Friedman Equations of Motion after substitution

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After substituting the known form of ϕ from the pre-substitution TEOM notes as well as the other constraints, the Friedman EOMs have been recalculated. $V(\vartheta_0) = 0$ is enforced now. The equations are presented:

Equation 1

$$\begin{aligned}
&\sin\theta\Big(4nfr^2\,\vartheta_0'\,nf\,\vartheta_0'(-e^{-\nu/2}) \\
&\qquad + e^{\nu/2-\lambda}\Big(1 + r\,\nu_r + e^{\lambda}\big(r^2((nf\,\vartheta_0'(-e^{-\nu/2}))^2 - 0^2) - 1\big)\Big)\Big) \\
&= \frac{1}{2}r^2\sin\theta\, e^{-\lambda/2-\nu}\sqrt{e^{\lambda+\nu}}\Big(2V(\vartheta_0)e^{\nu} + (\vartheta_0')^2\Big)
\end{aligned}$$

$$\begin{aligned}
&4n^2f^2r^2\,(\vartheta_0')^2(-e^{-\nu/2}) \\
&\qquad + e^{\nu/2-\lambda}\Big(1 + r\,\nu_r + e^{\lambda}\big(r^2((nf\,\vartheta_0'(-e^{-\nu/2}))^2) - 1\big)\Big) \\
&= \frac{1}{2}r^2e^{-\nu/2}(\vartheta_0')^2
\end{aligned}$$

$$\begin{aligned}
&-8n^2f^2r^2\,(\vartheta_0')^2 \\
&\qquad + 2e^{\nu-\lambda}\Big(1 + r\,\nu_r + e^{\lambda}\big(r^2((nf\,\vartheta_0'(-e^{-\nu/2}))^2) - 1\big)\Big) \\
&= r^2(\vartheta_0')^2
\end{aligned}$$

Focusing specifically on this term:

$$\begin{aligned}
&2e^{\nu-\lambda}\Big(1 + r\,\nu_r + e^{\lambda}\big(r^2((nf\,\vartheta_0'(-e^{-\nu/2}))^2) - 1\big)\Big) \\
&2e^{\nu-\lambda} + 2e^{\nu-\lambda}r\nu_r + 2e^{\nu}\big(r^2n^2f^2\,(\vartheta_0')^2(-e^{-\nu}) - 1\big) \\
&2e^{\nu-\lambda} + 2e^{\nu-\lambda}r\nu_r - 2r^2n^2f^2\,(\vartheta_0')^2 - 2e^{\nu}
\end{aligned}$$

Substituting back into the equation:

$$
\begin{gathered}
-8n^2f^2r^2(\vartheta_0')^2+2e^{\nu-\lambda}+2e^{\nu-\lambda}r\nu_r-2r^2n^2f^2\left(\vartheta_0'\right)^2-2e^{\nu}=r^2(\vartheta_0')^2\\
-10n^2f^2r^2(\vartheta_0')^2+2e^{\nu-\lambda}+2e^{\nu-\lambda}r\nu_r-2e^{\nu}=r^2(\vartheta_0')^2\\
2e^{\nu-\lambda}+2e^{\nu-\lambda}r\nu_r-2e^{\nu}=r^2(\vartheta_0')^2+10n^2f^2r^2(\vartheta_0')^2
\end{gathered}
$$

$$
2e^{\nu}\left(e^{-\lambda}+e^{-\lambda}r\nu_r-1\right)=\left(1+10n^2f^2\right)r^2(\vartheta_0')^2 \tag{1}
$$

Equation 2

$$
\begin{aligned}
e^{-\lambda/2}r\sin\theta\left(e^{\nu/2}r(0)(\nu_r)+\lambda_\tau\right)&=0\\
e^{-\lambda/2}r\sin\theta(\lambda_\tau)&=0\\
\lambda_\tau&=0 \qquad (2)
\end{aligned}
$$

This constraint will be applied to the rest of the equations.

Equation 3

$$
\begin{aligned}
&\frac{1}{4}e^{-\frac{1}{2}(\lambda+\nu)}\sin\theta\Big[-e^{\lambda}r\big(\lambda_\tau^2-\lambda_\tau\nu_\tau+2\lambda_{\tau\tau}\big)\\
&\quad -4e^{\lambda+\frac{\nu}{2}}r\big(0\lambda_\tau-4nf\vartheta_0'(-nf\vartheta_0'e^{-\frac{\nu}{2}})\big)\\
&\quad +e^{\nu}\Big(-4(0)^2re^{\lambda}+2\nu_r-\lambda_r(2+r\nu_r)+r(\nu_r^2+2\nu_{rr})\\
&\qquad +4e^{\lambda}(-nf\vartheta_0'e^{-\frac{\nu}{2}})^2\Big)\Big]\\
&=\frac{1}{2}r\sin\theta\, e^{-\nu}\sqrt{e^{\lambda+\nu}}\,\big(2V(\vartheta_0)e^{\nu}+(\vartheta_0')^2\big)
\end{aligned}
$$

$$
\begin{aligned}
&\frac{1}{2}e^{-\lambda}\Big[-4e^{\lambda+\frac{\nu}{2}}r\big(-4nf\vartheta_0'(-nf\vartheta_0'e^{-\frac{\nu}{2}})\big)\\
&\quad +e^{\nu}\Big(2\nu_r-\lambda_r(2+r\nu_r)+r(\nu_r^2+2\nu_{rr})+4e^{\lambda}(-nf\vartheta_0'e^{-\frac{\nu}{2}})^2\Big)\Big]\\
&=r(\vartheta_0')^2
\end{aligned}
$$

$$
\begin{aligned}
&\frac{1}{2}\Big[-4e^{\frac{\nu}{2}}r\big(4n^2f^2(\vartheta_0')^2e^{-\frac{\nu}{2}}\big)\\
&\quad +e^{\nu-\lambda}\Big(2\nu_r-\lambda_r(2+r\nu_r)+r(\nu_r^2+2\nu_{rr})+4e^{\lambda-\nu}n^2f^2(\vartheta_0')^2\Big)\Big]\\
&=r(\vartheta_0')^2
\end{aligned}
$$

$$
\begin{aligned}
&\frac{1}{2}\Big[-16rn^2f^2(\vartheta_0')^2\\
&\quad +e^{\nu-\lambda}\Big(2\nu_r-\lambda_r(2+r\nu_r)+r(\nu_r^2+2\nu_{rr})\Big)+4n^2f^2(\vartheta_0')^2\Big]\\
&=r(\vartheta_0')^2
\end{aligned}
$$

$$
\begin{aligned}
&-8rn^2f^2(\vartheta_0')^2+\frac{1}{2}e^{\nu-\lambda}\Big(2\nu_r-\lambda_r(2+r\nu_r)+r(\nu_r^2+2\nu_{rr})\Big)+2n^2f^2(\vartheta_0')^2\\
&=r(\vartheta_0')^2
\end{aligned}
$$

$$
\frac{1}{2}e^{\nu-\lambda}\Big(2\nu_r-\lambda_r(2+r\nu_r)+r(\nu_r^2+2\nu_{rr})\Big)=(r+8rn^2f^2-2n^2f^2)(\vartheta_0')^2 \quad (3)
$$

Equation 4

$$
\begin{aligned}
-\frac{1}{2}e^{\frac{1}{2}\nu}r\sin(\theta)\left(\nu_r\left(-e^{-\frac{1}{2}\nu}\vartheta_0'nf\right)+2\left(\frac{1}{2}e^{-\frac{1}{2}\nu}nf\vartheta_0'\nu_r\right)\right)&=0\\
-\nu_r e^{-\frac{1}{2}\nu}\vartheta_0'nf+e^{-\frac{1}{2}\nu}nf\vartheta_0'\nu_r&=0\\
0&=0
\end{aligned}
\tag{4}
$$

Equation 5

Equation 5 is functionally the same structure as Equation 4 and yields:

$$0=0 \tag{5}$$

Equation 6

Equation 7

Equation 7 is functionally the same structure as Equation 2 and yields:

$$\lambda_\tau=0 \tag{6}$$