Equations of Motion

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The original form of the equations from the mathematica notebook are given along with the simplifactions based on $V = 0$, $h = 0$, $h^{ij} = 0$, and $f^{ij} = 0$.

Eq.1

$$
\begin{aligned}
&-2r^2 \sin\theta \left(h' - 2n_f \left(f^{11} + \phi\right) \vartheta_0'\right) + \tfrac{1}{4} e^{\frac{1}{2}\nu - \lambda} \\
&\Big(e^{\lambda}\Big(r\Big(r \sin\theta \\
&\Big(-4h^2 + 2(f^{11} + \phi)(f^{22} + f^{33} + 2\phi) \\
&-2(f^{12})^2 - 2(f^{13})^2 + (h^{12})^2 + (h^{13})^2 + (h^{23})^2\Big) - 2f^{13}\cos\theta\Big) \\
&-4\sin\theta\Big) + 4\sin\theta\left(1 + r\nu_r\right)\Big) \\
&= \tfrac{1}{2} r^2 \sin\theta\, e^{-\frac{1}{2}\lambda - \nu} \sqrt{e^{\lambda+\nu}} \left(2V(\vartheta_0) e^{\nu} + (\vartheta_0')^2\right)
\end{aligned}
$$

Simplified:

$$
\begin{aligned}
&-4r^2 \sin\theta \left(n_f \phi \vartheta_0'\right) + \frac{e^{\frac{\nu}{2} - \lambda}}{4}\Big(e^{\lambda}\Big(r\Big(r\sin\theta\big(4\phi^2\big)\Big) - 4\sin\theta\Big) + 4\sin\theta\left(1 + r\nu_r\right)\Big) \\
&= \tfrac{1}{2} r^2 \sin\theta\, e^{-\frac{\lambda}{2} - \nu} \sqrt{e^{\lambda+\nu}} (\vartheta_0')^2
\end{aligned}
$$

Finally:

$$
-8r^2 \left(n_f \phi \vartheta_0'\right) + 2e^{\frac{\nu}{2} - \lambda}\Big(e^{\lambda}\Big(r^2\phi^2 - 1\Big) + r\nu_r + 1\Big) = r^2\, e^{-\frac{\nu}{2}} (\vartheta_0')^2
$$

Eq.2

$$
\begin{aligned}
\frac{r}{2}\Big[&e^{\frac{\nu}{2}}\Big(r\sin\theta\Big(-2h\, h^{12} + f^{12}(f^{33} + \phi) - f^{13} f^{23}\Big) - 2f^{23}\cos\theta\Big) \\
&- f^{13}\sin\theta\, e^{\frac{\nu - \lambda}{2}} - r\sin\theta\Big((h^{12})' - 8n_f f^{12} \vartheta_0'\Big)\Big] = 0
\end{aligned}
$$

Simplified:

$$
0 = 0
$$

Eq.3

$$\frac{r}{2}\Big[e^{\frac{\nu}{2}}\big(r\sin\theta\big(-2h\,h^{13}-f^{12}f^{23}+f^{13}(f^{22}+\phi)\big)+\cos\theta(f^{22}-f^{33})\big)$$
$$+f^{12}\sin\theta\,e^{\frac{\nu-\lambda}{2}}-r\sin\theta\big((h^{13})'-8n_f f^{13}\vartheta_0'\big)\Big]=0$$

Simplified:

$$0=0$$

Eq.4

$$\frac{1}{4}r\Big[e^{\frac{1}{2}\nu}\big(2h^{12}\cos\theta-r\sin\theta\big[h^{23}\big(2f^{11}+f^{22}+f^{33}+4\phi\big)-f^{12}h^{13}+f^{13}h^{12}\big]\big)$$
$$-8n_f r\,h^{23}\sin\theta\,\vartheta_0'+4e^{-\frac{1}{2}\lambda}\sin\theta\,e^{\frac{1}{2}\nu}\,rh(\nu_r+\lambda_\tau)\Big]=0$$

Simplified:

$$0=0$$

Eq.5

$$\frac{1}{4}\sin\theta\Big[e^{\frac{\nu}{2}}f^{13}\,(4+r\nu_r)+2e^{\frac{\lambda}{2}}r$$
$$\Big(e^{\frac{\nu}{2}}\big(2h\,h^{12}+f^{12}(f^{33}+\phi)-f^{13}f^{23}\big)+(h^{12})'+8n_f f^{12}\vartheta_0'+h^{12}\lambda_\tau\Big)\Big]$$
$$=0$$

Simplified:

$$0=0$$

Eq.6

$$
\begin{aligned}
\frac{1}{4}e^{-\frac{\lambda+\nu}{2}}\Big[&e^{\frac{\lambda+\nu}{2}}\sin\theta\, f^{23}\,(2+r\nu_r)-e^{\nu}\sin\theta\,(\lambda_r-\nu_r)\,(2+r\nu_r)-2r(\nu_r)_r\\
&+e^{\lambda}e^{\nu}\\
&\Big(r\sin\theta\big[-4h^2+2(f^{22}+\phi)(f^{11}+f^{33}+2\phi)\\
&-2(f^{12})^2-2(f^{23})^2+(h^{12})^2+(h^{13})^2+(h^{23})^2\big]+2f^{13}\cos\theta\Big)\\
&-4e^{\frac{\nu}{2}}r\sin\theta\Big(2h'-4n_f(f^{22}+\phi)\vartheta_0'+h\lambda_\tau\Big)\\
&-r\sin\theta\Big[(\lambda_\tau)_\tau-\lambda_\tau\nu_\tau+2(\lambda_\tau)_\tau\Big]\Big]\\
&=\frac{1}{2}r\sin\theta\, e^{-\nu}\sqrt{e^{\lambda+\nu}}\,\big(2V(\vartheta_0)e^{\nu}+(\vartheta_0')^2\big)
\end{aligned}
$$

Simplified:

$$
\begin{aligned}
\frac{1}{4}e^{-\frac{\lambda+\nu}{2}}\Big[&-e^{\nu}\sin\theta\,(\lambda_r-\nu_r)\,(2+r\nu_r)\\
&-2r(\nu_r)_r\\
&+4e^{\lambda}e^{\nu}r\phi^2\sin\theta\\
&+16e^{\frac{\nu}{2}}rn_f\phi\vartheta_0'\sin\theta\\
&-r\sin\theta\Big[(\lambda_\tau)_\tau-\lambda_\tau\nu_\tau+2(\lambda_\tau)_\tau\Big]\Big]\\
&=\frac{1}{2}r\sin\theta\, e^{-\nu}\sqrt{e^{\lambda+\nu}}\,\big((\vartheta_0')^2\big)
\end{aligned}
$$

Finally:

$$
\begin{aligned}
e^{-\frac{\lambda+\nu}{2}}\Big[&-\frac{e^{\nu}}{2}\,(\lambda_r-\nu_r)\left(\frac{2}{r}+\nu_r\right)-\frac{(\nu_r)_r}{\sin\theta}\\
&+2e^{\lambda}e^{\nu}\phi^2+8e^{\frac{\nu}{2}}n_f\phi\vartheta_0'-\frac{(\lambda_\tau)_\tau}{2}+\frac{\lambda_\tau\nu_\tau}{2}-(\lambda_\tau)_\tau\Big]=e^{\frac{\lambda-\nu}{2}}(\vartheta_0')^2
\end{aligned}
$$

Eq.7

$$\frac{1}{4}\Big[-e^{\frac{\nu}{2}}\sin\theta\Big(2(f^{11}-f^{33})+r(f^{11}+f^{22}+2\phi)\nu_r\Big)$$
$$+e^{\frac{\lambda}{2}}\Big(-2e^{\frac{\nu}{2}}\big[r\sin\theta\big(2h\,h^{23}-f^{23}(f^{11}+\phi)+f^{12}f^{13}\big)+f^{12}\cos\theta\big]$$
$$-r\sin\theta\big[2(h^{23})'-16n_f f^{23}\vartheta_0'+h^{23}\lambda_\tau\big]\Big)\Big]=0$$

Simplified:

$$\frac{1}{4}\Big[-e^{\frac{\nu}{2}}\sin\theta(2r\phi\nu_r)\Big]=0$$

Finally:

$$-\frac{r\phi\nu_r}{2}e^{\frac{\nu}{2}}\sin\theta=0$$

Eq.8

$$\frac{1}{4}\sin\theta\Big[-2e^{\frac{\nu}{2}}h^{12}\,(1+r\nu_r)$$
$$+e^{\frac{\lambda}{2}}r\Big(e^{\frac{\nu}{2}}\big[h^{13}(f^{11}+2f^{22}+f^{33}+4\phi)-f^{12}h^{23}-f^{23}h^{12}\big]$$
$$+8n_f\,h^{13}\,\vartheta_0'-f^{13}\lambda_\tau\Big)\Big]=0$$

Simplified:

$$0=0$$

Eq.9

$$\frac{1}{4}\Big[-e^{\frac{\nu}{2}}f^{12}\,(4+r\nu_r)$$
$$+2e^{\frac{\lambda}{2}}r\Big(e^{\frac{\nu}{2}}\big(2h\,h^{13}-f^{12}f^{23}+f^{13}(f^{22}+\phi)\big)$$
$$+(h^{13})'+8n_f f^{13}\vartheta_0'+h^{13}\lambda_\tau\Big)\Big]=0$$

Simplified:

$$0=0$$

Eq.10

$$\frac{1}{4}\Big[e^{\frac{\nu}{2}}\Big(2(f^{11}-f^{22})+r(f^{11}+f^{33}+2\phi)\nu_r\Big)$$
$$+e^{\frac{\lambda}{2}}r\Big(-2e^{\frac{\nu}{2}}\Big(-2h\,h^{23}-f^{23}(f^{11}+\phi)+f^{12}f^{13}\Big)$$
$$+2(h^{23})'+16n_f f^{23}\vartheta_0'+h^{23}\lambda_\tau\Big)\Big]=0$$

Simplified:

$$\frac{1}{4}\Big[e^{\frac{\nu}{2}}\Big(r(2\phi)\nu_r\Big)\Big]=0$$

Finally:

$$\frac{r\phi\nu_r}{2}e^{\frac{\nu}{2}}=0$$

Eq.11

$$\frac{1}{4}e^{-\frac{\lambda+\nu}{2}}\Big[e^{\nu}r\Big(-e^{\lambda}$$
$$\Big(4h^2-2(f^{33}+\phi)(f^{11}+f^{22}+2\phi)$$
$$+2(f^{13})^2+2(f^{23})^2-(h^{12})^2-(h^{13})^2-(h^{23})^2\Big)$$
$$-2\lambda_r+2\nu_r-e^{\frac{\lambda}{2}}f^{23}\left(2+r\nu_r\right)$$
$$+r\Big(-\lambda_r\nu_r+(\nu_r)^2+2(\nu_r)_r\Big)\Big)$$
$$-4e^{\lambda+\frac{\nu}{2}}r\Big(2h'-4n_f(f^{33}+\phi)\vartheta_0'+h\lambda_\tau\Big)$$
$$-e^{\lambda}r\Big[(\lambda_\tau)^2-\lambda_\tau\nu_\tau+2(\lambda_\tau)_\tau\Big]\Big]$$
$$=\frac{1}{2}re^{-\nu}\sqrt{e^{\lambda+\nu}}\left(2V(\vartheta_0)e^{\nu}+(\vartheta_0')^2\right)$$

Simplified:

$$\frac{1}{4}e^{-\frac{\lambda+\nu}{2}}\Big[e^{\nu}\Big(-4e^{\lambda}\phi^2-2\lambda_r+2\nu_r+r(-\lambda_r\nu_r+(\nu_r)^2+2(\nu_r)_r)\Big)$$
$$+16n_f\phi e^{\lambda+\frac{\nu}{2}}\vartheta_0'$$
$$-e^{\lambda}((\lambda_\tau)^2-\lambda_\tau\nu_\tau+2(\lambda_\tau)_\tau)\Big]$$
$$=\frac{1}{2}e^{\frac{\lambda-\nu}{2}}(\vartheta_0')^2$$

Finally:

$$e^{-\frac{\lambda+\nu}{2}}\Big[e^{\nu}\Big(-2e^{\lambda}\phi^2-\lambda_r+\nu_r+\frac{r}{2}(-\lambda_r\nu_r+(\nu_r)^2+2(\nu_r)_r)\Big)$$
$$+8n_f\phi e^{\lambda+\frac{\nu}{2}}\vartheta_0'-\frac{e^{\lambda}}{2}((\lambda_\tau)^2-\lambda_\tau\nu_\tau+2(\lambda_\tau)_\tau)\Big]=e^{\frac{\lambda-\nu}{2}}(\vartheta_0')^2$$

Eq.12

$$\frac{1}{4}\Big[-2e^{\frac{\nu}{2}}\,h^{13}\,(1+r\nu_r)$$
$$+e^{\frac{\lambda}{2}}r\Big(-e^{\frac{\nu}{2}}\big[h^{12}(f^{11}+f^{22}+2f^{33}+4\phi)+f^{13}h^{23}-f^{23}h^{13}\big]$$
$$-8n_f\,h^{12}\vartheta_0'+f^{12}\lambda_\tau\Big)\Big]=0$$

Simplified:

$$0=0$$

Eq.13

$$\frac{1}{2}e^{-\frac{\nu}{2}}\Big[re^{\frac{\lambda+\nu}{2}}\Big(r\sin\theta\big(-f^{12}h^{13}+f^{13}h^{12}-h^{23}(f^{22}+f^{33}+2\phi)\big)-h^{12}\cos\theta\Big)$$
$$-2r\sin\theta\lambda_\tau\Big]=0$$

Simplify:

$$\frac{1}{2}e^{-\frac{\nu}{2}}\Big[-2r\sin\theta\lambda_\tau\Big]=0$$

Finally:

$$-r\lambda_\tau e^{-\frac{\nu}{2}}\sin\theta=0$$

Eq.14

$$\frac{1}{2}r\sin\theta\Big[re^{\frac{\lambda}{2}}\big(h^{13}(f^{11}+f^{33}+2\phi)+f^{12}h^{23}+f^{23}h^{12}\big)+h^{12}\Big]=0$$

Simplify:

$$0=0$$

Eq.15

$$\frac{1}{2}r\sin\theta\Big[h^{13}-re^{\frac{\lambda}{2}}\big(h^{12}(f^{11}+f^{22}+2\phi)-f^{13}h^{23}+f^{23}h^{13}\big)\Big]=0$$

Simplify:

$$0=0$$

Eq.16

$$\begin{aligned}\frac{1}{4}e^{-\frac{\lambda+\nu}{2}}\sin\theta\Big[e^{\frac{\nu}{2}}\Big(4 &\\ &-e^{\lambda}\Big(r^2\big(12h^2-8\phi(f^{11}+f^{22}+f^{33})-4f^{33}(f^{11}+f^{22})\\ &\quad -4f^{11}f^{22}+4(f^{12})^2+4(f^{13})^2+4(f^{23})^2\\ &\quad +(h^{12})^2+(h^{13})^2+(h^{23})^2-12\phi^2\big)+4\Big)-4r\lambda_r\Big)\\ &-4e^{\lambda}r^2h\,\lambda_\tau\Big]\\ &=\frac{1}{2}r^2\sin\theta\,e^{-\frac{3}{2}\nu}\sqrt{e^{\lambda+\nu}}\Big(2V(\vartheta_0)e^{\nu}-(\vartheta_0')^2\Big)\end{aligned}$$

Simplify:

$$\begin{aligned}\frac{1}{4}e^{-\frac{\lambda+\nu}{2}}\sin\theta\Big[e^{\frac{\nu}{2}}\Big(4-e^{\lambda}\Big(r^2\big(-12\phi^2\big)+4\Big)-4r\lambda_r\Big)\Big]&\\ =\frac{1}{2}r^2\sin\theta\,e^{-\frac{3}{2}\nu}\sqrt{e^{\lambda+\nu}}\Big(-(\vartheta_0')^2\Big)&\end{aligned}$$

Finally:

$$2\Big(e^{\lambda}(3r^2\phi^2-1)-r\lambda_r+1\Big)=-r^2e^{\lambda-\nu}(\vartheta_0')^2$$

List of Equations

$$-8r^2\left(n_f\phi\vartheta_0'\right)+2e^{\frac{\nu}{2}-\lambda}\Big(e^{\lambda}\big(r^2\phi^2-1\big)+r\nu_r+1\Big)=r^2\,e^{-\frac{\nu}{2}}(\vartheta_0')^2$$

$$\begin{aligned}e^{-\frac{\lambda+\nu}{2}}\Big[&-\frac{e^{\nu}}{2}\left(\lambda_r-\nu_r\right)\left(\frac{2}{r}+\nu_r\right)-\frac{(\nu_r)_r}{\sin\theta}\\&+2e^{\lambda}e^{\nu}\phi^2+8e^{\frac{\nu}{2}}n_f\phi\vartheta_0'-\frac{(\lambda_\tau)_\tau}{2}+\frac{\lambda_\tau\nu_\tau}{2}-(\lambda_\tau)_\tau\Big]=e^{\frac{\lambda-\nu}{2}}(\vartheta_0')^2\end{aligned}$$

$$-\frac{r\phi\nu_r}{2}e^{\frac{\nu}{2}}\sin\theta=0$$

$$\frac{r\phi\nu_r}{2}e^{\frac{\nu}{2}}=0$$

$$\begin{aligned}e^{-\frac{\lambda+\nu}{2}}\Big[&e^{\nu}\Big(-2e^{\lambda}\phi^2-\lambda_r+\nu_r+\frac{r}{2}(-\lambda_r\nu_r+(\nu_r)^2+2(\nu_r)_r)\Big)\\&+8n_f\phi e^{\lambda+\frac{\nu}{2}}\vartheta_0'-\frac{e^{\lambda}}{2}((\lambda_\tau)^2-\lambda_\tau\nu_\tau+2(\lambda_\tau)_\tau)\Big]=e^{\frac{\lambda-\nu}{2}}(\vartheta_0')^2\end{aligned}$$

$$-r\lambda_\tau e^{-\frac{\nu}{2}}\sin\theta=0$$

$$2\Big(e^{\lambda}(3r^2\phi^2-1)-r\lambda_r+1\Big)=-r^2e^{\lambda-\nu}(\vartheta_0')^2$$