THE APPLICATION OF THE HOLLERITH TABULATING MACHINE TO BROWN'S TABLES OF THE MOON.

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Introduction.—Positions of the Moon derived from Brown's *Tables of the Motion of the Moon* were first introduced into national ephemerides in 1923. In these *Tables* over 1400 periodic terms with coefficients as large as 0″·003 are tabulated. Some idea of the extent of the *Tables* may be gathered from the fact that they contain 180 separate tables and cover 660 pages. Although

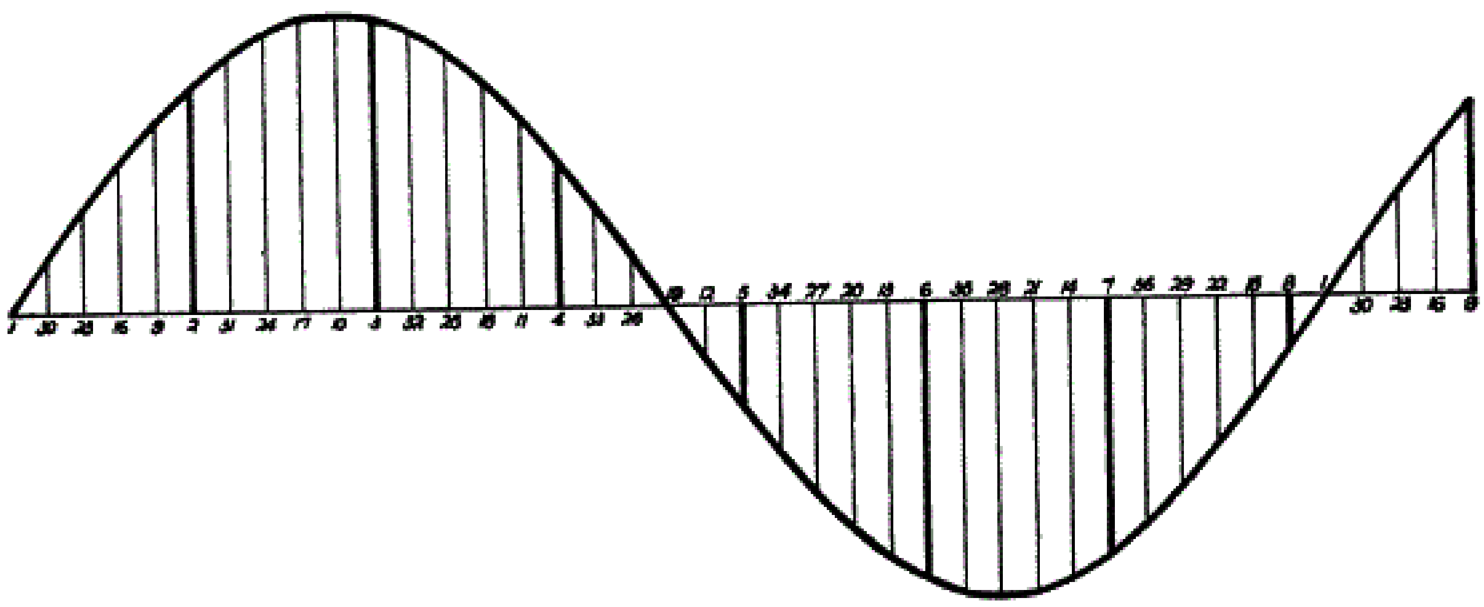

FIG. 1.

they are well arranged, the work of using them is laborious, and, before the advent of the Hollerith machine, represented the continuous work of two skilled computers. The mechanical methods that have been applied to certain portions of the work have eliminated much fatigue, increased tenfold the speed with which results can be obtained, and reduced the cost to one-quarter of its former amount.

The Tabulation of Harmonic Functions of the Time.—The inequalities arising in lunar theory are of the form $a \sin (b+ct)$, where a is the amplitude or coefficient, b is the phase angle at the moment when $t=0$, and c is the movement of the argument in a unit of time. This expression is called a harmonic; it represents a periodic term, of period $2\pi/c$. The *Tables* present values of these terms, and it is their selection and summation, or the synthesis of a series of harmonic terms, that constitutes the principal work of using the *Tables*. Where the periods of various terms are commensurable, for instance if they depend on $(b+ct)$, $2(b+ct)$, $3(b+ct)$, etc., they are combined into one table; it is this power of combination that has enabled 1400 terms to be tabulated in less than 200 tables. The fundamental periods of individual tables are not commensurable, so that, although the entries from each table are used over and over again, combinations of entries from any one group of tables do not recur. It is to be borne in mind that in preparing ephemerides the intervals of t are uniform, and that the period of each term is, in general, short when compared with the lifetime of the *Tables*.

Consider the tabulation of a term in which, for simplicity, $a=1$, $b=0°$, $c=50°$, and the unit of t is 1 day. The values required on successive days are represented by the ordinates 1, 2, 3 . . . in fig. 1. If we superimpose the values for day 9 and subsequent days on the first period of the curve, it is seen that this is eventually divided into 36 equidistant parts, after which the sequence repeats itself. This immediately suggests the procedure actually adopted in tabulation, namely, that a complete period of the harmonic should be subdivided into an integral number of parts, so chosen that the motion in a day (or other unit of time) is an integral number of these parts. In the illustration 1 day $=\frac{50°}{360°}=\frac{5}{36}$ revolutions, so that the constructor of the tables would divide the period into 36 integral parts, the movement per day being 5 of these parts. The order of the final tabular entries would be the order in which they are used.

In this simple illustration the daily motion in revolutions is represented by an exact fraction, but this is not the case in general. Consider the term $-1''\cdot272 \sin 2L$ in the long-period nutation in longitude, where L is the Sun's mean longitude. The daily motion of $2L$ (at epoch 1950) is $1°\cdot97129469$, or, in revolutions of 360°, $0^{r}\cdot0054758186$. This number of revolutions has to be represented by a proper fraction in its lowest terms. With the aid of a calculating machine one finds readily the figures in the accompanying table, the development being made in such a way that the residuals alternate in sign, each being numerically less than the preceding one.

d		r		r
182	=	1	−	0·003401
183	=	1	+	0·002075
365	=	2	−	0·001326
548	=	3	+	0·000749
913	=	5	−	0·000578
1461	=	8	+	0·000171
5296	=	29	−	0·000065

Hence $\frac{8}{1461}$ or $\frac{29}{5296}$ revolutions are close approximations to the daily motion of $2L$. If we adopt the former for our purpose, a table may be constructed giving $1\cdot272 \sin \frac{360° \times k}{1461}$ where k varies from 0 to 1460. By rearranging the 1461 entries in this table in the order, 1, 9, 17 . . . 1449, 1457, 4, 12 . . . 1452, 1460, 7, 15 . . . 1447, 1455, 2, 10 . . . 1450, 1458, 5, 13 . . . 1453, 1461, 8, 16 . . . 1448, 1456, 3, 11 . . . 1451, 1459, 6, 14 . . . 1446, 1454, 1, 9, 17 . . . they will represent the values of the term required at consecutive intervals of 1 day. It is seen that entry No. 1 (corresponding to $k=0$) is not used a second time until every other entry has been used once. The cycle of entries, when once started at the proper place, could be used indefinitely, except for the fact that $\frac{8}{1461}$ is not a perfect representation of the mean daily motion of $2L$, while even this mean motion is not constant. The entries are given permanent consecutive numbers representing the order in which they are used.

k	No. used above	$2L$	Order of use, or card number	Entry in units of $0''.001$
0	1	$0^\circ.000$	1	0
1	2	0·246	549	− 5
2	3	0·493	1097	− 11
3	4	0·739	184	− 16
4	5	0·986	732	− 22
5	6	1·232	1280	− 27
6	7	1·478	367	− 33
7	8	1·725	915	− 38
8	9	1·971	2	− 44

Hence the final table runs thus :—

No.	Entry
1	0
2	− 44
3	− 87
4	− 131
5	− 175

Since a revolution has been divided into 1461 parts, the interval between these parts is $0^\circ.2464$. The value of $2L$ will, in general, lie between two of those used in constructing the table. Thus if, at some given date, $2L = 1^\circ.000$, the entry required (see the line $k = 4$ in above table) may be expressed as $732^c.06$, since, as will be seen later, each tabular entry is represented by a card. Similarly the entry for $1^\circ.200$ would be $732^c.87$, or $1280^c - 0^c.13$. From the table in which the fraction $\frac{8}{1461}$ was developed, it is seen that the adopted value of $2L$ at the end of 1461 days is $0^r.000171$ too small. Thus the true interval between consecutive days is, in units of $\frac{1}{1461}$ revolutions, 8·000171. The advance in card number corresponding to 8 units is 1 ; hence the card number advances 1·000171 per day, the integral part, by virtue of the final numbering of the cards in the actual order of use, denoting 8 units, and the decimal part denoting units of $\frac{1}{1461}$ revolutions. If F be the fraction of the card number required to represent the true entry at any date, it is easily seen that, at any initial time, F need not exceed ± 0.5. It increases by 0·000171 per day, or by 1 every sixteen years. Hence once every sixteen years there must be a discontinuity in the sequence of card numbers to change a value of F just greater than 0·5 to a value just numerically less than -0.5. From the column "Order of use" it is seen that entries that are adjacent in numerical value, *e.g.* 732 and 1280, differ by 548 in their assigned numbers. Hence when F exceeds 0·5 an addition of 548 must be made to the card number to obtain the card number corresponding to the nearest tabular entry ; or, again, $732^c.50 = 1280^c - 0^c.50$.

The correction required to allow for the fraction or interpolating factor F is

$$-\frac{2\pi F \times 1''\cdot 272 \cos 2L}{1461} = -0''\cdot 0055 F \cos 2L$$

and, since F need not exceed $\pm 0\cdot 5$, this will not exceed $\pm 0''\cdot 003$. In computing nutation all terms with coefficients less than $0''\cdot 004$ are omitted, so that the omission of a correction or proportional part that may attain $\pm 0''\cdot 003$ is comparable with omitting a term whose coefficient is $0''\cdot 003$. The proportional parts omitted in the other nutation terms do not exceed $0''\cdot 001$. It would, of course, have been possible to reduce the proportional parts of this term to a maximum of $0''\cdot 001$ by adopting $\frac{29}{5296}$ revolutions as the daily motion of $2L$. But this would have involved the making of 5296 tabular entries, and the gain in accuracy would be inappreciable, as a value of the nutation to within $0''\cdot 01$ is ample for the present needs of astronomy.

The single-entry tables (*i.e.* those that depend on a single argument) of Brown's *Tables* are arranged in the manner described. Hence the user ascertains first the tabular entry corresponding to his initial date; the entries for subsequent consecutive dates are simply subsequent consecutive entries in the table, with occasional discontinuities. These entries may be copied in order from each table, and the transcribed entries added. This becomes laborious, mainly because of the number of entries to be copied and added—about 120,000 a year, as positions are computed for each noon and midnight. Also the risk of errors in copying and adding half a million figures is not inconsiderable. In the process about to be described the copying is done once only, the addition is done mechanically, and the results of the addition are printed.

*Hollerith Cards and Punches.**—The basic unit in Hollerith equipment is a card $7\frac{3}{8} \times 3\frac{1}{4}$ inches in size, divided into 45 vertical columns,† as shown in fig. 2. Each column contains 12 vertical positions, named, from top to bottom, Y, X, 0, 1 . . . 9.‡ A group of several columns constitutes a field, and in each field a number (within the capacity of the field) may be entered by punching a hole in each column. The key-punch is shown in fig. 3. The card is carried by a movable carriage, each key depression driving a knife through the card, and releasing the carriage so that it moves to the next column. Electrical punches are also available; in these the depression of any key merely actuates a magnet, which drives the appropriate knife through the card. In the electric duplicating punch (fig. 4) a completely punched master-card, if placed in the upper frame, will be automatically copied, at the rate of 10 holes a second, on a card in the lower part of the machine. If any columns of the master-card are unpunched, the machine will naturally

* A description of the Hollerith machines, containing more mechanical details, has been given by the writer in an article, "The Hollerith and Powers Tabulating Machines," in *Office Machinery Users' Association Transactions*, 1929–30.

† Machines using a card of the same size, but divided into 80 columns, are also in use.

‡ The letters Y and X are not printed on the card.

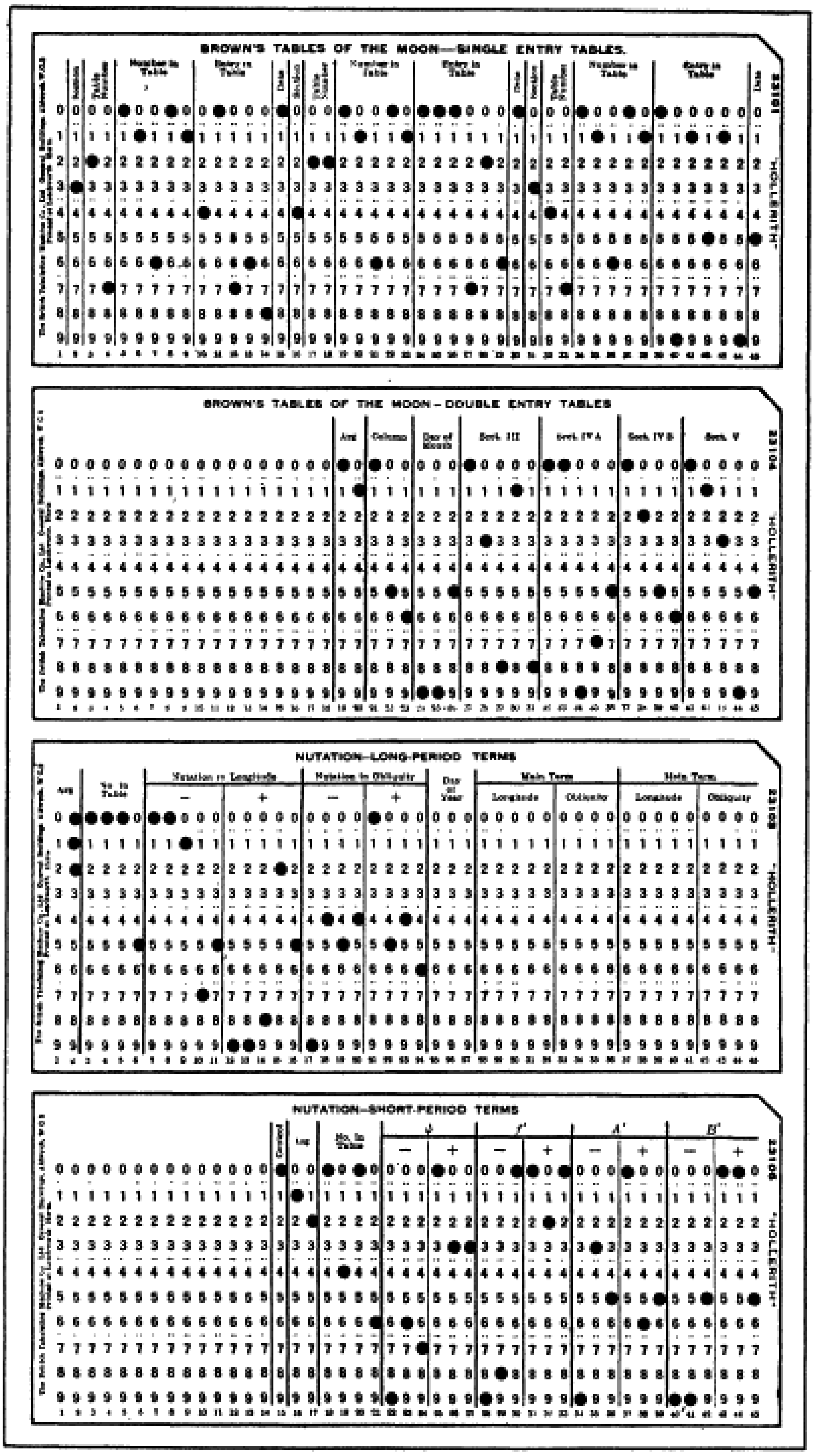

FIG. 2.—*Punched Cards.*

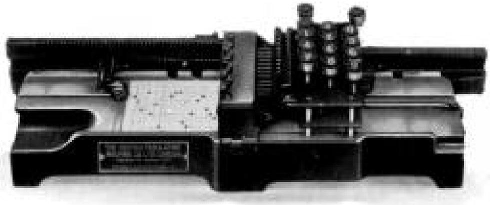

FIG. 3.—*Hand Key-punch.*

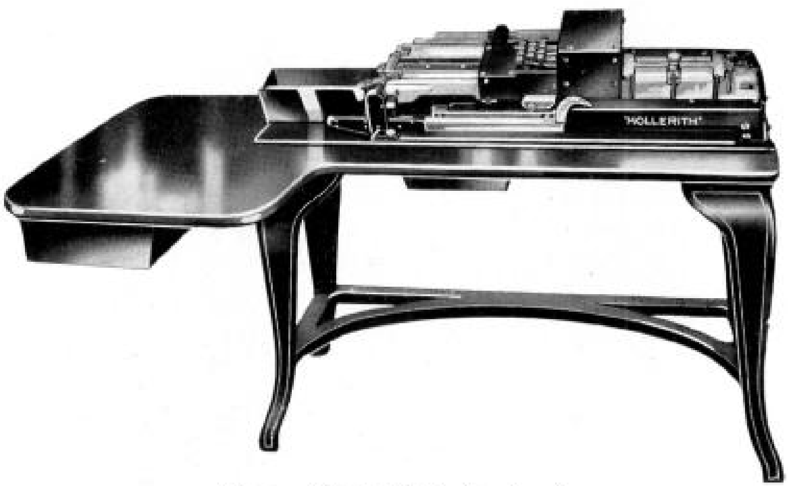

FIG. 4.—*Electric Duplicating Punch.*

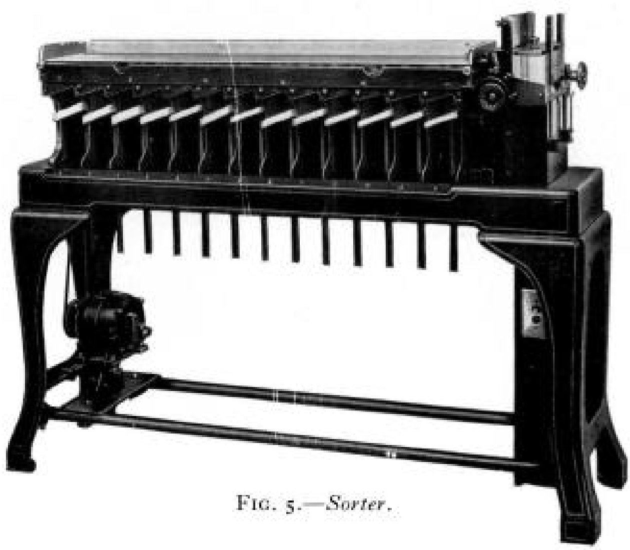

FIG. 5.—*Sorter.*

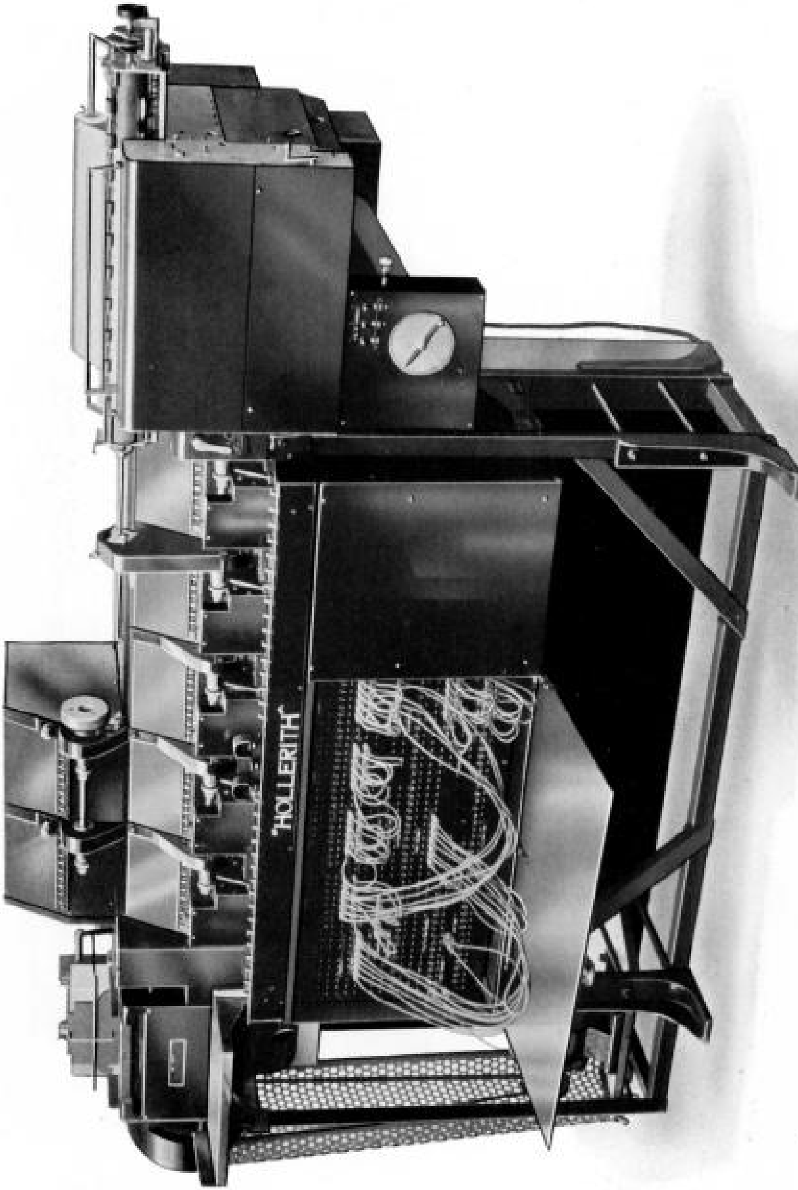

FIG. 6.—*Tabulator*.

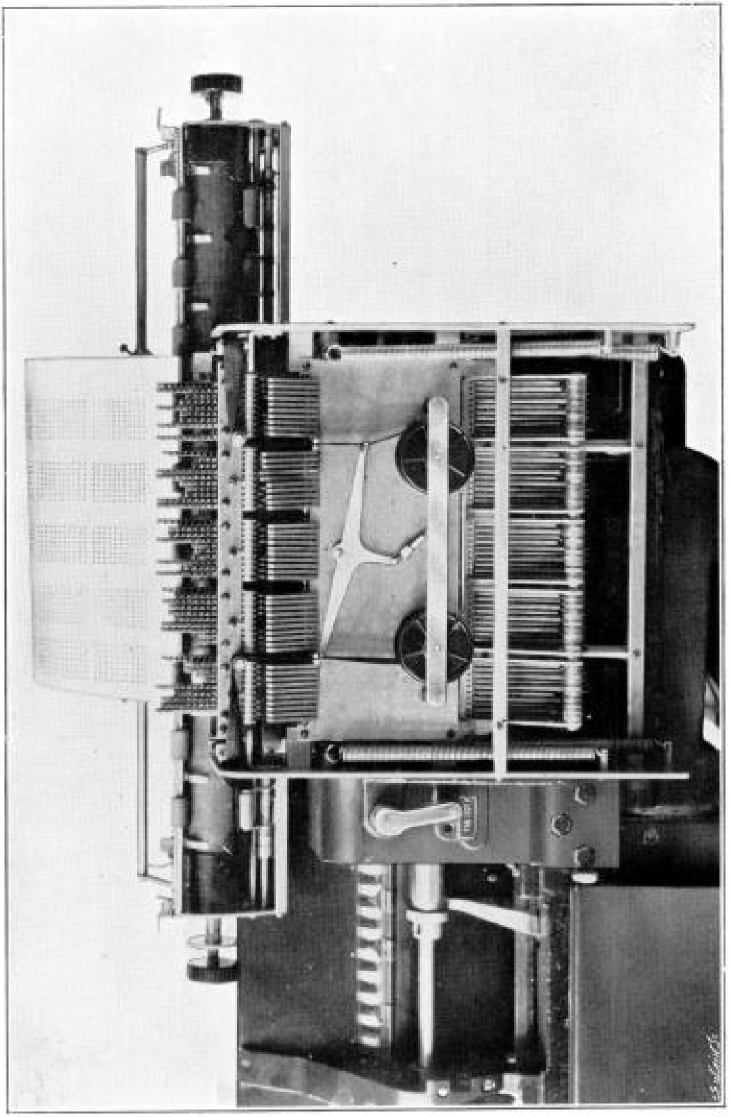

FIG. 7.—*Printing Unit.*