

# Friedman Equations of Motion

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The original form of the equations from the mathematica notebook are given along with the simplifications based on  $V = 0$ ,  $h = 0$ ,  $h^{ij} = 0$ , and  $f^{ij} = 0$ .

## Eq.1

$$\begin{aligned}
 & -2r^2 \sin \theta (h' - 2nf(f^{11} + \phi)\vartheta'_0) + \frac{1}{4}e^{\frac{1}{2}\nu - \lambda} \\
 & \left( e^\lambda (r(r \sin \theta \right. \\
 & (-4h^2 + 2(f^{11} + \phi)(f^{22} + f^{33} + 2\phi) \\
 & - 2(f^{12})^2 - 2(f^{13})^2 + (h^{12})^2 + (h^{13})^2 + (h^{23})^2) - 2f^{13} \cos \theta) \\
 & \left. - 4 \sin \theta) + 4 \sin \theta (1 + r\nu_r) \right) \\
 & = \frac{1}{2}r^2 \sin \theta e^{-\frac{1}{2}\lambda - \nu} \sqrt{e^{\lambda + \nu}} (2V(\vartheta_0)e^\nu + (\vartheta'_0)^2)
 \end{aligned}$$

Simplified:

$$\begin{aligned}
 & -4r^2 \sin \theta (nf\phi\vartheta'_0) + \frac{e^{\frac{\nu}{2} - \lambda}}{4} \left( e^\lambda (r(r \sin \theta (4\phi^2)) - 4 \sin \theta) + 4 \sin \theta (1 + r\nu_r) \right) \\
 & = \frac{1}{2}r^2 \sin \theta e^{-\frac{\lambda}{2} - \nu} \sqrt{e^{\lambda + \nu}} (\vartheta'_0)^2
 \end{aligned}$$

Finally:

$$-8r^2 (nf\phi\vartheta'_0) + 2e^{\frac{\nu}{2} - \lambda} \left( e^\lambda (r^2\phi^2 - 1) + r\nu_r + 1 \right) = r^2 e^{-\frac{\nu}{2}} (\vartheta'_0)^2$$

## Eq.2

$$\begin{aligned}
 & \frac{r}{2} \left[ e^{\frac{\nu}{2}} (r \sin \theta (-2h h^{12} + f^{12}(f^{33} + \phi) - f^{13} f^{23}) - 2f^{23} \cos \theta) \right. \\
 & \left. - f^{13} \sin \theta e^{\frac{\nu - \lambda}{2}} - r \sin \theta ((h^{12})' - 8nf f^{12} \vartheta'_0) \right] = 0
 \end{aligned}$$

Simplified:

$$0 = 0$$

### Eq.3

$$\begin{aligned} \frac{r}{2} \left[ e^{\frac{\nu}{2}} (r \sin \theta (-2h h^{13} - f^{12} f^{23} + f^{13} (f^{22} + \phi)) + \cos \theta (f^{22} - f^{33})) \right. \\ \left. + f^{12} \sin \theta e^{\frac{\nu-\lambda}{2}} - r \sin \theta ((h^{13})' - 8n f f^{13} \vartheta'_0) \right] = 0 \end{aligned}$$

Simplified:

$$0 = 0$$

### Eq.4

$$\begin{aligned} \frac{1}{4} r \left[ e^{\frac{1}{2}\nu} (2h^{12} \cos \theta - r \sin \theta [h^{23} (2f^{11} + f^{22} + f^{33} + 4\phi) - f^{12} h^{13} + f^{13} h^{12}]) \right. \\ \left. - 8n f r h^{23} \sin \theta \vartheta'_0 + 4e^{-\frac{1}{2}\lambda} \sin \theta e^{\frac{1}{2}\nu} r h (\nu_r + \lambda_r) \right] = 0 \end{aligned}$$

Simplified:

$$0 = 0$$

### Eq.5

$$\begin{aligned} \frac{1}{4} \sin \theta \left[ e^{\frac{\nu}{2}} f^{13} (4 + r \nu_r) + 2e^{\frac{\lambda}{2}} r \right. \\ \left. \left( e^{\frac{\nu}{2}} (2h h^{12} + f^{12} (f^{33} + \phi) - f^{13} f^{23}) + (h^{12})' + 8n f f^{12} \vartheta'_0 + h^{12} \lambda_r \right) \right] \\ = 0 \end{aligned}$$

Simplified:

$$0 = 0$$

## Eq.6

$$\begin{aligned}
& \frac{1}{4}e^{-\frac{\lambda+\nu}{2}} \left[ e^{\frac{\lambda+\nu}{2}} \sin \theta f^{23} (2 + r\nu_r) - e^\nu \sin \theta (\lambda_r - \nu_r) (2 + r\nu_r) - 2r(\nu_r)_r \right. \\
& \quad \left. + e^\lambda e^\nu \right. \\
& \quad \left( r \sin \theta \left[ -4h^2 + 2(f^{22} + \phi)(f^{11} + f^{33} + 2\phi) \right. \right. \\
& \quad \left. \left. - 2(f^{12})^2 - 2(f^{23})^2 + (h^{12})^2 + (h^{13})^2 + (h^{23})^2 \right] + 2f^{13} \cos \theta \right) \\
& \quad - 4e^{\frac{\nu}{2}} r \sin \theta \left( 2h' - 4nf(f^{22} + \phi)\vartheta'_0 + h\lambda_\tau \right) \\
& \quad \left. - r \sin \theta \left[ (\lambda_\tau)_\tau - \lambda_\tau \nu_\tau + 2(\lambda_\tau)_\tau \right] \right] \\
& = \frac{1}{2} r \sin \theta e^{-\nu} \sqrt{e^{\lambda+\nu}} \left( 2V(\vartheta_0)e^\nu + (\vartheta'_0)^2 \right)
\end{aligned}$$

Simplified:

$$\begin{aligned}
& \frac{1}{4}e^{-\frac{\lambda+\nu}{2}} \left[ -e^\nu \sin \theta (\lambda_r - \nu_r) (2 + r\nu_r) \right. \\
& \quad \left. - 2r(\nu_r)_r \right. \\
& \quad \left. + 4e^\lambda e^\nu r \phi^2 \sin \theta \right. \\
& \quad \left. + 16e^{\frac{\nu}{2}} r n f \phi \vartheta'_0 \sin \theta \right. \\
& \quad \left. - r \sin \theta \left[ (\lambda_\tau)_\tau - \lambda_\tau \nu_\tau + 2(\lambda_\tau)_\tau \right] \right] \\
& = \frac{1}{2} r \sin \theta e^{-\nu} \sqrt{e^{\lambda+\nu}} \left( (\vartheta'_0)^2 \right)
\end{aligned}$$

Finally:

$$\begin{aligned}
& e^{-\frac{\lambda+\nu}{2}} \left[ -\frac{e^\nu}{2} (\lambda_r - \nu_r) \left( \frac{2}{r} + \nu_r \right) - \frac{(\nu_r)_r}{\sin \theta} \right. \\
& \quad \left. + 2e^\lambda e^\nu \phi^2 + 8e^{\frac{\nu}{2}} n f \phi \vartheta'_0 - \frac{(\lambda_\tau)_\tau}{2} + \frac{\lambda_\tau \nu_\tau}{2} - (\lambda_\tau)_\tau \right] = e^{\frac{\lambda-\nu}{2}} (\vartheta'_0)^2
\end{aligned}$$

**Eq.7**

$$\begin{aligned} \frac{1}{4} \Big[ & -e^{\frac{\nu}{2}} \sin \theta \left( 2(f^{11} - f^{33}) + r(f^{11} + f^{22} + 2\phi)\nu_r \right) \\ & + e^{\frac{\lambda}{2}} \left( -2e^{\frac{\nu}{2}} \left[ r \sin \theta (2h h^{23} - f^{23}(f^{11} + \phi) + f^{12} f^{13}) + f^{12} \cos \theta \right] \right. \\ & \left. - r \sin \theta [2(h^{23})' - 16n f f^{23} \vartheta'_0 + h^{23} \lambda_\tau] \right) \Big] = 0 \end{aligned}$$

Simplified:

$$\frac{1}{4} \left[ -e^{\frac{\nu}{2}} \sin \theta (2r\phi\nu_r) \right] = 0$$

Finally:

$$-\frac{r\phi\nu_r}{2} e^{\frac{\nu}{2}} \sin \theta = 0$$

**Eq.8**

$$\begin{aligned} \frac{1}{4} \sin \theta \Big[ & -2e^{\frac{\nu}{2}} h^{12} (1 + r\nu_r) \\ & + e^{\frac{\lambda}{2}} r \left( e^{\frac{\nu}{2}} [h^{13}(f^{11} + 2f^{22} + f^{33} + 4\phi) - f^{12} h^{23} - f^{23} h^{12}] \right. \\ & \left. + 8n f h^{13} \vartheta'_0 - f^{13} \lambda_\tau \right) \Big] = 0 \end{aligned}$$

Simplified:

$$0 = 0$$

**Eq.9**

$$\begin{aligned} \frac{1}{4} \Big[ & -e^{\frac{\nu}{2}} f^{12} (4 + r\nu_r) \\ & + 2e^{\frac{\lambda}{2}} r \left( e^{\frac{\nu}{2}} (2h h^{13} - f^{12} f^{23} + f^{13} (f^{22} + \phi)) \right. \\ & \left. + (h^{13})' + 8n f f^{13} \vartheta'_0 + h^{13} \lambda_\tau \right) \Big] = 0 \end{aligned}$$

Simplified:

$$0 = 0$$

**Eq.10**

$$\begin{aligned} \frac{1}{4} \Big[ & e^{\frac{\nu}{2}} \left( 2(f^{11} - f^{22}) + r(f^{11} + f^{33} + 2\phi)\nu_r \right) \\ & + e^{\frac{\lambda}{2}} r \left( -2e^{\frac{\nu}{2}} \left( -2h h^{23} - f^{23}(f^{11} + \phi) + f^{12} f^{13} \right) \right. \\ & \left. + 2(h^{23})' + 16n f f^{23} \vartheta_0' + h^{23} \lambda_\tau \right) \Big] = 0 \end{aligned}$$

Simplified:

$$\frac{1}{4} \left[ e^{\frac{\nu}{2}} \left( r(2\phi)\nu_r \right) \right] = 0$$

Finally:

$$\frac{r\phi\nu_r}{2} e^{\frac{\nu}{2}} = 0$$

**Eq.11**

$$\begin{aligned} \frac{1}{4} e^{-\frac{\lambda+\nu}{2}} \Big[ & e^\nu r \left( -e^\lambda \right. \\ & \left( 4h^2 - 2(f^{33} + \phi)(f^{11} + f^{22} + 2\phi) \right. \\ & + 2(f^{13})^2 + 2(f^{23})^2 - (h^{12})^2 - (h^{13})^2 - (h^{23})^2 \Big) \\ & - 2\lambda_r + 2\nu_r - e^{\frac{\lambda}{2}} f^{23} (2 + r\nu_r) \\ & \left. + r \left( -\lambda_r \nu_r + (\nu_r)^2 + 2(\nu_r)_r \right) \right) \\ & - 4e^{\lambda+\frac{\nu}{2}} r \left( 2h' - 4n f (f^{33} + \phi) \vartheta_0' + h \lambda_\tau \right) \\ & - e^\lambda r \left[ (\lambda_\tau)^2 - \lambda_\tau \nu_\tau + 2(\lambda_\tau)_\tau \right] \Big] \\ & = \frac{1}{2} r e^{-\nu} \sqrt{e^{\lambda+\nu}} \left( 2V(\vartheta_0) e^\nu + (\vartheta_0')^2 \right) \end{aligned}$$

Simplified:

$$\begin{aligned} & \frac{1}{4}e^{-\frac{\lambda+\nu}{2}} \left[ e^\nu \left( -4e^\lambda \phi^2 - 2\lambda_r + 2\nu_r + r(-\lambda_r \nu_r + (\nu_r)^2 + 2(\nu_r)_r) \right) \right. \\ & \quad \left. + 16nf\phi e^{\lambda+\frac{\nu}{2}} \vartheta'_0 - e^\lambda ((\lambda_\tau)^2 - \lambda_\tau \nu_\tau + 2(\lambda_\tau)_\tau) \right] \\ & = \frac{1}{2}e^{\frac{\lambda-\nu}{2}} (\vartheta'_0)^2 \end{aligned}$$

Finally:

$$\begin{aligned} & e^{-\frac{\lambda+\nu}{2}} \left[ e^\nu \left( -2e^\lambda \phi^2 - \lambda_r + \nu_r + \frac{r}{2}(-\lambda_r \nu_r + (\nu_r)^2 + 2(\nu_r)_r) \right) \right. \\ & \quad \left. + 8nf\phi e^{\lambda+\frac{\nu}{2}} \vartheta'_0 - \frac{e^\lambda}{2} ((\lambda_\tau)^2 - \lambda_\tau \nu_\tau + 2(\lambda_\tau)_\tau) \right] = e^{\frac{\lambda-\nu}{2}} (\vartheta'_0)^2 \end{aligned}$$

## Eq.12

$$\begin{aligned} & \frac{1}{4} \left[ -2e^{\frac{\nu}{2}} h^{13} (1 + r\nu_r) \right. \\ & \quad \left. + e^{\frac{\lambda}{2}} r \left( -e^{\frac{\nu}{2}} [h^{12}(f^{11} + f^{22} + 2f^{33} + 4\phi) + f^{13}h^{23} - f^{23}h^{13}] \right. \right. \\ & \quad \left. \left. - 8nf h^{12} \vartheta'_0 + f^{12} \lambda_\tau \right) \right] = 0 \end{aligned}$$

Simplified:

$$0 = 0$$

## Eq.13

$$\begin{aligned} & \frac{1}{2}e^{-\frac{\nu}{2}} \left[ r e^{\frac{\lambda+\nu}{2}} \left( r \sin \theta (-f^{12}h^{13} + f^{13}h^{12} - h^{23}(f^{22} + f^{33} + 2\phi)) - h^{12} \cos \theta \right) \right. \\ & \quad \left. - 2r \sin \theta \lambda_\tau \right] = 0 \end{aligned}$$

Simplify:

$$\frac{1}{2}e^{-\frac{\nu}{2}} \left[ -2r \sin \theta \lambda_\tau \right] = 0$$

Finally:

$$-r \lambda_\tau e^{-\frac{\nu}{2}} \sin \theta = 0$$

### Eq.14

$$\frac{1}{2}r \sin \theta \left[ r e^{\frac{\lambda}{2}} (h^{13}(f^{11} + f^{33} + 2\phi) + f^{12}h^{23} + f^{23}h^{12}) + h^{12} \right] = 0$$

Simplify:

$$0 = 0$$

### Eq.15

$$\frac{1}{2}r \sin \theta \left[ h^{13} - r e^{\frac{\lambda}{2}} (h^{12}(f^{11} + f^{22} + 2\phi) - f^{13}h^{23} + f^{23}h^{13}) \right] = 0$$

Simplify:

$$0 = 0$$

### Eq.16

$$\begin{aligned} & \frac{1}{4}e^{-\frac{\lambda+\nu}{2}} \sin \theta \left[ e^{\frac{\nu}{2}} \left( 4 \right. \right. \\ & \quad - e^{\lambda} \left( r^2 (12h^2 - 8\phi(f^{11} + f^{22} + f^{33}) - 4f^{33}(f^{11} + f^{22}) \right. \\ & \quad \left. - 4f^{11}f^{22} + 4(f^{12})^2 + 4(f^{13})^2 + 4(f^{23})^2 \right. \\ & \quad \left. + (h^{12})^2 + (h^{13})^2 + (h^{23})^2 - 12\phi^2) + 4 \right) - 4r\lambda_r \Big) \\ & \quad \left. - 4e^{\lambda}r^2h\lambda_r \right] \\ & = \frac{1}{2}r^2 \sin \theta e^{-\frac{3}{2}\nu} \sqrt{e^{\lambda+\nu}} \left( 2V(\vartheta_0)e^{\nu} - (\vartheta'_0)^2 \right) \end{aligned}$$

Simplify:

$$\begin{aligned} & \frac{1}{4}e^{-\frac{\lambda+\nu}{2}} \sin \theta \left[ e^{\frac{\nu}{2}} \left( 4 - e^{\lambda} \left( r^2 (-12\phi^2) + 4 \right) - 4r\lambda_r \right) \right] \\ & = \frac{1}{2}r^2 \sin \theta e^{-\frac{3}{2}\nu} \sqrt{e^{\lambda+\nu}} \left( -(\vartheta'_0)^2 \right) \end{aligned}$$

Finally:

$$2 \left( e^{\lambda} (3r^2\phi^2 - 1) - r\lambda_r + 1 \right) = -r^2 e^{\lambda-\nu} (\vartheta'_0)^2$$

## List of Equations

$$-8r^2(nf\phi\vartheta'_0) + 2e^{\frac{\nu}{2}-\lambda}\left(e^\lambda(r^2\phi^2 - 1) + r\nu_r + 1\right) = r^2 e^{-\frac{\nu}{2}}(\vartheta'_0)^2 \quad (1)$$

$$e^{-\frac{\lambda+\nu}{2}}\left[-\frac{e^\nu}{2}(\lambda_r - \nu_r)\left(\frac{2}{r} + \nu_r\right) - \frac{(\nu_r)_r}{\sin\theta} + 2e^\lambda e^\nu \phi^2 + 8e^{\frac{\nu}{2}}nf\phi\vartheta'_0 - \frac{(\lambda_\tau)_\tau}{2} + \frac{\lambda_\tau\nu_\tau}{2} - (\lambda_\tau)_\tau\right] = e^{\frac{\lambda-\nu}{2}}(\vartheta'_0)^2 \quad (2)$$

$$-\frac{r\phi\nu_r}{2}e^{\frac{\nu}{2}}\sin\theta = 0 \quad (3)$$

$$\frac{r\phi\nu_r}{2}e^{\frac{\nu}{2}} = 0 \quad (4)$$

$$e^{-\frac{\lambda+\nu}{2}}\left[e^\nu\left(-2e^\lambda\phi^2 - \lambda_r + \nu_r + \frac{r}{2}(-\lambda_r\nu_r + (\nu_r)^2 + 2(\nu_r)_r)\right) + 8nf\phi e^{\lambda+\frac{\nu}{2}}\vartheta'_0 - \frac{e^\lambda}{2}((\lambda_\tau)^2 - \lambda_\tau\nu_\tau + 2(\lambda_\tau)_\tau)\right] = e^{\frac{\lambda-\nu}{2}}(\vartheta'_0)^2 \quad (5)$$

$$-r\lambda_\tau e^{-\frac{\nu}{2}}\sin\theta = 0 \quad (6)$$

$$2\left(e^\lambda(3r^2\phi^2 - 1) - r\lambda_r + 1\right) = -r^2 e^{\lambda-\nu}(\vartheta'_0)^2 \quad (7)$$

Looking at eq.3 and eq.4, we know exponential and sinusoidal terms cannot equal zero. Furthermore,  $r$  and  $\phi$  are coordinates. Therefore, both equations yield:

$$\nu_r = 0 \quad (8)$$



In the same vein, eq.6 yields:

$$\lambda_\tau = 0 \quad (9)$$

These constraints are consistent with the General Schwarzschild metric:

$$ds^2 = -e^{\nu(t,r)} dt^2 + e^{\lambda(t,r)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

Simplifying the equations even further, we are left with:

$$-8r^2 (nf\phi\vartheta'_0) + 2e^{\frac{\nu}{2}-\lambda} \left( e^\lambda (r^2\phi^2 - 1) + 1 \right) = r^2 e^{-\frac{\nu}{2}} (\vartheta'_0)^2 \quad (10)$$

$$e^{-\frac{\lambda+\nu}{2}} \left[ -\frac{e^\nu \lambda_r}{r} + 2e^\lambda e^\nu \phi^2 + 8e^{\frac{\nu}{2}} nf\phi\vartheta'_0 \right] = e^{\frac{\lambda-\nu}{2}} (\vartheta'_0)^2 \quad (11)$$

$$e^{-\frac{\lambda+\nu}{2}} \left[ e^\nu \left( -2e^\lambda \phi^2 - \lambda_r \right) + 8nf\phi e^{\lambda+\frac{\nu}{2}} \vartheta'_0 \right] = e^{\frac{\lambda-\nu}{2}} (\vartheta'_0)^2 \quad (12)$$

$$2 \left( e^\lambda (3r^2\phi^2 - 1) - r\lambda_r + 1 \right) = -r^2 e^{\lambda-\nu} (\vartheta'_0)^2 \quad (13)$$

Simplifying eq.10:

$$\begin{aligned} -8r^2 (nf\phi\vartheta'_0) + 2e^{\frac{\nu}{2}-\lambda} \left( e^\lambda (r^2\phi^2 - 1) + 1 \right) &= r^2 e^{-\frac{\nu}{2}} (\vartheta'_0)^2 \\ 2e^{\frac{\nu}{2}-\lambda} \left( e^\lambda (r^2\phi^2 - 1) + 1 \right) &= r^2 e^{-\frac{\nu}{2}} (\vartheta'_0)^2 + 8r^2 (nf\phi\vartheta'_0) \\ 2e^{\frac{\nu}{2}-\lambda} \left( e^\lambda \left( \phi^2 - \frac{1}{r^2} \right) + \frac{1}{r^2} \right) &= e^{-\frac{\nu}{2}} (\vartheta'_0)^2 + 8nf\phi\vartheta'_0 \end{aligned}$$

Next, eq.11 and eq.12 can be equated:

$$\begin{aligned}
e^{-\frac{\lambda+\nu}{2}} \left[ -\frac{e^\nu \lambda_r}{r} + 2e^\lambda e^\nu \phi^2 + 8e^{\frac{\nu}{2}} n f \phi \vartheta'_0 \right] &= \\
e^{-\frac{\lambda+\nu}{2}} \left[ e^\nu \left( -2e^\lambda \phi^2 - \lambda_r \right) + 8n f \phi e^{\lambda+\frac{\nu}{2}} \vartheta'_0 \right] & \\
-\frac{e^\nu \lambda_r}{r} + 2e^\lambda e^\nu \phi^2 + 8e^{\frac{\nu}{2}} n f \phi \vartheta'_0 &= e^\nu \left( -2e^\lambda \phi^2 - \lambda_r \right) + 8n f \phi e^{\lambda+\frac{\nu}{2}} \vartheta'_0 \\
-\frac{e^\nu \lambda_r}{r} + 2e^\lambda e^\nu \phi^2 + 8e^{\frac{\nu}{2}} n f \phi \vartheta'_0 &= -2e^\nu e^\lambda \phi^2 - e^\nu \lambda_r + 8n f \phi e^{\lambda+\frac{\nu}{2}} \vartheta'_0 \\
-e^\nu \lambda_r \left( \frac{1}{r} - 1 \right) + 4e^{\lambda+\nu} \phi^2 &= 8n f \phi \vartheta'_0 \left( e^{\lambda+\frac{\nu}{2}} - e^{\frac{\nu}{2}} \right) \\
\lambda_r \left( \frac{1}{r} - 1 \right) &= 4e^\lambda \phi^2 - 8n f \phi \vartheta'_0 \frac{(e^{\lambda+\frac{\nu}{2}} - e^{\frac{\nu}{2}})}{e^\nu}
\end{aligned}$$

Simplifying the exponents:

$$\begin{aligned}
\lambda_r \left( \frac{1}{r} - 1 \right) &= 4e^\lambda \phi^2 - 8n f \phi \vartheta'_0 \left( e^{\lambda-\frac{\nu}{2}} - e^{-\frac{\nu}{2}} \right) \\
\lambda_r \left( \frac{1}{r} - 1 \right) &= 4e^\lambda \phi^2 - 8n f \phi \vartheta'_0 e^{-\frac{\nu}{2}} (e^\lambda - 1)
\end{aligned}$$

Moving on to eq.13:

$$\begin{aligned}
2 \left( e^\lambda (3r^2 \phi^2 - 1) - r \lambda_r + 1 \right) &= -r^2 e^{\lambda-\nu} (\vartheta'_0)^2 \\
e^\lambda (3r^2 \phi^2 - 1) - r \lambda_r + 1 &= -\frac{1}{2} r^2 e^{\lambda-\nu} (\vartheta'_0)^2
\end{aligned}$$

**Therefore, the final equations of motion are:**

$$2e^{\frac{\nu}{2}-\lambda} \left( e^\lambda \left( \phi^2 - \frac{1}{r^2} \right) + \frac{1}{r^2} \right) = e^{-\frac{\nu}{2}} (\vartheta'_0)^2 + 8n f \phi \vartheta'_0 \quad (14)$$

$$\lambda_r \left( \frac{1}{r} - 1 \right) = 4e^\lambda \phi^2 - 8n f \phi \vartheta'_0 e^{-\frac{\nu}{2}} (e^\lambda - 1) \quad (15)$$

$$e^\lambda (3r^2 \phi^2 - 1) - r \lambda_r + 1 = -\frac{1}{2} r^2 e^{\lambda-\nu} (\vartheta'_0)^2 \quad (16)$$