Friedman Equations of Motion after substitution

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After substituting the known form of ϕ from the pre-substitution TEOM notes as well as the other constraints, the Friedman EOMs have been recalculated. $V = 0$ is enforced now. The equations are presented:

Equation 1

$$\begin{aligned}
&\sin\theta\Big(4nfr^2\,\vartheta_0'\,nf\,\vartheta_0'(-e^{-\nu/2}) \\
&\qquad + e^{\nu/2-\lambda}\Big(1 + r\,\nu_r + e^{\lambda}\big(r^2((nf\,\vartheta_0'(-e^{-\nu/2}))^2 - 0^2) - 1\big)\Big)\Big) \\
&= \frac{1}{2}r^2\sin\theta\,e^{-\lambda/2-\nu}\sqrt{e^{\lambda+\nu}}\Big(2V(\vartheta_0)e^{\nu} + (\vartheta_0')^2\Big)
\end{aligned}$$

$$\begin{aligned}
&4n^2f^2r^2\,(\vartheta_0')^2(-e^{-\nu/2}) \\
&\qquad + e^{\nu/2-\lambda}\Big(1 + r\,\nu_r + e^{\lambda}\big(r^2((nf\,\vartheta_0'(-e^{-\nu/2}))^2) - 1\big)\Big) \\
&= \frac{1}{2}r^2e^{-\nu/2}(\vartheta_0')^2
\end{aligned}$$

$$\begin{aligned}
&-8n^2f^2r^2\,(\vartheta_0')^2 \\
&\qquad + 2e^{\nu-\lambda}\Big(1 + r\,\nu_r + e^{\lambda}\big(r^2((nf\,\vartheta_0'(-e^{-\nu/2}))^2) - 1\big)\Big) \\
&= r^2(\vartheta_0')^2
\end{aligned}$$

Focusing specifically on this term:

$$\begin{aligned}
&2e^{\nu-\lambda}\Big(1 + r\,\nu_r + e^{\lambda}\big(r^2((nf\,\vartheta_0'(-e^{-\nu/2}))^2) - 1\big)\Big) \\
&2e^{\nu-\lambda} + 2e^{\nu-\lambda}r\nu_r + 2e^{\nu}\big(r^2n^2f^2\,(\vartheta_0')^2(-e^{-\nu}) - 1\big) \\
&2e^{\nu-\lambda} + 2e^{\nu-\lambda}r\nu_r - 2r^2n^2f^2\,(\vartheta_0')^2 - 2e^{\nu}
\end{aligned}$$

Substituting back into the equation:

$$-8n^2 f^2 r^2 (\vartheta_0')^2 + 2e^{\nu-\lambda} + 2e^{\nu-\lambda} r\nu_r - 2r^2 n^2 f^2 \, (\vartheta_0')^2 - 2e^{\nu} = r^2 (\vartheta_0')^2$$
$$-10n^2 f^2 r^2 (\vartheta_0')^2 + 2e^{\nu-\lambda} + 2e^{\nu-\lambda} r\nu_r - 2e^{\nu} = r^2 (\vartheta_0')^2$$
$$2e^{\nu-\lambda} + 2e^{\nu-\lambda} r\nu_r - 2e^{\nu} = r^2 (\vartheta_0')^2 + 10n^2 f^2 r^2 (\vartheta_0')^2$$

$$2e^{\nu} \left(e^{-\lambda} + e^{-\lambda} r\nu_r - 1 \right) = \left(1 + 10n^2 f^2 \right) r^2 (\vartheta_0')^2 \tag{1}$$