# Torsion EOMs before substitution

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Below are the Torsional Equations of Motion in the first half of the mathematica notebook. Only terms with both a left hand and right hand side are included. The rest of the Torsion EOMs trivially show $h = 0$, $h^{ij} = 0$, $f^{ij} = 0$. Further proof for this is in Adshead Appendix A.

$$\begin{aligned}
2nfre^{\lambda/2}\vartheta_0' &= -2r(f^{33}+\phi)e^{(\lambda+\nu)/2} \\
nf\vartheta_0' &= -(f^{33}+\phi)e^{\nu/2} \\
nf\vartheta_0' &= -\phi e^{\nu/2} \\
-e^{-\nu/2}nf\vartheta_0' &= \phi
\end{aligned} \tag{1}$$

$$\begin{aligned}
-2nfr\sin\theta e^{\lambda/2}\vartheta_0' &= 2r\sin\theta(f^{22}+\phi)e^{(\lambda+\nu)/2} \\
-nf\vartheta_0' &= (f^{22}+\phi)e^{\nu/2} \\
-nf\vartheta_0' &= \phi e^{\nu/2} \\
-e^{-\nu/2}nf\vartheta_0' &= \phi
\end{aligned} \tag{2}$$

$$\begin{aligned}
-2nfre^{\lambda/2}\vartheta_0' &= 2r(f^{33}+\phi)e^{(\lambda+\nu)/2} \\
-nf\vartheta_0' &= (f^{33}+\phi)e^{\nu/2} \\
-nf\vartheta_0' &= \phi e^{\nu/2} \\
-e^{-\nu/2}nf\vartheta_0' &= \phi
\end{aligned} \tag{3}$$

$$\begin{aligned}
2nfr^2\sin\theta\vartheta_0' &= -2r^2\sin\theta(f^{11}+\phi)e^{\nu/2} \\
nf\vartheta_0' &= -(f^{11}+\phi)e^{\nu/2} \\
-nf\vartheta_0' &= \phi e^{\nu/2} \\
-e^{-\nu/2}nf\vartheta_0' &= \phi
\end{aligned} \tag{4}$$

$$\begin{aligned}
2nfr\sin\theta e^{\lambda/2}\vartheta_0' &= -2r\sin\theta(f^{22}+\phi)e^{(\lambda+\nu)/2} \\
nf\vartheta_0' &= -(f^{22}+\phi)e^{\nu/2} \\
-nf\vartheta_0' &= \phi e^{\nu/2} \\
-e^{-\nu/2}nf\vartheta_0' &= \phi
\end{aligned} \tag{5}$$

$$\begin{aligned}
-2nfr^2\sin\theta\vartheta_0' &= 2r^2\sin\theta(f^{11}+\phi)e^{\nu/2} \\
-nf\vartheta_0' &= (f^{11}+\phi)e^{\nu/2} \\
-nf\vartheta_0' &= \phi e^{\nu/2} \\
-e^{-\nu/2}nf\vartheta_0' &= \phi
\end{aligned} \tag{6}$$

In all cases, $\phi$ simplifies to the same form:

$$\phi = -e^{-\nu/2}nf\vartheta_0' \tag{7}$$

Using this and $h = 0$, $h^{ij} = 0$, and $f^{ij} = 0$, the Torsion EOMs and Friedman EOMs can be recalculated with greater brevity in the notebook.