

# Vielbein of the General Schwarzschild Metric

8/7/2025

The General Schwarzschild Metric:

$$ds^2 = -e^{\nu(t,r)} dt^2 + e^{\lambda(t,r)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (1)$$

Note the the trace of the Minkowski metric is  $(-, +, +, +)$ . This gives the following Metric Tensor and its inverse:

$$g_{\mu\nu} = \begin{pmatrix} -e^{\nu(t,r)} & 0 & 0 & 0 \\ 0 & e^{\lambda(t,r)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{pmatrix} \quad (2)$$

$$g^{\mu\nu} = \begin{pmatrix} -e^{-\nu(t,r)} & 0 & 0 & 0 \\ 0 & e^{-\lambda(t,r)} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{1}{r^2 \sin^2 \theta} \end{pmatrix} \quad (3)$$

Calculating the Vielbeins:

$$g_{\mu\nu} = \eta_{ab} e_\mu^a e_\nu^b \quad (4)$$

Since the metric is diagonal, the equation can be simplified by saying  $a = b$ . I will also constrain the Vielbein to being a diagonal matrix, which allows me to simplify further by saying  $\mu = \nu$ . This also gives a correspondance between local and Schwarzschild coordinates, where  $a = 0, 1, 2, 3$  correspond to  $\mu = t, r, \theta, \phi$  respectively in the Vielbein. Anything else is 0.

So I am left with:

$$g_{\mu\mu} = \eta_{aa} (e_\mu^a)^2 \quad (5)$$

Solving this explicitly:

$$\begin{aligned} g_{tt} &= (-1)(e_t^0)^2 = -e^{\nu(t,r)} & e_t^0 &= e^{\frac{\nu(t,r)}{2}} \\ g_{rr} &= (e_r^1)^2 = e^{\lambda(t,r)} & e_r^1 &= e^{\frac{\lambda(t,r)}{2}} \\ g_{\theta\theta} &= (e_\theta^2)^2 = r^2 & e_\theta^2 &= r \\ g_{\phi\phi} &= (e_\phi^3)^2 = r^2 \sin^2 \theta & e_\phi^3 &= r \sin \theta \end{aligned}$$

Therefore, the Vielbeins:

$$e_{\mu}^a = \begin{pmatrix} e^{\frac{\nu(t,r)}{2}} & 0 & 0 & 0 \\ 0 & e^{\frac{\lambda(t,r)}{2}} & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & r \sin \theta \end{pmatrix}$$

$$e_a^{\mu} = g^{\mu\nu} \eta_{ab} e_{\nu}^b = \begin{pmatrix} e^{\frac{-\nu(t,r)}{2}} & 0 & 0 & 0 \\ 0 & e^{\frac{-\lambda(t,r)}{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ 0 & 0 & 0 & \frac{1}{r \sin \theta} \end{pmatrix}$$

$$e^{a\mu} = g^{\mu\nu} e_{\nu}^a = \begin{pmatrix} -e^{\frac{-\nu(t,r)}{2}} & 0 & 0 & 0 \\ 0 & e^{\frac{-\lambda(t,r)}{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ 0 & 0 & 0 & \frac{1}{r \sin \theta} \end{pmatrix}$$