relaxed by the Spline-GARCH model.

This paper is organized as follows. In Section 2, we describe a model of financial volatility in a macroeconomic environment. In Section 3, we introduce the Spline-GARCH model for low frequency volatility. In Section 4, we show estimation results for the Spline-GARCH model using time series of returns in a global context. Section 5 presents a description of the country specific data followed by a discussion on the definition and construction of the variables involved in the cross-sectional analysis. In this section, we motivate the econometric approach for the cross-sectional analysis and discuss the estimation results of the determinants of long run volatilities. In Section 6, we analyze the effects of country heterogeneity in our results. Section 7 presents a further robustness analysis with estimation of alternative models using other proxies for long term volatilities. Section 8 provides concluding remarks.

2. A Model of Financial Volatility in a Macroeconomic Environment

The now highly familiar log linearization of Campbell(1991) and Campbell and Shiller(1988) delivers an easy expression for the surprise in the return to a financial asset. Let r_t be the log return and d_t be the log dividend from owning the asset from time *t-1* through *t*. Then

$$(1) \qquad r_t - E_{t-1}(r_t) = (1-\rho)\sum_{j=0}^{\infty}\rho^j (E_t - E_{t-1})(d_{t+1+j}) - \sum_{j=0}^{\infty}\rho^j (E_t - E_{t-1})(r_{t+1+j}),$$

which can be written as

$$(2) \qquad r_t - E_{t-1}r_t = \eta_t^d - \eta_t^r$$

Unexpected returns can be described as innovations to future cash flows or expected returns. Shocks to dividends have a positive effect on returns while shocks to interest rates or risk premiums have a negative effect. Different news events may have very different impacts on returns depending on whether they have only a short horizon effect or a long horizon effect. As macroeconomic events in the future will influence dividends and profitability of required returns, the relevant macroeconomic variables are the innovations to predictions of the future. The variance of these innovations will be changing over time and can be forecast using current information.

In order to explain the size effects of these shocks, much research has decomposed unexpected returns into its news components. Equation (2) can be written as:

(3) $$r_t - E_{t-1}r_t = \sum_{i=1}^{K} \beta_i e_{t,i}$$

where there are K news sources. The magnitude of the news event is indicated by e which could be the difference between prior expected values and the announced value. It is clear that announcements cannot be the only source of news since the gradual accumulation of evidence prior to the actual announcement must also affect prices. This model is only useable if all news is observable. If it is not, then Equation (3) can be written with one innovation that represents all the remaining news. When no news announcements are identified this remains the only shock.

The innovation to stock returns will have a variance that changes over time. Two effects can be identified. This variance can be a result of constant news intensity with an impact on returns that varies over time. It is natural to think of this impact multiplier as dependent on the macroeconomic environment, which is characterized by a vector of state variables $\vec{z}_t$. For example, news about a firm may be more influential in a recession than in a fast growth period. Thus, the innovation to returns can be written as:

(4) $$r_t - E_{t-1}r_t = \sqrt{\tau_1(\vec{z}_t)}u_t ,$$

In addition, the magnitude and the intensity of the news may be varying in response to the macroeconomy and other unobserved variables. Then

(5) $$u_t = \sqrt{\tau_2(\vec{z}_t)g_t} \; \varepsilon_t ,$$

where g_t is a non-negative time series such as a GARCH with unconditional mean of one. In this expression, ε has constant variance of one. Hence,

(6) $$r_t - E_{t-1}r_t = \sqrt{\tau(\vec{z}_t)g_t} \; \varepsilon_t ,$$

where $\tau(\vec{z}_t) = \tau_1(\vec{z}_t)\tau_2(\vec{z}_t)$. Without more information, these components cannot be separately identified.

In this paper we will estimate (6) directly by specifying a relationship for $\tau(\vec{z}_t)$, the low frequency variance component. A second approach is to calculate the realized variance over a time period and then model the relation between this value and the macro variables. The realized variance is given by its expected value plus a mean zero error term with unspecified properties. This gives:

$$\hat{\sigma}_T^2 = \sum_{t=1}^{T} \left(r_t - E_{t-1} r_t\right)^2 = \sum_{t=1}^{T} \tau(\vec{z}_t) + w_T \tag{7}$$

It is clear that there is an error term in (7) that will make estimation of $\tau(\vec{z}_t)$ imprecise but still unbiased.

In practice, direct estimation of (6) is difficult as the macro variables are not defined on the same high frequency basis as the returns. Recognizing that the macroeconomy is slowly evolving, we use a partially non-parametric estimator to model the low frequency component of volatility. This has the great advantage that it can be used for any series without requiring specification of the economic structure. Then the estimated low frequency volatilities can be projected onto the macroeconomic variables:

$$\tau_t^{1/2} = \sum_k \beta_k z_{k,t} + u_t, \tag{8}$$

and this model can be entertained for forecasts or policy analysis. This Spline-GARCH model is introduced in the next section.

3. A New Time Series Model for High and Low Frequency Volatility

In this section, we introduce the Spline-GARCH model that extends the GARCH(1,1) model introduced by Bollerslev (1986) offering a more flexible specification of low frequency volatility based on a semi-parametric framework. To motivate our model, consider a specification for unexpected returns that follows the familiar GARCH(1,1) model:

(9) $$r_t - E_{t-1} r_t = \sqrt{h_t}\,\varepsilon_t,$$

(10) $$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1},$$

where ε_t is the innovation term assumed to be distributed with mean 0 and variance 1, the expectation E_{t-1} is conditional on an information set Φ_{t-1} including historical past returns up to time t-1, and h_t characterizes the corresponding conditional variance. Now, let us concentrate on the long run properties of this model. For example, we can rewrite Equation (10) in terms of the unconditional variance as follows:

(11) $$h_t = \sigma^2 + \alpha(\varepsilon_{t-1}^2 - \sigma^2) + \beta(h_{t-1} - \sigma^2),$$

where $\sigma^2 = \omega\left(1-\alpha-\beta\right)^{-1}$ is the unconditional variance. When $\alpha+\beta<1$, the conditional variance reverts to its mean value σ^2 at a geometric rate of $\alpha+\beta$. This structure allows mean reversion at a reasonable rate only if $\alpha+\beta$ is very close to unity. For a long horizon T, the T days ahead volatility forecast will be the same constant σ no matter if the forecast is made at day t or at day t-k, $k>0$. Therefore, despite the empirical success of this model in describing the dynamics of conditional volatility in financial markets (particularly in the short run), its ability to account for more permanent and/or slow moving patterns of volatility is limited.[1] This feature does not seem to be consistent with the time series behavior of realized (and implied) volatilities of stock market returns where volatility can be abnormally high or low for a decade. Consequently, we need a model flexible enough to generate an expected volatility that captures the low frequency patterns observed in the data. Allowing for "slow" time variation in σ seems to be the natural extension. However, this change induces a number of theoretical and practical questions. What are the statistical and economic properties of the new term? How can we identify it from the other elements describing the dynamics of volatility? What is the appropriate functional form?

The component GARCH model introduced by Engle and Lee (1999) provides a parametric approach to answer these questions. Their model involves a decomposition of the volatility process into two separate components. One describes the short run dynamics of conditional volatility associated with transitory effects of volatility innovations. The other characterizes slower variations in the volatility process associated with more permanent effects. An additive decomposition is motivated by replacing σ^2 in Equation (11) with a stochastic component describing the long memory features of the volatility process. This long memory component determines the unconditional volatility and might be interpreted as a trend around which the conditional volatility fluctuates. For identification, this component is assumed to have a much slower mean-reverting rate than the short run component.[2] In this regard, the component GARCH model relaxes parameter restrictions for the unconditional volatility and the speed of mean reversion in

[1] See Andersen and Bollerslev (1998b) for details on the empirical success of the GARCH(1,1) model in fitting and forecasting financial volatilities.
[2] Maheu (2002) finds that moderate to large datasets are needed to accurately identify the two components.

the standard GARCH(1,1) model; however, the slow moving trend is mean reverting to a fixed value and the conclusion that the volatility process reverts eventually to a constant level remains unchanged.[3]

In this paper, we go beyond and relax the assumption that the slow moving trend in the volatility process, named here low frequency volatility, reverts to a constant level. In addition, we take a non-parametric approach that allows the data to provide the functional form of this low frequency volatility. Moreover, instead of using an additive decomposition, we separate the high and low frequency components of the volatility process using a multiplicative decomposition motivated by the economic model of volatility presented in Section 2. Specifically, we modify the standard GARCH(1,1) model by introducing a trend in the volatility process of returns. This trend describes the low frequency component of the volatility process associated with slowly varying deterministic conditions in the economy, or random variables that are highly persistent and move slowly. We approximate this unobserved trend non-parametrically using an exponential quadratic spline, which generates a smooth curve describing this low frequency volatility component based exclusively on data evidence. The exponential functional form guarantees that the low frequency component of volatility is always positive. The quadratic form is motivated by the requirement to obtain smoothness through continuity of at least one derivative at a minimum cost in terms of degrees of

[3] Another interesting approach that allows for stochastic time variation in the parameters of a GARCH specification is the Markov Regime Switching GARCH approach introduced by Cai(1994) and Hamilton and Susmel (1994) for the ARCH case. This approach leads to time varying unconditional volatilities that change according to the volatility regime. However, the estimation process might become more complicated and data demanding.

freedom. Our Spline-GARCH model for stock returns implements Equation(6) as follows:

(12) $$r_t - E_{t-1}r_t = \sqrt{\tau_t g_t}\varepsilon_t, \text{ where } \varepsilon_t \mid \Phi_{t-1} \sim N(0,1)$$

(13) $$g_t = (1-\alpha-\beta) + \alpha\left(\frac{\left(r_{t-1} - E_{t-2}r_{t-1}\right)^2}{\tau_{t-1}}\right) + \beta g_{t-1}$$

(14) $$\tau_t = c\exp\left(w_0 t + \sum_{i=1}^{k} w_i\left((t-t_{i-1})_+\right)^2 + z_t\gamma\right),$$

where Φ_t denotes an extended information set including the history of returns up to time t and weakly exogenous or deterministic variables z_t,

$$(t-t_i)_+ = \begin{cases} (t-t_i) \text{ if } t > t_i \\ \quad 0 \qquad \text{otherwise} \end{cases}$$

and $\{t_0 = 0, t_1, t_2, ..., t_k = T\}$ denotes a partition of the time horizon T in k equally-spaced intervals. $\Theta = \{\alpha, \beta, c, w_0, w_1, ..., w_k\}$ includes the parameters estimated in the model. Since k, the number of knots in the spline model, is unspecified, we can use an information criterion to determine an "optimal" choice for this number, which in fact governs the cyclical pattern in the low frequency trend of volatility. Large values of k imply more frequent cycles. The "sharpness" of each cycle is governed by the coefficient, $\{w_i\}$. Notice that the normalization of the constant term in the GARCH equation implies that the unconditional volatility depends exclusively on the coefficients of the exponential spline. In fact, a special feature of this model is that the unconditional volatility coincides with the low frequency volatility, i.e.,

(15) $$E\left[(r_t - E_{t-1}r_t)^2\right] = \tau_t E(g_t) = \tau_t$$

Our semi-parametric approach has the potential to capture both short and long term dynamic behavior of market volatility. Equation (13) characterizes the short term dynamics keeping the nice properties of GARCH models in fitting and forecasting volatility processes at high and medium frequencies. Equation (14) describes non-parametrically low frequency volatility changes, which can be associated with volatility dynamics at longer horizons, using a smooth differentiable curve including *k-1* changes in curvature that (naturally) capture cyclical patterns.

Figure 1 and Table (1) illustrate the model with Gaussian innovations for the US, based on S&P500 data during the period 1955-2003. Table (1) reports the estimates for the Spline-GARCH specification with 7 knots, which is selected by the BIC among specifications with the number knots varying between 1 and 15. The coefficients of the GARCH component are statistically significant and standard in terms of magnitude. This will be discussed with more detail in the next section. The knot coefficients are also statistically significant for the six interior knots suggesting changes in the curvature of the time trend in February 1962, April 1969, April 1976, May 1983, May 1990 and June 1997. Figure 1 shows how this Spline-GARCH model fits high and low frequency patterns of volatility during the sample period. The volatility trend suggested by the data reveals a cyclical behavior that may be associated with the business cycle. In addition, the graph shows that the assumption that volatility reverts towards a constant is not appealing. More examples and further discussion on the specifics of the estimation of the Spline-GARCH model will be presented in the following section.

4. Time Series Estimation of Low Frequency Volatilities Using the Spline-GARCH Model

4.1 Returns Data

The first part of our empirical analysis considers stock market returns. Using the index associated with the main stock exchange, we collect daily data of several countries on stock market returns from Datastream and Global Financial Data.[4] Our sample includes all developed countries and most emerging markets that experienced significant liberalization during the 1980's and 1990's, as described in Bekaert and Harvey (2000). Table (2) lists these countries, the names of the exchanges and market indices, their IFC country classification as developed or emerging markets, as well as general exchange features, such as average values for the number of listed companies and market capitalization.

The sample windows vary for each exchange since we tried to maximize the number of daily observations used in the estimation. In other words, data availability, mainly associated with the age of each particular exchange, determined the sample periods. Columns 2 and 3 of Table (3) show the starting date and the number of observations used in the time series estimation for each country. In all the cases, the ending point is on June 25th, 2004.

4.2 Estimation of Low Frequency Volatilities Based on Global Equity Markets

[4] We only included countries for which daily stock market data and quarterly macroeconomic data are available.

For each country, we use its daily returns time series and estimate the Spline-GARCH model introduced in Section 3 assuming Gaussian innovations. We use the BIC to select the optimal number of knots associated with the spline component. Figure 2 presents some examples. These graphs illustrate the two volatility components associated with the short run conditional volatility and the slow moving trend that characterizes the low frequency volatility. In addition, annual realized volatilities are included to illustrate how realized volatility, as a consistent estimator of unconditional volatility, lies close to the estimated trend.

Table (3) summarizes the estimation results for all the countries included in our analysis. In column 1, the optimal number of knots in the Spline-GARCH model is presented. Variation in this number is associated with both country specific volatility patterns and the length of the sample period. The number of observations per knot, presented in column 4, is used as an indicator of the cyclical pattern observed in the low frequency volatility component for each country. Table (4) presents a more detailed description of the distributional features of this variable. The results indicate that the average number of observations per knot in developed markets is almost three times that number in emerging markets (including transition economies). Therefore, emerging markets show on average almost three times more cycles than developed economies.

To explore possible changes in the dependence structure of the Spline-GARCH model, we estimate a standard GARCH(1,1) model and compare the coefficients associated with temporal dependence in both models. The ARCH effects (alphas) in the Spline-GARCH and GARCH (1,1) models are presented in columns 5 and 6 of Table (3), respectively. The results suggest little variation between the two models in terms of these effects. In fact, the mean values are 0.17 and 0.16 for the Spline-GARCH and GARCH(1,1) models, respectively. Moreover, the first panel of Figure 3 shows that the number of knots does not seem to have an effect on this conclusion. Regarding the GARCH effects (betas), columns 7 and 8 of Table (3) present the estimated coefficients over the countries in our sample for the two models. The mean values suggest slightly less persistence in the Spline-GARCH model (0.73 compared with 0.80 of the GARCH(1,1)). The second panel of Figure 3 shows that this pattern is roughly independent of the number of knots. Overall, these results suggest that the Spline-GARCH model observes a slightly shorter memory ARMA structure in the squared innovations, which is a feature shared by other GARCH family models that relax the parameter restrictions for the unconditional variance, such as the component GARCH model described above.

Now, to show the improved performance of the Spline-GARCH model over the simple GARCH(1,1), we use the BIC and the likelihood ratio test. The two criteria suggest that the Spline-GARCH model is clearly preferred over the GARCH(1,1) model for all the countries where the optimal number of knots is larger than one. Moreover, even for the one-knot cases, where we would expect more difficulties in rejecting the assumption of mean reversion in volatility to a fixed value, we reject the GARCH(1,1) specification for all the cases but France. The BIC and LR statistics are shown in columns 11-13 of Table (3).

5. Economic Determinants of Low Frequency Volatilities

A second goal of this study is providing an explanation on what are the economic determinants of low frequency volatility. We approach this question by providing both cross-sectional and time series evidence along the countries included in our sample. We focus on macroeconomic fundamental variables and variables related to the market structure of each exchange. Economic theory and previous empirical evidence motivate the selection of such variables.

5.1 Data

The sources for our macroeconomic variables are Global Insight/WRDS, Global Financial Data, and the Penn World Tables. These variables include: GDP, inflation indices (Consumer Price Indices are used to measure inflation), exchange rates, and short term interest rates. The set of countries with available macroeconomic data is smaller than the set with available financial time series data. Thus, we are left with a reduced sample of 48 countries.

We also collect information for different years on the size and diversification of each market associated with the counties listed in Table (2), such as market capitalization and the number of listed companies. The former is obtained from Global Financial Data and the official web pages of the exchanges. The sources for the latter are: the World Federation of Exchanges, the Ibero-American Federation of Exchanges (FIAB), and official web pages of the exchanges.

5.2 Variables Discussion

We start with a description of the dependent variable. In this regard, given that volatilities are not directly observed, we need to define a measure of low frequency volatilities to construct our dependent variable.[5] For each country, we use the Spline-GARCH model introduced in Section 3 to fit its daily time series of market returns considering the sample periods described in Table (3). As mentioned in Section 4, we use the BIC to select the optimal number of knots associated with the spline component. In each case, we obtain the low frequency volatility component described in Equation (14). Thus, a measure of the low frequency volatility can be defined as the average of the daily low frequency volatilities over a long term horizon, namely one year.

We appeal to economic theory and previous empirical evidence to select the potential determinants of low frequency volatilities. Levels as well as fluctuations of fundamental variables are the natural candidates. Previous research has pointed out the relation between volatilities and the business cycle; for example, Schwert (1989) and Hamilton and Lin (1996) find economic recessions as the most important factor influencing the US stock return volatility. We consider the growth rate of real GDP as a variable accounting for changes in real economic activity.

Volatility and uncertainty about fundamentals are also potential factors affecting market volatility. For example, Gennotte and Marsh (1993) derive returns volatility and risk

[5] Andersen et. al (2003) argue that under suitable conditions, realized volatilities can be thought as the observed realizations of volatility. We present estimation results for this alternative measure of long term volatilities in Section 7.

premia based on stochastic volatility models of fundamentals; David and Veronesi (2004) identify inflation and earnings uncertainty as sources of stock market volatility and persistence. We consider measures of macroeconomic volatility to account for this uncertainty. Specifically, we construct a proxy for inflation volatility based on our CPI quarterly time series. We obtain the absolute values of the residuals from an AR(1) model, and then we compute their yearly average.

$$\begin{aligned} &\Delta \log\left(y_t\right) = c + u_t, \quad u_t = \rho u_{t-1} + e_t \\ &\sigma^2_{y,t} = \frac{1}{4}\sum_{j=t-2}^{t+1}\left|e_j\right| \end{aligned} \tag{16}$$

Following the same setup, we construct proxies for country economic uncertainty linked to fundamentals. In particular, we estimate volatilities of real GDP, interest rates (without logs) and exchange rates based on the residuals of fitted autoregressive models. Exchange rates are measured as US$ per unit, and interest rates are based on short term government bonds.

Some country-based empirical studies have suggested that market development is an important element in explaining differences in market volatilities across countries. For example, De Santis and Imrohoroglu (1997) find higher conditional volatilities, as well as larger probabilities of extreme events, in emerging markets relative to developed markets. Moreover; Bekaert and Harvey (1997) find that market liberalizations increase the correlation between the local market and the world market, but they do not find significant effects on market volatilities. In order to capture the effect of market development in our analysis we construct two dummy variables for emerging markets and transition economies. The emerging market classification comes from the IFC; we define transition economies as the former socialist economies, such as the Central European and Baltic countries in our sample.

To explain further variations in the cross-sectional stock market volatilities it is important to account for other factors associated with market liberalizations, for example macroeconomic reforms relevant for both increasing efficiency in risk sharing and increasing market liquidity. In emerging economies many macroeconomic reforms are intended to improve institutional control of inflation and to open the economies to international trade. Bekaert, Harvey, and Lundblad (2006) find that a larger inflation rate, as well as a larger external sector, is positively related to consumption and GDP growth volatility. Since we are interested in variables explaining volatility of fundamentals, we account for the effect of inflation rates, which are measured as the growth rate of the CPI.

Cross-sectional variation in market volatilities may also be related to the size of the markets and/or the size of the economies. We would expect that larger markets have advantages in terms of offering broader diversification opportunities and probably lower trading costs. We consider two different variables to account for these size effects. The first one is the log of the annual market capitalization of each exchange. The second one is the log of nominal GDP in US dollars. Having these variables in logs allows for testing the effect of the stock market size as a proportion of the overall value of the economy (ratio of market capitalization to GDP). This ratio can be used as a measure of how developed is the stock market and as a proxy for the degree of integration in terms of foreign investment.[6] All of these variables are converted to US dollars using annual exchange rates. Finally, we consider the number of listed companies on each exchange as a variable proxying the market size and the span of market diversification opportunities. Table (5) summarizes the variables of our analysis.

5.3 Cross-Sectional Analysis of Low Frequency Volatilities

In this subsection, we describe our cross-sectional analysis of expected long term market volatilities. Before describing the general setup, it is important to point out some data issues and conventions. First, we relate long term periods with annual intervals.[7] Thus, for each of the variables introduced above, we construct annual averages. Next, for each country, we have to match the annual low frequency volatility time series with several macroeconomic time series. This process leads to country-specific sample windows, and therefore to an unbalanced panel of countries. Moreover, the number of countries increases with time, since recent data is available for most of the countries, and also because many markets started operations during the 1990's (e.g. transition economies). Therefore, in order to keep a relatively large number of countries in the cross-sectional dimension, we consider a panel that covers 1990-2003.[8] This data structure can be summarized in a system of linear equations projecting, for each year, the low frequency volatility estimated from the Spline-GARCH model on the explanatory variables

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[6] Bekaert and Harvey (1997) consider the ratio market capitalization to GDP and the size of the trade sector as measures of the country's degree of financial and economic integration that affect the inter-temporal relation between domestic market volatilities and world factors.

[7] This convention has no effect in our framework. We could have taken a different horizon and followed the same process.

[8] Note that, for some countries, variables constructed from dynamic models, such as low frequency volatilities and macroeconomic volatilities, might have involved longer sample periods in the estimation process (see Table 3 for details).

described in Table (5). Following the discussion in Section 5.2, the annualized low frequency volatility for year t and country i is the following sample average:

$$(17) \qquad Lvol_{i,t} = \left(\frac{1}{M_{i,t}} \sum_{d=1}^{M_{i,t}} \tau_{i,t,d} \right)^{1/2},$$

where $M_{i,t}$ represents the number of trading days in country i at year t, and $\tau_{i,t,d}$ is the daily low frequency volatility in Equation (14) observed in country i at trading day d of year t.[9] Thus, the system of linear equations can be specified as follows:

$$(18) \qquad Lvol_{i,t} = \underline{z}'_{i,t}\,\beta_t + \mu_{i,t},\ t = 1,2,...,T,\ i = 1,2,...,N_t,$$

where $\underline{z}_{i,t}$ is a vector of explanatory variables associated with country i and year t, and $\mu_{i,t}$ is the error term assumed to be contemporaneously uncorrelated with $\underline{z}_{i,t}$.[10]

The next task is to find an econometric approach that efficiently accounts for the features observed in the structure of our data. We start by looking at the correlation structure of the data across time. In particular, we select a sub-panel from 1997-2003 to have an almost balanced structure. We look at the correlation across years of low frequency volatilities, regressors, and residuals coming from individual regressions for each year. Tables (6) and (7) present such correlations for low frequency volatilities and residuals, respectively. These tables show high correlation of the residuals, suggesting that unobservable factors affecting expected volatilities are likely to be serially correlated

[9] Note that in this section the sub-index t refers to years, not to days as in Sections 3 and 4.

[10] The assumption $E(\underline{z}'_{i,t}\,\mu_{i,t}) = 0,\ t = 1,2,...,T,\ i = 1,2,...,N_t$ does not rule out non contemporaneous correlation; so, the error term at time t may be correlated with the regressors at time t+1. Therefore, in this setup financial volatility can cause macroeconomic volatility, as is suggested in Schwert (1989). However when SUR estimation is used, the assumption of exogeneity will be maintained.

across time. In addition, even higher correlation is observed on the dependent variable suggesting little variation across time. Similarly, it is observed that many of the explanatory variables are also highly correlated across time, showing again little time variability. Some exceptions that show lower correlation across time are the real GDP growth rate and the exchange rate volatility.

The observation of these features motivates our econometric approach. As usual in cross sectional studies, we assume that the errors are uncorrelated in the cross-section.[11] However there is clear autocorrelation. A method that efficiently handles autocorrelation in the unobserved errors is appealing. The Seemingly Unrelated Regressions (SUR) model developed by Zellner (1962) provides a framework that imposes no assumptions on the correlation structure of the errors and easily incorporates restrictions on the coefficients. The presence of large autocorrelations across the disturbances, as suggested in Table (7), implies important gains in efficiency from using FGLS in a SUR system as well as improved standard errors. Standard panel data approaches that impose further restrictions could be considered; however, their underlying assumptions and estimation features seem to be less attractive based on the features of our data. For example, the low variation over time observed in many of the explanatory variables indicates that fixed effects models can lead to imprecise estimates (see Wooldridge, 2002). On the other hand, even though the standard random effects model allows for some time correlation, the structure of the covariances is restrictive in the sense that it comes exclusively from the variance of the individual effects, which is assumed to be constant across time. This

[11] Cross sectional dependence will generally not give inconsistency in our model, but inference and efficiency could be improved if a factor structure is assumed as in Pesaran(2005).

feature does not seem appealing based on the evidence in Table (7). Therefore, more general panel data approaches that deal more efficiently with serial correlation would be desirable. We will explore one possibility in the robustness section. Nevertheless, given that the SUR method allows for time fixed effects and flexible autocorrelation structure, we take this approach as our main specification for the cross sectional analysis. We assume that the coefficients, other than the intercept, remain constant over time.

Using this SUR modeling strategy, we start our cross sectional analysis by exploring the relationship between low frequency volatilities and each of the explanatory variables, one at a time. Table (8) presents the estimation results of the system of cross sectional regressions on single explanatory variables.[12] From this preliminary analysis, we observe positive relations among low frequency market volatilities and each of the following variables: emerging markets, log nominal GDP, inflation rate, and macroeconomic volatilities (associated with interest rates, exchange rates, GDP, and inflation). In contrast, the following variables show a negative relation with long term market volatility: transition economies, growth rate of GDP and market size variables, such as log market capitalization, and number of listed companies. The results are significant for most variables except for transition economies and log nominal GDP in current US dollars.

Next, we estimate the full system of equations described in (18), which includes all the explanatory variables. The corresponding results are presented in the first column of Table (9). From this analysis, we observe that emerging markets show larger expected

[12] The constant term is allowed to vary across years.

volatility compared to developed markets. The effect is significant and consistent with the empirical evidence about volatility of emerging markets (see Bekaert and Harvey, 1997). It is however much smaller than in the univariate regressions. Transition economies have only slightly larger volatility than developed economies. Market size variables show different results. Whereas log market capitalization has a significant negative effect (at the 10% level), log nominal GDP in current US dollars is positive and significant (at the 5% level). The positive effect dominates, since market size as a proportion of GDP has a negative effect on low frequency volatility, but larger economies are associated with larger volatilities. In contrast, the number of listed companies in the exchange has a negative effect on volatility. This suggests that markets with more listed companies may offer more diversification opportunities, reducing the overall expected volatility.

In regard to real economic activity variables, the results show that economic recessions increase low frequency volatilities, and inflation rates also affect them positively. These results indicate that countries experiencing low or negative economic growth observe larger expected volatilities than countries with superior economic growth. Similarly, countries with high inflation rates experience larger expected volatilities than those with more stable prices. Although the effect is not significant for real GDP growth, the effect is larger and highly significant for inflation rates.

In relation to volatility of macroeconomic fundamentals, the results suggest that volatility of inflation, as well as volatility of real GDP, are strong determinants of low frequency market volatility. Both variables are associated with significant positive effects. The coefficient on interest rate volatility is also positive and significant but small in magnitude. The effect of exchange rate volatility is negative, small and quite insignificant. This evidence encourages theoretical work relating volatility of fundamentals to causes of fluctuations in market volatility at long horizons.

We also consider plausible dimension reductions based on the significance of the explanatory variables. We estimate different model specifications based on a reduction process that drops the least significant variable one at a time. In this process, the goodness of fit in each model is given by the concentrated likelihood, and therefore by the determinant of the residual covariance. In addition, to select an optimal reduction, we take an information criterion approach; in particular, we select a BIC type of penalization for increasing the number of parameters. In column 2 of Table (9), we present the "best" reduction in which the BIC favors a specification for which volatility of exchange rates (first) and real GDP growth (second) are omitted. Therefore, the reduction process leads to a model with nine explanatory variables.

6. Country Heterogeneity

We start this section with a diagnostic analysis estimating the benchmark SUR model excluding from the sample one country at a time. Figures 4 and 5 show the coefficients associated with each regressor and the t-statistics respectively. Each point in the horizontal axis represents the country that is dropped from the sample following the order presented in Table (2). For instance, the first point corresponds to the estimation without Argentina, and the last point corresponds to the estimation without Venezuela. From Figure 5, we observe that the significance of some explanatory variables remains strong no matter which country is taken out of the sample. Indeed, this is the case for emerging, number of listings, log nominal GDP, and volatility of real GDP, which also preserve the same sign (see panels 1, 4, 5, and 10, Figures 4 and 5). In contrast, a surprising result arises with respect to real GDP growth and volatility of inflation. When we remove Argentina from the sample, volatility of inflation is no longer significant and changes sign (see panel 11, Figures 4 and 5); at the same time, real GDP growth becomes significant with a considerably larger negative sign (see panel 6, Figures 4 and 5).

Argentina seems to be an influential observation for other variables as well. For instance, volatility of interest rates becomes highly significant when this country is dropped from the sample. Moreover, although other observations such as Czech Republic and Russia seem to be influential for the significance of this variable (see panel 8, Figure 5).

In results not reported, the effect of these countries is no longer influential once Argentina is taken out of the sample. Thus, without Argentina, volatility of interest rate is significant at 5% level no matter which other country is omitted. Something similar occurs with inflation; indeed, the apparent influential effects on the significance of inflation of countries such as Lithuania, Peru, and Turkey are drastically diminished once Argentina is out of the sample.[13]

[13] Inflation remains significant at 5% when either Lithuania or Turkey is dropped from the sample without Argentina. For Peru, the variable is significant only at 13%.

Column 4 of Table (9) presents estimation results of the SUR model when Argentina is removed from the sample. As shown in Figures 4 and 5, the main differences with respect to column 1 include the loss of log market capitalization and volatility of inflation as significant explanatory variables, and the gain of real GDP growth as a significant variable. From these diagnostics we find that the results for six variables, namely emerging, log nominal GDP, number of listings, inflation, volatility of interest rates, and volatility of real GDP growth, are quite robust. Regarding real GDP growth and volatility of inflation, the results presented in the previous section should be taken with caution given the sensitivity of the corresponding estimates to the inclusion of Argentina in the sample.

However, dropping Argentina from the sample might be unsatisfactory not only because this country is an important emerging market in which the relation between macroeconomic environment and financial volatility might be of particular interest (especially during the period surrounding the recent Argentine crisis, 2001-2002), but also because looking at the macroeconomic time series of Argentina, we did not find a conclusive argument to support the deletion of this country.

Therefore, we explore the possibility of giving more structure to the unobserved individual country effects in order to evaluate their possible impacts in our results. Specifically, we estimate an alternative panel data model that accounts for individual country random effects, keeping the time fixed effects, and allows for serial correlation in the remainder error term using a simple first order autoregressive process.[14] In fact, this reflects the effect of unobserved variables that are serially correlated across time. Thus, the error term in Equation (18) is modeled as follows:

(19) $$\mu_{i,t} = \lambda_t + \eta_i + \nu_{i,t},$$

where

$$\begin{aligned}
&\lambda_t = \text{time fixed effects} \\
&\eta_i \sim iid(0, \sigma_\eta) \\
&\nu_{i,t} = \rho\nu_{i,t-1} + \varepsilon_{i,t} \\
&\varepsilon_{i,t} \sim iid(0, \sigma_\varepsilon) \\
&\varepsilon_{i,t} \perp \eta_i
\end{aligned}$$

Estimation results for this model are shown in the last column of Table (9). We confirm the robustness of our results with respect to the six variables mentioned above. Moreover, in this case neither real GDP growth nor volatility of inflation is significant. Interestingly, even though all countries were included in the sample, these results look quite similar to those in column 4, corresponding to the SUR model without Argentina. Therefore, modeling random country effects seems to account for the effect of unobservables associated with influential observations.[15]

7. Realized Volatility

We continue our robustness analysis by comparing the estimation results of the cross-sectional expected volatility model with alternative measures of long term volatilities.

[14] References for panel data models with serial correlation include Lillard and Willis (1978), Baltagi and Li (1991), and Chamberlain (1984).

[15] Specifications with fixed country effects were also considered; however, as we expected from our earlier discussion about the little time variability observed in most of our explanatory variables, the Hausman (1978) test rejected in general fixed effects specifications in favor of random effects models.

First, we estimate a system of equations using the annual realized volatility instead of the Spline-GARCH low frequency volatility. Following Equation (7), the annualized realized volatility can be expressed as:

(20) $$Rvol_{i,t} = \left(\sum_{d=1}^{M_{i,t}} r_{i,t,d}^2 \right)^{1/2},$$

where $M_{i,t}$ is the number of trading days observed in country i at year t, and $r_{i,t,d}^2$ denotes the daily square return observed in country i at day d of year t. Thus, we can specify the system of linear equations for annual realized volatilities as follows:

(21) $$Rvol_{i,t} = \underline{z}'_{i,t}\beta_t + \upsilon_{i,t},\ t = 1,2,...,T,\ i = 1,2,...,N_t,$$

where the same explanatory variables are included, and the error term $\upsilon_{i,t}$ satisfies the same conditions mentioned in Section 5. The estimation results for realized volatilities are presented in column 1 of Table (10). We observe the same signs for most of the variables with exception of volatility of inflation. Specifically, volatility of inflation shows a negative and insignificant effect on realized volatilities, contrasting with the low frequency volatility case, in which the effect was positive and highly significant.

Column 2 of Table (10) shows estimation results for the "best" reduction based on the same criterion described in the previous section. Specifically, for realized volatilities, the least significant variable is the indicator of transition, followed by volatility of inflation, and inflation rate. In this case, our information criterion suggests that omitting these three variables is optimal. Hence, in contrast with the low frequency volatility from the Spline-GARCH model, the realized volatility shows almost no responsiveness to inflation variables but is significantly negatively affected by the real GDP growth, a variable that is characterized by its low correlation across time with respect to other explanatory variables.

As in the case of low frequency volatilities, we perform a diagnostic analysis by reestimating the SUR model dropping from the sample one country at a time. Figures 6 and 7 present the estimates and t-statistics respectively. In this case, Argentina also seems to be an influential observation for volatility of inflation and real GDP growth (see panels 6 and 11, Figures 6 and 7). Nevertheless, volatility of inflation is never significant and real GDP growth is always significant. Figure 7 suggests that five variables, namely emerging, log nominal GDP, real GDP growth, volatility of interest rates, and volatility of real GDP growth, are always significant at 5% level no matter which country is deleted from the sample. On the other hand, number of listings is sensitive to the inclusion of the UK, and log market capitalization is sensitive to the inclusion of Chile, India, Poland, and South Africa. The last two columns of Table (10) confirm this description. The results from a SUR model without Argentina do not change too much with respect to the results in column 1 (including all countries). However, when random country effects are introduced, number of listings and log market capitalization are no longer significant. Just the five variables named above remain significant. Note that four of them, namely emerging, log nominal GDP, volatility of interest rates, and volatility of real GDP growth, coincide with the "robust" variables in the low frequency volatility case. Nevertheless, the main difference with respect to this case is maintained. Real GDP growth is always relevant for realized volatility but not for low frequency volatility; and inflation is always significant for low frequency volatility but never for realized volatility. Moreover, number of listings is also always significant for low frequency volatility, but it is not for realized volatility in the random effects model.

Furthermore, we observe that among the SUR specifications, the determinant of the residual covariance is smaller for the models with low frequency volatility as dependent variable. This may suggest that low frequency volatility fits better in terms of the concentrated likelihood. In addition, Table (11) shows the R-squares for each equation in the SUR system for both low frequency and realized volatility. The results point to the same direction that the model using low frequency volatility shows better fit than that using realized volatility. In summary, as it is illustrated in Figure 2, discrepancies in the results between the spline and realized volatility might be due to the fact that the latter is a noisier measure of low frequency volatility.

We also compare the results in levels from the previous sections with the results from a model in logs. Specifically, we estimate two systems of equations, in which the log of both the low frequency volatility from the Spline-GARCH model and the annual realized volatility are the dependent variables for each year, respectively. Column 3 in Tables (9) and (10) presents estimation results for these cases. Note that for most of the variables the signs do not change with respect to the models in levels. The only exception is the real GDP growth rate for low frequency volatility, whose coefficient turns positive, albeit it is the least significant variable. In fact, our reduction process suggests that omitting only this variable leads to the "best" specification.

8. Concluding Remarks

We introduce a new model to characterize the long term pattern of market volatility in terms of its low frequency component. Keeping the attractiveness of a GARCH framework, we model the slow moving trend of volatility taking a non-parametric approach that leads to a smooth curve that describes the low frequency volatility. A special feature of this model is that the unconditional volatility coincides with the low frequency volatility.

After proposing a method to estimate the low frequency volatility component, a deeper question arises: what influences this low frequency volatility? We answer this question empirically. We perform a cross-sectional analysis of low frequency volatility to explore its macroeconomic determinants by considering evidence from international markets.

Our empirical evidence suggests that long term volatility of macroeconomic fundamentals, such as GDP and interest rates, are primary causes of low frequency market volatility. These variables show a strong positive effect in the cross sectional analysis. In addition, volatility of inflation also presents a positive effect, but in this case, the result is sensitive to the inclusion of one country, Argentina. Countries with high inflation and countries with low real growth rate have higher volatility although the importance of real growth also depends on Argentina.

In line with other empirical studies, we find that market development is also a significant determinant. Emerging markets show higher levels of low frequency market volatilities. An explanation may be that emerging markets are typically associated with larger inflation rates.

Market size variables are also important. The number of listed companies, as an indicator of the span of local diversification opportunities, reduces low frequency market volatility. In addition, the size of the economies measured by the log of GDP in US dollars increases low frequency volatilities; bigger countries have more volatility.

After performing some diagnostic analyses, we conclude that the results are robust for all variables except volatility of inflation and real GDP growth for which statistical significance is sensitive to influential observations.

We compare our results with the results of annual realized volatility as an alternative measure of low frequency volatility. We find changes in significance due to the fact that realized volatility is a noisier measure of low frequency volatility than the spline volatility. Inflation variables are no longer good predictors of annual realized volatilities.

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Figure 1

High and Low frequency Volatility S&P500

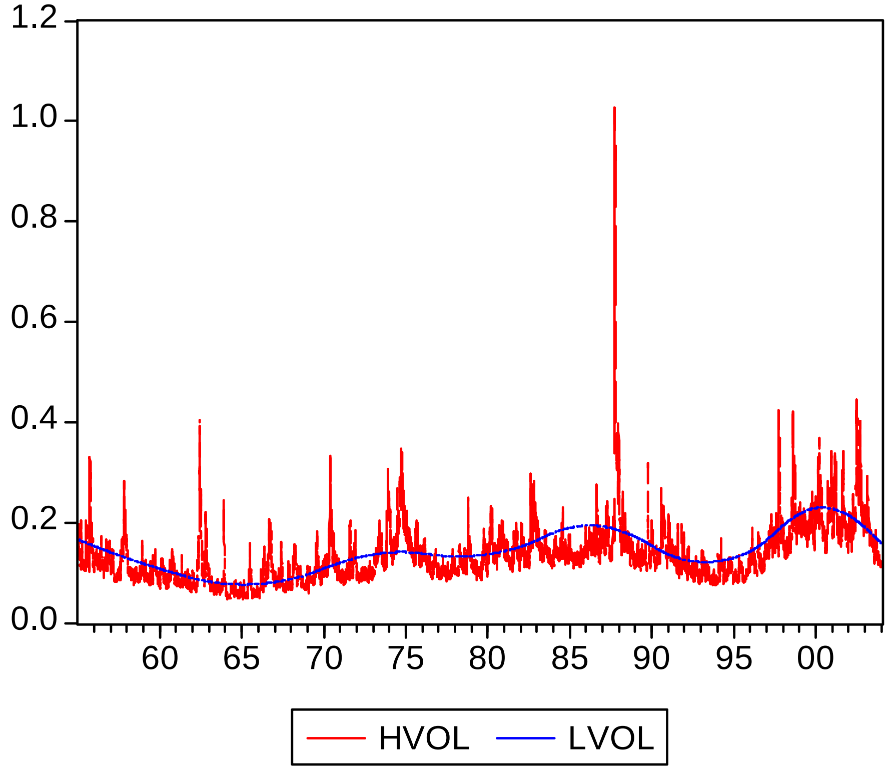


Figure 2
High Frequency, Low frequency, and Annual Realized Volatilities of Selected Countries

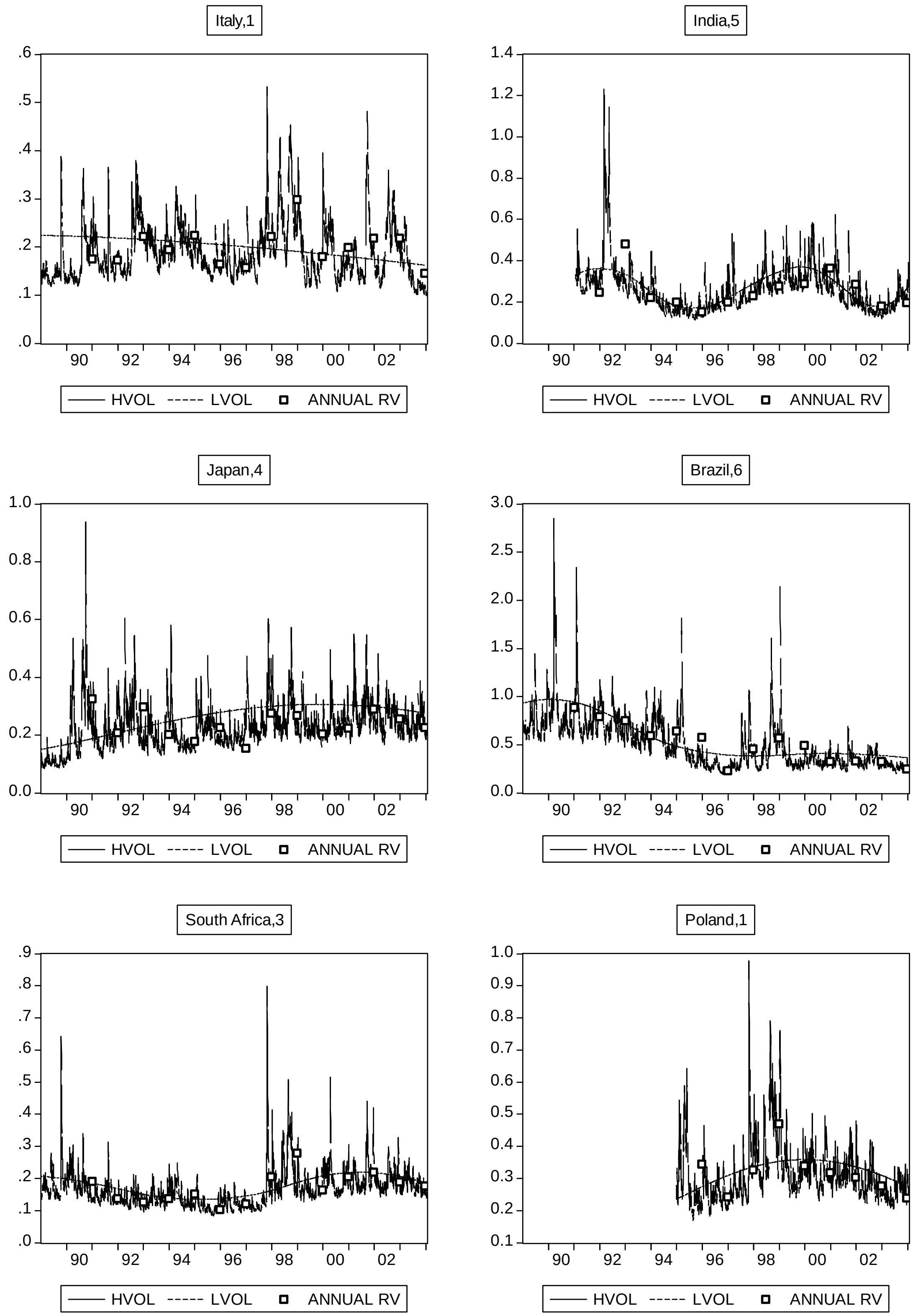
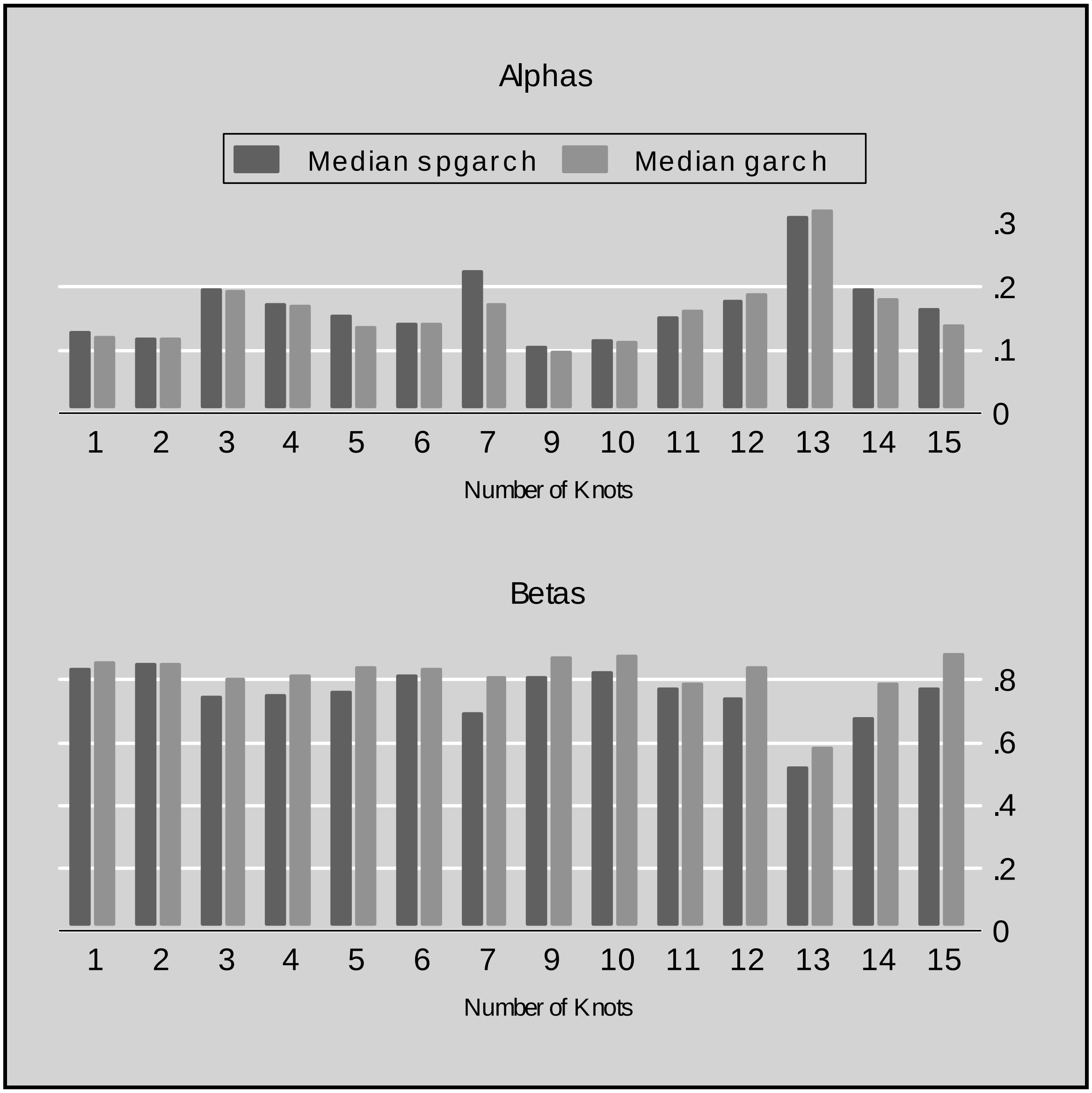


Figure 3
Dependence Structure in the Spline-GARCH and GARCH(1,1) Models[a]



a) In the Spline-GARCH model (spgarch), the "alphas" and "betas" correspond to the specification in Equation (13). In the GARCH(1,1) model (garch), these values correspond to the specification in Equation (10).

Figure 4
Estimates for Low Frequency Volatility: Droping One Country at a Time

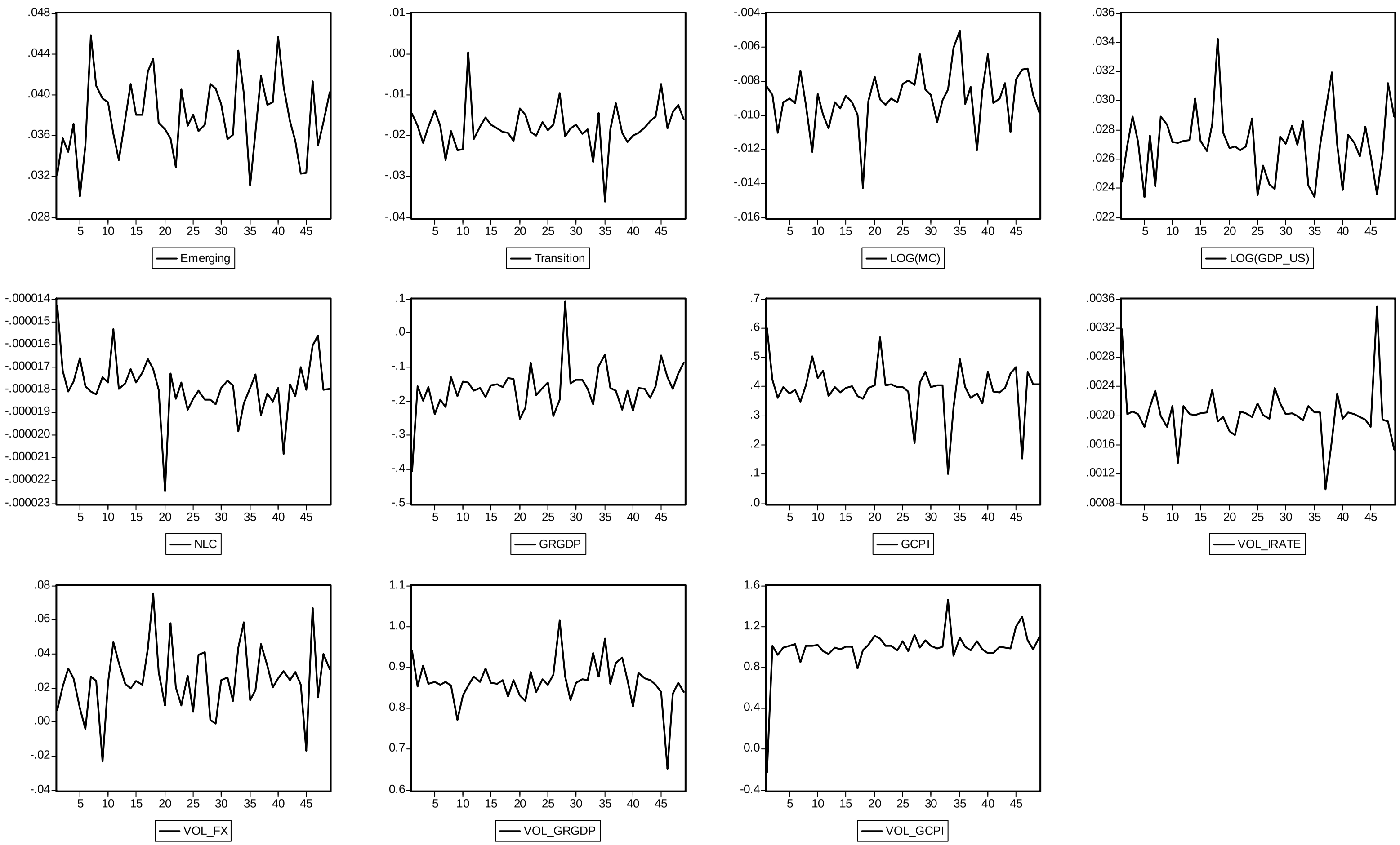


Figure 5
T-Statistics for Low Frequency Volatility: Droping One Country at a Time

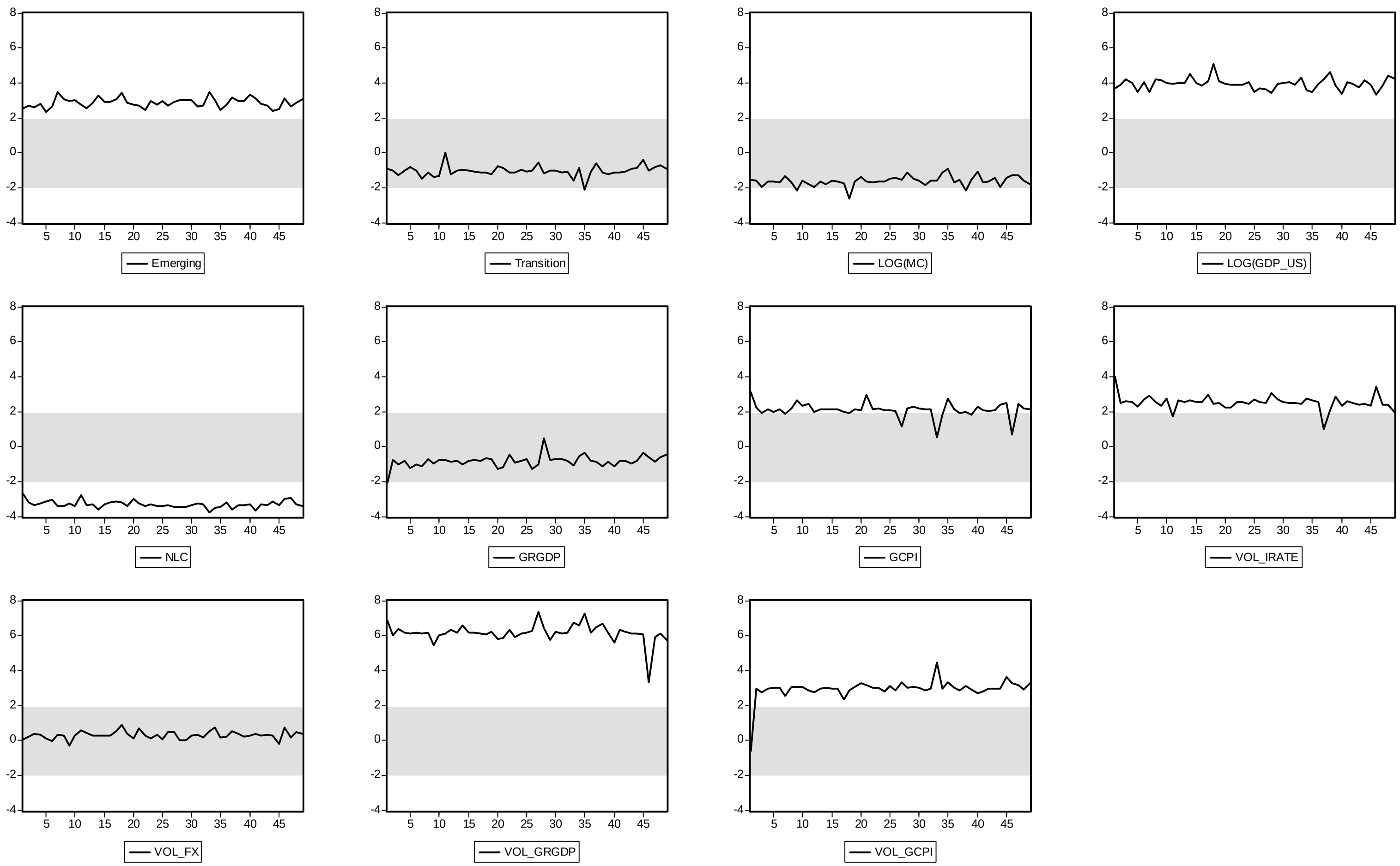


Figure 6
Estimates for Realized Volatility: Droping One Country at a Time

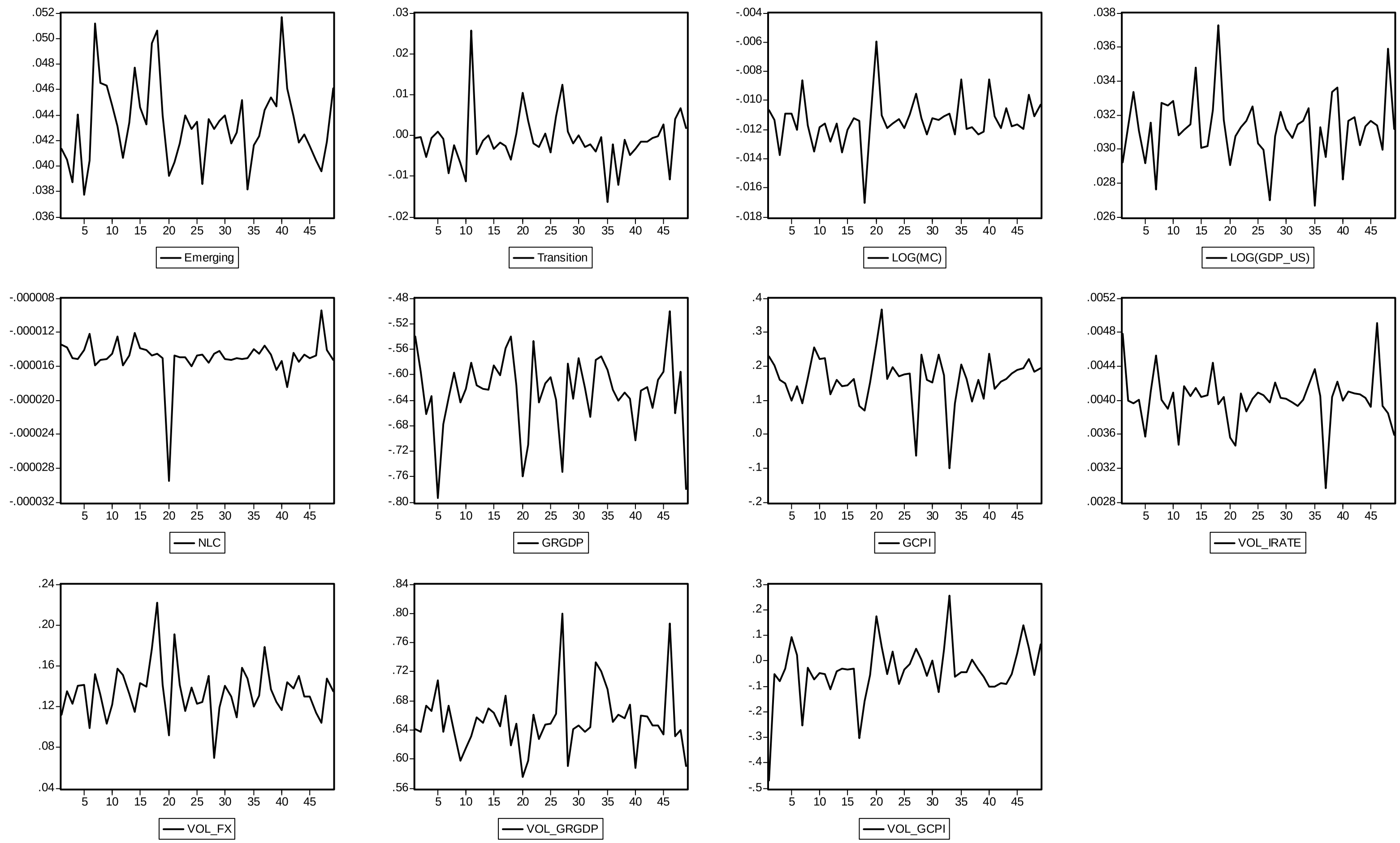


Figure 7
T-Statistics for Realized Volatility: Droping One Country at a Time

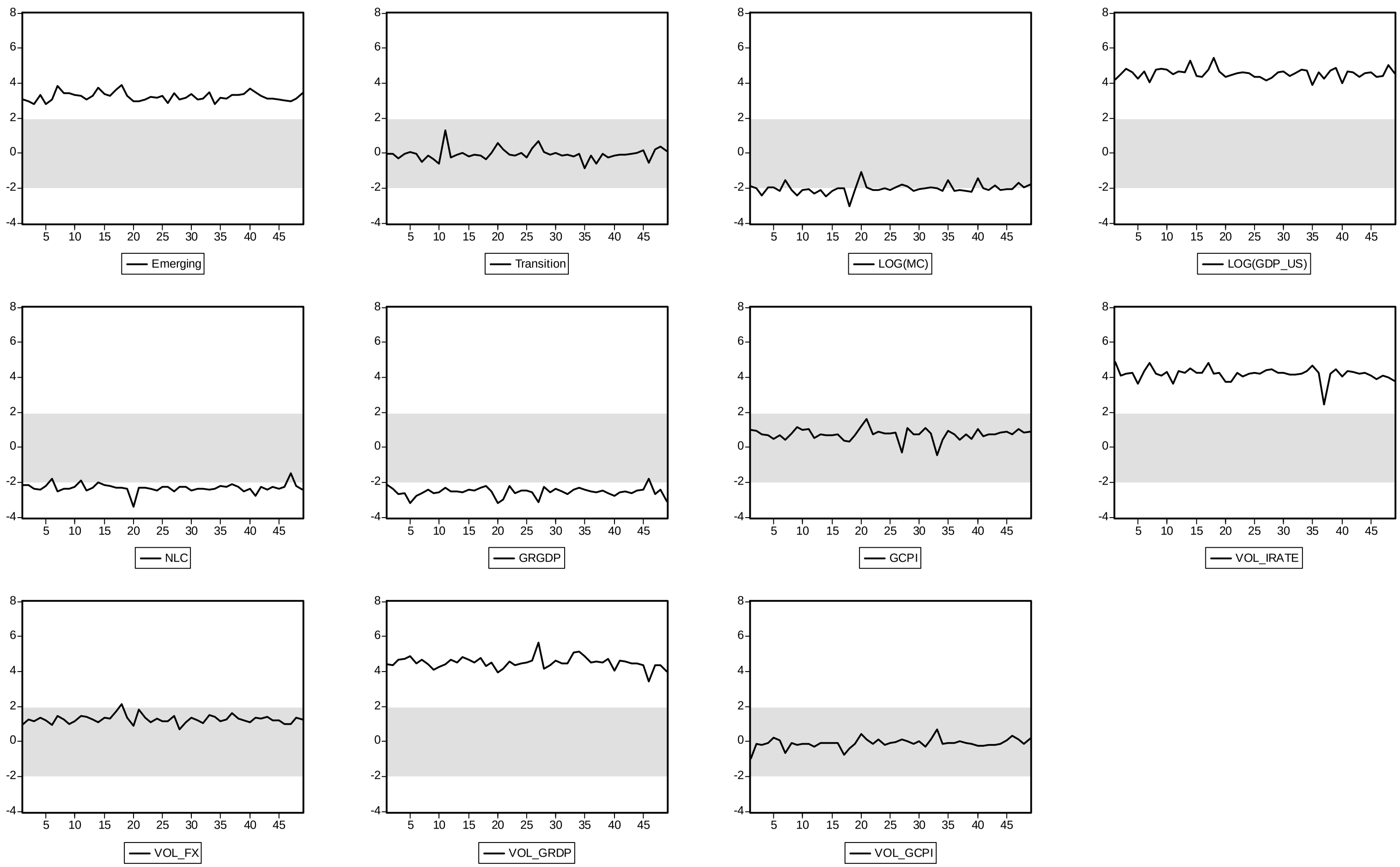


Table (1)

Estimation Results for the S&P500 (1955-2004)[a]

	Coefficient	Std. Error
c	1.1373	0.0436
w_0	-0.0003	7.5E-05
w_1	-1.9E-08	2.6E-08
w_2	2.7E-07	2.9E-08
w_3	-4.4E-07	3.9E-08
w_4	3.3E-07	5.4E-08
w_5	-4.0E-07	5.4E-08
w_6	6.0E-07	5.9E-08
w_7	-8.0E-07	9.9E-08
α	0.0895	0.0024
β	0.8810	0.0046
Log likelihood	-15733.51	
BIC	2.5348	

a) Estimation based a model with Gaussian Innovations. See model Specification in Equations (12), (13) and (14).

Table (2)

Country	Market Clasification	Exchange	Name of the Index	Average No. of Listings	Average Market Capitalization
Argentina	emerging	Buenos Aires	IVBNG	143	35352.96
Australia	developed	Australian	ASX	1236	295354.2
Austria	developed	Wiener Börse	ATX	137	31104.35
Belgium	developed	Euronext	CBB	1229	128803.2
Brazil	emerging	Sao Paulo	BOVESPA	513	155037
Canada	developed	TSX Group	S&P/TXS 300	1633	501122.3
Chile	emerging	Santiago	IGPAD	261	54529.27
China	emerging	Shanghai Stock Exchange	SSE-180	370	216199.3
Colombia	emerging	Bogota	IGBC	109	11480.09
Croatia	emerging	Zagreb	CROBEX	57	2406
Czech Republic	emerging	PSE	SE PX-50 Index	563	13319.22
Denmark	developed	Copenhagen	KAX All-Share Index	241	72720.3
Finland	developed	Helsinki	HEX	106	113409
France	developed	Euronext	CAC-40*	1229	752041.9
Germany	developed	Deutsche Börse	DAX	880	759628.3
Greece	developed	Athens	Athens SE General Index	224	56050.52
Honk Kong	developed	Hong Kong	Hang Seng Composite Index	637	389810
Hungary	emerging	Budapest	Budapest SE Index*	53	9728.453
India	emerging	Mumbai	Mumbay SE-200 Index	5696	128732.4
Indonesia	emerging	Jakarta	Jakarta SE Composite Index	243	36744.79
Ireland	developed	Irish	ISEQ Overall Price Index	89	69934.38
Israel	emerging	Tel-Aviv	TA SE All-Security Index	563	41720.75
Italy	developed	Borsa Italiana	Milan MIB General Index	263	374715.4
Japan	developed	Tokyo	Nikkei 225	1911	2930639
Korea	emerging	Korea	KOSPI	708	163264.7
Lithuania	emerging	National SE of Lithuania	Lithuania Litin-G Stock Index	174	3190.185
Malaysia	emerging	Bursa Malaysia	KLSE Composite	610	141464.6
Mexico	emerging	Mexico	IPC	208	119904.7
Netherlands	developed	Euronext	AEX	1229	366983.1
New Zealand	developed	New Zealand	New Zealand SE All-Share Capital Index	190	23119.93
Norway	developed	Oslo	Oslo SE All-Share Index	175	50232.67
Peru	emerging	Lima	Lima SE General Index	235	8892.879
Philippines	emerging	Philippine	Manila SE Composite Index	205	33072.59
Poland	emerging	Warsaw	Poland SE Index (Zloty)	129	15687.93
Portugal	developed	Euronext	Portugal PSI General Index*	1229	32279.57
Russia	emerging	Russian Exchange	Russia AKM Composite	169	52182.45
Singapore	developed	Singapore	SES All-Share Index	336	114633.9
Slovak Republic	emerging	Bratislava	SAX Index	764	3909.196
South Africa	emerging	JSE South Africa	FTSE/JSE All-Share Index	618	200916.7
Spain	developed	Spanish Exchanges (BME)	Madrid SE General Index	3119	315363.5
Sweden	developed	Stockholmsbörsen	SAX All-Share index	242	206177.8
Switzerland	developed	Swiss Exchange	Switzerland Price Index	431	463321.4
Taiwan	emerging	Taiwan	Taiwan SE Capitalization Weighted Index	410	237885.5
Thailand	emerging	Thailand	SET General Index	369	68325.18
Turkey	emerging	Istanbul	Istanbul SE IMKB-100 Price Index	227	41548.86
United Kingdom	developed	London	FTSE-250*	2497	1739880
United States	developed	NYSE	S&P500	2298	6805999
Venezuela	emerging	Caracas	Caracas SE General Index	71	7718.482

Source: Global Financial Data and Datastream*
Yearly Averages over the period 1990-2003
Units market capitalization: USD millions

Table (3)

Estimation Results: Spline-GARCH and GARCH(1,1) Models

	1	2	3	4	5	6	7	8	9	10	11	12	13
Country	Knots[a]	Starting Year[b]	Obs	obs/knot[c]	Alpha[d]		Beta[e]		Log likelihood		BIC		LRT[f]
					spgarch	garch	spgarch	garch	spgarch	garch	spgarch	garch	
ARGENTINA	3	Jan-67	9,240	3,080	0.21	0.19	0.76	0.83	-8785.2	-8879.7	1.9085	1.9252	189.0
AUSTRALIA	1	Jan-58	11,682	11,682	0.23	0.17	0.71	0.84	-14244.6	-14396.8	2.4427	2.4674	304.4
AUSTRIA	11	Jan-86	4,574	416	0.15	0.12	0.77	0.87	-5733.3	-5816.8	2.5346	2.5495	166.9
BELGIUM	2	Jan-91	3,370	1,685	0.12	0.12	0.85	0.85	-4153.7	-4167.6	2.4796	2.4813	27.6
BRAZIL	6	Jan-72	8,220	1,370	0.14	0.14	0.82	0.87	-9705.7	-9775.2	2.3724	2.3820	139.0
CANADA	10	Jan-76	7,182	718	0.11	0.11	0.82	0.87	-8892.1	-8957.4	2.4897	2.4946	130.7
CHILE	4	May-76	7,003	1,751	0.36	0.37	0.52	0.55	-8819.5	-8963.8	2.5289	2.5638	288.6
CHINA	7	Jan-95	2,266	324	0.25	0.17	0.59	0.81	-2786.2	-2927.2	2.4966	2.5950	282.0
COLOMBIA	13	Jan-92	2,971	229	0.46	0.49	0.30	0.36	-3752.1	-3854.5	2.5715	2.6037	205.0
CROATIA	3	Jan-97	1,723	574	0.20	0.21	0.64	0.76	-2020.2	-2072.5	2.3752	2.4201	104.7
CZECHREP	1	Sep-94	2,405	2,405	0.15	0.13	0.78	0.86	-3143.9	-3168.1	2.6307	2.6443	48.3
DENMARK	5	Jan-79	6,344	1,269	0.22	0.16	0.65	0.81	-8220.0	-8305.9	2.6038	2.6231	171.8
FINLAND	4	Jan-87	4,379	1,095	0.15	0.12	0.76	0.88	-4979.5	-5069.3	2.2896	2.3216	179.6
FRANCE	1	Sep-87	4,385	4,385	0.09	0.09	0.88	0.89	-5715.2	-5716.4	2.6163	2.6136	2.6
GERMANY	6	Sep-59	11,208	1,868	0.14	0.14	0.82	0.84	-13953.2	-14022.9	2.4982	2.5050	139.4
GREECE	7	Oct-88	3,926	561	0.20	0.19	0.69	0.81	-4910.6	-4978.9	2.5247	2.5433	136.7
HONGKONG	1	Nov-69	8,528	8,528	0.15	0.15	0.84	0.85	-10237.0	-10249.5	2.4061	2.4072	25.1
HUNGARY	4	Feb-91	3,496	874	0.22	0.18	0.66	0.79	-4224.4	-4292.2	2.4354	2.4632	135.6
INDIA	5	Jan-91	3,157	631	0.14	0.13	0.78	0.85	-3994.5	-4038.8	2.5536	2.5671	88.4
INDONESIA	15	Apr-83	5,204	347	0.20	0.17	0.75	0.87	-4539.6	-4779.5	1.7759	1.8421	479.6
IRELAND	9	Jan-87	4,348	483	0.11	0.10	0.80	0.87	-5539.7	-5602.2	2.5732	2.5833	125.1
ISRAEL	11	Jun-81	5,665	515	0.14	0.16	0.77	0.79	-7423.5	-7510.1	2.6437	2.6565	173.3
ITALY	1	Jan-75	7,421	7,421	0.09	0.09	0.89	0.89	-9702.5	-9712.2	2.6209	2.6214	19.3
JAPAN	4	Jan-55	13,759	3,440	0.17	0.16	0.78	0.84	-16702.2	-16824.7	2.4334	2.4479	245.0
KOREA	15	Jan-62	12,136	809	0.13	0.11	0.80	0.90	-11875.8	-12034.8	1.9718	1.9858	318.0
LITHUANIA	6	Jun-98	1,536	256	0.16	0.17	0.64	0.52	-2081.3	-2126.4	2.7578	2.7831	90.2
MALAYSIA	14	Jan-80	6,057	433	0.19	0.19	0.67	0.78	-6942.0	-7050.7	2.3158	2.3305	217.4
MEXICO	12	Jan-85	4,859	405	0.14	0.12	0.74	0.85	-5940.6	-6010.4	2.4731	2.4797	139.7
NETHERLANDS	1	Jan-83	5,433	5,433	0.11	0.11	0.87	0.88	-6607.8	-6613.7	2.4404	2.4398	11.7
NEWZEALAND	3	Jul-86	4,512	1,504	0.19	0.20	0.73	0.78	-5708.5	-5745.5	2.5434	2.5529	73.9
NORWAY	4	Jan-83	5,385	1,346	0.18	0.19	0.73	0.76	-6886.8	-6928.7	2.5705	2.5786	83.9
PERU	11	Jan-82	5,580	507	0.27	0.30	0.65	0.70	-6349.4	-6451.1	2.2990	2.3173	203.4
PHILIPPINES	13	Jan-86	4,580	352	0.16	0.15	0.74	0.80	-5693.5	-5820.3	2.5143	2.5444	253.6
POLAND	1	Jan-95	2,338	2,338	0.11	0.11	0.83	0.84	-3121.4	-3127.5	2.6867	2.6865	12.3
PORTUGAL	7	May-88	4,216	602	0.28	0.09	0.56	0.90	-5133.7	-5315.6	2.4571	2.5282	363.8
RUSSIA	14	Jan-95	2,338	167	0.20	0.17	0.68	0.79	-2825.9	-2870.8	2.3374	2.3560	89.9
SINGAPORE	7	Jul-65	9,917	1,417	0.22	0.21	0.74	0.79	-11694.1	-11851.3	2.3686	2.3931	314.4
SLOVAKREP	5	Oct-93	2,507	501	0.16	0.14	0.74	0.82	-2942.7	-3000.9	2.3757	2.4043	116.4
SOUTHAFRICA	3	May-86	4,618	1,539	0.12	0.11	0.82	0.86	-5988.7	-6011.4	2.6064	2.6095	45.6
SPAIN	5	Aug-71	7,454	1,491	0.14	0.11	0.81	0.86	-9477.8	-9559.3	2.5538	2.5688	163.0
SWEDEN	4	Jun-86	4,525	1,131	0.12	0.12	0.82	0.85	-5737.8	-5765.6	2.5509	2.5545	55.6
SWISS	6	Jan-69	8,862	1,477	0.14	0.14	0.81	0.83	-11011.8	-11099.1	2.4954	2.5082	174.7
TAIWAN	3	Jan-67	10,650	3,550	0.10	0.09	0.88	0.91	-12893.4	-12949.8	2.4260	2.4334	112.7
THAILAND	12	May-75	7,271	606	0.18	0.19	0.75	0.84	-7852.8	-7992.7	2.1778	2.2007	279.7
TURKEY	3	Nov-87	4,143	1,381	0.22	0.20	0.72	0.76	-5433.3	-5450.4	2.6370	2.6378	34.1
UK	1	Jan-87	4,563	4,563	0.17	0.17	0.76	0.80	-5742.2	-5799.8	2.5261	2.5482	115.1
US	7	Jan-55	12,455	1,779	0.09	0.08	0.88	0.92	-15733.5	-15811.2	2.5348	2.5412	155.3
VENEZUELA	12	Jan-94	2,492	208	0.35	0.33	0.34	0.64	-3103.2	-3203.7	2.5407	2.5817	201.1

a) Optimal number of knots in the Spline-GARCH model.

b) Starting date in the Sample Period. Ending date is June 31, 2006.

c) Number of Observations per Knot in the Spline-GARCH model (Ratio of Column 3 to Column 1).

d) ARCH effects in the Spline-GARCH model (spgarch) and the GARCH(1,1) model (garch).

e) GARCH effects in the Spline-GARCH model (spgarch) and the GARCH(1,1) model (garch).

f) Statistic of Likelihood Ratio Test: GARCH(1,1) vs Spline-GARCH.

Table (4)

Descriptive Statistics on the Distribution of the Number of Observations per Knot in the Spline-GARCH Model[a]

	Country Classification		
	Developed	Emerging[b]	Transition Econ.
Number of Countries	23	18	7
Minimum	415.82	207.67	167.00
Maximum	11682.00	3550.00	2405.00
Mean	2795.39	1002.03	1016.53
Standard Deviation	2951.17	969.33	953.54
Quantiles			
25%	1094.75	352.31	256.00
50%	1490.80	560.46	574.33
75%	4385.00	1381.00	2338.00

a) The variable "Observations per Knot" is presented in column 4 of Table (3).

b) Emerging markets excluding emerging transition economies.

Table (5)

Explanatory Variables

Name	Description
emerging	Indicator of Market Development (1=Emerging, 0=Developed)
Transition	Indicator of Transition Economies (Central European and Baltic Countries)
log(mc)	log Market Capitalization ($US)
log(gdp_dll)	Log Nominal GDP in Current $US
nlc	Number of Listed Companies in the Exchange
grgdp	GDP Growth Rate
gcpi	Inflation Rate
vol_irate	Volatility of Short Term Interest Rate*
vol_forex	Volatility of Exchange Rates*
vol_grgdp	Volatility of GDP*
vol_gcpi	Volatility of Inflation*

*Volatilities are obtained from the residuals of AR(1) models

Table (6)

Correlation Low Frequency Volatilities Across Years							
	LVOL1997	LVOL1998	LVOL1999	LVOL2000	LVOL2001	LVOL2002	LVOL2003
LVOL1997	1	0.76800	0.79614	0.71752	0.64246	0.66100	0.74651
LVOL1998	0.76800	1	0.91144	0.71398	0.52270	0.49749	0.58763
LVOL1999	0.79614	0.91144	1	0.88333	0.72605	0.68825	0.70021
LVOL2000	0.71752	0.71398	0.88333	1	0.93833	0.87955	0.84312
LVOL2001	0.64246	0.52270	0.72605	0.93833	1	0.94249	0.87678
LVOL2002	0.66100	0.49749	0.68825	0.87955	0.94249	1	0.91471
LVOL2003	0.74651	0.58763	0.70021	0.84312	0.87678	0.91471	1

Table (7)

Correlation of Residuals from Yearly Regressions (1997-2003)							
	RES97	RES98	RES99	RES00	RES01	RES02	RES03
RES97	1	0.72148	0.58690	0.63573	0.52845	0.51425	0.66501
RES98	0.72148	1	0.76567	0.70793	0.50636	0.46868	0.49255
RES99	0.58690	0.76567	1	0.76222	0.49994	0.54647	0.47898
RES00	0.63573	0.70793	0.76222	1	0.90622	0.82757	0.78706
RES01	0.52845	0.50636	0.49994	0.90622	1	0.89690	0.82175
RES02	0.51425	0.46868	0.54647	0.82757	0.89690	1	0.85353
RES03	0.66501	0.49255	0.47898	0.78706	0.82175	0.85353	1

Table (8)

Individual SUR Regressions[a]					
	Coefficient	Std. Error	t-Statistic	Prob.	Det Residual Covariance
emerging	0.0957	0.0176	5.4528	0.0000	6.45E-39
transition	-0.0077	0.0180	-0.4284	0.6685	1.53E-38
log(mc)	-0.0093	0.0032	-2.9345	0.0035	3.76E-38
log(gdp_dll)	0.0015	0.0055	0.2740	0.7842	2.18E-37
nlc	-1.29E-05	0.0000	-2.3706	0.0181	1.23E-37
grgdp	-0.6645	0.1255	-5.2945	0.0000	3.89E-38
gcpi	0.6022	0.0418	14.4181	0.0000	1.64E-38
vol_irate	0.0089	0.0006	14.4896	0.0000	8.59E-39
vol_forex	0.5963	0.0399	14.9468	0.0000	2.47E-38
vol_grgdp	1.1192	0.1008	11.1056	0.0000	8.71E-39
vol_gcpi	0.9364	0.0848	11.0375	0.0000	2.84E-38

a) SUR estimation of annual low frequency volatilities on each individial variable (see Equation 18).

Table (9)

Estimation Results for Low Frequency Volatilities

	SUR Models				Panel Specification[a]
	All Countries	Opt. Reduction	Logs	Without Arg	Random Country Effects
emerging	0.0376	0.0387	0.2079	0.0322	0.0478
	(0.0131)**	(0.0128)**	(0.0592)**	(0.0128)**	(0.0212)**
transition	-0.0178	-0.0164	-0.0332	-0.0147	-0.0258
	(0.0171)	(0.0167)	(0.0741)	(0.0163)	(0.0304)
log(mc)	-0.0092	-0.0085	-0.0345	-0.0083	-0.0046
	(0.0055)*	(0.0053)	(0.0235)	(0.0054)	(0.0067)
log(gdpus)	0.0273	0.0271	0.1156	0.0245	0.0175
	(0.0068)**	(0.0066)**	(0.0302)**	(0.0067)**	(0.0099)*
nlc	-1.8E-05	-1.8E-05	-8.1E-05	-1.4E-05	-1.7E-05
	(5.4E-06)**	(5.3E-06)**	(2.3E-05)**	(5.2E-06)**	(8.6E-06)**
grgdp	-0.1603		0.0962	-0.4046	-0.2094
	(0.1930)		(0.7474)	(0.1984)**	(0.2258)
gcpi	0.3976	0.3915	1.1459	0.5985	0.6114
	(0.1865)**	(0.1641)**	(0.7755)	(0.1939)**	(0.2229)**
vol_irate	0.0020	0.0022	0.0061	0.0032	0.0034
	(0.0008)**	(0.0008)**	(0.0031)*	(0.0008)**	(0.0009)**
vol_gforex	0.0222		0.0185	0.0068	-0.0221
	(0.0844)		(0.3383)	(0.0878)	(0.0959)
vol_grgdp	0.8635	0.8373	2.5808	0.9392	0.9019
	(0.1399)**	(0.1352)**	(0.6138)**	(0.1371)**	(0.1862)**
vol_gcpi	0.9981	1.0983	3.1467	-0.2243	-0.0849
	(0.3356)**	(0.3208)**	(1.3431)**	(0.3627)	(0.3917)
d1990	0.1532	0.1471	-1.8546	0.1638	0.0252
	(0.04835)**	(0.0472)**	(0.2068)**	(0.0470)**	(0.0185)
d1991	0.1488	0.1427	-1.8687	0.1569	0.0160
	(0.0480)**	(0.0468)**	(0.2058)**	(0.0465)**	(0.0173)
d1992	0.1314	0.1245	-1.9539	0.1407	0.0004
	(0.0472)**	(0.0459)**	(0.2037)**	(0.0457)**	(0.0170)
d1993	0.1435	0.1362	-1.9398	0.1447	0.0000
	(0.0498)**	(0.0485)**	(0.2118)**	(0.0480)**	(0.0159)
d1994	0.1244	0.1169	-2.0181	0.1314	-0.0138
	(0.0498)**	(0.0484)**	(0.2144)**	(0.0481)**	(0.0152)
d1995	0.1230	0.1150	-2.0304	0.1320	-0.0236
	(0.0490)**	(0.0477)**	(0.2115)**	(0.0476)**	(0.0141)*
d1996	0.1177	0.1087	-2.0580	0.1274	-0.0276
	(0.0491)**	(0.0479)**	(0.2120)**	(0.0476)**	(0.0134)**
d1997	0.1371	0.1284	-1.9570	0.1483	-0.0068
	(0.0495)**	(0.0482)**	(0.2124)**	(0.0479)**	(0.0124)
d1998	0.1831	0.1763	-1.7804	0.1951	0.0455
	(0.0506)**	(0.0493)**	(0.2150)**	(0.0490)**	(0.0121)**
d1999	0.2028	0.1938	-1.7047	0.2164	0.0648
	(0.0517)**	(0.0503)**	(0.2197)**	(0.0502)**	(0.0114)**
d2000	0.1941	0.1851	-1.7241	0.2049	0.0562
	(0.0499)**	(0.0486)**	(0.2135)**	(0.0484)**	(0.0104)**
d2001	0.1762	0.1683	-1.7837	0.1866	0.0406
	(0.0493)**	(0.0479)**	(0.2110)**	(0.0477)**	(0.0094)**
d2002	0.1619	0.1540	-1.8487	0.1701	0.0242
	(0.0487)**	(0.0473)**	(0.2090)**	(0.0471)**	(0.0076)**
d2003	0.1358	0.1272	-1.9588	0.1456	0.0213
	(0.0505)**	(0.0490)**	(0.2167)**	(0.0487)**	(0.1032)
Det residual covariance	2.3E-38	3.8E-39	4.2E-22	1.6E-39	
BIC	-88.067	-88.15	-48.89	-89.00	

Standard errors reported in parentheses.
* Denotes significance at 10%.
**Denotes significance at 5%.
a) Estimated autocorrelation coefficient: **ρ** = 0.4731 (See Equation 19 for assumptions on the error term).

Table (10)

Estimation Results for Realized Volatilities

	SUR Models				**Panel Specification**
	All Countries	Opt. Reduction	Logs	Without Arg	Random Country Effects
emerging	0.0434	0.0408	0.0964	0.0413	0.0373
	(0.0134)**	(0.0124)**	(0.0317)**	(0.0136)**	(0.0199)*
transition	-0.0013		-0.0084	-0.0007	0.0018
	(0.0182)		(0.0417)	(0.0183)	(0.0282)
log(mc)	-0.0116	-0.0112	-0.0256	-0.0107	-0.0042
	(0.0055)**	(0.0052)**	(0.0130)**	(0.0056)*	(0.0074)
log(gdpus)	0.0314	0.0309	0.0730	0.0292	0.0245
	(0.0068)**	(0.0066)**	(0.0162)**	(0.0069)**	(0.0101)**
nlc	-1.5E-05	-1.4E-05	-3.8E-05	-1.3E-05	-1.3E-05
	(6.4E-06)**	(6.2E-06)**	(1.5E-05)**	(6.2E-06)**	(8.8E-06)
grgdp	-0.6222	-0.6568	-0.9639	-0.5400	-1.0773
	(0.2442)**	(0.2322)**	(0.5277)*	(0.2517)**	(0.2939)**
gcpi	0.1598		0.2366	0.2286	0.4299
	(0.2159)		(0.4840)	(0.2312)	(0.2630)
vol_irate	0.0040	0.0043	0.0059	0.0048	0.0056
	(0.0010)**	(0.0008)**	(0.0021)**	(0.0010)**	(0.0011)**
vol_gforex	0.1329	0.1649	0.2807	0.1120	0.1040
	(0.1057)	(0.0894)*	(0.2247)	(0.1105)	(0.1203)
vol_grgdp	0.6500	0.7002	1.3278	0.6414	0.6728
	(0.1437)**	(0.1277)**	(0.3378)**	(0.1463)**	(0.1989)**
vol_gcpi	-0.0432		-0.1124	-0.4683	-0.5073
	(0.3978)		(0.9042)	(0.4700)	(0.4799)
d1990	0.4158	0.4133	-0.9029	0.4187	0.0640
	(0.0512)**	(0.0471)**	(0.1172)**	(0.0515)**	(0.0193)**
d1991	0.3726	0.3702	-0.9944	0.3751	0.0189
	(0.0489)**	(0.0447)**	(0.1142)**	(0.0491)**	(0.0180)
d1992	0.3583	0.3551	-1.0306	0.3610	0.0045
	(0.0493)**	(0.0451)**	(0.1156)**	(0.0494)**	(0.0179)
d1993	0.3492	0.3457	-1.0560	0.3492	0.0008
	(0.0500)**	(0.0455)**	(0.1172)**	(0.0501)**	(0.0168)
d1994	0.3616	0.3570	-1.0243	0.3584	0.0187
	(0.0502)**	(0.0454)**	(0.1173)**	(0.0504)**	(0.0163)
d1995	0.3439	0.3403	-1.0681	0.3406	-0.0083
	(0.0513)**	(0.0464)**	(0.1193)**	(0.0514)**	(0.0151)
d1996	0.3194	0.3186	-1.1212	0.3202	-0.0368
	(0.0502)**	(0.0452)**	(0.1176)**	(0.0504)**	(0.0145)**
d1997	0.4102	0.4090	-0.9139	0.4127	0.0503
	(0.0509)**	(0.0458)**	(0.1184)**	(0.0511)**	(0.0135)**
d1998	0.4656	0.4630	-0.8042	0.4693	0.1095
	(0.0515)**	(0.0464)**	(0.1190)**	(0.0517)**	(0.0134)**
d1999	0.4136	0.4117	-0.9067	0.4168	0.0527
	(0.0524)**	(0.0471)**	(0.1218)**	(0.0526)**	(0.0128)**
d2000	0.4276	0.4259	-0.8772	0.4330	0.0630
	(0.0512)**	(0.0460)**	(0.1191)**	(0.0513)**	(0.0121)**
d2001	0.4157	0.4131	-0.8969	0.4193	0.0481
	(0.0505)**	(0.0454)**	(0.1177)**	(0.0507)**	(0.0114)**
d2002	0.4068	0.4048	-0.9206	0.4088	0.0415
	(0.0504)**	(0.0456)**	(0.1173)**	(0.0506)**	(0.0097)**
d2003	0.3616	0.3589	-1.0160	0.3657	-0.0904
	(0.0518)**	(0.0467)**	(0.1209)**	(0.0521)**	(0.0978)
Det residual covariance	3.6E-37	3.6E-37	1.8E-27	3.0E-37	
BIC	-83.58	-83.63	-61.25	-83.75	

Standard errors reported in parentheses.

* Denotes significance at 10%.

**Denotes significance at 5%.

a) Estimated autocorrelation coefficient: **ρ** = 0.4731 (See Equation 19 for assumptions on the error term).

Table (11)

R-Squared Statistics for Each Equation in the SUR System Including All Countries

	Low Frequency Vol[a]	Realized Vol[b]
1990	0.5816	0.4019
1991	0.6435	0.5786
1992	0.7293	0.3640
1993	0.6463	0.5102
1994	0.5798	0.5577
1995	0.6689	0.4982
1996	0.7040	0.7218
1997	0.5700	0.4172
1998	0.5608	0.4835
1999	0.4481	0.3878
2000	0.3908	0.2442
2001	0.3477	0.2556
2002	0.3636	0.0985
2003	0.3968	0.2026
Average	**0.5451**	**0.4087**

a) Values correspond to system in Equation (18).

b) Values correspond to system in Equation (21).